Introduction to ‘Generalized Hydrodynamics’ (GHD) in the Lieb-Liniger gas

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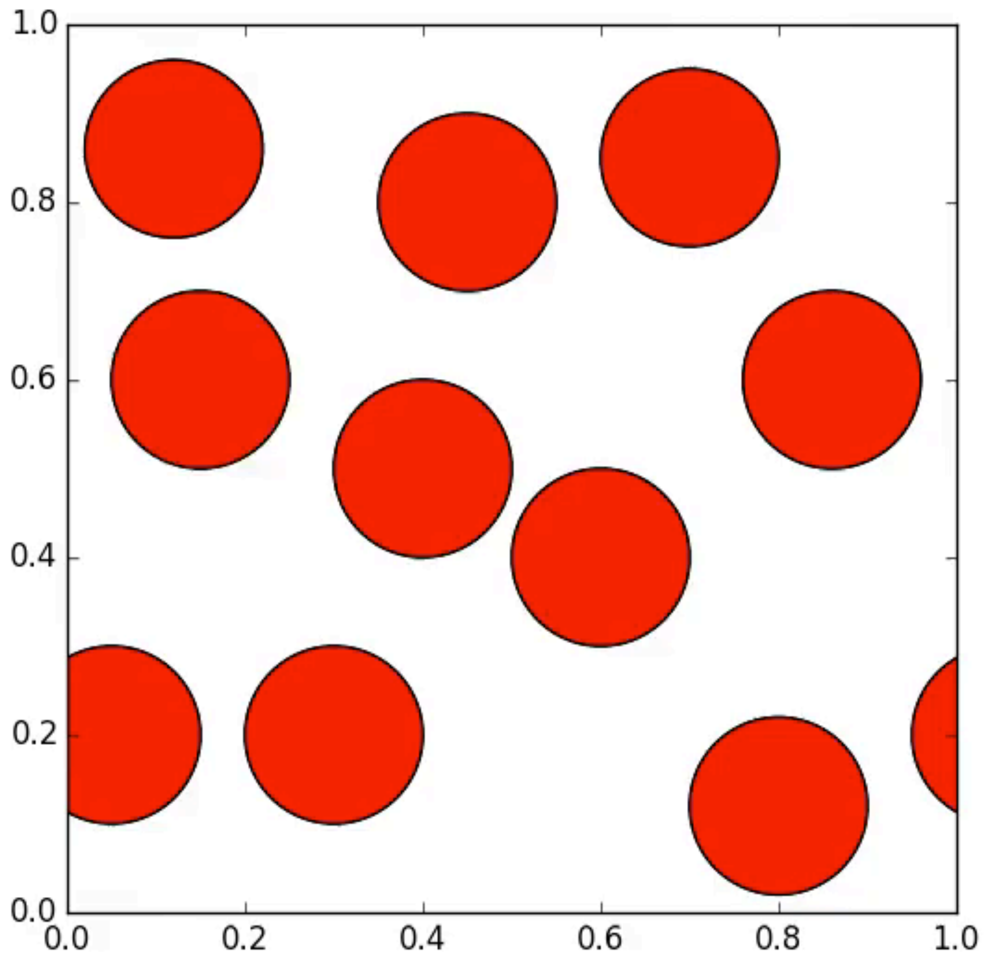
Talk based on works with:

Benjamin Doyon (King’s College London, UK)
Takato Yoshimura (Oxford, UK)
Robert Konik (Brookhaven, US)
Jean-Sébastien Caux (Amsterdam, NL)
Pasquale Calabrese (SISSA, IT)
Paola Ruggiero (King’s College, London, UK)
Stefano Scopa (SISSA, IT)
Alvise Bastianello (Munich, GER)
Jean-Marie Stéphan (Lyon, FR)
Isabelle Bouchoule (Laboratoire Charles Fabry, FR)
Max Schemmer (Laboratoire Charles Fabry, FR)
David Weiss (Penn State, US)
Neel Malvania (Penn State, US)
Yuan Le (Penn State, US)
Marcos Rigol (Penn State, US)
Yicheng Zhang (Penn State, US)

and on review article [I. Bouchoule, JD, 2022] in Special Volume of J. Stat. Mech. edited by A. Bastianello, B. Bertini, B. Doyon, R. Vasseur

Randomness, Integrability and Universality, GGI Florence, May 2022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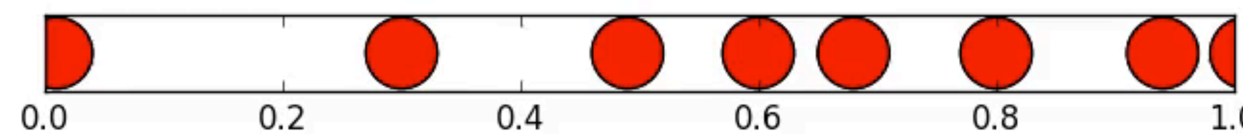
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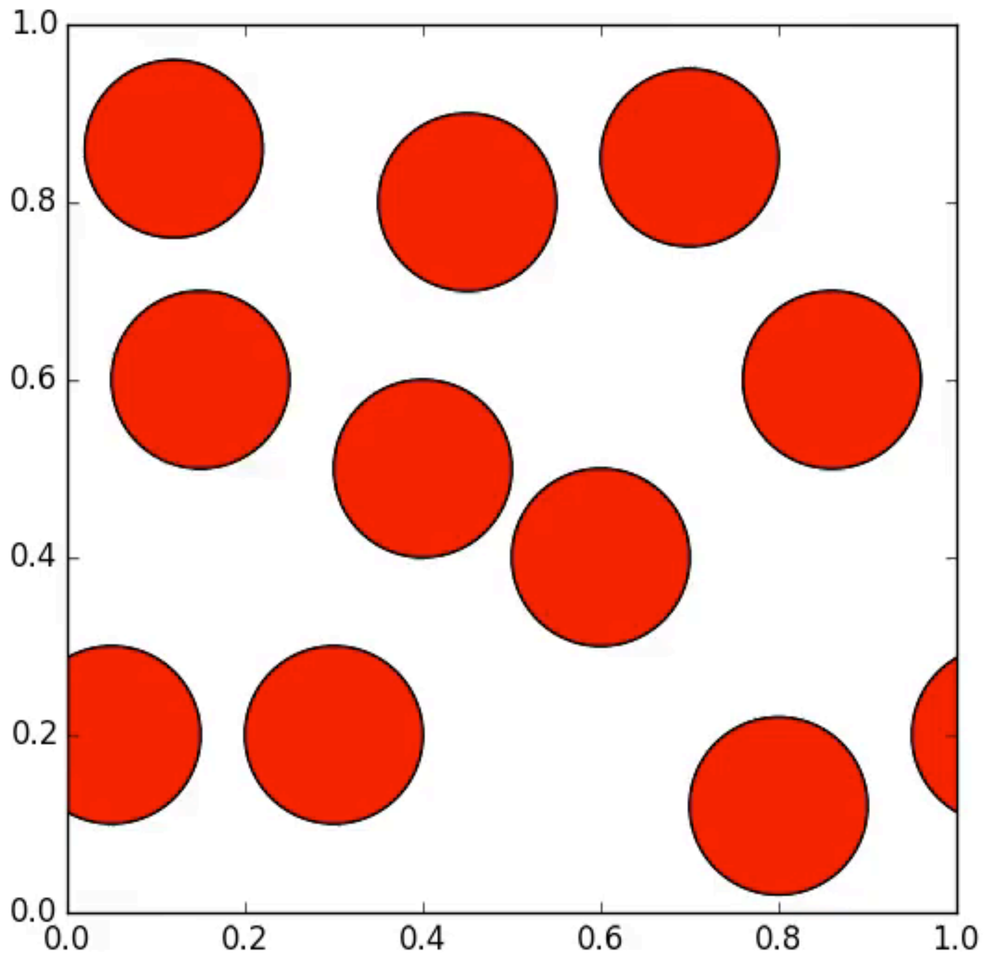
(Classical) Newton's Cradle on YouTube

