Decomposition of the gauge field in the maximal abelian gauge

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The work is completed in collaboration with

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1. Maximal abelian projection and abelian dominance
2. Decomposition of static potential in SU(2) gluodynamics and SU(2) QCD
3. Decomposition in SU(3)
4. Other observables

Dual superconductor scenario of confinement

t'Hooft '75, Mandelstam '76

- In a dual superconductor roles of the electric and magnetic fields are interchanged
- Formation of the Abrikosov-Nilsen-Olesen string is due condensation of color-magnetic monopoles
- Described by dual Abelian Higgs model
- Yet unsolved task to rigorously prove that infrared QCD is dual to Abelian Higgs model

Maximal abelian gauge

MA gague condition (SU(2) case)

$$(\partial_\mu \delta_{kl} + \epsilon_{k3l} A^3_\mu(x)) A^l_\mu(x) = 0$$

Functional

$$F[A] = \int d^4x \quad \left[(A^1_\mu)^2 + (A^2_\mu)^2 \right]$$

Abelian projection

$$A^a_\mu T^a \to A^3_\mu T^3 \qquad \text{(in observables)}$$

Abelian dominance hypothesis

Abelian projection (in terms of lattice gauge fields):

$$U_\mu(x) = C_\mu(x) u_\mu(x)$$

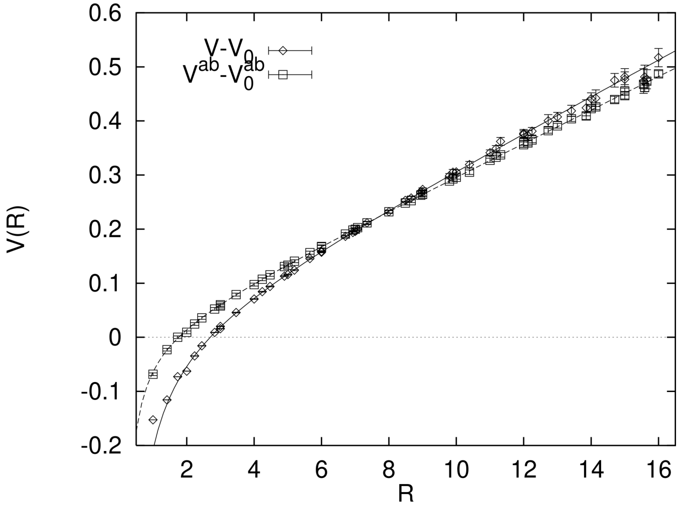
$$< O >= \frac{1}{Z} \int e^{-S} O(U_\mu) DU_\mu$$

and

$$< O >^{Ab}= \frac{1}{Z} \int e^{-S} O(u_\mu) DU_\mu$$

give rise to approximately equal values of infrared physical observables like string tension.

First numerical results:
Suzuki and Yotsuyanagi, 1990



G. Bali et al., 1996

Monopoles

We use the DeGrand-Toussaint definition:

$$u_\mu(x) = e^{i\theta_\mu(x)T_3}$$

$$\theta_{\mu\nu}(x) = \partial_\mu\theta_\nu(x) - \partial_\nu\theta_\mu(x)$$

$$\theta_{\mu\nu}(x) = \bar{\theta}_{\mu\nu}(x) + 2\pi m_{\mu\nu}(x)$$

$$k_\mu(x) = \frac{1}{2}\epsilon_{\mu\nu\alpha\beta}\partial_\nu m_{\alpha\beta}(x)$$

Monopole decomposition

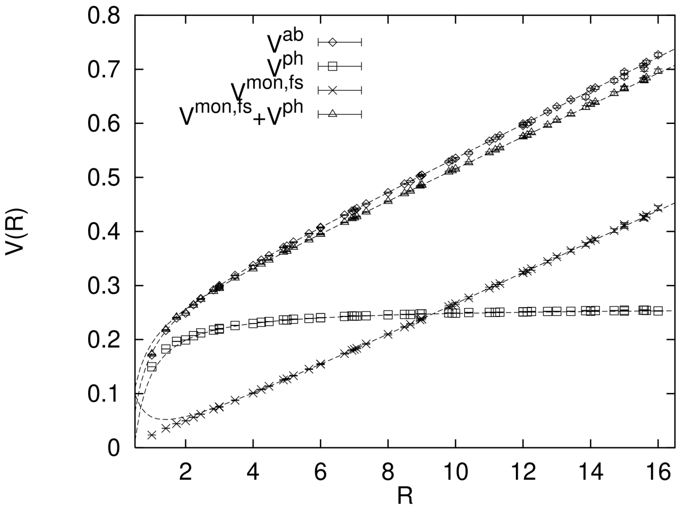
$$\theta_\mu^{mon}(x) = -2\pi \sum_y D(x-y)\partial'_\nu m_{\nu\mu}(y)$$

