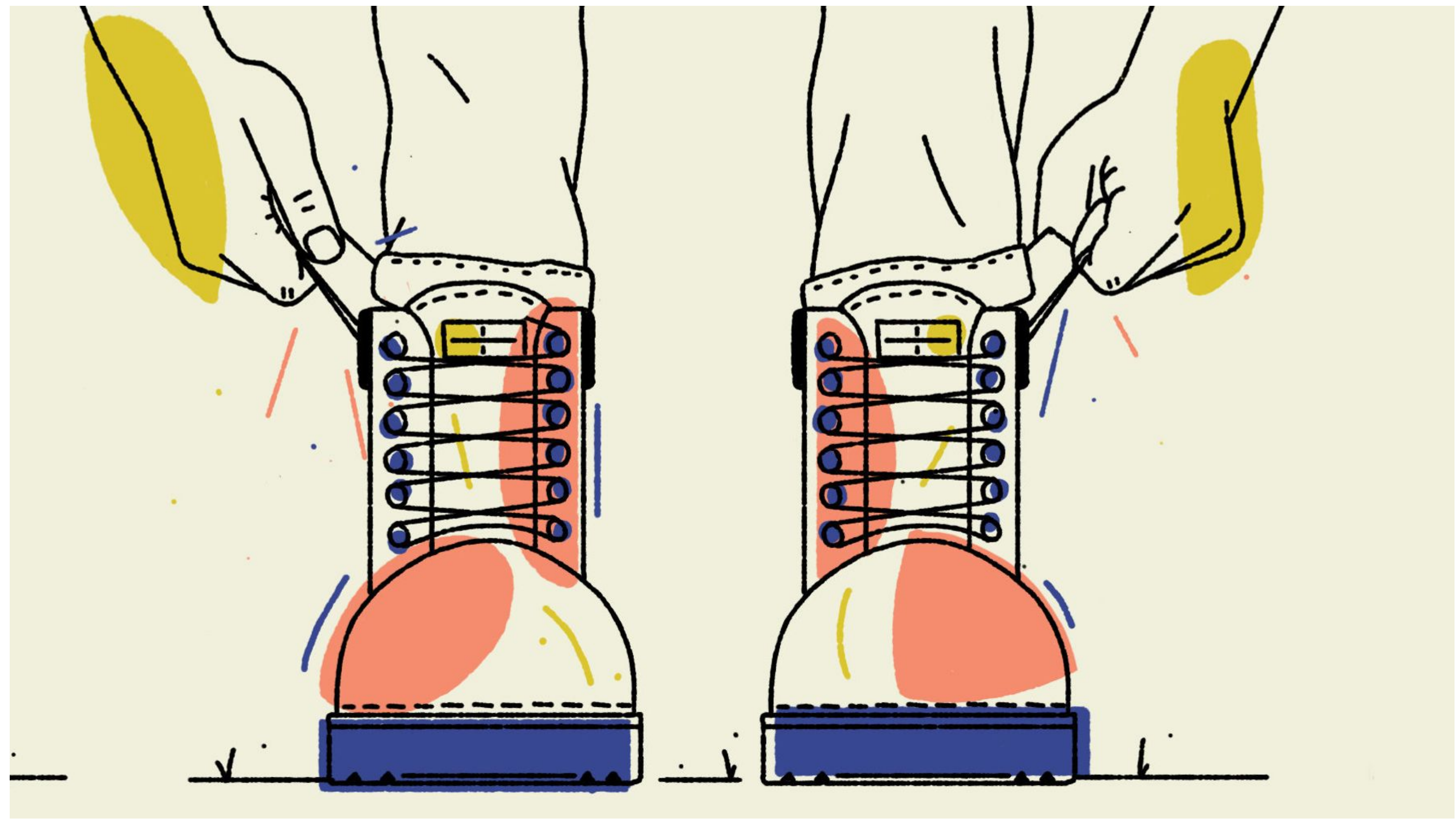
Pulling yourself up by your Bootstraps in Quantum Gravity

Leonardo Rastelli

Yang Institute, Stony Brook



Galileo Galileo Institute
Colloquium
April 27 2022

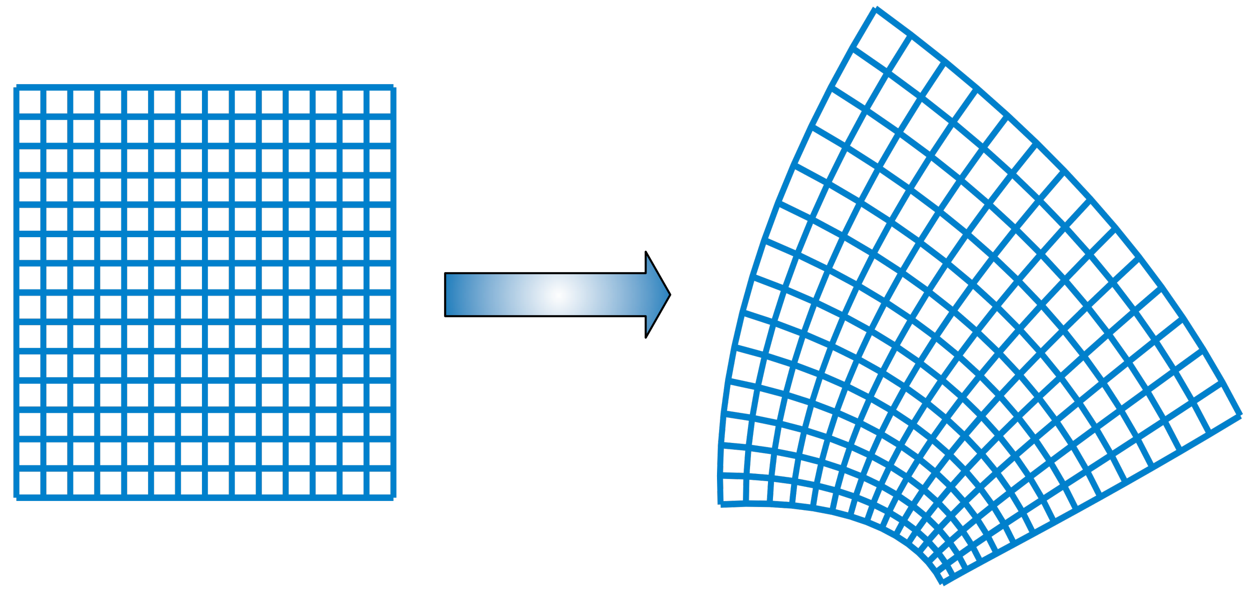
Bootstrap

Find the space of *possible theories* just using consistency:
Symmetries and Quantum Mechanics

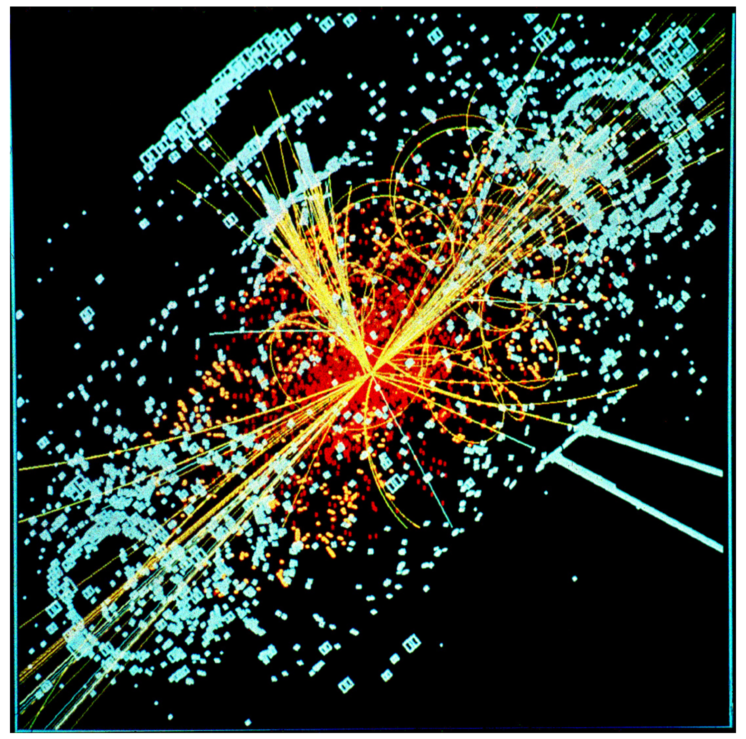
From Wikipedia: "A self-starting process that is supposed to proceed without external input"