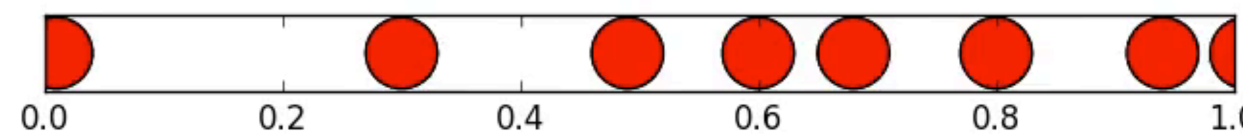
chaotic / ergodic **vs.** **integrable / non-ergodic**



after short relaxation time τ_{relax}, the macrostate in the box is entirely characterized by

$$n,\ u,\ \varepsilon$$

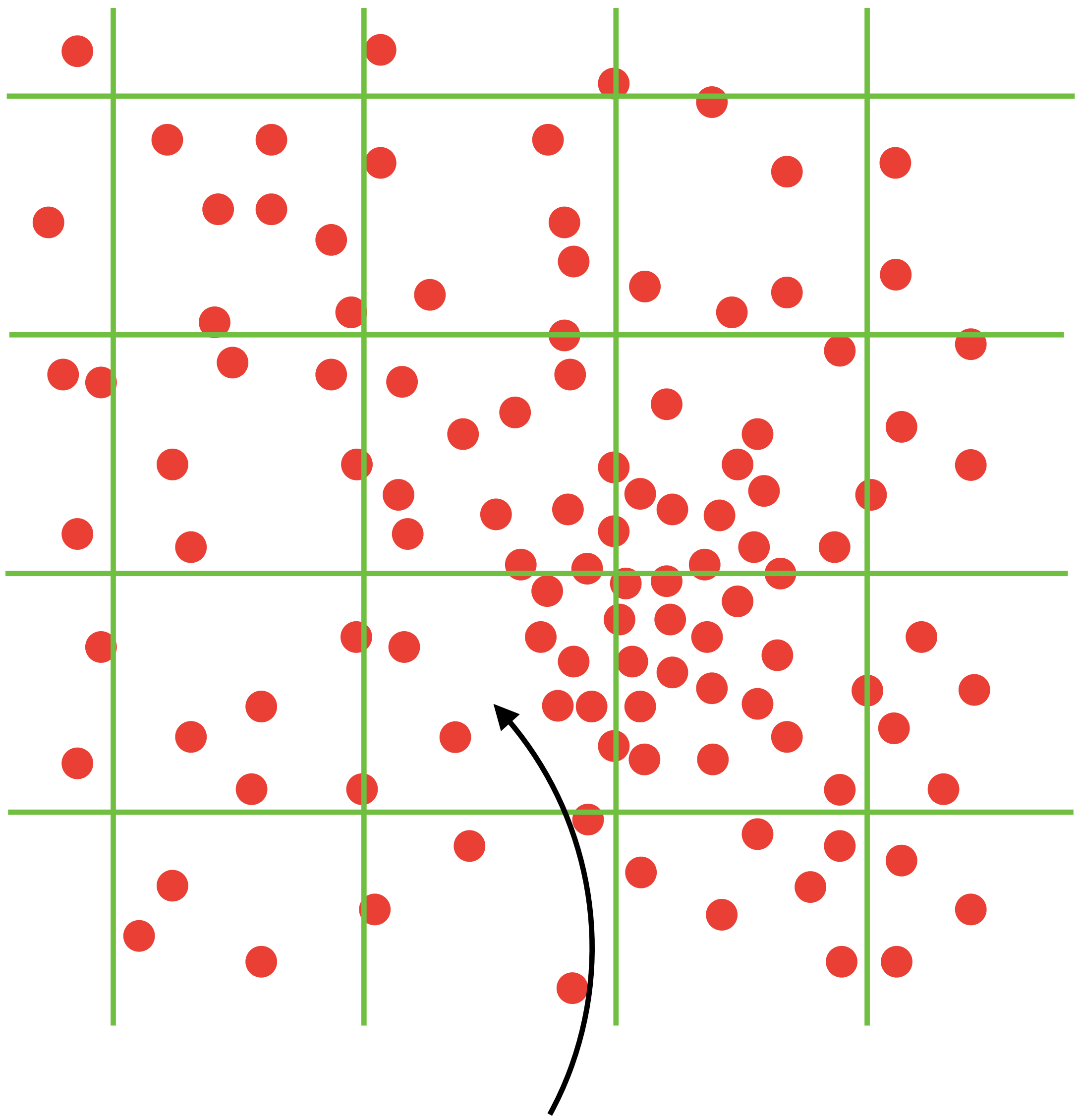
particle density, mean velocity, energy density



to characterize a macrostate, one needs the entire distribution of velocities

$$\rho(v) = \frac{1}{L} \sum_{i=1}^{N} \delta(v - v_i)$$

chaotic / ergodic **vs.** **integrable / non-ergodic**

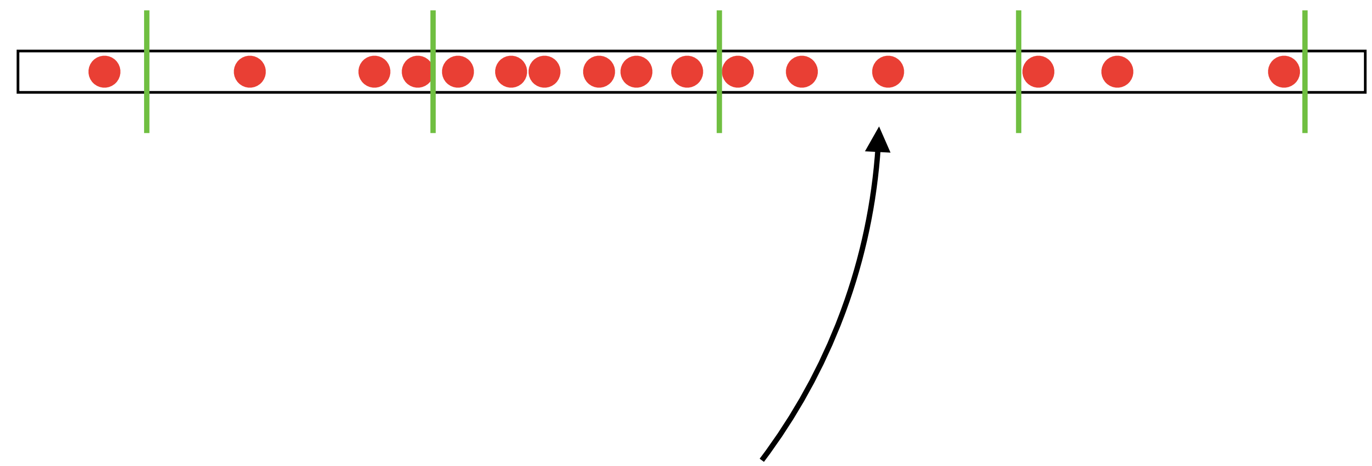


state in each 'fluid cell' characterized by

$$n(x,t), u(x,t), \varepsilon(x,t)$$

whose evolution is governed by continuity eqns

$$\begin{cases} \partial_t n + \partial_x (nu) & = & 0 \\ \partial_t (nu) + \partial_x j_P & = & -n \frac{\partial_x V}{m} \\ \partial_t \varepsilon + \partial_x j_E & = & 0 \end{cases}$$



is there a coarse-grained or hydrodynamic description?

This is what 'Generalized HydroDynamics' (GHD) is about.

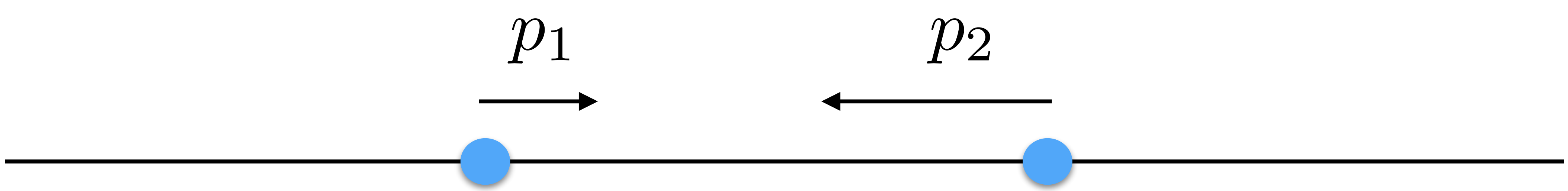
This talk

Generalized Hydrodynamics (GHD) in the quantum one-dimensional Bose gas.

