Charged fixed points in 3D U(1) scalar gauge theories

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Work in collaboration with C. Bonati and E. Vicari (Pisa U)

arXiv:2201.01082 Critical behaviors of lattice U(1) gauge models and three-dimensional Abelian-Higgs gauge field theory

arXiv:2011.04503 Higher-charge three-dimensional compact lattice Abelian-Higgs models

arXiv:2010.06311 Lattice Abelian-Higgs model with noncompact gauge fields

Why studying gauge theories in three dimensions

Most studies of gauge theories strictly connected with High Energy Physics and fundamental interactions.

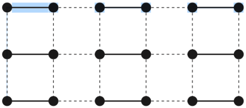
Gauge theories are also very important in condensed matter physics.

1) U(1) gauge symmetry: charged systems...

2) Discrete gauge symmetries: $\mathbb{Z}_2$ gauge models appear in several contexts, giving rise to topological transitions (they control the dynamics of classes of topological defects)

3) Emergent gauge symmetries: a new concept that is now used to interpret many exotic 2D quantum transitions.

A simple case of emergent U(1) gauge symmetry



Spin-1/2 antiferromagnet on a square lattice

$$H = \sum_{\langle xy \rangle} J_{xy} \boldsymbol{S}_x \cdot \boldsymbol{S}_y$$

with two types of bond couplings J_{xy} (thick lines and dashed lines)

If all couplings are identical, we have a standard antiferromagnet.
Order parameter: $N = \sum_x (-1)^P S_x$.
Gapless excitations: spin waves.

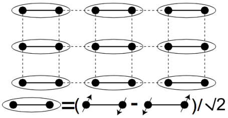


Fig. 2. Schematic of the quantum paramagnet ground state for small λ. The ovals represent singlet valence bond pairs.

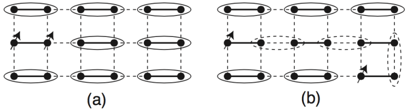


Fig. 3. (a) Cartoon picture of the bosonic $S = 1$ excitation of the paramagnet. (b) Fission of the $S = 1$ excitation into two $S = 1/2$ spinons. The spinons are connected by a "string" of valence bonds (denoted by dashed ovals) which lie on weaker bonds; this string costs a finite energy per unit length and leads to the confinement of spinons.

If the couplings along the dashed lines are zero, the ground state is formed by spin-zero singlets on the thick bonds.
Gapped excitations: Spin-one bonds (triplons, excitons).

There is a quantum phase transition between the two states.
Is there an order parameter for both phases?

Deconfined criticality

The solution: fractionalization and emergent U(1) symmetry

1) We first move from the quantum theory to a classical model in 2+1 d.
2) The order parameter is the spinon, a two-dimensional complex spinor z related to the spin S by $\boldsymbol{S}_x = \bar{z}_x \sigma z_x$,, where σ is a Pauli matrix.
3) The relation between z and S is defined modulo a U(1) transformation: emergence of a U(1) gauge invariance
4) The order parameter of the paramagnetic dimer phase is density of defects.

Field theory: (2+1) Abelian-Higgs model.
A detailed analysis of the path-integral formulation shows that one should consider the formulation with noncompact gauge fields.

Abelian-Higgs model

We consider an N_f-component unit-length scalar field $\boldsymbol{z}_x$ and a real U(1) field $A_{x,\mu}$ defined on the lattice bonds.

Matter action [$\lambda_{x,\mu} = \exp(iA_{x,\mu})$]

$$S_m = J \sum_{x\mu} \text{Re}\, \bar{\boldsymbol{z}}_x \lambda_{x,\mu} \boldsymbol{z}_{x+\mu}$$

Gauge compact action:

$$S_{g,c} = \kappa \sum_P \text{Re}\, \lambda_{x,\mu} \lambda_{x+\mu,\nu} \bar{\lambda}_{x+\nu,\mu} \bar{\lambda}_{x,\nu}$$

Gauge non compact action:

$$S_{g,nc} = -\frac{\kappa^2}{2} \sum_P (\nabla_\mu A_{x,\nu} - \nabla_\nu A_{x,\mu})^2$$

In the compact case, there excitations connected to monopole configurations: in the absence of matter fields, these excitations make the theory always confining (Polyakov).

Critical behavior: the field-theory point of view

First approach: Landau-Ginzburg-Wilson framework

The critical dynamics is ONLY controlled by the dynamics of the gauge-invariant matter sector. Gauge fields are only relevent for selecting the gauge-invariant sector.

