A template for Composite Higgs models
SU(2) gauge theory with N_f=2 fundamental fermions

Antonio Rago
University of Plymouth & CERN

with V. Drach, P. Fritzsch, F. Romero-Lopez

[2107.09974] & [2012.09761]

Scattering observables as a probe of New Physics

LHC Run II :

- Study of the properties of the Higgs boson Measure Vector Boson Scattering

Vector Boson Scattering :

- Probe the structure of electroweak interactions in the SM
- Very sensitive to new physics.

[Covarelli et al, Int.J.Mod.Phys.A 36 (2021) 16, 2130009, 2102.10991]

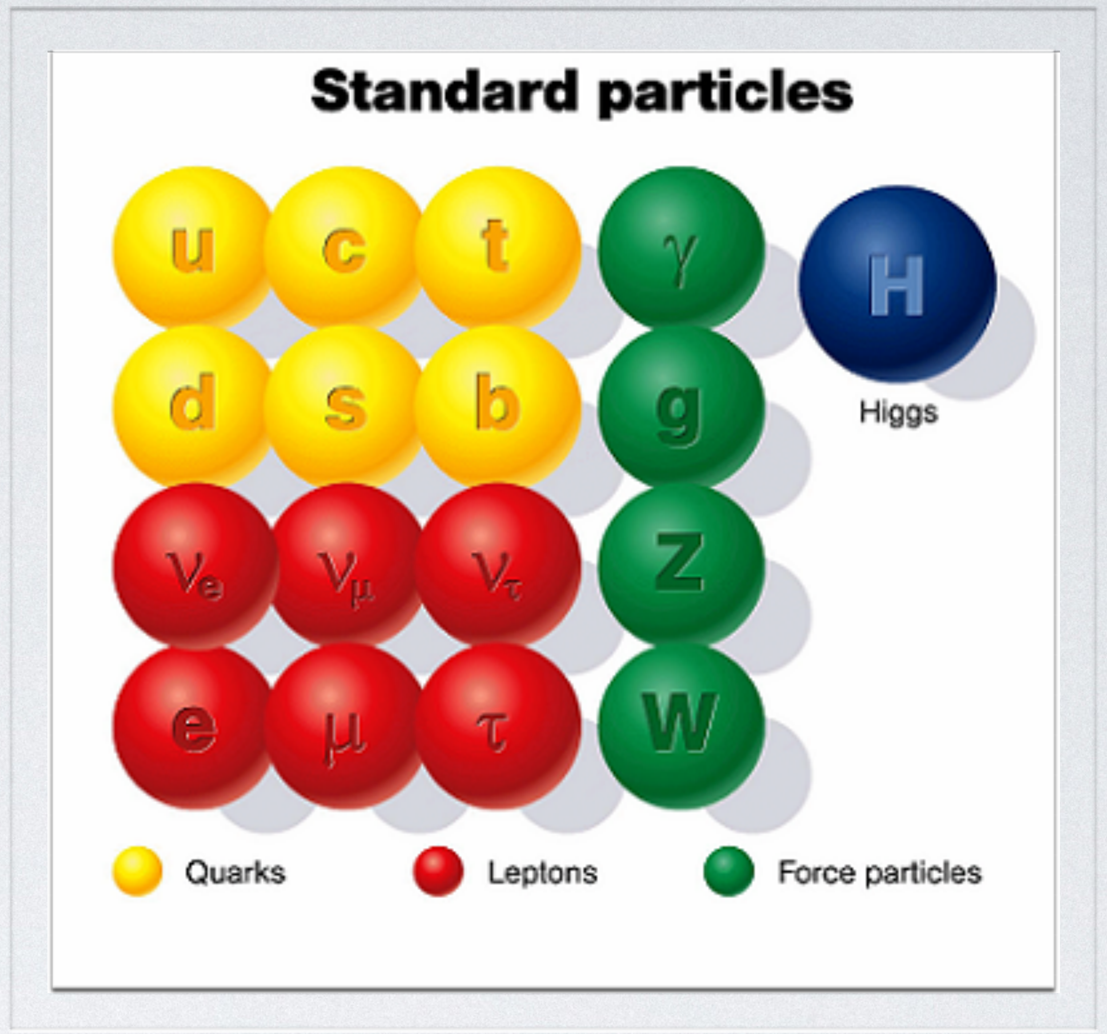
New strong dynamics Beyond the Standard Model:

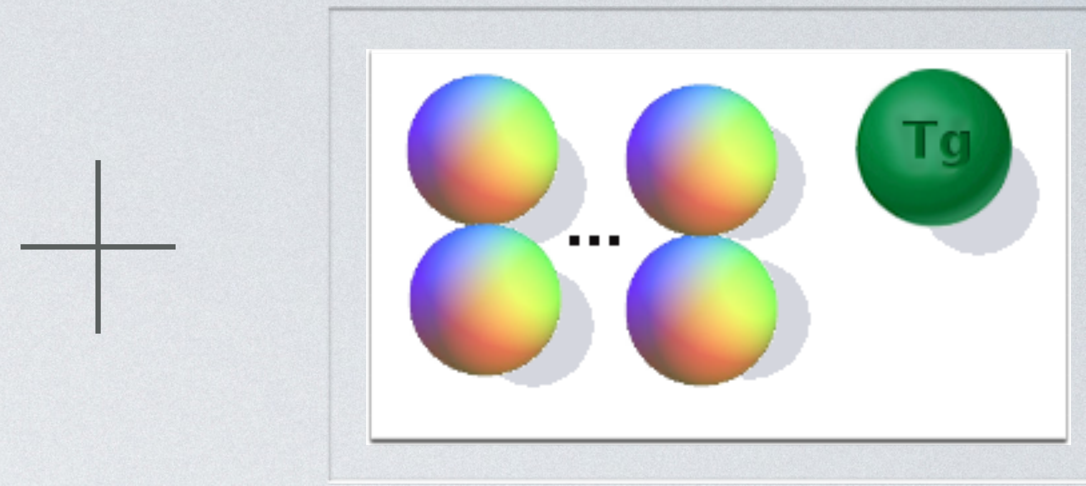
- Dynamical origin to the EWSB
- Scattering processes of the new strong sector contribute to SM processes investigated at the LHC.