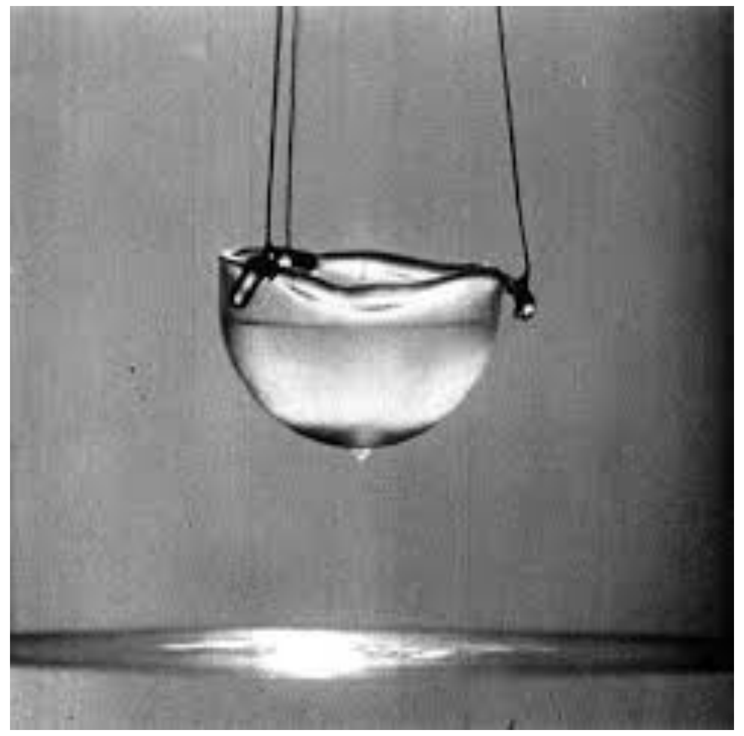
Conformal Field Theories

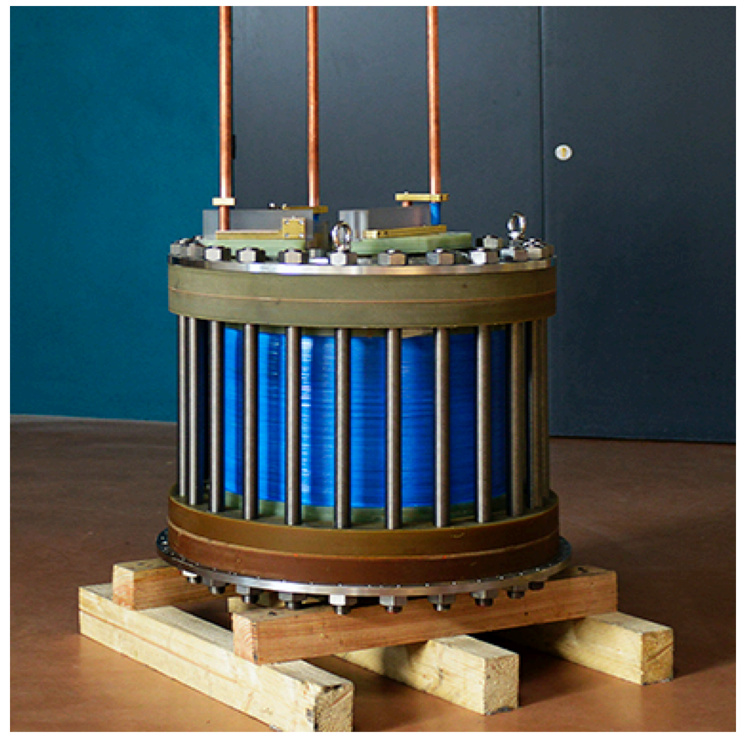
CFTs are quantum field theories invariant under angle-preserving transformations of space (or spacetime)



They are central objects in physics. Mathematically rigid structures.







Conformal Bootstrap

Compile a catalogue of all consistent CFTs in various dimensions

Early 70s: Ferrara Gatto Grillo, Polyakov
bootstrap axioms

Early 80s: Belavin Polyakov Zamolodchikov^2
exact solution of some 2d CFTs, e.g. 2d Ising

Modern era (since 2008): Rattazzi Rychkov Tonni Vichi
Carve out *theory space* by the linear functional method

The bootstrap oracle

Not anything goes in CFT!

Tentative CFT data are constrained by rigorous bootstrap inequalities

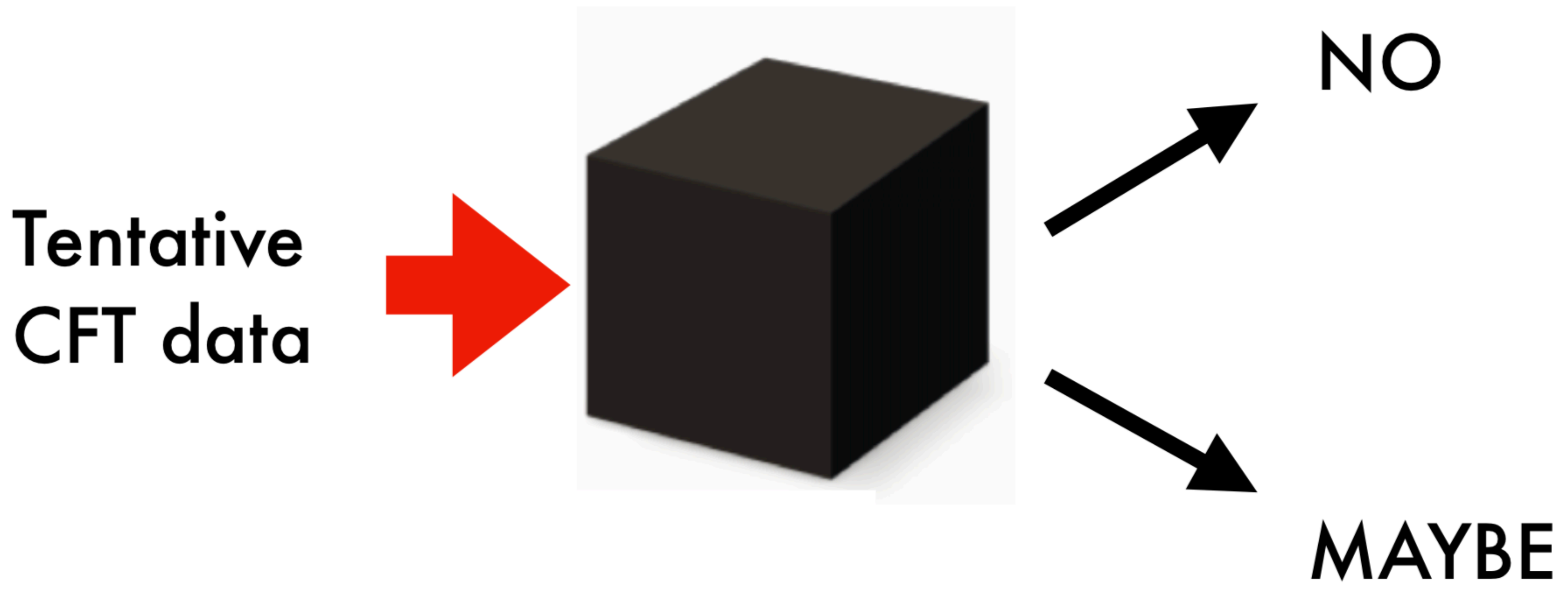
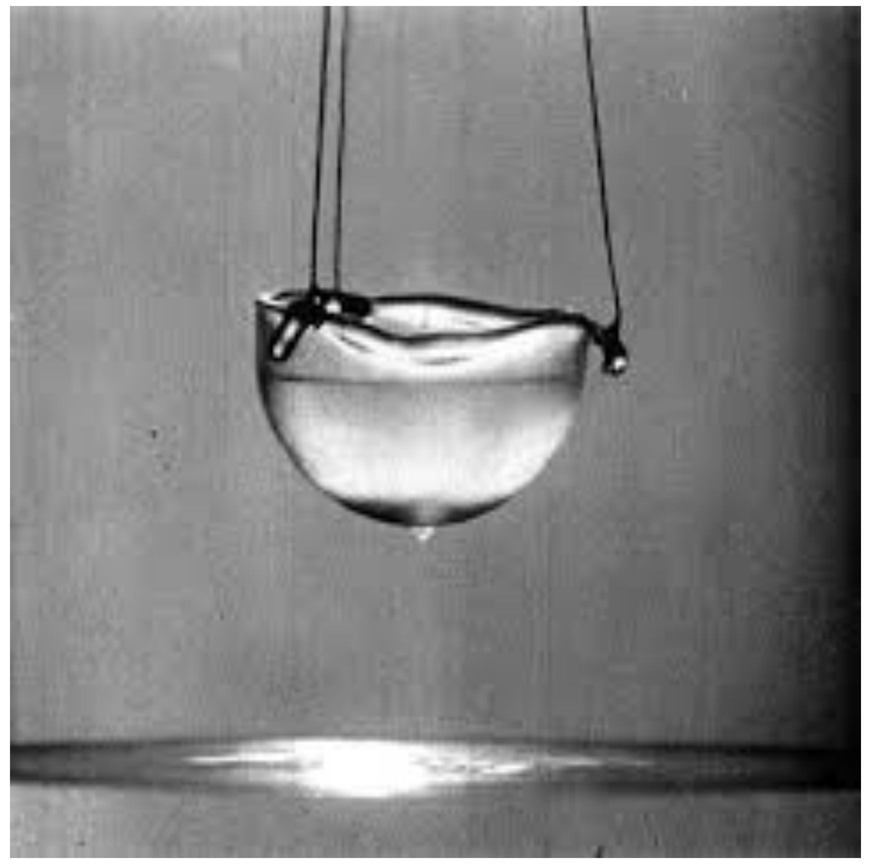
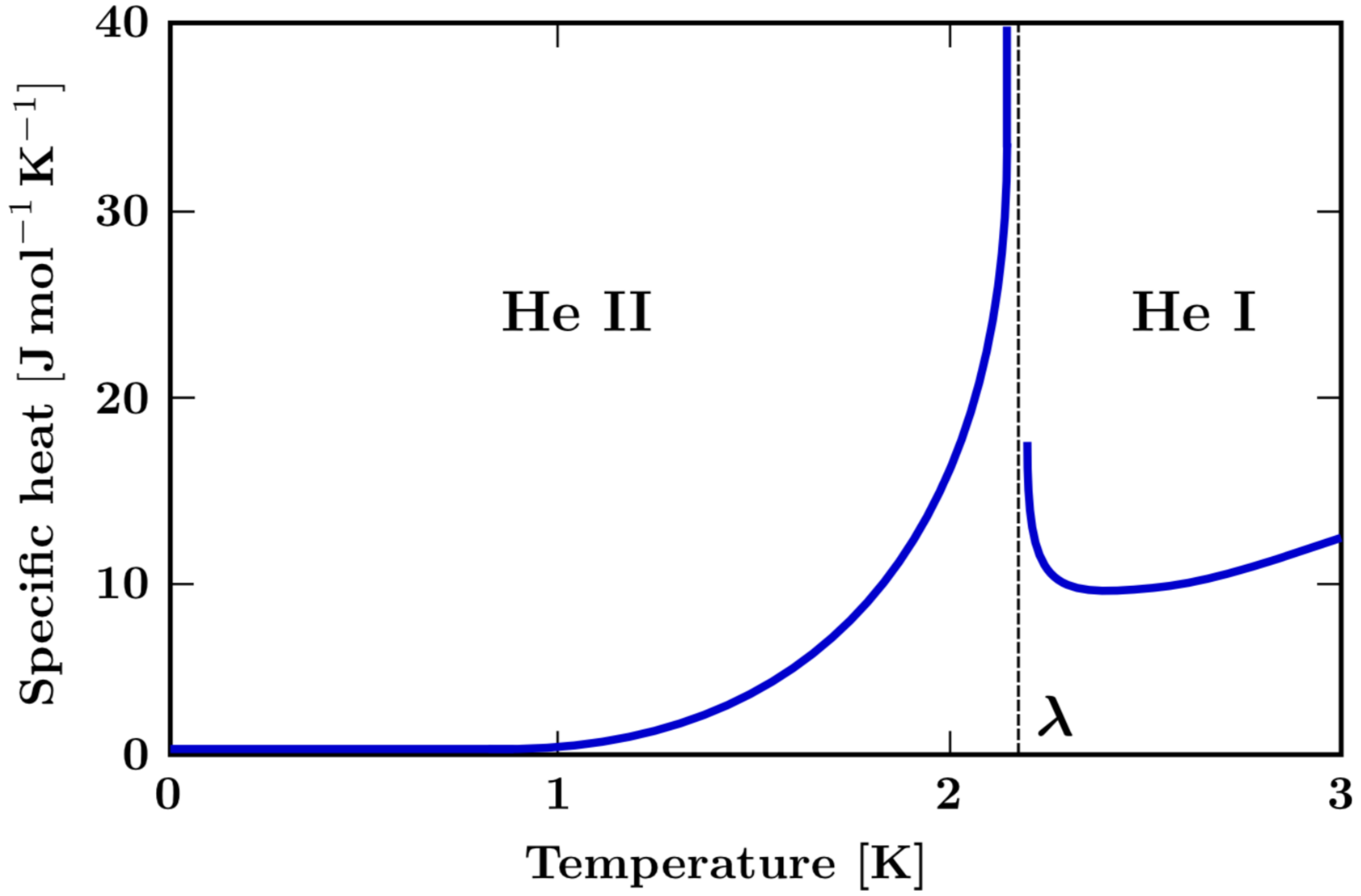