1. Generalized Hydrodynamics (GHD) of the 1D Bose gas: standard theory
2. Checks in cold atoms experiments
3. Re-introducing quantum fluctuations

1. Crash course on GHD

The 2-body problem



As we've seen, the eigenstates of

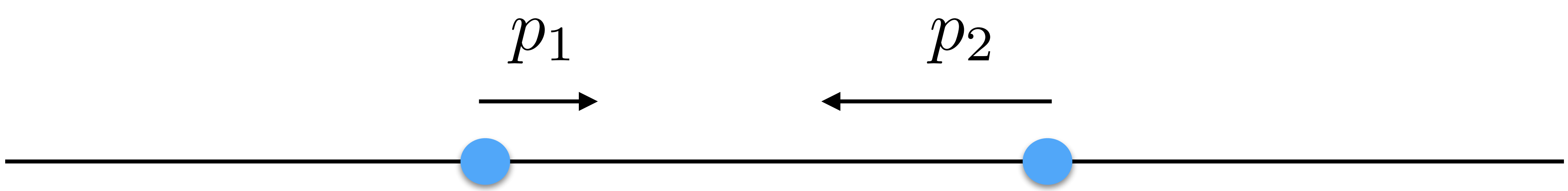
$$H = -\frac{\hbar^2}{2m}(\partial_{x_1}^2 + \partial_{x_2}^2) + g\,\delta(x_1 - x_2)$$

in the limit of infinite repulsion $g \to \infty$ were

$$\psi(x_1, x_2) = \begin{cases} e^{\frac{i}{\hbar}(p_1 x_1 + p_2 x_2)} - e^{\frac{i}{\hbar}(p_2 x_1 + p_1 x_2)} & \text{if} \quad x_1 < x_2 \\ (x_1 \leftrightarrow x_2) & \text{if} \quad x_2 < x_1 \end{cases}$$

1. Crash course on GHD

The 2-body problem: the scattering phase



Now, as we go away from that limit

$$H = -\frac{\hbar^2}{2m}(\partial_{x_1}^2 + \partial_{x_2}^2) + g\,\delta(x_1 - x_2)$$

the eigenstates change continuously with g

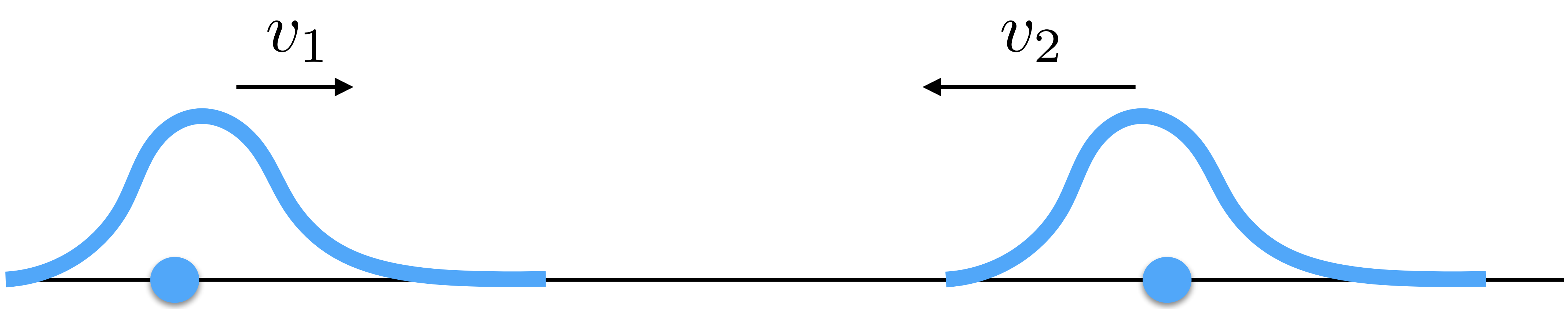
$$\psi(x_1, x_2) = \begin{cases} e^{\frac{i}{\hbar}(p_1 x_1 + p_2 x_2)} - e^{i\varphi} e^{\frac{i}{\hbar}(p_2 x_1 + p_1 x_2)} & \text{if} \quad x_1 < x_2 \\ (x_1 \leftrightarrow x_2) & \text{if} \quad x_2 < x_1 \end{cases}$$

$$e^{i\varphi} = \frac{mg/\hbar - i(p_2 - p_1)}{mg/\hbar + i(p_2 - p_1)}$$

1. Crash course on GHD

The 2-body problem: the scattering phase

One physical consequence of this scattering phase is the following. Take two wave packets with semiclassical velocities $v_1 = p_1/m$ and $v_2 = p_2/m$



$$\psi(x_1, x_2) = \begin{cases} e^{\frac{i}{\hbar}(p_1 x_1 + p_2 x_2)} - e^{i\varphi} e^{\frac{i}{\hbar}(p_2 x_1 + p_1 x_2)} & \text{if} \quad x_1 < x_2 \\ (x_1 \leftrightarrow x_2) & \text{if} \quad x_2 < x_1 \end{cases}$$

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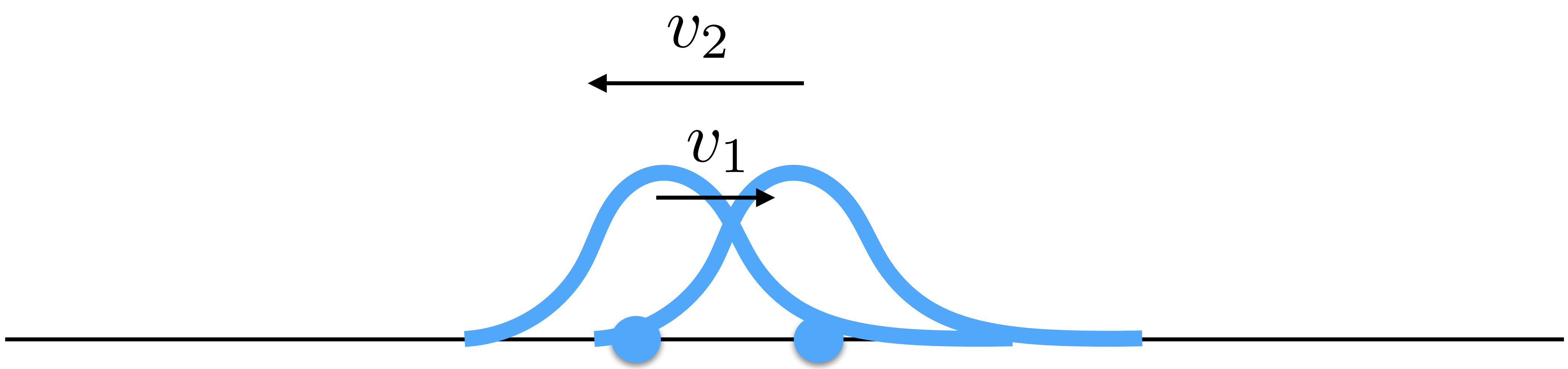

$$\psi(x_1, x_2) = \begin{cases} e^{\frac{i}{\hbar}(p_1 x_1 + p_2 x_2)} - e^{i\varphi} e^{\frac{i}{\hbar}(p_2 x_1 + p_1 x_2)} & \text{if} \quad x_1 < x_2 \\ (x_1 \leftrightarrow x_2) & \text{if} \quad x_2 < x_1 \end{cases}$$

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1. Crash course on GHD

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1. Crash course on GHD

The 2-body problem: the scattering phase

One physical consequence of this scattering phase is the following. Take two wave packets with semiclassical velocities $v_1 = p_1/m$ and $v_2 = p_2/m$