Composite Higgs models in a nutshell

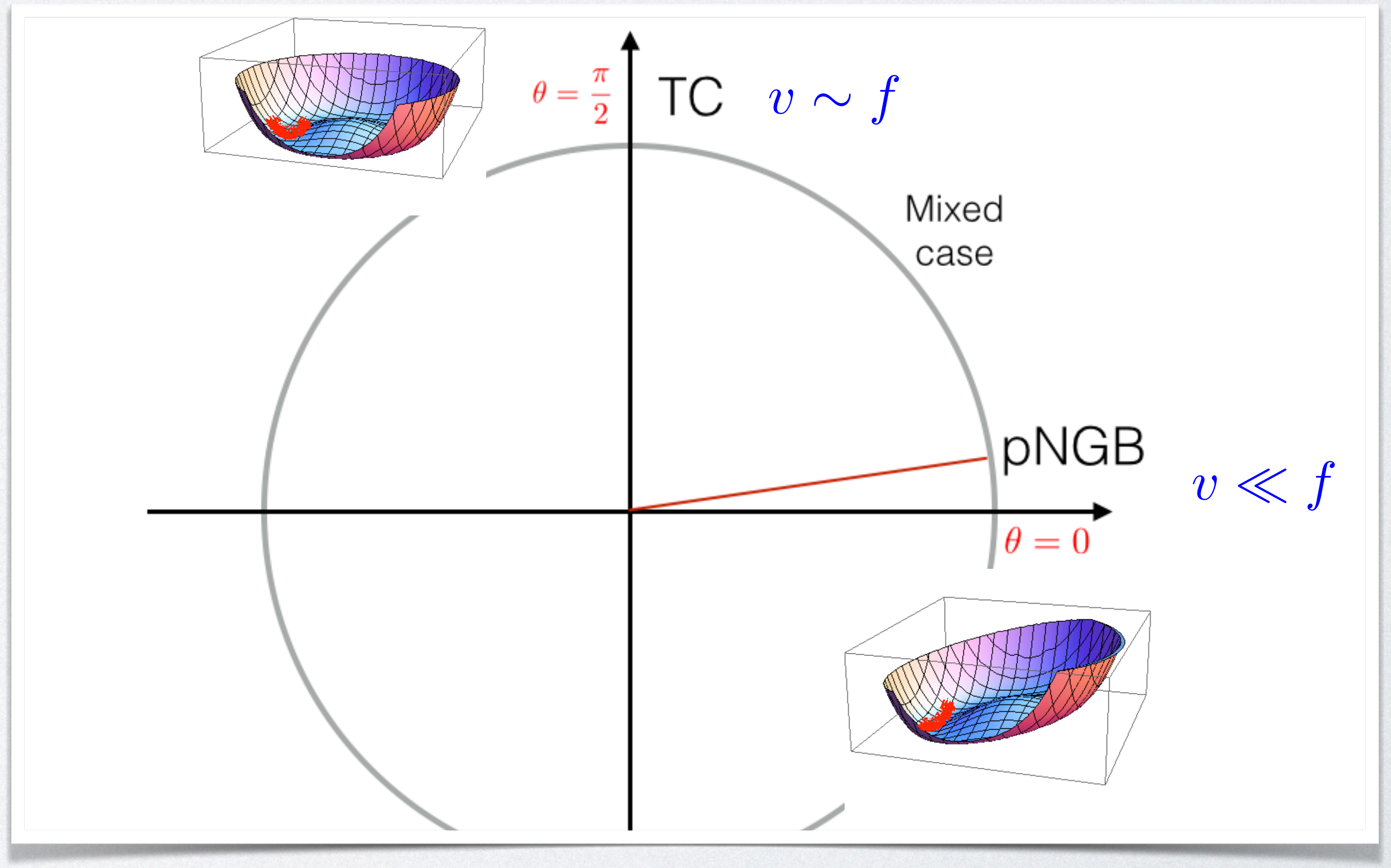
Symmetry broken by a condensate
(new sector fermions)

Higgs and longitudinal Z/W emerge as mesons





A **composite Higgs boson** can arise as a pNGB or as a light resonance based on the misalignment of the condensate with respect to the EW gauge symmetry



f: Higgs decay constant
v: EW scale

Sigma observables as a probe of New Physics

- One feature of a strongly interacting sector is the inevitable presence of a flavour singlet state of positive parity.
- In QCD-like theories, the σ is expected to be a resonance of two Goldstone bosons in the limit of massless underlying fermions.
- In these models, aside from the Goldstone bosons, also the presence of new resonances like the σ can affect the predictions for the LHC.
- In general, the phenomenological implications of the new scalar resonance in a composite Higgs scenario will depend on the underlying dynamics and on the details of the electroweak embedding.

Sigma observables as a probe of New Physics

Phenomenological perspectives in the context of Composite Higgs models:

0^{++} state mixes with the Higgs boson: alter its physical properties
0^{++} is expected to show up at the LHC as a heavy resonance

Such a resonance is expected to be produced at the LHC similarly to the SM Higgs, i.e. via gluon fusion and vector boson fusion mechanisms

Phenomenological implication for theories based on $SU(4) \to Sp(4)$ breaking considered

[Buarque Franzosi, Cacciapaglia et al *Eur.Phys.J.C* 80 (2020) 1, 28, 1809.09146]

$SU(2)_c$ with Nf=2 fundamental Dirac flavours

Fundamental representation of SU(2) is pseudo-real

Chiral symmetry breaking pattern : SU(4) → Sp(4)

[Buarque Franzosi, Cacciapaglia et al *Eur.Phys.J.C* 80 (2020) 1, 28, 1809.09146]

UV completion of a Minimal composite Higgs model

[G. Cacciapaglia & F. Sannino, *JHEP* 04 (2014) 111 1402.0233]

The Higgs is a linear combination of GBs and of the 0^+ state

Not excluded by experimental data

[Arbey *et al*, *Phys.Rev.D* 95 (2017) 1, 015028 1502.04718]

GBs in SU(2)c with Nf=2

Goal: build a correlation of operator with flavour singlet quantum numbers

$$Q = \begin{pmatrix} u_L \\ d_L \\ \widetilde{u}_L \\ \widetilde{d}_L \end{pmatrix} = \begin{pmatrix} u_L \\ d_L \\ (-i\sigma_2)C\bar{u}_R^T \\ (-i\sigma_2)C\bar{d}_R^T \end{pmatrix} \qquad E = \begin{pmatrix} 0 & 1_2 \\ -1_2 & 0 \end{pmatrix}$$

With the above convention we can define the multiplet and the singlet as

$$\Pi^i = \frac{1}{2}\left[Q^T(-i\sigma_2)C\gamma_5 X^i E Q + \text{h.c}\right] \ ,$$

$$\mathcal{O}_\sigma = \frac{1}{\sqrt{2}}\left[Q^T(-i\sigma_2)CEQ + \text{h.c}\right] .$$

and $\mathrm{X}^{1,\ldots,5}$ are the generators of SU(4)/Sp(4) .

GBs in SU(2)c with Nf=2 (cont.)

$$\begin{aligned}
\Pi_{ud}(x) &= u^T(x)(-i\sigma_2)C\gamma_5 d(x),\\
\Pi_{\bar{u}\bar{d}}(x) &= \bar{u}(x)(-i\sigma_2)C\gamma_5 \bar{d}(x)^T,\\
\pi^-(x) &= \bar{u}(x)\gamma_5 d(x),\\
\pi^+(x) &= -\bar{d}(x)\gamma_5 u(x),\\
\pi^0(x) &= \frac{1}{\sqrt{2}}\left[\bar{u}(x)\gamma_5 u(x) - \bar{d}(x)\gamma_5 d(x)\right],
\end{aligned}$$

The two GBs flavour singlet operator is

$$\mathcal{O}_{\Pi\Pi} = -\frac{4}{\sqrt{5}}\sum_{i=1}^{5}\Pi^i\Pi^i = \frac{1}{\sqrt{5}}\Big[+\pi^+\pi^- + \pi^-\pi^+ - \pi^0\pi^0 \quad +\Pi_{ud}\Pi_{\bar{u}\bar{d}} + \Pi_{\bar{u}\bar{d}}\Pi_{ud}\Big].$$