Figure credit: S. Rychkov

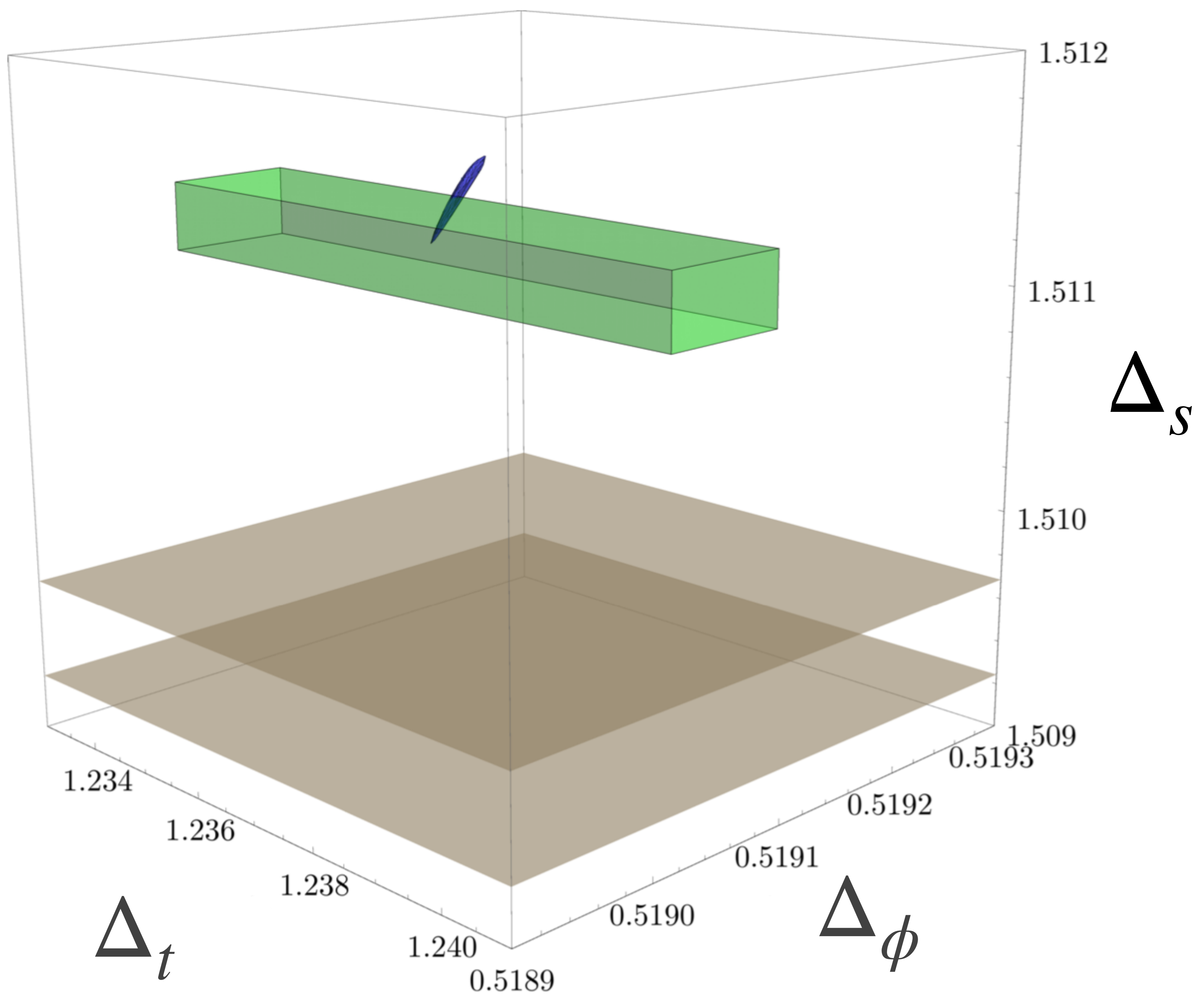
The λ-point experiment





Specific heat $C \sim |T - T_c|^{\alpha}$ with $\alpha = \dfrac{2\Delta_s - 3}{3 - \Delta_s}$

Microgravity experiment in space (1992): 8σ discrepancy with Montecarlo



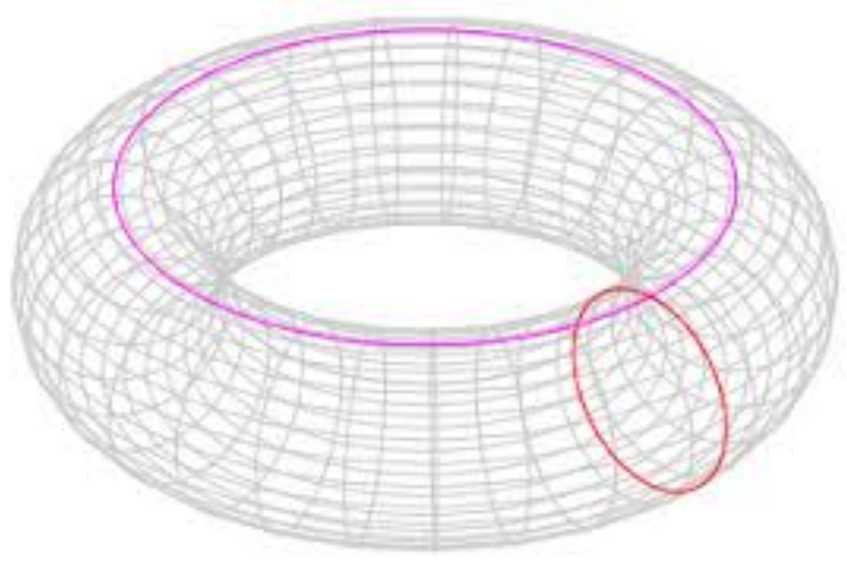
[Chester, Landry, Liu, Poland, Simmons-Duffin, Su, Vichi]

Modular Bootstrap

Hellerman, ….

