QCD thermal phase transition, its scaling window and novel order parameter

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JÜLICH Forschungszentrum

Phase transitions in particle physics, 2022

Symmetries of QCD with n quarks

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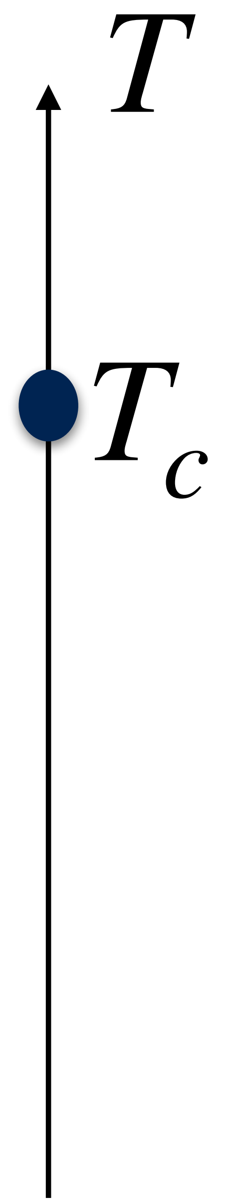

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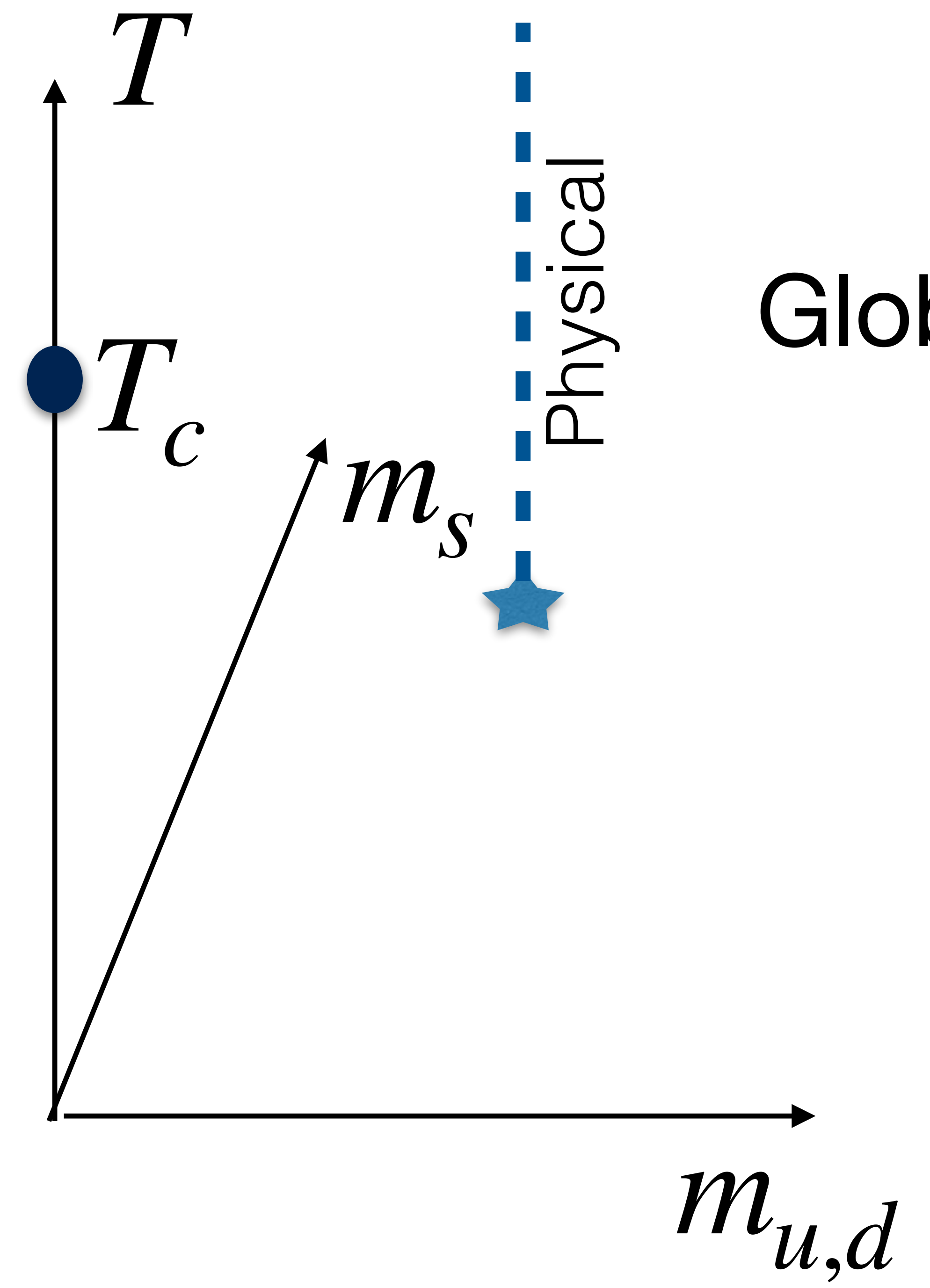

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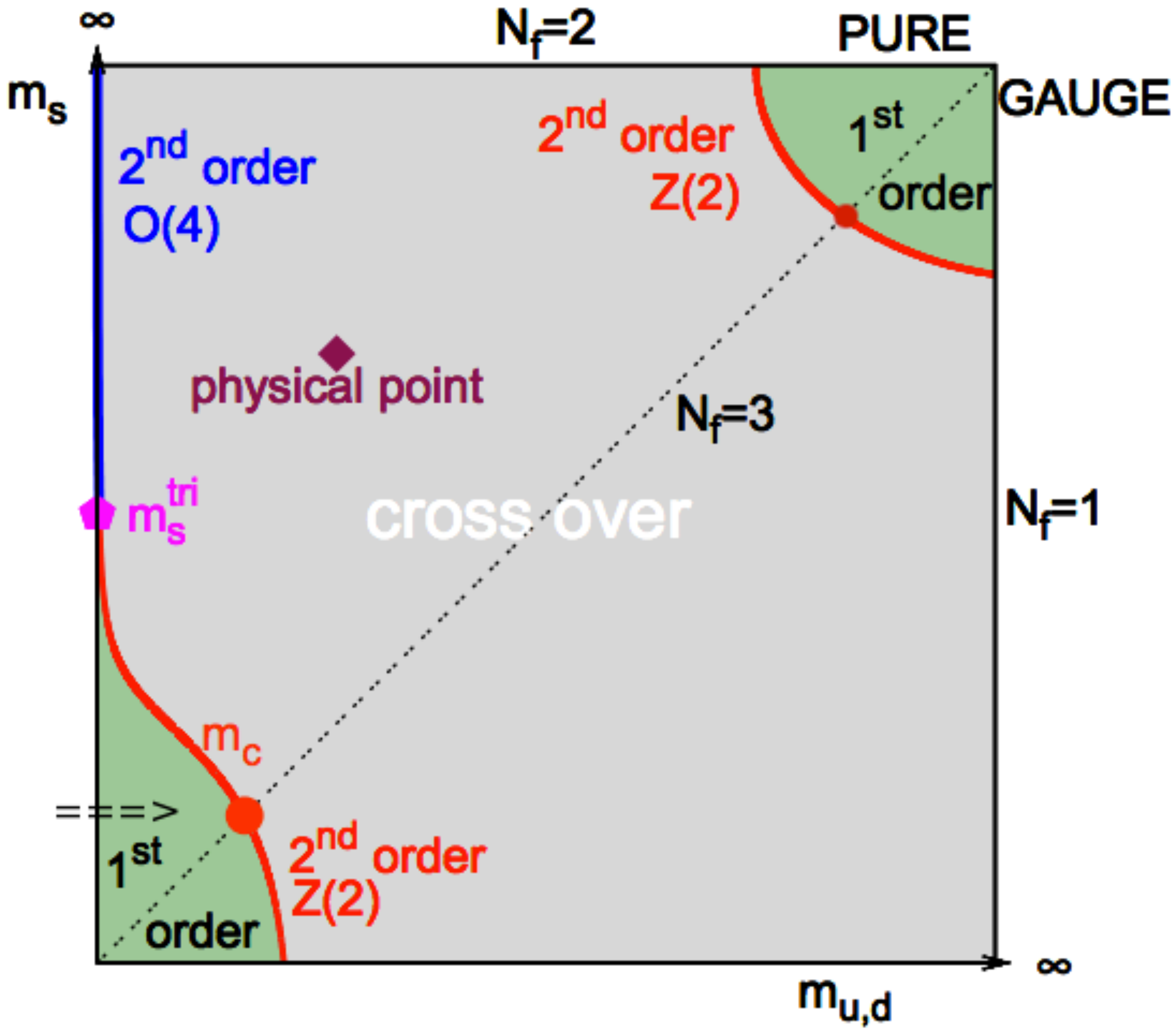
$m \neq 0$: explicit symmetry breaking

Columbia plot



[de Forcrand, D'Elia, 1702.00330]

Order of the phase transition at quark masses m_s, m_l

[talks by O. Philipsen, A. Lahiri, S. Sharma]

