Phase Transitions in Particle Physics
Mini Workshop
Galileo Galilei Institute, Firenze – Spring 2022

Interfaces near criticality: results from field theory

Marianna Sorba
SISSA, Trieste

ma per seguir virtute e canoscenza
SISSA

Interfaces

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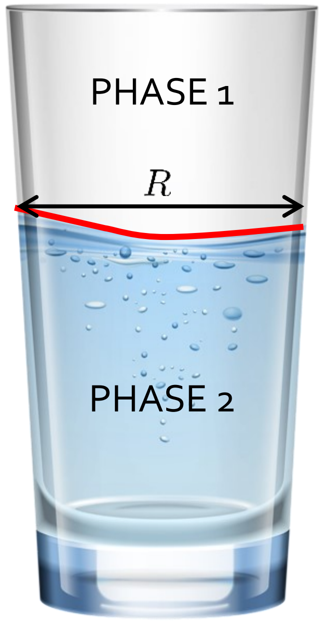

- Discrete internal symmetry
- $T < T_c$
- Suitable boundary conditions on the bulk (homogeneous) system
- Interface linear size $R \gg \xi$

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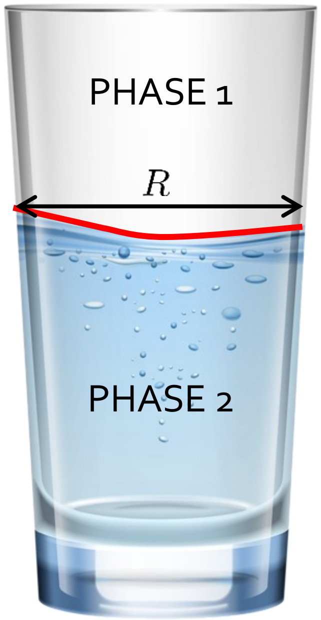

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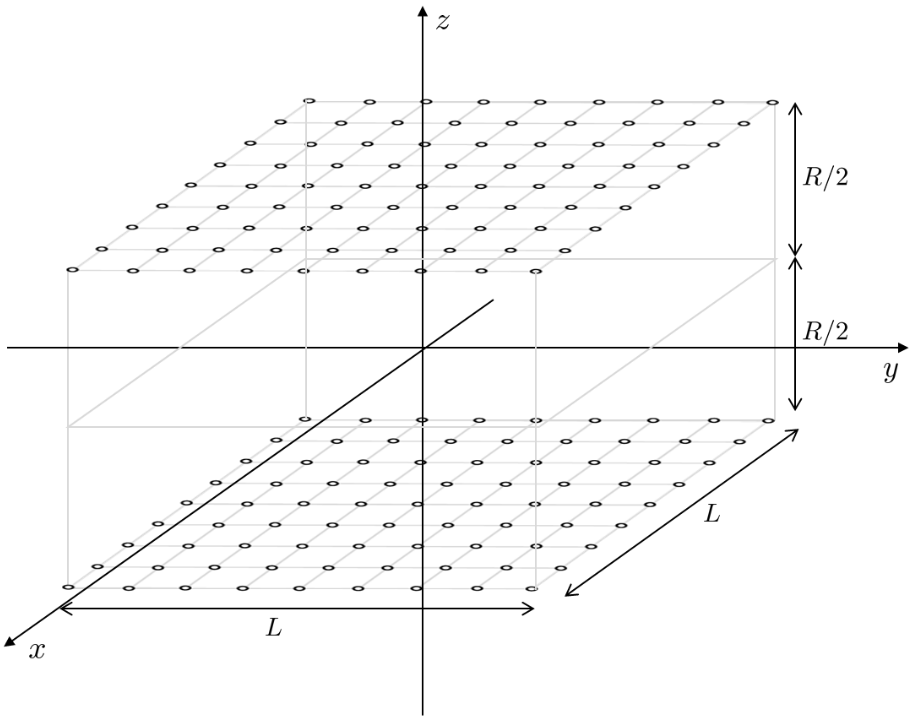
Simplest implementation:
three-dimensional Ising model

Ising model

$$\mathcal{H} = -\frac{1}{T} \sum_{\langle i,j \rangle} s_i s_j \,, \quad s_i = \pm 1$$ on a cubic lattice $(R \gg \xi,\, L \to \infty)$



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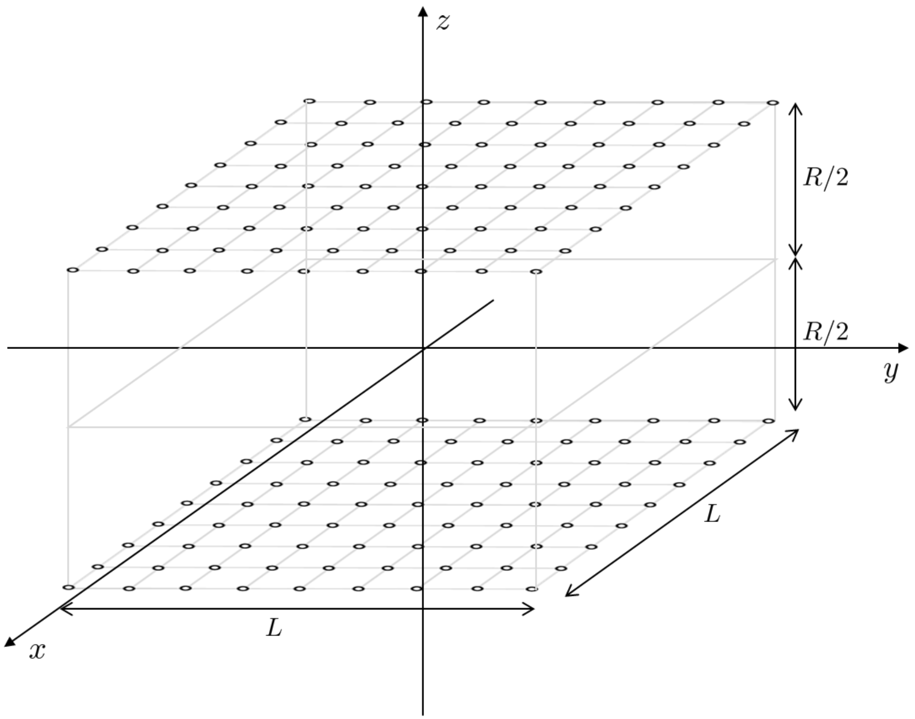

