

# **Influence of a phase transition on the transport properties of QCD matter**

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Mini Workshop «Phase transitions in particle physics»

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Galilei Galileo Institute



# Properties of QGP: transport coefficients

! One has to specify transport and microscopic properties as well as EoS for theoretical simulations of HICs (hydro / transport approaches)



EoS( $\epsilon, n$ )  
 $\sigma(\sqrt{s}, m_q, m_q, T, \mu_B)$   
 $m(T, \mu_B)$

On practice: effective models  
for QGP

## Transport simulations with QGP phase:

### Catania transport – QuasiParticle Model

F. Scardina, S. K. Das, V. Minissale, S. Plumari, and V. Greco,  
PRC 96, 044905 (2017).



### – Dynamical QPM for partonic phase

W. Cassing, E. Bratkovskaya, PRC 78 (2008) 034919  
P. Moreau, O. S, L. Oliva, T. Song, W. Cassing, E. Bratkovskaya,  
PRC 100 (2019) , 014911;  
O. S, P. Moreau, L. Oliva, V. Voronyuk, V. Kireyeu, T. Song,  
E. Bratkovskaya, Particles 3 (2020), 178-192

### AMPT – PNJL EoS (Mean field potentials)

K.J. Sun, C. M. Ko, and Z.-W. Lin, PRC 103(2021)

## Hybrid simulations with QGP: vHLL/UrQMD

Iu.A. Karpenko, P. Huovinen, H. Petersen and M. Bleicher  
PRC 91 (2015), 064901  
S. Ryu, J.F.Paquet, C. Shen, G.S. Denicol, B. Schenke  
PRL 115 (2015), 132301

Today:

Transport coefficients at finite  $T$  and  $\mu_B$

- 1.) crossover, CEP and 1st order phase transition ( $N_f = 3$  PNJL model)
- 2.) crossover + CEP ( $N_f = 3$  DQPM)

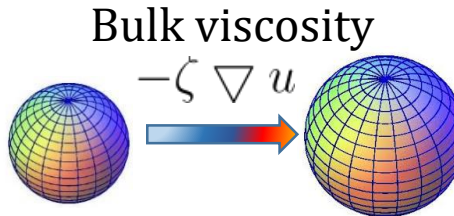
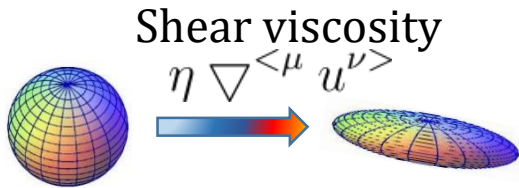
# Properties of QGP: transport coefficients

Hydrodynamics

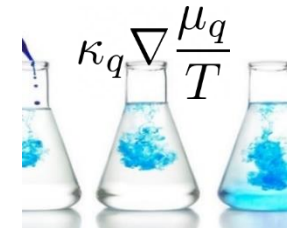
$$\begin{cases} \partial_\mu T^{\mu\nu} = 0 & T^{\mu\nu} = -Pg^{\mu\nu} + wu^\mu u^\nu + \Delta T^{\mu\nu} \\ \partial_\mu J_B^\mu = 0 & J_B^\mu = n_B u^\mu + \Delta J_B^\mu \end{cases}$$

$\eta (D^\mu u^\nu + D^\nu u^\mu + \frac{2}{3} \Delta^{\mu\nu} \partial_\rho u^\rho) - \zeta \Delta^{\mu\nu} \partial_\rho u^\rho$   
 $\Delta J_B^\mu = \kappa_B D^\mu (\frac{\mu_B}{T})$   
 input for hydro

Transport coefficients:

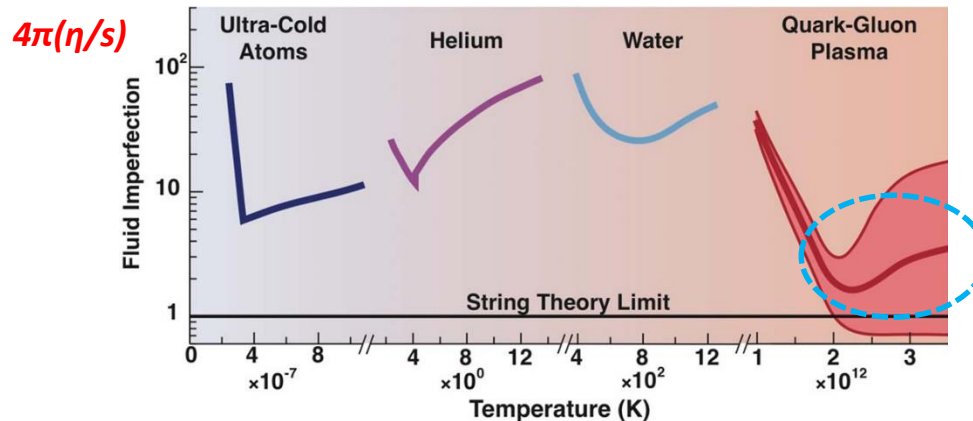


(B, Q, S) diffusion coefficients



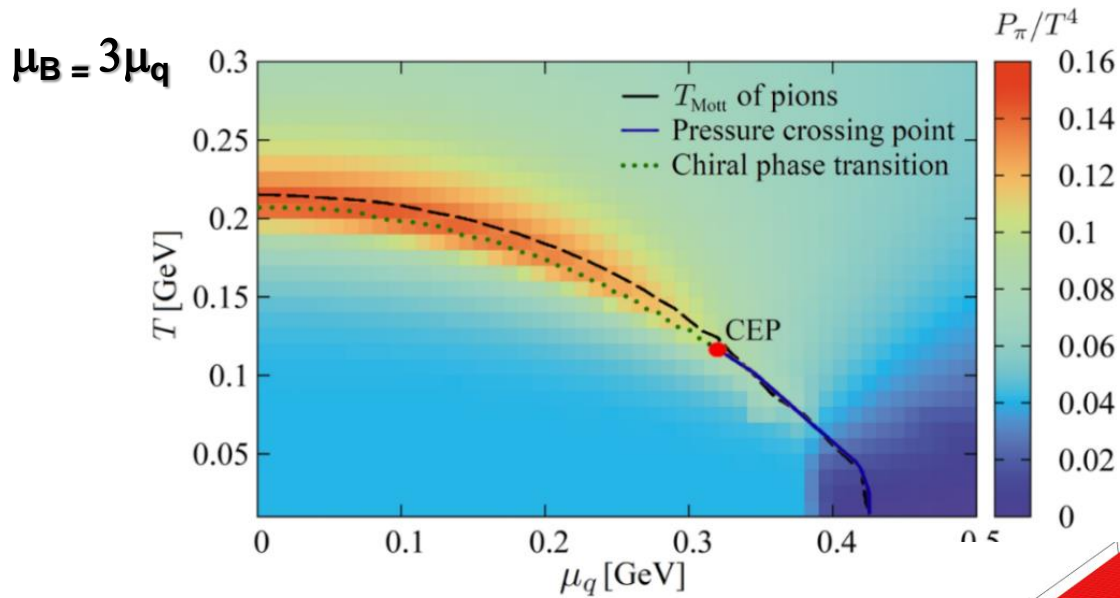
Model predictions: ! same EoS but different transport coefficients

QGP is the most perfect fluid



# QGP in the Polyakov extended NJL model

- PNJL allows for prediction of macroscopic properties of QGP at finite  $T$  and  $\mu_B$
- & QGP transport coefficients for  $0 \leq \mu_B \leq 1.2$  GeV

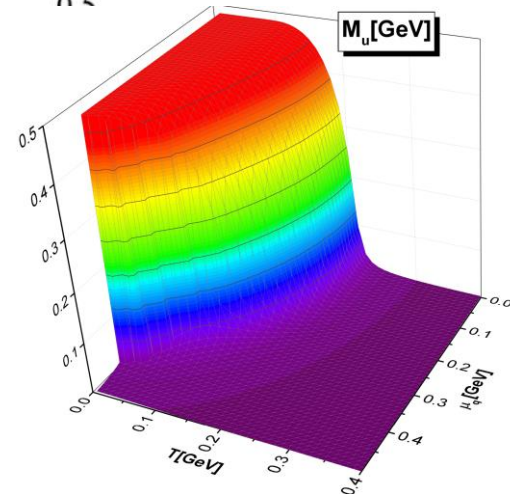


D. Fuseau, T. Steinert, J. Aichelin PRC 101 (2020) 6 065203

- **CEP**:  $(T, \mu_B) = (110, 960)$  MeV,  $\mu_B/T = 8.73$
- 1st order PT at high  $\mu_B$
- **same symmetries** for the quarks as QCD

Chiral masses ( $M_L, M_S$ )

$$m_i = m_{0i} - 4G \langle \langle \bar{\psi}_i \psi_i \rangle \rangle + 2K \langle \langle \bar{\psi}_j \psi_j \rangle \rangle \langle \langle \bar{\psi}_k \psi_k \rangle \rangle$$



# QGP in the Polyakov extended NJL model

- PNJL model based on effective Lagrangian with the same symmetries for the quark dof as QCD

$$\begin{aligned} \mathcal{L}_{PNJL} = & \sum_i \bar{\psi}_i (iD - m_{0i} + \mu_i \gamma_0) \psi_i \\ & + G \sum_a \sum_{ijkl} \left[ (\bar{\psi}_i i\gamma_5 \tau_{ij}^a \psi_j) (\bar{\psi}_k i\gamma_5 \tau_{kl}^a \psi_l) + (\bar{\psi}_i \tau_{ij}^a \psi_j) (\bar{\psi}_k \tau_{kl}^a \psi_l) \right] \\ & - K \det_{ij} [\bar{\psi}_i (-\gamma_5) \psi_j] - K \det_{ij} [\bar{\psi}_i (+\gamma_5) \psi_j] \\ & - \mathcal{U}(T; \Phi, \bar{\Phi}) \end{aligned}$$

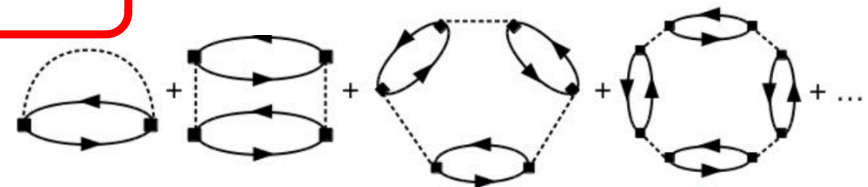
← Polyakov-loop effective potential fitted to the YM

5 parameters fixed by vacuum values  $K, \pi$  masses,  $\eta$ - $\eta'$  mass splitting,  $\pi$  decay constant, Chiral condensate

## Improvements:

- Next to leading order in  $N_c(O(1/N_c)^0)$  of the grand-canonical potential : presence of the mesons below  $T_c$

$$\Omega_{PNJL}(T, \mu_i) = \Omega_q^{(-1)}(T, \mu_i) + \sum_{M \in J^\pi = \{0^+, 0^-\}} \Omega_M^{(0)}(T, \mu_M(\mu_i)) + \mathcal{U}_{glue}(T),$$



J. M. Torres-Rincon, J. Aichelin PRC 96 (2017) 4 045205  
 D. Fuseau, T. Steinernert, J. Aichelin PRC 101 (2020) 6 065203

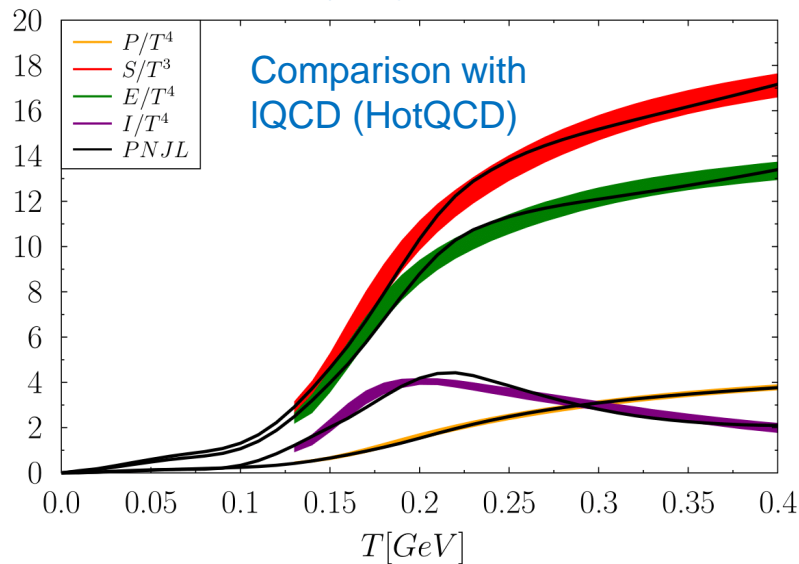
- Modification of the gluon potential due to the presence of the quark

# QGP in the Polyakov extended NJL model

- PNJL allows for prediction of macroscopic properties of QGP at finite  $T$  and  $\mu_B$
- & QGP transport coefficients for  $0 \leq \mu_B \leq 1.2$  GeV

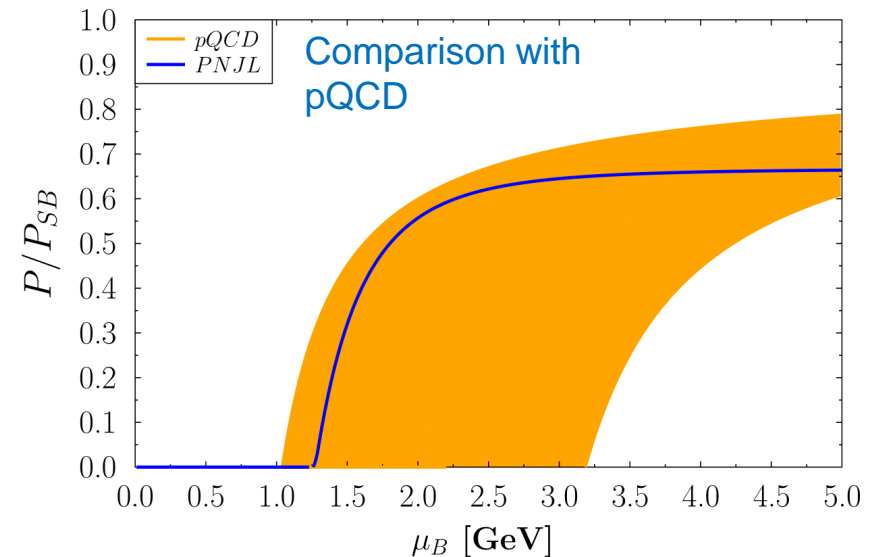
➤ Parameters fixed, EoS at  $\mu_B = 0$ :

