Computation of QCD meson screening masses at high temperature

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Outline

Introduction

Screening masses m characterize spatial length scale of a perturbation by O

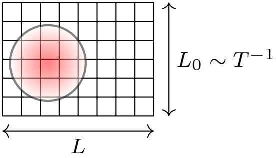
$$C(x_3) = \int \mathrm{d}x_0 \mathrm{d}x_1 \mathrm{d}x_2 \, \langle O(x) O(0) \rangle \sim \mathrm{e}^{-m x_3} \qquad \text{as} \qquad x_3 \to \infty$$

$\rightsquigarrow$ probe restoration of global symmetries

$\rightsquigarrow$ bilinear (meson) correlators of phenomenological interest

Screening masses are good lattice observables

- static
- RGI
- high precision
- compared with perturbative EFT predictions



Compute mesonic screening masses in high-temperature regime $T \sim 100\,\mathrm{GeV}$ using lattice QCD

[1] C. DeTar and J. Kogut. In: *Phys. Rev. Lett.* 59 (4 July 1987), pp. 399–402.
[2] B. B. Brandt et al. In: *JHEP* 05 (2014), p. 117. arXiv: 1404.2404 [hep-ph].
[3] A. Bazavov et al. In: *Phys. Rev. D* 100.9 (2019), p. 094510. arXiv: 1908.09552 [hep-lat].

Lattice QCD at very large temperatures

Scale setting using a hadronic quantity

$$a = (aM^{\text{latt.}})/M^{\text{exp.}},$$

requires physically large volumes

$$a \ll 1/M \ll L.$$

At high temperatures we must also satisfy

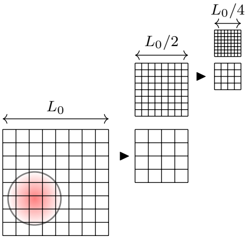
$$a \ll 1/T \ll L$$

$\rightarrow$ double scale hierarchy if $T \gg M$.

Instead, using a finite-volume coupling $\mu = 1/L_0$

$$\bar{g}_{\text{SF}}(g_0^2, a\mu) = \bar{g}_{\text{SF}}(\mu)$$

$\rightsquigarrow$ link scale to temperature $\mu = \sqrt{2}T$



allows us to reach high temperatures $T \sim 100\,\text{GeV}$

[4] M. Lüscher et al. In: *Nucl. Phys. B* 413 (1994), pp. 481–502. arXiv: hep-lat/9309005.

[5] L. Giusti and M. Pepe. In: *Phys. Lett. B* 769 (2017), pp. 385–390. arXiv: 1612.00265 [hep-lat].

Fixing the Lines of Constant Physics (LCP)

Knowledge of the renormalized coupling at given scale

$$\bar{g}^2_{\mathrm{SF}}(\sqrt{2}T) = 2.0120 \quad \longleftrightarrow \quad T\sqrt{2} = 4.30(11)\mathrm{GeV}$$

and non-perturbative running we have the relation $T \leftrightarrow \bar{g}^2_{\mathrm{SF}}$

Bare parameters are set for each L_0/a by interpolating

$$\frac{1}{\bar{g}^2_{\mathrm{SF}}} = \frac{1}{g_0^2} + \sum_k c_k g_0^{2k}$$

	$\bar{g}^2_{\mathrm{SF}}(\mu = T\sqrt{2})$	T (GeV)
T_0	–	164.6(5.6)
T_1	1.11000	82.3(2.8)
T_2	1.18446	51.4(1.7)
T_3	1.26569	32.8(1.0)
T_4	1.3627	20.63(63)
T_5	1.4808	12.77(37)
T_6	1.6173	8.03(22)
T_7	1.7943	4.91(13)
T_8	2.0120	3.040(78)
	$\bar{g}^2_{\mathrm{GF}}(\mu = T/\sqrt{2})$	T (GeV)
T_9	2.7359	2.833(68)
T_{10}	3.2029	1.821(39)
T_{11}	3.8643	1.167(23)

⇝ LCP fixed by $L_0 = 1/\sqrt{2}T$ and $m_{\mathrm{q}} = 0$

⇝ For each L_0/a keep L/a fixed

[6] M. Bruno et al. In: *Phys. Rev. Lett.* 119.10 (2017), p. 102001. arXiv: 1706.03821 [hep-lat].

[7] M. Dalla Brida et al. In: *Eur. Phys. J. C* 78.5 (2018), p. 372. arXiv: 1803.10230 [hep-lat].

Finite volume effects on thermal correlators

Why can we simulate in such small physical volumes?