After they have scattered, the two packets are not quite where you would expect them. Compared to the non-interacting case, they are shifted by a distance

$$\Delta = \hbar \frac{d\varphi}{dp}$$

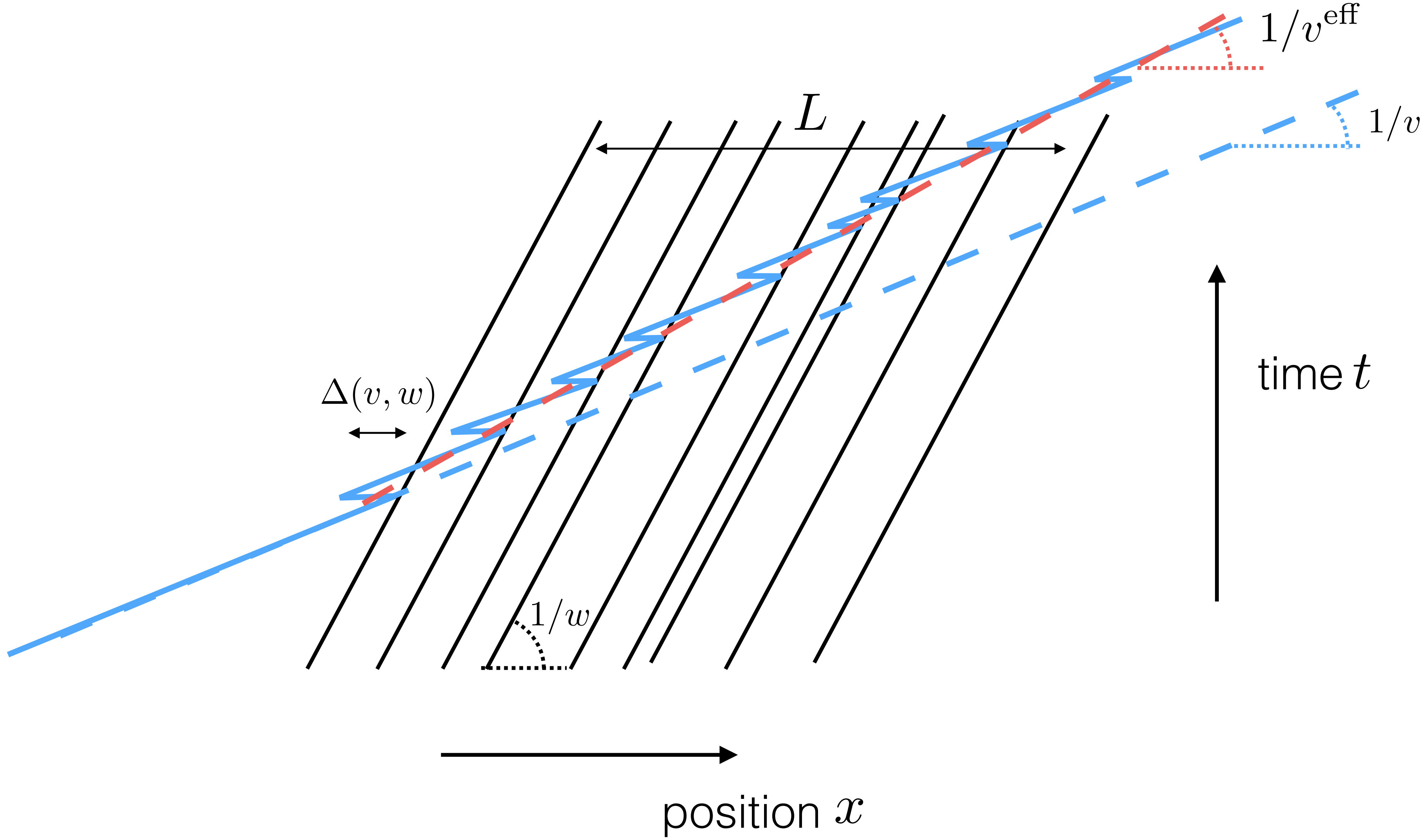
In the Lieb-Liniger model this is a lorentzian

$$\Delta(v_2 - v_1) \;=\; \frac{2g/m}{(g/\hbar)^2 + (v_2 - v_1)^2}$$

1. Crash course on GHD

The ‘effective velocity’ caused by 2-body scattering

$$v^{\rm eff}_{[\rho]}(v) = v + \int dw\, \Delta(v-w)\, \rho(w)\, \left(v^{\rm eff}_{[\rho]}(w) - v^{\rm eff}_{[\rho]}(v)\right)$$



See e.g. **[Caux Doyon, Yoshimura, 2017]**. This effective velocity had appeared previously in **[Bonnes, Essler, Läuchli 2014]**, and for the 1d billiard or hard gas in **[Percus, 1969]**, **[Boldrighini, Dobrushin, Sukhov 1983]**.

1. Crash course on GHD

The rapidities

For N particles, the eigenstates on the infinite line are Bethe states labeled by their rapidities **[Lieb, Liniger, 1963]**

$$\psi(\{x_j\}) = \begin{cases} \sum_{\text{perm. } \sigma} (-1)^{|\sigma|} e^{i\phi_\sigma} e^{\frac{im}{\hbar}(v_{\sigma(1)} x_1 + \cdots + v_{\sigma(N)} x_N)} & \text{if} \quad x_1 < \cdots < x_N \\ (x_i \leftrightarrow x_k, x_j \leftrightarrow x_l, etc.) & \text{otherwise} \end{cases}$$

(the total phase breaks down into a sum of 2-body scattering phases)

$$e^{i\phi_\sigma} = \prod_{\text{transp. } \tau_{ij}} e^{i\varphi(p_j - p_i)}$$

Each eigenstate is labeled by its set of rapidities $\{v_1, v_2, \ldots, v_N\}$**.**

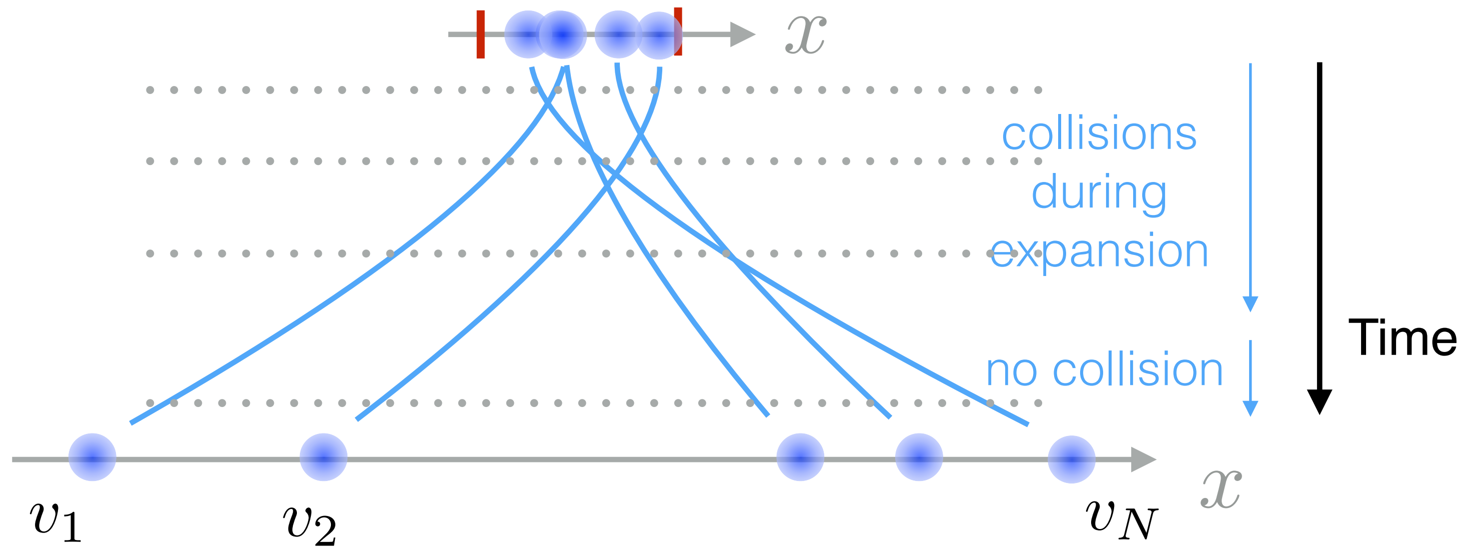
One can define the distribution of rapidities as:

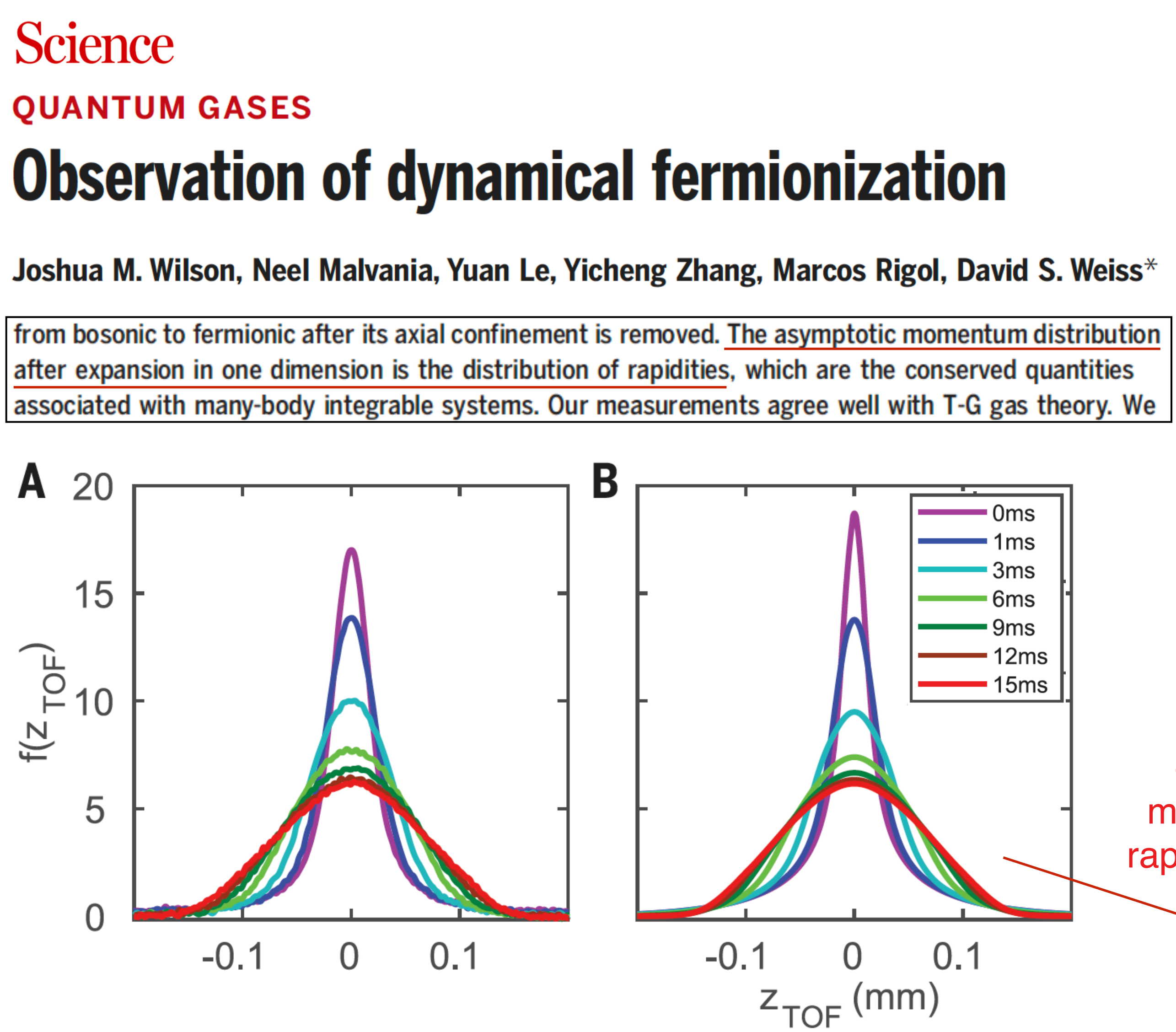
$$\rho(v) = \frac{1}{L} \sum_{j=1}^{N} \delta(v - v_j)$$

1. Crash course on GHD

The rapidities

The rapidities (i.e. asymptotic velocities) can be measured by letting the gas expand in 1D [Rigol-Muramatsu, PRL 94, 2005; Minguzzi-Gangardt, PRL 94, 2005; Jukic-Pezer-Gasenzer-Buljan, PRA 78, 2008; Bolech-Heidrich-Meisner-Langer-McCulloch-Orso-Rigol, PRL 109, 2012; Bolech-Heidrich-Meisner-Langer-McCulloch-Orso-Rigol, J.o. Physics: Conference Series 414 2013, Campbell-Gangardt-Kheruntsyan, PRL 114, 2015; Caux-Doyon-JD-Konik-Yoshimura, SciPost 6, 2019, …]




Science
QUANTUM GASES
Observation of dynamical fermionization
Joshua M. Wilson, Neel Malvania, Yuan Le, Yicheng Zhang, Marcos Rigol, David S. Weiss*

from bosonic to fermionic after its axial confinement is removed. The asymptotic momentum distribution after expansion in one dimension is the distribution of rapidities, which are the conserved quantities associated with many-body integrable systems. Our measurements agree well with T-G gas theory. We

experimental measurement of rapidity distribution

$$\rho(v) = \frac{1}{L}\sum_{i=1}^{N}\delta(v - v_i)$$

1. Crash course on GHD

The people who made the discovery

Breakthrough from 2016!

PRL **117,** 207201 (2016) PHYSICAL REVIEW LETTERS week ending 11 NOVEMBER 2016

Transport in Out-of-Equilibrium *XXZ* Chains: Exact Profiles of Charges and Currents

Bruno Bertini,[1] Mario Collura,[1,2] Jacopo De Nardis,[3] and Maurizio Fagotti[3]
[1]*SISSA and INFN, via Bonomea 265, 34136 Trieste, Italy*
[2]*The Rudolf Peierls Centre for Theoretical Physics, Oxford University, Oxford, OX1 3NP, United Kingdom*
[3]*Département de Physique, École Normale Supérieure/PSL Research University, CNRS, 24 rue Lhomond, 75005 Paris, France*
(Received 17 June 2016; published 8 November 2016)

We consider the nonequilibrium time evolution of piecewise homogeneous states in the *XXZ* spin-1/2 chain, a paradigmatic example of an interacting integrable model. The initial state can be thought of as the result of joining chains with different global properties. Through dephasing, at late times, the state becomes locally equivalent to a stationary state which explicitly depends on position and time. We propose a kinetic
and derive a continuity equation which fully characterizes the
estrict ourselves to the gapless phase and consider cases where the
mperatures, (2) in the ground state of two different models, and (3) in
xcellent agreement (any discrepancy is within the numerical error)
umerical simulations of time evolution based on time-evolving block
ry, we unveil an exact expression for the expectation values of the
ry state.

Selected for a Viewpoint in *Physics*
PHYSICAL REVIEW X **6,** 041065 (2016)

Emergent Hydrodynamics in Integrable Quantum Systems Out of Equilibrium

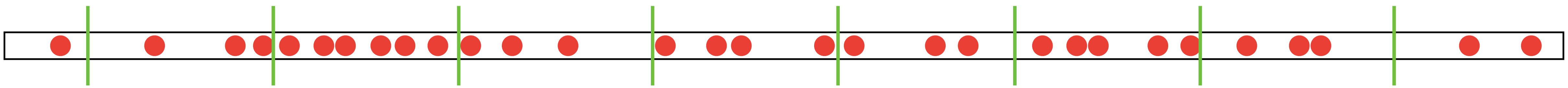
Olalla A. Castro-Alvaredo,[1] Benjamin Doyon,[2] and Takato Yoshimura[2]
[1]*Department of Mathematics, City, University of London, Northampton Square, London EC1V 0HB, United Kingdom*
[2]*Department of Mathematics, King's College London, Strand, London WC2R 2LS, United Kingdom*
(Received 12 July 2016; revised manuscript received 22 September 2016; published 27 December 2016)

Understanding the general principles underlying strongly interacting quantum states out of equilibrium is one of the most important tasks of current theoretical physics. With experiments accessing the intricate dynamics of many-body quantum systems, it is paramount to develop powerful methods that encode the emergent physics. Up to now, the strong dichotomy observed between integrable and nonintegrable evolutions made an overarching theory difficult to build, especially for transport phenomena where space-time profiles are drastically different. We present a novel framework for studying transport in integrable systems: hydrodynamics with infinitely many conservation laws. This bridges the conceptual gap between integrable and nonintegrable quantum dynamics, and gives powerful tools for accurate studies of space-time profiles. We apply it to the description of energy transport between heat baths, and provide a full description of the current-carrying nonequilibrium steady state and the transition regions in a family of models including the Lieb-Liniger model of interacting Bose gases, realized in experiments.