HotQCD PRD 90 (2014) 094503



➤ EoS at high  $\mu_B$  :

pQCD: A.Kurkela, A.Vuorinen, PRL 117 (2016)4 042501



# PNJL relaxation times

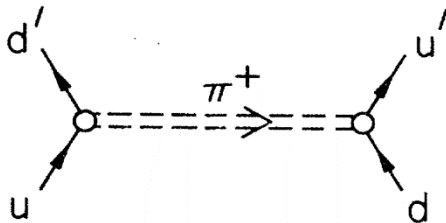
$$\tau_i(\mathbf{p}, T, \mu_B) = \frac{1}{\Gamma_i(\mathbf{p}, T, \mu_B)}$$

$$\Gamma_i^{\text{non}}(\mathbf{p}_i, T, \mu_q) = \frac{1}{2E_i} \sum_{j=q, \bar{q}, g} \int \frac{d^3 p_j}{(2\pi)^3 2E_j} d_j f_j(E_j, T, \mu_q) \int \frac{d^3 p_3}{(2\pi)^3 2E_3} \int \frac{d^3 p_4}{(2\pi)^3 2E_4} (1 \pm f_3)(1 \pm f_4)$$

$$|\bar{\mathcal{M}}|^2(p_i, p_j, p_3, p_4) (2\pi)^4 \delta^{(4)}(p_i + p_j - p_3 - p_4)$$

qq - interactions:

4 point interaction -> meson exchange ( $\pi, \sigma, \eta, \eta', K, \dots$  for s, t, u channels)



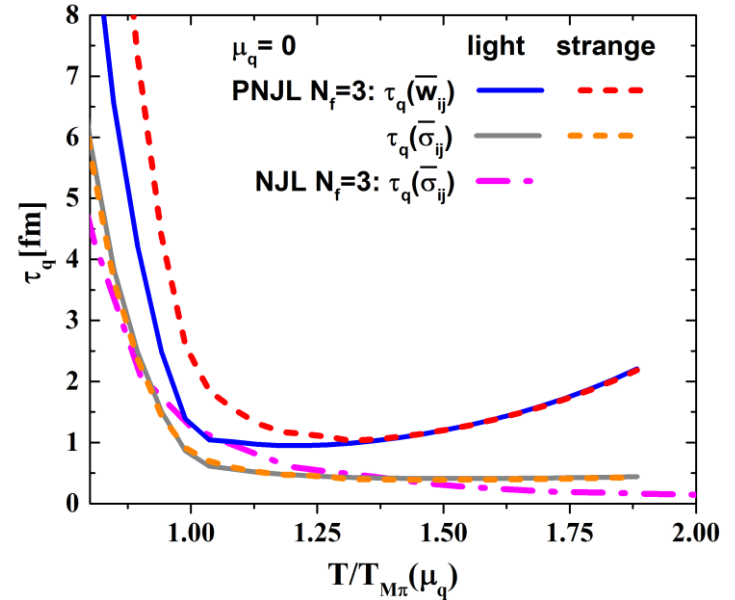
$$\boxed{\text{---}} \Rightarrow \boxed{\text{---}} = (i\gamma_5)\tau^{(-)} \frac{-ig^2_{\pi qq}}{k^2 - m_\pi^2} (i\gamma_5)\tau^{(+)}$$

meson propagator  $\mathcal{D} = \frac{2ig_m}{1 - 2g_m \Pi_{ff'}^\pm(k_0, \vec{k})}$

Effective interaction in RPA

$$\text{---} \approx \text{---} + \text{---} + \text{---} + \dots = \frac{\text{---}}{1 - \text{---}}$$

Relaxation times (PNJL vs NJL)

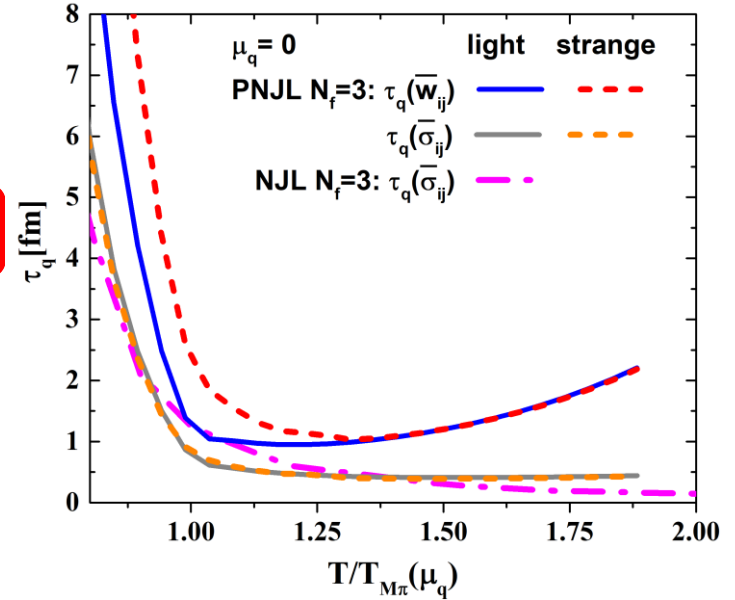




# PNJL relaxation times

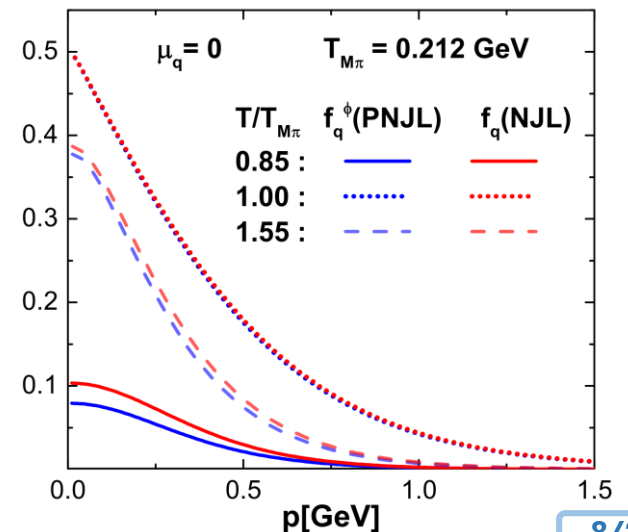
$$\tau_i(\mathbf{p}, T, \mu_B) = \frac{1}{\Gamma_i(\mathbf{p}, T, \mu_B)}$$

$$\Gamma_i^{\text{on}}(\mathbf{p}_i, T, \mu_q) = \frac{1}{2E_i} \sum_{j=q, \bar{q}, g} \int \frac{d^3 p_j}{(2\pi)^3 2E_j} d_j f_j(E_j, T, \mu_q) \int \frac{d^3 p_3}{(2\pi)^3 2E_3} \int \frac{d^3 p_4}{(2\pi)^3 2E_4} (1 \pm f_3)(1 \pm f_4) |\bar{\mathcal{M}}|^2(p_i, p_j, p_3, p_4) (2\pi)^4 \delta^{(4)}(p_i + p_j - p_3 - p_4)$$



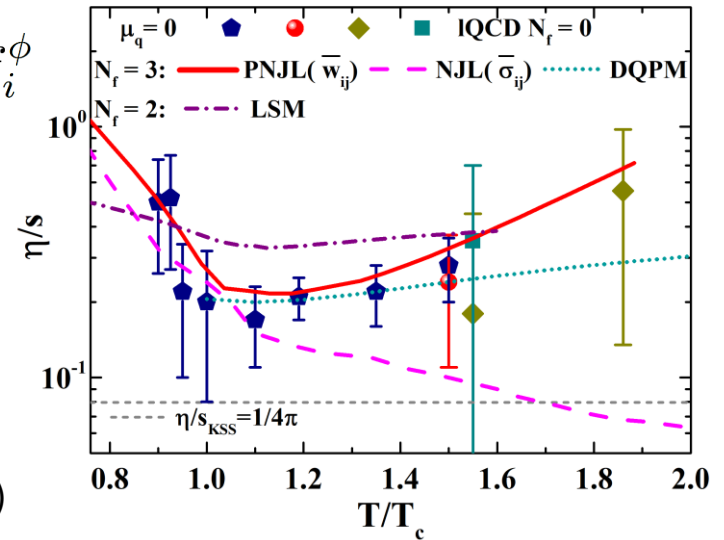
Modified distribution functions:  
Polyakov loop contributions

$$f_q \rightarrow f_q^\Phi(\mathbf{p}, T, \mu) = \frac{(\bar{\Phi} + 2\Phi e^{-(E_p - \mu)/T}) e^{-(E_p - \mu)/T} + e^{-3(E_p - \mu)/T}}{1 + 3(\bar{\Phi} + \Phi e^{-(E_p - \mu)/T}) e^{-(E_p - \mu)/T} + e^{-3(E_p - \mu)/T}}$$

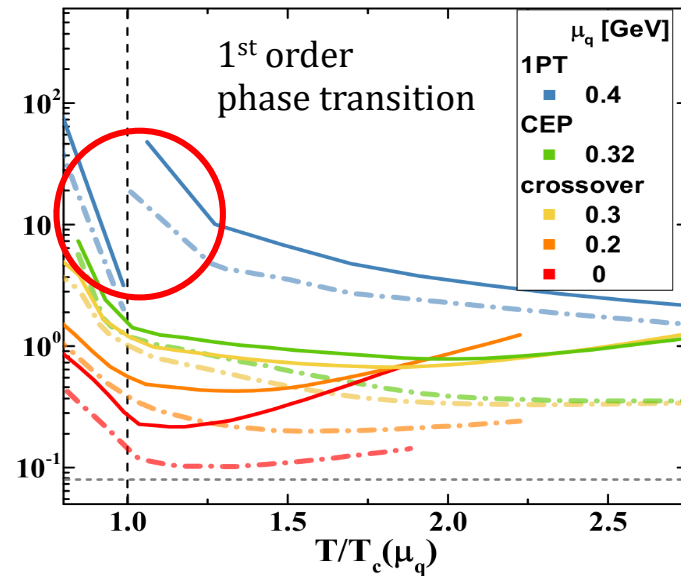
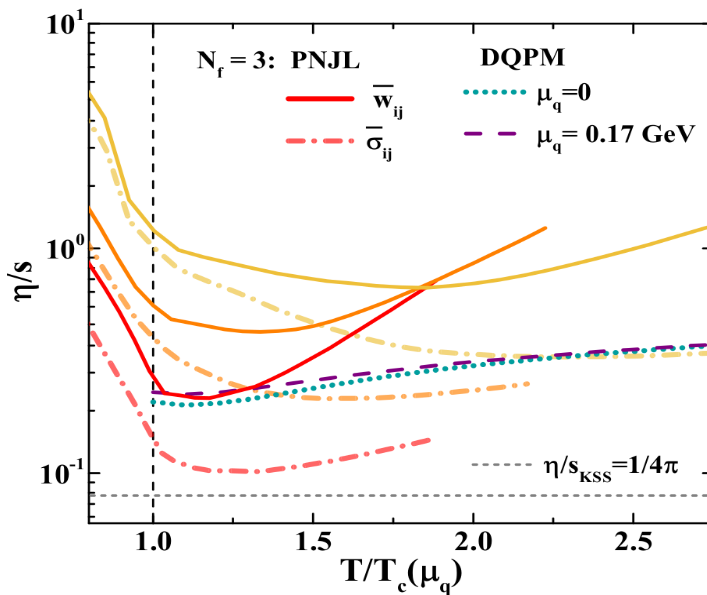


# Specific shear viscosity at high $\mu_B$

$$\eta^{\text{RTA}}(T, \mu_B) = \frac{1}{15T} \sum_{i=q, \bar{q}, g} \int \frac{d^3p}{(2\pi)^3} \frac{\mathbf{p}^4}{E_i^2} \tau_i(\mathbf{p}, T, \mu_B) d_q f_i^\phi$$

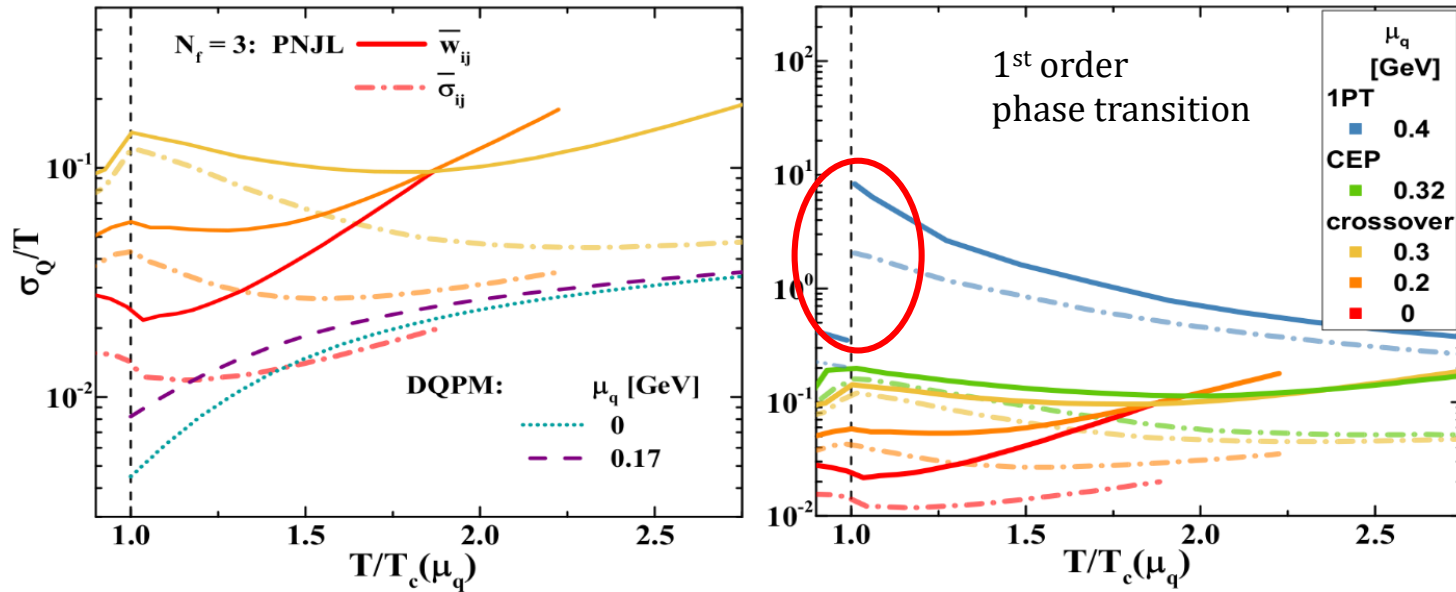


In agreement w Nf=2 NJL results C. Sasaki et al, NPA 832 (2010)