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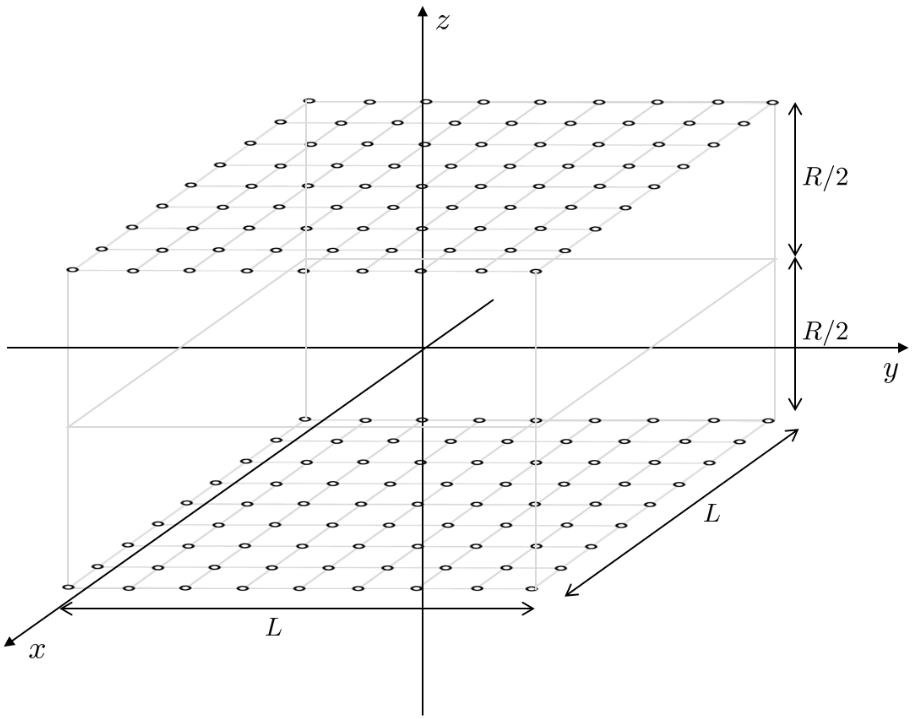
Construct a non-effective theory:

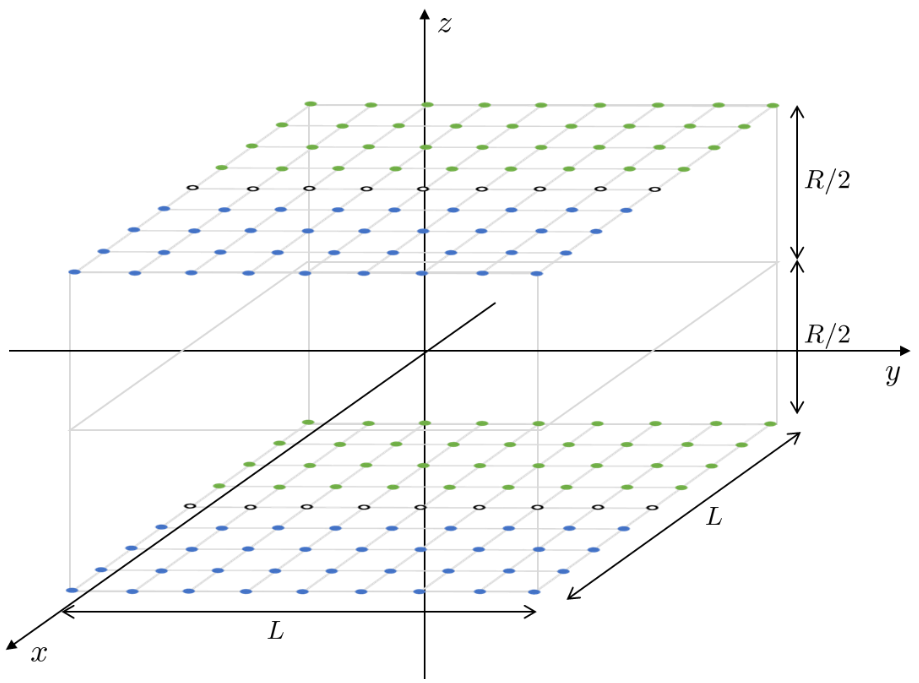
study interfaces analytically at fundamental level, starting from the bulk field theory.



Standard interface setting

Boundary conditions at $z = \pm R/2$:

$$s_i = \begin{cases} +1 & x < 0 \\ -1 & x > 0 \end{cases}$$



G. Delfino, W. Selke and A. Squarcini, Nucl. Phys. B 958 (2020) 115139.

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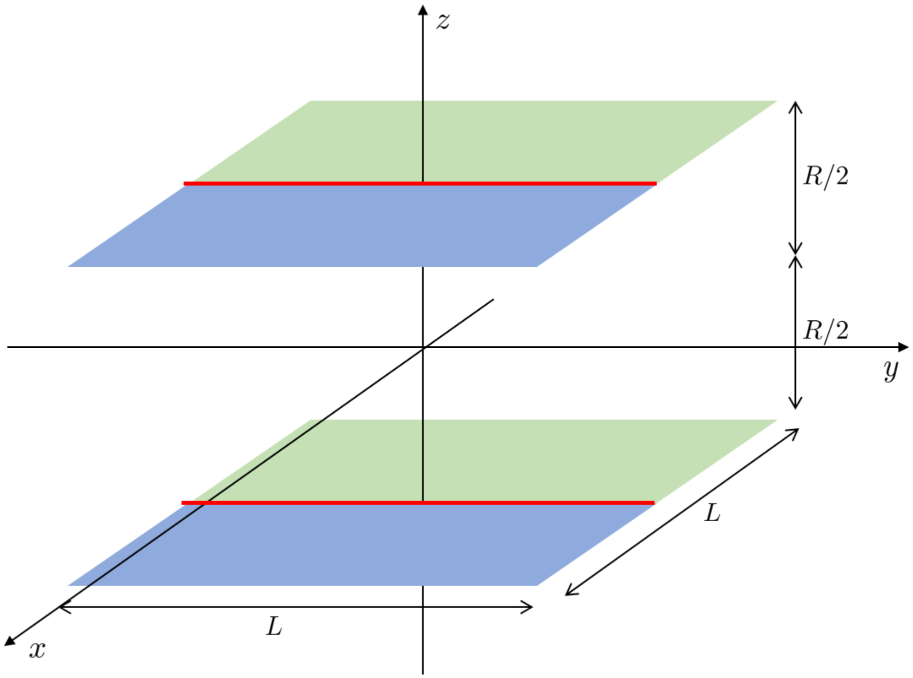

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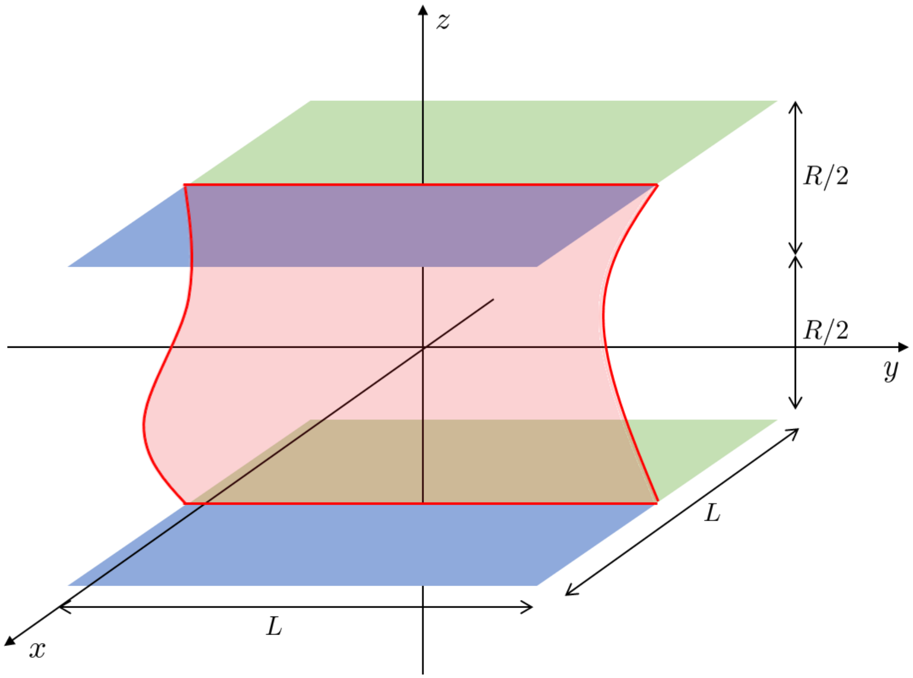
Interface running between the pinning axes $x = 0,\ z = \pm R/2$.