$$\theta_\mu^{ph}(x) = \theta_\mu(x) - \theta_\mu^{mon}(x)$$

$$u_\mu^{mon}(x) = exp(i\theta_\mu^{mon}(x))$$

$$u_\mu^{ph}(x) = exp(i\theta_\mu^{ph}(x))$$

$$U_\mu^{mod}(x) = U_\mu(x) u_\mu^{mon,\dagger}(x)$$



G. Bali et al., 1996

Observables

$$W(C) = \frac{1}{2}\mathrm{Tr}\left(\prod_{l\in C} U(l)\right)$$

$$W_{mon}(C) = \frac{1}{2}\mathrm{Tr}\left(\prod_{l\in C} u^{mon}(l)\right)$$

$$W_{mod}(C) = \frac{1}{2}\mathrm{Tr}\left(\prod_{l\in C} U^{mod}(l)\right)$$

$$V(R) = \lim_{T\to\infty} -\log\left(\frac{W(R,T+1)}{W(R,T)}\right)$$

Usual representation:

$$U_\mu(x) = C_\mu(x) u_\mu(x)$$

$$u_\mu(x) = u_\mu^{mon}(x) u_\mu^{ph}(x)$$

We suggest to consider:

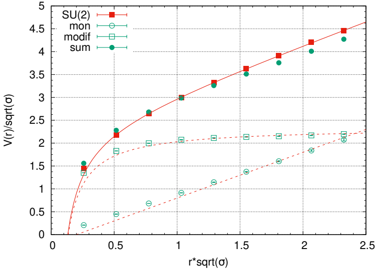
$$U_\mu(x) = U_\mu^{mod}(x) u_\mu^{mon}(x)$$

Then we demonstrate that:

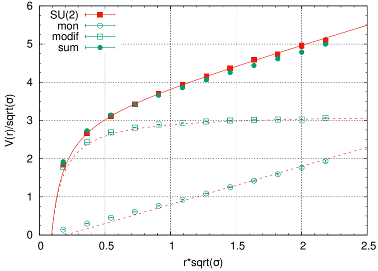
$$V(r) \approx V_{mon}(r) + V_{mod}(r) \qquad (*)$$

