Integrable Domain Walls in N=4 SYM and ABJM theory

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Based on:

- C.K, D.L. Vu & K. Zarembo, arXiv:2112.10438[hept-th], JHEP 02 (2022) 070
- C.K., D. Müller & K. Zarembo, arXiv:2106.08116[hep-th], JHEP 09 (2021) 004, arXiv:2011.12192, JHEP 03 (2021) 100

GGI Workshop on Randomness, Integrability and Probability

April 19th, 2022

AdS/CFT

QFT in lower D* $\longleftrightarrow$ String theory in 10D

- Conformal symmetry
- Supersymmetry
- Planar integrability

AdS/dCFT

Domain wall $\longleftrightarrow$ Probe D-brane

- Conformal symmetry partially broken
- Supersymmetry partially or completely broken

In the planar limit, $N \to \infty$:

* $\mathcal{N} = 4$ SYM in 4D, ABJM-theory in 3D

Motivation

- Gain insight on the interplay between conformal symmetry, supersymmetry and integrability
- Test the AdS/CFT dictionary for set-ups with supersymmetry partially or completely broken (all tests positive)
- Exact results for novel types of observables such as one-point functions
- Produce input data for the boundary conformal bootstrap program.
- Interesting connections to statistical physics and QI: matrix product states and quantum quenches
- Novel examples of integrable boundary states, novel characterization at the discrete level
- Novel microscopic duality relations for correlation functions (Strong predictive/constraining power)

Plan of the talk

I. Overlaps and correlation functions in AdS/dCFT

II. Integrable boundary states in AdS/dCFT

III. Exact results for overlaps in N=4 SYM

IV. Duality relations for overlaps

V. Predicting new overlap formulas for ABJM theory

VI. Future directions

AdS/CFT and Overlaps

Conformal operators $\longleftrightarrow$ String states

$\updownarrow$ $\updownarrow$

Eigenstates of integrable super spin chain: $|\mathbf{u}\rangle$

Minahan. Zarembo '02

Beisert, Staudacher '03

Karch. Randall '01

Co-dimension one defect $\longleftrightarrow$ Karch-Randall probe brane

$|\Psi_0\rangle$ (integrable) boundary state describing defect / probe brane

$\langle\Psi_0|\mathbf{u}\rangle$ is a one-point function

De Leeuw, C.K. Zarembo '15

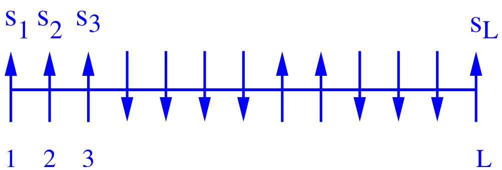
Determinant operator $\longleftrightarrow$ Giant graviton

Similar idea: $|\Psi_0\rangle \sim$ determinant operators/giant graviton

Jiang, Komatsu Vescovi '19

$\langle\Psi_0|\mathbf{u}\rangle$ is a three-point function

Integrable boundary states



$s_{L+m} = s_m$ $\qquad |\Psi\rangle = |s_1 s_2 s_3 \ldots s_L\rangle$

Eigenstates: $H_0|\mathbf{u}\rangle = E_0|\mathbf{u}\rangle$

Integrable boundary state $\langle\Psi_0|$: $\langle\Psi_0|\mathbf{u}\rangle$ computable in closed form

Identified types of relevance for AdS/dCFT:

Matrix product states: $|B\rangle = |\mathrm{MPS}\rangle = \sum_{\{s_i\}} \mathrm{Tr}(t_{s_1} \ldots t_{s_L})|s_1 \ldots s_L\rangle$

De Leeuw, C.K., Zarembo '15

Valence Bond States: $|\mathrm{VBS}\rangle = |K\rangle^{\otimes \frac{L}{2}}$, $\qquad K = \sum_{s_1, s_2} K_{s_1, s_2}|s_1 s_2\rangle$

C.K., Müller, Zarembo '20

Of possible relevance for AdS/CFT:

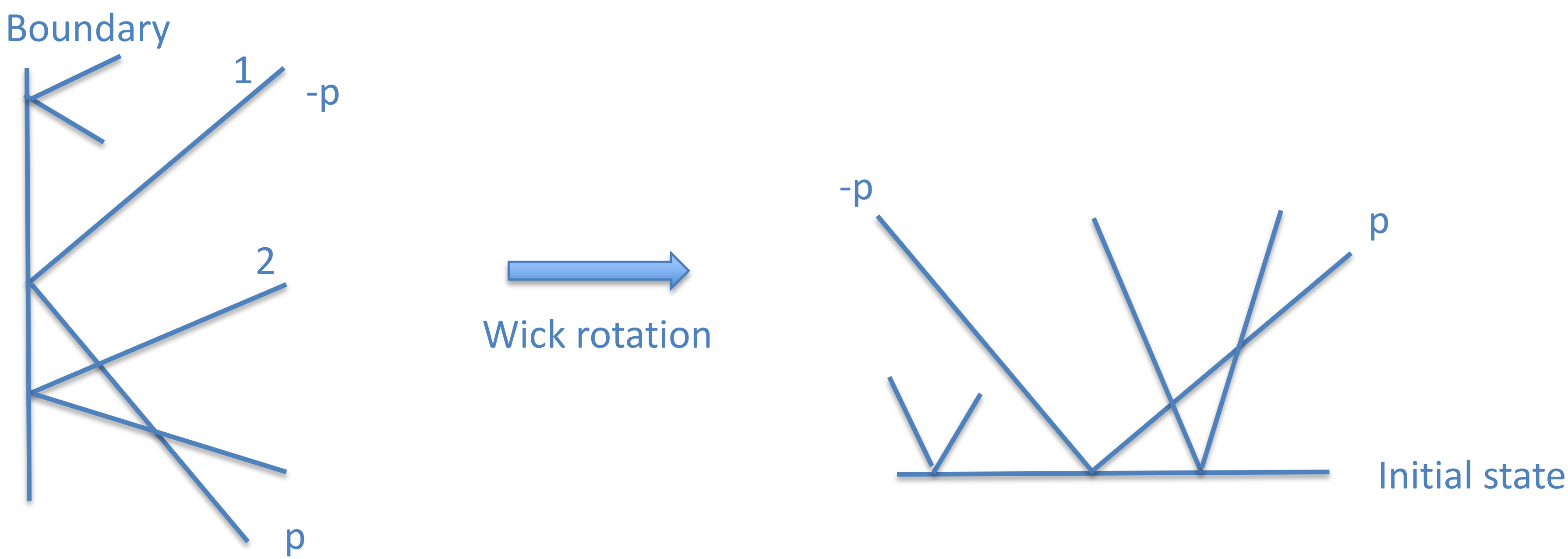
Cross cap states: $|C\rangle = |c\rangle\rangle^{\otimes L/2}$, where $|c\rangle\rangle = |\uparrow\rangle_j |\uparrow\rangle_{\frac{L}{2}+j} + |\downarrow\rangle_j |\downarrow\rangle_{\frac{L}{2}+j}$