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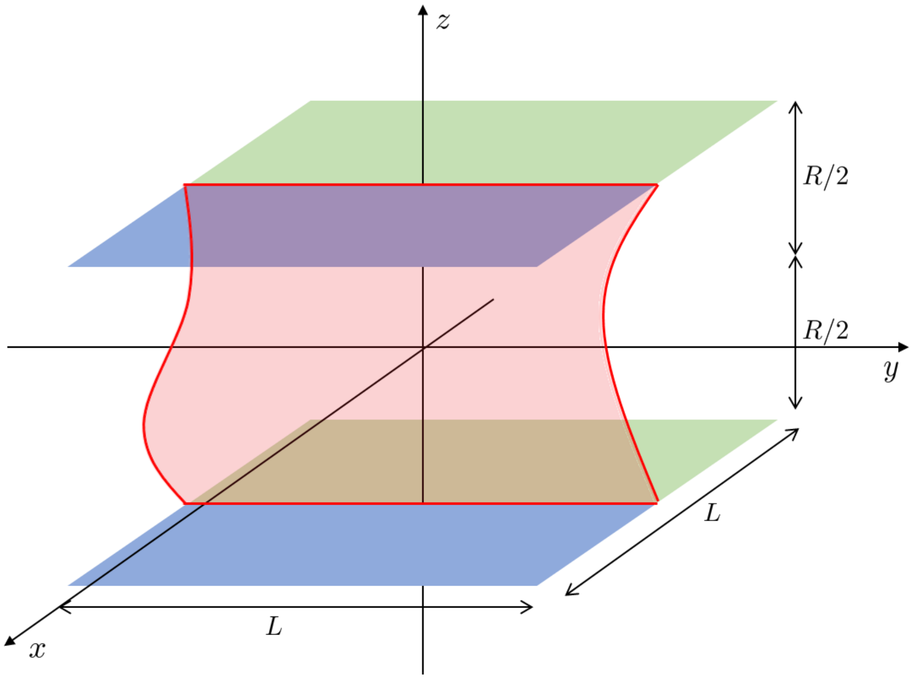

Analytic predictions about observables (e.g. $\langle s(r) \rangle_{+-}$) starting from first principles ?

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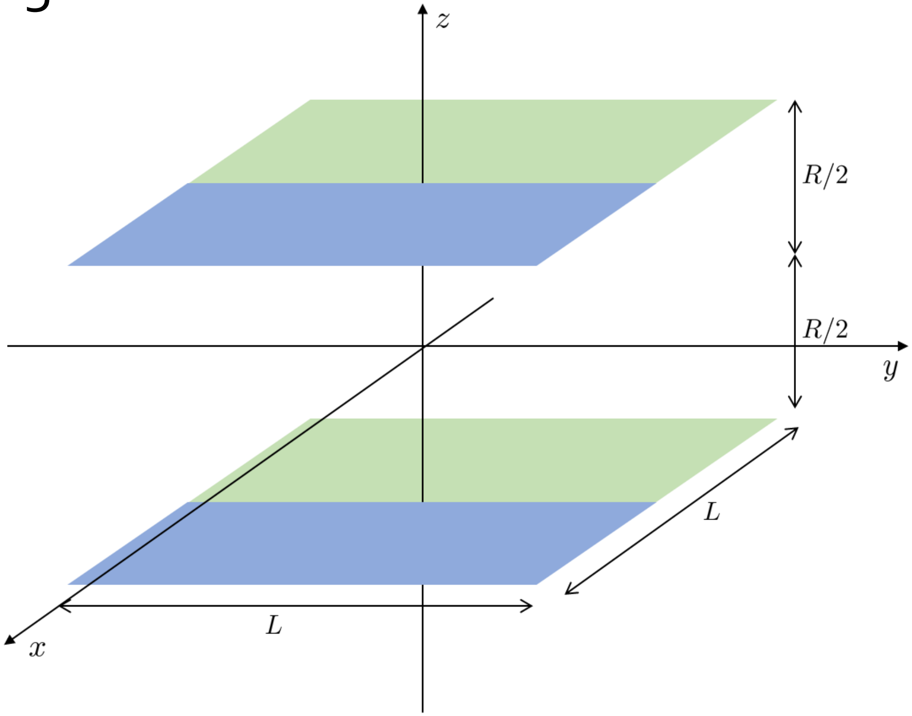

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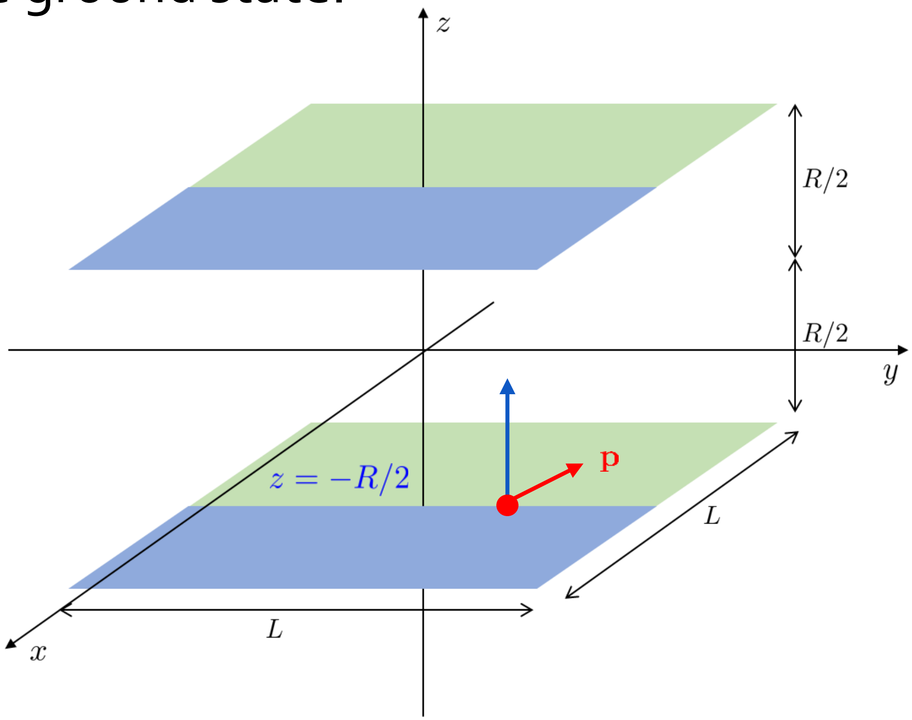
lightest particle