Finite volume effect on the correlator $C(x_3)$ in an $L_0 \times L_1 \times L \times L$ volume

$$\mathcal{I}(x_3, L) = C(x_3) - \lim_{L_1 \to \infty} C(x_3)$$

Spectral decomposition in the compact $\hat{1}$ direction

$$\mathcal{I}(x_3, L) = \sum_{M_0 < E_n < \pi T} \mathrm{e}^{-LE_n} \{\tilde{G}_n(x_3) - \tilde{G}_0(x_3)\} + \dots$$

in terms of some matrix element $\tilde{G}_n \sim \int \langle n | O^a O^a | n \rangle$.

High-temperature EFT guarantees mass gap $M_0 > 0$

$$\boxed{M_0 \sim g^2 T \Rightarrow \text{exponentially suppressed in } LT \sim 20 - 50}$$

[8] M. Laine and M. Vepsalainen. In: *JHEP* 09 (2009), p. 023. arXiv: 0906.4450 [hep-ph].

[9] M. T. Hansen and A. Patella. In: *Phys. Rev. Lett.* 123 (2019), p. 172001. arXiv: 1904.10010 [hep-lat].

Lattice set-up

- $N_f = 3$ chiral limit $m_q = 0$
- wide temperature range

  $164.6(5.6) \ldots 1.167(23)$ GeV
- $O(a)$-improved Wilson fermions
- four lattice resolutions

  $L_0/a = 4, 6, 8, 10$
- large spatial size $L/a = 288$

  very large aspect ratios $LT = 20 - 50$
- very high T allows us to fix topology $Q = 0$
- shifted boundary conditions

$$U_\mu(x_0 + L_0, \boldsymbol{x}) = U_\mu(x_0, \boldsymbol{x} - L\boldsymbol{\xi}), \qquad \text{with shift} \qquad \boldsymbol{\xi} = (1,0,0),$$
$$\psi(x_0 + L_0, \boldsymbol{x}) = -\psi(x_0, \boldsymbol{x} - L\boldsymbol{\xi})$$

T	L_0/a	β	κ_{cr}	c_{sw}
T_0 164.6(5.6) GeV	4	8.7325	0.131887597685602	1.224666388699756
	6	8.9950	0.131885781718599	1.214293680665697
T_1	4	8.3033	0.132316223701646	1.244443949720750
	6	8.5403	0.132336064110711	1.233045285565058
	8	8.7325	0.132133744093735	1.224666388699756
	10	8.8727	0.131984877002653	1.218983546266290
T_2	4	7.9794	0.132672230374640	1.262303345977765
	6	8.2170	0.132690343212428	1.248924515099129
	8	8.4044	0.132476707113024	1.239426196162344
	10	8.5534	0.132305706323476	1.232451001338001
⋮	⋮	⋮	⋮	⋮
⋮	⋮	⋮	⋮	⋮
T_9	4	4.764900	0.134885548000448	1.335350323996506
	6	4.938726	0.134507608658235	1.308983384364439
	8	5.100000	0.134168886219319	1.288203306487197
T_{10}	4	4.457600	0.135606746160064	1.39574103127591
	6	4.634654	0.135199857298424	1.358462476494125
	8	4.800000	0.134821158536685	1.329646151978636
T_{11} 1.167(23) GeV	4	4.151900	0.136325892438363	1.482418125298923
	6	4.331660	0.135926636004668	1.427424655158656
	8	4.500000	0.135525721037715	1.386110343557152

[10] L. Giusti and H. B. Meyer. In: *JHEP* 11 (2011), p. 087. arXiv: 1110.3136 [hep-lat].

Definition of lattice correlators

Flavour non-singlet static screening correlators

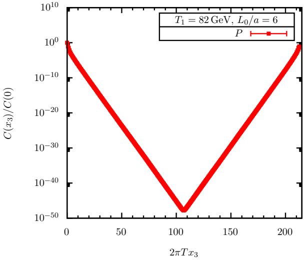
$$C(x_3 - y_3) = a^3 \sum_{x_0, x_1, x_2} \langle O^a(x) O^a(y) \rangle$$