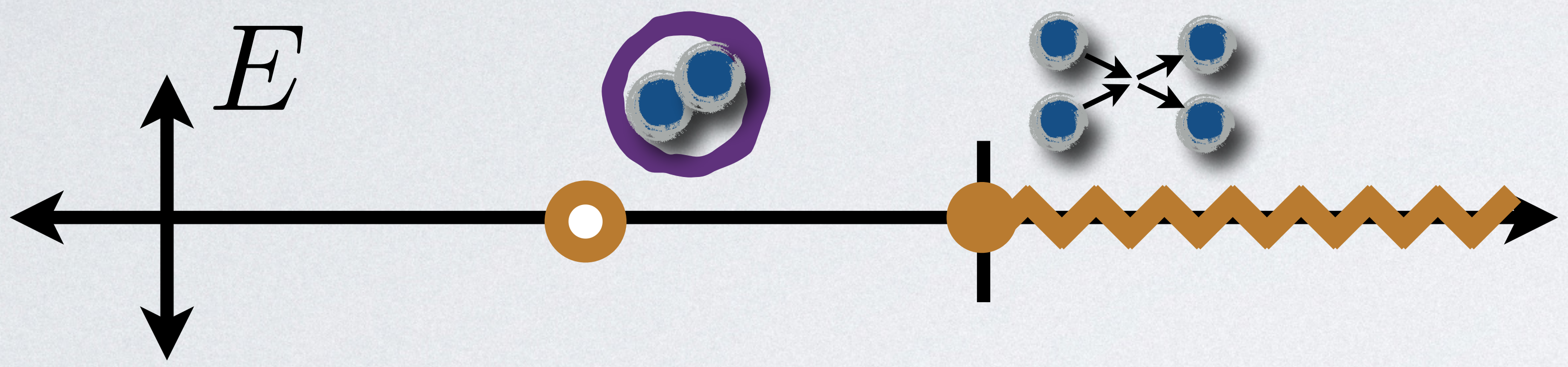
The fermion bilinear operator with the flavour singlet quantum number reads:

$$\mathcal{O}_\sigma = \frac{1}{\sqrt{2}}\left[Q^T(-i\sigma_2)CEQ + \text{h.c}\right]. = \frac{1}{\sqrt{2}}\left[\bar{u}(x)u(x) + \bar{d}(x)d(x)\right].$$

S-matrix and scattering

In our lattice simulations the only rigorous approach to reveal the nature of a resonance is to estimate the scattering amplitude.

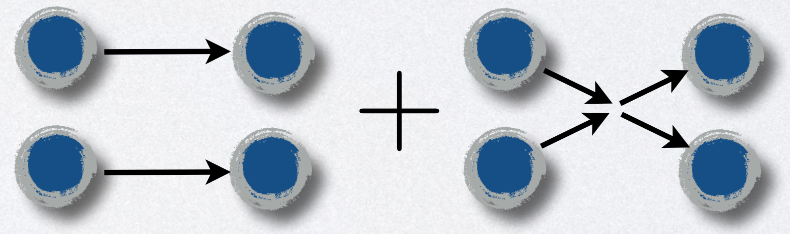
For scattering states, listing allowed energies is no longer useful



All above-threshold energies appear

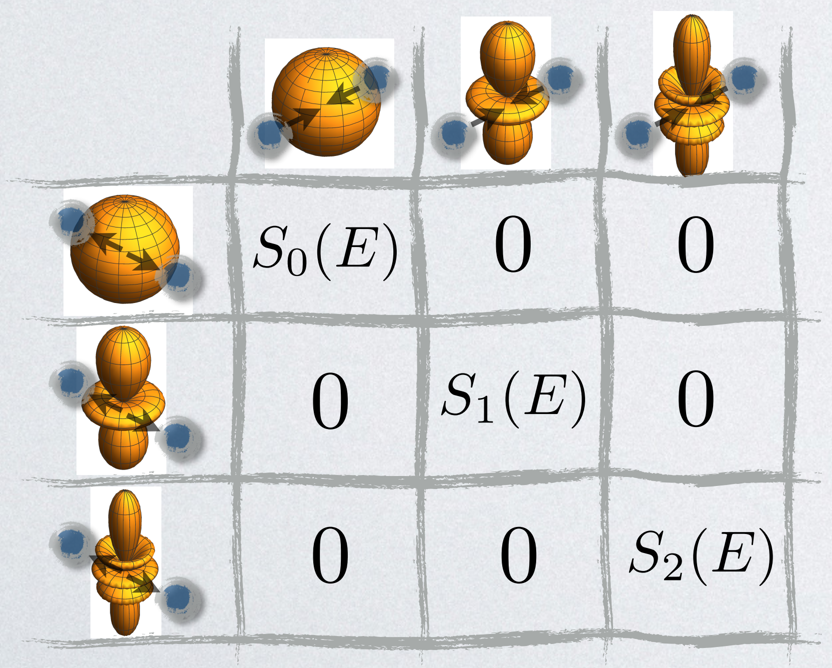
Instead, physical information is in the matrix elements: $S(E) = \langle \pi\pi, \text{out} | \pi\pi, \text{in} \rangle$

Relation to the scattering amplitude

$S(E) =$ 

$$S_0(E) = e^{2i\delta_0(E)} \longrightarrow \mathcal{M}_0(E) \propto e^{2i\delta_0(E)} - 1$$