The relevant symmetry group is the global symmetry group. More precisely, the universality class of the transition is determined uniquely by the the global symmetry breaking pattern and the transformation properties (representation) of the fhe matter field under global transformations. The gauge symmetry group is largely irrelevant.

This is the approach used by Pisarski and Wilczek to study the finite-T transition in QCD. They consider the order parameter $\phi_{ij} = \bar{\psi}_{i,L}\psi_{j,R}$ and the most general scalar theory for ϕ (without gauge fields) with $SU(N_f)_L \otimes SU(N_f)_R$ global symmetry.
PW's assumtion is that, at criticality, the universal behavior of the finite-T QCD and of the scalar theory is the same.

Second approach: Gauge-field theory

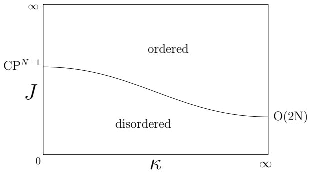
One can consider the gauge field theory with the same field content and use the usual Wilson-FIsher approach, i.e., determine the stable fixed points of the renormalization-group flow. The fixed points that correspond to non-zero values of the gauge couplings are charged fixed points (CFPs).

Charged fixed points cannot be obtained in the LGW approach.

Computationally, one can determine them using the Wilson-Fisher ϵ-expansion, i.e. perturbatively in powers of $\epsilon = 4 - d$.

AT PRESENT: all physical systems studied have LGW transitions
CFPs have been identified in Abelian-Higgs models for $N_f \geq 10$.
Little evidence of CFPs in non-Abelian systems. A candidate has been found in a system with $N_f = 40$ flavors, SU(N_f) global symmetry and SU(2) gauge invariance (it does not exist for $N_f = 20$).

The compact Abelian-Higgs model



Scalar matter: scalar field $\boldsymbol{z}$ with N_f components and $|z| = 1$.
$U(1)$ gauge fields with standard Wilson action.
$SU(N_f)$ global invariance.
κ is the plaquette coupling $S = \kappa \Pi_{x,\mu\nu}$,
J is the matter field coupling $S = J\boldsymbol{z}_x \boldsymbol{z}_{x+\mu} U_{x,\mu}$.

Single transition line, independent of κ (κ is irrelevant).
It is a standard order-disorder transition. For $N_f = 2$, the order parameter is the Neél order parameter of the antiferromagnet, $\bar{\boldsymbol{z}}_x \sigma \boldsymbol{z}_x$. For generic N_f, we should use the $SU(N_f)$ adjoint (gauge-invariant) combination $\bar{z}^a z^b - \delta^{ab}/N_f$.

The compact Abelian-Higgs field theory: LGW approach

Field: a hermitian traceless scalar field ϕ^{ab} that is a coarse-grained representation of the order parameter $\bar{z}^a z^b - \delta^{ab}/N_f$.
Lagrangian: the most general up to four powers of the field.

$$L = \frac{1}{2}\mathrm{Tr}\, \partial_\mu \phi \partial_\mu \phi + \frac{r}{2}\mathrm{Tr}\, \phi^2 + g_3 \mathrm{Tr}\, \phi^3 + g_{4,1} \mathrm{Tr}\, \phi^4 + g_{4,2} (\mathrm{Tr}\, \phi^2)^2$$

Because of the presence of a ϕ^3 term we predict a first-order transition, independently of N_f.

$N_f = 2$ is peculiar, simce $\mathrm{Tr}\, \phi^3 = 0$.
$\mathrm{Tr}\, \phi = 0 = \lambda_1 + \lambda_2$, implies $\lambda_2 = -\lambda_1$.
$\mathrm{Tr}\, \phi^3 = \lambda_1^3 + \lambda_2^3 = 0$

Moreover, if we write $\phi = \psi \cdot \sigma$, we can rewrite L as the LGW for an O(3) real field ψ. For $N_f = 2$ we predict O(3) behavior.

The compact Abelian-Higgs model: numerical results

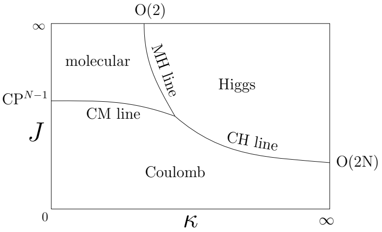
Numerical simulations:
1) O(3) behavior for $N_f = 2$
2) first-order transition for $N_f \geq 3$, with a latent heat $E_{HT} - E_{LT}$ and a surface tension ($e^{\beta\sigma L^2}$ is the height of the barrier between the two phases) that increase with N_f.

A first-order transition is predicted in the limit $N_f \to \infty$.
Textbook results for $N_f \to \infty$: Continuum and lattice calculations predict a continuous transition, *under the usual technical assumptions*.