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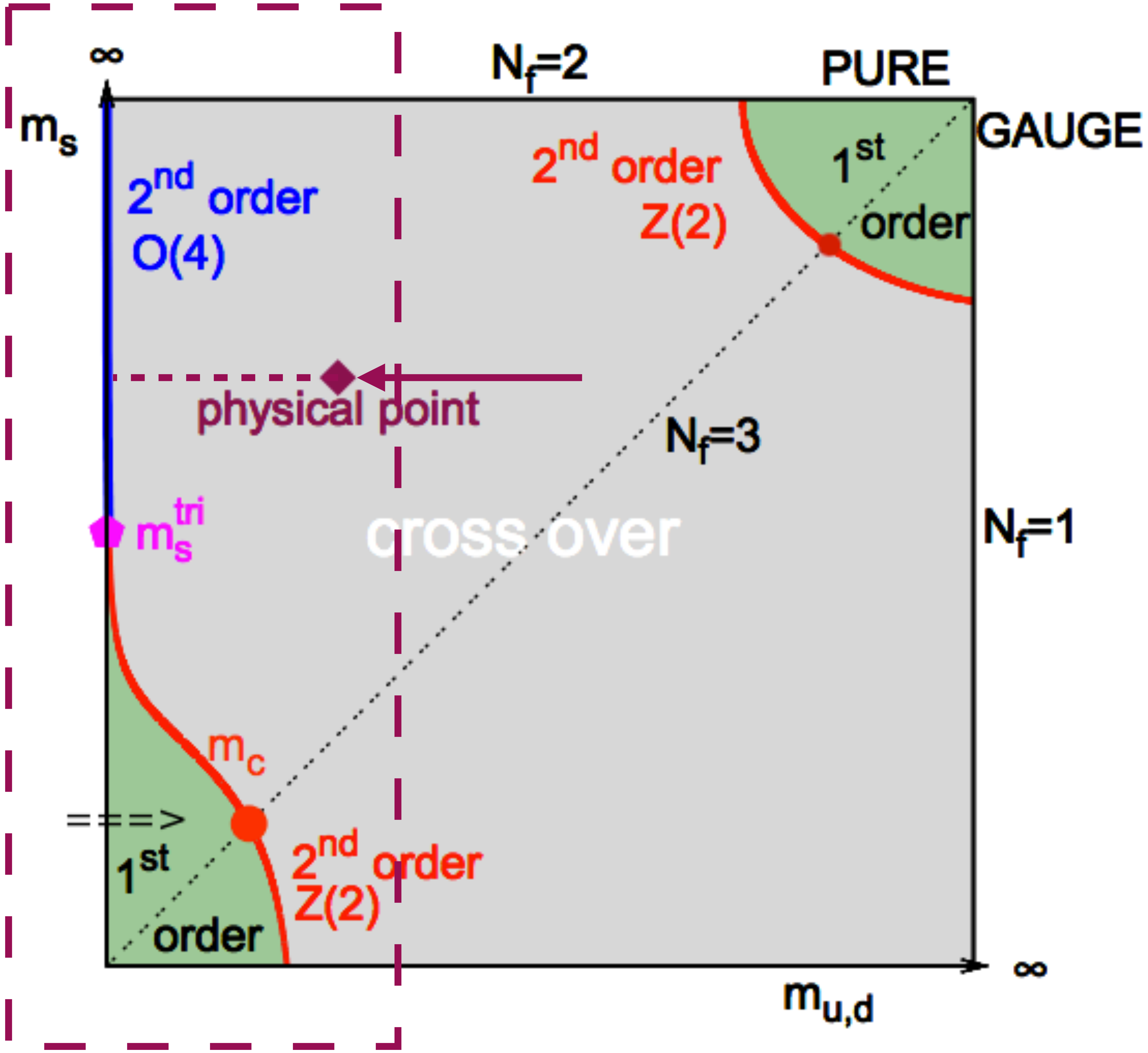

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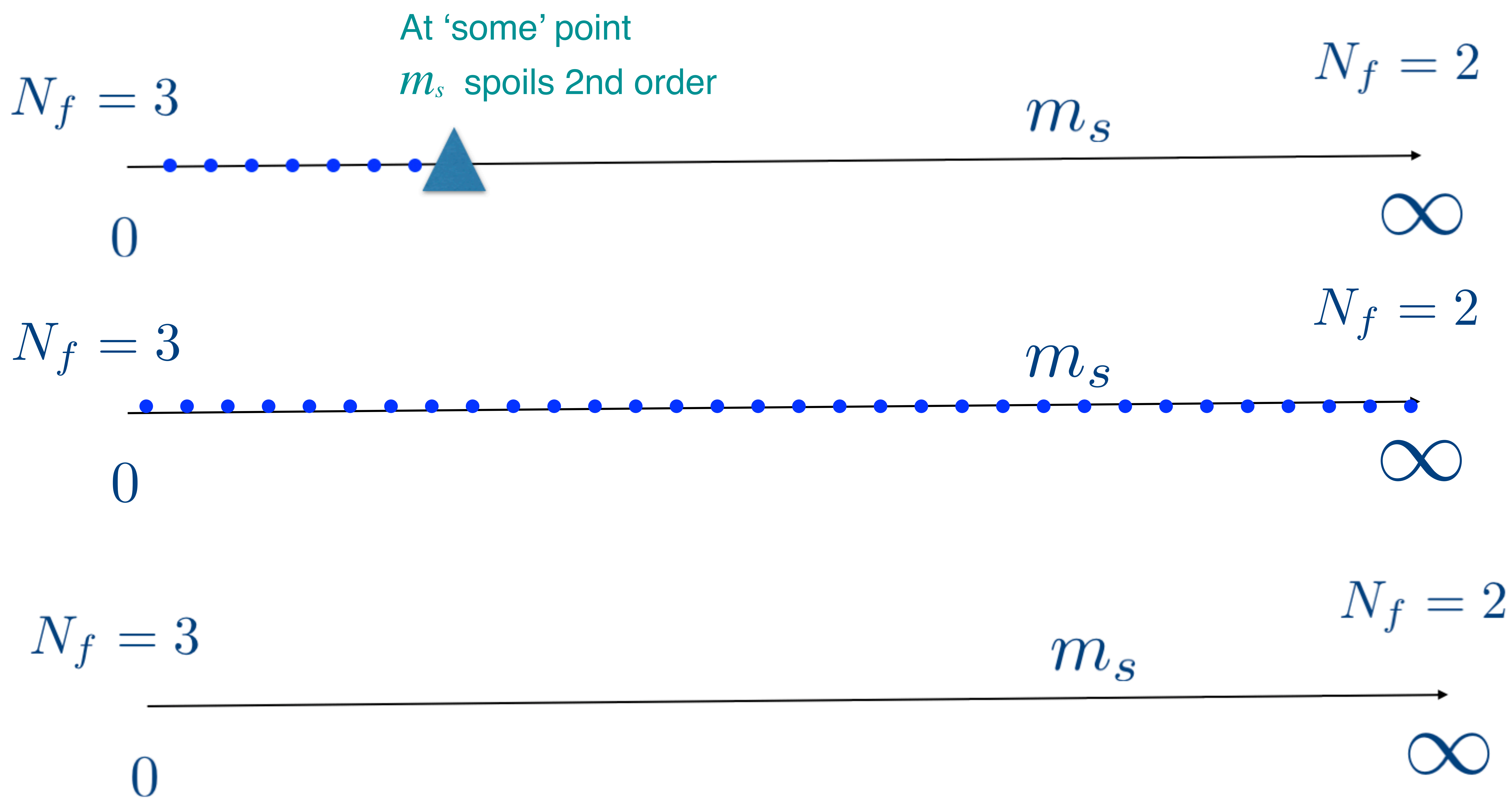
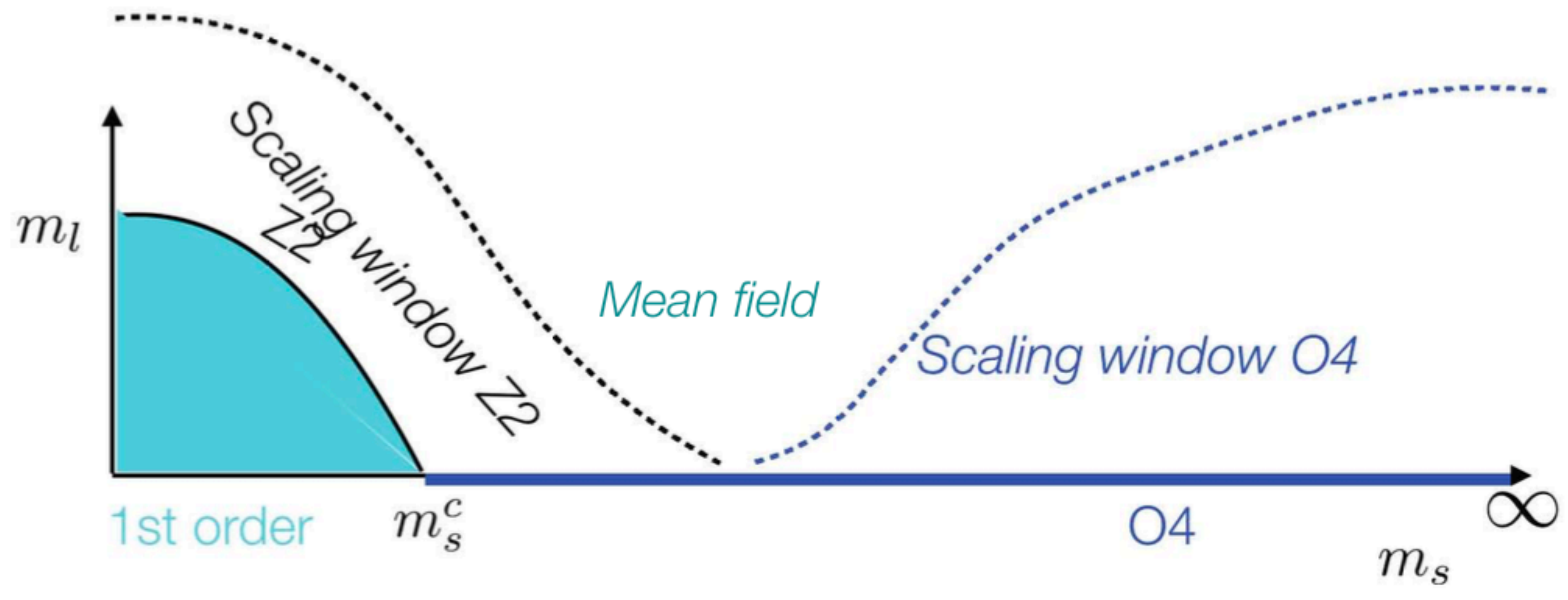
Order of the phase transition at
quark masses m_s, m_l
[talks by O. Philipsen, A. Lahiri, S. Sharma]