real function contains the scattering information



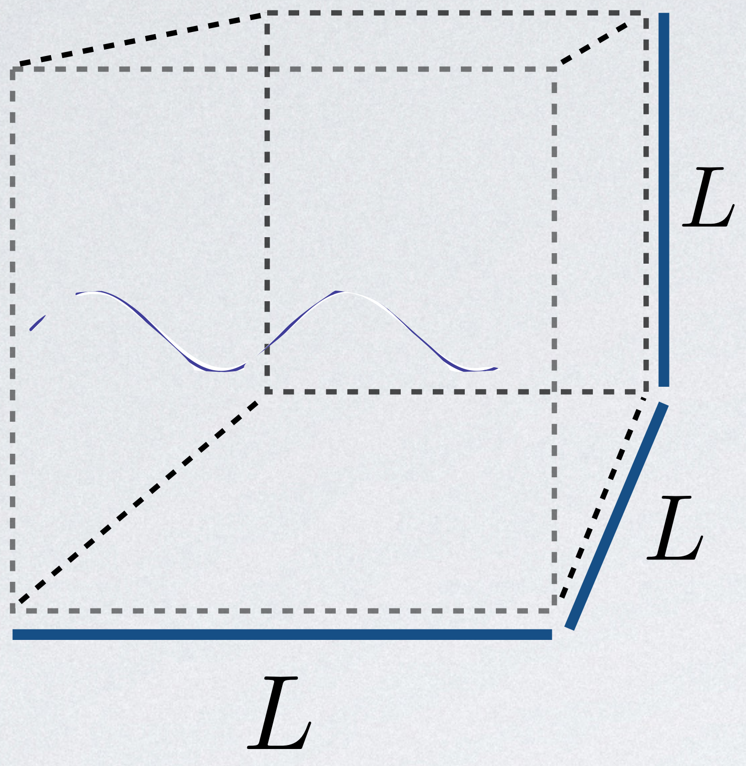
S-matrix properties

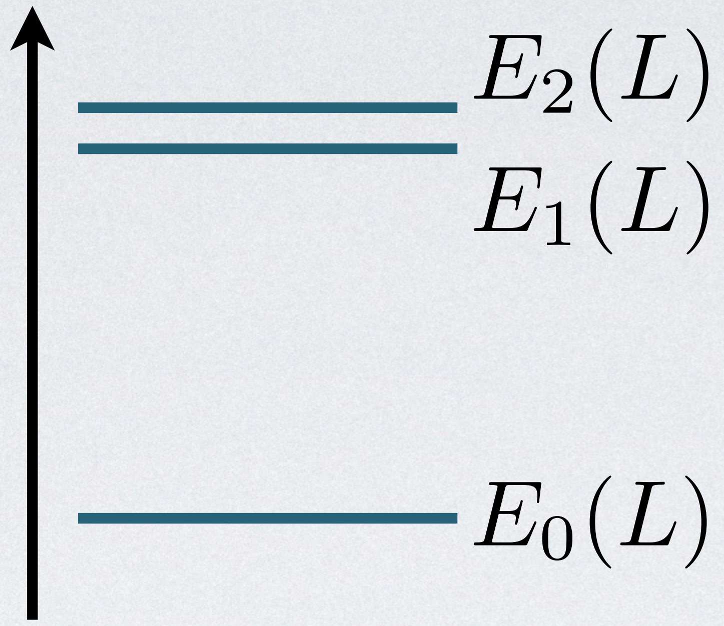
- Diagonal in angular momentum
- S-matrix unitarity

$$S^\dagger(E) S(E) = \sum_\alpha \langle \pi\pi, \text{in} | \alpha \rangle \langle \alpha | \pi\pi, \text{in} \rangle = \mathbb{I}$$

Lüscher's method

- Finite-volume set-up





- cubic, spatial volume (extent *L*)
- periodic boundary conditions

$$\vec{p} = \frac{2\pi}{L}\vec{n}\,, \qquad \vec{n} \in \mathbb{Z}^3$$

- *L* is large enough to neglect $e^{-M_\pi L}$

- Scattering observables leave an ***imprint*** on finite-volume quantities

E.g. in a weakly-interacting, two-body system with no bound states

$$\mathcal{M}_{\ell=0}(2M_\pi) = -32\pi M_\pi a$$

Information is in the scattering amplitude

Finite-volume ground state

$$E_0(L) = 2M_\pi + \frac{4\pi a}{M_\pi L^3} + \mathcal{O}(1/L^4)$$

[Huang, Yang (1958)]

- All results are contained in a generalized quantization condition

$$\det\left[\mathcal{M}_2^{-1}(E_n^*) + F(E_n, \vec{P}, L)\right] = 0$$

scattering amplitude — known geometric function

Matrices in angular momentum, spin and channel space

Lüscher's method in s wave

In s wave life is simple.
Quantisation condition for s-wave scattering:

$$k \cot \delta_0(k) = \frac{2}{\sqrt{\pi} L} \mathcal{Z}_{00}\left(\frac{(Lk)^2}{4\pi^2}\right)$$

k : relative momentum of the two GBs in the c.m.f obtained from the energy levels.

$\mathcal{Z}_{00}$: Lüscher zeta function

Bound-state condition: $k \cot \delta_0(k) = -\sqrt{-k^2}$

i.e. must cross the bound state condition from below for decreasing k

Simulation details:

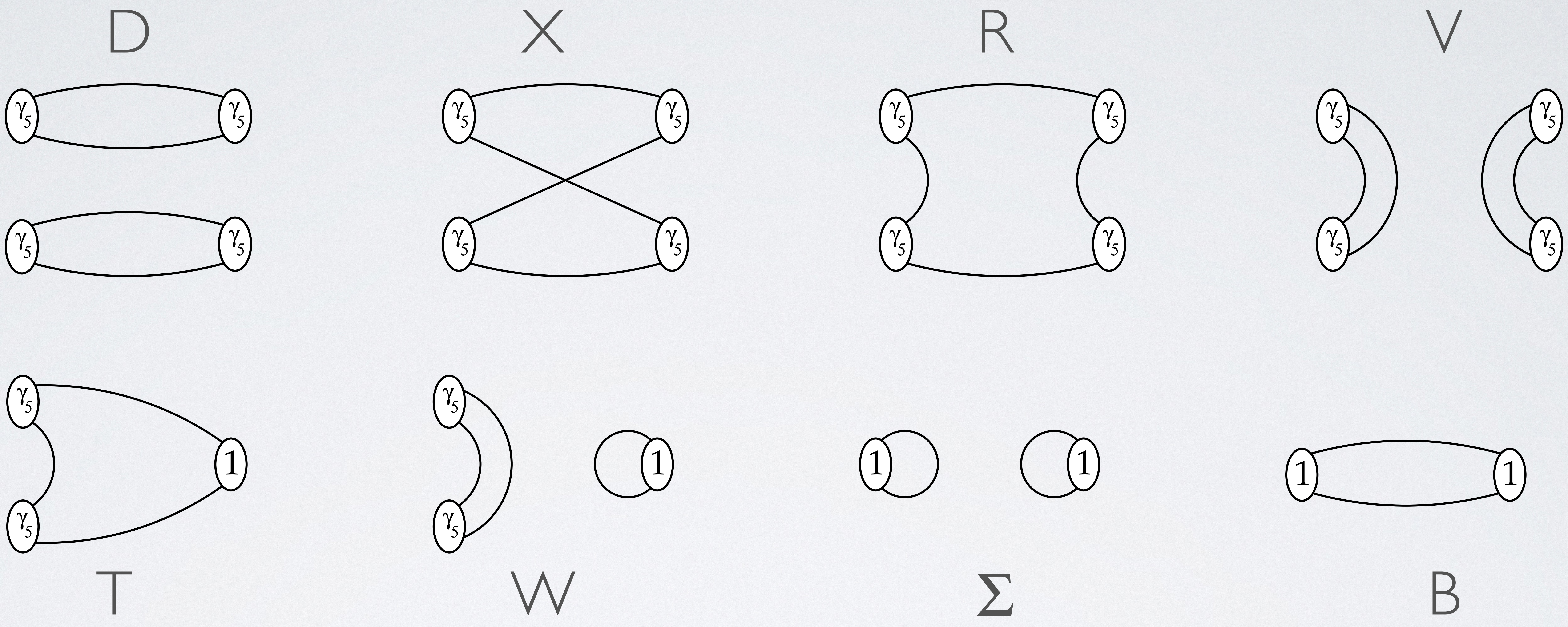
We used the **HiRep** suite to simulate an SU(2) gauge theory with N_f=2.
Fermions: Wilson action with tree-level O(a)-improvement clover term.
Gauge: tree-level Symanzik improved action.

