Hexagons, QSC and the Bottom bridge

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Summery

- Hexagons are a powerfull tool for computing structure constants.
- As it happens for the anomalous dimensions finite size effects (Wrapping) are difficult to control
- We will mainly focus on a particular contribution to the hexagons (Bottom bridge)

OverwieV

- $\mathcal{N}=4$ Super Yang-Mills (SYM)
- A Bit of Quantum Spectral Curve (QSC)
- Hexagons, Wrapping & some conjectures
- Results (5-loops from QSC)
- Conclusions

Integrability in $\mathcal{N}=4$ SU(N) SYM

- $d=4$ interacting CFT. Composed of:

  $$\left\{ \underset{1}{A^\mu}, \underset{6}{\Phi^I}, \underset{4}{\psi^a_{\dot\alpha}}, \underset{4}{\psi_{\dot\alpha a}} \right\}$$ all transforming in the Adjoint SU(N)

- Theory is dual to String theory on $AdS_5 \times S^5$

- Planar limit $(N \to \infty)$ the theory is integrable

- Supersymmetry + conformal invariance $\Rightarrow$ $PSU(2,2|4)$

$$\boxed{\gamma \text{ and } C_{IJK}}$$

- Given conformal invariance, we are interested into two quantities: anomalous dimensions & structure const.

- $\mathcal{O}(x) \xrightarrow{x \to \lambda x} \lambda^{-\Delta} \mathcal{O}(\lambda x)$

scaling dimension

$$\boxed{\text{if op is not protected this gets corrections!}}$$

- OPE expansion

$$\mathcal{O}_I(x)\, \mathcal{O}_J(y) \simeq \sum_K C_{IJK}\, g(\Delta, x-y)\, \mathcal{O}_K(x-y)$$

structure constants $\langle O_I O_J O_K \rangle = C_{IJK} / \ldots$

- Higher points correlation functions can be reduced with OPE

Spectrum

- In the planer limit we need to focus on single trace operators only: $\mathcal{O}(x) = \mathrm{Tr}[\dots\dots]$ → field + derivatives of $\mathcal{N}=4$

- Single trace operators, can be seen as spin-chains with periodic boundery conditions

$$\mathrm{Tr}[Z X Z Z Z Z X] \longrightarrow \uparrow\downarrow\uparrow\uparrow\uparrow\uparrow\downarrow$$

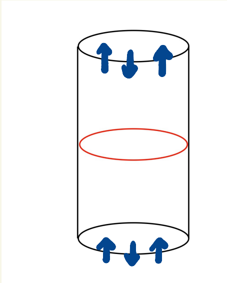
$$iD \longrightarrow H$$

$$\gamma_n \longrightarrow E_n$$

non-protected can have γ_n

protected, Δ is correct ($\frac{1}{2}$BPS - $\mathrm{Tr}[Z^L]$)

Spectrum of $\mathcal{N}=4$

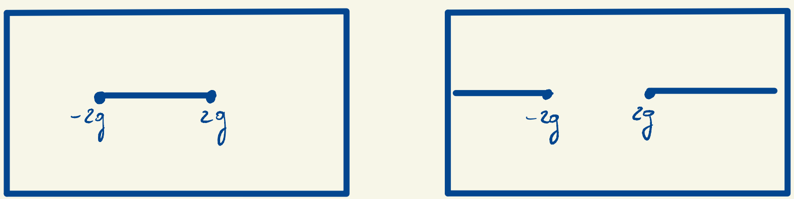


- Problem in trying to move up in perturbative order (away from large chains) are finite size corrections.
- TBA first solution for obtaining γ_n at finite coupling (complicated set of nested integral eq.)
- TBA $\longrightarrow$ Y-System $\longrightarrow$ T (hirota) $\longrightarrow$ QSC