at all distances r

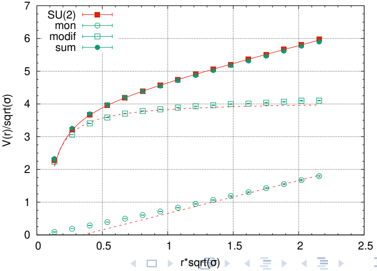
beta 2.4



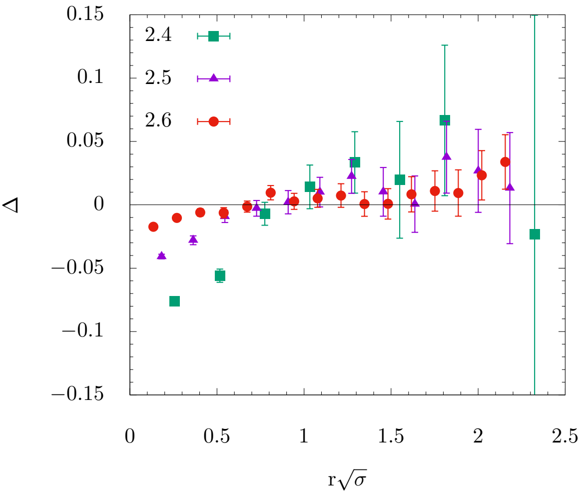
beta 2.5



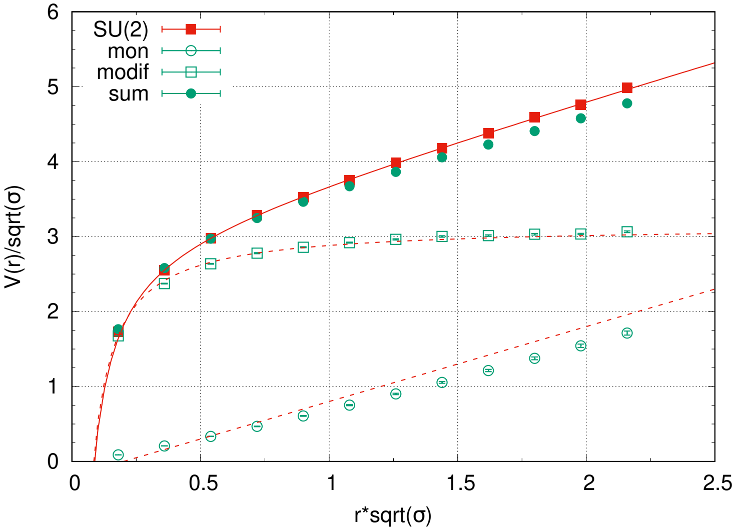
beta 2.6



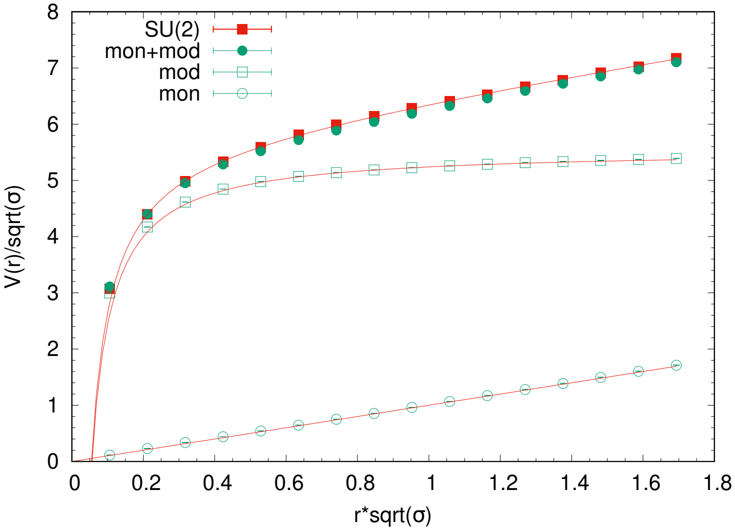
Relative deviation



Universality check

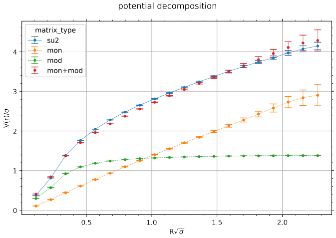


QC_2D

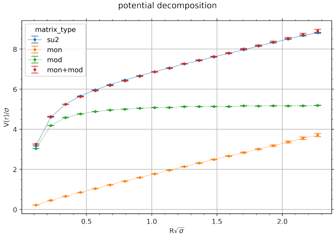

SU(2)
mon+mod
mod
mon
V(r)/sqrt(σ)
r*sqrt(σ)
0
1
2
3
4
5
6
7
8
0.2
0.4
0.6
0.8
1.2
1.4
1.6
1.8