# Electric conductivity at high $\mu_B$

$$\sigma_0^{\text{RTA}}(T, \mu_B) = \frac{e^2}{3T} \sum_{i=q, \bar{q}} q_i^2 \int \frac{d^3 p}{(2\pi)^3} \frac{\mathbf{p}^2}{E_i^2} \tau_i(\mathbf{p}, T, \mu_B) d_q f_i^\phi$$



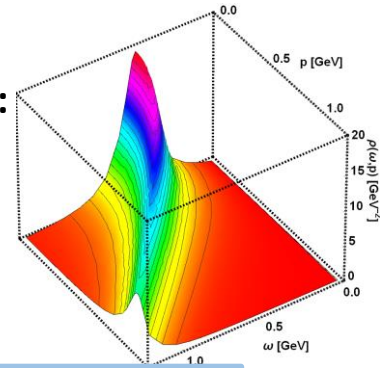
- Two different models have similar increase with  $\mu_B$  -dependence in the crossover region
- Drastic change of T-dependence for all transport coefficients after 1st order phase transition

# Dynamical Quasi-Particle Model

- The QGP phase is described in terms of strongly-interacting quasiparticles - quarks and gluons with Lorentzian spectral functions:

$$\rho_j(\omega, \mathbf{p}) = \frac{\gamma_j}{\tilde{E}_j} \left( \frac{1}{(\omega - \tilde{E}_j)^2 + \gamma_j^2} - \frac{1}{(\omega + \tilde{E}_j)^2 + \gamma_j^2} \right)$$

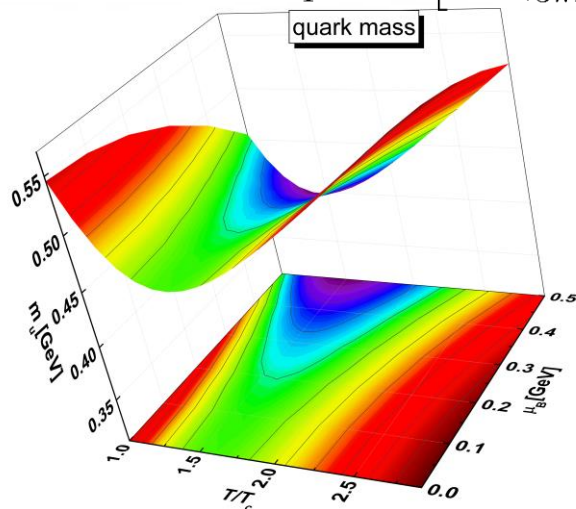
$$\equiv \frac{4\omega\gamma_j}{(\omega^2 - \mathbf{p}^2 - M_j^2)^2 + 4\gamma_j^2\omega^2}$$



resummed propagators:  $\Delta_i(\omega, \mathbf{p}) = \frac{1}{\omega^2 - \mathbf{p}^2 - \Pi_i}$  & self-energies:  $\Pi_i = m_i^2 - 2i\gamma_i\omega$

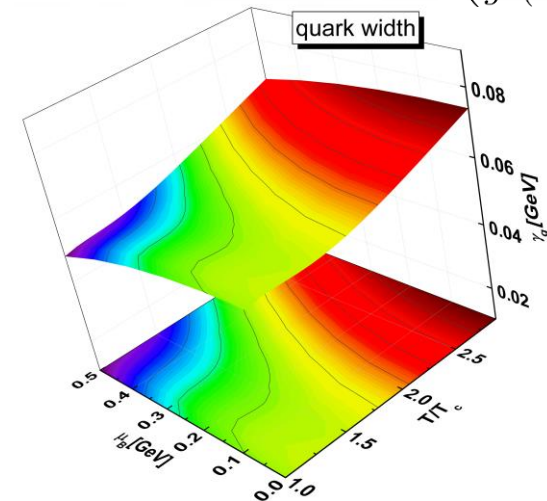
Re  $\Pi_i$ : thermal mass ( $M_g, M_q$ )

$$m_{q(\bar{q})}^2(T, \mu_B) = C_q \frac{g^2(T, \mu_B)}{4} T^2 \left[ 1 + \left( \frac{\mu_B}{3\pi T} \right)^2 \right]$$



Im  $\Pi_i$ : interaction width ( $\gamma_g, \gamma_q$ )

$$\gamma_j(T, \mu_B) = \frac{1}{3} C_j \frac{g^2(T, \mu_B) T}{8\pi} \ln \left( \frac{2c_m}{g^2(T, \mu_B)} + 1 \right)$$



# DQPM: EoS

- Entropy and baryon density in the quasiparticle limit (G. Baym 1998, Blaizot et al. 2001 ):

$$s^{dqp} = - \int \frac{d\omega}{2\pi} \frac{d^3p}{(2\pi)^3} \left[ d_g \frac{\partial n_B}{\partial T} (\text{Im}(\ln -\underline{\Delta}^{-1}) + \text{Im} \underline{\Pi} \text{Re} \underline{\Delta}) + \sum_{q=u,d,s} d_q \frac{\partial n_F(\omega - \mu_q)}{\partial T} (\text{Im}(\ln -\underline{S}_q^{-1}) + \text{Im} \underline{\Sigma}_q \text{Re} \underline{S}_q) + \sum_{\bar{q}=\bar{u},\bar{d},\bar{s}} d_{\bar{q}} \frac{\partial n_F(\omega + \mu_q)}{\partial T} (\text{Im}(\ln -\underline{S}_{\bar{q}}^{-1}) + \text{Im} \underline{\Sigma}_{\bar{q}} \text{Re} \underline{S}_{\bar{q}}) \right]$$

$$n^{dqp} = - \int \frac{d\omega}{2\pi} \frac{d^3p}{(2\pi)^3} \left[ \sum_{q=u,d,s} d_q \frac{\partial n_F(\omega - \mu_q)}{\partial \mu_q} (\text{Im}(\ln -\underline{S}_q^{-1}) + \text{Im} \underline{\Sigma}_q \text{Re} \underline{S}_q) + \sum_{\bar{q}=\bar{u},\bar{d},\bar{s}} d_{\bar{q}} \frac{\partial n_F(\omega + \mu_q)}{\partial \mu_q} (\text{Im}(\ln -\underline{S}_{\bar{q}}^{-1}) + \text{Im} \underline{\Sigma}_{\bar{q}} \text{Re} \underline{S}_{\bar{q}}) \right]$$

- Input: entropy density as a  $f(T, \mu_B = 0)$

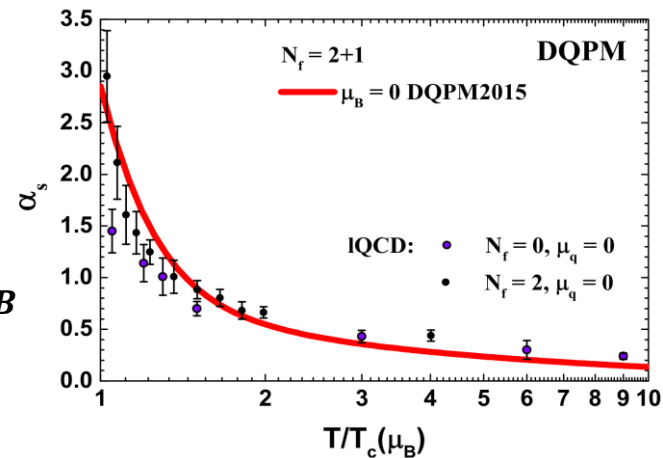
$$g^2(s/s_{SB}) = d((s/s_{SB})^e - 1)^f$$

$$s^{DQPM}(\Pi, \Delta, S_q, \Sigma) = s^{lattice}$$

➔ fix the model parameters

- Scaling hypothesis for the **crossover region** at finite  $\mu_B$

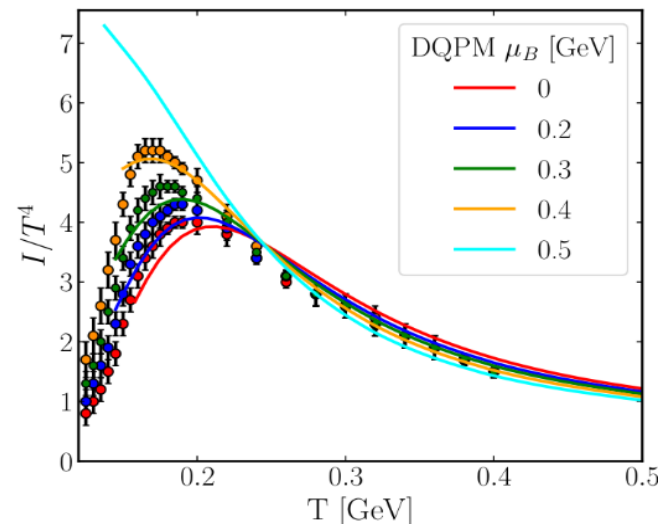
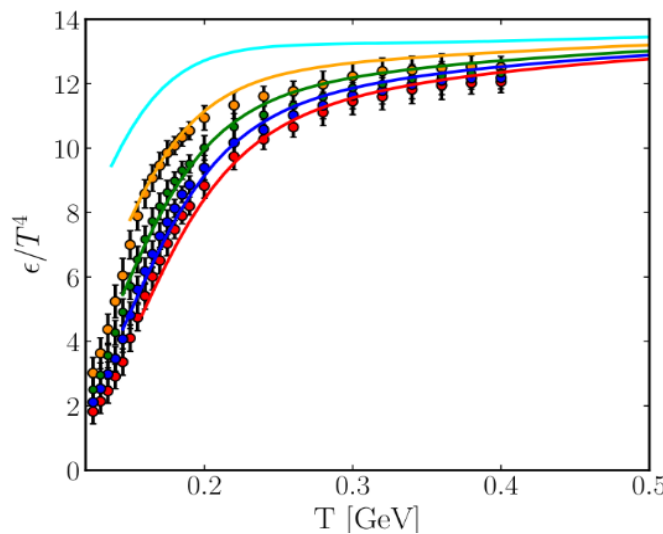
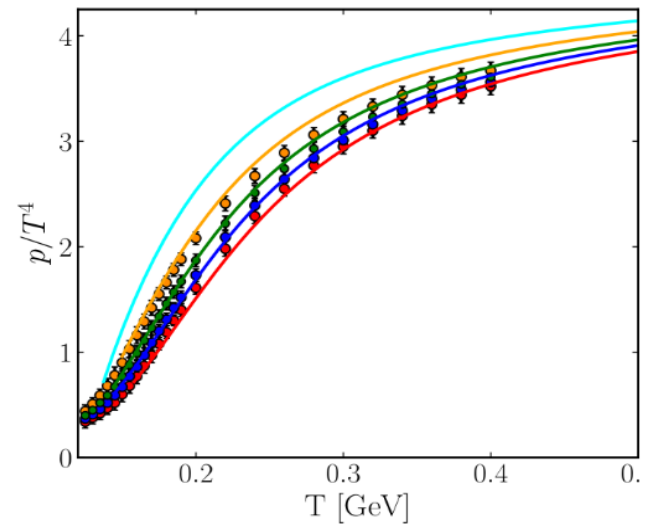
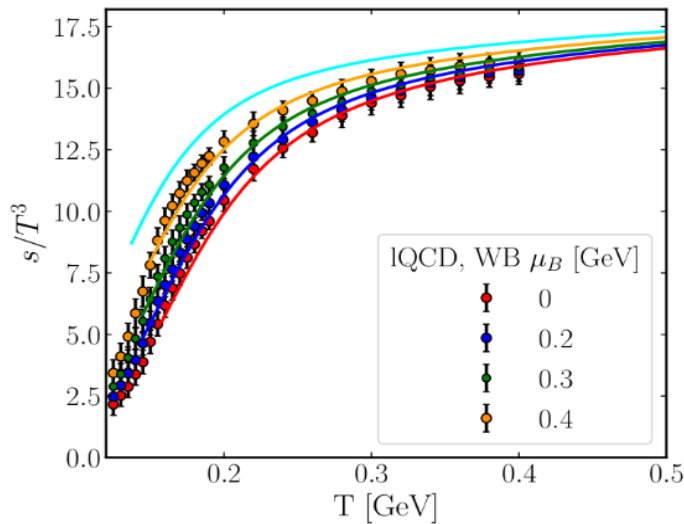
$$g^2(T/T_c, \mu_B) = g^2\left(\frac{T^*}{T_c(\mu_B)}, \mu_B = 0\right) \quad \text{with} \quad T^* = \sqrt{T^2 + \mu_q^2/\pi^2}$$



