The topological susceptibility in high-T full QCD from staggered spectral projectors

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The QCD Axion and Dark Matter

The QCD axion, being weakly coupled to the Standard Model, has been considered as a Dark Matter candidate.
The behavior of the axion effective potential $V_{eff}(T)$ at high temperatures is extremely relevant for cosmology (e.g., axion relic abundance, axion mass) $\rightarrow$ essential input for present and future experimental searches.

Axion effective parameters related to QCD topological observables (χ, b_2, ...) $\rightarrow$ great interest around QCD topology at high-T for axion cosmology:

$$\chi = \frac{\langle Q^2 \rangle}{V}\bigg|_{\theta=0}, \qquad b_2 = -\frac{1}{12}\frac{\langle Q^4 \rangle - 3\langle Q^2 \rangle^2}{\langle Q^2 \rangle}\bigg|_{\theta=0}, \ldots$$

$$m_a^2 \propto \chi, \qquad \lambda_{4a} \propto b_2, \ldots$$

Non-chiral fermions and would-be-zero modes

In the QCD path-integral, field configurations are weighted with the determinant of the Dirac operator:

$$\det\{\not{D} + m_q\} = \prod_{\lambda \in \mathbb{R}} (i\lambda + m_q) .$$

The Index Theorem relates the presence of zero-modes in the spectrum of $\not{D}$ to the topological charge of the gluon field:

$$Q = \mathrm{Index}\{\not{D}\} = \mathrm{Tr}\{\gamma_5\} = n_+ - n_- .$$

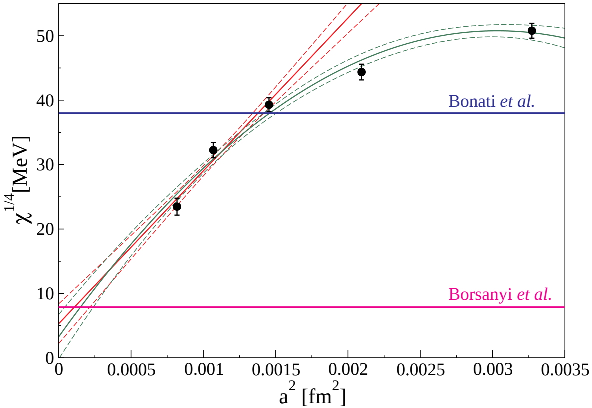
If a configuration has $Q \neq 0$, lowest eigenvalues are $\lambda_{min} = m_q$.

On the lattice, however, some fermionic discretizations (e.g., staggered) do not have exact zero-modes. $\implies$ The determinant fails to efficiently suppress non-zero charge configurations.

$$\lambda_{min} = m_q \longrightarrow m_q + i\lambda_0, \qquad \lambda_0 \underset{a \to 0}{\to} 0.$$

Non-chiral fermions and large lattice artifacts

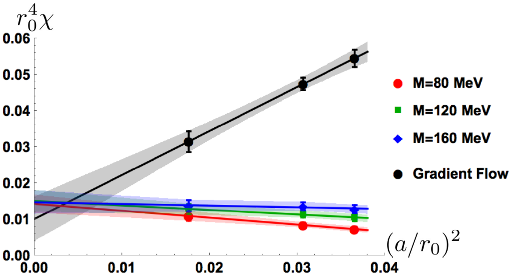
Bad suppression of non-zero charge configurations $\Longrightarrow$ large discretization corrections $\Longrightarrow$ continuum extrapolation not under control (Bonati et al., 2018):



In (Borsanyi et al., 2016) lattice artifacts affecting χ at high-T have been suppressed *a posteriori* by reweighting configurations with the corresponding continuum lowest eigenvalues of $\not{D}$.

Fermionic topological charge

Another possible solution, which does not require any ad hoc assumption, could be to switch, through the Index Theorem, to fermionic definitions of Q. Using the same "bad" operator to weight configurations and to count eigenmodes to measure Q may introduce smaller lattice artifacts.



Idea supported by results at $T = 0$ (Alexandrou et al., 2017): twisted mass Wilson fermions employed for the MC evolution and for the measure of χ through spectral projectors $\longrightarrow$ improved scaling of χ towards the continuum!

Goal: use staggered fermions spectral projectors definition (CB et al., 2019) to study χ at high-T from full QCD simulations with staggered fermions.