DOI: 10.1103/PhysRevX.6.041065 Subject Areas: Nonlinear Dynamics, Quantum Physics, Statistical Physics

1. Crash course on GHD

Long story short, **GHD** is a fluid-like picture where each 'cell' is in a macrostate characterized by its distribution of rapidities.



Then one writes a transport equation for the rapidities:

$$\partial_t \rho + \partial_x \left(v^{\mathrm{eff}}_{[\rho]} \, \rho \right) \; = \; \frac{\partial_x V}{m} \partial_v \rho$$

where $\rho(v, x, t)$ is the local distribution of rapidities, and the effective velocity is the one caused by the 2-body scattering:

$$v^{\mathrm{eff}}_{[\rho]}(v) = v + \int dw \, \Delta(v - w) \, \rho(w) \, \left(v^{\mathrm{eff}}_{[\rho]}(w) - v^{\mathrm{eff}}_{[\rho]}(v) \right)$$

1. Crash course on GHD

This was the 'handwaving' intodoction of GHD. Another, more well-defined, way to arrive at it is to derive it like a standard hydrodynamic theory.

Namely, to **postulate local relaxation** and **express the expectation values of the currents in terms of those of the local charges**.

1. Crash course on GHD

GHD from the formula for the expectation value of currents

Let $|\mathbf{v}\rangle = |\{v_a\}_{1\leq a\leq N}\rangle$ be the Bethe state with rapidities $v_a,\ a = 1,\ldots,N$.

The conserved charges of the Lieb-Liniger model are the operators $Q[f]$ diagonal in the basis of Bethe states and with eigenvalues:

$$Q[f]\,|\mathbf{v}\rangle \;=\; \left(\sum_{a=1}^{N} f(v_a)\right)|\mathbf{v}\rangle$$

The current operator $j[f](x)$ associated to the charge $Q[f] = \int_0^L q[f](x)dx$ is defined by the continuity equation

$$\partial_t q[f] + \partial_x j[f] \;=\; i\,[H, q[f]] + \partial_x j[f] \;=\; 0$$

Then the key problem is to compute the expectation value of the current

$$\boxed{\frac{\langle\mathbf{v}|\, j[f]\,|\mathbf{v}\rangle}{\langle\mathbf{v}|\mathrm{v}\rangle} = ?}$$

1. Crash course on GHD

GHD from the formula for the expectation value of currents

Note: before 2016, this looked like a **totally hopeless problem**. Several issues:

First, writing the charges $Q[f] = \int_0^L q[f](x)dx$ in second quantization is usually not possible, as it gives rise to regularization issues in the Lieb-Liniger model **[Davies, Korepin 1989].**

Second, even if one has a good regularization for the charges, one needs to compute the current operators. This looks like a non-trivial problem (to me, at least)

Third, even if one has an expression for the currents, computing their expectation value in Bethe states may well turn out to be an intractable problem…

1. Crash course on GHD

GHD from the formula for the expectation value of currents

The big 'guess' of **[Castro-Alvaredo, Doyon, Yoshimura, 2016]** and **[Bertini, Collura, de Nardis, Fagotti, 2016]** is that, in the thermodynamic limit, one should have

$$\lim_{N,L\to\infty} \frac{\langle \mathbf{v}|\, q[f]\, |\mathbf{v}\rangle}{\langle \mathbf{v}|\mathbf{v}\rangle} = \int f(v)\rho(v)dv$$

$$\lim_{N,L\to\infty} \frac{\langle \mathbf{v}|\, j[f]\, |\mathbf{v}\rangle}{\langle \mathbf{v}|\mathbf{v}\rangle} = \int f(v) v^{\rm eff}_{[\rho]}(v)\rho(v)dv$$

Since 2016, several works have increased our understanding of that formula for the current **[Vu, Yoshimura, 2019]**, **[Spohn, 2020]**, **[Cubero, Panfil, 2020]**, **[Bajnok, 2020]**, **[Spohn, Yoshimura 2020]**, and the level of rigor in its derivation.

Remarkably, a **new fundamental Bethe Ansatz formula** was discovered in finite size **[Pozsgay 2020]**, **[Borsi, Pozsgay, Pristyak, 2020]**, **[Pozsgay 2020]**. It is proved using new developments in the Algebraic Bethe Ansatz. Its thermodynamic limit gives the formula above.

$$\frac{\langle \mathbf{v}|\, j[f]\, |\mathbf{v}\rangle}{\langle \mathbf{v}|\mathbf{v}\rangle} = \sum_{1\leq a,b\leq N} v_a \,[G^{-1}]_{ab}\, f(v_b)$$

inverse of Gaudin matrix

This talk

Generalized Hydrodynamics (GHD) in the quantum one-dimensional Bose gas.

1. Generalized Hydrodynamics (GHD) of the 1D Bose gas: standard theory

2. **Checks in cold atoms experiments**

3. Re-introducing quantum fluctuations

2. Applying the theory to cold atom experiments

Why should experimentalists care about GHD?

It is much easier to numerically solve the GHD equation than to solve the full many-body Schrödinger equation for $N \sim 10^1 - 10^4$ atoms.

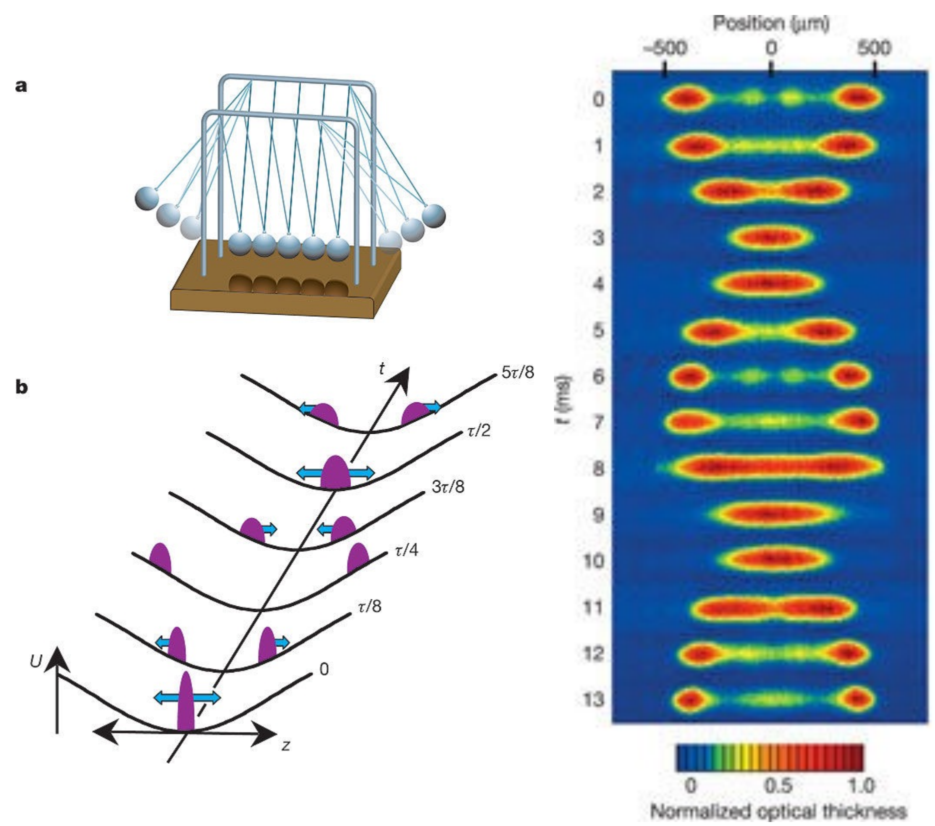
For instance: how hard is it to simulate the quantum Newton's cradle numerically?


nature

LETTERS

A quantum Newton's cradle

Toshiya Kinoshita[1], Trevor Wenger[1] & David S. Weiss[1]



Before 2016, no one knew how to do this.

Now, with GHD, it can be done on a laptop in a few minutes (at least for a single tube).