# DQPM: EoS

**Input:**  
lattice EoS  
 $\mu_B = 0$   
(red dots)

**Output:**  
DQPM EoS  
 $\mu_B \geq 0$   
(lines)

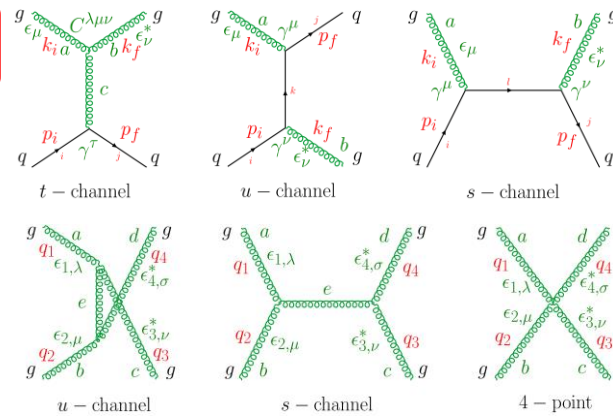


# Transport coefficients at finite $\mu_B$

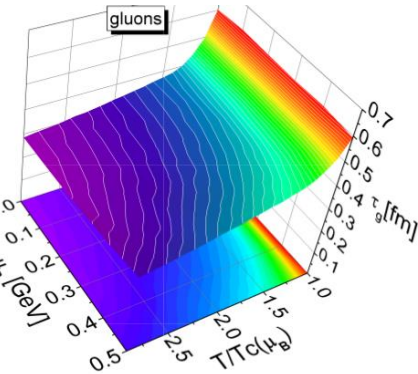
$$\eta^{\text{RTA}}(T, \mu_B) = \frac{1}{15T} \sum_{i=q,\bar{q},g} \int \frac{d^3p}{(2\pi)^3} \frac{\mathbf{p}^4}{E_i^2} \tau_i(\mathbf{p}, T, \mu_B) d_i(1 \pm f_i) f_i$$

$$\tau_i(\mathbf{p}, T, \mu_B) = \frac{1}{\Gamma_i(\mathbf{p}, T, \mu_B)}$$

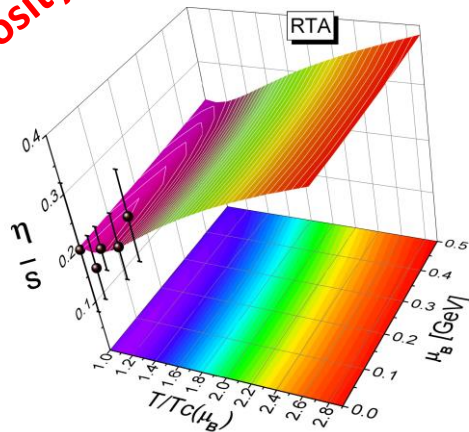
2  $\leftrightarrow$  2 scatterings



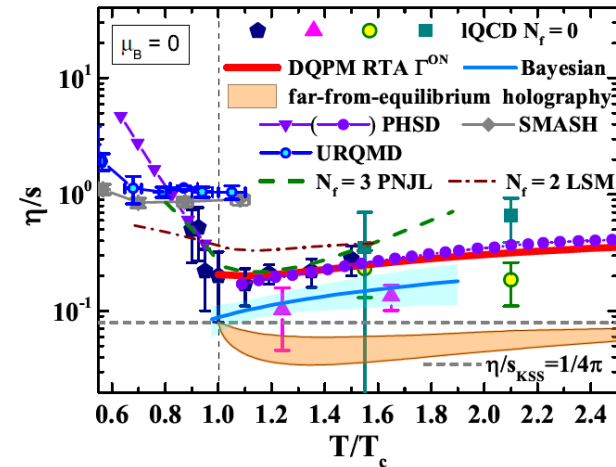
Relaxation times



Specific shear viscosity



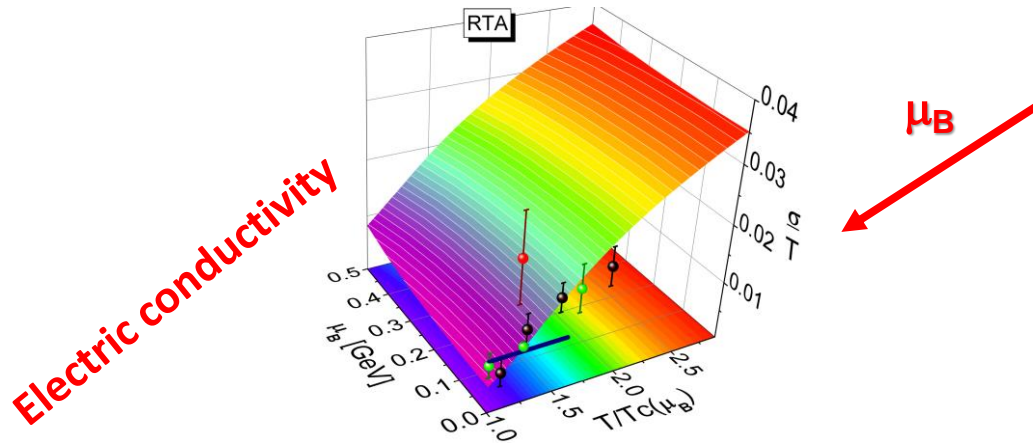
$\mu_B$



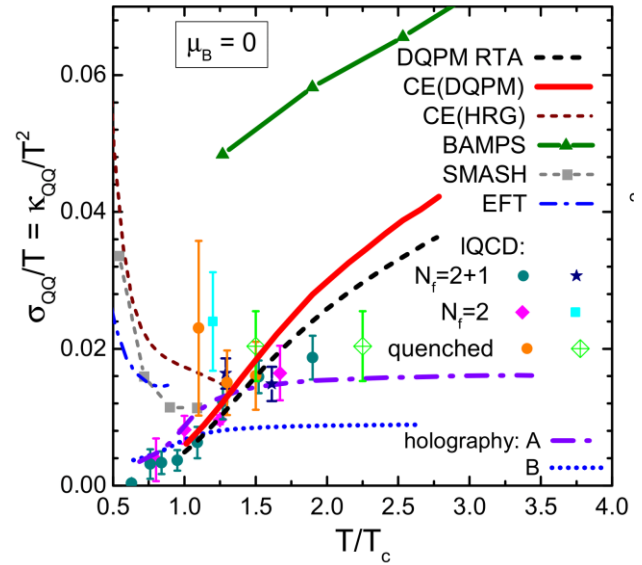
O. Soloveva, P. Moreau and E. Bratkovskaya, PRC 101 (2020), 045203

- Good agreement with IQCD predictions and Bayesian estimates
- Light increase with  $\mu_B$  in the crossover region for viscosities and electric conductivity

# Transport coefficients at finite $\mu_B$



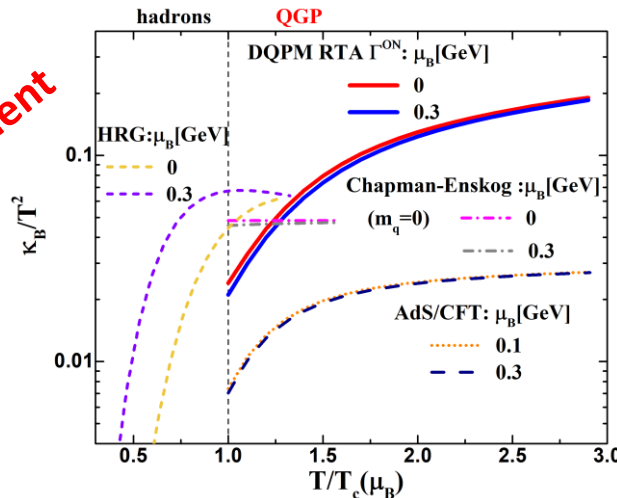
Electric conductivity



+ Full diffusion coefficient matrix  
has been evaluated  $\sigma_q = \kappa_q/T$

$$\begin{pmatrix} j_B^\mu \\ j_Q^\mu \\ j_S^\mu \end{pmatrix} = \begin{pmatrix} \kappa_{BB} & \kappa_{BQ} & \kappa_{BS} \\ \kappa_{QB} & \kappa_{QQ} & \kappa_{QS} \\ \kappa_{SB} & \kappa_{SQ} & \kappa_{SS} \end{pmatrix} \cdot \begin{pmatrix} \nabla^\mu \alpha_B \\ \nabla^\mu \alpha_Q \\ \nabla^\mu \alpha_S \end{pmatrix}$$

J. A. Fotakis, O. Soloveva, C. Greiner, O. Kaczmarek  
and E. Bratkovskaya PRD 104 (2021), 034014



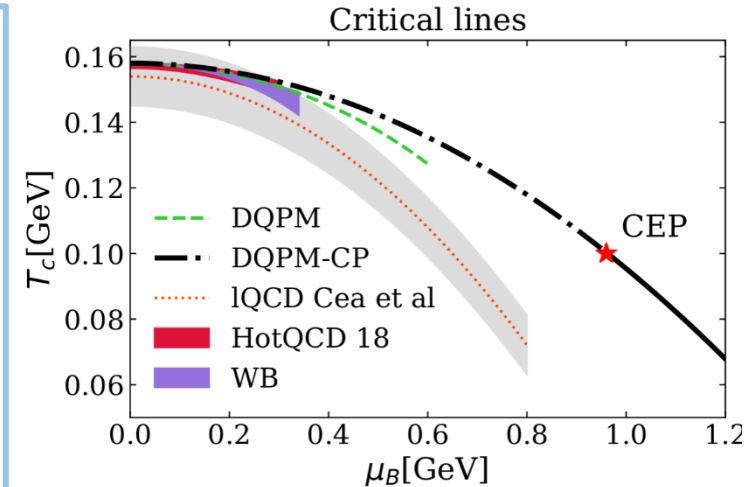
Baryon diffusion coefficient

- Light increase with  $\mu_B$  in the crossover region for shear and bulk viscosities and electric conductivity
- Baryon diffusion coefficients decrease with  $\mu_B$



# Quasiparticle model with CEP at high $\mu_B$

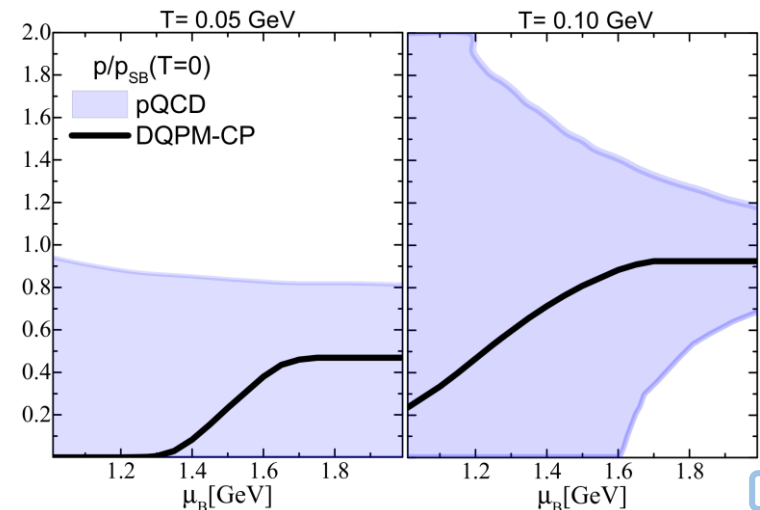
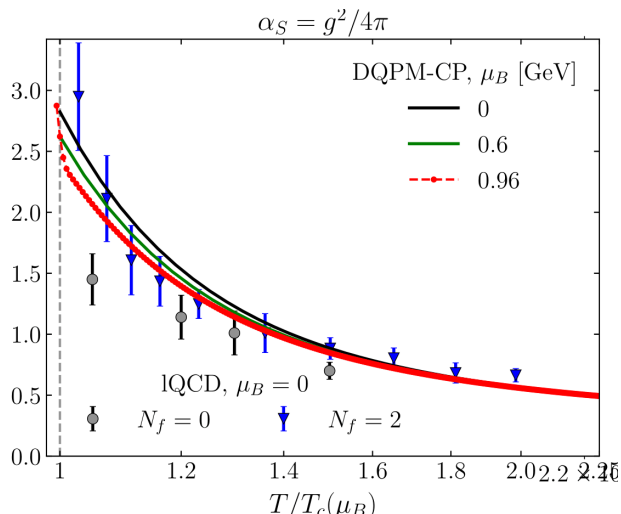
- DQPM-CP for high  $\mu_B$ , including the CEP region based on the scaling properties of the entropy density from the PNJL model
- DQPM-CP interpolates EoS and microscopic properties between two asymptotics - high  $T \gg T_c, \mu_B = 0$  and  $T > T_c, \mu_B \gg T$
- EoS and transport coefficients of the QGP phase for the wide range of  $T > T_c, \mu_B$



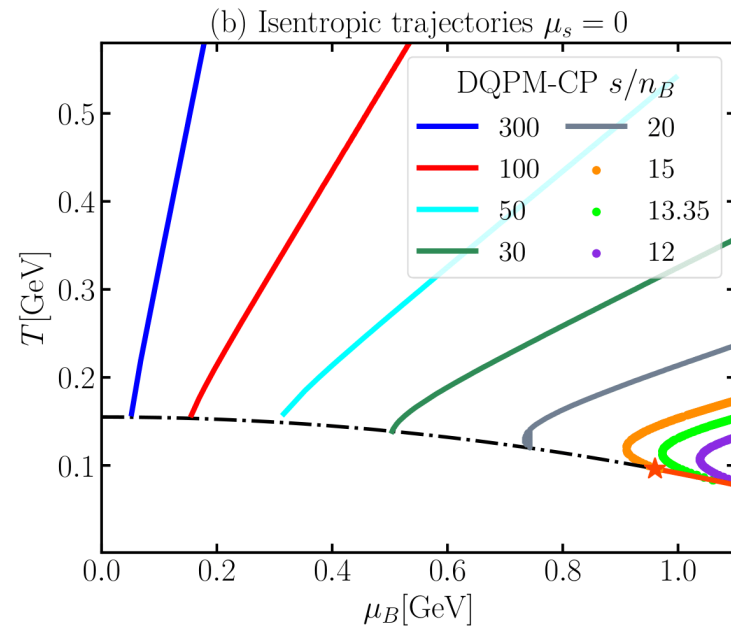
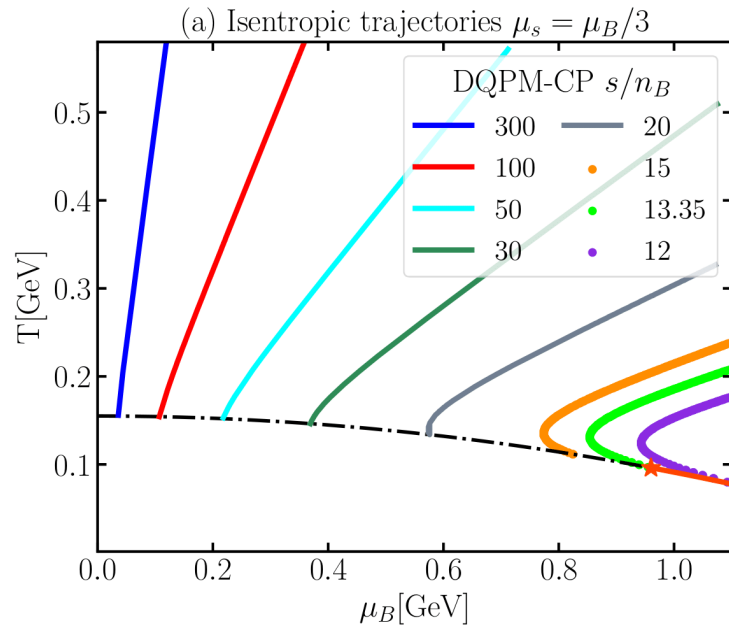
➤ **CEP:**  $(T, \mu_B) = (100, 960)$  MeV,  $\mu_B/T = 9.6$