Spectral projectors with staggered fermions

In the continuum, only zero-modes contribute to Q. This is not true on the lattice for staggered fermions, due to the absence of exact zero-modes:

$$Q = \mathrm{Tr}\{\gamma_5\} \longrightarrow \mathrm{Tr}\{\Gamma_5 \mathbb{P}_M\},$$

$$\mathbb{P}_M = \sum_{|\lambda_k| \leq M} u_k u_k^\dagger, \qquad i\not{D}_{stag} u_k = \lambda_k u_k.$$

To avoid a mode over-counting, taste degeneration has to be considered ($n_t = 2^{d/2}$):

$$Q_{0,stag} = \frac{1}{n_t} \mathrm{Tr}\{\Gamma_5 \mathbb{P}_M\}.$$

Lattice charge gets a renormalization $Z_Q^{stag} = \frac{Z_P}{Z_S}$, which can be derived from Ward identities for the flavor-singlet axial current:

$$Q_{stag} = \frac{Z_P}{Z_S} Q_{0,stag}, \qquad \left(\frac{Z_P}{Z_S}\right)^2 = \frac{\langle \mathrm{Tr}\{\mathbb{P}_M\}\rangle}{\langle \mathrm{Tr}\{\Gamma_5 \mathbb{P}_M \Gamma_5 \mathbb{P}_M\}\rangle}.$$

Choice of the cut-off mass M

The choice of the cut-off mass M is irrelevant in the continuum limit. Its renormalized value $M_R = M/Z_S$ must be kept constant as $a \to 0$ to guarantee $O(a^2)$ corrections:

$$\chi_{\mathrm{SP}}(a, M_R) = \chi + c_{\mathrm{SP}}(M_R) a^2 + o(a^2).$$

To avoid the direct computation of Z_S for each lattice spacing, one can observe that, for staggered fermions:

$$m_{q,R} = m_q / Z_S.$$

If a Line of Constant Physics is known, it is sufficient to keep

$$M/m_q = M_R/m_{q,R}$$

constant as $a \to 0$ to have M_R constant too.
Is there an optimal choice for M_R? One would like to have small corrections, i.e., $c_{\mathrm{SP}}(M_R) \ll c_{\mathrm{gluo}}$.

Optimal choice for the cut-off mass M/m_q

Guiding principle: choose M/m_q to include all relevant Would-Be Zero-Modes (WBZMs) in spectral sums. E.g., look at chirality: $r_\lambda = |u_\lambda^\dagger \Gamma_5 u_\lambda|$ vs λ/m_q.

However, distinguishing between WBZMs and non-chiral modes is ambiguous → choose cut-off to include "chiral enough" modes.

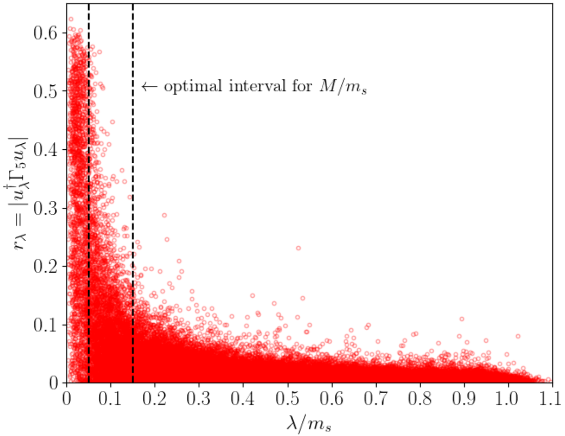
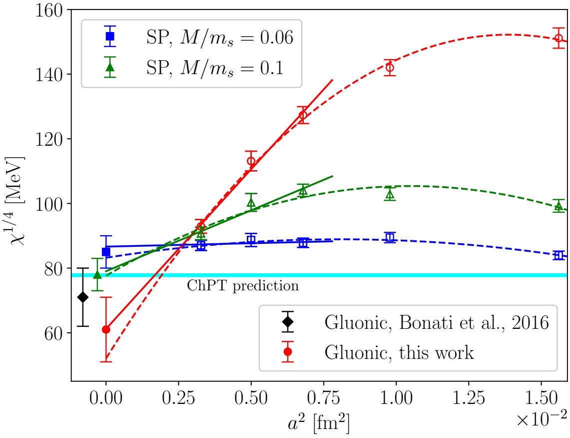


Figure refers to: $N_f = 2+1$ QCD, $T \simeq 0$, $V = 48^4$, $a \simeq 0.057$ fm, $m_q = m_s$.

Vertical lines: optimal choices for $M/m_s \in [0.05, 0.15]$.