Caetano, Komatsu '21

Integrable boundaries in integrable QFTs

Ghoshal, Zamolodchikov '93

- No particle production or annihilation
- Pure reflection, possibly change of internal quantum numbers
- Yang-Baxter relations fulfilled (order of reflection does not matter)



Pure reflection
+BYB for reflection matrix

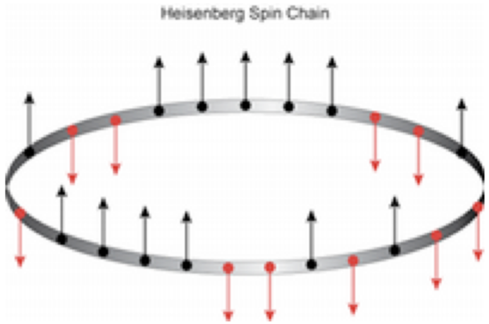
Entangled (p,-p) pairs
+BYB for initial state

Integrable spin chain boundary states

Piroli, Pozsgay Vernier '17

Example

$$H = \sum_{n=1}^{L} (1 - P_{n,n+1})$$



Excited states with K excitations (and momenta): $|\{p_i\}_{i=1}^K\rangle$

Eigenstates: p_i have to fulfil the Bethe equations

$u_i = \frac{1}{2}\coth(p_i/2)$, rapidities or Bethe roots

L conserved charges, $\hat{Q}_n$, with eigenvalues Q_n

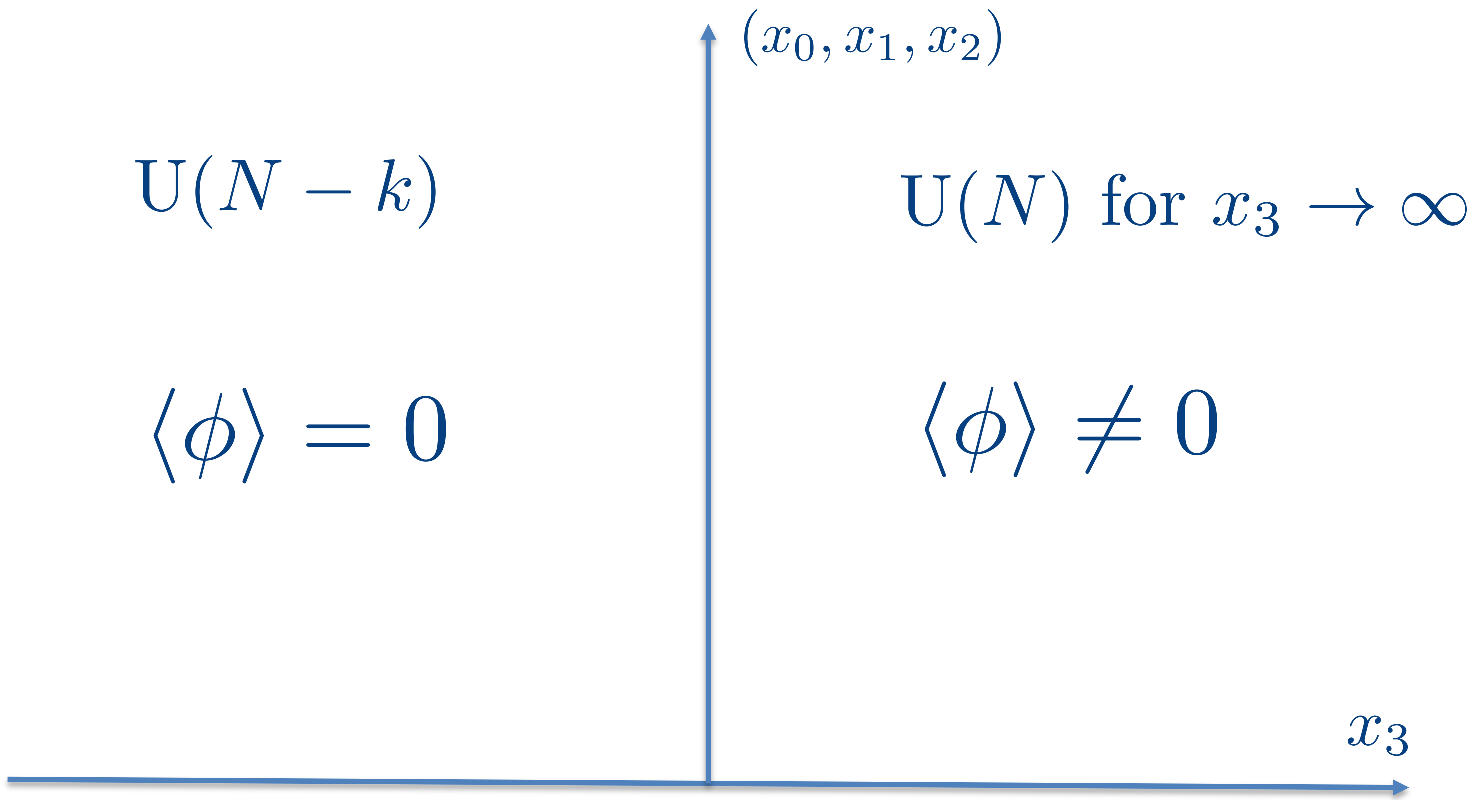
$$Q_n(\{p_i\}) = (-1)^n Q_n(\{-p_i\})$$

Integrable initial state: $\widehat{Q}_{2m+1}|\Psi_0\rangle = 0, \quad \forall\, m$

(BYB observed to be fulfilled for all cases considered)

The defect set-up of $|\mathrm{MPS}\rangle$

$$\mathcal{N} = 4 \quad \mathrm{SYM}$$



Classical Fields (simplest case)

Assume only x_3 -dependence and $x_3 > 0$, $A_\mu^{\rm cl} = 0, \qquad \Psi_A^{\rm cl} = 0$

Classical e.o.m.:

(x_3 is distance to defect)

$$\frac{d^2\phi_i^{\rm cl}}{dx_3^2} = \left[\phi_j^{\rm cl}, \left[\phi_j^{\rm cl}, \phi_i^{\rm cl}\right]\right].$$

