Chiral Phase Transition in 3-flavor QCD from Lattice QCD

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in collaboration with

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under extreme conditions

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Outline

- ▶ Motivation : Pisarski and Wilczek remarks.
- ▶ Past Work.
- ▶ Universal properties near a critical point.
- ▶ Definitions of chiral observables and simulation details.
- ▶ Results for order parameter and its susceptibility.
- ▶ Finite size scaling analysis.
- ▶ Conclusions & Outlook.

Motivation

▶ $SU(N_f)_L \times SU(N_f)_R \to SU(N_f)_V$

Figure: Columbia Plot
[A.Peikert, Ph.D. Thesis, 2000]
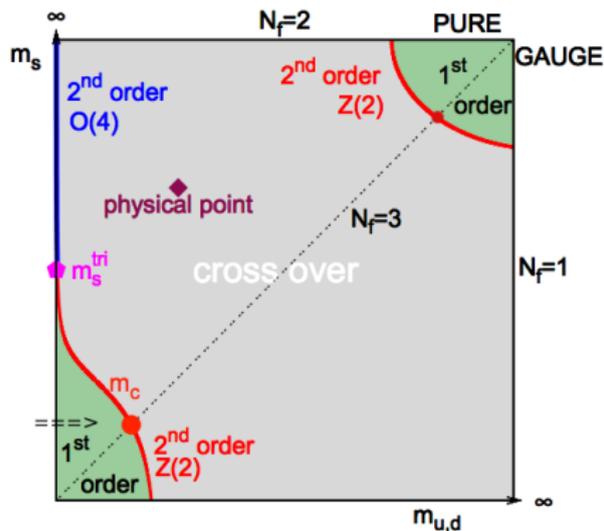
▶ In the chiral limit the order of the QCD phase transition depends on the number of quark flavours that become massless. For $N_f \geq 3$ massless quark flavors, this phase transition is first order.

[Pisarski & Wilczek, 1983]

Past work

▶ For calculations performed with unimproved gauge and fermion actions, bounds on the critical parameters show a strong cut-off and discretization dependence.

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▶ Wilson-Clover fermion action was used to put an upper bound on critical pion mass which was found to be equal to 110 MeV. T_E was quoted to be equal to 134(3) MeV.

[Y. Kuramashi et al., 2020]

Universal properties near a critical point

▶ In the vicinity of a second order critical point, singular part of the free energy density dominates.

$$f_s(t, h, ...) = b^{-d} f_s(b^{y_t} t, b^{y_h} h, ...)$$

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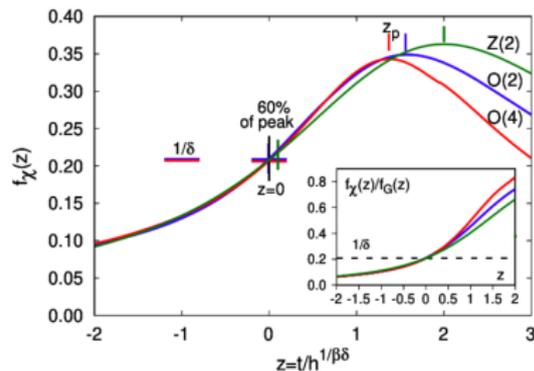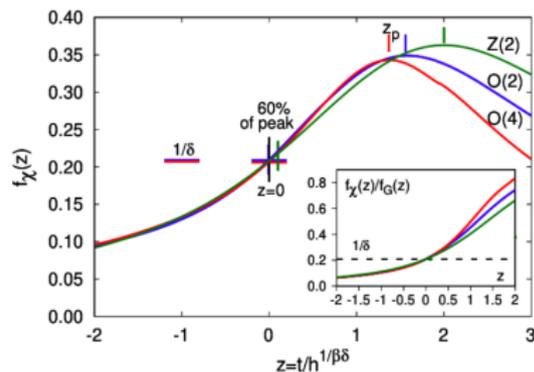
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$$\frac{f_\chi(z)}{f_G(z)} = \frac{1}{\delta}\left(1 - \frac{z}{\beta}\frac{f_G'(z)}{f_G(z)}\right)$$