Lattice Setup: $N_f = 2+1$ rooted stout staggered fermions at physical point.
Expected continuum scaling for Spectral Projectors (SP):

$$\chi_{\rm SP}^{1/4}(a, M/m_s) = \chi^{1/4} + c_{\rm SP}(M/m_s)a^2 + o(a^2).$$



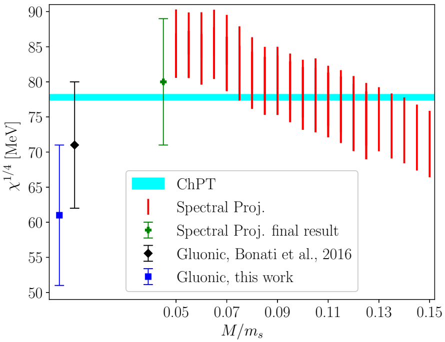
M/m_s inside optimal interval $\rightarrow$ reduction of lattice artifacts:
$c_{\rm SP}(0.06)/c_{\rm gluo} \sim 1 \cdot 10^{-2}$,
$c_{\rm SP}(0.1)/c_{\rm gluo} \sim 3 \cdot 10^{-1}$.

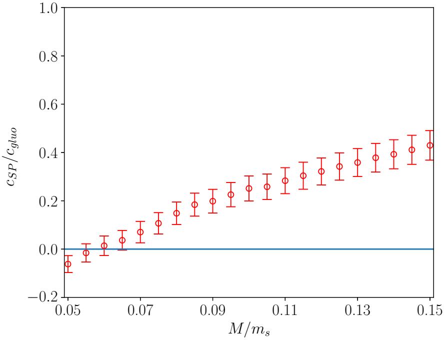
Spectral determination: very good agreement with gluonic and leading order Chiral Perturbation Theory (ChPT) determinations.

Continuum extrapolation $T = 0$ vs M/m_s

Choosing M/m_s inside the optimal range we observe:

- good agreement within the errors for determinations obtained for different values of M/m_s (Fig. on the left)
- significant reduction of lattice artifacts compared to the standard gluonic computation (Fig. on the right)





Chirality vs M/m_s at finite T

Same strategy as $T = 0$: consider $r_\lambda = |u_\lambda^\dagger \Gamma_5 u_\lambda|$ vs λ/m_s to estimate optimal range for M/m_s.

At finite T more clear separation of WBZMs compared to $T = 0$ case, although some ambiguity is still present.

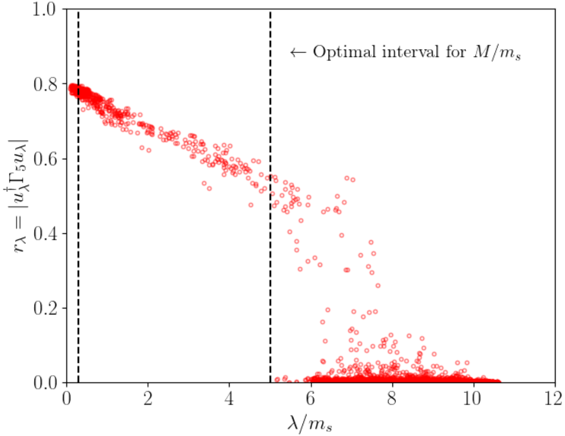


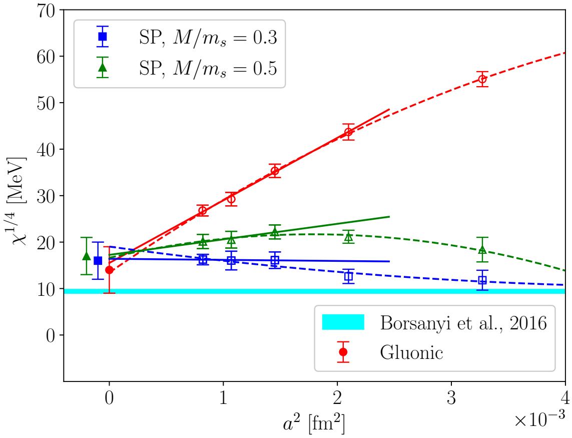
Figure refers to: $N_f = 2+1$ QCD, $T \simeq 430$ MeV, $V = 48^3 \times 16$, $a \simeq 0.0286$ fm, $m_q = m_s$.

Vertical lines: optimal choices for $M/m_s \in [0.3, 5]$.