There is a publicly available GHD code: iFluid [Moller, Schmiedmayer, Scipost 2020]

2. Applying the theory to cold atom experiments

Example: GHD numerical simulation of the Newton's cradle **[Caux, Doyon, JD, Konik, Yoshimura, 2017]**

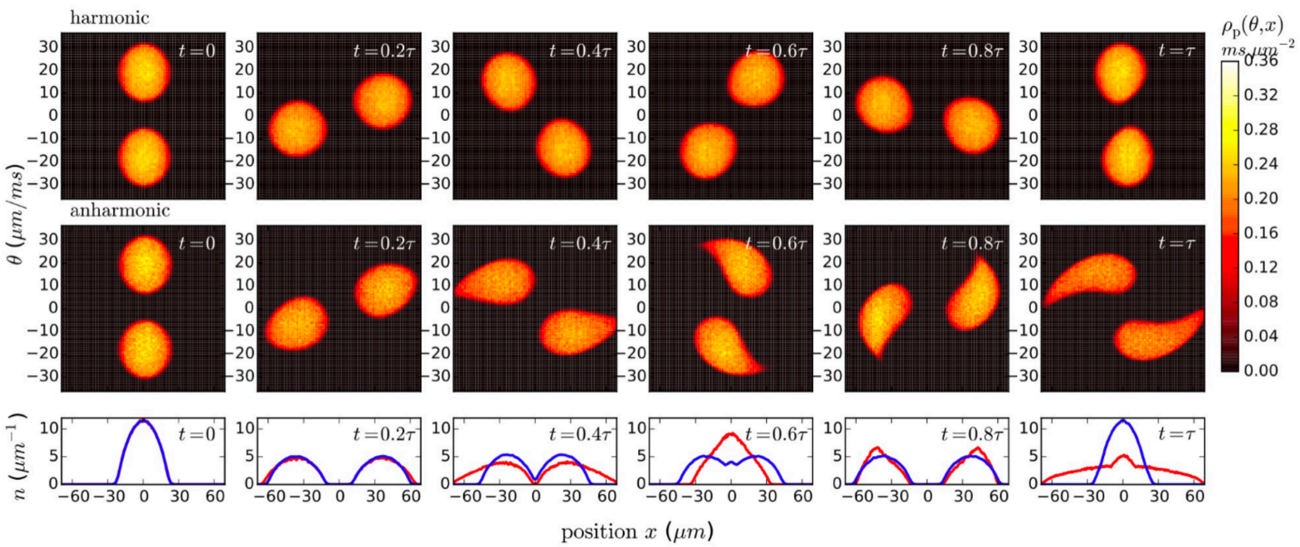


Figure 1: Evolution of the density of quasi-particles $\rho_p(\theta,x,t)$ —here plotted in the (x,θ)-plane— in the QNC setup, with parameters given in the text. The solution of the GHD equations are obtained from the flea gas [35]. The results are displayed for the harmonic trap (top row) and the weakly anharmonic one (middle row), on one period of the (quasi-)harmonic trap. (Bottom row) Corresponding density of particles $n(x,t)$, for the harmonic trap (blue) and the anharmonic one (red).

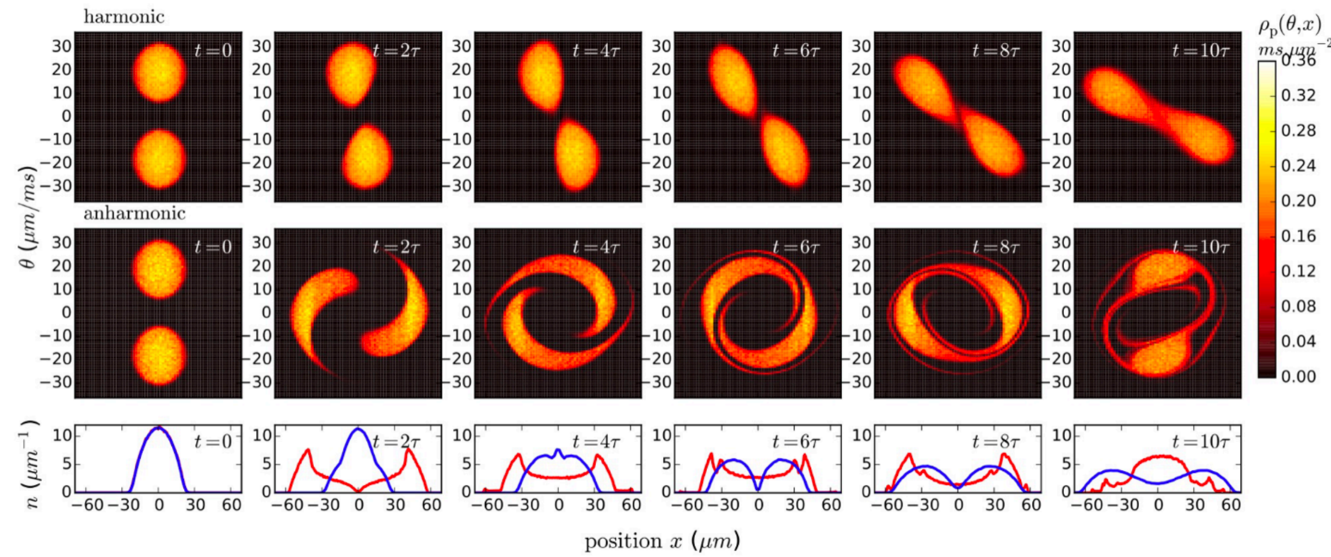


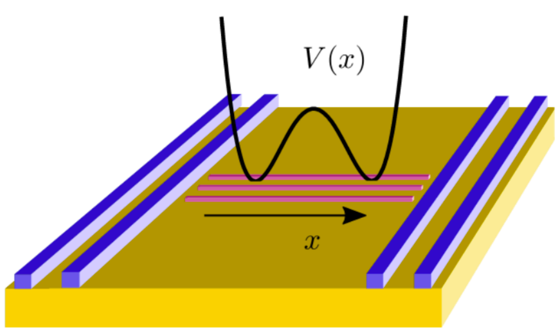
Figure 2: Same as Fig. 1, on a larger time window. In the harmonic case, the two blobs in the (x,θ)-plane keep rotating around each other after several trap periods. In the anharmonic case, the distribution $\rho_p(\theta,x)$ is strongly stirred up after a few trap periods, and it goes to stationary state that looks rotationally invariant in the (x,θ)-plane.

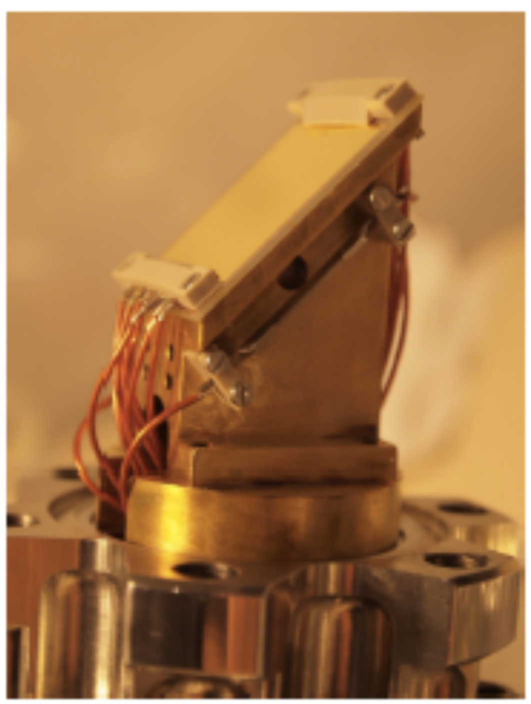
2. Applying the theory to cold atom experiments

Two classes of experiments on 1D gases

Atom chip: atoms trapped by the magnetic field created above an electronic chip.