Specific to 2d CFTs. Consider the theory at finite temperature $T = 1/\beta$

$$Z(\beta) = \sum_i e^{-\beta E_i} \qquad Z(\beta) = Z\left(\frac{1}{\beta}\right)$$



Posivity and crossing: *not anything goes*

$$\sum_n \alpha_n \left(\frac{\partial}{\partial \beta}\right)^{2n+1} \Bigg|_{\beta=1} \left[\sum_i e^{-\beta E_i} - e^{-E_i/\beta}\right] = 0$$

Cleverly constructed linear functional constrains possible spectra $\{E_i\}$

Bootstrapping quantum gravity?

Not anything goes in quantum gravity!

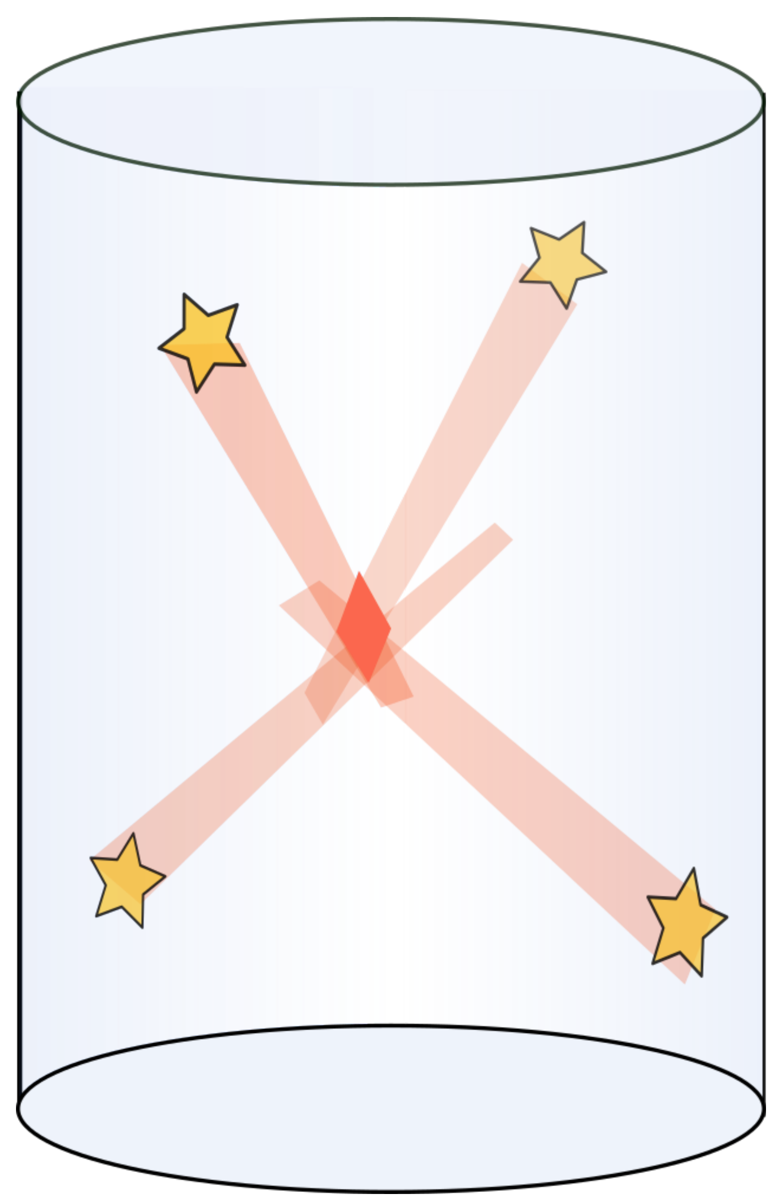
Give a low-energy effective theory (particle content and interactions), does it admit a UV completion or is part of the *Swampland*?

$$S = \frac{1}{8\pi G}\int d^D x \sqrt{g}\,\left(R + \alpha_2 R^2 + \ldots + (\nabla\phi)^2 + \lambda\phi^4 + g_2(\nabla\phi)^4 + \ldots\right)$$

Bootstrapping quantum gravity?

Bootstrap methods apply most directly to gravity in Anti de Sitter space

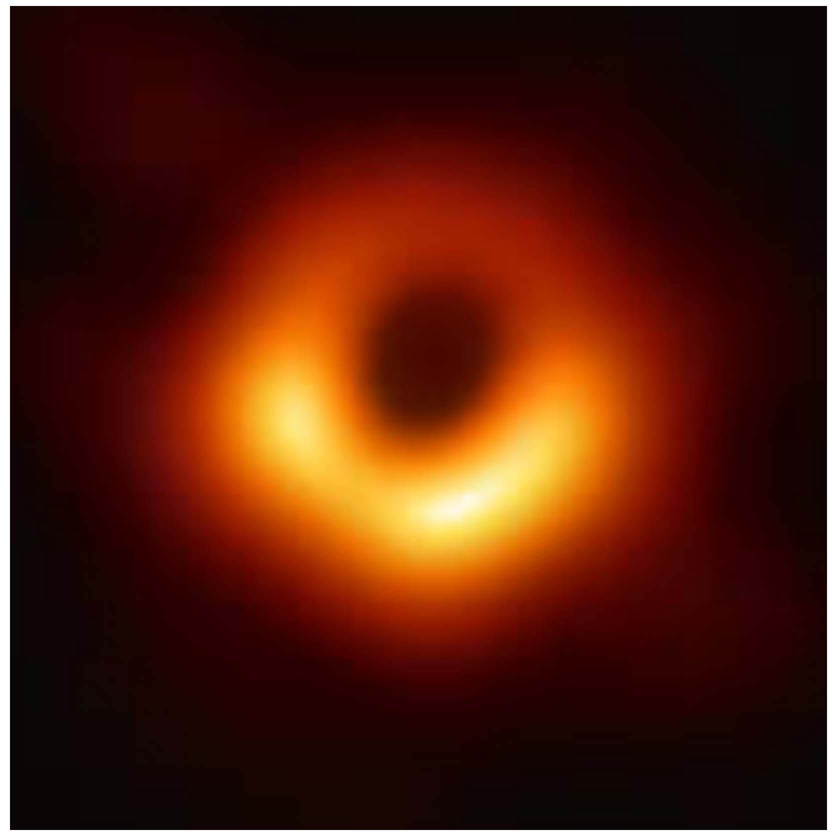
CFTs are holograms for quantum gravity theories in AdS



Black hole thermodynamics

The original hint for holography

$$\text{Entropy} = \frac{A}{4G\hbar} = \frac{A}{4\ell^2_{\text{Planck}}}$$



For M87 black hole, $S \sim 10^{96}$

What is the simplest theory of 3D gravity?

Unlike in 4D, no propagating bulk gravitons. There are however boundary gravitons.

$$S = \frac{1}{8\pi G}\int d^3x \sqrt{g}\,(R - \frac{1}{\ell^2}) + \ldots ?$$

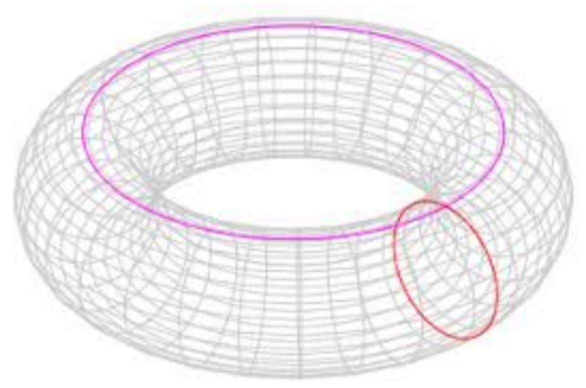
Is there a theory of "pure" gravity? Or do we need strings, supersymmetry, . . . ?