[H.-T. Ding et al., 2019]

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▶ $M - H\chi_M$ does not receive regular contribution at $\mathcal{O}(H)$.
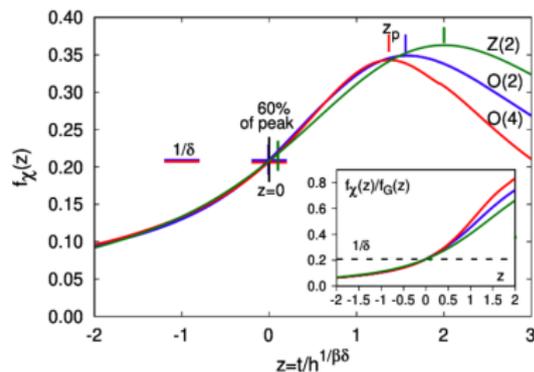
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1/4 is the staggered rooting factor, $N_\tau = (aT)^{-1}$, and $V = (aN_\sigma)^3$

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▶ $H \to m_\ell/m_s$, $T \to T$

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▶ Number of saved trajectories:

m_s/m_ℓ	27	40	60	80
	O(1000)	O(1000)	O(3500)	O(5000)

Chiral condensate in f_K normalization
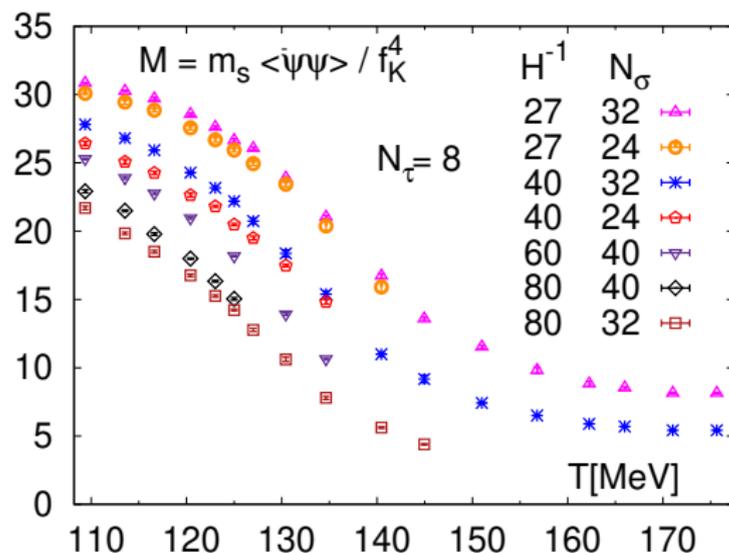
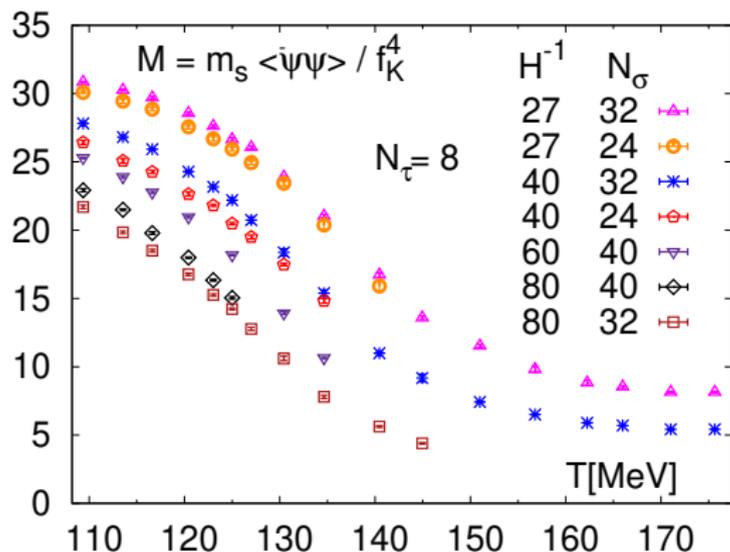Similar to m_s, f_K is an external parameter used to set scale for T.