This talk:
QCD phase transition at $m_s \neq m_l \to 0$

Between N_f=2 to N_f=3: m_s as an interpolator



$m_l \neq 0$, possible scenarios



Scaling window: universal behaviour given by EoS

$$M = h^{1/\delta} f(t/h^{1/\beta\delta})$$

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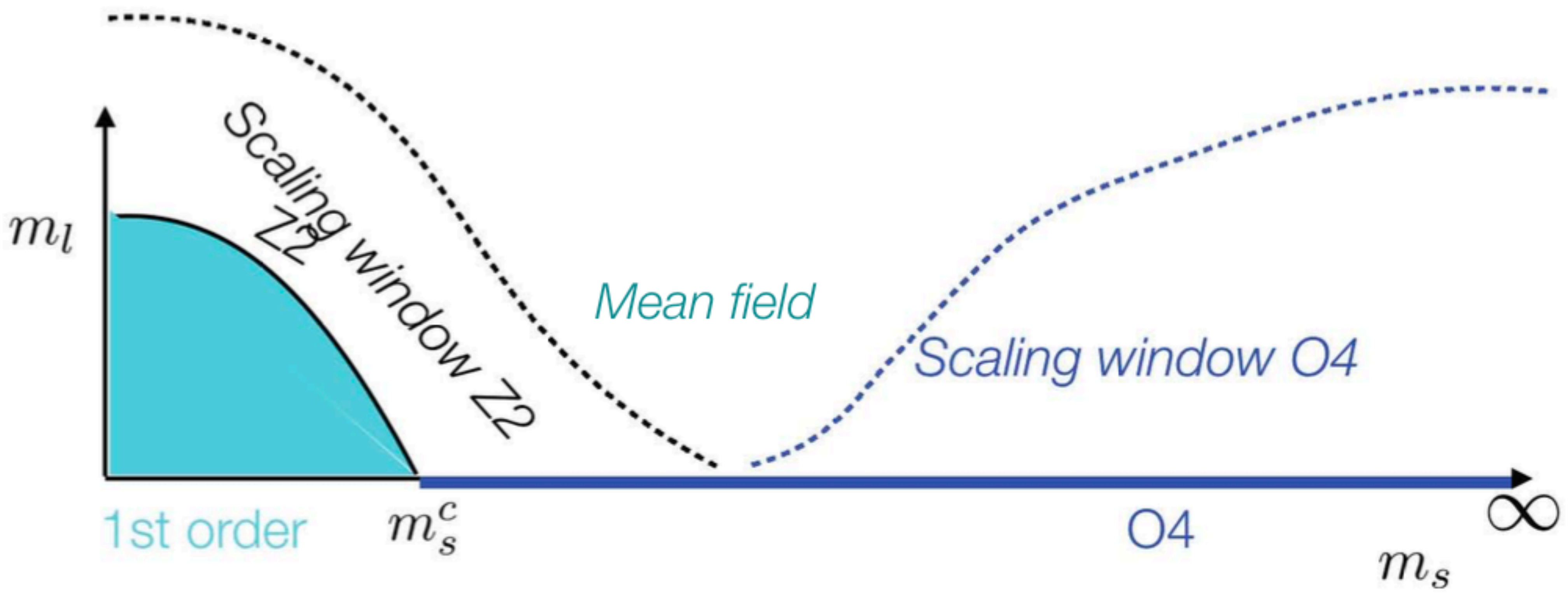

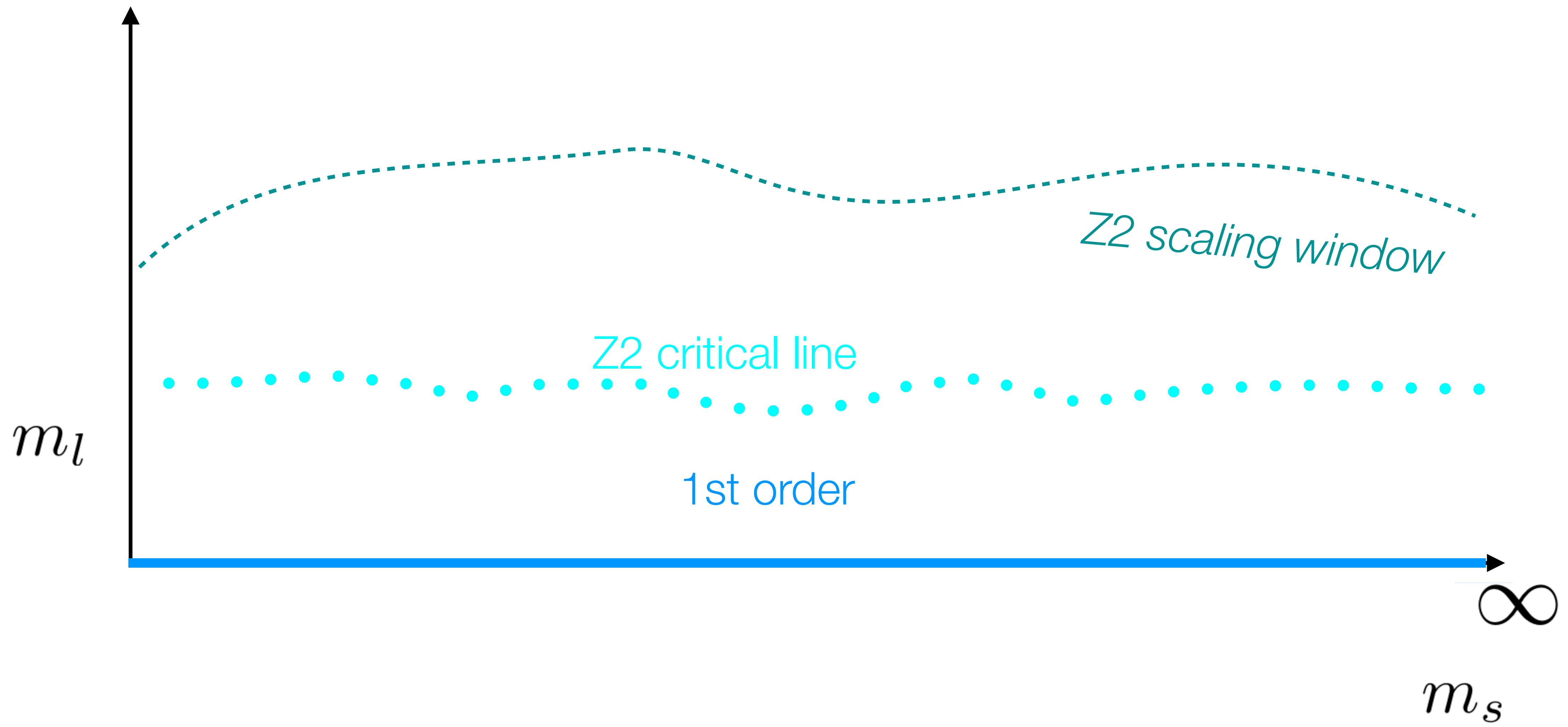

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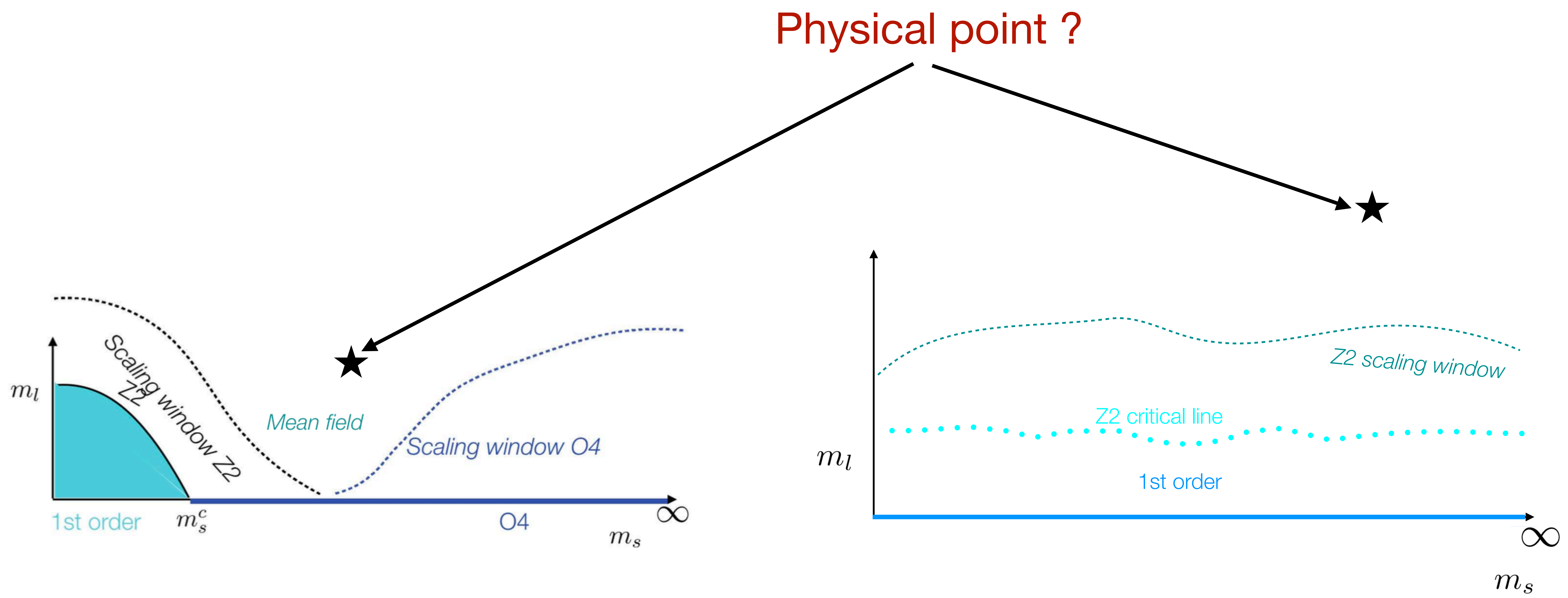

