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(Non-)perturbative jet dispersion in hot QCD

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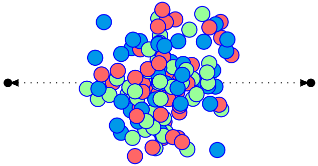
Phase transitions in particle physics, GGI, 2022

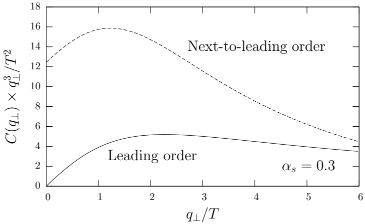
J. Ghiglieri, G. D. Moore, P. Schicho, and N. Schlusser, *The force-force-correlator in hot QCD perturbatively and from the lattice,* JHEP **02** (2022) 58 [2112.01407]

Motivation

Heavy-ion collisions



- Hard particles carry most of the stress-energy tensor $P \sim T$.
- Medium soft modes at scale $P \sim gT$.
- Jet-medium interactions in the Quark-Gluon Plasma (QGP) can receive large non-perturbative infrared contributions.[1]



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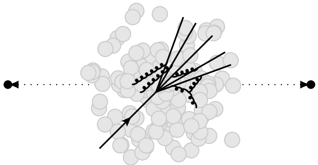

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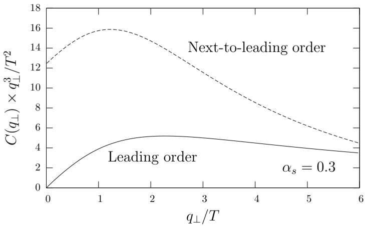

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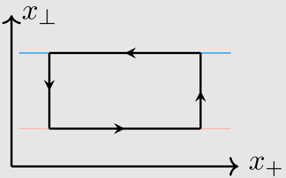
Important quantities

Collision kernel

$$C(q_\perp) = \frac{\mathrm{d}\Gamma}{\mathrm{d}^2 q_\perp \mathrm{d}L}$$

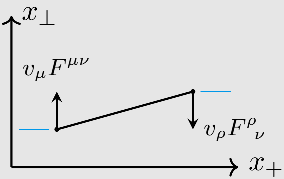
Wilson loop[2]



Asymptotic masses

$$m_\infty^2 = C_\mathrm{R}(Z_\mathrm{g} + Z_\mathrm{f})$$

Force-force-correlator[3]



Non-perturbative gluon-zero-mode contributions → calculate in lattice "electrostatic QCD" (EQCD).[4]

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[2] J. Casalderrey-Solana and D. Teaney, *Transverse Momentum Broadening of a Fast Quark in a N=4 Yang Mills Plasma*, JHEP **04** (2007) 039 [hep-th/0701123]

[3] E. Braaten and R. D. Pisarski, *Simple effective Lagrangian for hard thermal loops*, Phys. Rev. D **45** (1992) R1827

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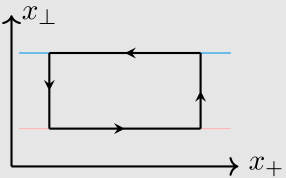

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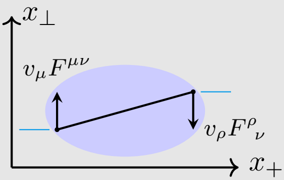

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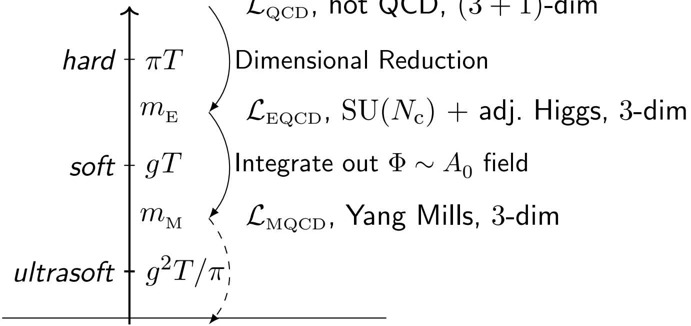
Dimensional Reduction (DR)

Integrate out fast (hard) modes perturbatively → EFT for static modes.[5]

All order thermal resummation to by-pass IR problem. Applied for thermodynamics of non-Abelian gauge theories such as (EW) phase transitions[6] and QCD.