Black hole entropy $\sim \ell/\ell_{\text{Planck}} = \ell/G$, which is very large for weakly coupled gravity

By "pure" gravity we mean *just* boundary gravitons and black holes

$c = \dfrac{3\ell}{2G}$ is also the central charge (# of degrees of freedom) of the dual CFT

Bootstrapping 3D gravity



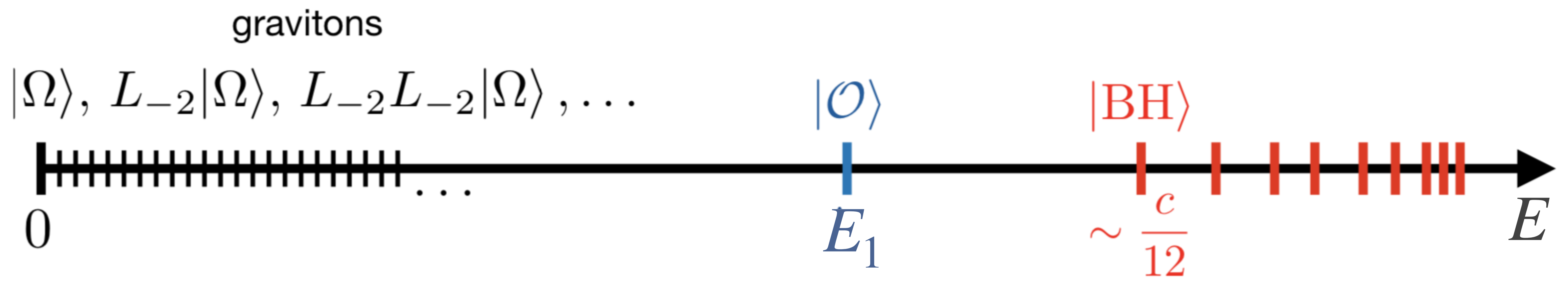
$=$

$$Z_{\text{CFT}} = Z_{AdS}$$

Look for the theory with the highest possible E_1

For given c, $E_1 \leq E_{\text{gap}}(c)$, or else we cannot satisfy $Z(\beta) = Z(1/\beta)$

Hellerman



Numerical bounds

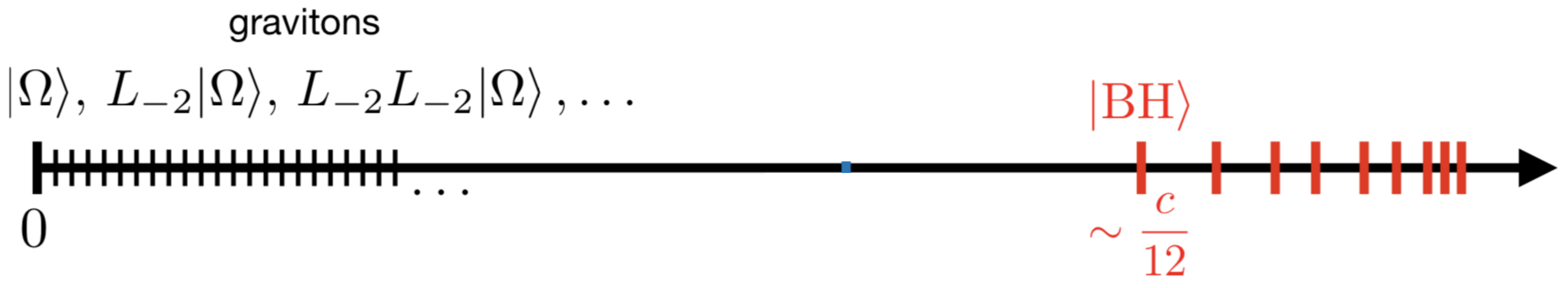
Afkhami-Jeddi Hartman Tajdini

Two numerical surprises at $c = 4$ and $c = 12$:
spectrum and degeneracies converge to integers, for no good apparent reason

E.g. for $c = 12$:
$E_{\mathrm{gap}} \leq 2 + 10^{-30}$ and $n_1 = 196884 = 196883 + 1, \quad Z^{c=12} \to j(\tau) - 744$

Modularity and monster symmetry! But *why*?

From $c \lesssim 2000$, extrapolated asymptotics $E_{\mathrm{gap}}(c) \sim \dfrac{c}{9.08}$ as $c \to \infty$



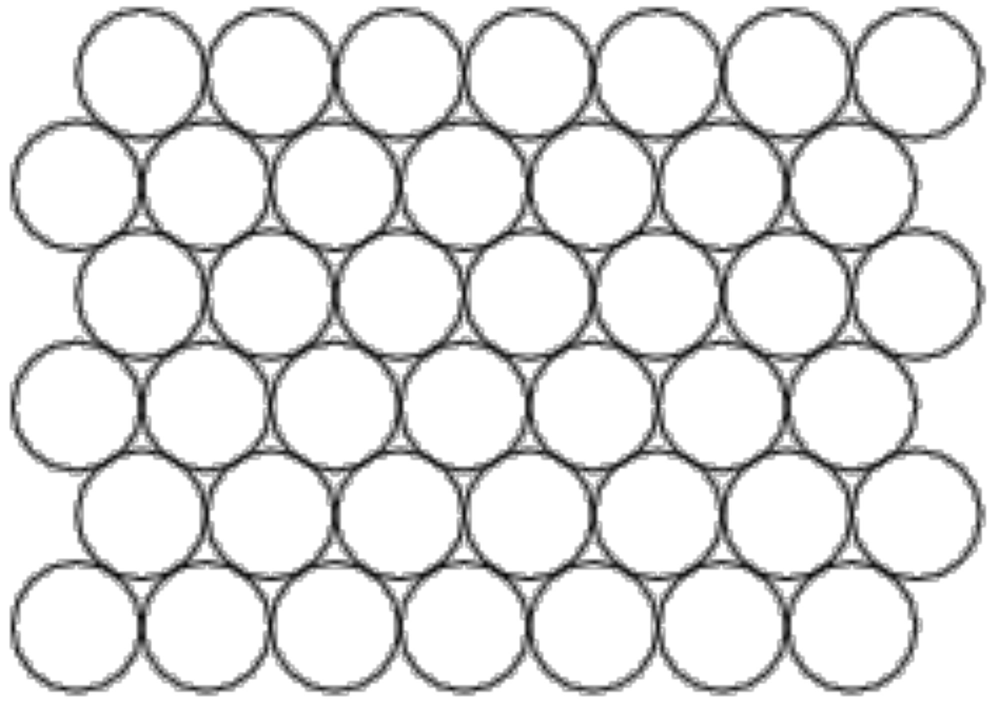
Sphere Packing Problem

What is the densest configuration of identical spheres in d dimensions?

Deep question, with connections to modular forms, number theory, cryptography, …

Solved only in $d = 1, 2, 3, 8, 24$.

In $d = 2$, honeycomb lattice (Toth, 1940)



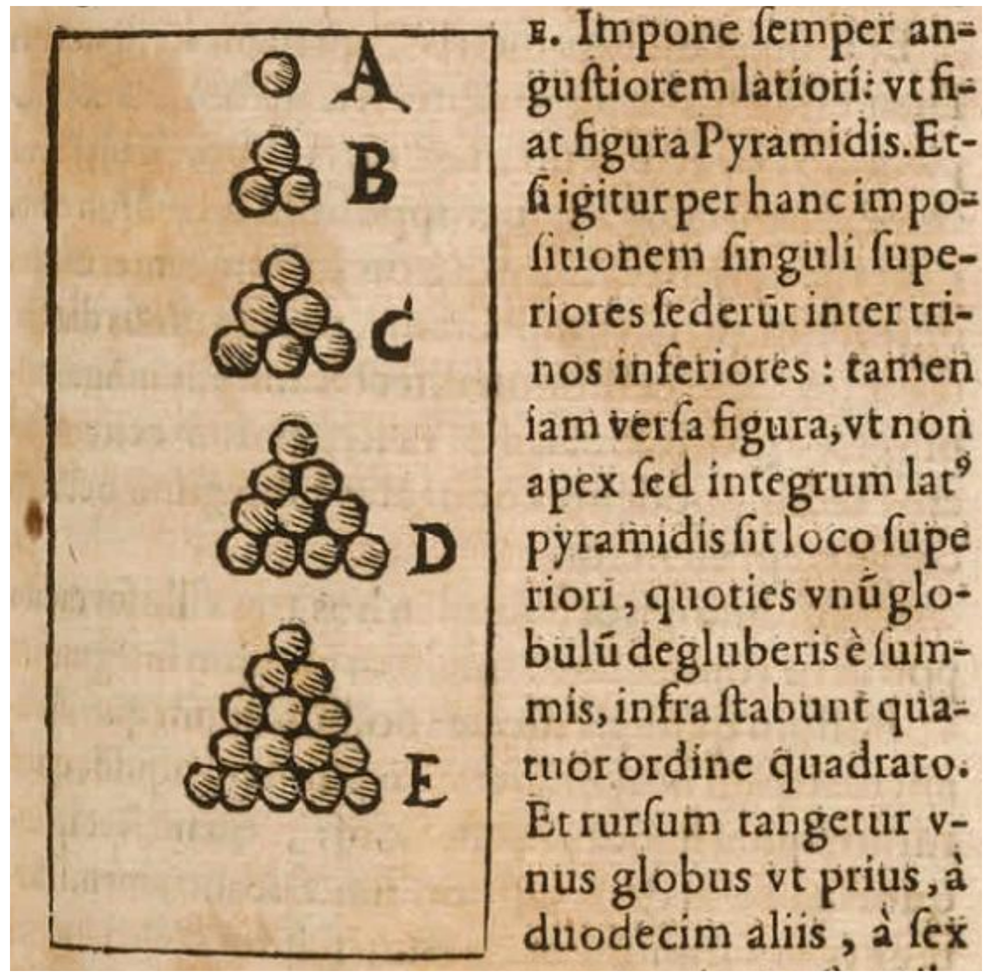
Sphere Packing Problem

What is the densest configuration of identical spheres in d dimensions?