A Bit of QSC

- A non-perturbative approach to computing Δ.
- 8 basic Q-functions

$$(P_1, P_2, P_3, P_4 \mid Q_1, Q_2, Q_3, Q_4)$$



- Global information about the state is encoded into asymptotics ($u \to \infty$)
- Obtaining these functions is a R-H probem + finite difference eq.
- Can obtain T-matrices from combinations of Q-functions

- QQ-Relations: $Q_i = -P^a Q^+_{a|i}$, $Q^i = P_a Q^{a|i+}$

$$Q^+_{a|i} - Q^-_{a|i} = P_a Q_i \qquad P_a P^a = 0 = Q_i Q^i \qquad Q^{a|i} = -\left(Q^+_{a|i}\right)^{-1}$$

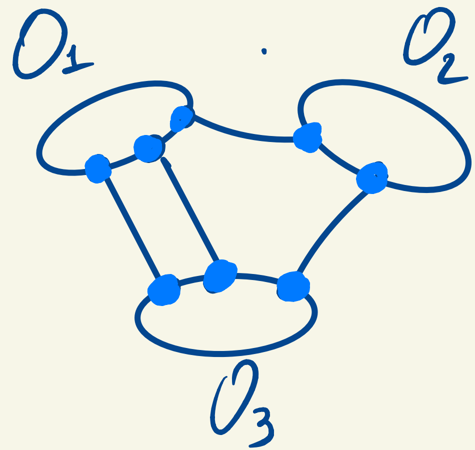
- two extra i-periodic functions

μ_{ab}, ω_{ij} $\longleftarrow$ used for analytical continuation

P-μ System		**Q-ω System**
$P_a, P^a, \tilde{P}_a, \tilde{P}^a$	$\xleftrightarrow{\;Q_{a\|i}\;}$	$Q_i, Q^i, \tilde{Q}_i, \tilde{Q}^i$
$\mu_{ab}, \tilde{\mu}_{ab}$		$\omega_{ij}, \tilde{\omega}_{ij}$

Structure Constants

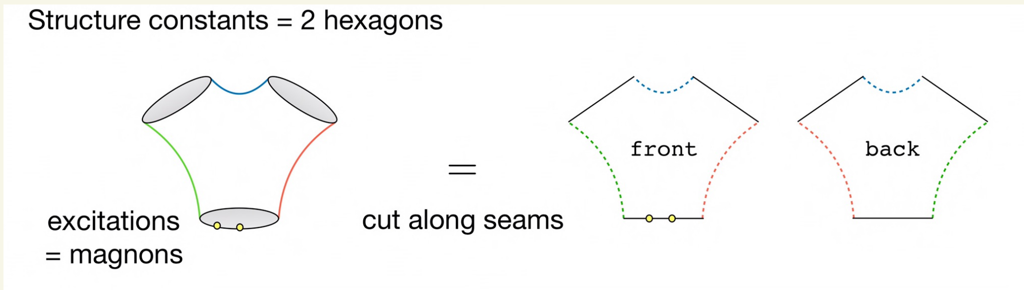
- As we have seen structure constants are associated with 3-point function:



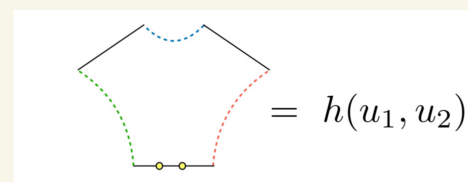
- We will focus on the simplest case where only $\mathcal{O}_3$ is non-protected