Ensemble	L/a	T/a	β	$a\,m_0$	c_{sw}	# configs
Heavy	24	48	1.45	-0.6050	1.0	1980
Light	32	48	1.45	-0.6077	1.0	1160

Ensemble	aM_π	aM_ρ	aF_π^{bare}	$M_\pi/F_\pi^{\text{bare}}$	$M_\pi L$
Heavy	0.2065(12)	0.438(27)	0.0395(9)	5.24(11)	4.95
Light	0.1597(18)	0.3864(30)	0.0357(9)	4.36(11)	5.11

Contractions

Wick contractions read :

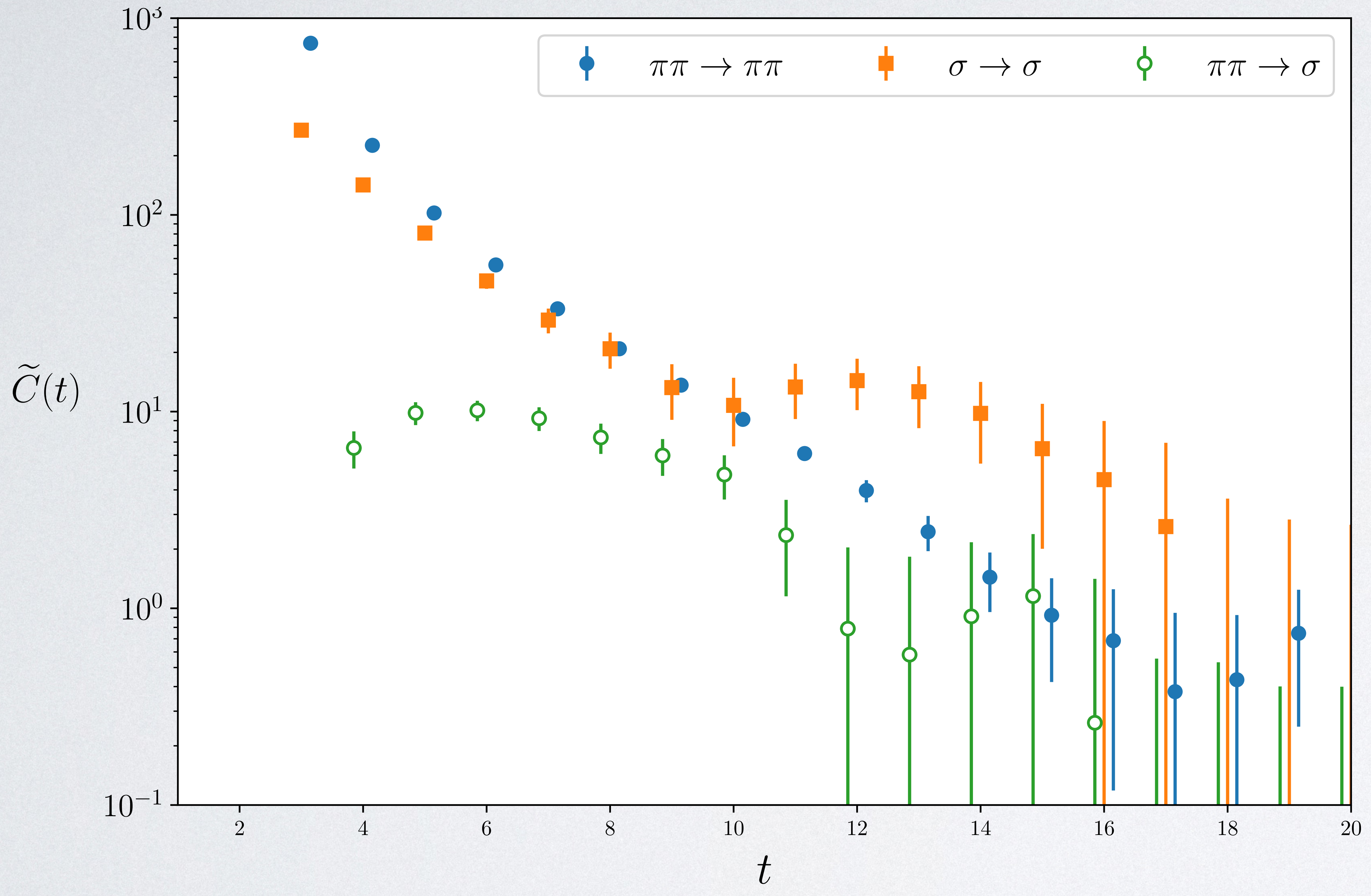


$$C_{\sigma\to\sigma}(t) = -B(t) + 2\Sigma(t),$$
$$C_{\pi\pi\to\pi\pi}(t) = 2D(t) + 3X(t) - 10R(t) + 5V(t),$$
$$C_{\pi\pi\to\sigma}(t) = \sqrt{10}\,(T(t) - W(t))\,.$$

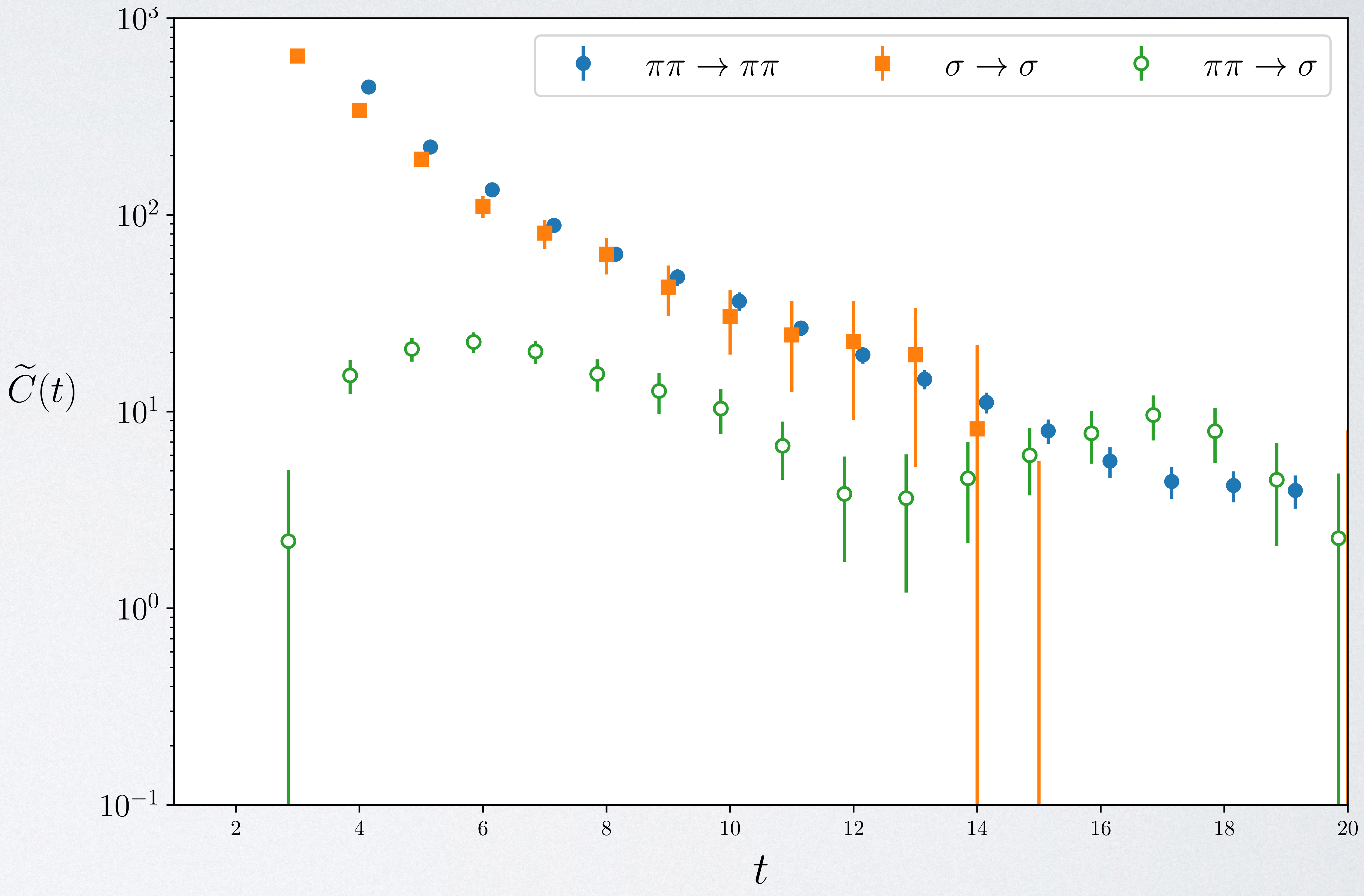
$$C_{X\to Y}(\delta t) = \frac{1}{T}\sum_t \langle O_X(t+\delta t) O_Y(t)^\dagger \rangle.$$