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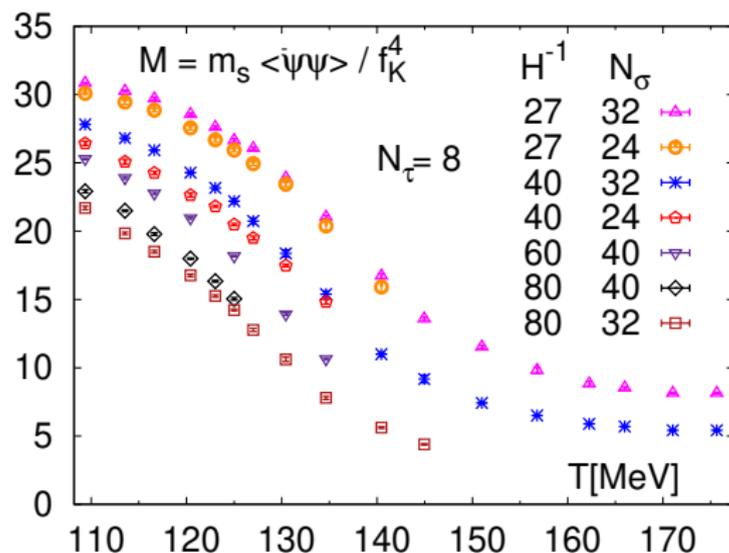
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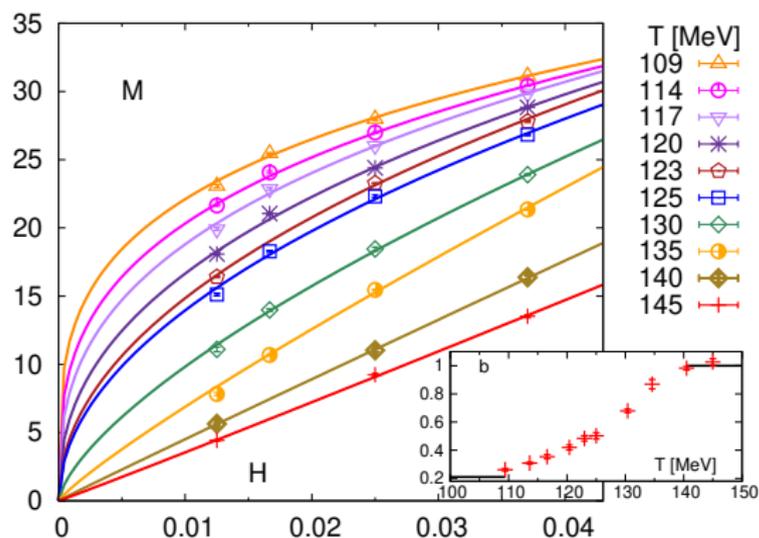
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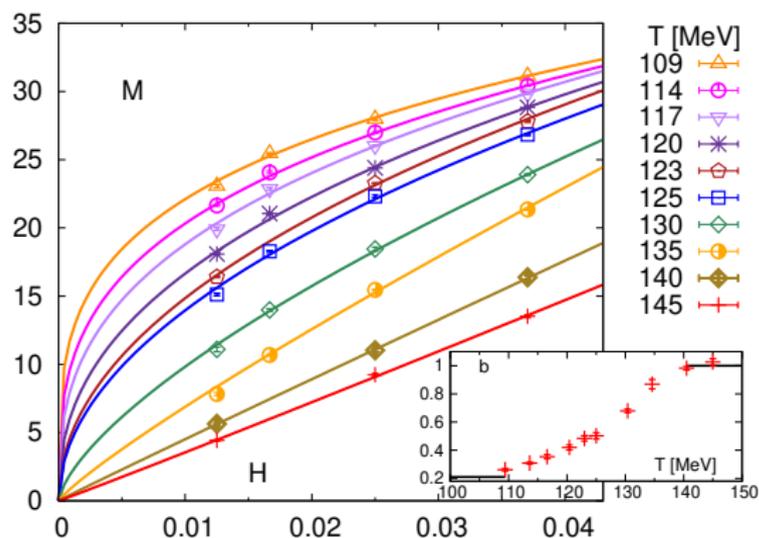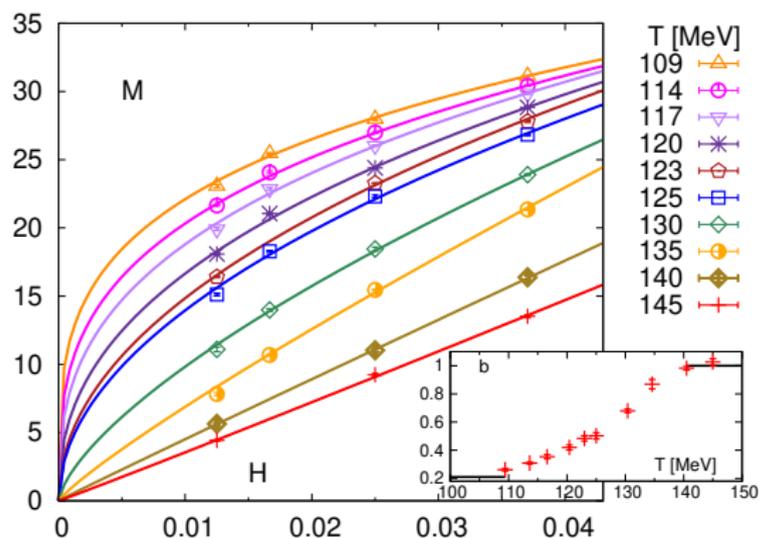
Order parameter at fixed T