[5] cf. talk by J. Ghiglieri and also D. Bödeker, M. Sangel, and M. Wörmann, *Equilibration, particle production, and self-energy,* Phys. Rev. D **93** (2016) 045028 [1510.06742]

[6] K. Kajantie, M. Laine, K. Rummukainen, and M. E. Shaposhnikov, *Generic rules for high temperature dimensional reduction and their application to the standard model,* Nucl. Phys. B **458** (1996) 90 [hep-ph/9508379], K. Kajantie, M. Laine, K. Rummukainen, and M. E. Shaposhnikov, *The Electroweak phase transition: A Nonperturbative analysis,* Nucl. Phys. B **466** (1996) 189 [hep-lat/9510020]

Dimensionally reduced effective theory for hot QCD

QCD described by 3-dimensional super-renormalisable theory

$$S_{\rm EQCD} = \frac{1}{T}\int_{\mathbf{x}} \left\{ \mathcal{L}_{\rm EQCD} + \sum_{n\geq 5} \frac{\mathcal{O}_n}{(\pi T)^n} \right\} .$$

"Electrostatic QCD" (EQCD) at high-T ($A_0^a \to \Phi^a$)

$$\mathcal{L}_{\rm EQCD} \equiv \frac{1}{2}{\rm Tr}\, F_{ij}F_{ij} + {\rm Tr}\,[D_i,\Phi][D_i,\Phi] + m_{\rm D}^2\,{\rm Tr}\,\Phi^2 + \lambda_{\rm E}({\rm Tr}\,\Phi^2)^2 \ .$$

Developed to study high-T thermodynamics[7], but also used for soft light-cone observables[8].

[7] P. Ginsparg, *First and second order phase transitions in gauge theories at finite temperature,* Nucl. Phys. B **170** (1980) 388, T. Appelquist and R. D. Pisarski, *High-temperature Yang-Mills theories and three-dimensional quantum chromodynamics,* Phys. Rev. D **23** (1981) 2305

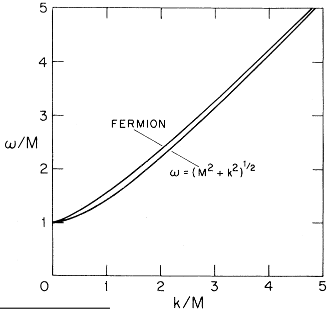
[8] S. Caron-Huot, *O(g) plasma effects in jet quenching,* Phys. Rev. D **79** (2009) 065039 [0811.1603], J. Ghiglieri, J. Hong, A. Kurkela, E. Lu, G. D. Moore, and D. Teaney, *Next-to-leading order thermal photon production in a weakly coupled quark-gluon plasma,* JHEP **2013** (2013) 10 [1302.5970]

Asymptotic masses

Integrate out jet energy scale $E \gg T$. Truncate $\frac{T}{E}$-series: LO correlators[9]

$$Z_{\rm f} \equiv \frac{1}{2d_{\rm R}} \left\langle \overline{\psi} \frac{v_\mu \gamma^\mu}{v \cdot D} \psi \right\rangle$$

$$Z_{\rm g} \equiv -\frac{1}{d_{\rm A}} \left\langle v_\mu F^{\mu\nu} \frac{1}{(v \cdot D)^2} v_\rho F^{\rho}{}_{\nu} \right\rangle$$



[9] E. Braaten and R. D. Pisarski, *Simple effective Lagrangian for hard thermal loops,* Phys. Rev. D **45** (1992) R1827, S. Caron-Huot, *O(g) plasma effects in jet quenching,* Phys. Rev. D **79** (2009) 065039 [0811.1603]

figure by H. A. Weldon, *Effective Fermion Masses of Order gT in High Temperature Gauge Theories with Exact Chiral Invariance,* Phys. Rev. D **26** (1982) 2789