Hexagons

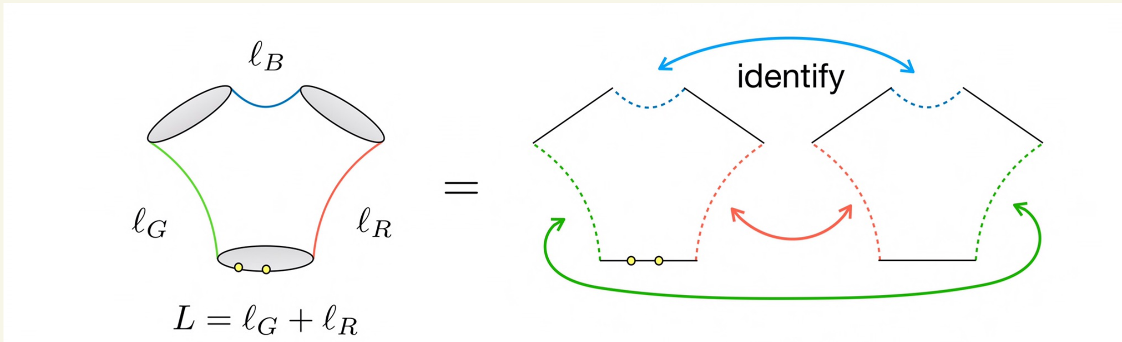
- Building blocks for correlation functions (single trace Op)



- Hexagon form factor conjecturally known to all loops, fixed by symmetry and form-factors axioms

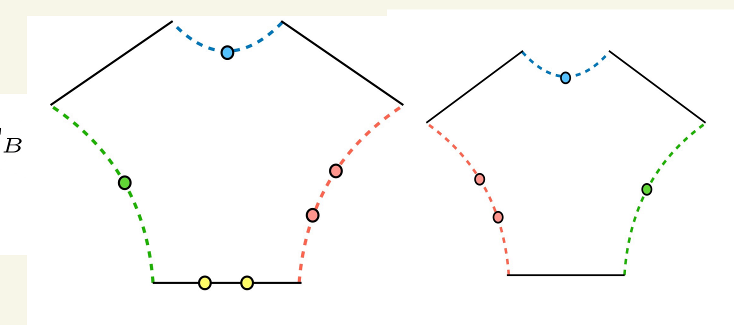


- How does it work?

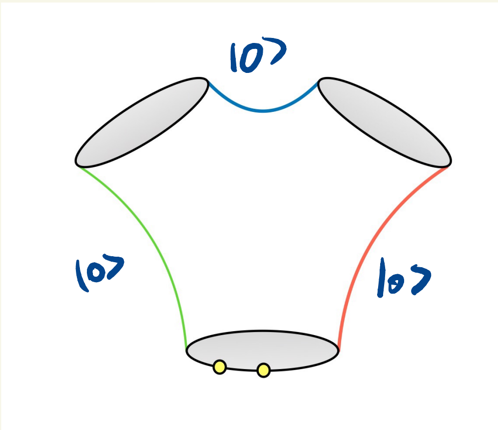


To obtain C_{ijk}, we need to sum over the 3-mirror channels

$$\sum_{N_{\circ}, N_{\circ}, N_{\circ}} e^{-\ell_G E_G - \ell_R E_R - \ell_B E_B}$$



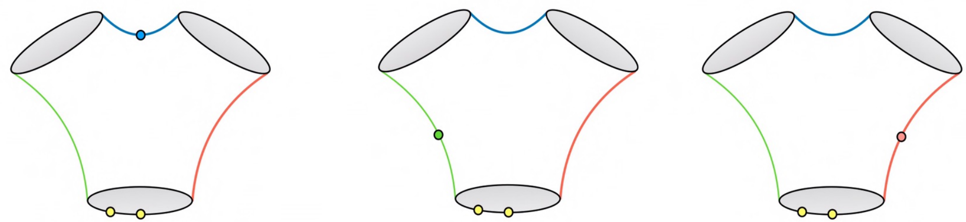
The asymptotic contribution, when we have 3-vacuums



$\approx$ Gaudin^{-1} $\times$ Sum over partitions of Bethe roots

- for large enough bridges ($\ell_g, \ell_r, \ell_b \gg 1$) we can suppress the other contribution.

