QCD towards the chiral limit – where are we?

Anirban Lahiri

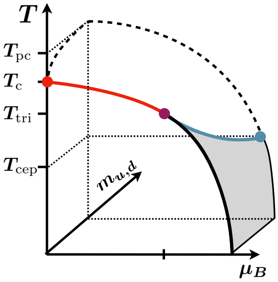
Overview

1. Adding the mass axis to the phase diagram
2. Energy-like direction
3. Summary and Outlook

Overview

Qualitative discussion

- For massless quarks chiral symmetry is exact and symmetry breaking can only happen through a phase transition.



Karsch, arXiv:1905.03936.

- $T = 0$ transition is of first order.
- Phase transition at $\mu_B = 0$ is expected to be of second order belonging to $SU(2) \times SU(2) \simeq O(4)$ universality class.
  [Pisarski and Wilczek. Phys. Rev. D29, 338, 1983.]
- Tricritical point in the $T - \mu_B$ plane: three phase (two broken with opposite sign of magnetization and on restored) coexistence ends and second order line also terminates from the other side. [Phys. Rev. D58, 096007 (1998).]
- CEP shifts to larger μ_B and smaller T with increasing mass.
  [Hatta and Ikeda. Phys. Rev. D67, 014028, 2003.]

- In case effective restoration of anomalous $U_A(1)$, the chiral transition can be of first order. [Pisarski and Wilczek. Phys. Rev. D29, 338, 1983.]

Effective restoration of $U_A(1)$ @ T_c

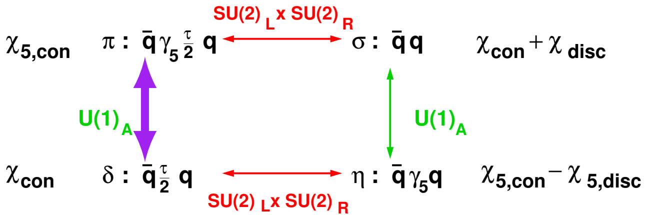
$$
\begin{tikzcd}
\chi_{5,\mathrm{con}} & \pi : \bar{q}\gamma_5 \frac{\tau}{2} q \arrow[r, leftrightarrow, "SU(2)_L \times SU(2)_R"] \arrow[d, leftrightarrow, "U(1)_A"'] & \sigma : \bar{q} q \arrow[d, leftrightarrow, "U(1)_A"] & \chi_{\mathrm{con}} + \chi_{\mathrm{disc}} \\
\chi_{\mathrm{con}} & \delta : \bar{q} \frac{\tau}{2} q \arrow[r, leftrightarrow, "SU(2)_L \times SU(2)_R"'] & \eta : \bar{q}\gamma_5 q & \chi_{5,\mathrm{con}} - \chi_{5,\mathrm{disc}}
\end{tikzcd}
$$

[HotQCD, Phys. Rev. D86, 094503 (2012).]

$$\chi_H = \int d^4x \, [\langle O_H(x) O_H(0)\rangle - \langle O_H(x)\rangle\langle O_H(0)\rangle]$$

"Order parameter" for effective restoration of $U_A(1)$.

Effective restoration of $U_A(1)$ @ T_c



[HotQCD, Phys. Rev. D86, 094503 (2012).]

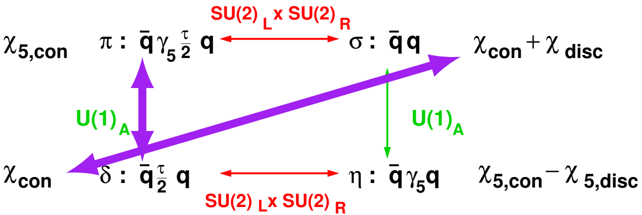
$$\chi_H = \int d^4x \left[\langle O_H(x) O_H(0) \rangle - \langle O_H(x) \rangle \langle O_H(0) \rangle \right]$$

"Order parameter" for effective restoration of $U_A(1)$.

- $m_\pi \stackrel{?}{=} m_{a_0}$ or more generally $\chi_\pi \stackrel{?}{=} \chi_{a_0}$.