Deep question, with connections to modular forms, number theory, cryptography, …

Solved only in $d = 1, 2, 3, 8, 24$.

In $d = 3$, Kepler's conjecture (1611): stack hexagonal layers. Proved by Hales in 1998.



E. Impone ſemper an-
guſtiorem latiori: vt fi-
at figura Pyramidis. Et-
ſi igitur per hanc impo-
ſitionem ſinguli ſupe-
riores ſederũt inter tri-
nos inferiores : tamen
iam verſa figura, vt non
apex ſed integrum lat⁹
pyramidis ſit loco ſupe
riori, quoties vnũ glo-
bulũ degluberis è ſum-
mis, infra ſtabunt qua-
tuor ordine quadrato.
Et rurſum tangetur v-
nus globus vt prius, à
duodecim aliis, à ſex

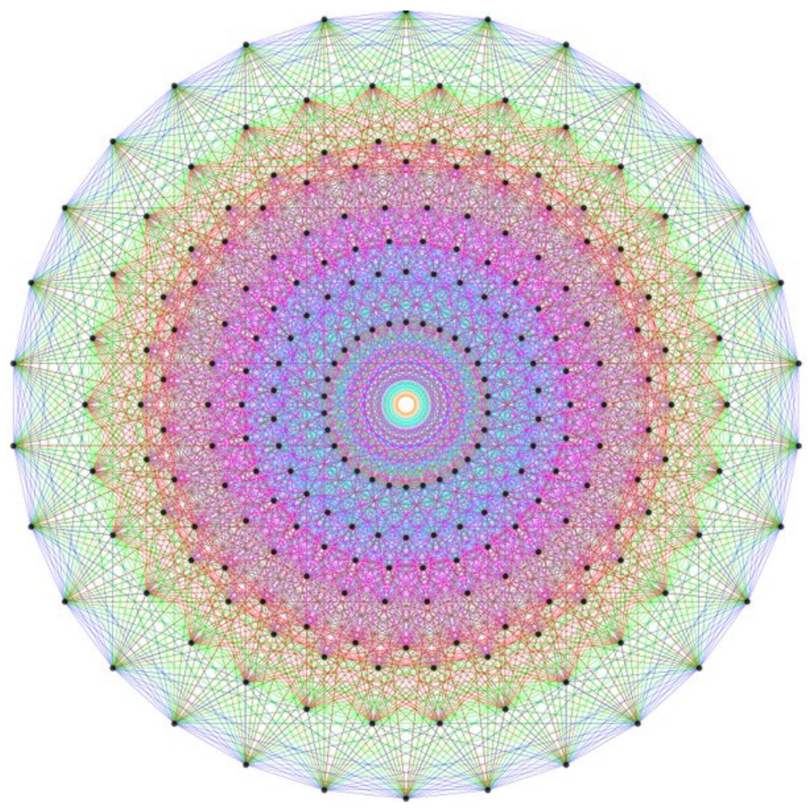
Sphere Packing Problem

What is the densest configuration of identical spheres in d dimensions?

Deep question, with connections to modular forms, number theory, cryptography, …

Solved only in $d = 1, 2, 3, 8, 24$.

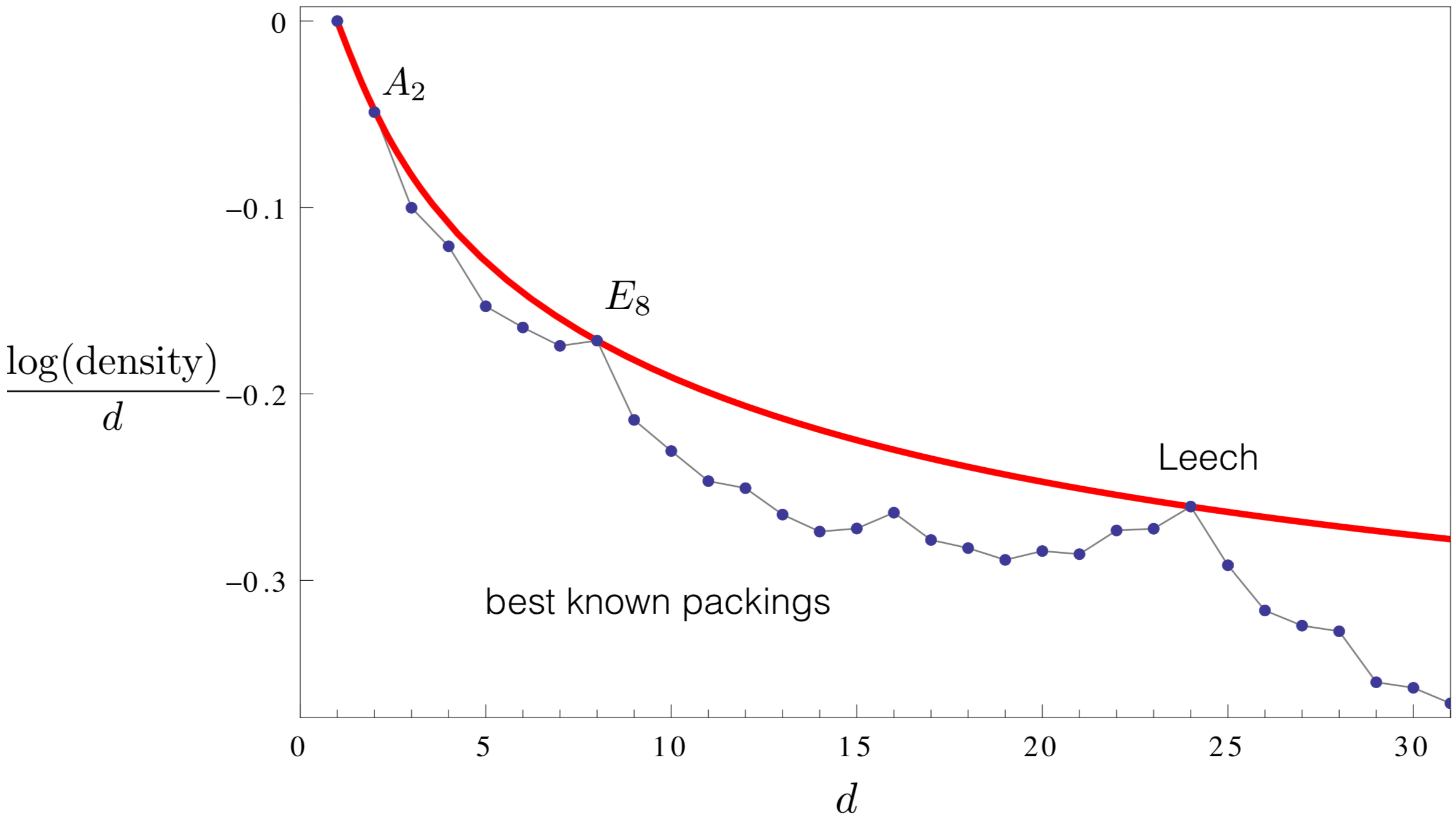
In $d = 8$, E_8 lattice.
Solved by Viazovska on 3/14/2016. Short elegant proof.



In d=24, Leech lattice.
Solved a week later with the same technique.

Linear programming bounds

Viazovska built on the Cohn-Elkies approach.
A function $f(r)$ with certain positivity and Fourier properties gives an upper bound.
Find the best $f(r)$ by linear optimization.



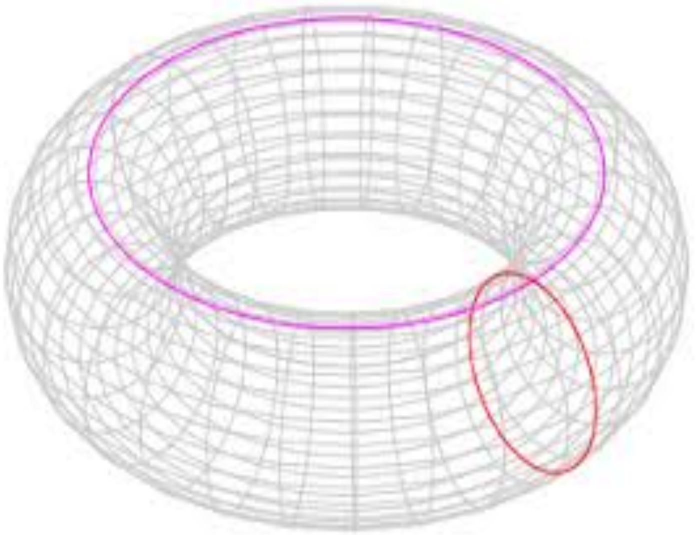
This amounts to a numerical proof in $d = 8\,, 24$. But an analytic proof was elusive.