Solution:

$$\phi_i^{\rm cl} = \frac{1}{x_3}\begin{pmatrix} (t_i)_{k\times k} & 0 \\ 0 & 0 \end{pmatrix}, \quad i = 1, 2, 3$$

Constable, Myers & Tafjord '99

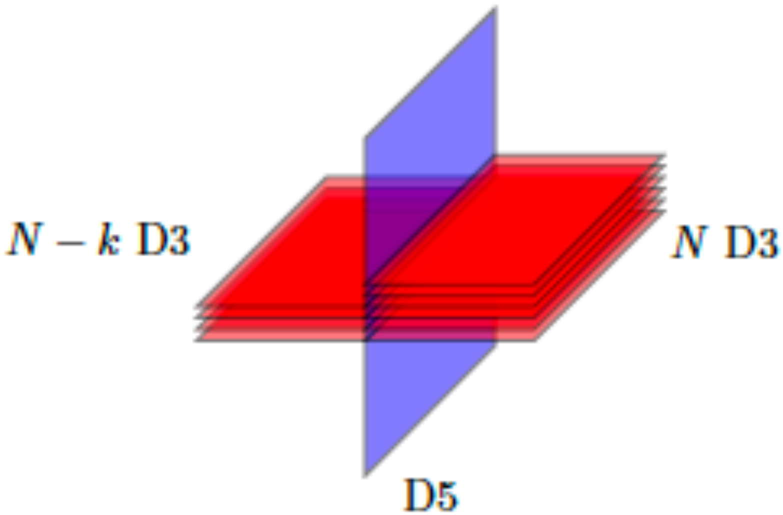
$$\phi_4^{\rm cl} = \phi_5^{\rm cl} = \phi_6^{\rm cl} = 0$$

where t_i, i=1,2,3, constitute a k-dimensional irreducible repr. of SU(2). (Nahm eqns. also fulfilled.)

Set-up ½ BPS (for appropriate choice b.c. for zero-modes, Gaiotto & Witten '08)

AdS/dCFT — The string theory side

	x^0	x^1	x^2	x^3	x^4	x^5	x^6	x^7	x^8	x^9
D3	×	×	×	×						
D5	×	×	×		×	×	×			



Karch & Randall '01,

Geometry of D5-brane: $AdS_4 \times S^2$

Background gauge field: k units of magnetic flux on S^2

One-point functions and $|\mathrm{MPS}_k\rangle$

$$\langle \mathcal{O}_\Delta^{\mathrm{bulk}}(x)\rangle = \frac{C}{|x_3|^\Delta}$$

Cardy ´84
McAvity & Osborn '95

Due to vevs scalar operators can have non-zero 1-pt fcts at tree-level

$$\langle \mathcal{O}_\Delta(x)\rangle = (\mathrm{Tr}(\phi_{i_1}\ldots\phi_{i_\Delta}) + \ldots)\,|_{\phi_i \to \phi_i^{\mathrm{cl}} = \frac{t_i}{x_3}}$$

$\mathcal{O}_\Delta(x) \sim$ eigenstate of integrable $SO(6)$ spin chain

Minahan & Zarembo '02

$$\mathrm{Tr}(\phi_{i_1}\phi_{i_2}\ldots\phi_{i_L}) \sim |s_{i_1}s_{i_2}\ldots s_{i_L}\rangle$$

Matrix Product State associated with the defect:

deLeeuw, C.K. & Zarembo '15,

$$|\mathrm{MPS_k}\rangle = \sum_{\vec{i}} \mathrm{tr}[t_{i_i}\ldots t_{i_L}]|s_{i_1}\ldots s_{i_L}\rangle,$$

Bethe eigenstate $\mathbf{u} = \{u_1^i, u_2^j, u_3^k\}$

Object to calculate:

$$C_k\,(\mathbf{u}) = \frac{\langle \mathrm{MPS}_k\,|\mathbf{u}\rangle}{\langle \mathbf{u}\,|\mathbf{u}\,\rangle^{\frac{1}{2}}}$$

Overlap Formulas — Experience form $\mathcal{N} = 4$ SYM

Selection rule

$\langle \Psi_0 | \mathbf{u} \rangle \neq \mathbf{0} \iff \{\mathbf{u_j}\} = \{-\mathbf{u_i}, \mathbf{u_i}\}$ Parity invariance

Ingredients:

de Leeuw, C.K. & Zarembo '15 de Leeuw, C.K. & Mori '16 de Leeuw, C.K. & Linardopoulos '18 de Leeuw, Gombor, C.K., Linardopoulos, Pozsgay '19

For $|\mathrm{MPS}_k\rangle$:

- Superdeterminant of Gaudin matrix: $\mathbb{D} = S\det G = \frac{\det(G_+)}{\det(G_-)}$

  $\langle \mathbf{u} | \mathbf{u} \rangle = \det G = \det G_+ \det G_-$

- Ratios of Baxter polynomiums (reduced): $Q(u) = \prod_i (u^2 - u_i^2)$
- "Transfer matrices": Sums of ratios of Baxter polynomials: $\sum_{a=-\frac{q}{2}}^{a=\frac{q}{2}} \ldots$

For $|\mathrm{VBS}\rangle$:

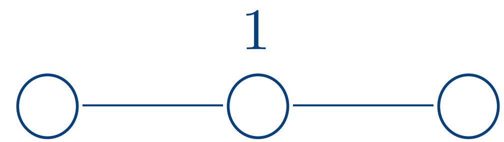
- No sums involved Poszgay '18 Gombor '21 (Analytical proof for bosonic chains)

$|\mathrm{VBS}\rangle$ more fundamental, starting point for deriving $|\mathrm{MPS}\rangle$ overlaps

$|\text{VBS}\rangle$ overlaps in $\mathcal{N}=4$ SYM

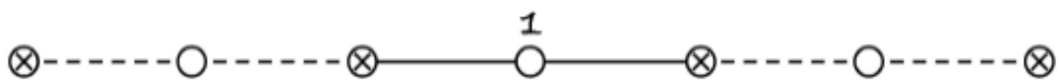
$SO(6)$: $\quad |\text{VBS}\rangle = (|XX\rangle + |YY\rangle + |ZZ\rangle + |\bar{X}\bar{X}\rangle + |\bar{Y}\bar{Y}\rangle + |\bar{Z}\bar{Z}\rangle)^{\otimes L/2}$,

$$C = \frac{Q_1(0)Q_2(0)Q_3(0)}{Q_1\left(\frac{i}{2}\right)Q_2\left(\frac{i}{2}\right)Q_3\left(\frac{i}{2}\right)} S \det G$$