UNIVERSITÄT BIELEFELD
Faculty of Physics

Effective restoration of $U_A(1)$ @ T_c



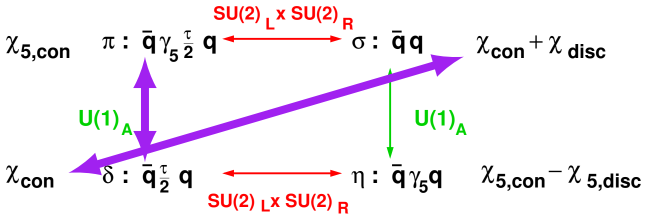
[HotQCD, Phys. Rev. D86, 094503 (2012).]

$$\chi_H = \int d^4x \, [\langle O_H(x) O_H(0)\rangle - \langle O_H(x)\rangle\langle O_H(0)\rangle]$$

"Order parameter" for effective restoration of $U_A(1)$.

- $m_\pi \overset{?}{=} m_{a_0}$ or more generally $\chi_\pi \overset{?}{=} \chi_{a_0}$.
- In the chirally symmetric phase, $\chi^{\text{disc}}_{\bar{\psi}\psi} \overset{?}{=} 0$.

Effective restoration of $U_A(1)$ @ T_c



[HotQCD, Phys. Rev. D86, 094503 (2012).]

$$\chi_H = \int d^4x\,[\langle O_H(x) O_H(0)\rangle - \langle O_H(x)\rangle\langle O_H(0)\rangle]$$

"Order parameter" for effective restoration of $U_A(1)$.

- $m_\pi \stackrel{?}{=} m_{a_0}$ or more generally $\chi_\pi \stackrel{?}{=} \chi_{a_0}$.
- In the chirally symmetric phase, $\chi^{\text{disc}}_{\bar\psi\psi} \stackrel{?}{=} 0$.

- Even using same observable, contradictory findings between different fermion formulation, for high T. Ding *et. al.*, Phys. Rev Lett. 126, 082001, 2021. JLQCD, arXiv:2103.05954 [hep-lat].
- Effective restoration of $U_A(1)$ needs more attention near the chiral transition temperature!
- For temperatures higher but close to the transition temperature, the chiral extrapolation is more subtle and yet inconclusive.

Kaczmarek *et. al.*, arXiv:2003.07920 [hep-lat]. Dentinger *et. al.*, arXiv:2102.09916 [hep-lat].

Effective restoration of $U_A(1)$ @ T_c

CRC-TR 211 Strong-interaction matter under extreme conditions

- For temperatures significantly higher than the transition temperature, $U_A(1)$ breaking is small, which is consistent with dilute instanton gas picture.

$U_A(1)$ broken	$U_A(1)$ restored
DWF $N_\tau = 8,\ 16$ HotQCD, Phys. Rev. D86, 094503, (2012).	Overlap $N_\tau = 8$ Cossu *et. al.*, Phys. Rev. D87, 114514, (2013). Erratum: Phys. Rev. D88, 019901(2013).
DWF $N_\tau = 8$ LLNL/RBC, Phys. Rev. D89, 054514, (2014).	$\mathcal{O}(a)$-improved Wilson $N_\tau = 16$ Brandt *et. al.*, JHEP 12 (2016) 158.
HISQ $N_\tau = 6,\ 8$ Dick *et. al.*, Phys. Rev. D91, 094504, (2015).	DW/Overlap on DWF $N_\tau = 8,\ 12$ JLQCD, Phys. Rev. D96, 034509, (2017).
HISQ continuum Ding *et. al.*, Phys. Rev Lett. 126, 082001, 2021.	DW/Overlap on DWF fixed a JLQCD, arXiv:2103.05954 [hep-lat].
Overlap on HISQ $N_\tau = 8$ Mazur *et. al.*, arXiv:2102.06136 [hep-lat].	

UNIVERSITÄT BIELEFELD Faculty of Physics

Effective restoration of $U_A(1)$ @ T_c

- For temperatures significantly higher than the transition temperature, $U_A(1)$ breaking is small, which is consistent with dilute instanton gas picture.

