Nonperturbative IR finiteness in super-renormalisable scalar quantum field theory.

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Paper

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**Nonperturbative Infrared Finiteness
in a Superrenormalizable Scalar Quantum Field Theory**

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Broader **LatCos collaboration** including Valentin Nourry, Lizzie Dobson.

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Inflation

Inflation Solves

- The Horizon Problem
- The Flatness Problem
- The formation of struture in the universe

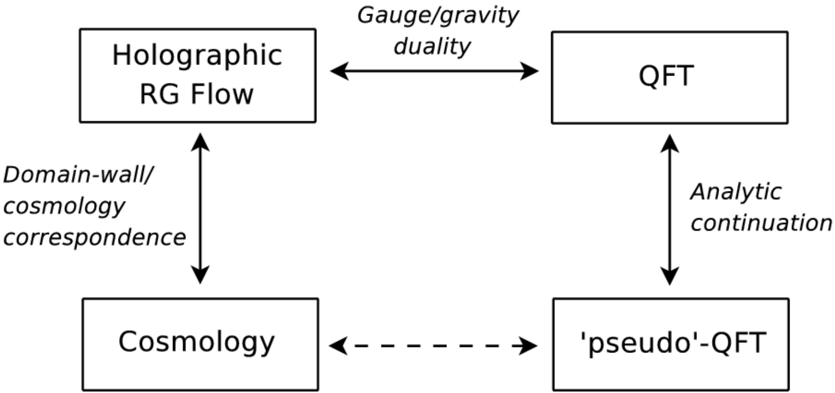
Testable

$$\Delta_R^2(q) = \Delta_0^2 \left(\frac{q}{q_*}\right)^{n_s - 1} \quad (\Lambda_{CDM})$$

Problems

- Fine-tuning of the inflation potential/inflaton
- Requires specific initial conditions
- Is an Effective field theory, leaving the initial singularity as an unresolved problem

- A full **Quantum Gravity** description of the early universe through a Holographic dual QFT in 3-dimensions
- Built upon the **dS/CFT correspondence** (Strominger, 2001; Maldacena, 2003)



The Holographic Dual QFT

Action

$$S = \frac{1}{g_{YM}^2} \int d^3x \, \mathrm{Tr} \left[(D\phi)^2 + \frac{1}{2} F_{ij} F^{ij} + \lambda \phi^4 + \bar{\psi} D_i \gamma^i \psi + \mu \bar{\psi} \psi \phi \right]$$

- The fields ϕ are $SU(N)$-valued
- In 3-dimensions, with a **dimensional coupling**, $[g_{YM}^2] = 1$.
- See (McFadden and Skenderis, 2010) for more

Promote g_{YM} to a field with appropriate conformal transformation properties $\rightarrow$ CFT (**generalalized conformal invariance**).

IR Behavior

- ▷ Time in the bulk is mapped to **Inverse RG-Flow** in the dual
- ▷ Therefore the IR of the dual corresponds to the **initial singularity**

Holographic Dictionary

EMT two-point functions

Through symmetry arguments have

$$\langle T_{ij}(\bar{q})T_{kl}(-\bar{q})\rangle = A(\bar{q})\Pi_{ijkl} + B(\bar{q})\pi_{ij}\pi_{kl}$$

We can relate A and B to the **scalar** and **tensor power spectra** of the CMB (Δ_s and Δ_t respectively)

Holographic Dictionary

$$\Delta^2_S(q) = \frac{-q^3}{16\pi^2 ImB(-iq)}, \qquad \Delta^2_T(q) = \frac{-2q^3}{\pi^2 ImA(-iq)}$$

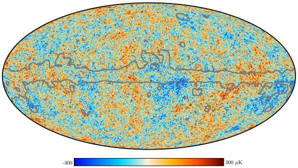


Figure: The Cosmic Microwave Background as observed by Planck (Akrami et al., 2018)

Perturbation Theory in the dual

- Corrections to the mass ($m^2\phi^2$-term) from renormalization.
- Theory IR-finite at one-loop order

Two-loop

$$\delta_{m^2} \propto g^2 D(p)\mathcal{N}(N)$$

$$D(p) = \int_{-\pi/a}^{\pi/a} \frac{d^3k}{(2\pi)^3}\frac{d^3q}{(2\pi)^3}\frac{1}{k^2\,q^2\,(k+p+q)^2} \xrightarrow{p\to 0} D_{\mathrm{IR}}(p) = -\frac{\log(|pa|)}{(4\pi)^2}$$

- At higher loop order there are polynomial IR divergences

A Finiteness Conjecture

Divergences are an **artifact** of perturbation theory (Jackiw and Templeton, 1981; Appelquist and Pisarski, 1981)
g takes the role of an **IR-regulator** in the log. (e.g. $D(p) \propto \log(g)$ as $p \to 0$). Accounts for higher-loops also!