➤ **EoS:** for  $\mu_B/T < 2$  agreement with IQCD for  $\mu_B/T > 6$  agreement with pQCD

Near CEP:  $g^2 = f(s^{PNJL}(T/T_c)) \rightarrow g^2(T/T_c)$



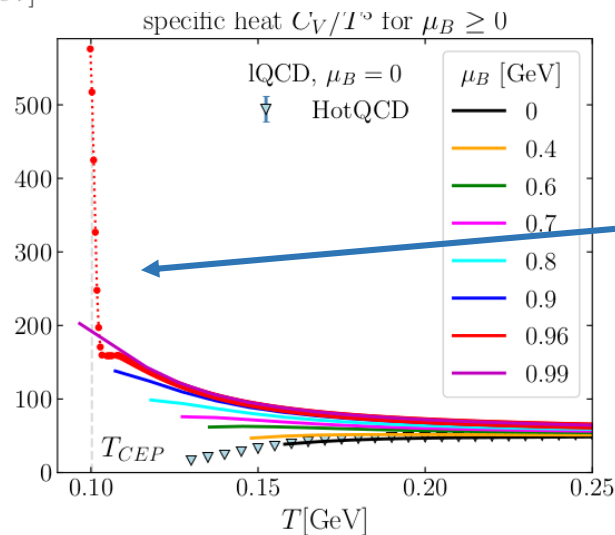
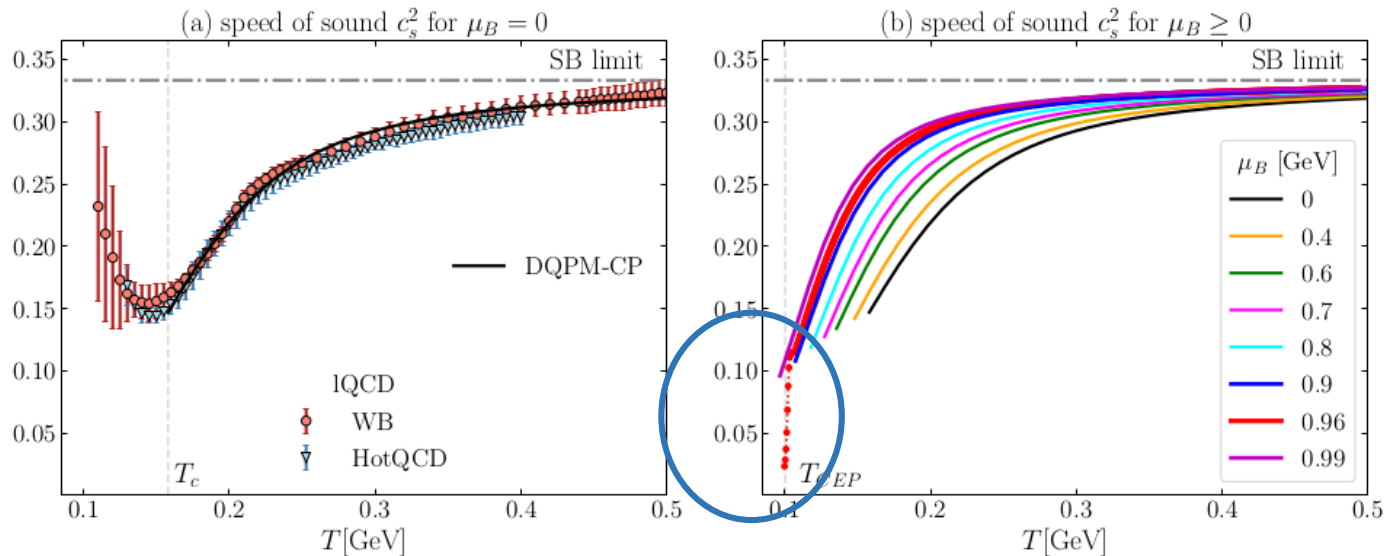
# Isentropic trajectories



- CEP acts as an attractor of isentropic trajectories (Chiho Nonaka and Masayuki Asakawa PRC 71 (2005), 044904)
- Trajectories of  $s/n_B = \text{const}$  for  $\langle ns \rangle = 0$  are shifted towards higher  $\mu_B$

# Speed of sound

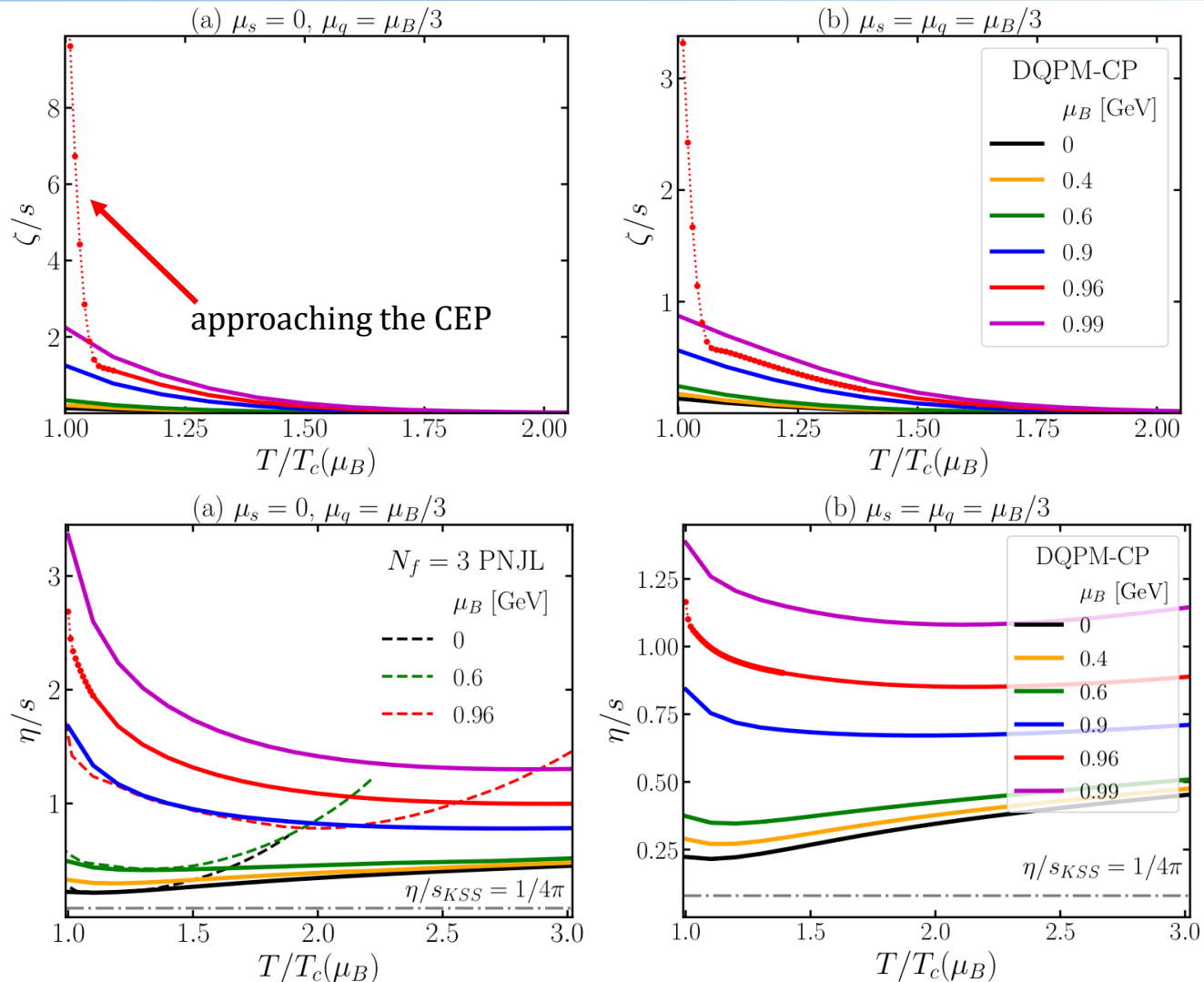
- **EoS**: for  $\mu_B/T < 2$  agreement with IQCD for  $\mu_B/T > 6$  agreement with pQCD



Near CEP critical scaling  
can be seen:

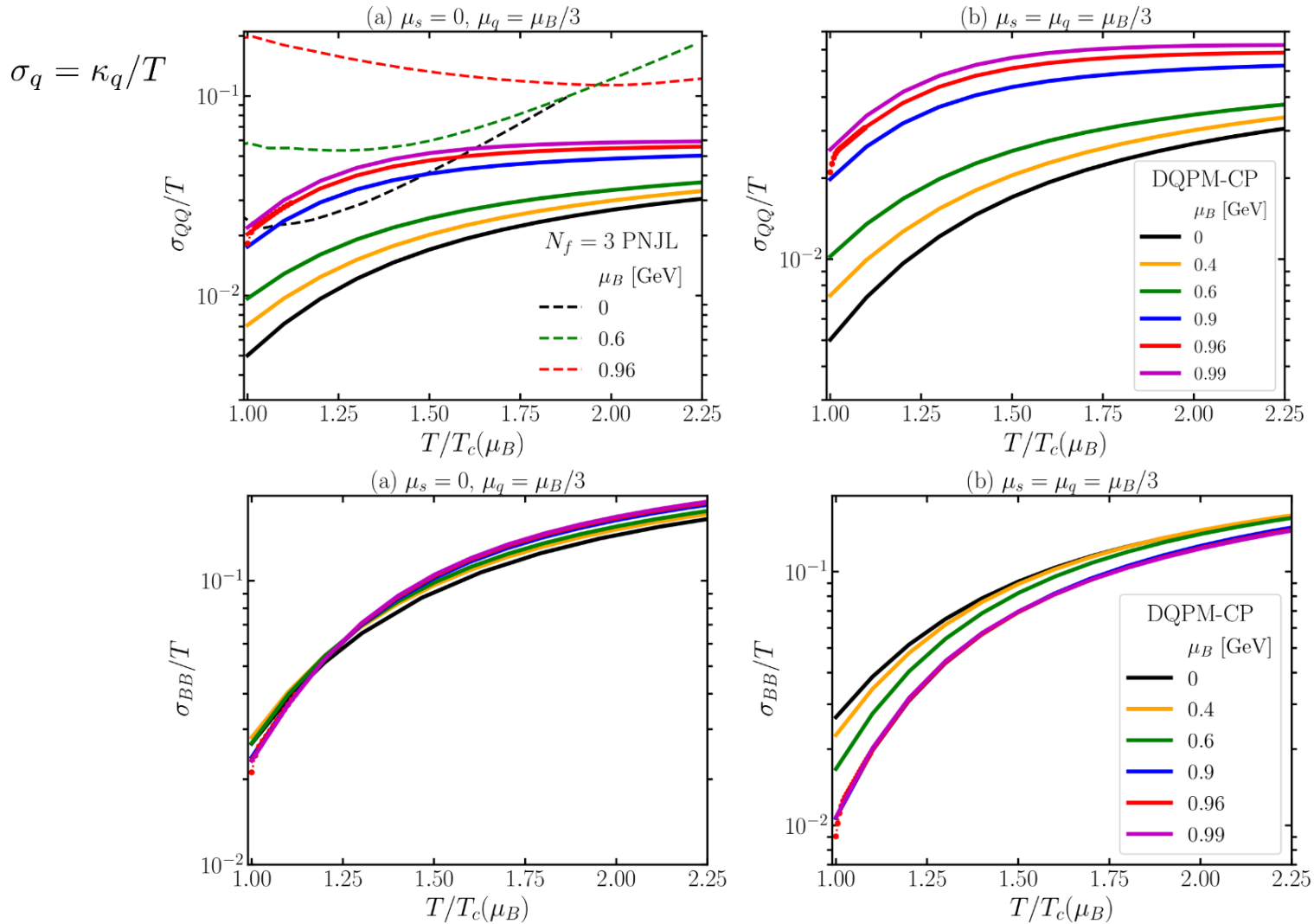
$$\ln(C_V) = -\alpha \cdot \ln(T - T_{CEP}) + const$$

# Shear and bulk viscosities near the CEP



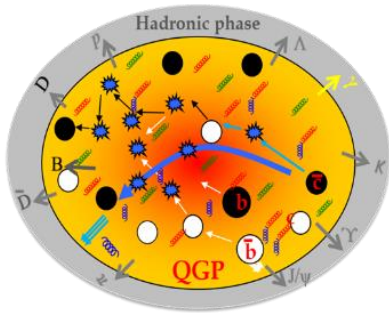
- Sudden rise of specific bulk viscosity approaching the CEP