$U_A(1)$ broken	$U_A(1)$ restored
DWF $N_\tau = 8,\ 16$ HotQCD, Phys. Rev. D86, 094503, (2012).	Overlap $N_\tau = 8$ Cossu *et. al.*, Phys. Rev. D87, 114514, (2013). Erratum: Phys. Rev. D88, 019901(2013).
DWF $N_\tau = 8$ LLNL/RBC, Phys. Rev. D89, 054514, (2014).	$\mathcal{O}(a)$-improved Wilson $N_\tau = 16$ Brandt *et. al.*, JHEP 12 (2016) 158.
HISQ $N_\tau = 6,\ 8$ Dick *et. al.*, Phys. Rev. D91, 094504, (2015).	DW/Overlap on DWF $N_\tau = 8,\ 12$ JLQCD, Phys. Rev. D96, 034509, (2017).
HISQ continuum Ding *et. al.*, Phys. Rev Lett. 126, 082001, 2021.	DW/Overlap on DWF fixed a JLQCD, arXiv:2103.05954 [hep-lat].
Overlap on HISQ $N_\tau = 8$ Mazur *et. al.*, arXiv:2102.06136 [hep-lat].	

- Effective restoration of $U_A(1)$ needs more attention near the chiral transition temperature!

Effective restoration of $U_A(1)$ @ T_c

- For temperatures significantly higher than the transition temperature, $U_A(1)$ breaking is small, which is consistent with dilute instanton gas picture.

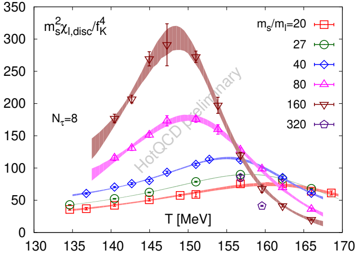
$U_A(1)$ broken	$U_A(1)$ restored
DWF $N_\tau = 8,\ 16$ HotQCD, Phys. Rev. D86, 094503, (2012).	Overlap $N_\tau = 8$ Cossu *et. al.*, Phys. Rev. D87, 114514, (2013). Erratum: Phys. Rev. D88, 019901(2013).
DWF $N_\tau = 8$ LLNL/RBC, Phys. Rev. D89, 054514, (2014).	$\mathcal{O}(a)$-improved Wilson $N_\tau = 16$ Brandt *et. al.*, JHEP 12 (2016) 158.
HISQ $N_\tau = 6,\ 8$ Dick *et. al.*, Phys. Rev. D91, 094504, (2015).	DW/Overlap on DWF $N_\tau = 8,\ 12$ JLQCD, Phys. Rev. D96, 034509, (2017).
HISQ continuum Ding *et. al.*, Phys. Rev Lett. 126, 082001, 2021.	DW/Overlap on DWF fixed a JLQCD, arXiv:2103.05954 [hep-lat].
Overlap on HISQ $N_\tau = 8$ Mazur *et. al.*, arXiv:2102.06136 [hep-lat].	

- Effective restoration of $U_A(1)$ needs more attention near the chiral transition temperature!
- For temperatures higher but close to the transition temperature, the chiral extrapolation is more subtle and yet inconclusive.

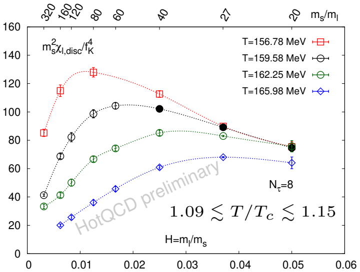
Kaczmarek *et. al.*, arXiv:2003.07920 [hep-lat].
Dentinger *et. al.*, arXiv:2102.09916 [hep-lat].

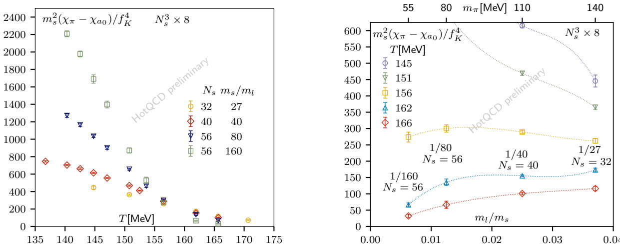
Effective restoration of $U_A(1)$ @ T_c

CRC-TR 211 Strong-interaction matter under extreme conditions



- $T \leqslant T_c$: divergence in the chiral limit, monotonically.
- $T \gtrsim T_c$: highly non-monotonic approach to chiral limit.
- Towards the chiral limit the aspect ratio has to be increased to keep the finite volume effect under control.
- Continuum extrapolation for $T \gtrsim T_c$ is tricky.
- Similar finding through studies of $\chi_\pi - \chi_{a_0}$.