Analysis of the correlation matrix

Constant contribution to the correlation functions removed by defining shifted-correlators:



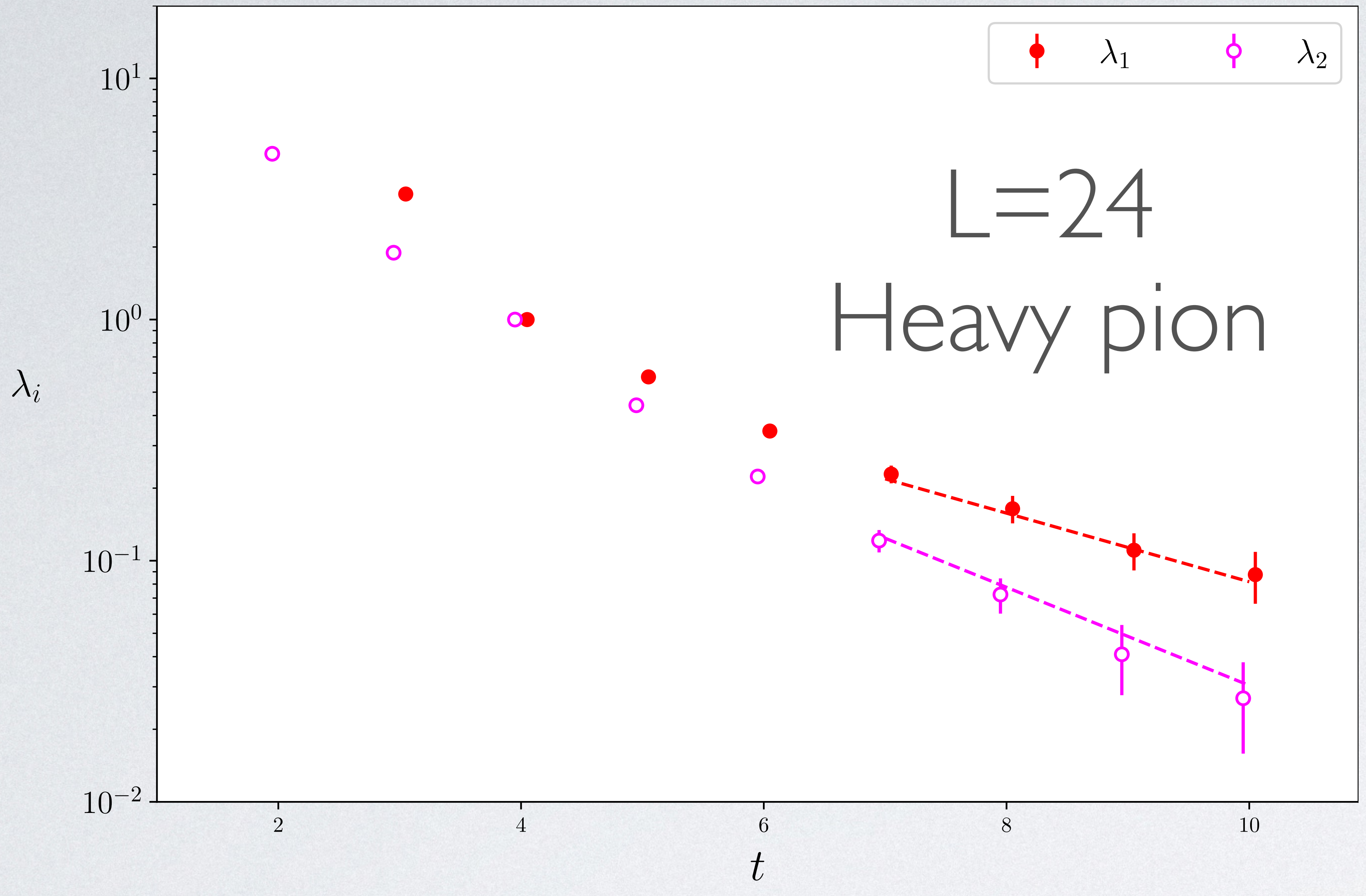
L=24