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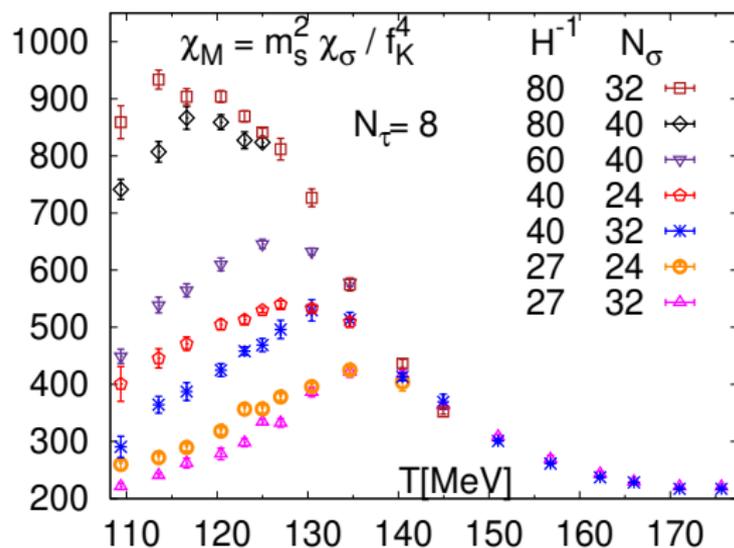