Dentinger *et. al.*, arXiv:2102.09916 [hep-lat].

UNIVERSITÄT BIELEFELD Faculty of Physics

Overview

Gluonic observables towards chiral limit

CRC-TR 211
Strong-interaction matter under extreme conditions

- Wilson's RG approach: thermodynamics in the vicinity of a critical point can be described by an effective Hamiltonian.
- Two types of operators: ones which respect the symmetry and others don't; termed as energy-like and magnetization-like.
- Being gluonic, Polyakov loop (PL) and heavy quark free energy (HQFE) are both expected to be energy-like operators w.r.t. chiral phase transition.

$$F_q(T,H)/T = AH^{(1-\alpha)/\beta\delta} f'_f(z) + f_{\text{reg}}(T,H)$$

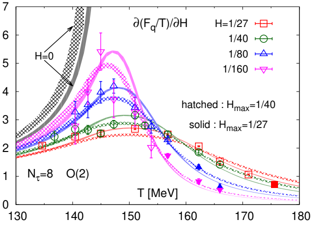
- HQFE doesn't diverge at chiral critical point, so importance of the regular terms could be higher. Let's calculate the mixed susceptibility

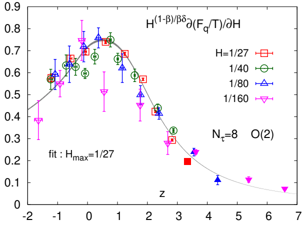
$$\frac{\partial F_q(T,H)/T}{\partial H} = -AH^{(\beta-1)/\beta\delta} f'_G(z) + \frac{\partial f_{\mathrm{reg}}(T,H)}{\partial H}$$

which has a divergent behavior.

UNIVERSITÄT BIELEFELD
Faculty of Physics

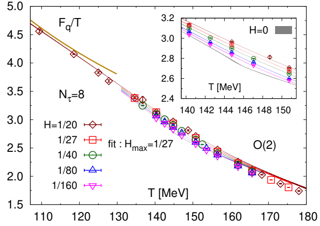
Gluonic observables towards chiral limit





$$\frac{F_q(T,H)}{T} \sim \begin{cases} a^-(T) + A p_s^-(T)\ H & , \ T < T_c \\ a^r_{0,0} + A a_1\ H^{(1-\alpha)/\beta\delta} & , \ T = T_c \\ a^+(T) + p^+(T)\ H^2 & , \ T > T_c \end{cases}$$

- Fit with singular terms only.
- Determined singular part compares well with other determinations.

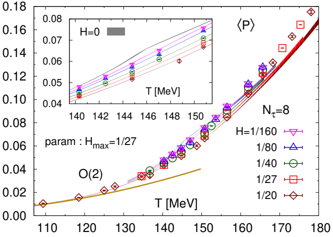


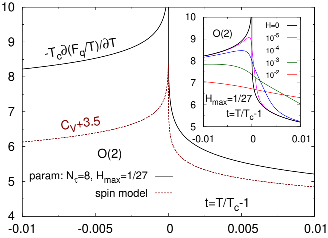
- For $T < T_c$, linear H dependence of HQFE is general, due to Goldstone effect.

Bazavov *et. al.*, Phys. Rev. D87, 094505 (2013).
Brambilla *et. al.*, Phys. Rev. D97, 034503 (2018).
Megías *et. al.*, Phys. Rev. Lett. 109, 151601 (2012).

- Regular part is then determined from HQFE keeping the singular part fixed at the value determined from $\partial(F_q/T)/\partial H$ fit. Clarke *et. al.*, Phys. Rev. D103, L011501 (2021).