Gombor '21
de Leeuw, Gombor, C.K., Linardopoulos, Pozsgay '19

$PSU(2,2|4)$:



$$C = \frac{Q_1(0)Q_3(0)Q_4(0)Q_5(0)Q_7(0)}{Q_2(0)Q_2(\frac{i}{2})Q_4(\frac{i}{2})Q_6(0)Q_6(\frac{i}{2})} S \det G$$

Bajnok & Gombor '20

Found by "bootstrap" (S-matrix known, BYB, unitary, crossing) and subsequent analytical continuation

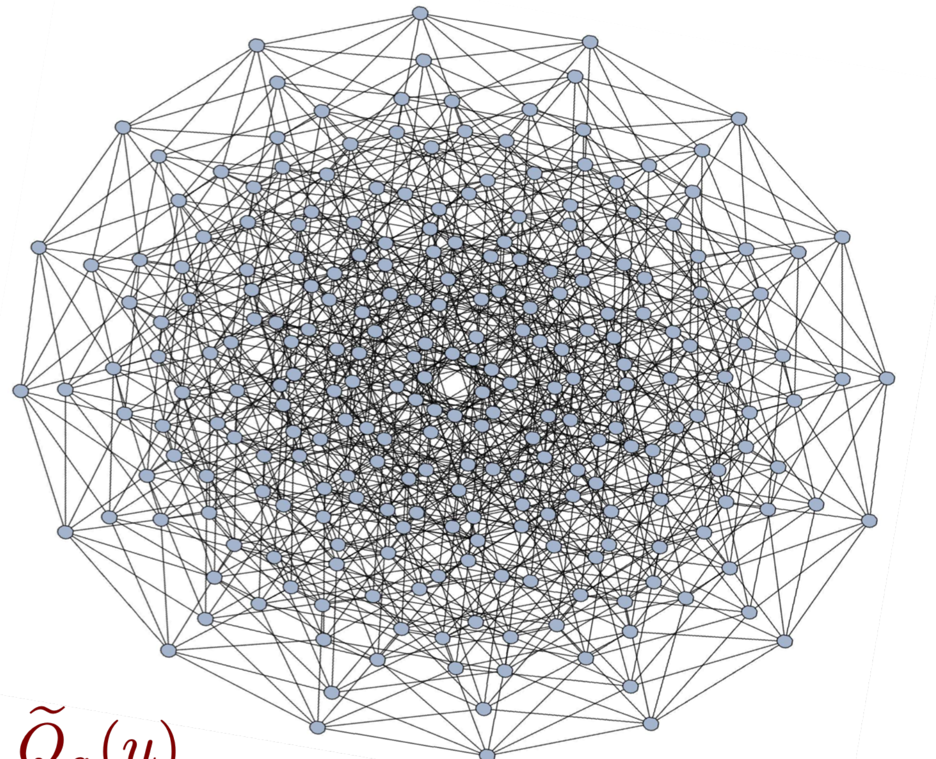
$|\text{VBS}\rangle$ overlap of relevance for AdS/dCFT singled out by transforming covariantly under fermionic duality

C.K., Müller & Zarembo '20

QQ-system

Many equivalent ways of writing the Bethe equations

For $\mathcal{N}=4$ SYM, # different choices of Q-functions $= 2^8$



Connected via dualities

- Fermionic (Change of Dynkin diagram)
- Bosonic

Dualities = Change of variables in the Bethe equations: $Q_a(u) \to \widetilde{Q}_a(u)$

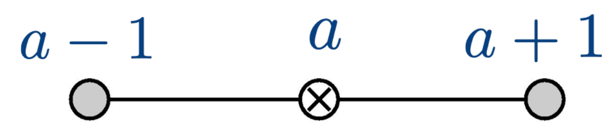
Tsuboi '98
Kazakov '18

$|\text{VBS}\rangle$ overlap of relevance for AdS/dCFT singled out by transforming covariantly under fermionic duality

Fermionic dualities in general

- Allow one to move between any two Dynkin diagrams of a super Lie algebra (of type $SU(N|M)$)

- Involve a fermionic node and its neighbours only



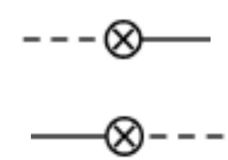
$$Q_a \to \widetilde{Q}_a \ : \quad Q_a \widetilde{Q}_a = Q^-_{a-1} Q^+_{a+1} - Q^+_{a-1} Q^-_{a+1}$$

- Changes the nature of neighbouring nodes $\bigotimes \longleftrightarrow \bigcirc$ and the connections —— $\longleftrightarrow$ - - -

- Dualized node non-momentum carrying $\implies$ Dynkin labels unchanged

- Dualized node momentum carrying $\implies$ Dynkin labels change

$$\begin{bmatrix} 0 \\ V \\ 0 \end{bmatrix} \longrightarrow \begin{bmatrix} V \pm 1 \\ -V \\ V \mp 1 \end{bmatrix} \quad \text{for}$$



Transformation rule for Gaudin determinant

Fermionic duality after node a: $Q_a \to \widetilde{Q}_a$

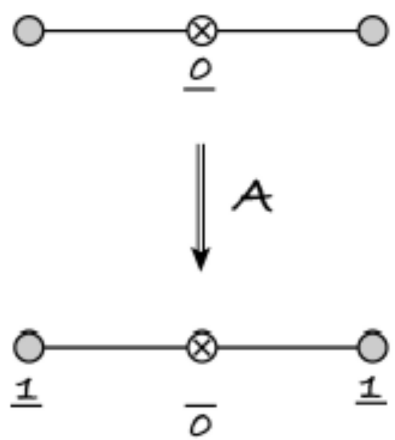
$$Q_a(0)\mathbb{D} = Q_{a-1}(i/2)\, Q_{a+1}(i/2)\; \frac{\widetilde{\mathbb{D}}}{\widetilde{Q}_a(0)}$$

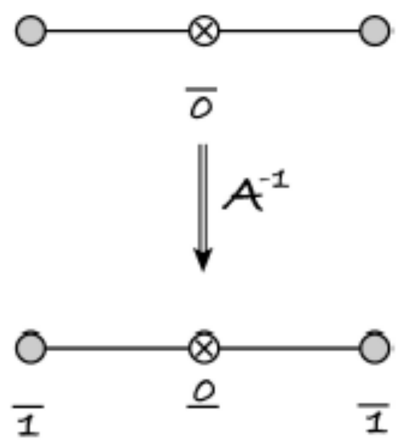
Found numerically

Analytical proof in progress

C.K., Müller, Zarembo '20

OBS: Covariance of overlap formula which involves $Q_a(0)\mathbb{D}$ or $\widetilde{\mathbb{D}}/\widetilde{Q}_a(0)$