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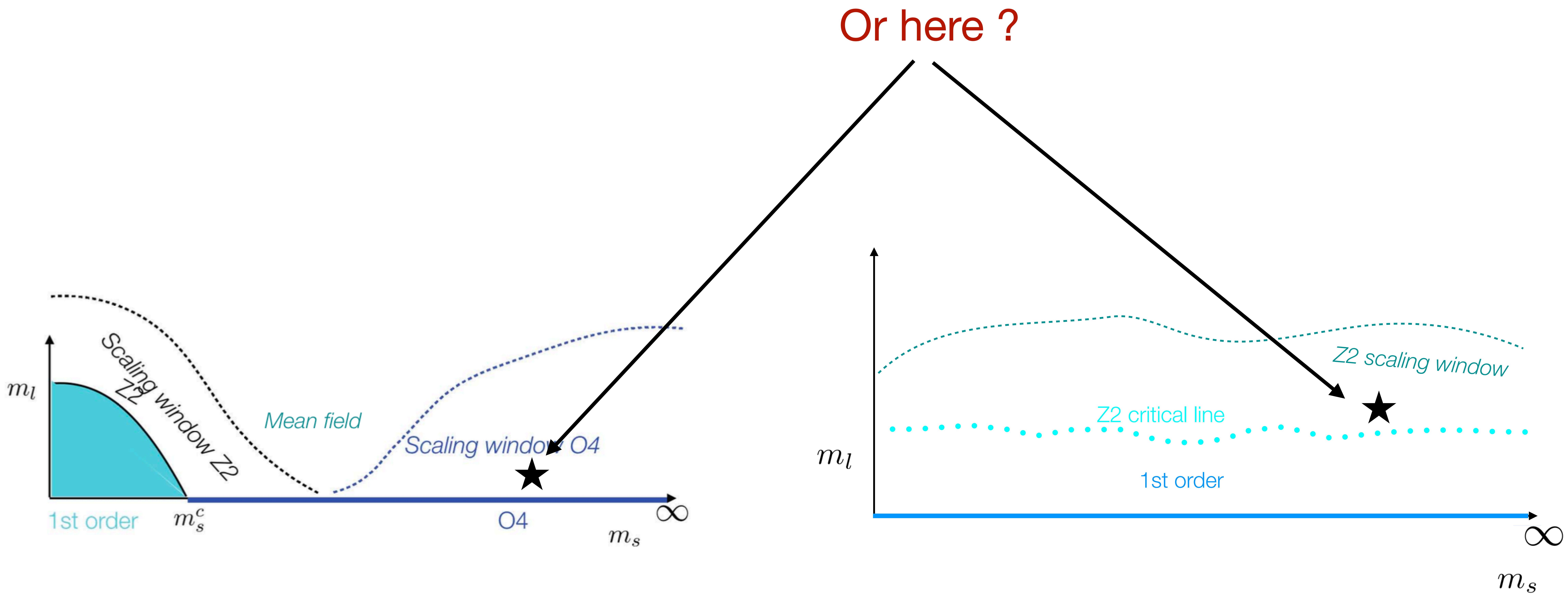

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- Mainly: O(4) universality class, other possible scenarios: Z_2 scaling, mean field
- Byproduct: estimation of $T_0 = T_c(m_\pi \to 0)$

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- Chiral susceptibility $\chi = \partial\langle\bar{\psi}\psi\rangle/\partial m$

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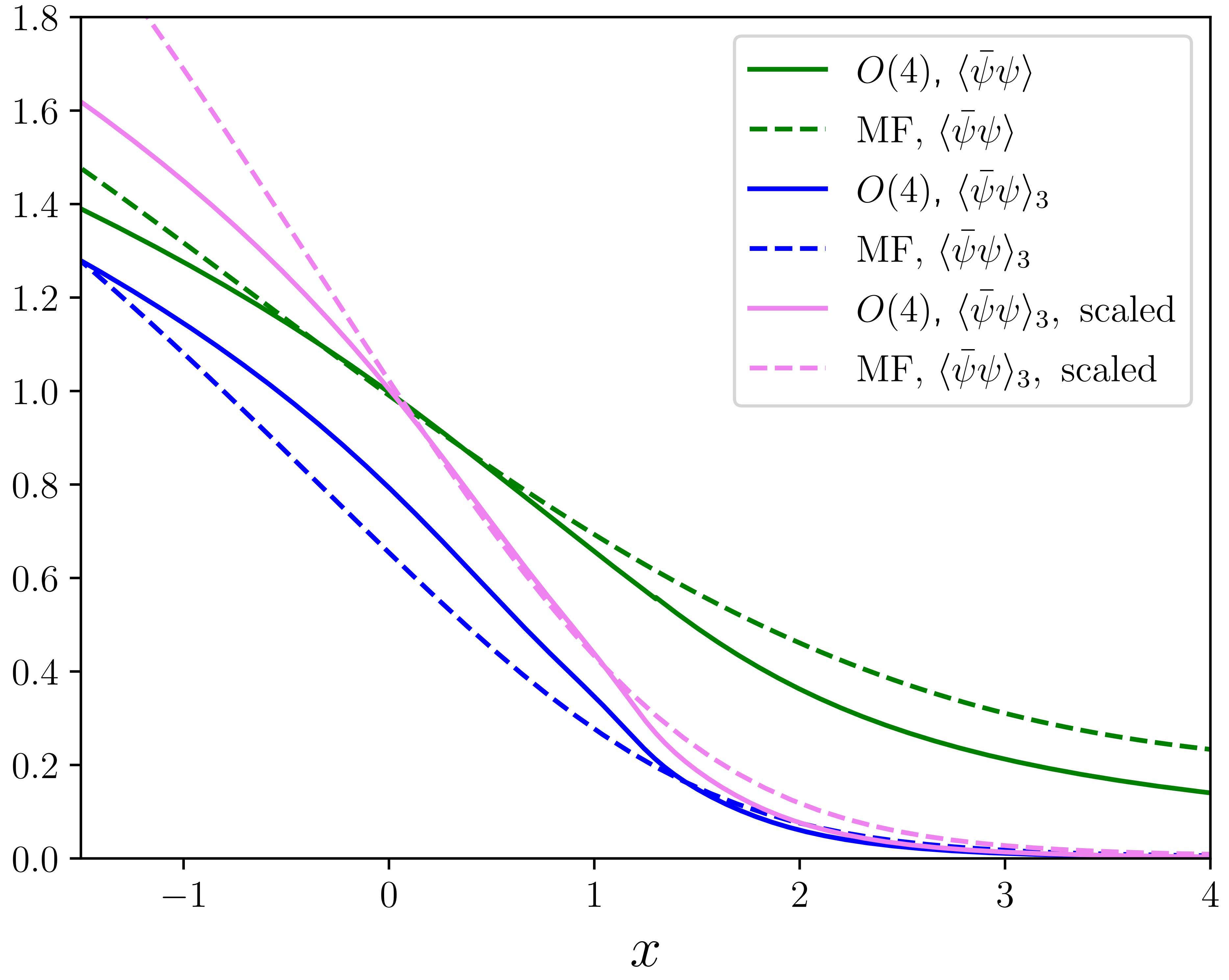
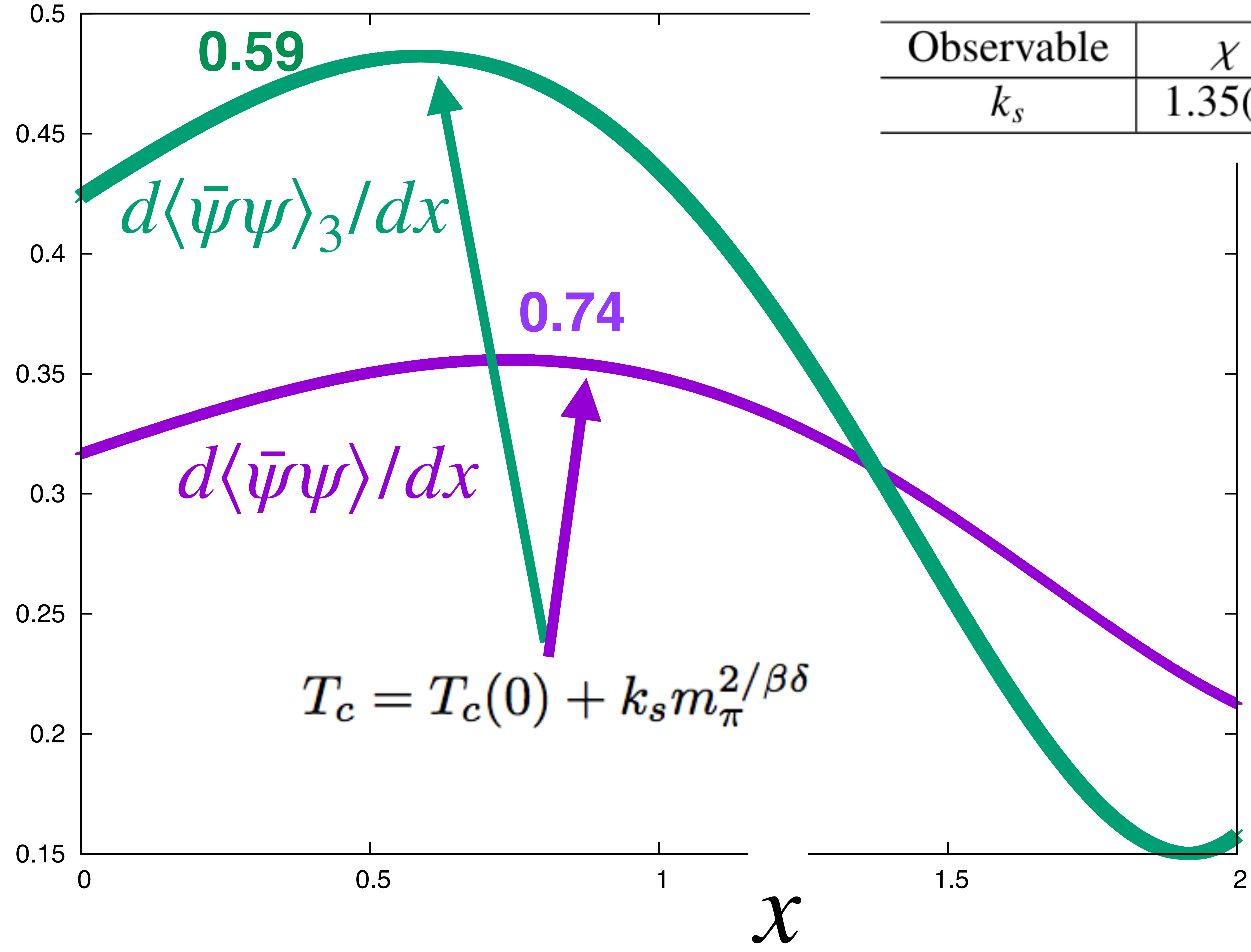
- Chiral susceptibility $\chi = \partial\langle\bar{\psi}\psi\rangle/\partial m$
- Novel order parameter: $\langle\bar{\psi}\psi\rangle_3 = \langle\bar{\psi}\psi\rangle - m\chi$
    - ~m^3 (symmetric phase)
    - $1/a^2$ divergences cancel
    - $\langle\bar{\psi}\psi\rangle_3 \sim t^{-\gamma-2\beta\delta}$ vs $\langle\bar{\psi}\psi\rangle \sim t^{-\gamma}$ as $t \to \infty$