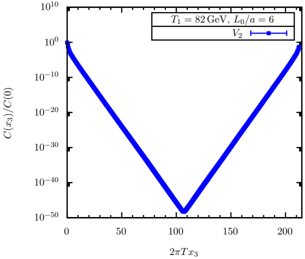
using four sources y per configuration

for (bare) densities and transverse currents

$$\begin{aligned} P^a &= \bar{\Psi}\gamma_5 T^a \Psi, \\ S^a &= \bar{\Psi} T^a \Psi, \\ V_2^a &= \bar{\Psi}\gamma_2 T^a \Psi, \\ A_2^a &= \bar{\Psi}\gamma_2\gamma_5 T^a \Psi \end{aligned}$$

only "quark-connected" contraction





Distance preconditioning

Dirac operator D has large gap $\sim \omega_0 = \pi T$

$D^{-1}(x,y)$ becomes small at large $|x-y| \sim L$

$\Rightarrow$ poor accuracy using global residuum

solution "distance preconditioning"

$$D\psi = \eta \to M^{-1}DM \cdot M^{-1}\psi = M^{-1}\eta$$

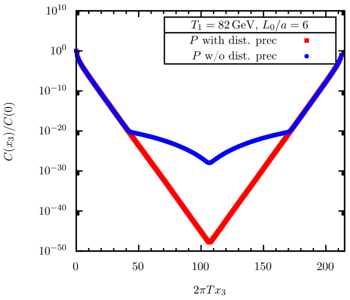
where

$$M(x,y) = \cosh\{m_M(x_3 - y_3 - L/2)\}$$

Choose $m_M \sim \pi T$ makes all components of

$$M^{-1}\psi$$

to have similar magnitude.



[11] G. M. de Divitiis et al. In: *Phys. Lett. B* 692 (2010), pp. 157–160. arXiv: `1006.4028 [hep-lat]`.

Effective mass

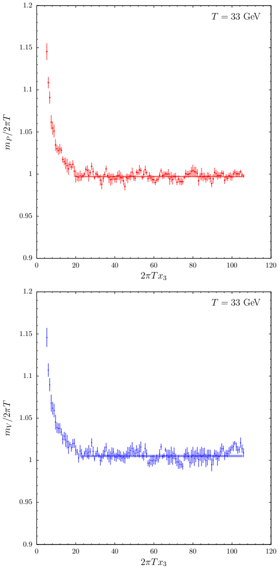
$$am_{\text{eff}}(x_3) = \text{arccosh}\left[\frac{C(x_3+a)+C(x_3-a)}{2C(x_3)}\right]$$

Very small $P-V$ mass splitting

$\Rightarrow$ no signal-to-noise ratio problem

High accuracy requires systematic control:

- excited-state contamination
- finite volume
- continuum limit

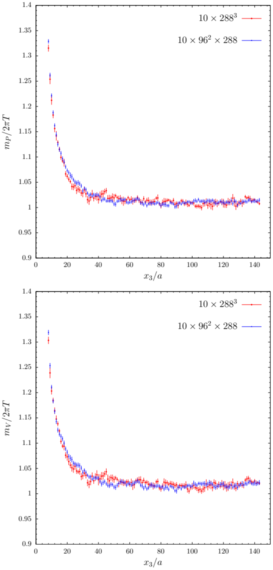


Finite volume check

Finite-volume effects expected to be very small

Change transverse volume by $\div 2$ or $\div 3$

Good agreement within statistical precision



Continuum limit

For each L_0/a perform replacement

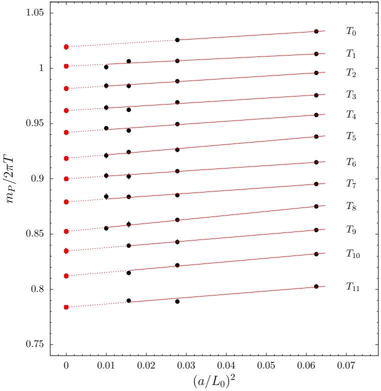
$$m \to m - \Big[m_{\mathrm{latt.}} - m_{\mathrm{cont.}}\Big]_{\mathrm{leading-order}}$$

leading-order (parameter-free) improvement

Continuum estimate using Ansatz

$$m = \hat{m}_{\mathrm{cont.}} + \mathrm{const.} \times \left(\frac{a}{L_0}\right)^2$$

also with a^3 and $a^2 \ln(a/L_0)$ terms



(T_i shifted down by $0.02 \times i$)

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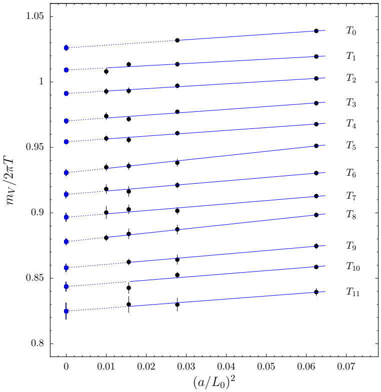


(T_i shifted down by $0.02 \times i$)