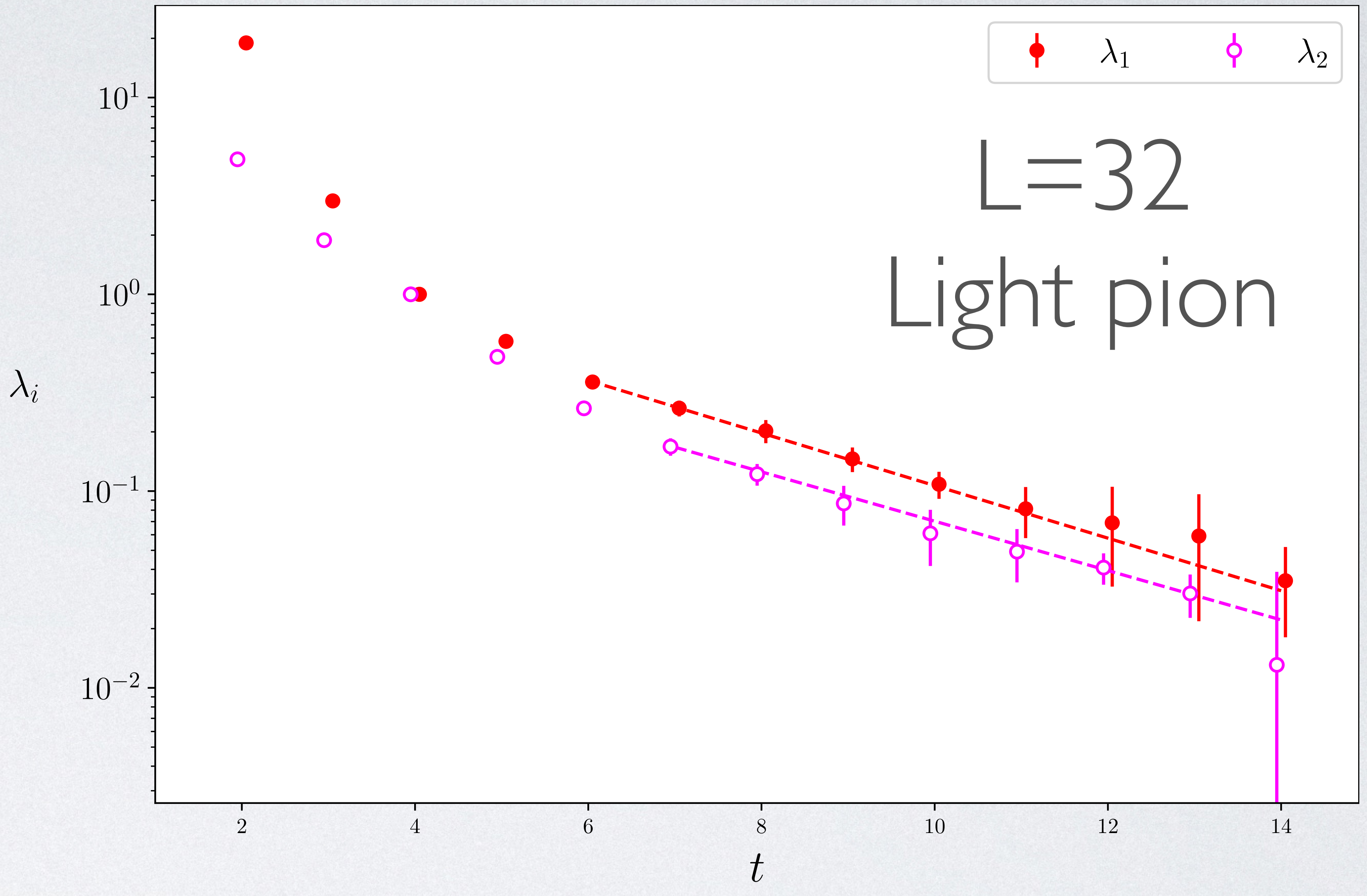


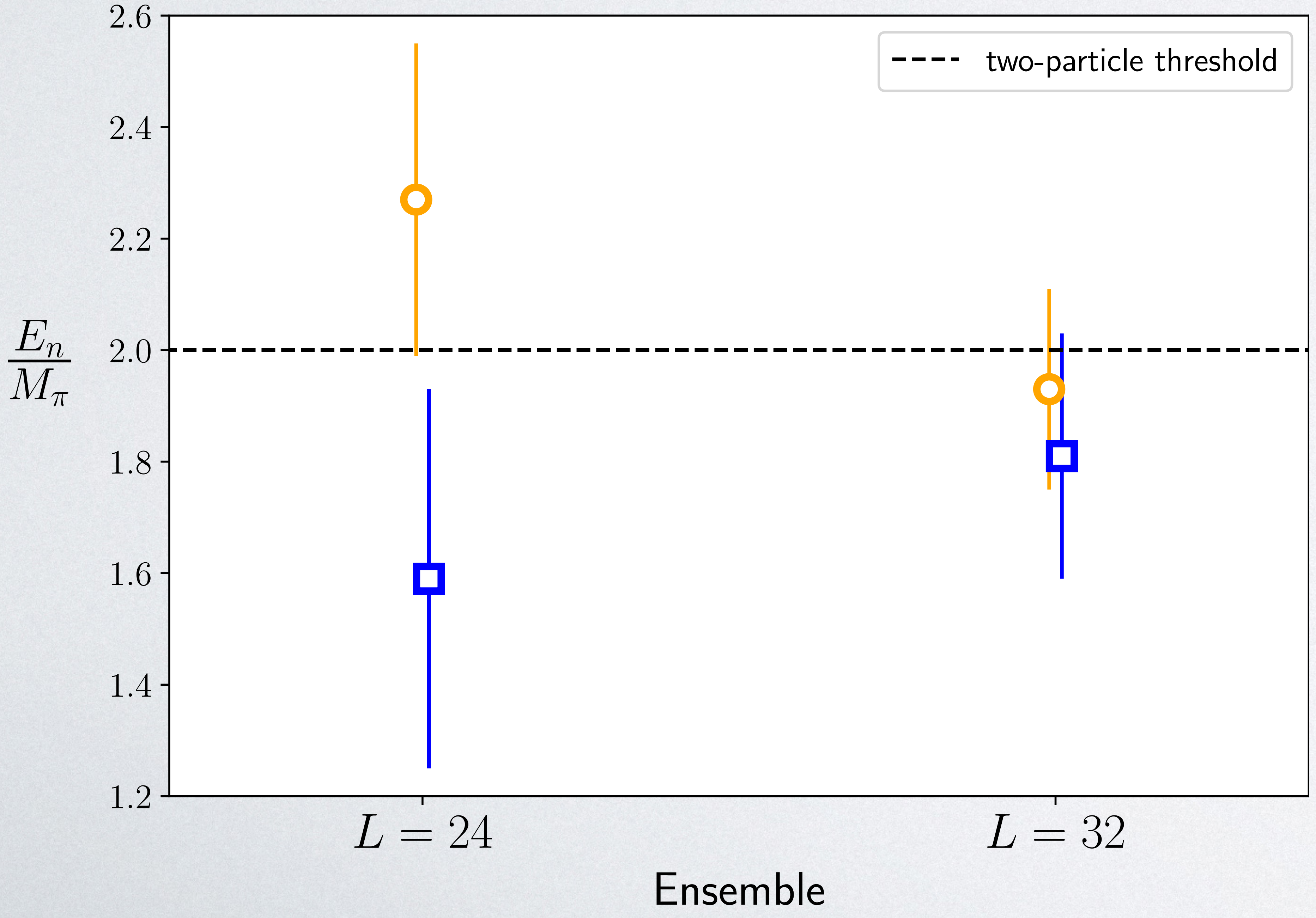
L=32

GEVP from the shifted correlators $\tilde{C}(t) = \frac{1}{2}\left[C(t-1) - C(t+1)\right]$