Scaling of T_c with pion mass



Observable	χ	$\bar{\psi}\psi$	Δ_3
k_s	1.35(3)	0.74(4)	0.59(1)

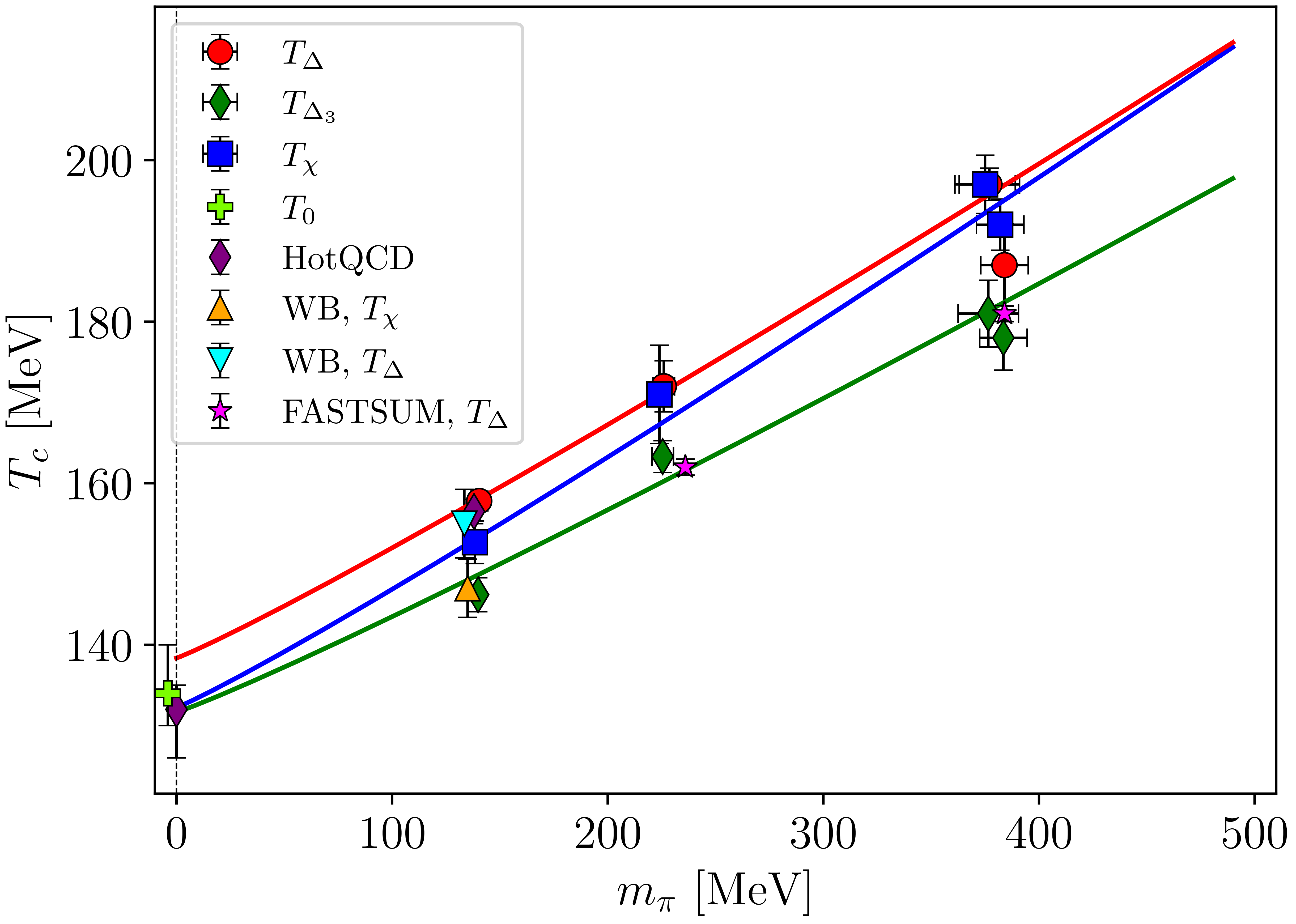
A couple of words about parameters

- $N_f = 2 + 1 + 1$ twisted mass Wilson fermions at maximal twist
- Fixed scale approach: $a =$ fixed, $T \leftrightarrow N_t$
- Based on ETMC T=0 parameters

[C. Alexandrou et al., 2018]

m_π [MeV]	a [fm]
139.7(3)	0.0801(4)
225(5)	0.0619(18)
383(11)	0.0619(18)
376(14)	0.0815(30)

Critical temperature and the chiral limit

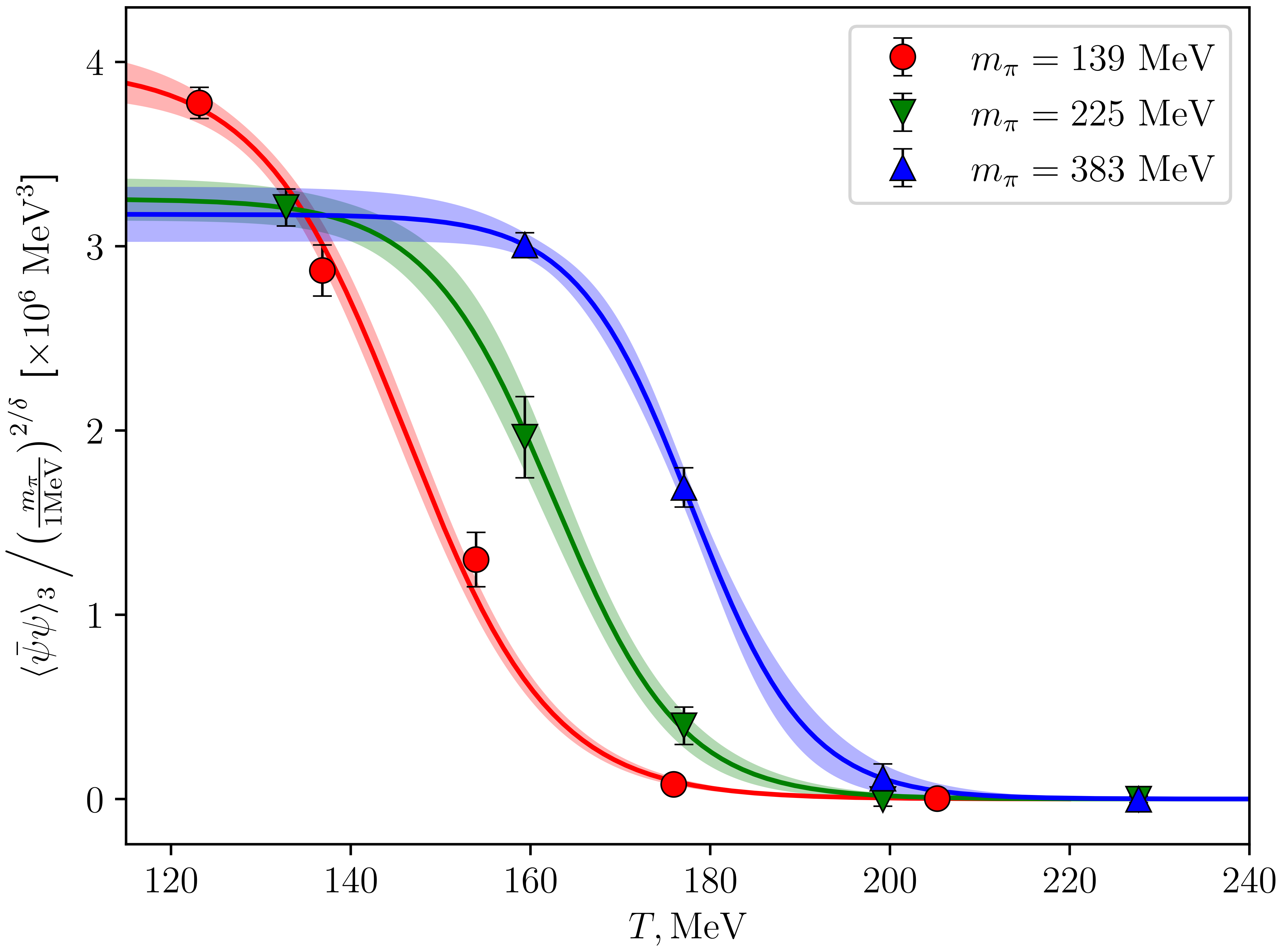


	$T(m_\pi = 139$ MeV) [MeV]	$T(m_\pi = 0)$ [MeV]
$\langle\bar{\psi}\psi\rangle$	157.8(12)	138(2)
χ	153(3)	132(4)
$\langle\bar{\psi}\psi\rangle_3$	146(2)	132(3)