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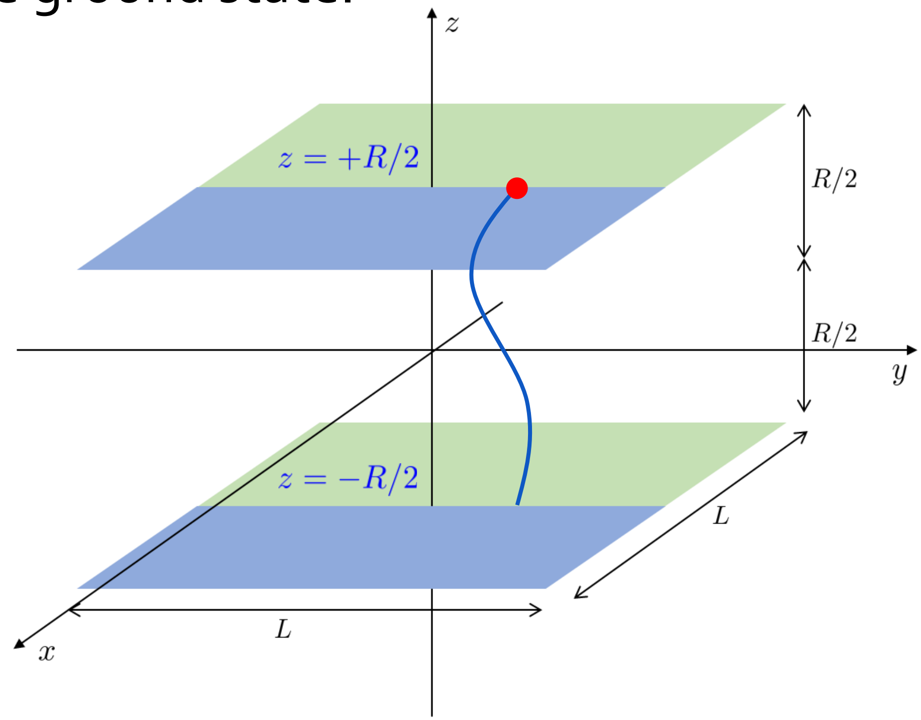

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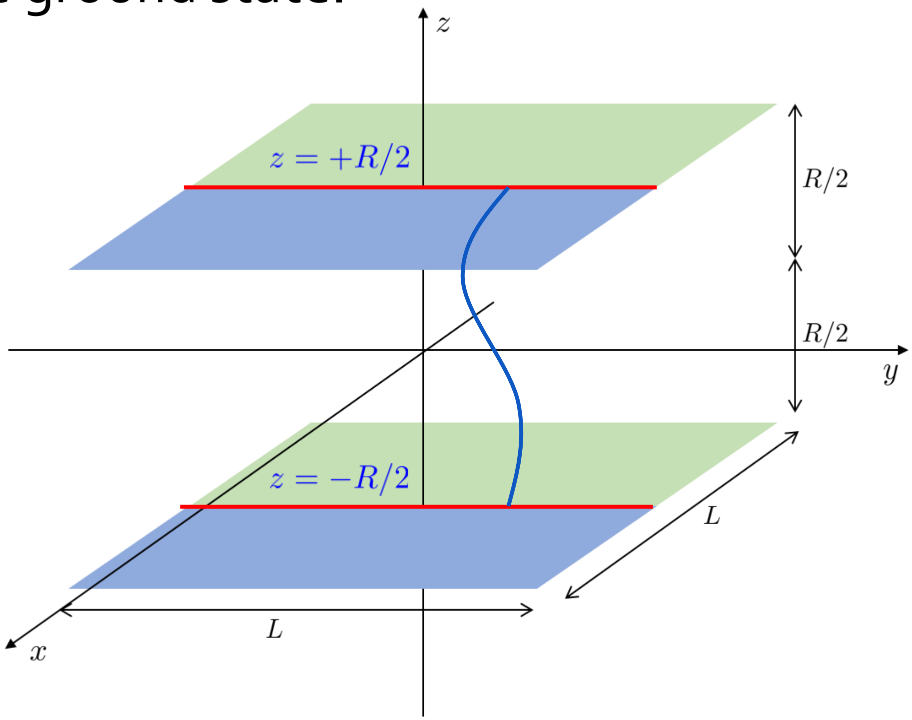

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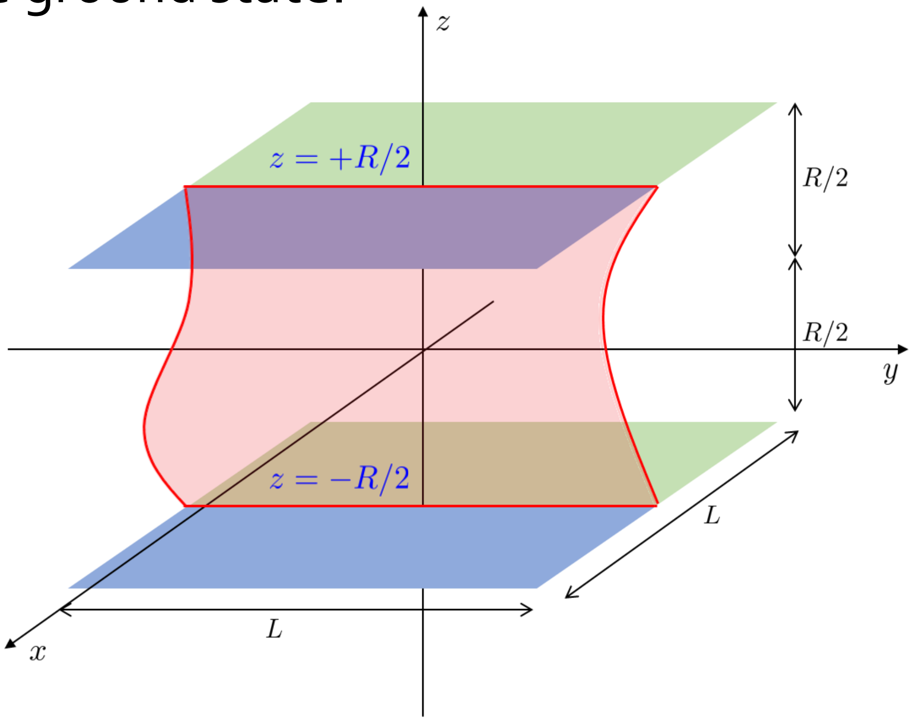

The interface is generated by the imaginary time propagation of an excitation (**string**) made of $N \propto L \to \infty$ particles.

Boundary states

Boundary states $|B(\pm R/2)\rangle$ specify that particles are emitted/absorbed at $z = \pm R/2$.