- Fit ansatz, aH^b.
- At high temperatures, M is linear in H.
- At the lowest T, b is slightly greater than the critical exponent of the 3-d $O(2)$ universality class. We have bracketed the scaling window.
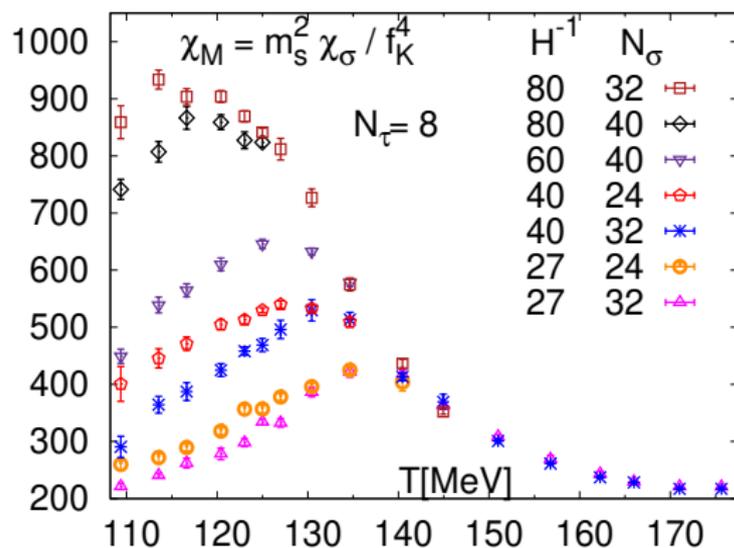
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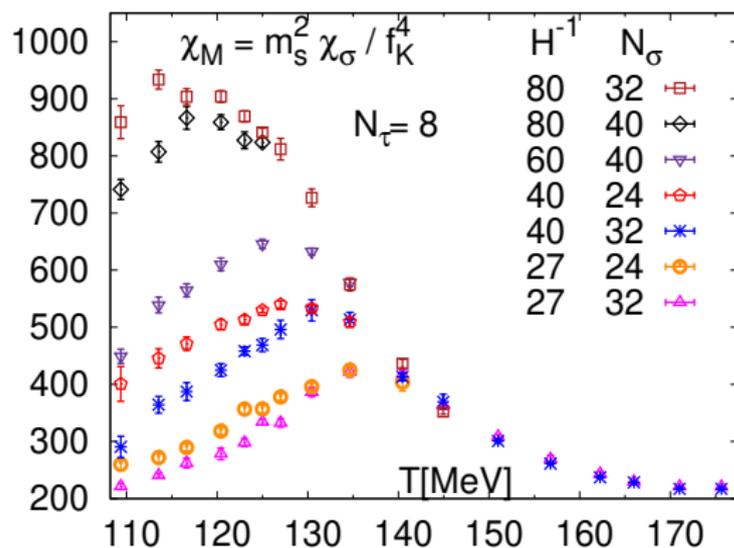
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- Peak height for a fixed mass doesn't increase with the volume. No evidence of a first order transition.
- Peak gets less resolved for the lowest volume of the lowest quark mass. Finite-size effects need to be accounted for.

Finite size dependence of scaling functions

▶ One additional relevant scaling field $\ell = L_0/L$ can be added to the singular part of the free energy density :

$$f_s(t, h, \ell, ...) = b^{-d} f_s(b^{y_t} t, b^{y_h} h, b\ell...)$$

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▶ f_s can be brought to a form such that it depends upon two scaling variables, $z = t/h^{1/\beta\delta}$ and $z_L = \ell/h^{\nu/\beta\delta}$

$$M = -\frac{\partial f_s}{\partial H} = h^{1/\delta} f_G(z, z_L) + H \times reg$$