$$T_c = T_c(0) + k_s m_\pi^{2/\beta\delta}$$

$$T_0 = 134^{+6}_{-4}\ \text{MeV}$$

Simple estimation of T_0 from EOS



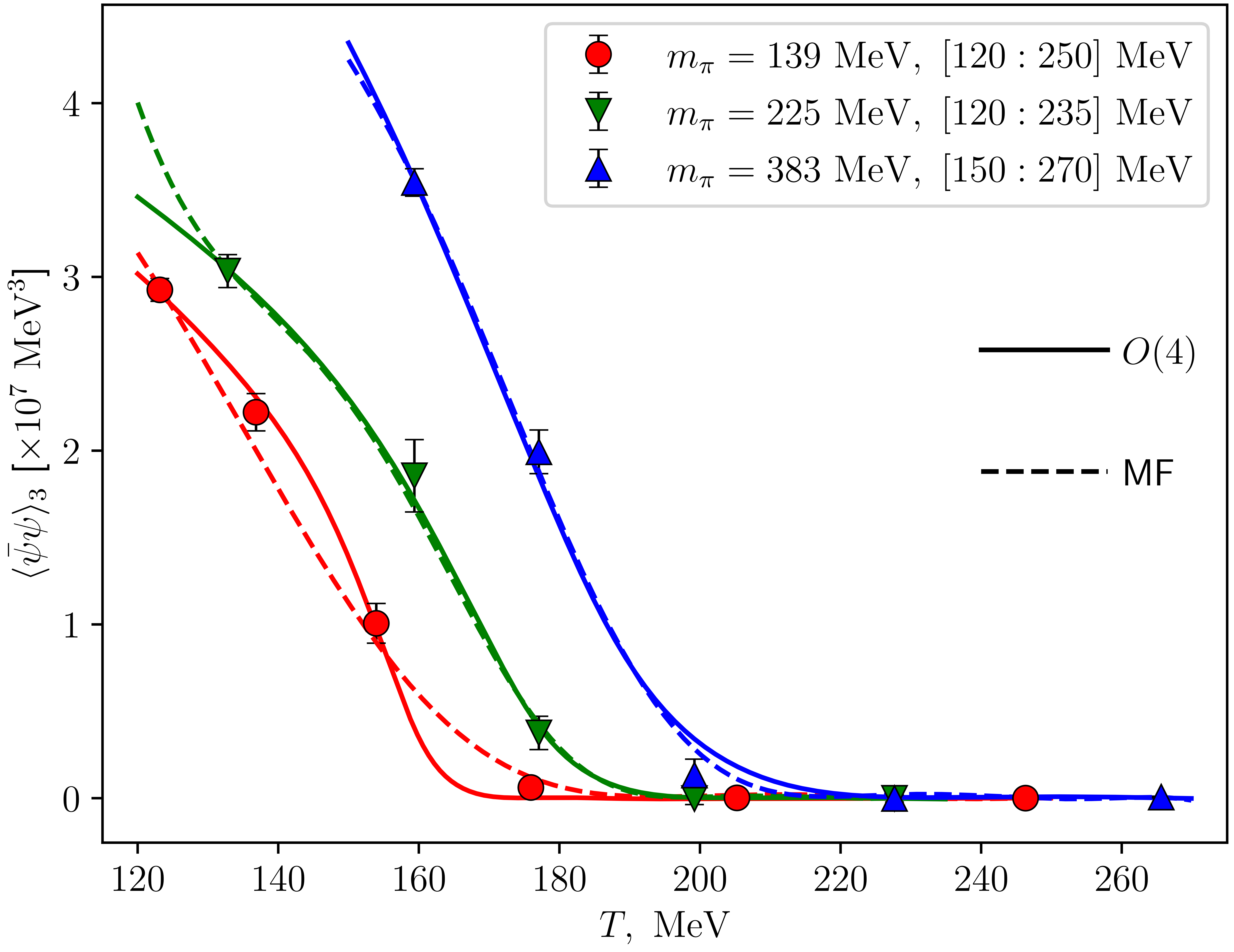
Prediction of EoS:

$$\frac{\langle\bar{\psi}\psi\rangle_3}{m^{1/\delta}} \sim \frac{\langle\bar{\psi}\psi\rangle_3}{m_\pi^{2/\delta}} = \text{const}$$

at

$$T = T_0(m_\pi = 0) = 138(2) \text{ MeV}$$

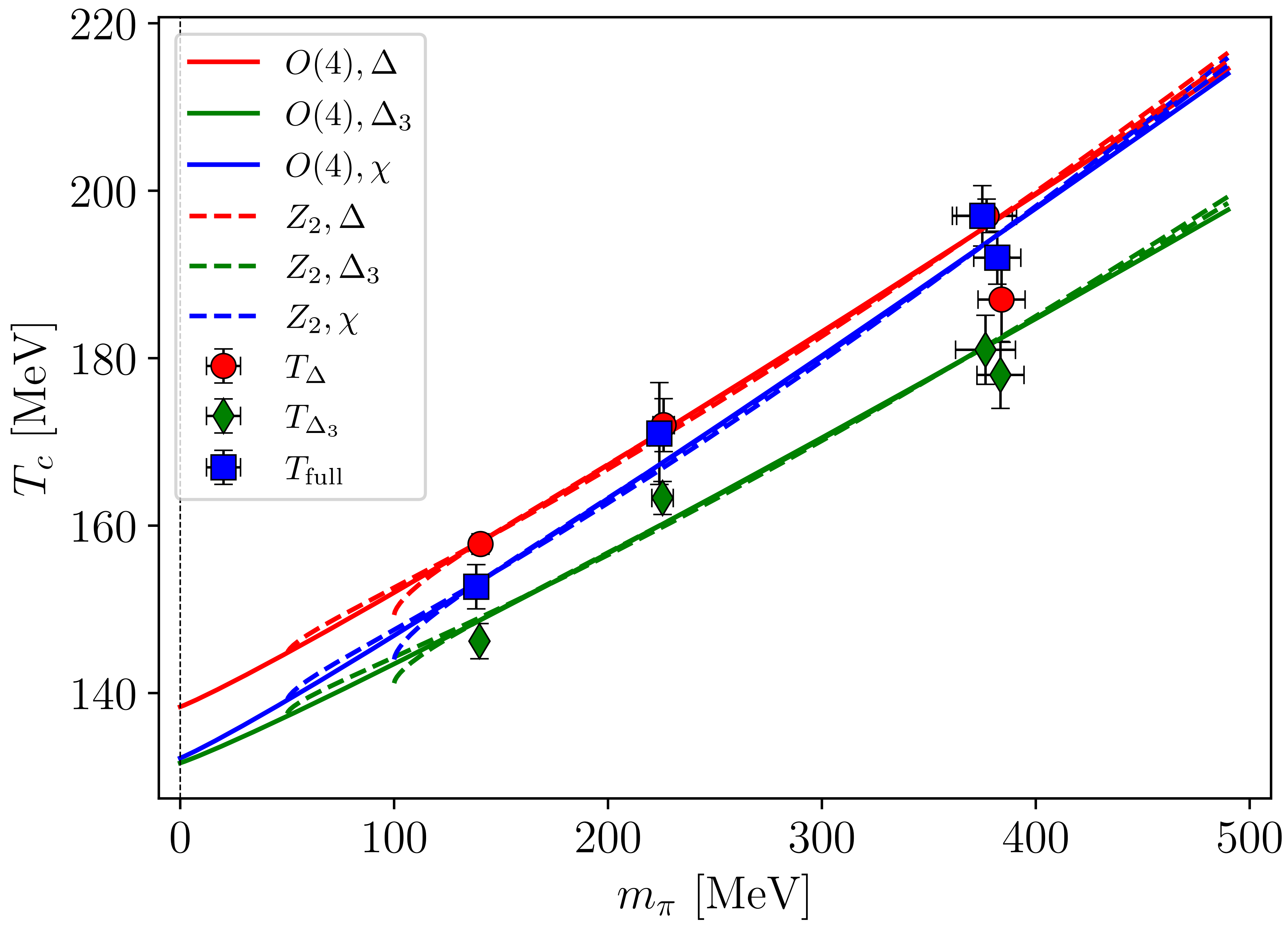
O(4) vs mean field



Mild tension between data and MF for m_π=139 MeV

m_π [MeV]	T_0 [MeV]
139	142(2)
225	159(3)
383	174(2)

Z_2 vs O(4) scaling



$$T_0 = T_c(m_\pi \to 0) = 134^{+6}_{-4} \text{ MeV}$$

O(4) scaling:

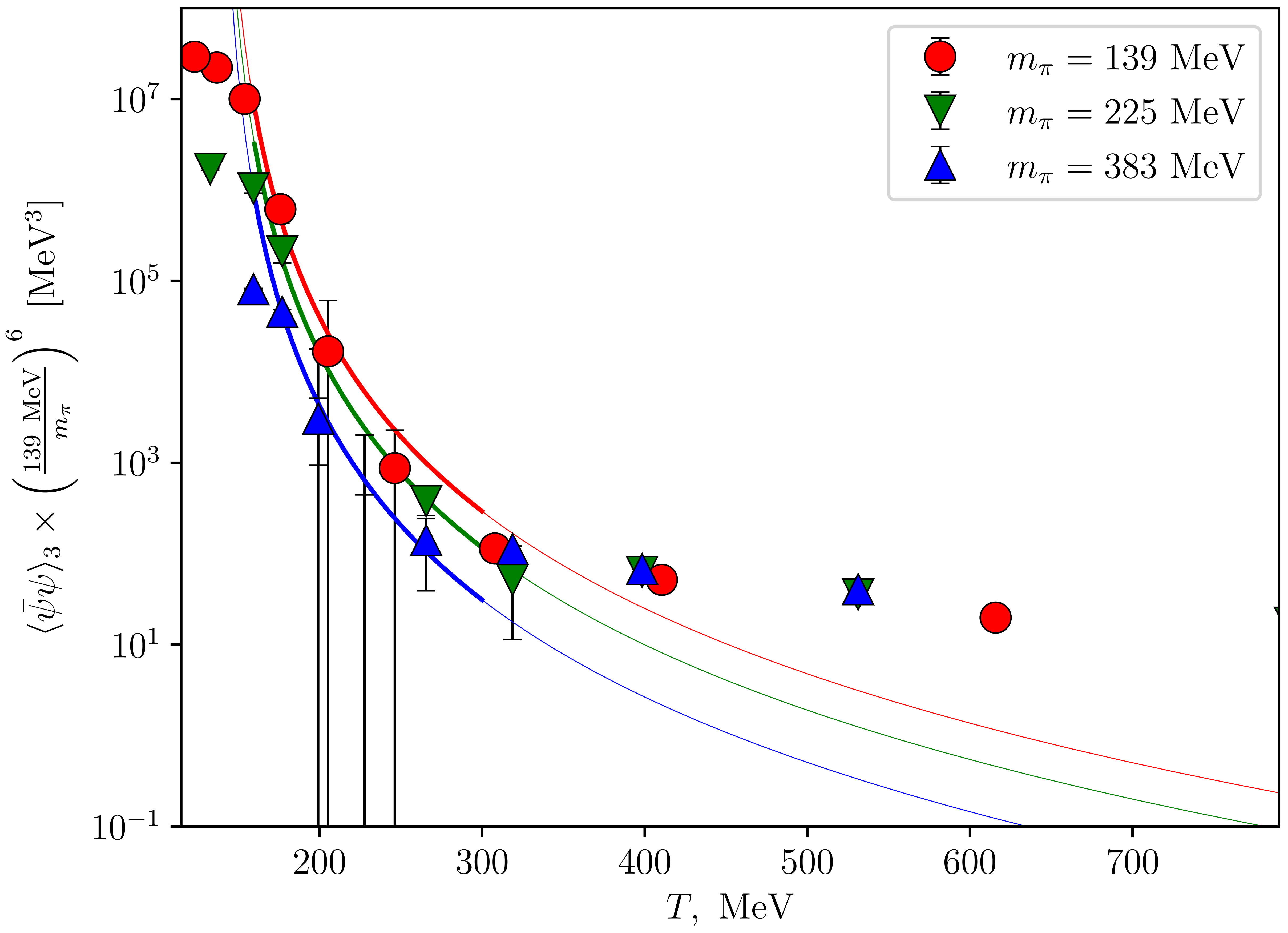
Observable	T_0 [MeV]	$z_p/z_{\bar\psi\psi_3}$	$z_p/z_{\bar\psi\psi_3}$ $O(4)$	z_p $O(4)$
χ	132(4)	1.24(17)	2.45(4)	1.35(3)
$\langle\bar\psi\psi\rangle$	138(2)	1.15(24)	1.35(7)	0.74(4)
$\langle\bar\psi\psi\rangle_3$	132(3)	1	1	0.55(1)

Z_2 scaling:

$m_\pi^c = 100$ MeV is still ok

$m_\pi^c = 0$ MeV is indistinguishable from O(4)

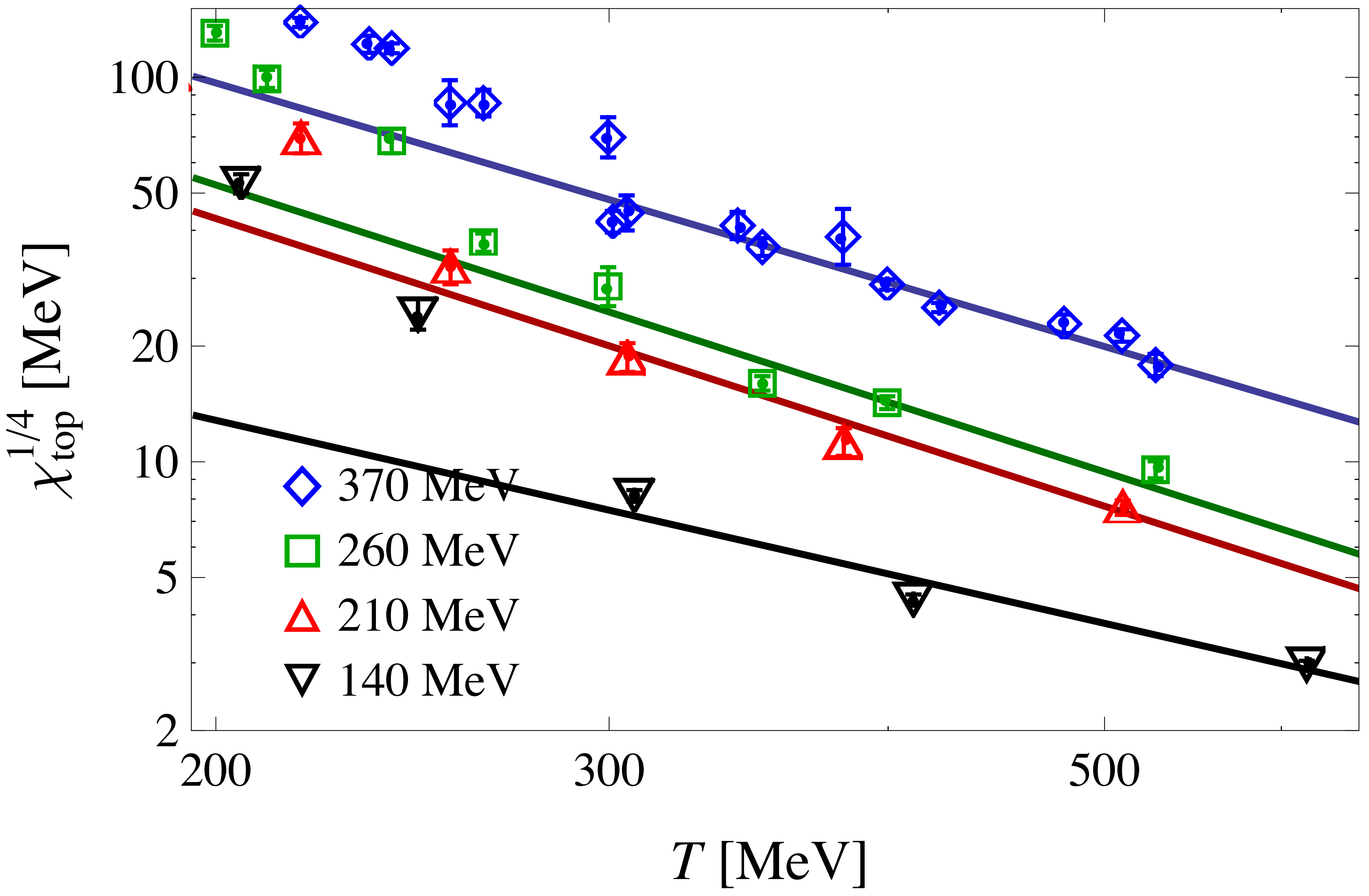
Large temperature behaviour



- O(4): $\langle\bar\psi\psi\rangle_3 \sim t^{-\gamma-2\beta\delta}$
- Griffith analyticity: $\langle\bar\psi\psi\rangle_3 \sim m^3 \sim m_\pi^6$
- $T \sim 300$ MeV