Which technical assumption fails?
Analytic calculations consider a saddle point with $U_{x,\mu} = 1$ modulo gauge transformations. This is correct in the ordered phase, but not in the disordered phase, where monopole configurations are also relevant for $N_f \to \infty$.

We have verified that $U_{x,\mu}$ does not order for $N_f \to \infty$ in the (matter) disordered phase (J small).

The noncompact Abelian-Higgs model



The gauge field is a real field $A_{x,\mu}$ with gauge action $\frac{\kappa}{2}(\nabla_\mu A_{x,\nu} - \nabla_\nu A_{x,\mu})^2$ on each plaquette [∇_μ is the lattice derivative].
Scalar matter and interactions: as before with $U_{x,\mu} = \exp(iA_{x,\mu})$.
Gauge group: (noncompact) $\mathbb{R}$ instead of (compact) $U(1)$.

There is a new transition line (line MH) that divides the (matter) ordered phase (large-J phase).

There are two different large-J phases: "molecular" (compact/noncompact does not play any role), "Higgs" (peculiar of the noncompact model).

MH line

For $J \to \infty$ we should maximize the matter-field interaction on each link $S \sim J\bar{\mathbf{z}}_x \cdot \mathbf{z}_{x+\mu} U_{x,\mu}$.
It implies $z_x = U_{xy} z_y$ for nearest neighbors xy.
If $x \to z \to y$ (z is a n.n. of x, y is a n.n. of z:

$$z_x = U_{xz} z_z = U_{xz} U_{zy} z_y = [\prod_{\text{path } x \to y} U_p]\, z_y$$

This result can be generalized to any lattice path. In particular to a plaquette: $z_x = \Pi_x z_x$. Therefore, for $J \to \infty$, we have $\Pi_x = 1$. Modulo gauge transformations, this implies $U_{x,\mu} = 1$, which implies

$$A_{x,\mu} = 2\pi n_{x,\mu} \qquad n_{x,\mu} \in \mathbb{Z}$$

For $J \to \infty$ we have a discrete gauge theory with gauge (n.c.) group $\mathbb{Z}$.
Duality can be used to prove that the model has a topological transition in the XY/O(2) universality class.

MH line

Interaction: $2\pi^2\kappa\sum_P(\nabla_\mu n_{x,\nu} - \nabla_\nu n_{x,\mu})^2$
Two phases:
1) small κ; fields are disordered and $n_{x,\nu}$ fluctuate wildly.
2) large κ; gauge fields are ordered. Modulo gauge transformations we have $n_{x,\mu} = 0$ at most of the lattice sites.

For κ small and any J the model is equivalent to the compact one (one can imagine the compact one as a model with fields $A_{x,\mu}$ in which changes by $2\pi n_{x,\mu}$ are unconstrained).
For large J and large κ, we have $A_{x,\mu} = 0$ at most of the lattice sites (periodicity is completely lost). This phase is only possible in the noncompact model.
If one is looking at new physics (DQC), one should consider the CH line.

Numerical results

We have studied the CM and the CH line.

CM line: as in the compact model, we have $O(3)$ continuous transitions for $N_f = 2$, first-order transitions for $N_f \geq 3$.

CH line: first-order transition for $N_f = 2, 4$, continuous transitions for $N_f = 10, 15, 25$. These transitions are not LGW transitions. Are they associated with charged fixed points?

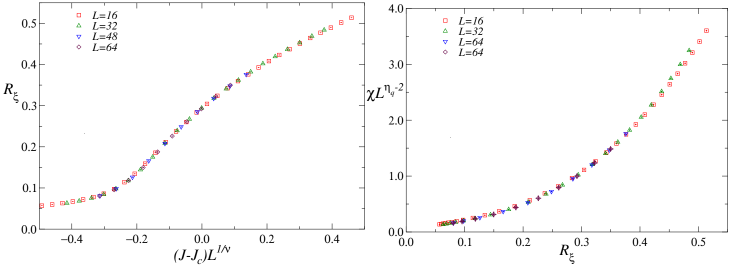
Results from the Abelian-Higgs field theory in ϵ expansion [$\epsilon = 4 - d$].
A CFP exists for $N_f > N_f^*$, with (at 4 loops)

$$N_f^* = 183 \left[1 - 1.752\,\epsilon + 0.789\,\epsilon^2 + 0.362\,\epsilon^3 + O(\epsilon^4) \right].$$

This expansion does not look to be very predictive for $d = 3$, i.e., $\epsilon = 1$.
Anyway it indicates that a CFP exists for N_f large enough!