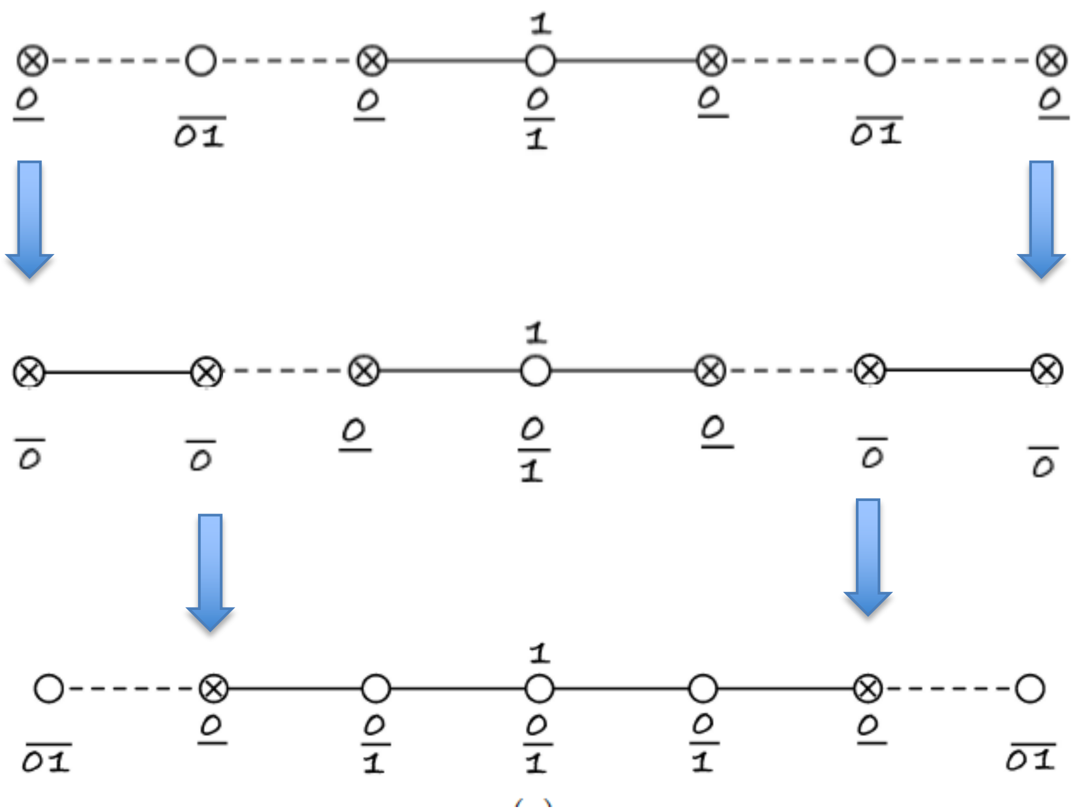


Covariance of overlap formulas very constraining

Dualizing overlap formulas

$PSU(2,2|4)$ overlap formula, alternating grading

Gombor & Bajnok '20



Agrees with field theory result in $SO(6)$ sector

C.K., Müller, Zarembo '20

Covariance requirement fixes the overlap formula from $SO(6)$ result

ABJM theory

ABJM theory in 3D $\longleftrightarrow$ Type IIA strings on $AdS_4 \times CP^3$

$\mathcal{N} = 6$ susy

Field content: $A_\mu, \hat{A}_\mu, \Psi_A, Y^A, \;\; A = 1, 2, 3, 4$

Gauge symmetry: $U(N)_k \times \hat{U}(N)_{-k}$

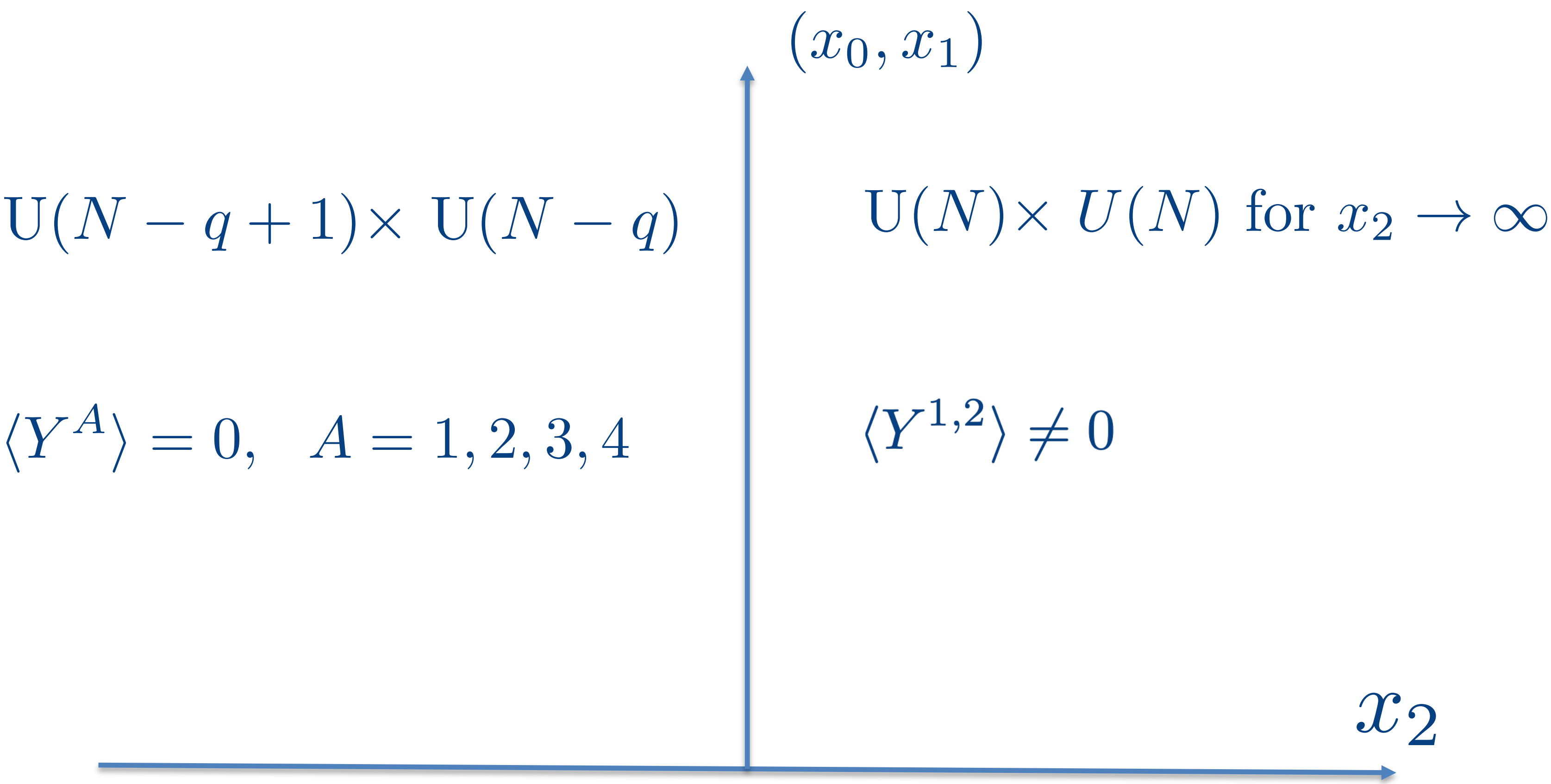
Planar 't Hooft limit: $N, k \to \infty, \; \lambda = \frac{N}{k}$ fixed