# Transport coefficients near the CEP



- B,Q,S diffusion coefficients have pronounced  $\mu_B, \mu_S$ -dependence
- Only small increase approaching the CEP

# Modelling HICs: PHSD

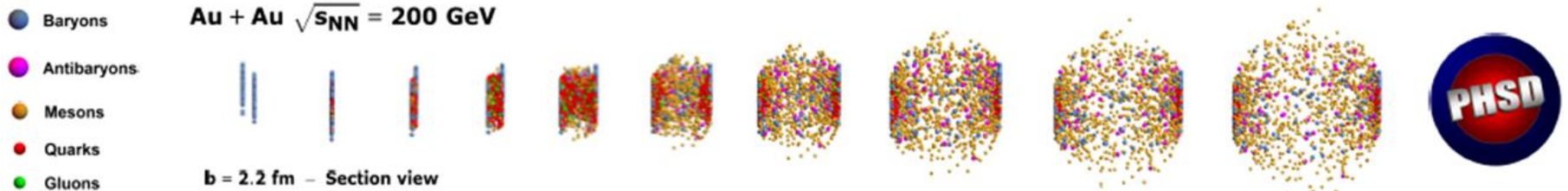


**QGP out-of equilibrium  $\leftrightarrow$  HIC**

## Parton-Hadron-String-Dynamics (PHSD)

Non-equilibrium **microscopic transport approach** for the description of strongly-interacting **hadronic** and **partonic** matter created in heavy-ion collisions

**Dynamics:** based on the solution of generalized off-shell transport equations derived from Kadanoff-Baym many-body theory



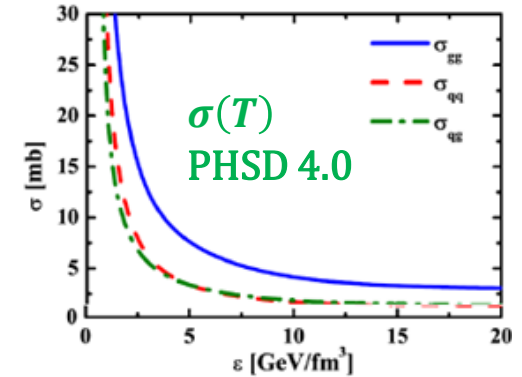
W. Cassing, E. Bratkovskaya, PRC 78 (2008) 034919; NPA831 (2009) 215; W. Cassing, EPJ ST 168 (2009) 3; .....;  
P. Moreau, O. Soloveva, L. Oliva, T. Song, W. Cassing, E. Bratkovskaya, PRC 100 (2019), 014911;  
O. Soloveva, P. Moreau, L. Oliva, V. Voronyuk, V. Kireyeu, T. Song, E. Bratkovskaya, Particles 3 (2020), 178-192;....

# PHSD

➤ PHSD 4.0 : only isotropic  $\sigma(T)$  and  $\rho(T)$

parton cross sections

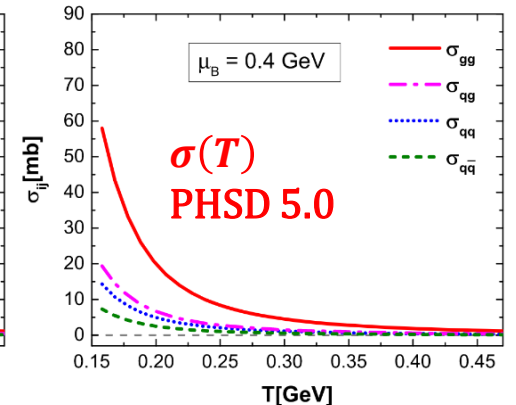
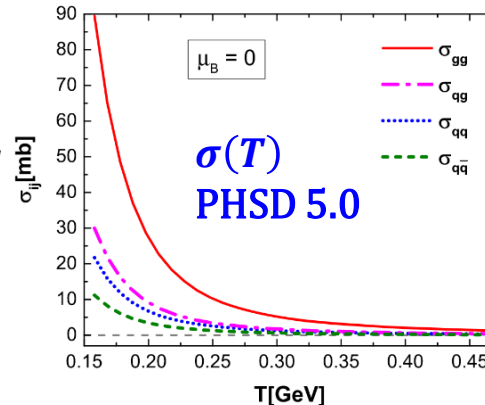
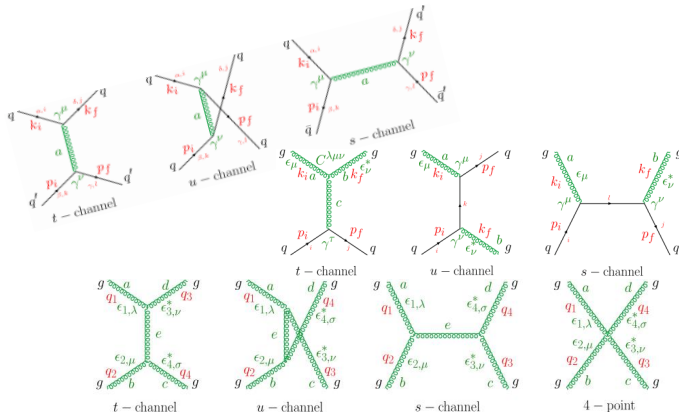
parton spectral function (masses and widths)



new PHSD 5 : angular dependence of  $d\sigma/d\cos\theta$

➤ PHSD 5.0 : with  $\sigma(\sqrt{s}, m_1, m_2, T, \mu_B = 0)$  and  $\rho(T, \mu_B = 0)$

➤ PHSD 5.0 : with  $\sigma(\sqrt{s}, m_1, m_2, T, \mu_B)$  and  $\rho(T, \mu_B)$



# PHSD: QGP evolution in HICs

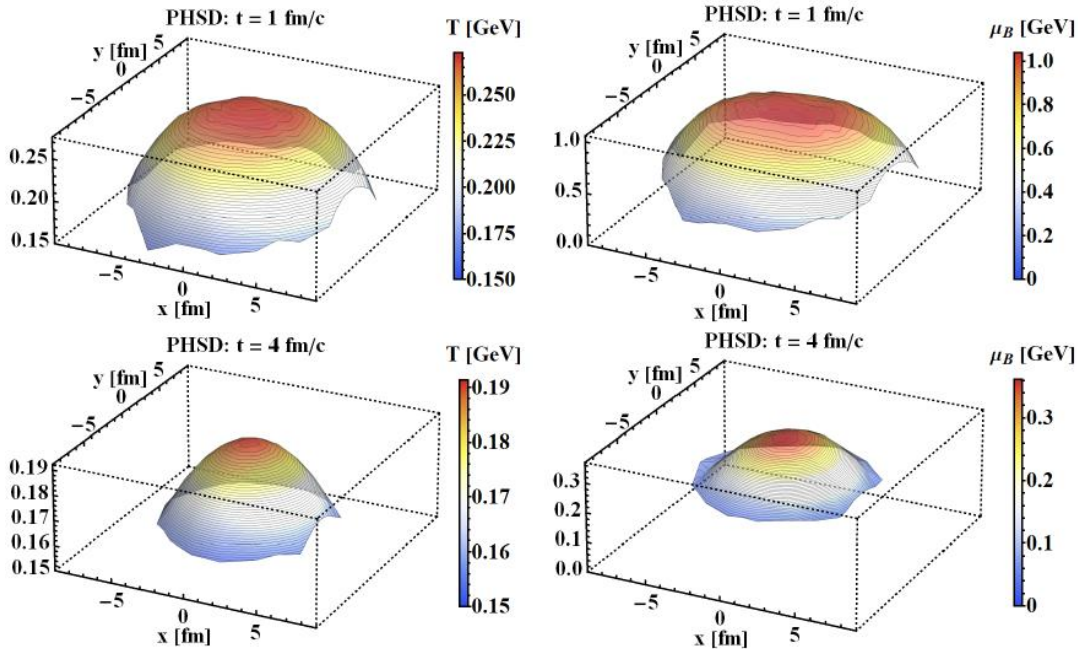
**Input:**  
 $\epsilon^{\text{PHSD}}$  and  $n_B^{\text{PHSD}}$



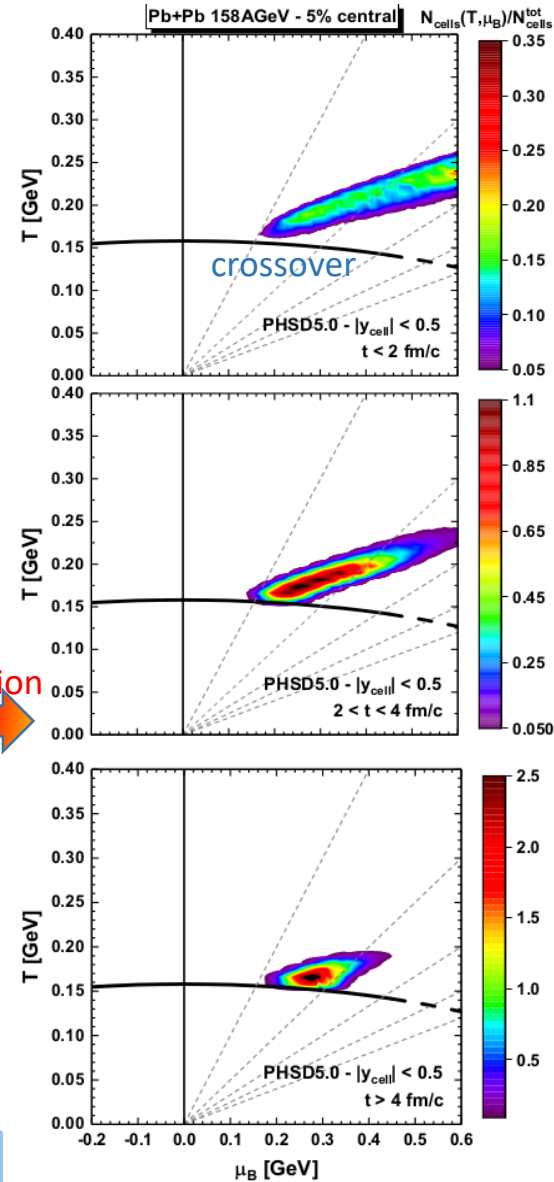
**Output:**  
 $T, \mu_B$

The  $T$ -profile in  $(x;y)$  &  $\mu_B$  profile in  $(x;y)$   
 at midrapidity ( $|y_{\text{cell}}| < 1$ ) at fixed times (1 and 4 fm/c)

Pb+Pb 158A GeV - 5% central



time evolution



Path through the phase diagram is not trivial and localized



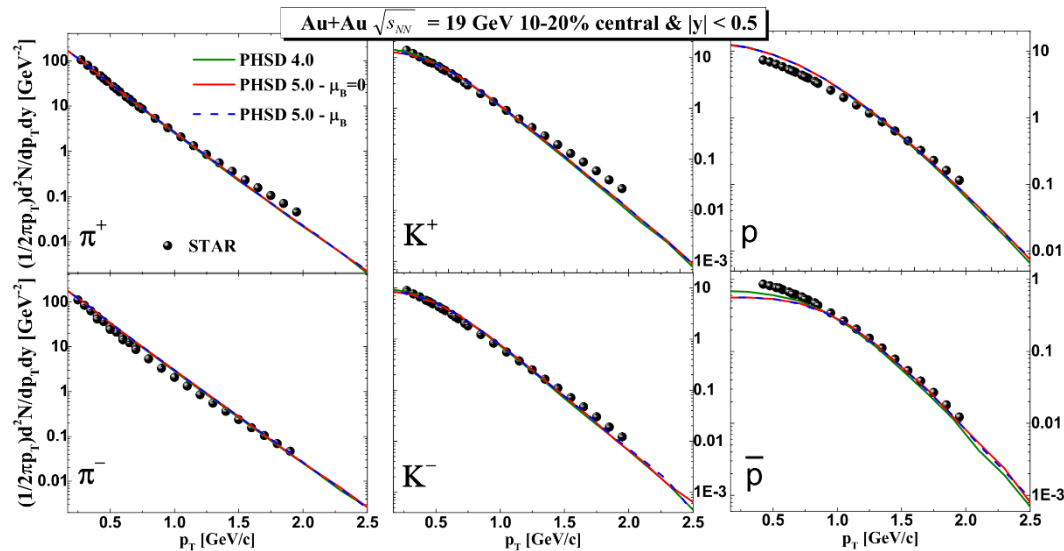
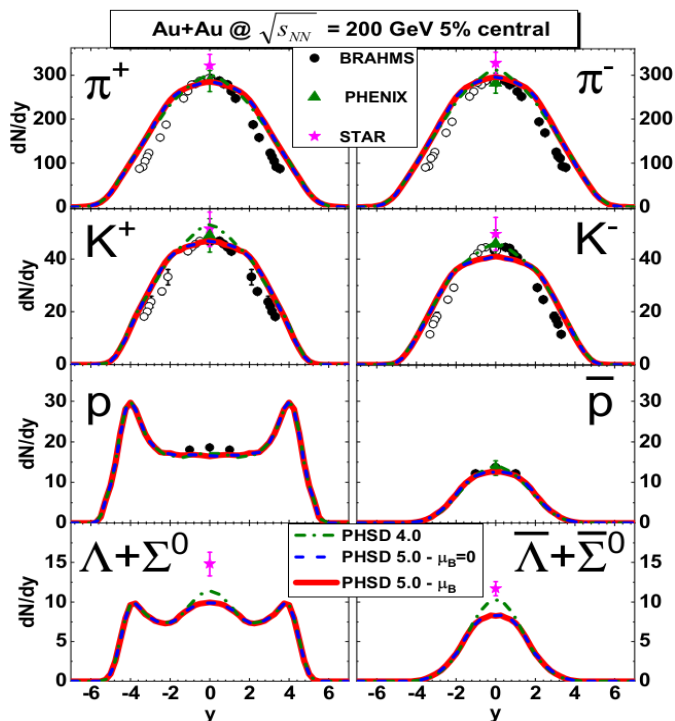
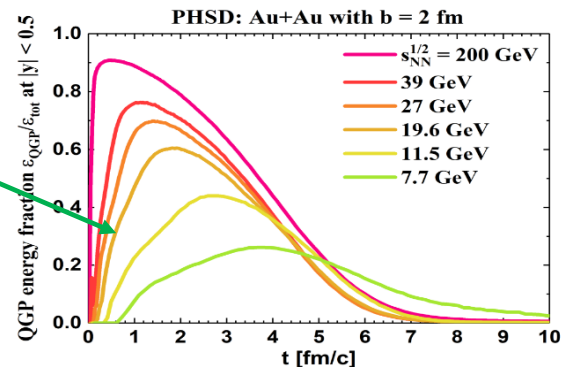
# Results for ( $\sqrt{s_{NN}} = 200 \text{ GeV} - 7 \text{ GeV}$ )