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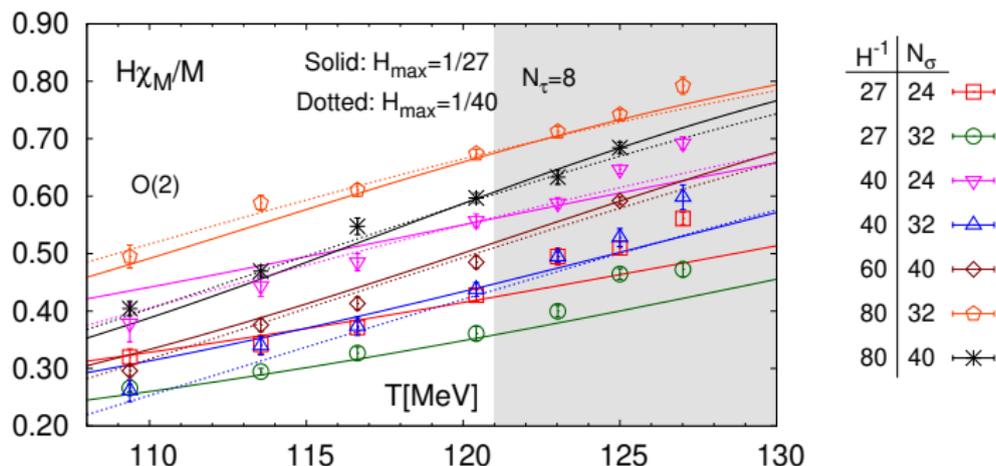
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▶ We used finite size scaling functions to fit data corresponding to quantities such as $H\chi_M/M$ and $M - H\chi_M$ for various available combinations of quark masses and volumes. T_c can be extracted as a fit parameter.
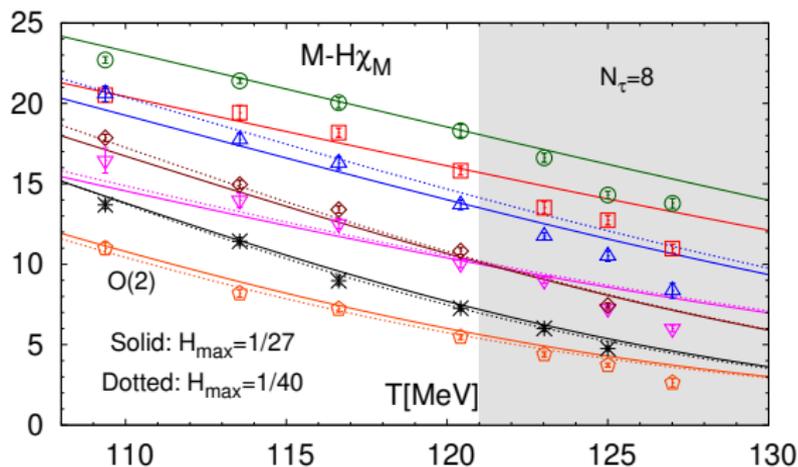
Joint fit to Hχ_M/M (Ratio)

H_{max}^{-1}	27	40
T_c [MeV]	101(2)	93(4)
χ^2/dof	2.97	1.32

▶ Grey region is not included in the fit.

▶ Ratio should have a unique crossing point in the thermodynamic limit, which in this case is below the temperature range explored and gets spoiled by the regular contribution (Linear in tt_0).

Joint fit to M – Hχ_M (Difference)

H_{max}^{-1}	27	40
T_c [MeV]	100(1)	97(1)
χ^2/dof	4.52	2.44

▶ Grey region is not included in the fit.
▶ This quantity scales as $\mathcal{O}(H)^3$ at high temperatures.

Comparison of the fits results

▶ The parameters of the singular part of the fits from both the quantities are in excellent agreement with each other.

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▶ The parameters of the regular term seem to be quite sensitive to the upper bound (H_{max}) for the set of quark masses used in the fit. They are not well determined in the small temperature interval used for these fits.

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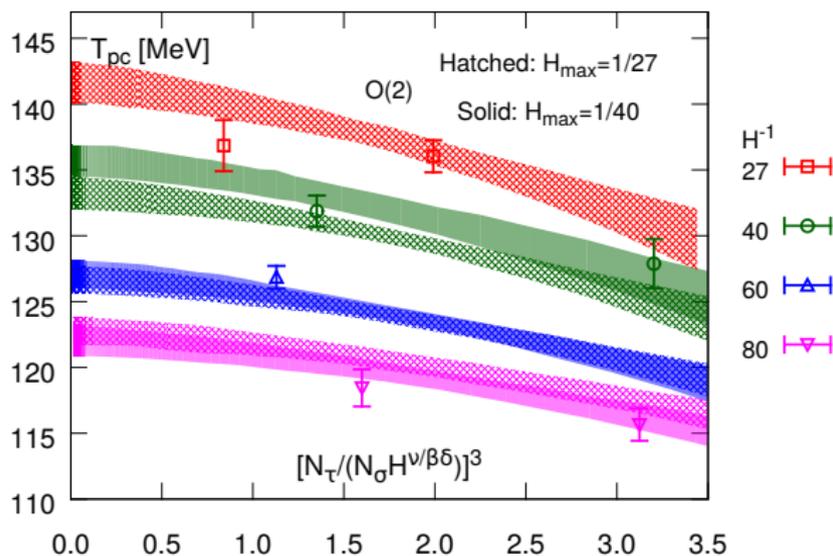
▶ T_{pc} can be fitted to the scaling expectation,