Integrable in the planar limit

$$\begin{aligned}\mathcal{L} = \quad & \frac{k}{4\pi} \operatorname{tr} \Big[\varepsilon^{\mu\nu\lambda} \Big(A_\mu \partial_\nu A_\lambda + \frac{2}{3} A_\mu A_\nu A_\lambda - \hat{A}_\mu \partial_\nu \hat{A}_\lambda - \frac{2}{3} \hat{A}_\mu \hat{A}_\nu \hat{A}_\lambda \Big) \\ & + D_\mu Y_A^\dagger D^\mu Y^A + \frac{1}{12} Y^A Y_A^\dagger Y^B Y_B^\dagger Y^C Y_C^\dagger + \frac{1}{12} Y^A Y_B^\dagger Y^B Y_C^\dagger Y^C Y_A^\dagger \\ & - \frac{1}{2} Y^A Y_A^\dagger Y^B Y_C^\dagger Y^C Y_B^\dagger + \frac{1}{3} Y^A Y_B^\dagger Y^C Y_A^\dagger Y^B Y_C^\dagger + \text{fermions} \Big] .\end{aligned}$$

The defect set-up of $|\mathrm{MPS}\rangle$

ABJM theory



Classical fields

Classical e.o.m.: $\frac{d^2Y^\alpha}{dx_2^2} = \ldots$

$\langle Y^3\rangle = \langle Y^4\rangle = 0$

$A_\mu = \hat{A}_\mu = 0,\ \Psi^A = 0$

$\Uparrow$

BPS eqns: $\frac{dY^\alpha}{dx_2} = \frac{1}{2}Y^\alpha Y^\dagger_\beta Y^\beta - \frac{1}{2}Y^\beta Y^\dagger_\beta Y^\alpha, \qquad \alpha,\beta = 1,2$

Basu-Harvey eqns.

$$\mathcal{L} = \frac{k}{4\pi}\,\mathrm{tr}\Big[\varepsilon^{\mu\nu\lambda}\Big(A_\mu\partial_\nu A_\lambda + \frac{2}{3}A_\mu A_\nu A_\lambda - \hat{A}_\mu\partial_\nu\hat{A}_\lambda - \frac{2}{3}\hat{A}_\mu\hat{A}_\nu\hat{A}_\lambda\Big) + D_\mu Y^\dagger_A D^\mu Y^A + \frac{1}{12}Y^AY^\dagger_AY^BY^\dagger_BY^CY^\dagger_C + \frac{1}{12}Y^AY^\dagger_BY^BY^\dagger_CY^CY^\dagger_A - \frac{1}{2}Y^AY^\dagger_AY^BY^\dagger_CY^CY^\dagger_B + \frac{1}{3}Y^AY^\dagger_BY^CY^\dagger_AY^BY^\dagger_C + \text{fermions}\Big].$$

Classical fields

Terashima '08

$$\langle Y^\alpha \rangle = \frac{1}{\sqrt{x_2}} \begin{pmatrix} S^\alpha_{(q-1)\times q} & 0 \\ 0 & 0_{(N-q+1)\times(N-q)} \end{pmatrix}, \quad \alpha = 1, 2$$

$$\langle Y^3 \rangle = \langle Y^4 \rangle = 0$$

$$S^1_{ij} = \delta_{i,j-1}\sqrt{i}, \qquad S^2_{ij} = \delta_{ij}\sqrt{q-i}, \qquad i = 1, \ldots, q-1, \;\; j = 1, \ldots, q$$

$$S^1_{(q-1)\times q} = \begin{pmatrix} 0 & 1 & 0 & \ldots & 0 & 0 \\ 0 & 0 & \sqrt{2} & 0 & \ldots & 0 \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ 0 & . & . & . & 0 & \sqrt{q-1} \end{pmatrix} \quad S^2_{(q-1)\times q} = \begin{pmatrix} \sqrt{q-1} & 0 & 0 & \ldots & 0 & 0 \\ 0 & \sqrt{q-2} & 0 & \ldots & 0 & 0 \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ 0 & . & . & . & 1 & 0 \end{pmatrix}$$

One-point functions and MPS

$$\langle \mathcal{O}_\Delta(x) \rangle = \frac{C}{|x_2|^\Delta}$$

Cardy ´84

McAvity & Osborn ’95

Due to vevs scalar operators can have non-zero 1-pt fcts at tree-level

$$\langle \mathcal{O}_\Delta(x) \rangle = \Big(\mathrm{Tr}(Y^{\alpha_1} Y^\dagger_{\beta_1} \ldots Y^{\alpha_L} Y^\dagger_{\beta_L}) + \ldots \Big) \,|_{Y^{\alpha_i} \to \langle Y^{\alpha_i} \rangle}$$

$\mathcal{O}_\Delta(x) \sim$ eigenstate of integrable alternating $SU(4)$ spin chain

$$\mathrm{Tr}(Y^{\alpha_1} Y^\dagger_{\beta_2} \ldots Y^{\alpha_L} Y^\dagger_{\beta_L}) \sim |s^{\alpha_1} \bar{s}_{\beta_2} \ldots s^{\alpha_L} \bar{s}_{\beta_L} \rangle$$

Minahan & Zarembo ’08

$$H = \lambda^2 \sum_{l=1}^{2L} \left(1 - P_{l,l+2} + \frac{1}{2} P_{l,l+2} K_{l,l+1} + \frac{1}{2} K_{l,l+1} P_{l,l+2} \right),$$