Expansion over the basis of **asymptotic particle states**:

$\{|\mathbf{p}_1, ..., \mathbf{p}_N\rangle\}$ with eigenvalues $\sum_{i=1}^{N} E_{\mathbf{p}_i}, \sum_{i=1}^{N} \mathbf{p}_i$

normalization $\langle \mathbf{p}|\mathbf{q}\rangle = (2\pi)^2\, E_{\mathbf{p}}\, \delta(\mathbf{p} - \mathbf{q})$

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$$|B(\pm R/2)\rangle = e^{\pm\frac{R}{2}H}\,|B(0)\rangle$$

$$= \frac{1}{\sqrt{N!}}\int\prod_{i=1}^N \frac{d\mathbf{p}_i}{(2\pi)^2 E_{\mathbf{p}_i}} f(\mathbf{p}_1,...,\mathbf{p}_N) e^{\pm\frac{R}{2}\sum\limits_{i=1}^N E_{\mathbf{p}_i}}\,\delta\left(\sum_{i=1}^N p_{y,i}\right)|\mathbf{p}_1,...,\mathbf{p}_N\rangle + ...$$

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emission/absorption amplitude

translation invariance in y-direction

Partition function

$$Z_{+-} = \langle B(R/2) | B(-R/2) \rangle$$
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Monte Carlo estimate $\kappa = 0.1084(11)$

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widely separated particles

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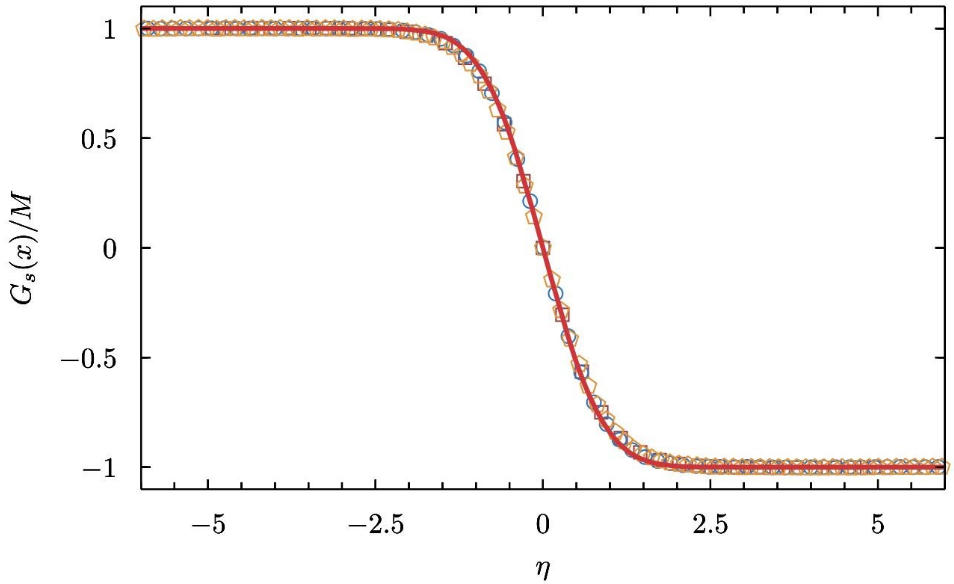

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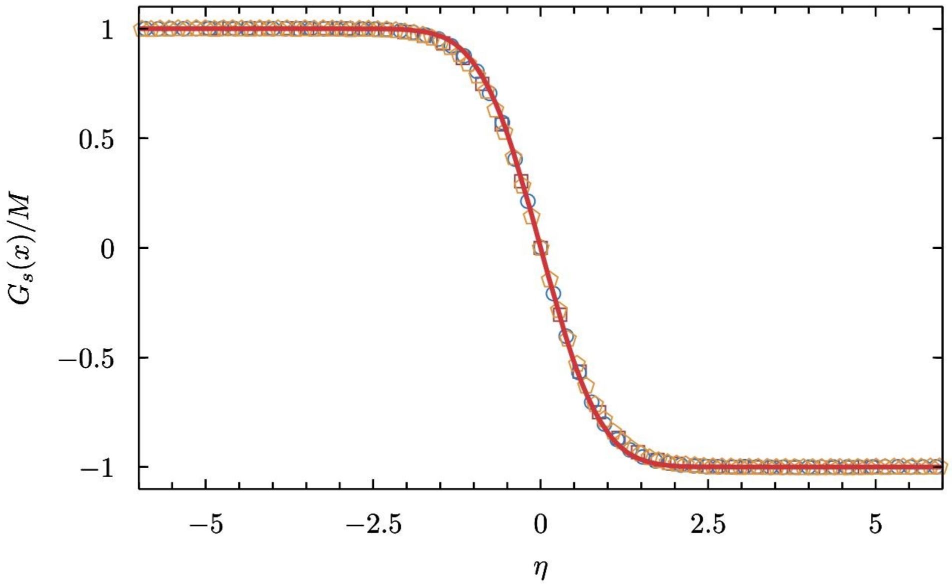

$$\xi \simeq \xi_0\,(T_c - T)^{-\nu}$$

$$\xi_0 \simeq 0.668$$
$$\nu = 0.6310(15)$$
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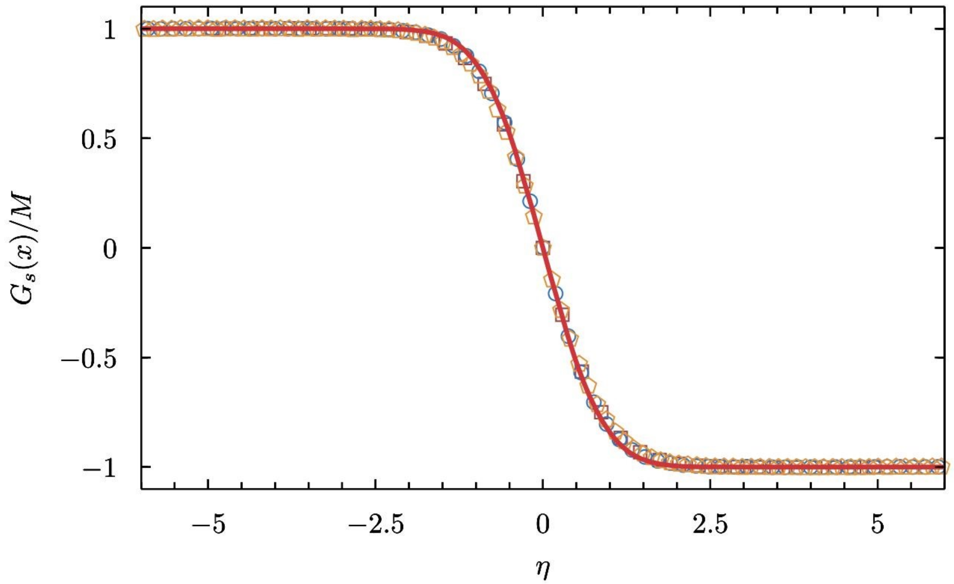

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First analytical determination and numerical confirmation of the profile.