Viazovska constructed analytically the “magic function” $f(r)$

Punchline

Hartman Mazac LR

$=$

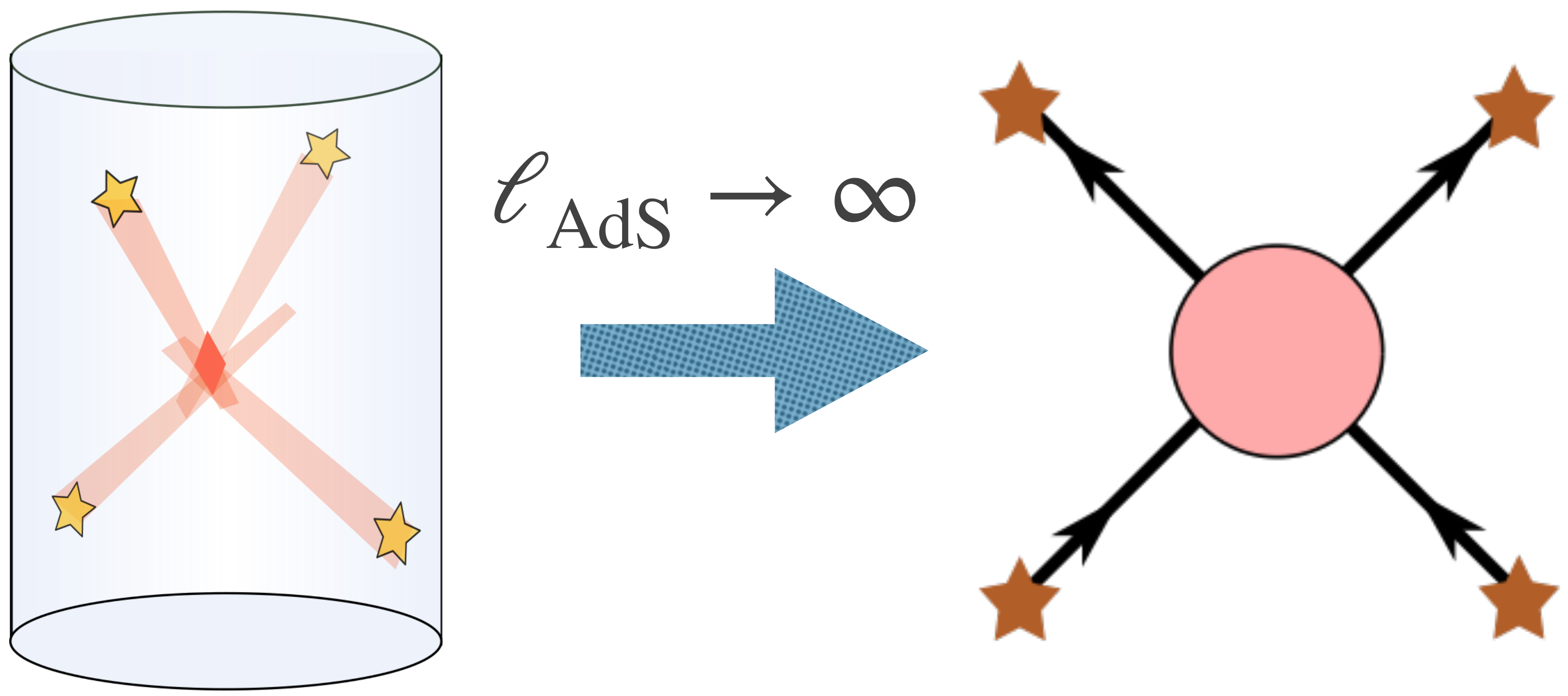


Cohn-Elkies $f(r)$ $\equiv$ linear functional for the modular bootstrap $Z(\beta) = Z(1/\beta)$!

More precisely, the Cohn-Elkies problem is equivalent to the modular bootstrap for $U(1)^c$

Sphere packing	*Modular bootstrap*
d	$c = \frac{d}{2}$
density ρ	$E_1 = \rho^{2/d}$
Viazovska \`16	Mazáč\`16

Bootstrapping graviton scattering?

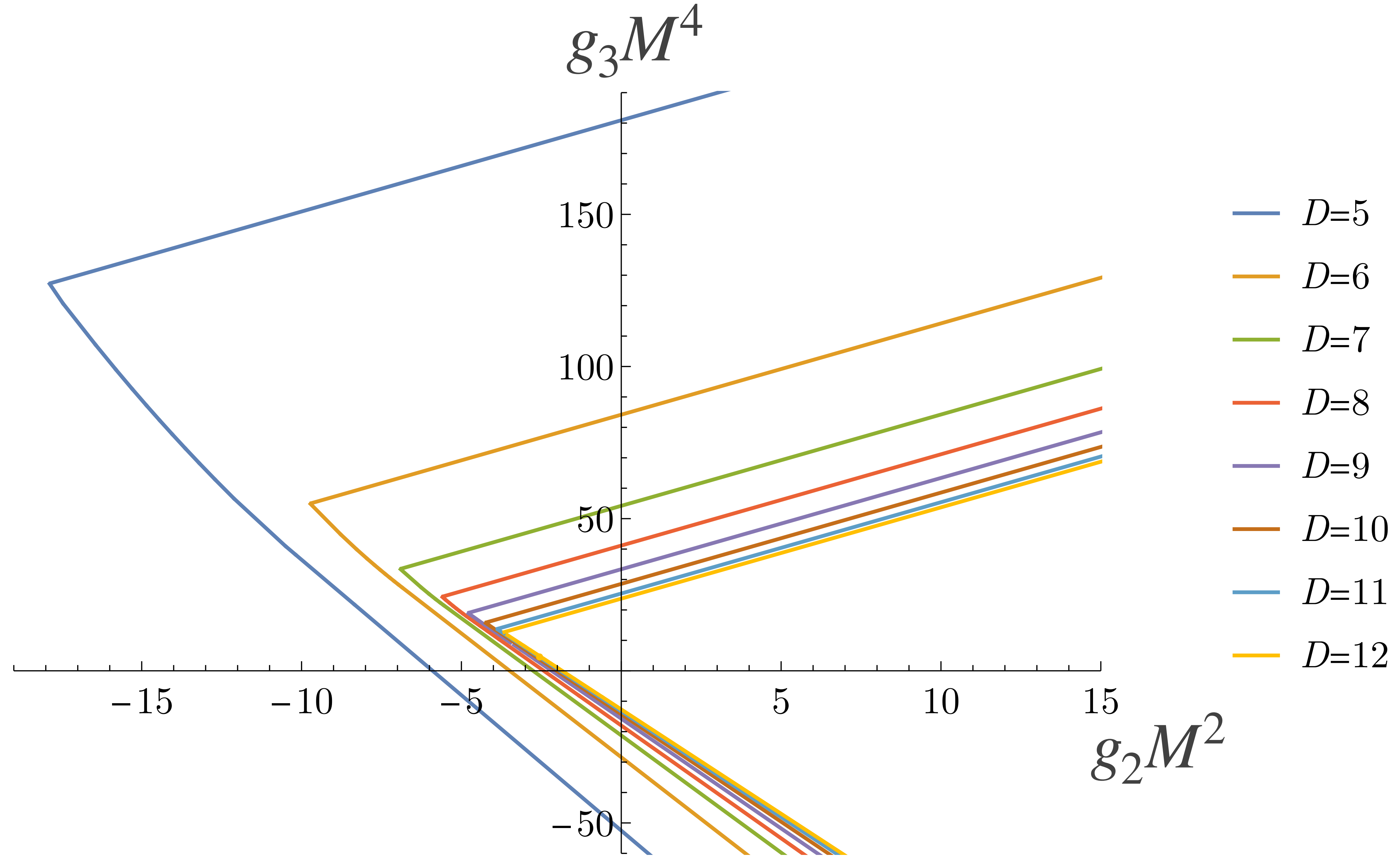


CFT bootstrap → S-matrix bootstrap

Constrain $2 \to 2$ amplitude $\mathscr{M}(s, t)$ subject to analyticity, crossing, boundedness

Bounds in a weakly coupled low-energy theory of a scalar + gravity

$$S = \frac{1}{8\pi G}\int d^D x\sqrt{g}\,\left(R + (\nabla\phi)^2 + \lambda\phi^4 + g_2(\nabla^4\phi^4) + g_3(\nabla^6\phi^4) + g_4(\nabla^6\phi^4)\ldots\right)$$