One-point functions and $|\mathrm{MPS}\rangle$

$$\langle \mathcal{O}_\Delta(x)\rangle = \Big(\mathrm{Tr}(Y^{\alpha_1}Y^\dagger_{\beta_1}\ldots Y^{\alpha_1}Y^\dagger_{\beta_1}+\ldots\Big)\,\Big|_{Y^{\alpha_i}\to\frac{S^{\alpha_i}}{\sqrt{x_2}}}$$

C.K., Vu
& Zarembo '21,

Two Matrix Product States associated with the defect:

$$|\mathrm{MPS}_{\mathrm{q}-1}\rangle = \sum_{\vec\alpha,\vec\beta}\mathrm{Tr}[S^{\alpha_1}S^\dagger_{\beta_1}\ldots S^{\alpha_L}S^\dagger_{\beta_L}]|s^{\alpha_1}\bar{s}_{\beta_1}\ldots s^{\alpha_L}\bar{s}_{\beta_L}\rangle,$$

$$|\widehat{\mathrm{MPS}}_q\rangle = \sum_{\vec\alpha,\vec\beta}\mathrm{Tr}[S^\dagger_{\alpha_1}S^{\beta_1}\ldots S^\dagger_{\alpha_L}S^{\beta_L}]|\bar{s}_{\alpha_1}s^{\beta_1}\ldots \bar{s}_{\alpha_L}s^{\beta_L}\rangle,$$

Bethe eigenstate

Object to calculate: $$C_q(\mathbf{u}) = \frac{\langle \mathrm{MPS}_{q-1}|\mathbf{u}\rangle}{\langle\mathbf{u}|\mathbf{u}\rangle^{\frac{1}{2}}} = \frac{\langle \widehat{\mathrm{MPS}}_q|\mathbf{u}\rangle}{\langle\mathbf{u}|\mathbf{u}\rangle^{\frac{1}{2}}}$$

Connection to SU(2) reps.

C.K., Müller, Zarembo '20

Nastase '09

For $\alpha = 1, 2$:

$$\Phi^{\alpha}_{\beta} = Y^{\alpha} Y^{\dagger}_{\beta} \equiv \Phi^{i} (\sigma_i)^{\alpha}_{\beta} + \Phi \delta^{\alpha}_{\beta}, \qquad (q-1) \times (q-1) \text{ matrix}$$

$$\frac{d\Phi^{i}}{dx} = \frac{i}{2} \epsilon^{ijk} \left[\Phi^{j}, \Phi^{k}\right] \qquad \text{Nahm's equation} \qquad \frac{d\Phi}{dx} = \Phi^{i}\Phi^{i} - \Phi^{2}$$

Solution:

$$\Phi^{i} = \frac{t^{i}}{x}, \qquad (\{t^{i}\} = (q-1)\text{-dim. irrep. of SU}(2))$$

$$\Phi = \frac{q}{2x} I_{q-1}$$

For $q = 2$: $\Phi^{\alpha}_{\beta} = \delta^{\alpha}_{\beta}$, i.e. $|\text{MPS}_1\rangle = |\text{VBS}\rangle$

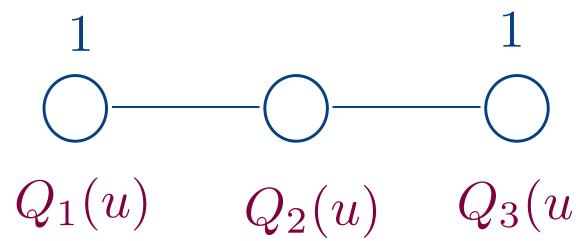
Similarly for $\widehat{\Phi}^{\alpha}_{\beta} = Y^{\dagger}_{\beta} Y^{\alpha}$, with q-dimensional rep. of $SU(2)$

The alternating integrable $SU(4)$ spin chain of ABJM theory

Vacuum state: $\mathrm{Tr}(Y^1 Y_2^\dagger)^L$

Excited states described in terms of Bethe roots $\{u_1^{(i)}\}$, $\{u_2^{(j)}\}$, $\{u_3^{(k)}\}$

Roots of Baxter polynomials

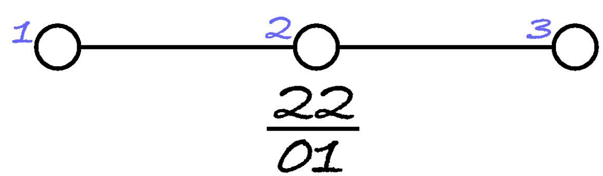


Overlaps non-vanishing for states with Z_2 symmetry: $\Omega : u_1^{(k)} \leftrightarrow -u_3^{(k)}$

Result for $|\mathrm{VBS}\rangle$: $C = Q_2(i)\sqrt{\frac{S\det G}{Q_2(0)Q_2(\frac{i}{2})}}$

Gombor '21,

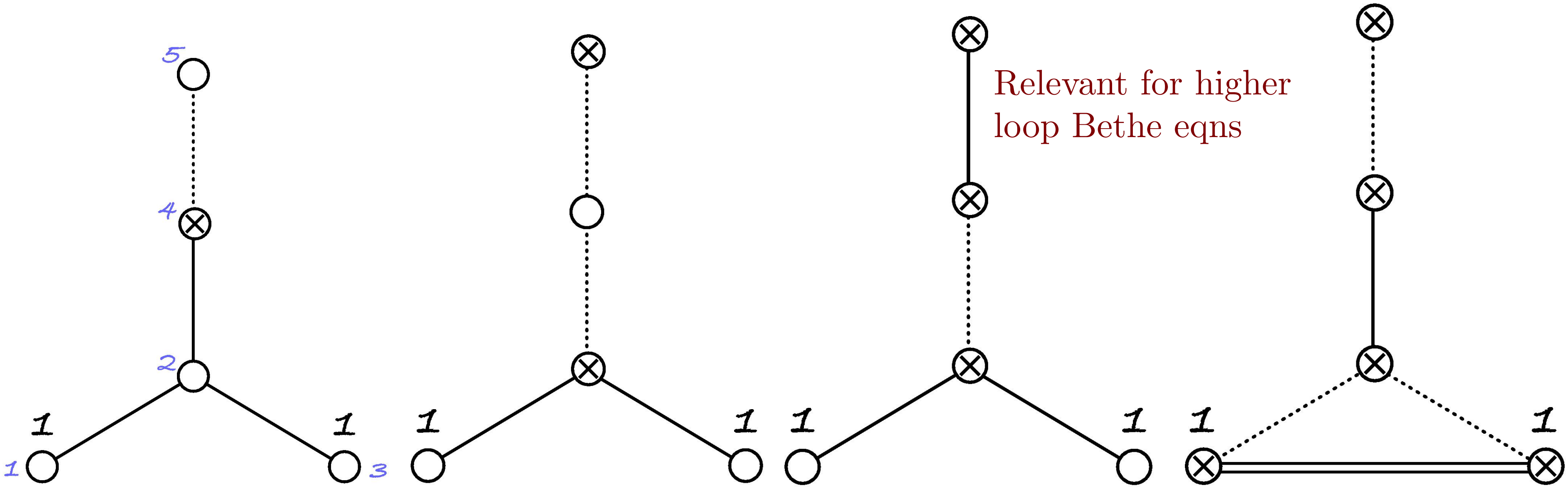
C.K., Vu
& Zarembo '21,



What about the full ABJM theory ?