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Interface in presence of a wall

Boundary conditions on the wall $x = 0$:

$$s_i = \begin{cases} +1 & |z| < R/2 \\ -1 & |z| > R/2 \end{cases}$$

$\Downarrow$

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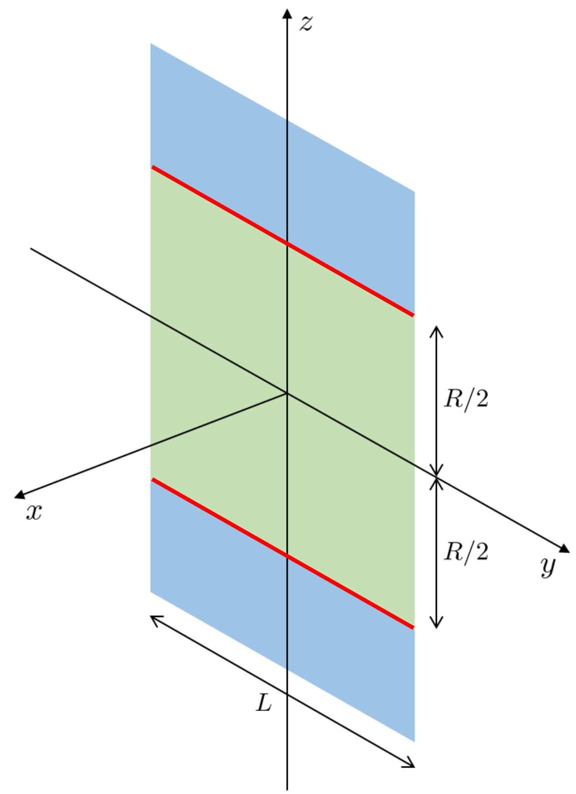

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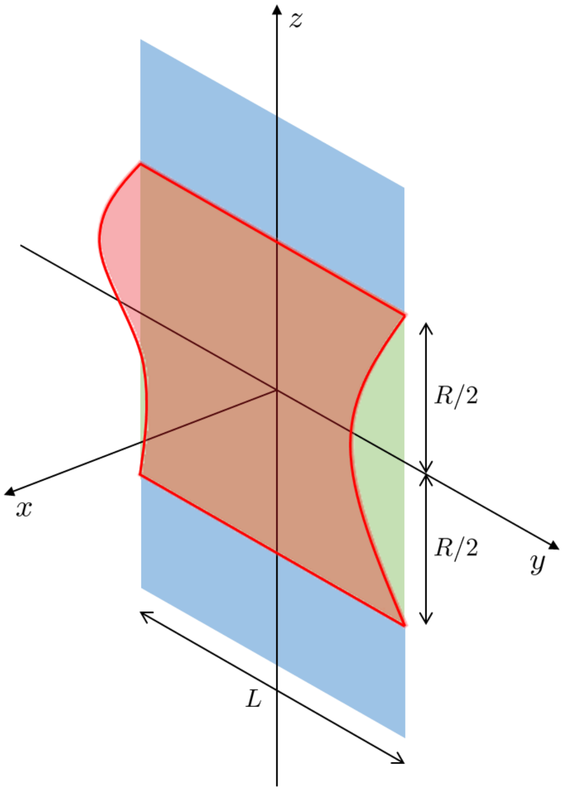

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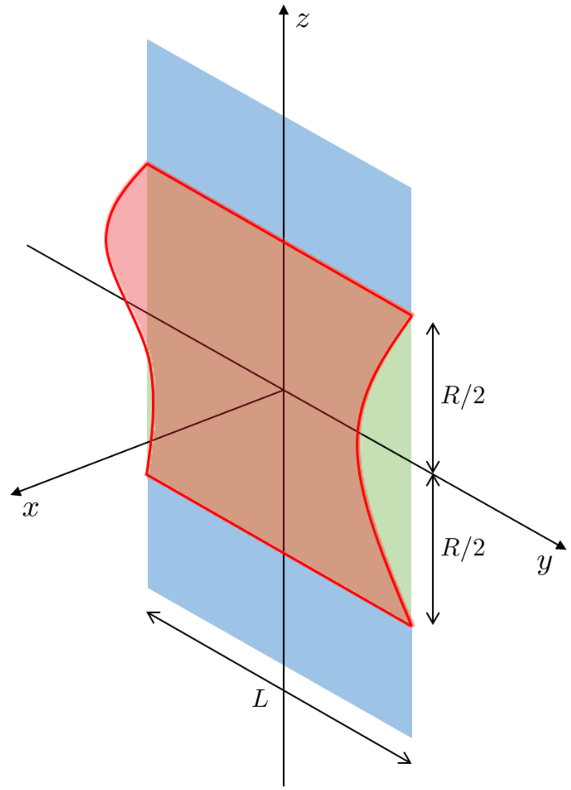

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Field theoretical description ?

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None of the particles can stay along the **impenetrable wall** $x = 0$ (i.e. $f = 0$ when $\exists p_{x,i} = 0$).

$$Z_{+-} \sim \frac{L\,|f_0|^2\,e^{-RNm}}{(2\pi)^{2(N+1)}} \left(\frac{2\pi}{R^2}\right)^N \sqrt{\frac{2\pi R}{Nm}}$$

Interfacial tension:

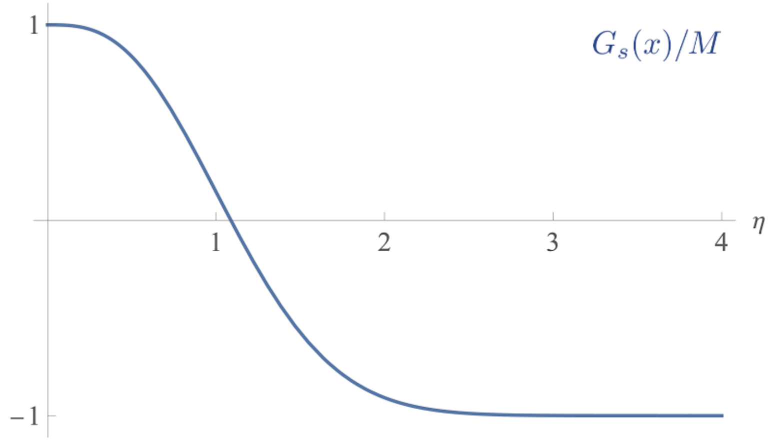
$$\sigma = -\lim_{R\to\infty} \frac{1}{LR} \ln Z_{+-} = \frac{\kappa}{\xi^2}$$

same as in absence of the wall

Order parameter profile

$$G_s(x) \sim M + 2M \left\{ \frac{2}{\sqrt{\pi}}\,\eta\, e^{-\eta^2} - \mathrm{erf}(\eta) \right\}$$

$$\eta = \sqrt{\frac{2}{R\xi}}\, x$$



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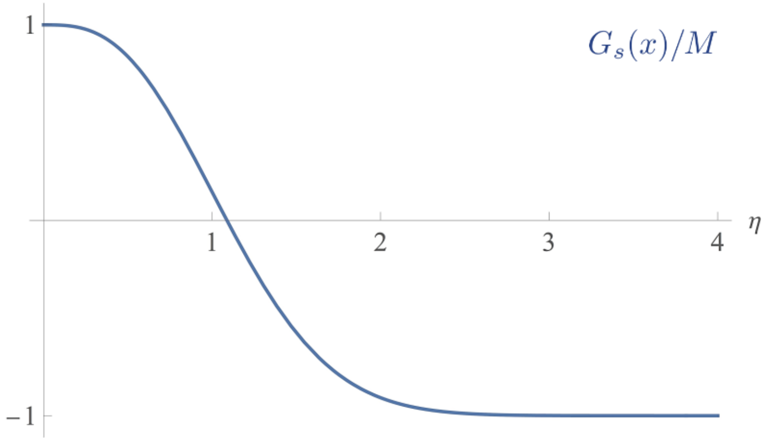