Condensates of the asymptotic masses

In QCD rewrite detour through the medium as[10]

$$Z_{\rm g} = -\frac{1}{d_{\rm A}} \int_0^\infty {\rm d}x^+ x^+ \Big\langle v_\mu F_a^{\mu\nu}(x^+)\, U_{\rm A}^{ab}(x^+;0)\, v_\rho F^{\rho}_{b\,\nu}(0) \Big\rangle \;,$$

and match also operator onto EQCD

$$Z_{\rm g}^{\rm 3d} = -\frac{4T}{d_{\rm A}} \int_0^\infty {\rm d}L\, L \left(-\langle EE \rangle + \langle BB \rangle + i\langle EB \rangle \right) \,.$$

Correlator splits into electro- and magneto-static contributions:

$$\langle EE \rangle \equiv \frac{1}{2} \Big\langle (D_x \Phi(L))^a\, \tilde{U}_{\rm A}^{ab}(L,0)\, (D_x \Phi(0))^b \Big\rangle \;,$$

$$\langle BB \rangle \equiv \frac{1}{2} \Big\langle F^a_{xz}(L)\, \tilde{U}_{\rm A}^{ab}(L,0)\, F^b_{xz}(0) \Big\rangle \;,$$

$$i\langle EB \rangle \equiv \frac{i}{2} \Big\langle (D_x \Phi(L))^a\, \tilde{U}_{\rm A}^{ab}(L,0)\, F^b_{xz}(0) \Big\rangle + [BE] \,.$$

[10] $U_{\rm A}(x^+;0)$ is an adjoint, light-like Wilson line.

EFT matching with full QCD

Strategy:

$$C_{\mathrm{QCD}}(x) = \underbrace{(C_{\mathrm{QCD}}(x) - C_{\mathrm{EQCD}}(x))}_{\text{UV dominated}} + \underbrace{C_{\mathrm{EQCD}}(x)}_{\text{lattice}}$$

- Done[11] for $C(q_\perp)$.
- Partially done[12] for m_∞^2. Missing full QCD contribution.

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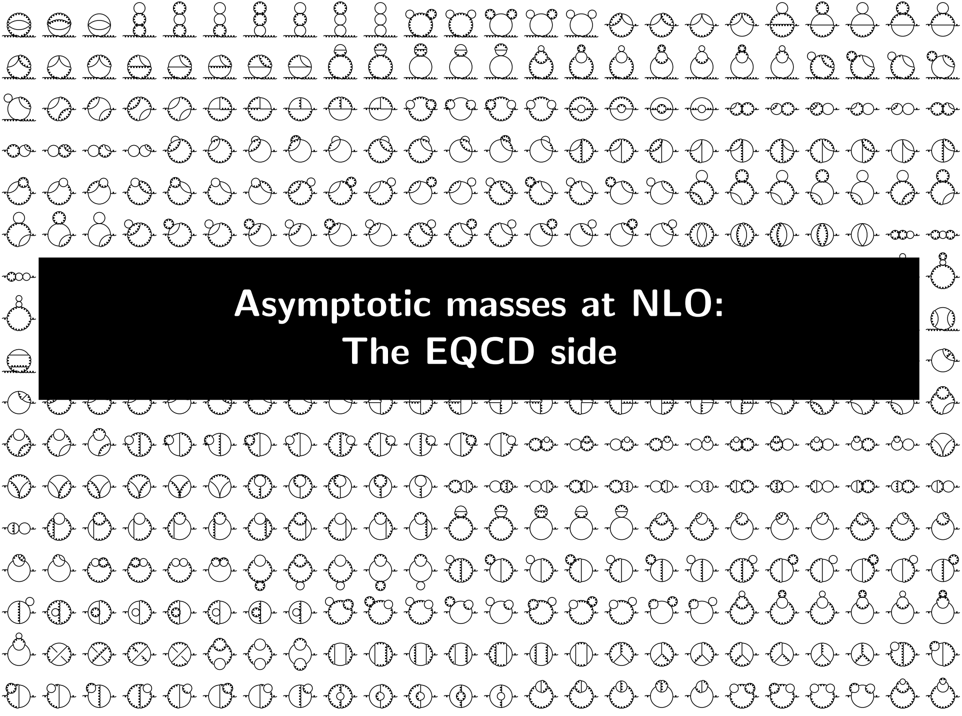
[11] P. Arnold and W. Xiao, *High-energy jet quenching in weakly coupled quark-gluon plasmas,* Phys. Rev. D **78** (2008) 125008 [0810.1026], J. Ghiglieri and H. Kim, *Transverse momentum broadening and collinear radiation at NLO in the $\mathcal{N} = 4$ SYM plasma,* JHEP **2018** (2018) 49 [1809.01349], G. D. Moore, S. Schlichting, N. Schlusser, and I. Soudi, *Non-perturbative determination of collisional broadening and medium induced radiation in QCD plasmas,* JHEP **10** (2021) 059 [2105.01679]