The full Osp(6|4) spin chain of ABJM theory

Possible Dynkin diagrams



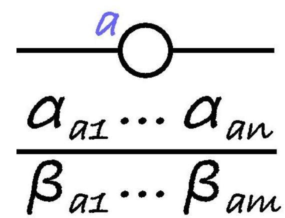
All connected via fermionic dualities

Idea: Determine the complete overlap formula by requiring covariance under fermionic duality

Fixing overlap by covariance requirement

Assume factorized formula (possibly a sum of such terms)

$$C = \sqrt{\prod_n \frac{\prod_j Q_n(i\alpha_{aj}/2)}{\prod_k Q_n(i\beta_{ak}/2)}\,\mathbb{D}}$$



Fermionic duality transformation after node a

Compatible with all data

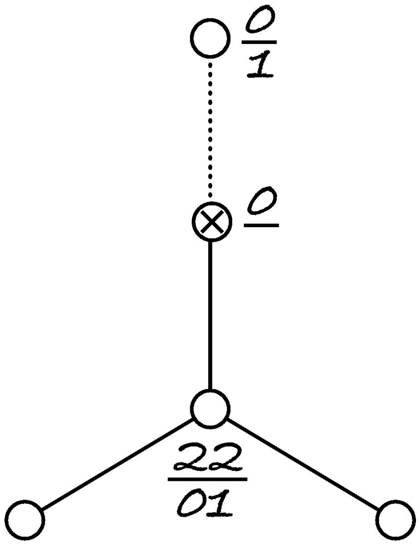
$$Q_a(0)\;\mathbb{D} = \prod_{\text{b:neigbour}} Q_b(i/2)\;\frac{\widetilde{\mathbb{D}}}{\widetilde{Q}_a(0)}$$

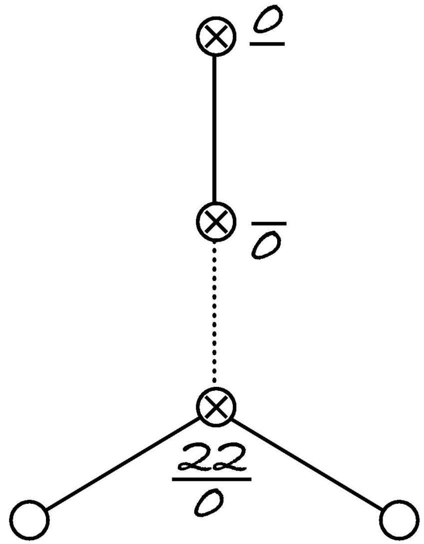
Shown numerically, holds semi-on-shell

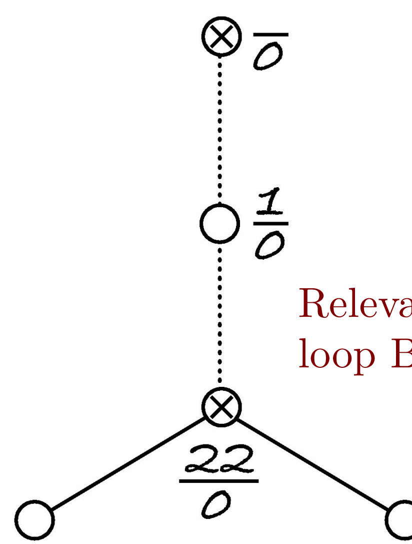
C.K., Müller & Zarembo ‘21, C.K., Vu & Zarembo ‘21,

Overlap formula in different gradings

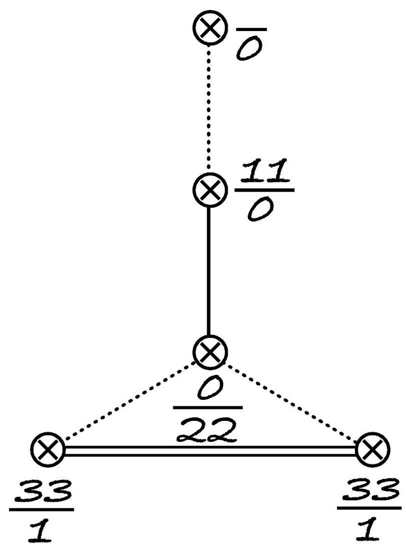
C.K., Vu & Zarembo '21,

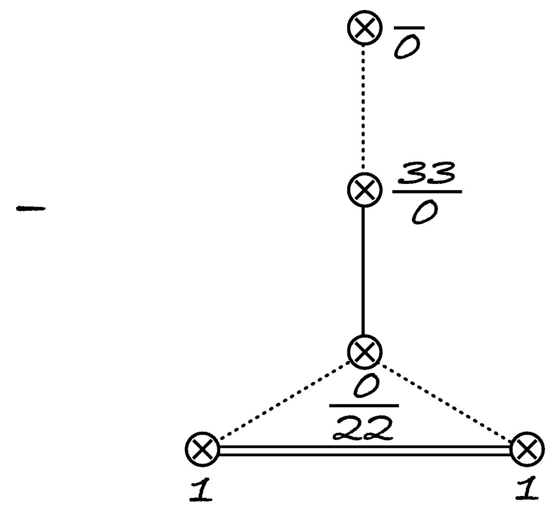






Relevant for higher loop Bethe eqns





Future directions

- Bootstrap the formula to higher loop orders (has been done for N=4 SYM)
- Consider MPS with higher bond dimension
- Other integrable defect set-ups ? (Coulomb branch, co-dimension-2 defects….)
- Classification of integrable boundary states in AdS/CFT (VBS, MPS, cross-cap states (?))
- Proof of predicted ABJM overlap formula
- Proof of the duality transformation formula
- Derive the TBA for overlaps (Finite size effects).

Thank you