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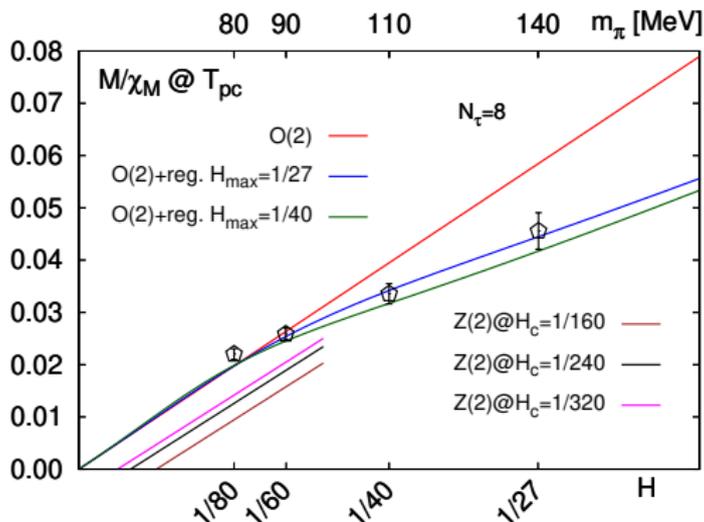[H.-T. Ding et al., 2019]

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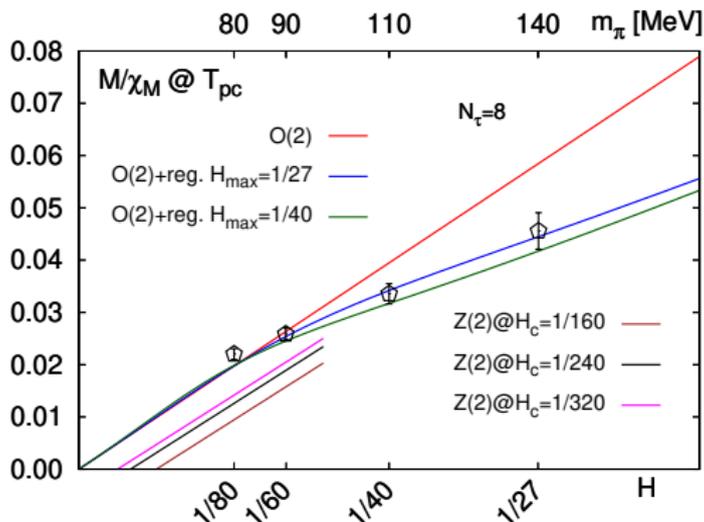$$[N_\tau/N_\sigma H^{\nu/\beta\delta}]^3 \propto z_L{}^3.$$

H^{-1}_{max}	27	40
T_c [MeV]	102(2)	95(3)
χ^2/dof	3.58	2.68

M/χ_M as a function of H at T_{pc}

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- $H\chi_M/M$ at T_{pc} is a constant given by the $f_\chi(z_p)/f_G(z_p)$, and it is almost volume independent. Regular terms are quadratic in tt_0.
- Non-vanishing critical pion mass can not describe the data. $H_c = 1/320$ corresponds to a pion mass of 40 MeV in the continuum.

Conclusions & Outlook

▶ We varied the quark mass range as well as the regular contribution in the finite size scaling fits to take care of the systematics. Fit parameters obtained from all the fits are combined to get a final value for T_c in the chiral limit,

$$T_c = 98^{+3}_{-6} \text{MeV}.$$

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▶ Next step would be to take the continuum limit. In doing so, careful consideration of the correct ordering of continuum limit and chiral limit is required. It is worthwhile to mention that in the $(2+1)$-flavor study, both these limits turned out to be interchangeable.