Eigenvalues

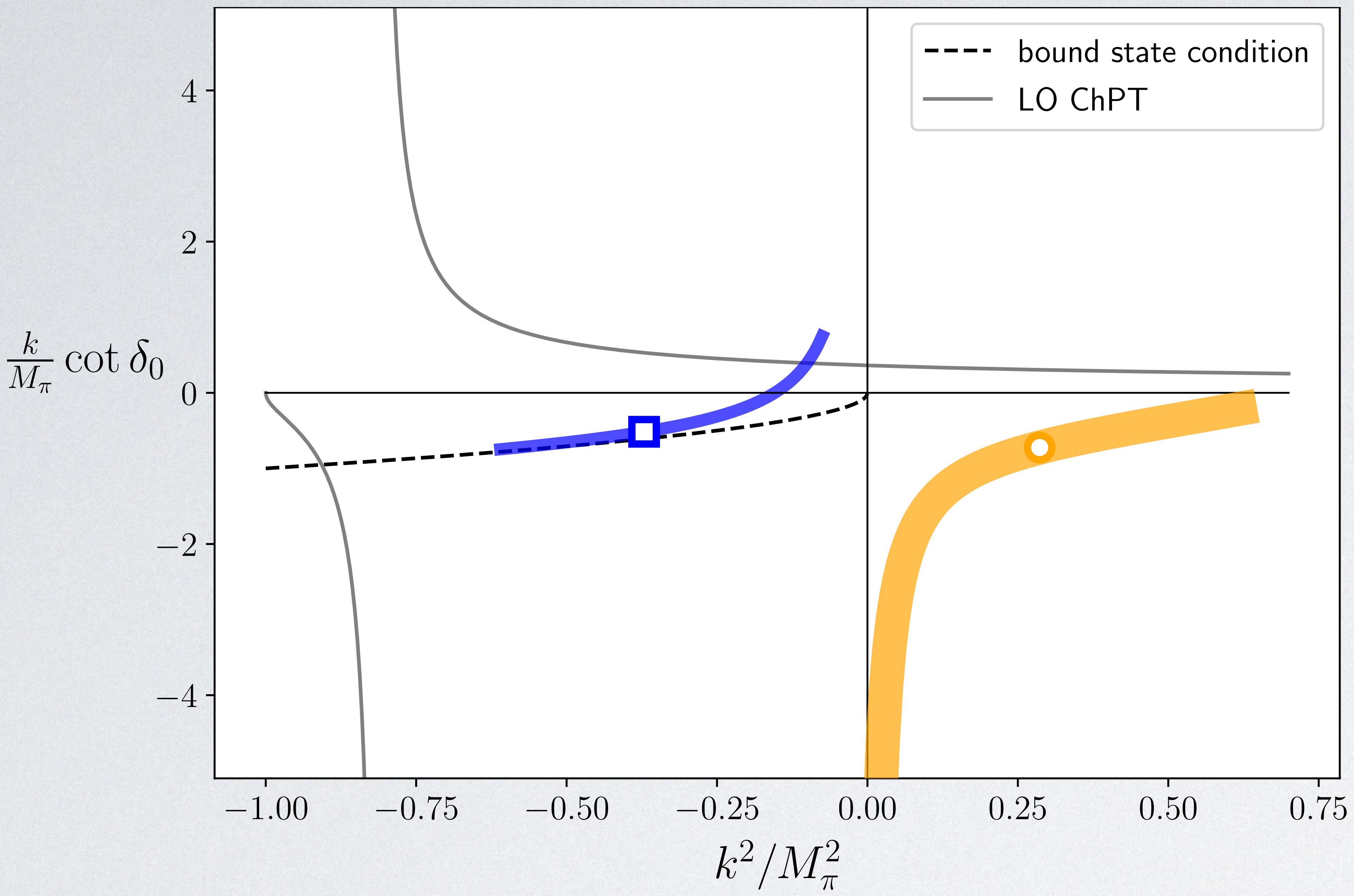


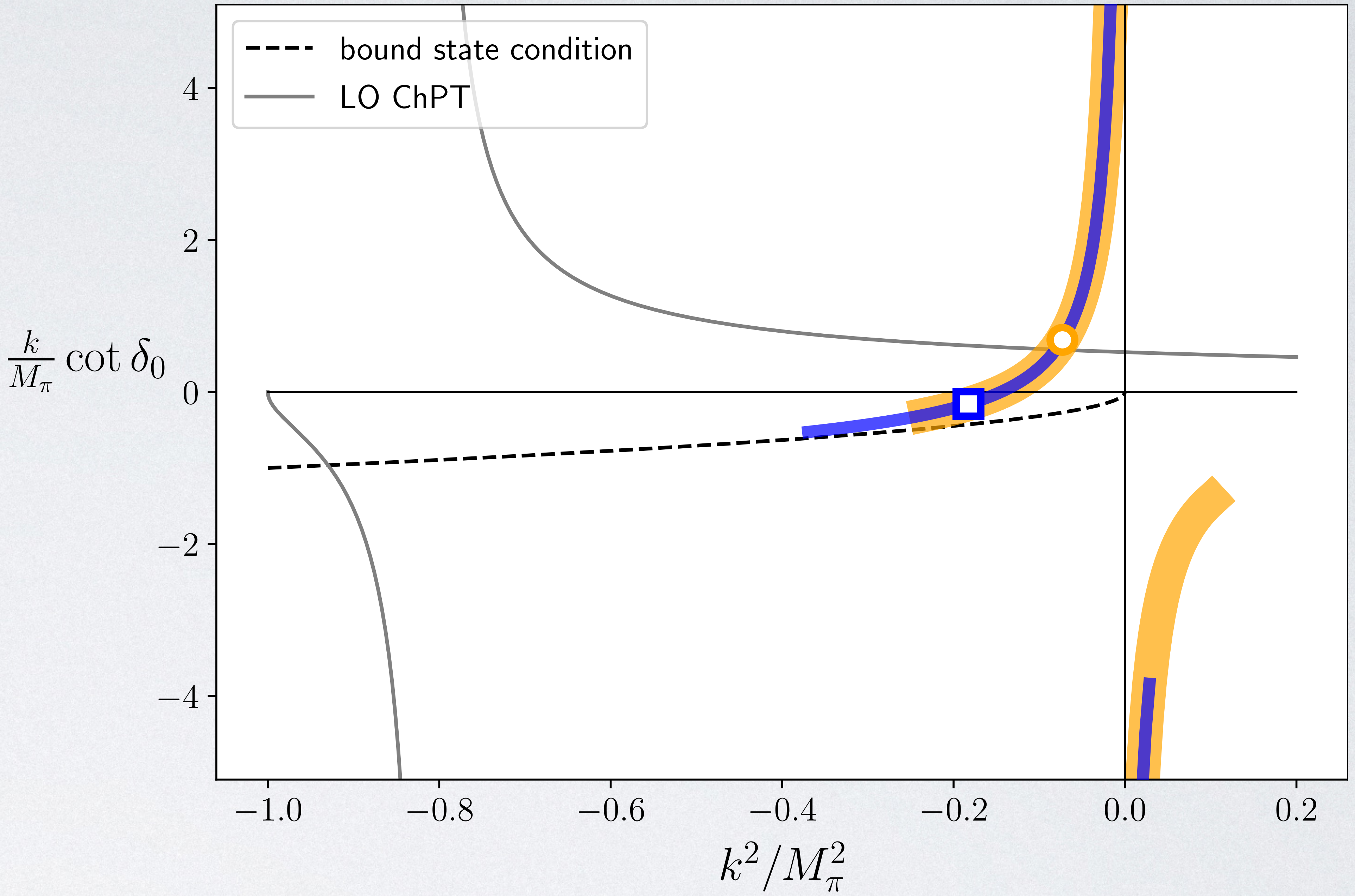




The ensemble with the lightest fermion mass is close to threshold

Scattering amplitude





- ☑ **In the explored region of fermion masses the sigma is most likely a stable particle, that is, a two-pion bound state**
- ☑ **We are able to put non-perturbative constraints on the singlet scattering amplitude.**
- ☐ Interestingly, we find that leading-order chiral perturbation theory does not seem to describe the amplitude correctly, and fails in predicting a bound state around the region where we observe it
- ☐ In our two ensembles, we find that $\frac{M_\sigma}{M_\pi} \sim 1.5 - 1.8$. We however expect this feature to depend strongly upon the pion mass.

Conclusion

First study of the singlet channel in four-dimensional gauge theories beyond QCD

Study of scattering processes is crucial to constrain underlying dynamics of Pseudo-Nambu Goldstone Bosons Composite Higgs models

We use :

- Two ensembles below vector channel threshold at fixed lattice spacing 2x2 GEVP (including all disconnected contributions)
- Lüscher's method

Results are compatible with a bound state in the singlet channel.

Results complement recent calculation of the scattering amplitude in the vector channel and our prediction of its coupling to two GBs.

More chiral setup could change the picture (suggested by the discrepancy with LO ChPT)

Continuum limit ongoing