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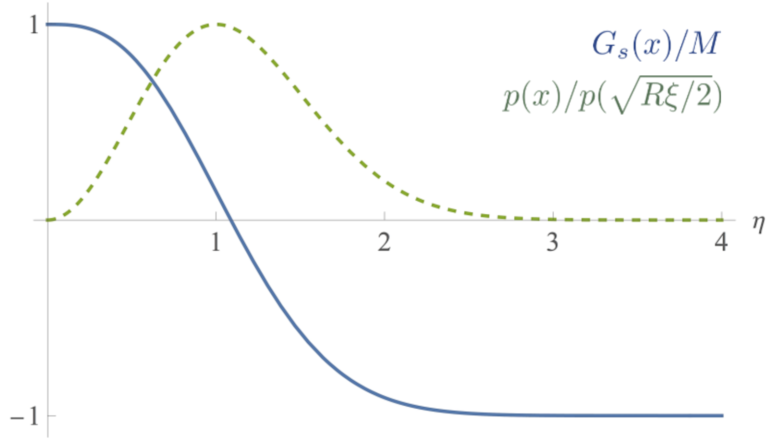

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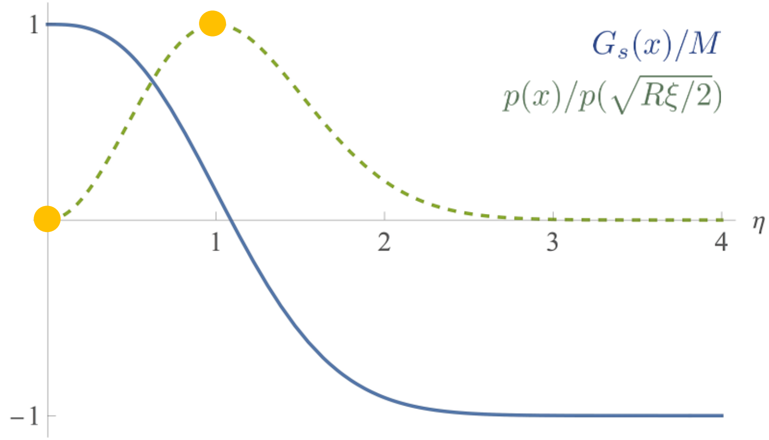

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- Correct normalization
- Impenetrability of the wall
- Average distance $x \propto \sqrt{R}$

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The wall at $x = 0$ contributes to the theory with a boundary Hamiltonian $h \int dydz\, \Phi_B(0, y, z)$.
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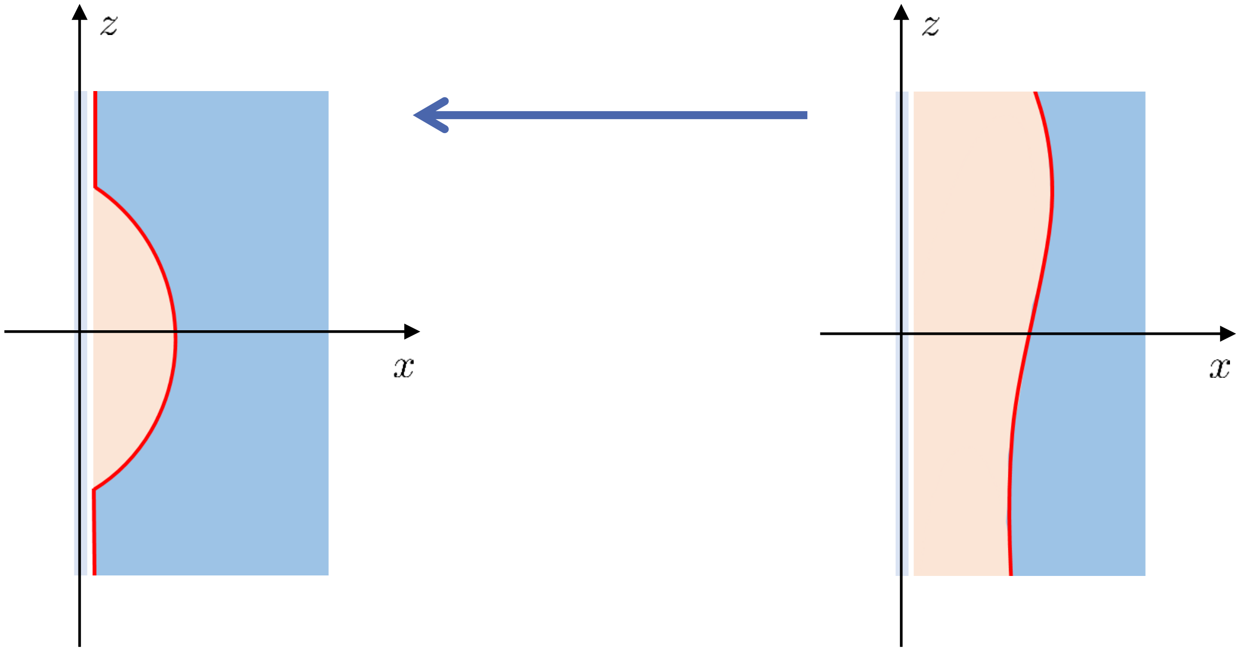

Binding regime

Fluctuating regime

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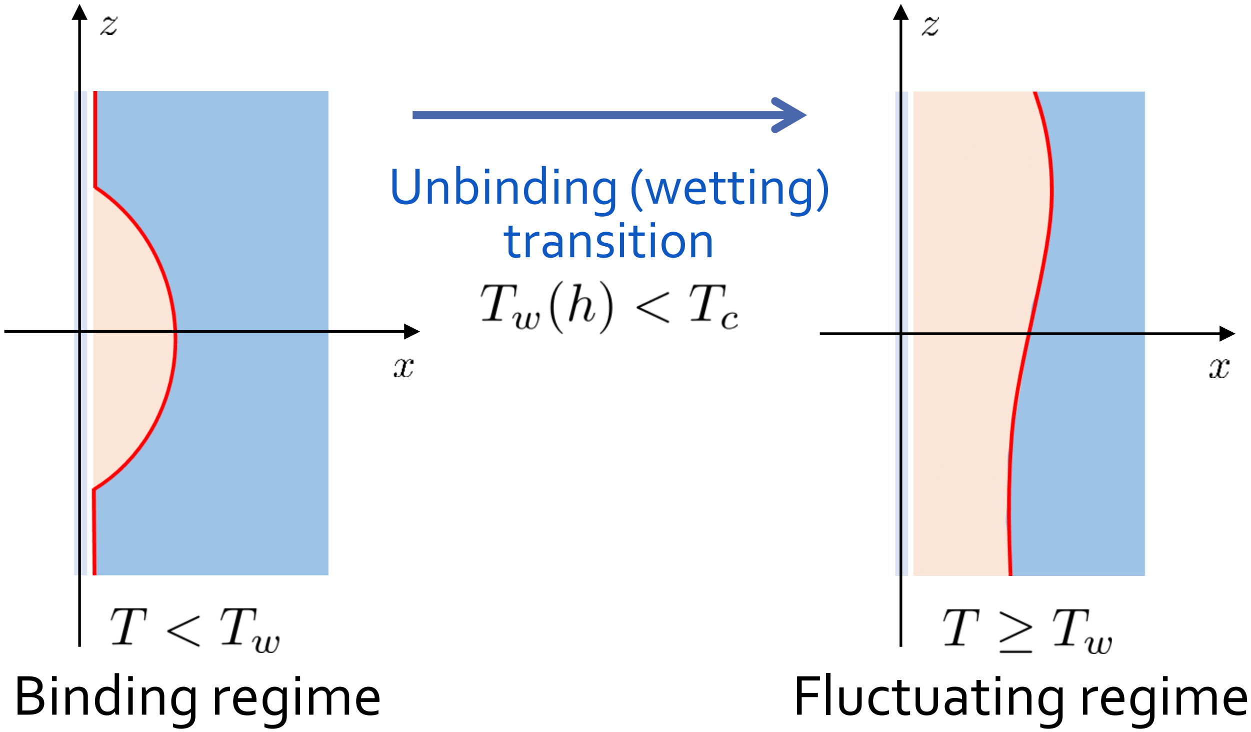