[12] J. Ghiglieri, G. D. Moore, P. Schicho, and N. Schlusser, *The force-force-correlator in hot QCD perturbatively and from the lattice,* JHEP **02** (2022) 58 [2112.01407]



Asymptotic masses at NLO: The EQCD side

Contributions to Z_g

$$
\begin{aligned}
& \quad \text{scale } T \qquad\qquad\qquad \text{scale } gT \qquad\qquad\qquad\qquad\qquad\qquad\qquad \text{scale } g^2T \\
Z_\mathrm{g} = & \Big[\frac{T^2}{6} - \frac{T\mu_\mathsf{h}}{\pi^2} \Big] \\
& + \Big[-\frac{Tm_\mathrm{D}}{2\pi} + \frac{T\mu_\mathsf{h}}{\pi^2} \Big] \\
& + \Big[c^{\ln}_\mathsf{hard} \ln\frac{T}{\mu_\mathsf{h}} + c_T \quad + c^{\ln}_\mathsf{hard} \ln\frac{\mu_\mathsf{h}}{m_\mathrm{D}} + c^{\ln}_\mathsf{soft} \ln\frac{m_\mathrm{D}}{\mu_\mathsf{s}} + c_{gT} \quad + c^{\ln}_\mathsf{soft} \ln\frac{\mu_\mathsf{s}}{g^2T} + c_{gT^2} \Big] \\
& + \mathcal{O}(g^3) \, ,
\end{aligned}
$$

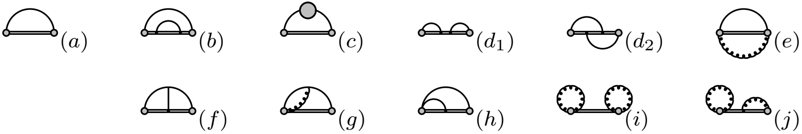
Z_g receives IR contributions already at $\mathcal{O}(g)$.[13]

Scheme-dependent at NLO; use intermediate regulators $T \gg \mu_\mathsf{h} \gg gT$ and $gT \gg \mu_\mathsf{s} \gg g^2T$.

[13] S. Caron-Huot, *O(g) plasma effects in jet quenching*, Phys. Rev. D **79** (2009) 065039 [0811.1603]

$Z_{\rm g}$ in EQCD perturbatively

Diagrams contributing at LO and NLO to the EQCD force-force correlator $Z_{\rm g}$:



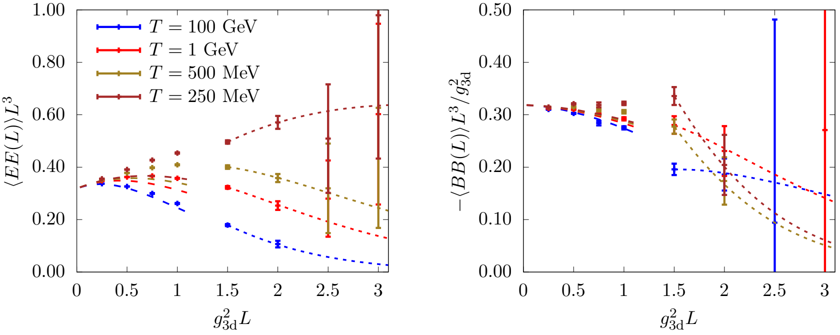
Example: LO colour-electric condensate $\langle EE \rangle$ – free solution

$$= 2 \times (a)^{\rm EE} = \partial_x \partial_{x'} {\rm Tr} \left\langle \Phi^a(x, L) \Phi^a(x', 0) \right\rangle \Big|_{x,x' \to 0}$$

$$= \frac{2 C_{\rm A} C_{\rm F}}{4\pi L^3} \epsilon^{-m_{\rm D} L} \left(1 + m_{\rm D} L\right)$$