Caron-Huot Mazáč LR Simmons-Duffin

g_2 allowed to be a bit negative thanks to gravitational time delay

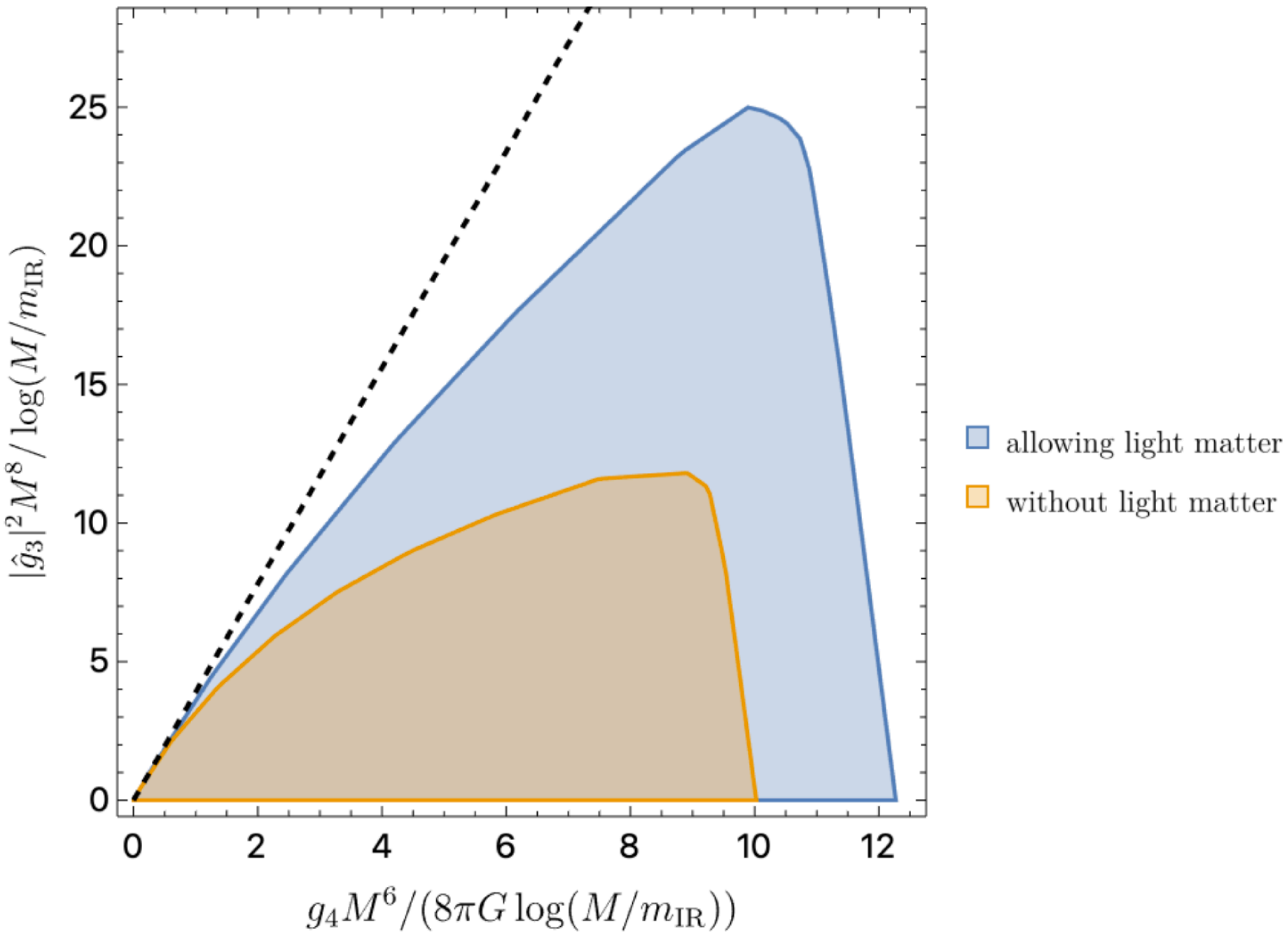
A bound in maximal sugra

$$S = \frac{1}{8\pi G} \int d^D x \sqrt{g} \left(R + g_0 R^4 + \ldots \right)$$

$0 \leq g_0 M^6 \leq 3.000$. Compatible with type II string, $g_0 M^6 = 2\zeta(3) \cong 2.40$

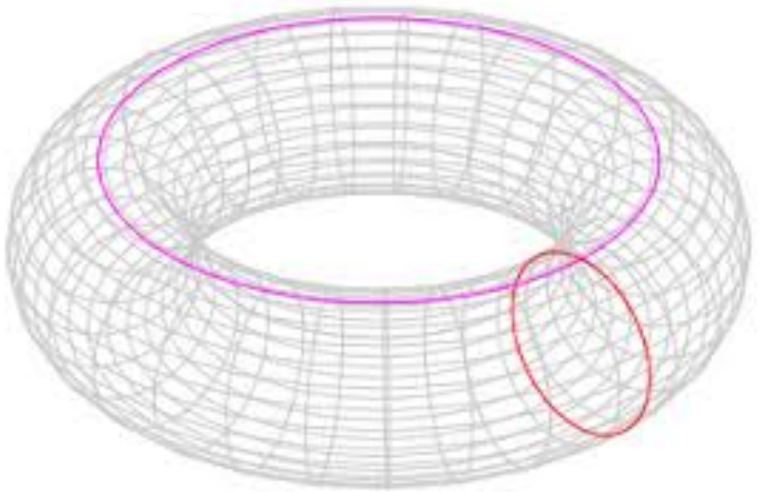
Bounds from graviton scattering in D=4

Caron-Huot Li Parra-Martinez Simmons-Duffin



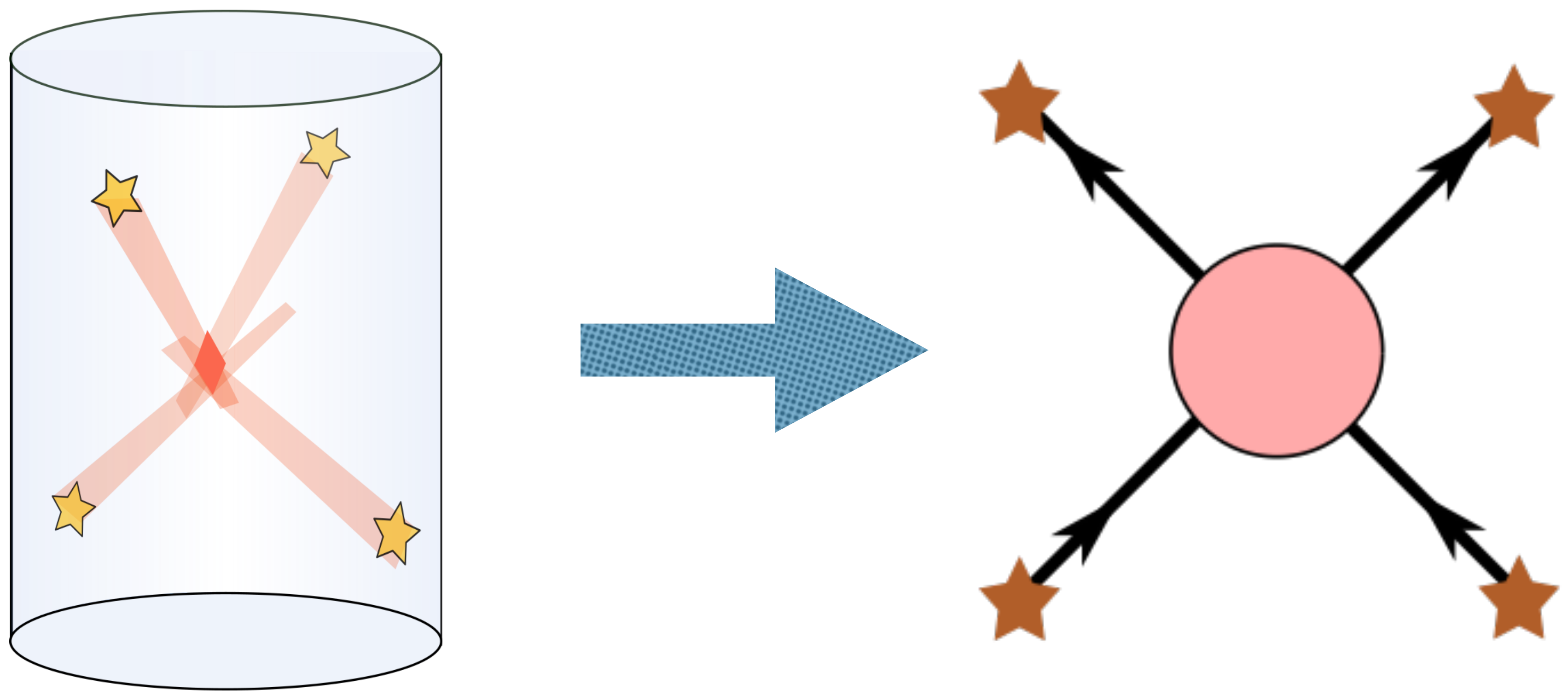
(See also Chiang Huang Li Rodina Weng)

Outlook



- Simplest theory of 3D gravity? Still an open question
- New insights bootstrap $\leftrightarrow$ sphere packing
  E.g. improved numerics for large d: $\rho \lesssim 2^{-0.6044\,d}$ Afkhami-Jeddi et al
- Is there a more general relation between the constraints on both sides?

Outlook



- In asymptotic Minkowski, beginning of the quantum gravity S-matrix bootstrap
  Must make plausible physical assumptions

- In asymptotic AdS, a corner of the CFT bootstrap
  Fully rigorous

  Proof that large N CFTs with large gap have a local AdS dual, with sharp inequalities
  Caron-Huot Mazáč LR Simmons-Duffin

MUST WE NOT THEN RENOUNCE THE OBJECT ALTOGETHER, THROW IT TO THE WINDS AND INSTEAD LAY BARE THE PURELY ABSTRACT?

VASILY KANDINSKY 1911