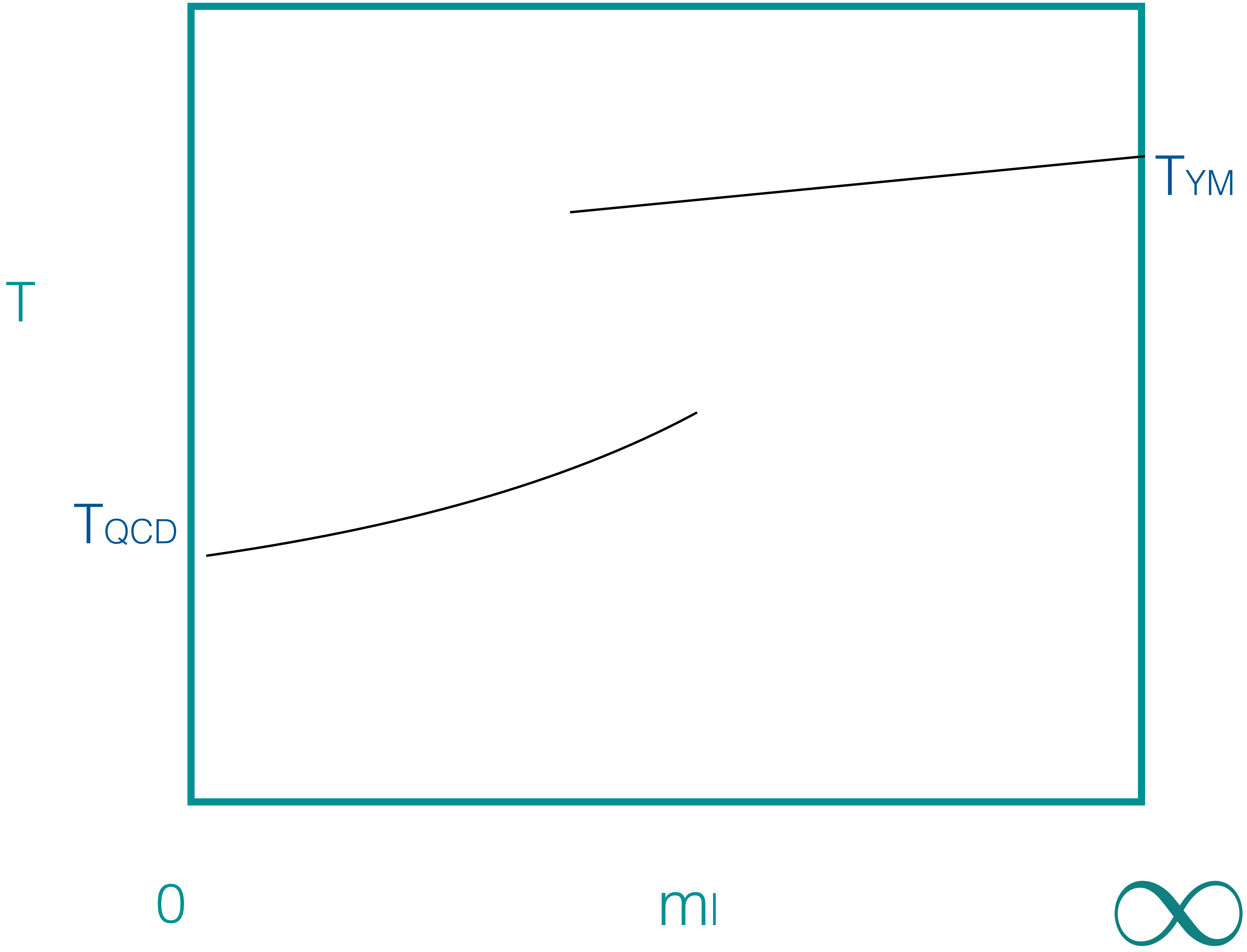
Thresholds in QGP

$T \sim 300$ **MeV**

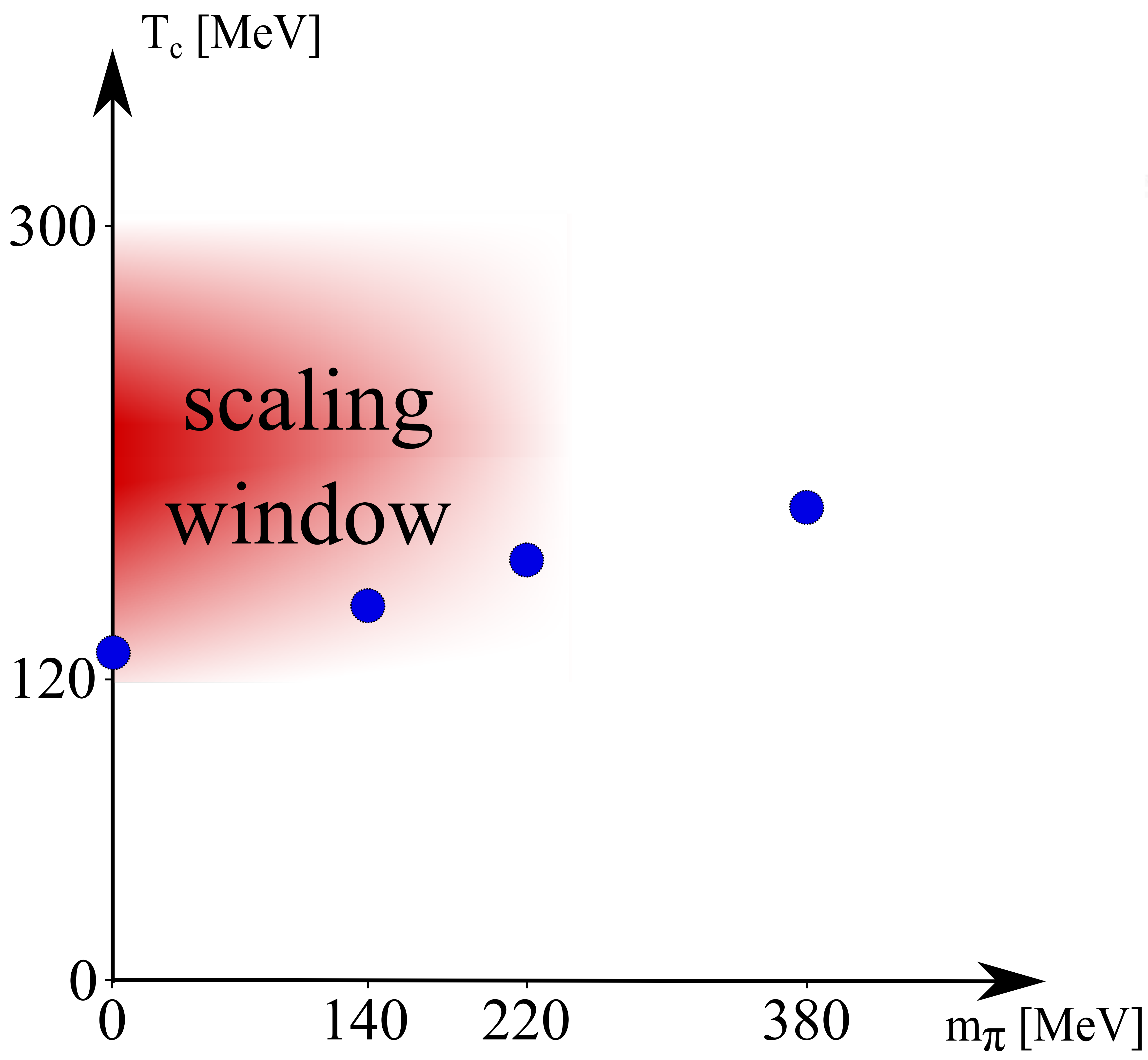


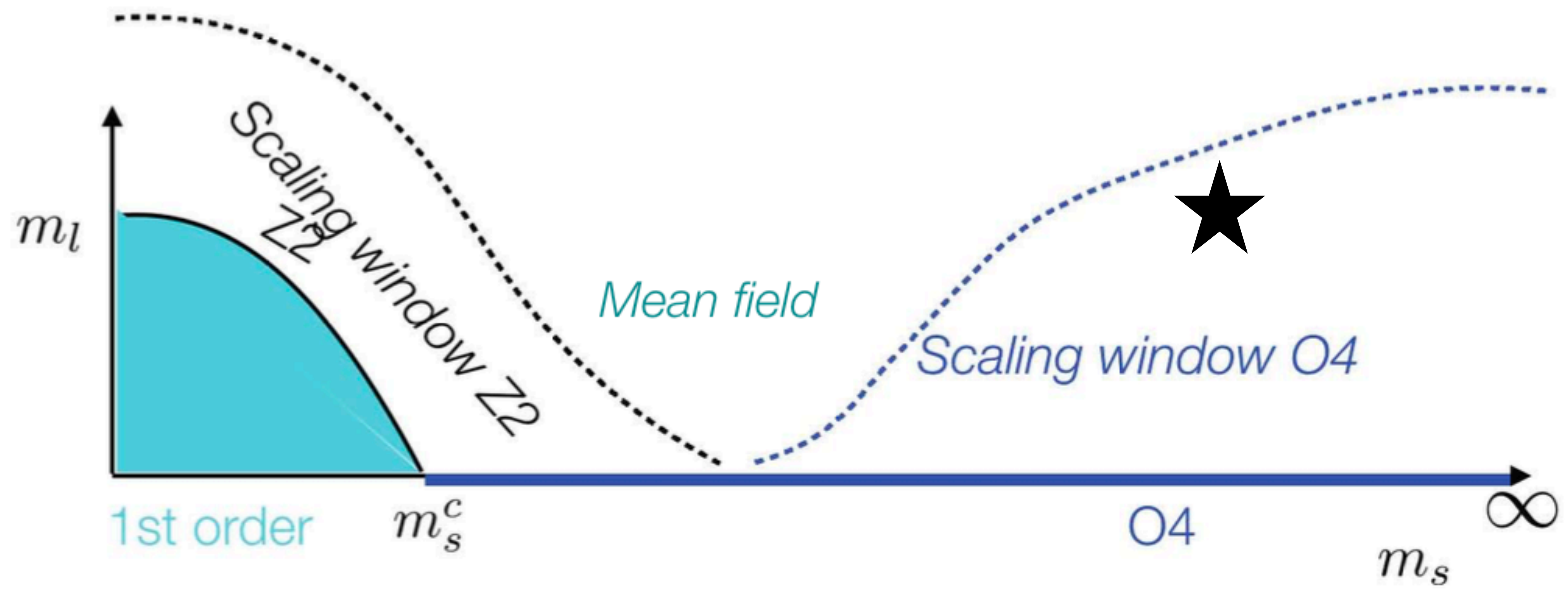
- Onset of DIGA behaviour
- Monopole condensation [Cardinali, D'Elia, Pasqui, 2021]
- Spectrum of Dirac operator [Alexandru, Horvath, 2019]
- Chiral-spin symmetry [Glozman, 2020, …]

Sketch of possible phase diagram



Scaling window





FRG: Tiny scaling window ($m_\pi < 1$MeV) ?

[Talk by J. Pawlowski]

Conclusions

- $\langle \bar{\psi}\psi \rangle_3 = \langle \bar{\psi}\psi \rangle - m\chi$ is useful to study scaling
- $T_0 = 134^{+6}_{-4}$ MeV in the chiral limit
- $O(4)$ scaling for $m_\pi \lesssim 140$ MeV, $T \in [120,\ 300]$ MeV
- Z_2 scaling cannot be excluded
- $T \sim 300$ MeV: threshold(s) in QGP