Scattering theory

Interaction of a particle with the wall:

- Rapidity parametrization $\begin{cases} E = m\cosh\beta \\ |\mathbf{p}| = m\sinh\beta \end{cases}$
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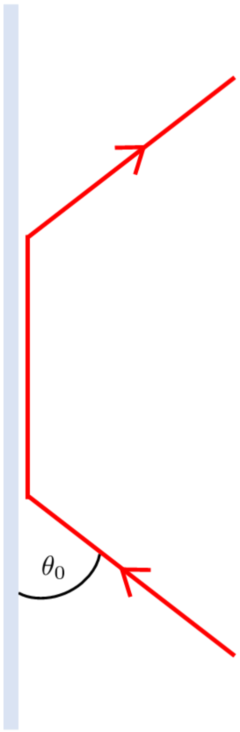

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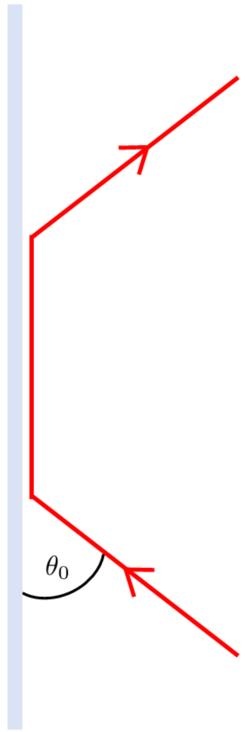

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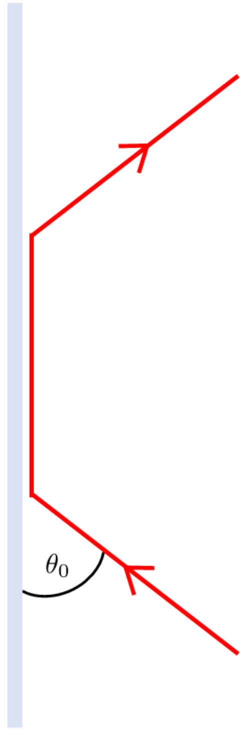

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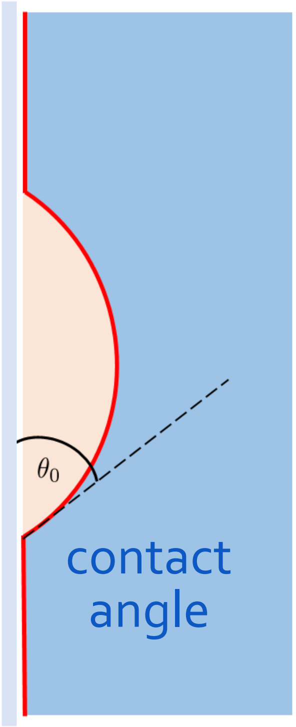

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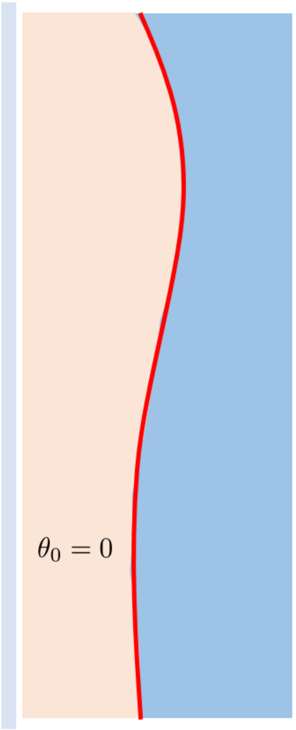

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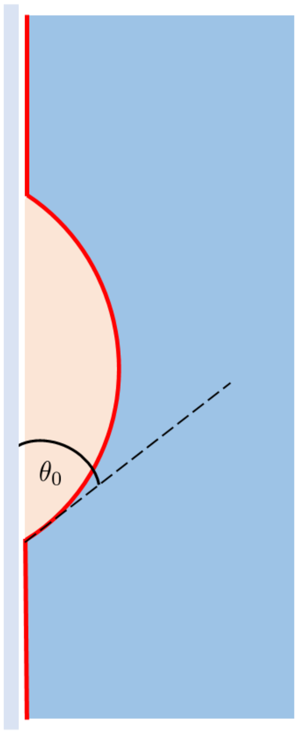

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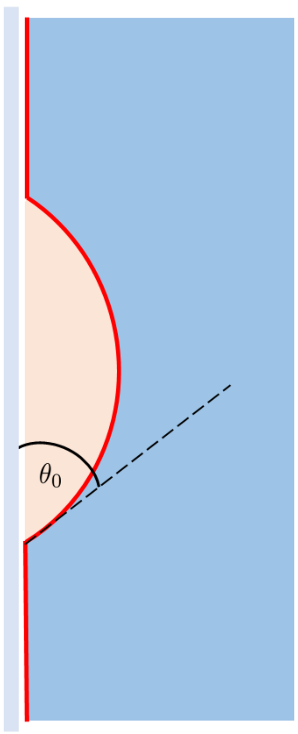

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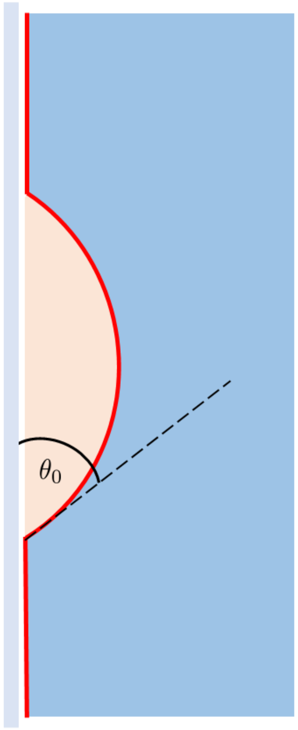

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in agreement with Monte Carlo simulations.

K. Binder, D.P. Landau and D.M. Kroll, Phys. Rev. Lett. 56 (1986) 2272.

Summary

- Formulation of a field theory of interfaces near criticality, giving analytic universal predictions with no need of free parameters.

- Interfacial tension, magnetization profile, passage probability density from the definition of the partition function.

- Implementing in momentum space the presence of an impenetrable wall.

- Binding transition and its key features using the particle description of the bulk field theory.

Thank you
for the attention !

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