A first test of holographic cosmology

- Data suggest **no fermions** in theory just scalars and gauge fields (Afshordi et al., 2017)
- Holographic Cosmology fits data with $l \geq 30$ as well as Λ_{CDM} (perturbative regime)
- Λ_{CDM} tensions at **low multi-poles**.
- To fully test the model need to go into the non-perturbative regime

Need a for a **lattice simulation**.

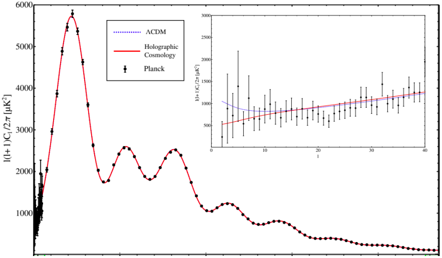


Figure: CMB power spectrum, with Λ_{CDM} and Holographic cosmology fits.

Fitting functions

$$\Delta_R^2(q) = \Delta_0^2 \left(\frac{q}{q_*}\right)^{n_s - 1} \qquad (\Lambda_{CDM})$$

$$\Delta_R^2(q) = \frac{\Delta_0^2}{1 + \frac{gq_*}{q}\log\left|\frac{q}{\beta g q_*}\right|} (HC)$$

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Strategy

We start with a simple **pure-scalar** SU(N) theory. UV understood (see Laine, 1995) For $N = 2$ the model is equivalent to $O(3)$ and $N = 3$ in same universality as $O(8)$ (see Campostrini et al., 2002; Pelissetto and Vicari, 2015; Hasenbusch, 2022 for example for $O(N)$.)

Scalar Lattice Action

$$S = \frac{N}{g} \sum_{\mathrm{x} \in \Lambda} \mathrm{Tr} \left[(\partial_\mu \phi(\mathrm{x}))^2 + (m^2 - m_c^2) \phi(\mathrm{x})^2 + \phi(\mathrm{x})^4 \right]$$

where a **bare-mass** parameter has been added so that it can be tuned to sit at the phase-transition

Goals

- Test IR-finiteness of the theory
- Find the bare mass parameter value in the continuum limit

Ensembles

- Implemented with the GRID framework (github.com/paboyle/Grid)
- Generated on SKL Cluseters (STFC DiRAC CSD3, Cambridge and Iridis5, Southampton)
- $\mathcal{O}(10^5)$ configurations per ensemble

N	2, 4
g	0.1, 0.2, 0.3, 0.5, 0.6
L	8, 16, 24, 32, 48, 64, 96, 128
m^2	Many points in the vicinity of the transition

Open Source

- **Code** DOI: 10.5281/zenodo.4290508
- **Data** DOI: 10.5281/zenodo.4266114

Phase Transitions and the Binder Cumulant

We can tune the parameter m

Consider the zero-mode (EFT)

In $m \to \infty$ limit ground state is symmetric

In $m \to -\infty$ limit there is SSB

Therefore the theory contains a phase transition.

The holographic map can be used when the theory is at the critical mass point.

We therefore need something to quantify the phase transition:

Binder Cumulant

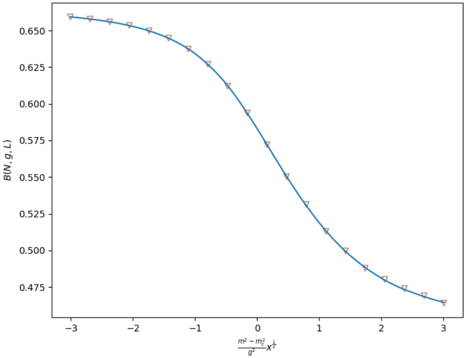
$$B = 1 - \frac{N}{3} \frac{\langle \mathrm{Tr}\left[M^4\right] \rangle}{\langle \mathrm{Tr}\left[M^2\right] \rangle^2}$$