Chiral symmetry restoration

Probe symmetries using spectrum

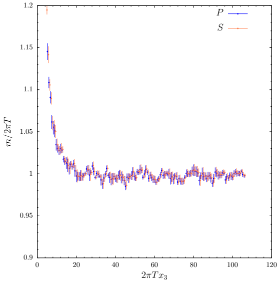
Classically, under singlet axial $\mathrm{U}(1)$ transformations

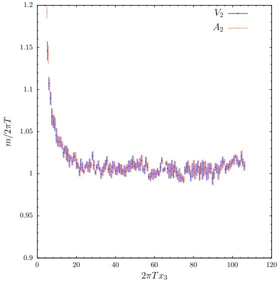
$$P^a \quad \longleftrightarrow \quad S^a$$

while under non-singlet axial $\mathrm{SU}(N_\mathrm{f})$

$$V_2^a \quad \longleftrightarrow \quad A_2^a$$

no violation of anomalous axial or spontaneously-broken chiral symmetry at any temperature, as expected





Temperature dependence I

Smooth dependence on $1/\ln(2\pi T/\Lambda_{\overline{\rm MS}})$

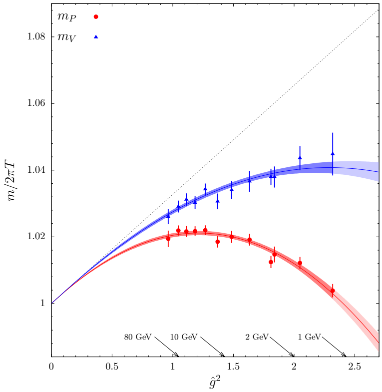
Choose to parameterize in terms of

$$\frac{1}{\hat{g}^2(T)} = \frac{9}{8\pi^2}\ln\frac{2\pi T}{\Lambda_{\overline{\rm MS}}} + \frac{4}{9\pi^2}\ln\left(2\ln\frac{2\pi T}{\Lambda_{\overline{\rm MS}}}\right)$$

Convenient to compare with NLO prediction

$$\frac{m_{\rm NLO}}{2\pi T} = 1 + 0.03273996\, g^2$$

- few percent deviation from PT
- $P - V$ splitting observed at all temperature
- same sign as NLO correction



12 M. Laine and M. Vepsalainen. In: *JHEP* 02 (2004), p. 004. arXiv: hep-ph/0311268.

Temperature dependence II

Polynomial parameterization in $\hat{g}$

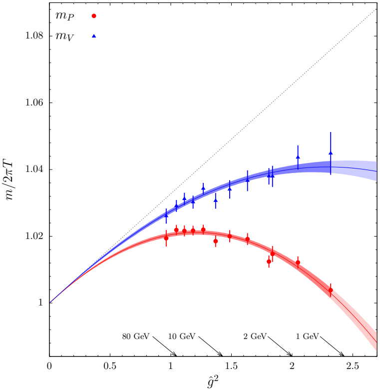
$$\frac{m}{2\pi T} = p_0 + p_2\hat{g}^2 + p_3\hat{g}^3 + (p_4 + s_4)\hat{g}^4$$

Pseudoscalar P

- $\hat{g}^4$ contribution negative
- cancels $\hat{g}^2$ contribution at $T \sim 1\,\mathrm{GeV}$

Vector V

- large spin-dependent contribution at $T \sim 1\,\mathrm{GeV}$



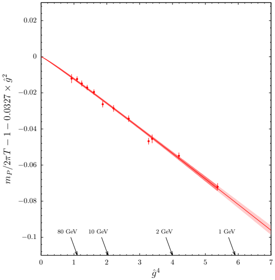
Temperature dependence III

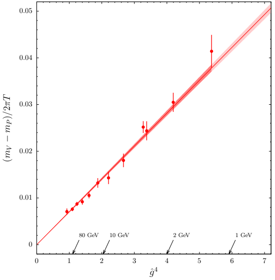
Difference with NLO

- consistency at $T \to \infty$ ($\hat{g} \to 0$)
- consistent with $\hat{g}^4$ scaling at all T
- $\hat{g}^4$ relevant even at $T \sim 160\,\mathrm{GeV}$

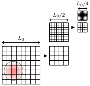
$P - V$ splitting

- consistent with $\mathrm{O}(\hat{g}^4)$
- spin-dependent term relevant at all T





Conclusions



First non-perturbative results from QCD at $T \to 160\,\mathrm{GeV}$

- scale-setting through finite-volume coupling
- validated simulation strategy
- precise results for non-singlet static correlators

⇝ higher-order corrections relevant!

Outlook

- thermodynamic and fermionic observables
- probe EFT at very large T