Numerical results

Just to give an example of the quality of the numerical results. Here $N_f = 25$.



The transition is clearly continuous. We estimate $\nu = 0.802(8)$, $\eta_q = 0.883(7)$.

Numerical results

How can we be sure that the transitions we have found are those predicted by the continuum Abelian-Higgs field theory?

Large-N_f expressions:

$$\nu = 1 - \frac{48}{\pi^2 N_f} + O(N_f^{-2}) \qquad \eta_q = 1 - \frac{32}{\pi^2 N_f} + O(N_f^{-2}),$$

N_f	ν	ν_{ln}	η_q	$\eta_{q,ln}$
25	0.802(8)	0.805	0.883(7)	0.870
15	0.721(3)	0.676	0.815(10)	0.784
10	0.64(2)	0.514	0.74(2)	0.678

Comparison: quite good agreement for $N_f = 25$, larger differences for $N_f = 15, 10$ that can be explained in terms of $1/N_f^2$ corrections

The conjecture that the transition is controlled by the field-theory CFP is supported by the numerical results

A CFP exists for $N_f > N_f^*$ with $4 < N_f^* < 10$.

Unfortunately the CFP is not relevant for the physical case $N_f = 2$.

Charge-Q compact model

$$S = J \sum_{\langle xy \rangle} \mathrm{Re}\, \bar{z}_x \cdot z_y U_{xy}^Q + \kappa \sum_P \mathrm{Re}\, \Pi_P .$$

The matter fields belong to the charge-Q rep. of $U(1)$.
We can rewrite the model as follows. Define

$$U_{xy} = \exp\left(\frac{iA_{xy}}{Q}\right) \qquad V_{xy} = e^{iA_{xy}} \qquad -\pi Q \le A_{xy} \le \pi Q$$

The action becomes

$$S = J \sum_{\langle xy \rangle} \mathrm{Re}\, \bar{z}_x \cdot z_y V_{xy} + \kappa \sum_{P(\mu,\nu)} \cos\left[\frac{1}{Q}(\nabla_\mu A_{x\nu} - \nabla_\nu A_{x\mu})^2\right]$$

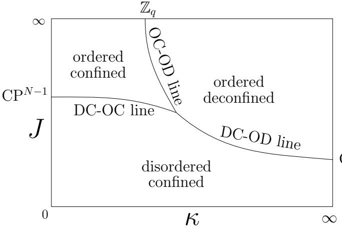
It interpolates between the compact ($Q = 1$) and the n.c. model $Q \to \infty$.

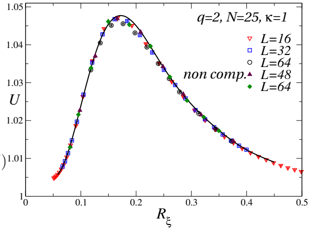
For $J \to \infty$ we again obtain a discrete gauge theory. Since $V_{x,\mu} = 1$ modulo gauge transformations

$$A_{x\mu} = 2\pi n_{x,\mu} \qquad n_{x,\mu} \in \mathbb{Z}_Q$$

$\mathbb{Z}_Q$: additive group of integers modulo Q.

Charge-Q compact model





Same phase diagram as in the n.c. case.
Along the analogue of the CH line (DC-OD line) we find a transition controlled by the field theory CFP for N_f large (verified for $N_f = 25$).

Conclusions

We have studied three models:
1) compact AH model
2) noncompact AH model
3) compact charge-Q AH model (it interpolates between 1 and 2)

For $N_f = 2$ we only find a LGW FP [$O(3)$ behavior] or first-order behavior.
Ongoing work: this is is still true if we replace the U(1) gauge group with the subgroup $\mathbb{Z}_p$ with $p \geq 3$. This is agreement with the LGW hypothesis on the irrelevance of the gauge group.

Transitions controlled by the field-theory CFP are observed for $N_f > N_f^*$, with $N_f^* = 5$-9.
A CFP transition requires the presence of a topological transition that kills some of the low-energy topological excitations (monopoles) that are present in the compact model.
The ϵ expansion is not predictive for N_f^*. In 4D we get $N_f^* = 183$, to be compared with the single-digit result in 3D.

Conclusions

What does it happen with fermions?

In 4D, the SU(3) gauge theory with N_f Dirac fermions has a CFP for $N_f > N_f^* = 33/2 = 16.5$.

Speculations. What is the critical value in 3D? If a CFP exists for physically interesting values of N_f, we should revisit our predictions for the finite-temperature transition of QCD.

Pisarski and Wilczek assume that the relevant fixed point is a LGW fixed point, while instead the physics may be controlled by a CFP.