Gluonic observables towards chiral limit





- Polyakov loop behaves as an energy-like observable towards chiral limit.
- No inflection point can be identified in the chiral crossover region.
- In the chiral limit: Clarke *et. al.*, Phys. Rev. D103, L011501 (2021).

$$T_c \frac{\partial(F_q(T,0)/T)}{\partial T} = a^r_{1,0}\left(1 + R^{\pm}|t|^{-\alpha}\right)$$

- Peak develops only in a very tiny interval around T_c towards chiral limit.
- Peak height is non-universal.
- Identifying a peak in C_V is hard because of the rising regular background in QCD.

Gupta and Sharma, PoS CPOD2014 (2015) 011.

What about the chemical potentials?

- QCD Lagrangian, written in flavor basis, contains flavor chemical potentials μ_f with $f = u, d, s$.
- In the isospin symmetric limit it is fair to talk about $\mu_l \equiv \mu_u = \mu_d$ and μ_s.
- None of μ_l and μ_s break the chiral symmetry $\Rightarrow$ μ_l and μ_s should appear in the $T-$like scaling field or so called reduced temperature.

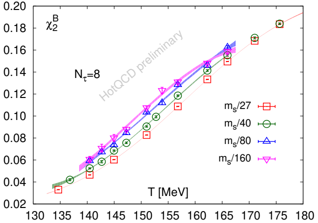
$$t = \frac{1}{t_0}\left(\frac{T - T_c}{T_c} + \kappa_2^l \left(\frac{\mu_l}{T}\right)^2 + \kappa_2^s \left(\frac{\mu_s}{T}\right)^2 + 2\kappa_{11}^{ls}\frac{\mu_l \mu_s}{T^2}\right)$$

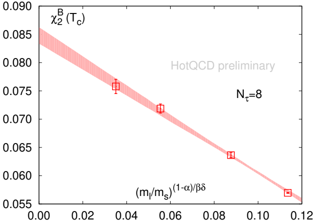
- Same for the chemical potentials in the conserved charge basis?
- Let's start with μ_B which does not break chiral symmetry explicitly.

Conserved charge fluctuations towards chiral limit

- For finite μ_B, to the leading order: $t = \frac{1}{t_0}\left(\frac{T-T_c^0}{T_c^0} + \kappa_B \left(\frac{\mu_B}{T}\right)^2\right)$ and $h = \frac{1}{h_0}\frac{m_l}{m_s}$.
- Close to chiral limit: $\frac{\partial}{\partial T} \sim \frac{\partial}{\partial \mu_B^2} \Rightarrow \chi_2^B$ is expected to be energy-like observable.

$$\chi_2^B(T,H) = -A\kappa_2^B H^{(1-\alpha)/\beta\delta} f_f'(z) + \text{regular terms}.$$





- Linear fit in $H^{(1-\alpha)/\beta\delta}$ works quite well.
- Singular part vanishes at the chiral limit.

Sarkar *et. al.*, arXiv:2011.00240.

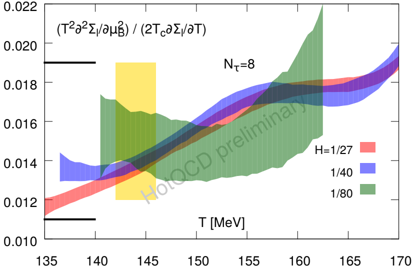
- $\chi_2^B(T_c,0) - \chi_2^B(T_c,H)$ gives the singular part for any finite mass.
- Ratio of singular parts of of conserved charge X and Y is same as κ_2^X/κ_2^Y.

Curvature of the (pseudo-)critical line

- Joint scaling fit to any mixed susceptibility for various small masses $\Rightarrow \kappa^{H=0}$ is a fit parameter; *e.g.* fit $\frac{\partial^2 \Sigma_l}{\partial \mu_B^2}$ to the form proportional to $H^{(1-\beta)/\beta\delta} f'_G(z)$ and the proportionality constant is related to $\kappa^{H=0}$.