Non-zero chemical potential

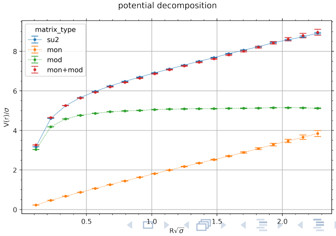
$\mu = 0$

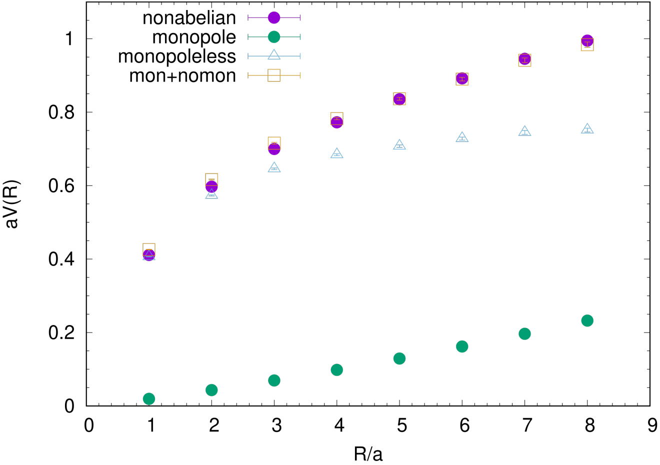


$\mu = 0.2$



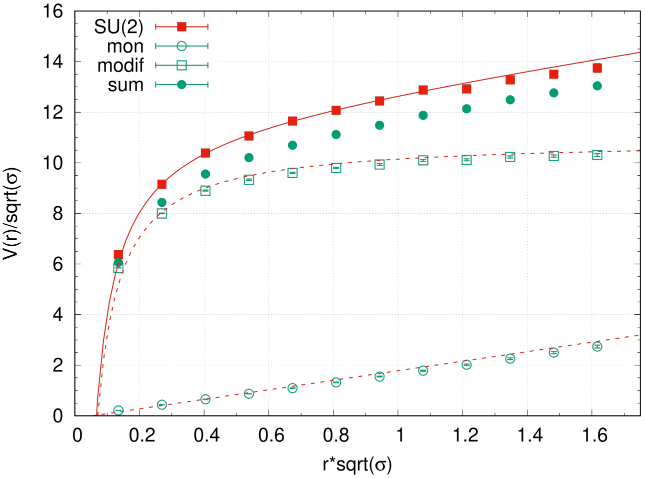
$\mu = 0.3$





$\beta = 6.0, \quad 16^4$ lattice, $\quad a \approx 0.1\text{fm}$

Adjoint representation, $\beta = 2.6$



Flux tube

$$< f_R^{\mu\nu}(r) >= \frac{< W(R,T) P^{\mu\nu}(r) >}{< W(R,T) >} - < P^{\mu\nu}(r) >$$

$$< E_i^2 >= \lim_{T\to\infty} < f_R^{0i} >$$

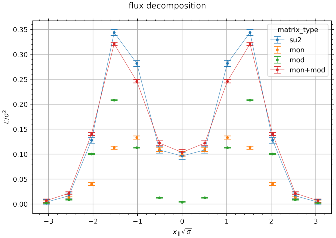
$$< B_i^2 >= \lim_{T\to\infty} < f_R^{jk} >$$

$$< \epsilon >=< E^2 > - < B^2 >$$

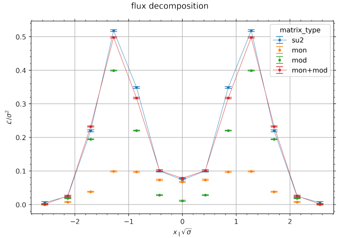
$$< \mathscr{L} >=< E^2 > + < B^2 >$$

Longitudinal flux tube

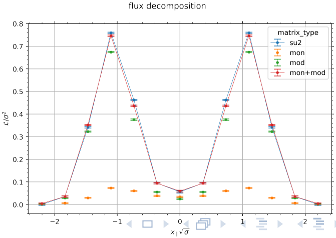
beta 2.4



beta 2.5

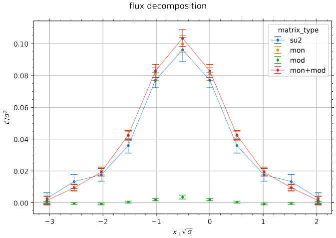


beta 2.6

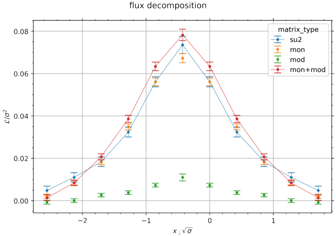


Transversal flux tube

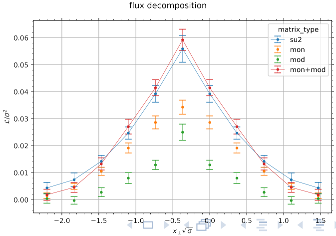
beta 2.4



beta 2.5



beta 2.6



Conclusions and outlook

We demonstrated that the relation

$$V(r) \approx V_{mon}(r) + V_{mod}(r) \qquad (*)$$

- holds in various theories: SU(2) and SU(3) gluodynamics with Wilson and improved action; in SU(2) QCD
- might become exact in the continuum limit !
- similar relations for other observables: flux tube, adjoint representation
- our next steps:
- to check the relation in full QCD
- to search for explanation. We will study the correlations between U^{mod} and u^{mon}.