Where M is the **Magnetisation**

Reweighting

We use **multi-histogram reweighting** which allows you to interpolate between simulated mass points (Ferrenberg and Swendsen, 1988)

$$
\begin{aligned}
\langle \mathcal{O}_m \rangle &= \frac{\int \mathcal{D}\phi e^{-S_m[\phi]} \mathcal{O}}{\int \mathcal{D}\phi e^{-S_m[\phi]}} \\
&= \frac{\int \mathcal{D}\phi e^{-(m^2 - m_0^2)\phi^2 + S'_{m_0}[\phi]} \mathcal{O}}{\int \mathcal{D}\phi e^{-(m^2 - m_0^2)\phi^2 + S'_{m_0}[\phi]}} \\
&= \frac{\langle e^{-(m^2 - m_0^2)\phi^2} \mathcal{O} \rangle_{m_0}}{\langle e^{-(m^2 - m_0^2)\phi^2} \rangle_{m_0}}
\end{aligned}
$$



$$
S_m = \frac{N}{g} \int d^3x \left[\mathrm{Tr} \left[(\partial_\mu \phi(x))^2 + (m^2 - m_c^2)\phi(x)^2 + \phi(x)^4 \right] \right]
$$

Scaling laws

The Binder Cumulant, like all critical quantities, follows a **Scaling Law**

Binder Cumulant Scaling Law

$$B = f\left(\frac{1}{g^2}\left(m^2(g,L) - m_c^2(g)\right)(gL)^{1/\nu}\right)$$
$$\implies \bar{B} = f\left(\frac{1}{g^2}\left(\overline{m}^2(\bar{B},g,L) - m_c^2(g)\right)(gL)^{1/\nu}\right)$$

We can Taylor expand f near the critical point. To first order this gives

General fit anzatz

$$\overline{m}^2(\bar{B},g,L) = m_c^2(g,L) + g^2(gL)^{-1/\nu}\frac{\bar{B} - f(0)}{f'(0)}$$

Competing Anzatz

We can express the critical mass in different ways

Critical Bare-mass parameter

$$m_c^2(g, L) = m_c^2(g)|_{1-\mathrm{loop}} + \beta D_{IR}(\Lambda_{IR}(g, L))\mathcal{N}(N)$$

with

$$D(p) = D_{IR}(p) = -\frac{\log(\Lambda_{IR})}{(4\pi)^2}$$

Competing Anzatz

- **MODEL 1:** IR Divergences, $\Lambda_{IR} = 1/L$
- **MODEL 2:** IR-finite a. la. Jackiw and Templeton, 1981; Appelquist and Pisarski, 1981, $\Lambda_{IR} = g/(4\pi N)$

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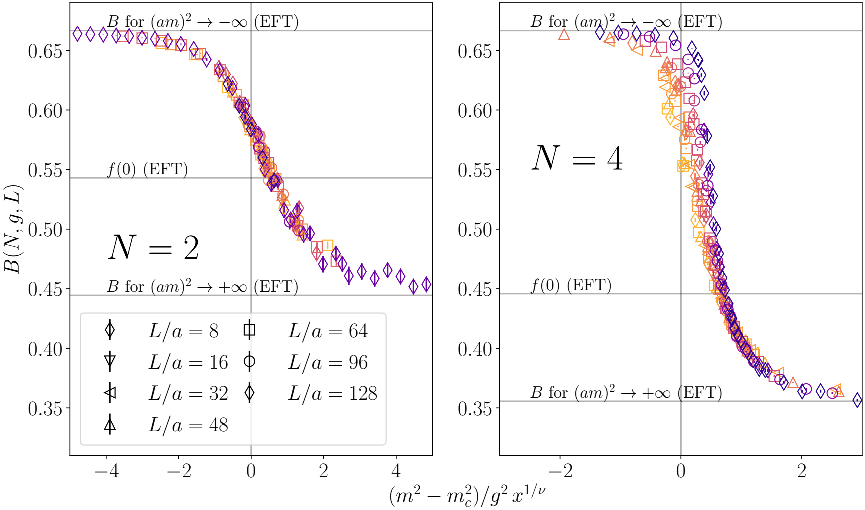
A first look at the data



Determining the preferred anzatz (*)

How do we determine the preferred anzatz?