- Exploit $\frac{\partial}{\partial T} \sim \frac{\partial}{\partial \mu_B^2}$ and calculate ratio *e.g.* $\frac{T}{2} \frac{\partial^2 \Sigma_l / \partial \mu_B^2}{\partial \Sigma_l / \partial T}\Big|_{T=T_c^0}$ to get the estimators of $\kappa_B^{H=0}$ at finite masses.
- Follow some physical condition over the $T - \mu_B$ plane; *e.g.* following the inflection point of Σ_l gives

$$\kappa_B^H = \frac{T}{2} \frac{\frac{\partial^2}{\partial T^2} \frac{\partial^2 \Sigma_l}{\partial \mu_B^2}}{\frac{\partial^3 \Sigma_l}{\partial T^3}}\Bigg|_{T=T_{\text{pc}}^H} .$$



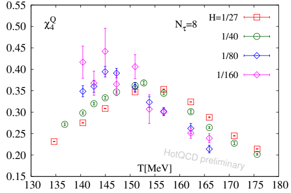
Lahiri, arXiv:2112.08164.

Sarkar *et. al.*, arXiv:2112.15398.

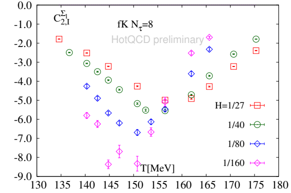
Isospin and electric charge directions? Work in progress ..

CRC-TR 211
Strong-interaction matter under extreme conditions

- Non-vanishing isospin chemical potential or electric charge chemical potential breaks the the isospin sector of the chiral symmetry explicitly.
- How to incorporate and interpret these directions?
- For $N_f = 2$, $Q = \frac{1}{2}B + I_3 \Rightarrow$ any criticality in χ_2^B is going to reflect on the χ_2^Q given there is no criticality in the isospin channel. Hatta and Stephanov, PRL 91, 102003 (2003).
- Is it going to be the same for chiral criticality?
- χ_4^Q develops the characteristic peak towards the chiral limit.
- μ_I seems to produce similar effect in mixed suceptibility as μ_B.
- Behavior of χ_4^I towards the chiral limit will be interesting.
- Details of operator topology in non-perturbative QCD is also under investigation.



Sarkar *et. al.*, arXiv:2011.00240.



UNIVERSITÄT BIELEFELD
Faculty of Physics

Role of strange quark mass for 2+1-flavor chiral limit

- m_s does not break the chiral symmetry in the light sector $\Rightarrow$ m_s appears in the temperature like scaling field.

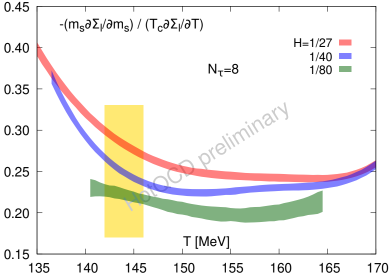
$$t = \frac{1}{t_0}\left[\frac{T}{T_c(m_s)} - 1\right] = \frac{1}{t_0}\left[\left(\frac{T}{T_c(m_s^{phy})} - 1\right) - \kappa^{(m_s)}\frac{m_s - m_s^{phy}}{m_s^{phy}}\right]$$

- Close to the chiral limit: $\frac{\partial}{\partial T} \sim \frac{\partial}{\partial m_s} \Rightarrow \langle\bar{\psi}\psi\rangle_s$ is expected to be energy-like observable.
- $\langle\bar{\psi}\psi\rangle_s \sim H^{(1-\alpha)/\beta\delta} f_f'(z) \Rightarrow$ Singular contribution vanishes in the chiral limit, like any other energy-like observables.
- Relevance in MEoS analysis because the Subtracted condensate $M = \frac{m_s}{f_K^4}\left(\langle\bar{\psi}\psi\rangle_l - \frac{m_l}{m_s}\langle\bar{\psi}\psi\rangle_s\right)$ can't be associated to a purely order parameter scaling function $f_G(z)$.