Continuum limit of $\chi^{1/4}$ at finite T ($T = 430$ MeV)

Same lattice setup of the $T = 0$ case. Also in this case, we consider the following continuum-scaling function for Spectral Projectors (SP):

$$\chi_{\mathrm{SP}}^{1/4}(a, M/m_s) = \chi^{1/4} + c_{\mathrm{SP}}(M/m_s)a^2 + o(a^2).$$

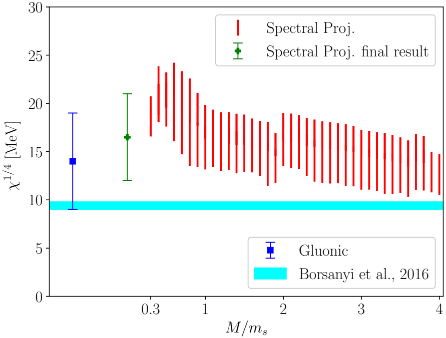


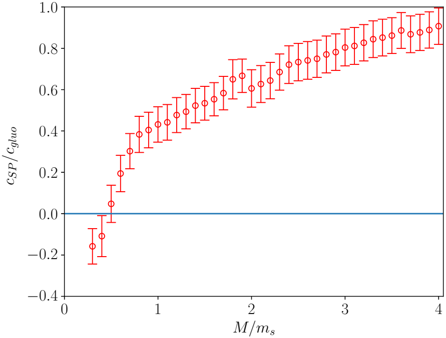
Spectral lattice artifacts are suppressed compared to the gluonic case when M/m_s is chosen in the previously determined optimal interval:
$c_{\mathrm{SP}}(0.3)/c_{\mathrm{gluo}} \sim 5 \cdot 10^{-2}$,
$c_{\mathrm{SP}}(0.5)/c_{\mathrm{gluo}} \sim 10^{-1}$.

Continuum extrapolation $T = 430$ vs M/m_s

Choosing M/m_s inside the optimal range we observe:

- good agreement within the errors for determinations obtained for different values of M/m_s (Fig. on the left)
- significant reduction of lattice artifacts compared to the standard gluonic computation can be achieved with suitable choice of M/m_s (Fig. on the right)

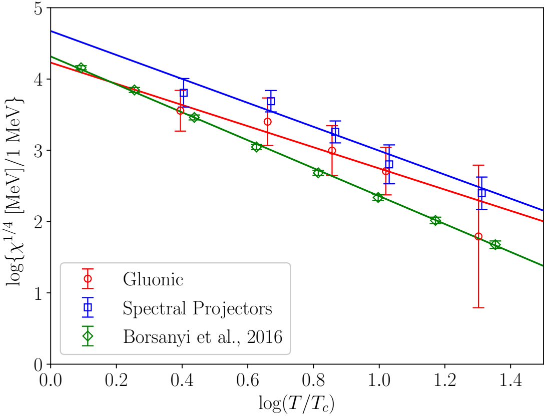




$\chi(T)$ for $T > T_c$ from Spectral Projectors

The Dilute Instanton Gas Approximation (DIGA) predicts the power-law:

$$\chi^{1/4}(T) \sim T^{-b}, \qquad T \gg T_c, \qquad b_{\text{DIGA}} \simeq 2.$$



Our data are in very good agreement with a decaying power-law, with exponents:

$$b_{\text{SP}} = 1.68(31)$$
$$b_{\text{gluo}} = 1.48(62)$$

Compare also with result from WB collab. (Borsanyi at al., 2016):

$$b_{\text{WB}} = 1.96(2).$$

Conclusions

Summary of the talk:

- Spectral Projectors (SP) provide a theoretically well-posed method to define the topological susceptibility
- Spectral definition of χ allows to control the magnitude of lattice artifacts through a smart choice of the cut-off mass M
- Systematics related to the continuum extrapolation and to the choice of M are well under control
- Good agreement among SP data and DIGA prediction: $\chi_{\mathrm{SP}}^{1/4}(T) \sim T^{-b}$, $b_{\mathrm{SP}} = 1.68(31)$ VS $b_{\mathrm{DIGA}} \simeq 2$

Future outlooks

- it would be interesting to explore higher temperatures, where Spectral Projectors are expected to provide major improvements
- to reach $T \gtrsim 700$ MeV on typical lattices, $a \sim 0.01$ fm needed $\implies$ severe Topological Slowing Down with standard RHMC. Promising candidate for a viable solution: **Parallel Tempering on Boundary Conditions** (Hasenbusch, 2017; CB, Bonati, D'Elia, 2021)

Thank you for your attention!