The other contributions look like (we specialize to the "lowest" operator)



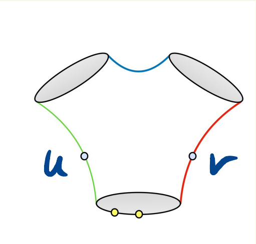
1 blue mirror magnon @2-loop

1 red/green @3-loop

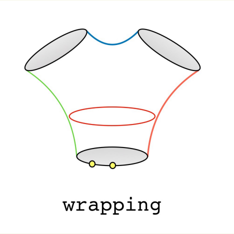
- Still under control, for example the first term

$$\text{Asymptotic contribution} \times \sum_{a=1}^{\infty} \int du \, \frac{T_a(u^{\gamma})\, \mu_a(u^{\gamma})}{\left(y^{[a]} y^{[-a]}\right)^{\ell} \prod_j h_a(u^{\gamma}, u_j)} \Longrightarrow 36\,\zeta_3 \text{ for Konishi}$$

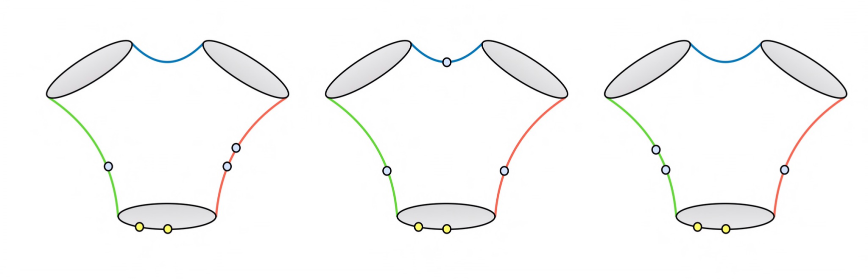
- At 4-loops we encounter the first divergence

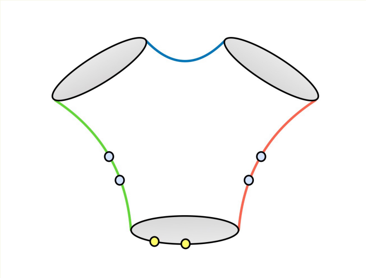


pole $u=v$ $\Longrightarrow$

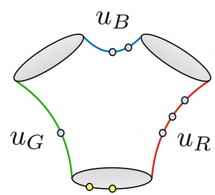


- We get way more terms at the next orders.



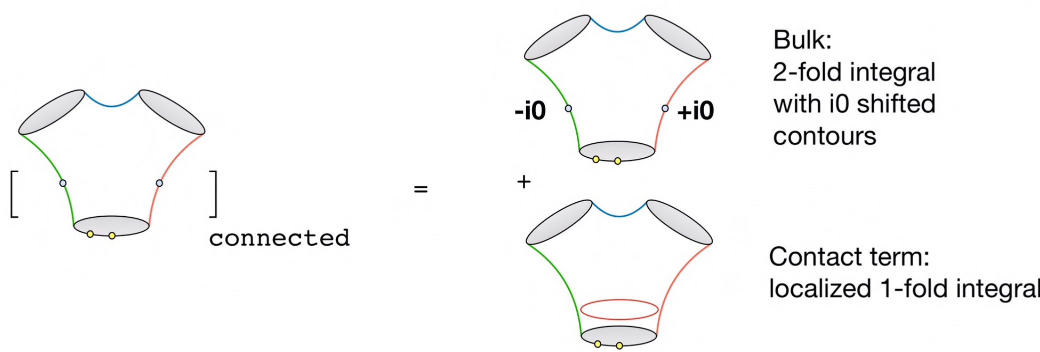


- Let us look at the bare integrand:

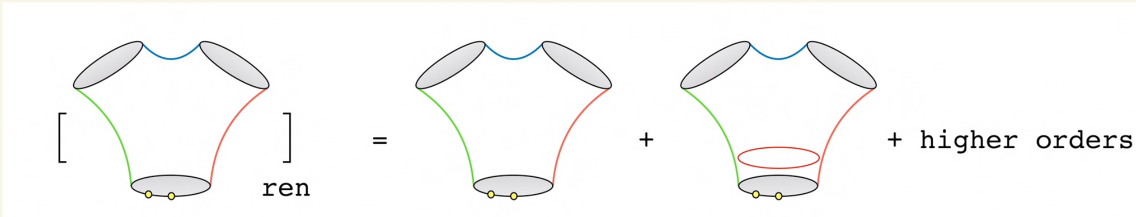


$$\frac{e^{-\ell_G E_G} W_G(u_G) e^{-\ell_B E_B} W_B(u_B) e^{-\ell_R E_R} W_R(u_R)}{\Delta(u_G, u_R)}$$

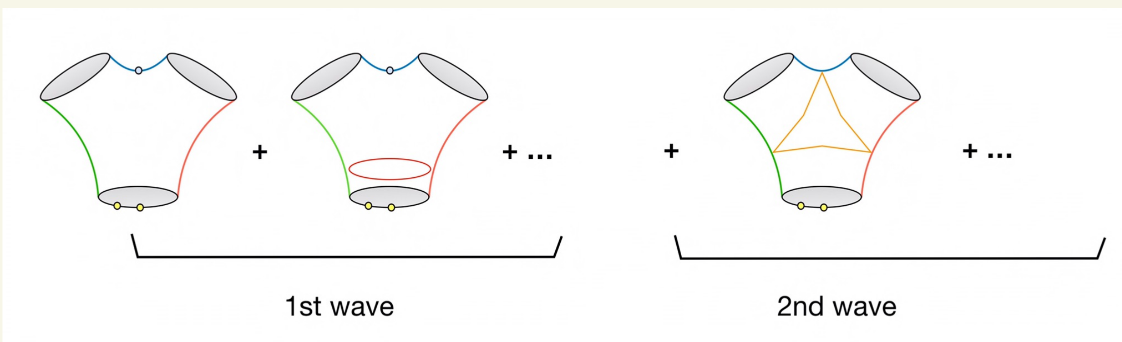
- divergence comes from $\Delta(u_G, u_r)$, we could shift the integration contour and add a contact term (proper derivation using form-factors)



- In this process the contact term modifies the asymptotic contribution



- By doing the same procedure @ 5-loops discovered a new contribution



Renormalized Integrand

We can now write down a renormalized version of our bare integrand

$$\boxed{C_{ijk} = \mathcal{N} \sum \int \frac{W_G\, W_B\, W_R}{\Delta_{GR}^{\text{tio}}}}$$

$$W_{G,R}(u) = e^{-\ell_{G,R}E}\,\mathbb{T}_{+,-}(u^\gamma)$$

$$W_B(u) = e^{-\ell_B E}\,\mathbb{T}_-(u^{-\gamma}) + e^{-\ell_D E}\,\mathbb{T}_+(u^{+\gamma})$$

dressed Transf. M.

$$e^{-LE}\,\mathbb{T}_+(u^\gamma)\mathbb{T}_-(u^\gamma) = \frac{Y(u^\gamma)}{1+Y(u^\gamma)}$$

$$\mathbb{T}_+(u^\gamma)/\mathbb{T}_-(u^\gamma) = e^{i\int \partial_v \ln \Delta(v^\gamma, u^\gamma)\log(1+Y(v^\gamma))}$$

QSC-Integrand

- Using a relation between the different $\mathbb{W}_i$ we can write down:

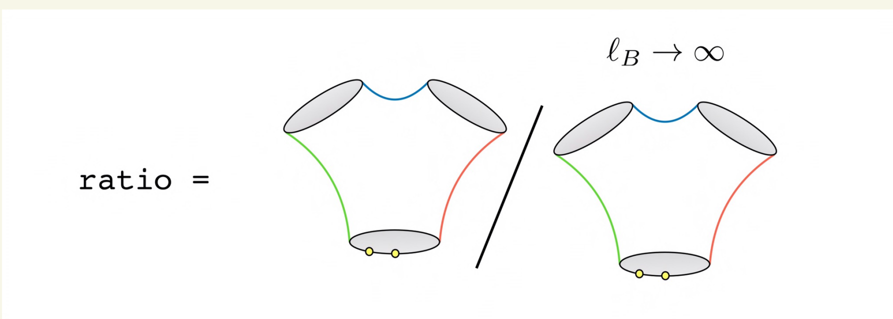
$$\mathbb{W}_B \simeq - \frac{Q_i^{[+a]} Q^{i[-a]}}{y^{[+a]} y^{[-a]}} \mu_a(v^r).$$

- $Q_i^{[+a]} Q^{i[-a]}$ can be extracted perturbatively from the QSC.

Match with perturbative data

C_{ijk} for our "lowest" operator (Konishi) with two protected ones $(\langle \mathrm{Tr}[Z^2]\,\mathrm{Tr}[D^2 Z^2]\,\mathrm{Tr}[Z^2]\rangle)$ has been computed using perturbation theory (Feynman diagrams) up to 5-loops.

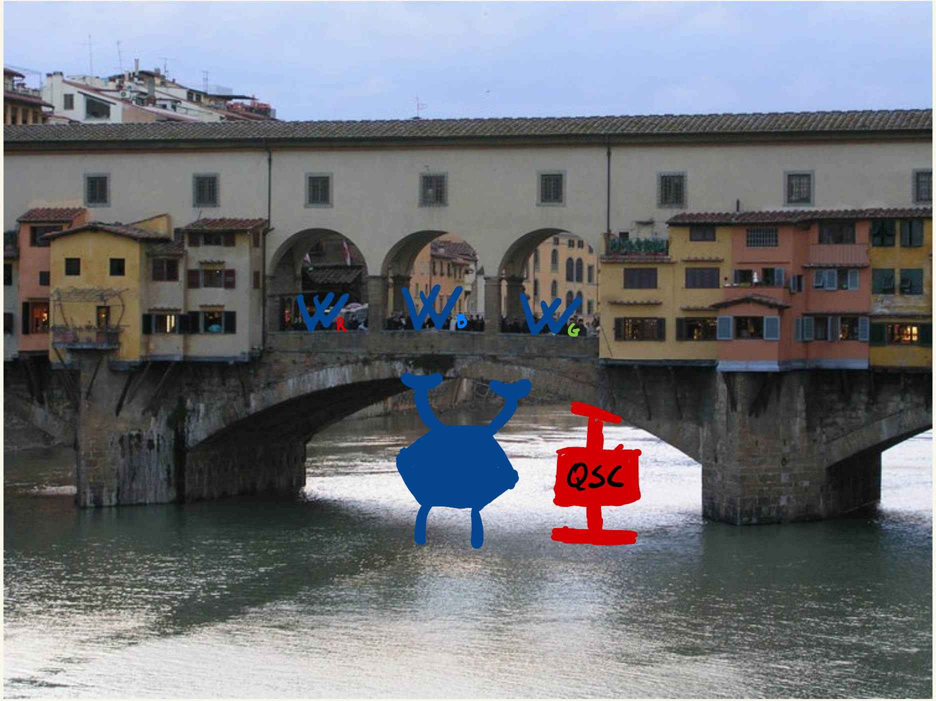
- We can extract the bottom contribution by:



$\Longrightarrow$ Matches with $\mathcal{W}_B$!

Conclusions

- Interpret "triple-wrapping" as new foundamental object in the Bottom bridge
- Constructed corrected integrand to account for new effect using QSC building blocks (Non trivial 5-loop check with perturbative data)
- We also have several strong coupling checks that support our conjectures. Same ideas also can be applied to Adjacent bridges.



Thank you!