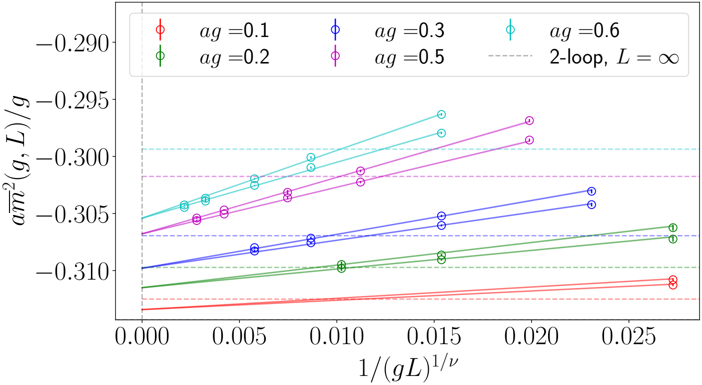
- **Frequentist Analysis**: Through determining which anzatz can fit the most data (up to a significatance level α.)
- **Bayesian Analysis**: Through directly comparing the likelihoods of the models

Pros & Cons

- **Frequentist** Is conceptually, and often computationally simpler, but doesn't answer the question directly
- **Bayesian** Directly answers the question "Which model do the data prefer?", but has ambiguities

Method 1: Frequentist

N	2	4
gL$_{min}$ (IR Finite)	12.8	12.8
gL$_{min}$ (IR Infinite)	32	24



Critical scaling parameter, ν agrees with Hasenbusch, 2001 for $N=2$.

Bayesian Evidence

We can apply Bayes Theorem:

$$\begin{aligned} p(M|\mathrm{data}) &= \int d\alpha\, p(M(\alpha)|\mathrm{data})\, p(\alpha|M), \\ &= \int d\alpha\, \frac{p(\mathrm{data}|M(\alpha))\, p(M(\alpha))}{p(\mathrm{data})}\, p(\alpha|M), \\ &= \frac{p(M)}{p(\mathrm{data})} \int d\alpha\, L(M(\alpha))\, p(\alpha|M), \end{aligned}$$

We can then define the Bayes Factor

Bayes Factor

$$K(\mathrm{data}) = \frac{p(M_1|\mathrm{data})}{p(M_2|\mathrm{data})} = \frac{\int d\alpha_1\, L(M(\alpha_1))\, p(\alpha_1|M)}{\int d\alpha_2\, L(M(\alpha_2))\, p(\alpha_2|M)}\, \frac{p(M_1)}{p(M_2)}.$$

Jeffreys' Scale

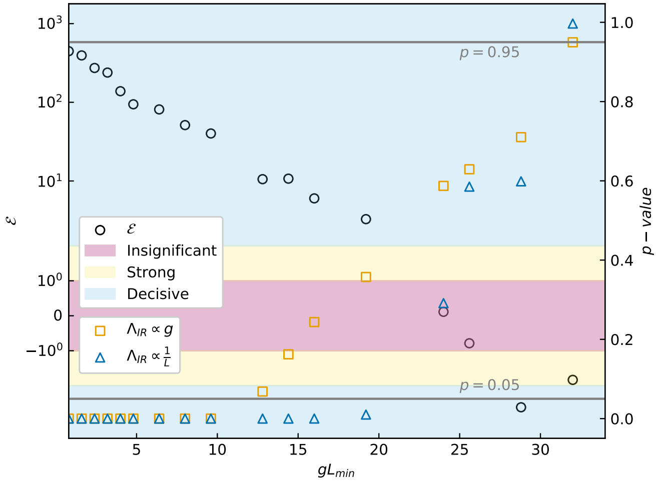
- First written in **Harold Jeffreys' book Theory of Probability** in 1939
- Along with variants it is popular to this day

$\log_{10}(K)$	Interpretation
$x < 0$	Support for the alternative model
$0 < x < 1/2$	Barely worth mentioning
$1/2 < x < 1$	Substantial
$1 < x < 1.5$	Strong
$1.5 < x < 2$	Very Strong
$x > 2$	Decisive

How can we choose priors? (*)

- For a problem with **Gaussian errors**, we choose a **uniformly** distributed prior
- **Physics** can give bounds on prior (e.g. $f_0 \in [0, 1]$)
- Or at least a **natural** parameterisation
- If not, ideally we would use as large a prior as possible
- However, the integration above is non-trivial, so we have to compromise.
- Priors here chosen to be large enough to contain distribution within 10 sigma.

Results



We conclude that the scalar $SU(N)$ theory is **IR-finite**

Updates & Future Works

Updates

- Code has been adapted to run on NVidea GPU's for new Tursa machine in Edinburgh
- Gauge Fields have been implemented in GRID code
- Renormalization of T-$Tr[\phi^2]$ correlator complete (Del Debbio et al., 2021)

Ongoing work

- $N = 3$ and $N = 5$ datasets have been produced, fitting ongoing
- We want to determine if we can take a large-N limit
- Renormalization of the TT-correlator is underway. Once complete this will allow fitting to CMB data

Thank you for listening!