- No visible effects on  $p_T$ -spectra,  $dN/dy$  of  $\mu_B$ -dependence
- Small effect of the angular dependence of  $d\sigma/d\cos\theta$

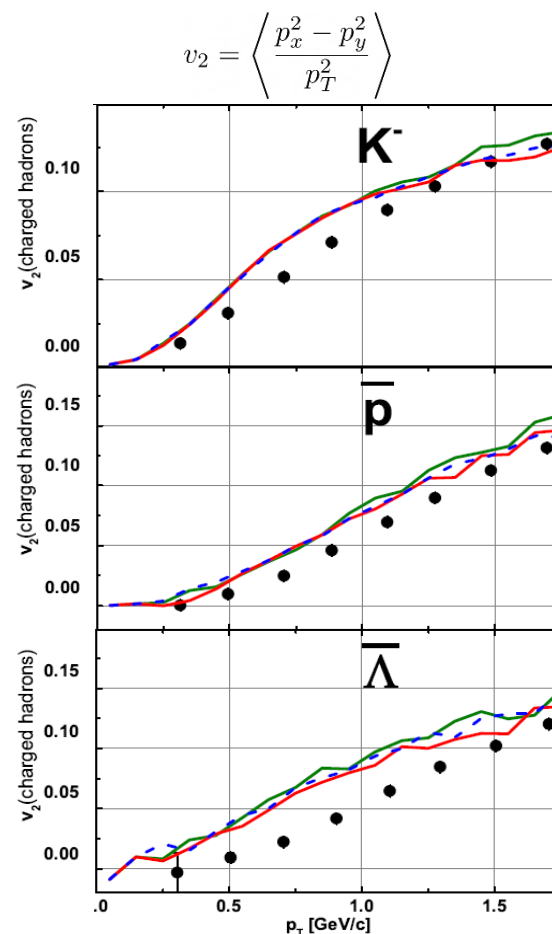
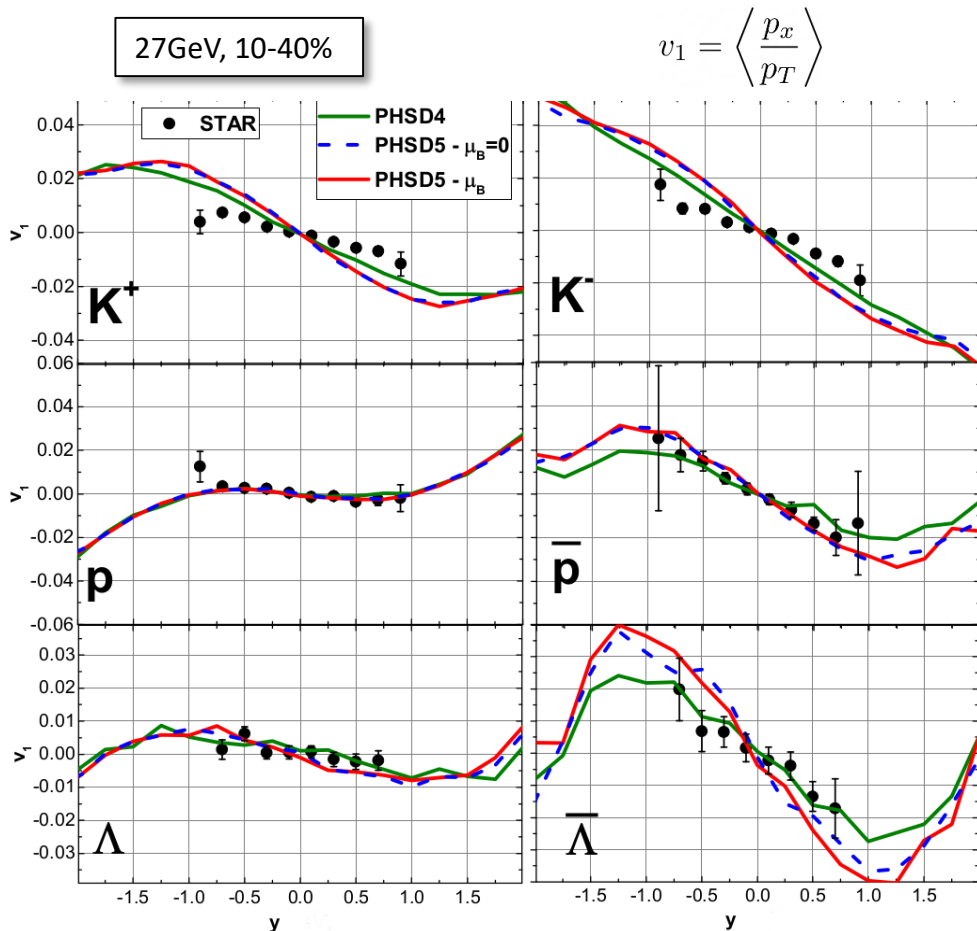
at high  $\sqrt{s_{NN}}$  - low  $\mu_B$

! QGP fraction is **small** at low  $\sqrt{s_{NN}}$



# Elliptic flow ( $\sqrt{s_{NN}} = 200 \text{ GeV} - 27 \text{ GeV}$ )

- Weak  $\mu_B$  -dependence - small fraction of QGP or low  $\mu_B$
- Small effect of the angular dependence of  $d\sigma/d\cos\theta$
- Strong flavor dependence



# Summary

---

Transport properties of the strongly-interacting QGP matter at finite  $T$  and  $\mu_B$  have been investigated.

Influence of an order of a phase transition on thermodynamic and transport properties has been studied.

- Transport coefficients can differ among the models, which have similar phase structures and EoS
- 

Evolution of the QGP matter created in HICs and the sensitivity of the bulk and flow observables on the QGP interactions and transport properties have been explored by the simulations within the PHSD transport approach

- **High- $\mu_B$**  regions are probed at **low  $\sqrt{s_{NN}}$**  or high rapidity regions  
Moreover, **QGP** fraction is **small** at **low  $\sqrt{s_{NN}}$**  : small effect seen in observables
  - **$\mu_B$ -dependence** of QGP interactions is more pronounced in observables for strange hadrons and antiprotons
-

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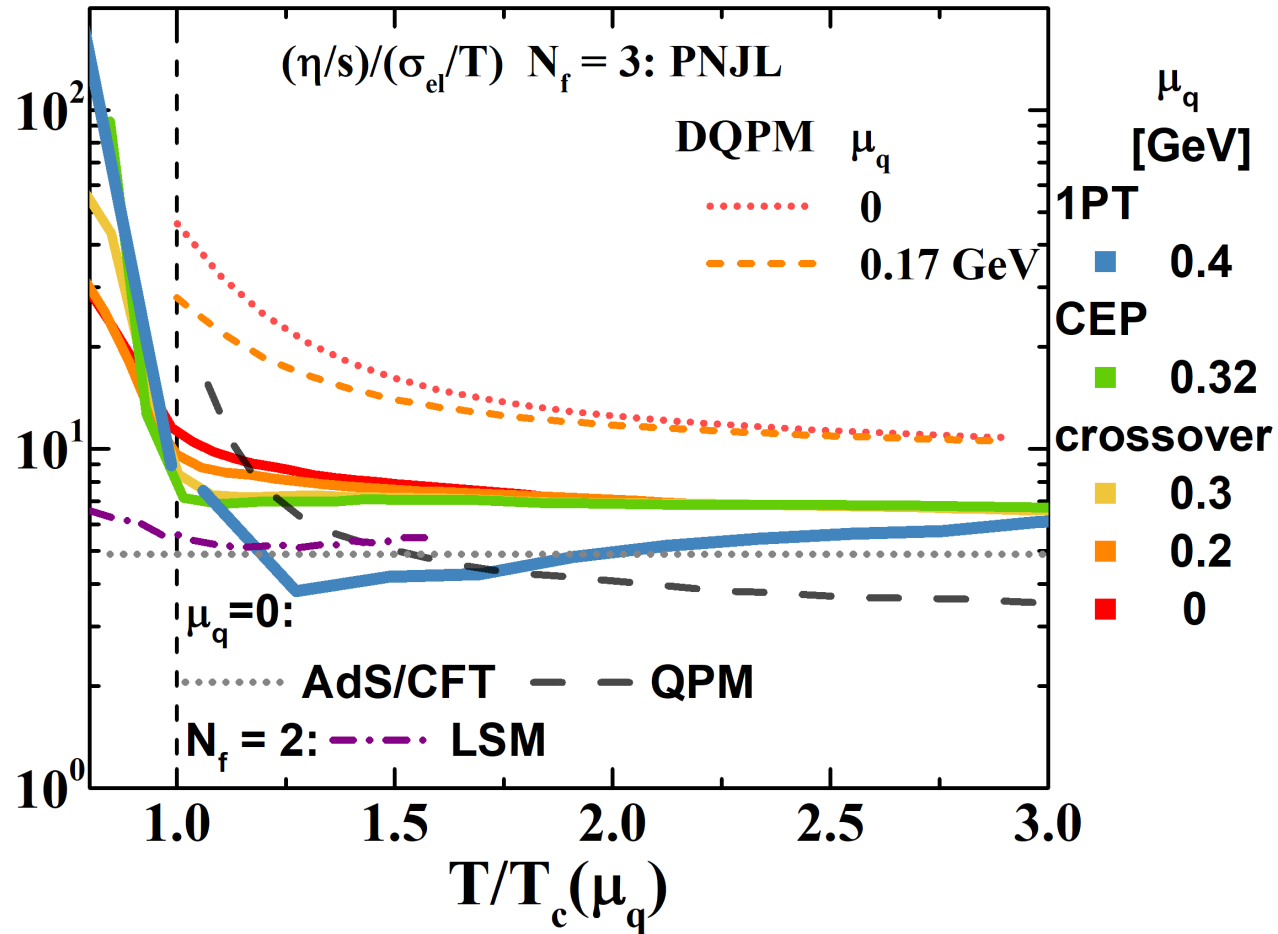
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- 

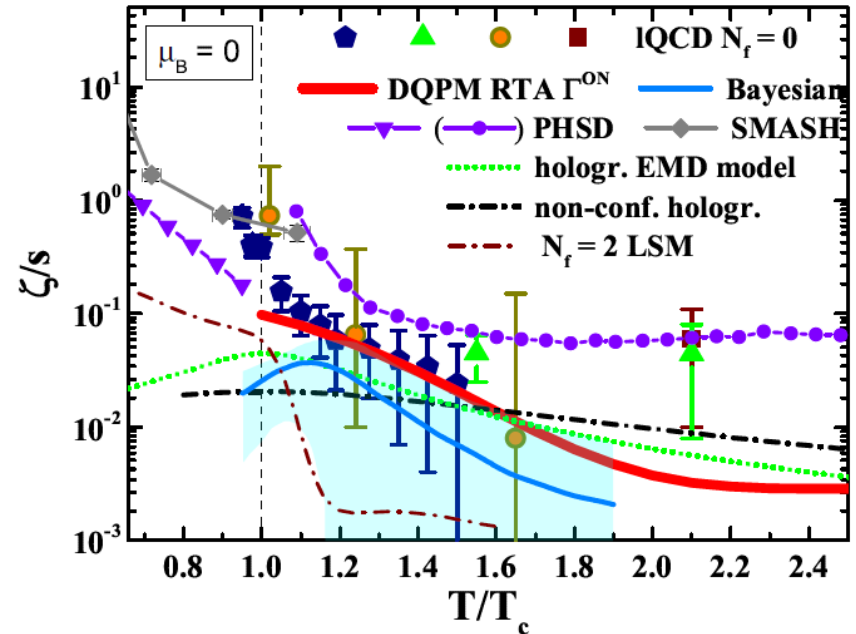
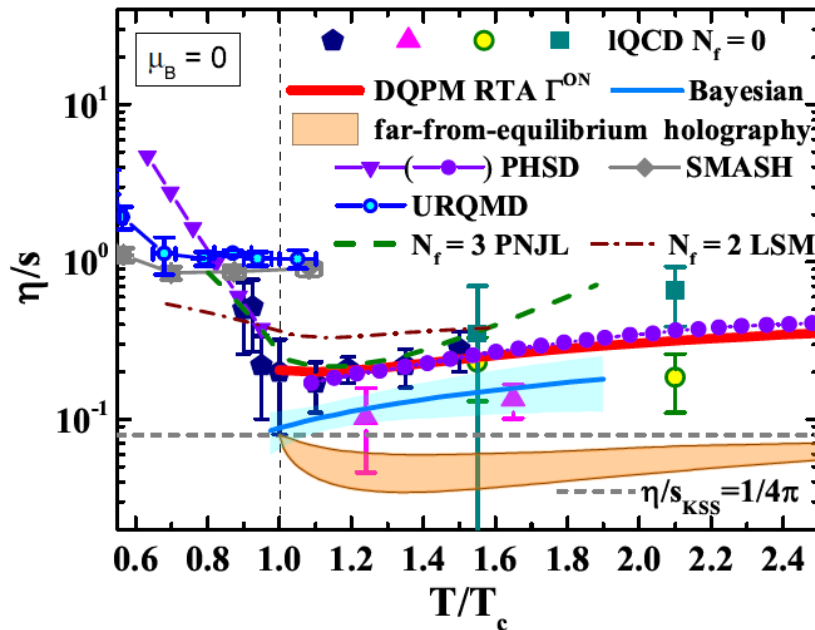
**Thank you for your attention!**

# Specific shear viscosity to conductivity



# Specific shear viscosity compilation

Model predictions:



**!** Different models using the same EoS can have completely different transport coefficients!

# Specific shear viscosity compilation

- **Kubo formalism: transport coefficients are expressed through correlation functions of stress-energy tensor**

used in **lattice QCD, transport approaches(hadrons), effective models**

$$\eta = \frac{1}{20} \lim_{\omega \rightarrow 0} \frac{1}{\omega} \int d^4x e^{i\omega t} \langle [S^{ij}(t, \mathbf{x}), S^{ij}(0, \mathbf{0})] \rangle \theta(t) \quad S^{ij} = T^{ij} - \delta^{ij} \mathcal{P}$$

$$\zeta = \frac{1}{2} \lim_{\omega \rightarrow 0} \frac{1}{\omega} \int d^4x e^{i\omega t} \langle [\mathcal{P}(t, \mathbf{x}), \mathcal{P}(0, \mathbf{0})] \rangle \theta(t) \quad \mathcal{P} = -\frac{1}{3} T^i_i$$

R. Lang and W. Weise, EPJ. A 50, 63 (2014) (NJL model)

A. Harutyunyan et al, PRD 95, 114021, (2017)

## Kinetic theory:

- **Relaxation time approximation(RTA) : consider relaxation time**  $\frac{df_a^{\text{eq}}}{dt} = C_a = -\frac{f_a^{\text{eq}} \phi_a}{\tau_a}$

P. Chakraborty and J. I. Kapusta, PRC 83,014906 (2011)

- **Chapman-Enskog : expand the distribution in terms of the Knudsen number**

J. A. Fotakis et al, PRD 101 (2020) 7, 076007 (HRG)

**And more!**

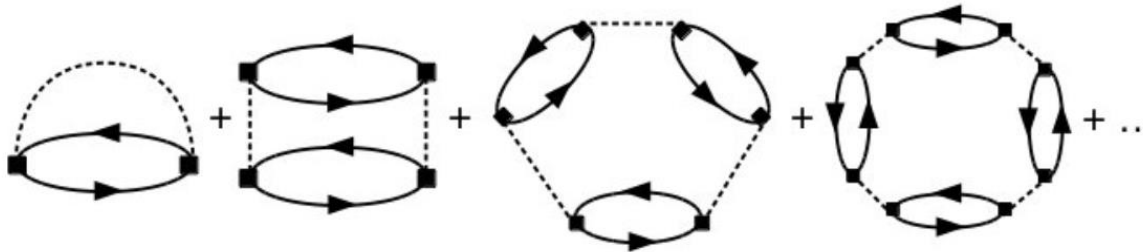
## Holographic models: AdS/CFT correspondence

D. T. Son and A. O. Starinets, JHEP 0603, 052 (2006)

M. Attems et al , JHEP 10 (2016), 155.

# PNJL improvements

- Next to leading order in  $N_c$  ( $O(1/N_c)^0$ ) of the grand-canonical potential : **presence of the mesons below  $T_c$**



J. M. Torres-Rincon, J. Aichelin PRC 96 (2017) 4 045205

- **Modification of the gluon potential due to the presence of the quark**

$$\frac{U(\phi, \bar{\phi}, T)}{T^4} = -\frac{b_2(T)}{2} \bar{\phi}\phi - \frac{b_3}{6} (\bar{\phi}^3 + \phi^3) + \frac{b_4}{4} (\bar{\phi}\phi)^2$$

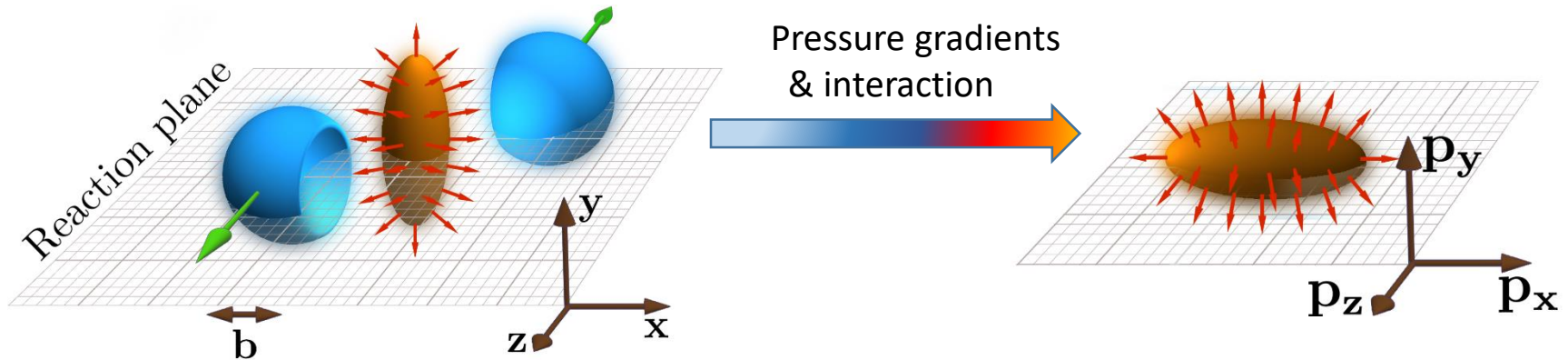
$$b_2(T) = a_0 + \frac{a_1}{1 + \tau} + \frac{a_2}{(1 + \tau)^2} + \frac{a_3}{(1 + \tau)^3} \quad \text{where} \quad \tau_{\text{phen}} = 0.57 \frac{T - T_{\text{phen}}^{\text{cr}}(T)}{T_{\text{phen}}^{\text{cr}}(T)}$$

$$T_{\text{phen}}(T) = a + bT + cT^2 + dT^3 + \boxed{e \frac{1}{T}}$$

D. Fuseau, T. Steinernert, J. Aichelin PRC 101 (2020) 6 065203



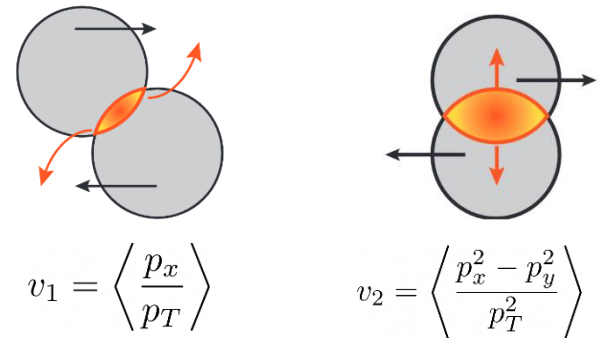
# Anisotropic flow coefficients



Quantify the anisotropic flow using Fourier expansion

$$\frac{dN}{d\varphi} \propto \left( 1 + 2 \sum_{n=1}^{+\infty} v_n \cos[n(\varphi - \psi_n)] \right)$$

$$v_n = \left\langle \cos n(\varphi - \psi_n) \right\rangle, \quad n = 1, 2, 3, \dots$$



## Anisotropic flow

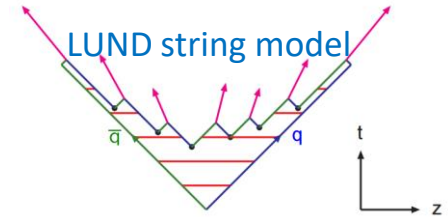
- Assess the transport properties of the QGP
- Sensitive to the QGP EoS and initial state
- Validate models of bulk evolution that are used in the computation of other observables

# Stages of collisions in PHSD

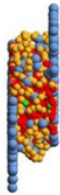
## Initial A+A collision



- String formation in primary NN collisions  
→ decays to pre-hadrons (baryons and mesons)

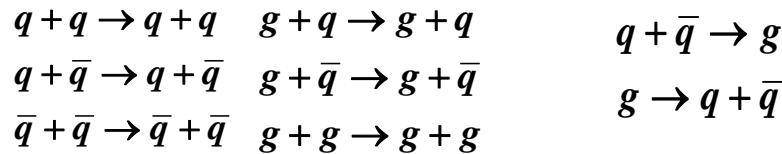


## Partonic phase

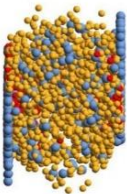


- Formation of a QGP state if  $\epsilon > \epsilon_{critical}$  :  
Dissolution of pre-hadrons → DQPM  
→ massive quarks/gluons and mean-field energy

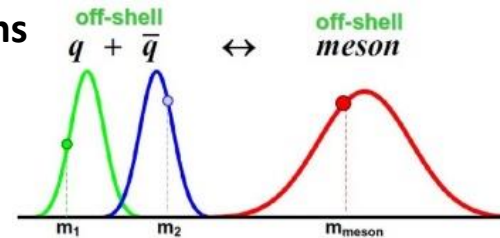
(quasi-)elastic collisions :                      inelastic collisions:



## Hadronization



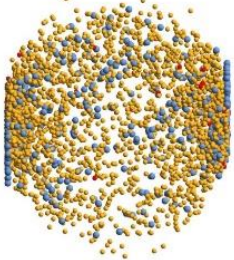
- Hadronization to colorless off-shell mesons and baryons  
 $g \rightarrow q + \bar{q}$ ,     $q + \bar{q} \leftrightarrow meson$  ('string')  
 $q + q + q \leftrightarrow baryon$  ('string')



Strict 4-momentum and quantum number conservation

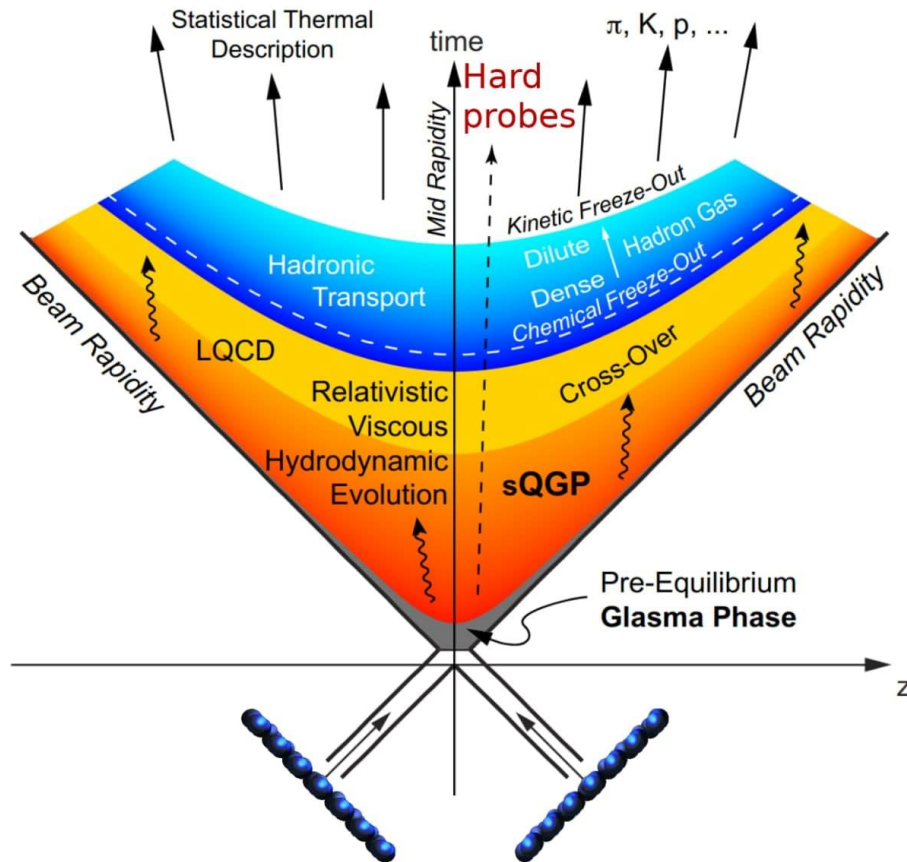
- Hadron-string interactions – off-shell HSD

## Hadronic phase



W. Cassing, E. Bratkovskaya, PRC 78 (2008) 034919; NPA831 (2009) 215;  
W. Cassing, EPJ ST 168 (2009) 3

# Stages of HIC



$t > 10 \text{ fm}$  - hadronisation and free stream to detectors

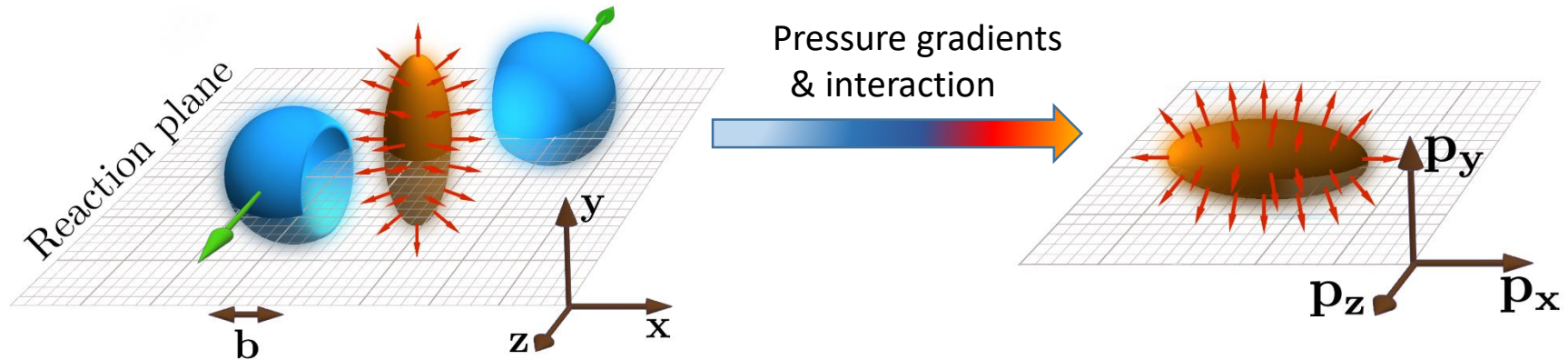
$10 \text{ fm} > t > 1 \text{ fm}$  - QGP expansion

$t \approx 1 \text{ fm}$  - Equilibration

$t \ll 1 \text{ fm}$  - Initial state

**QGP out-of equilibrium  $\leftrightarrow$  HIC**

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