Asymptotic masses (non-)perturbatively

Three different correlators contribute to $Z_\mathrm{g} \subset m_\infty^2$ in EQCD:



- small-L: NLO perturbative estimate
- large-L: Fit long L-tail to model[14]

[14] M. Laine and O. Philipsen, *Gauge-invariant scalar and field strength correlators in three dimensions*, Nucl. Phys. B **523** (1998) 267 [9711022]

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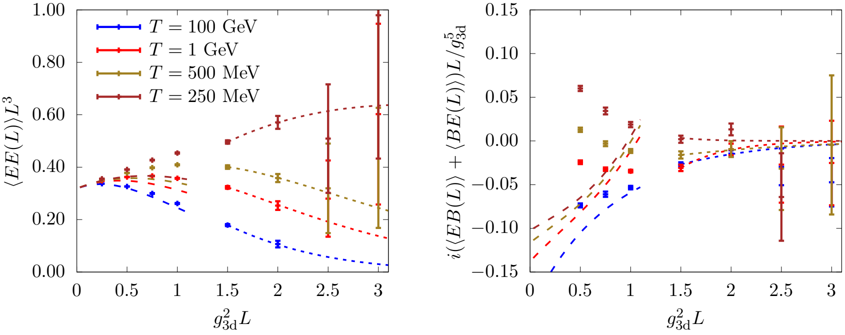

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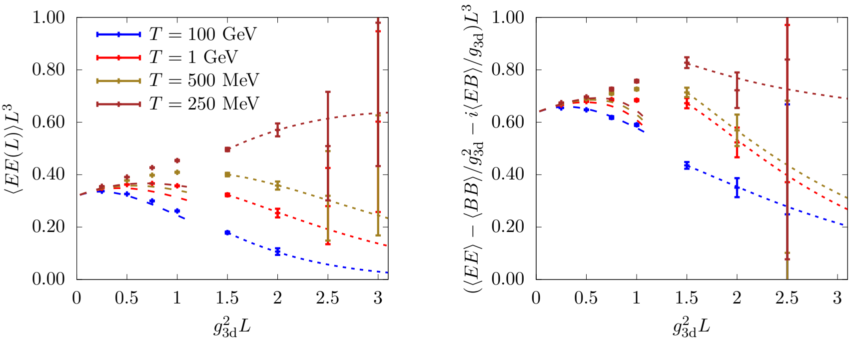

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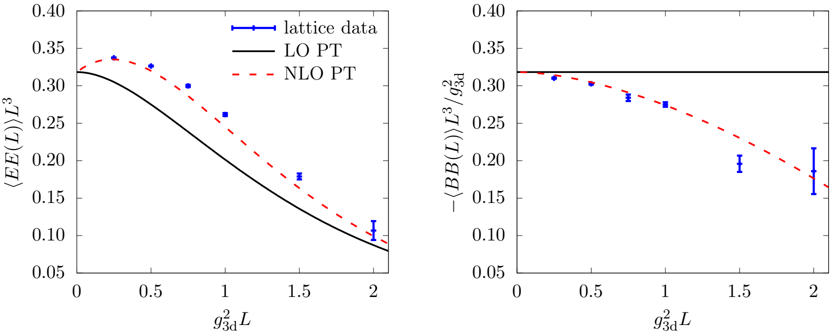
Asymptotic masses (non-)perturbatively

For $T = 100$ GeV and $N_\mathrm{f} = 5$, strong agreement between perturbative and non-perturbative Z_g.



Conclusions

- Jet modifications (+other transport) involves soft IR QCD $\to$ (lattice) QCD
- Key quantities are $C(b_\perp)$ and asymptotic mass m_∞^2 from lattice EQCD

What's next for m_∞^2?

☆ Finalise matching computation to full QCD

☆ Input to effective kinetic theory AMY[15] $\to$ GMT[16]

☆ Ingredients for NNLO-transport

☆ Feed into event generator

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[15] P. B. Arnold, G. D. Moore, and L. G. Yaffe, *Effective kinetic theory for high temperature gauge theories*, JHEP **01** (2003) 030 [hep-ph/0209353]

[16] J. Ghiglieri, G. D. Moore, and D. Teaney, *Jet-medium interactions at NLO in a weakly-coupled quark-gluon plasma*, JHEP **2016** (2016) 95 [1509.07773]