Curvature in the $T - m_s$ plane

$$t = 0 = \frac{1}{t_0}\left[\left(\frac{T}{T_c(m_s^{phy})} - 1\right) - \kappa^{(m_s)}\frac{m_s - m_s^{phy}}{m_s^{phy}}\right] \Rightarrow T_c(m_s) = T_c(m_s^{phy})\left[1 + \kappa^{m_s}\frac{m_s - m_s^{phy}}{m_s^{phy}}\right]$$

- Joint scaling fit to the mixed susceptibility $\frac{\partial^2 \Sigma_l}{\partial m_s}$ for various small masses $\Rightarrow \kappa^{(m_s)}_{H=0}$ is a fit parameter.
- Exploit $\frac{\partial}{\partial T} \sim \frac{\partial}{\partial m_s}$ and calculate ratio *e.g.* $\left.\frac{\partial^2 \Sigma_l/\partial m_s}{\partial \Sigma_l/\partial T}\right|_{T=T_c^0}$ to get the estimators of $\kappa^{(m_s)}_{H=0}$ at finite masses.



- Crude estimate of $\kappa^{(m_s)}_{H=0} = 0.2$ for $N_\tau = 8$ along with a input of $T_c^{N_f=2+1} = 144$ MeV gives an estimate of $T_c^{N_f=3} = 115$ MeV!

'Mixed' susceptibilities and Specific-heat like observables

Mixed susceptibilities

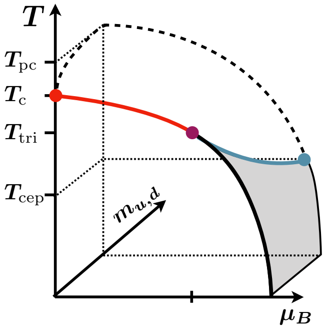
- Quantities defined through $\frac{\partial^2 \ln Z}{\partial h \partial t}$.
- Expected to diverge in the chiral limit following $H^{(1-\beta)/\beta\delta} f'_G(z)$.
- $\frac{\partial \Sigma_l}{\partial T}$, $\frac{\partial^2 \Sigma_l}{\partial \mu_B^2}$, $\frac{\partial \Sigma_l}{\partial m_s}$, $\frac{\partial F_Q}{\partial H}$ – all shows the characteristic peaks related to (negative of) $f'_G(z)$.

C_v-like observables

- Quantities defined through $\frac{\partial^2 \ln Z}{\partial t^2}$.
- Expected to produce a finite cusp in the chiral limit following $H^{-\alpha/\beta\delta} f''_f(z)$.
- Apart from actual C_v, $\chi_4^{B/S} \propto \frac{\partial^4 \ln Z}{\partial \mu_{B/S}^4}$, $\frac{\partial \chi_2^{B/S}}{\partial T} \propto \frac{\partial^3 \ln Z}{\partial \mu_{B/S}^2 \partial T}$, $\chi_{m_s} \propto \frac{\partial^2 \ln Z}{\partial m_s^2}$, $\frac{\partial F_Q}{\partial T}$, $\frac{\partial F_Q}{\partial \mu_{B/S}^2}$, PL susceptibility are expected to behave as specific heat w.r.t. scaling perspective.
- Some of these shows the characteristic cusps and others don't $\Rightarrow$ Strong regular background?

Overview

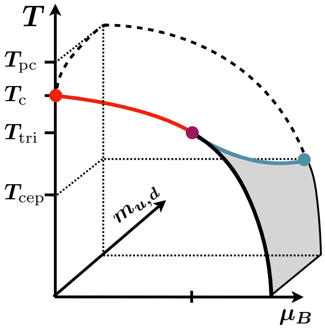
Summary and Outlook



- The chiral critical temperature for (2+1)-flavor is found to be $\sim$ 130 MeV.
- CEP is unlikely to be found for $T > 130$ MeV and correspondingly for $\mu_B < 400$ MeV.
- Ongoing calculations about the curvatures of the (pseudo-)critical lines is going to be important.
- Calculations of energy-like observables seems to provide important information.
- Results in the isospin direction is expected to provide interesting facts.

- $U_A(1)$ restoration at T_c is one of the deciding calculation.

Summary and Outlook



- The chiral critical temperature for (2+1)-flavor is found to be $\sim$ 130 MeV.
- CEP is unlikely to be found for $T > 130$ MeV and correspondingly for $\mu_B < 400$ MeV.
- Ongoing calculations about the curvatures of the (pseudo-)critical lines is going to be important.
- Calculations of energy-like observables seems to provide important information.
- Results in the isospin direction is expected to provide interesting facts.

- $U_A(1)$ restoration at T_c is one of the deciding calculation.