Allows to trap a single 1D cloud, usually with weak repulsion between the atoms

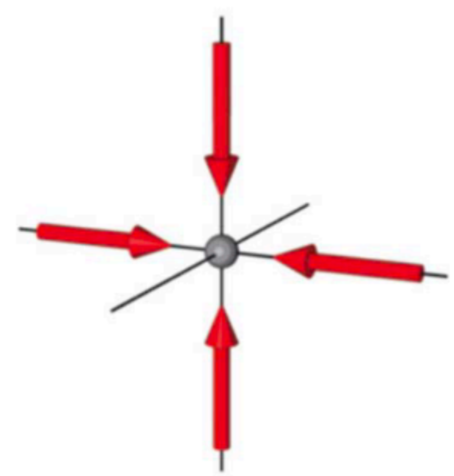


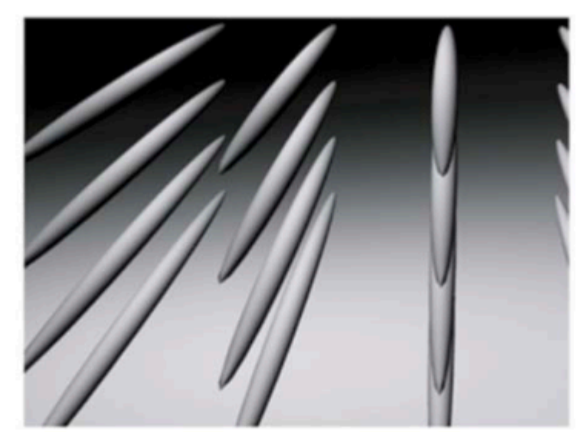


Optical trapping: atoms trapped by counter propagating lasers.

Allows to create a bundle of 1D clouds, with slightly different parameters for each 1D tube: observables are averaged over all tubes

Strong repulsion between the atoms.





2. Applying the theory

Two classes of experiments

PRL 100, 090402 (2008) PHYSICAL REVIEW LETTERS week ending 7 MARCH 2008

Yang-Yang Thermodynamics on an Atom Chip

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[2]*ARC Centre of Excellence for Quantum-Atom Optics, School of Physical Sciences, University of Queensland, Brisbane, Queensland 4072, Australia*

(Received 12 September 2007; revised manuscript received 24 January 2008; published 3 March 2008)

We investigate the behavior of a weakly interacting nearly one-dimensional trapped Bose gas at finite temperature. We perform *in situ* measurements of spatial density profiles and show that they are very well described by a model based on exact solutions obtained using the Yang-Yang thermodynamic formalism, in a regime where other, approximate theoretical approaches fail. We use Bose-gas focusing [I. Shvarchuck *et al.*, Phys. Rev. Lett. **89**, 270404 (2002)] to probe the axial momentum distribution of the gas and find good agreement with the *in situ* results.

DOI: 10.1103/PhysRevLett.100.090402 PACS numbers: 05.30.Jp, 03.75.Hh, 05.70.Ce

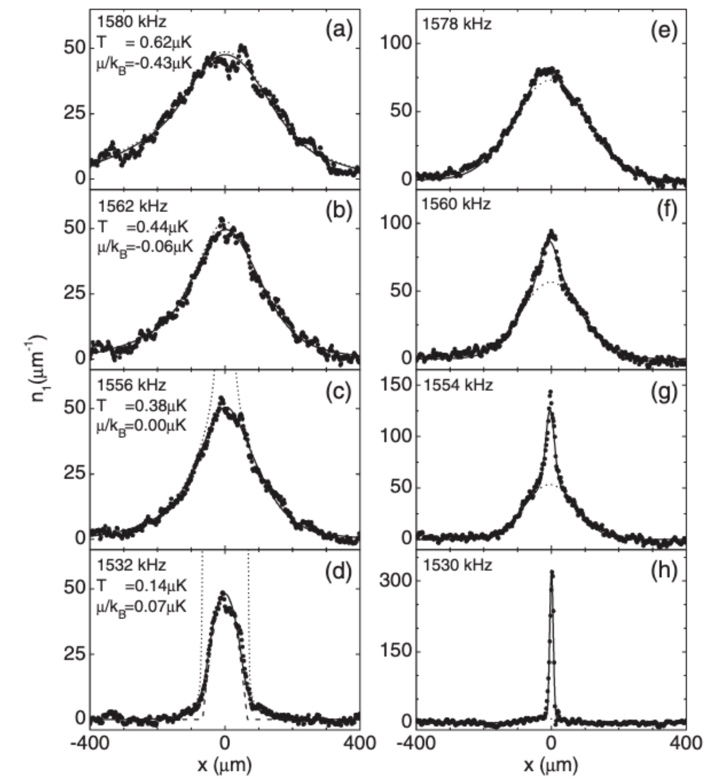

FIG. 1. Linear atomic density from absorption images obtained *in situ* (a)–(d) and *in focus* (e)–(h) by lowering (from top to bottom as indicated) the final rf evaporation frequency. *In situ*: solid lines are fits using Yang-Yang thermodynamic equations (see text). The values of μ and T resulting from the fits are shown in the figure. Dotted line: ideal Bose-gas profile showing divergence for $\mu(x) = 0$. Dashed line in (d): quasicon-

PHYSICAL REVIEW A **88**, 031603(R) (2013)

Thermodynamics of strongly correlated one-dimensional Bose gases

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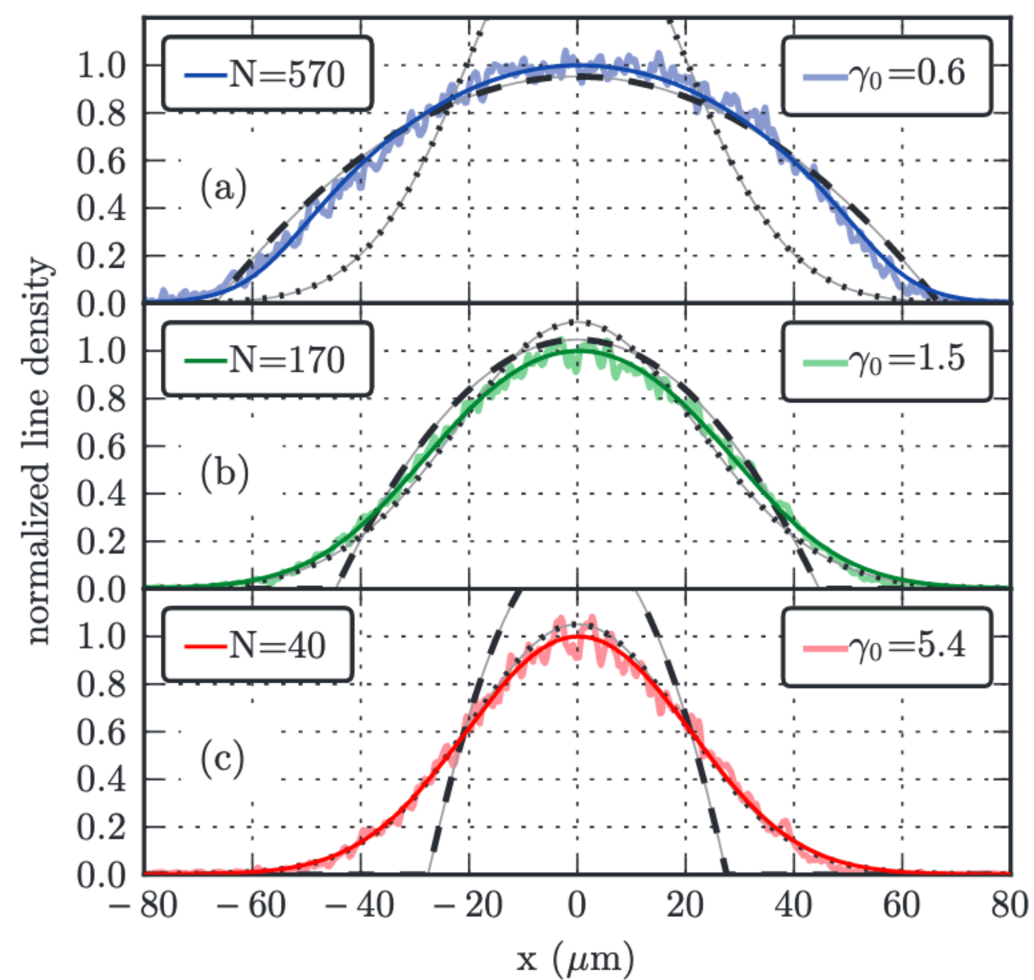
(Received 8 April 2013; revised manuscript received 17 June 2013; published 11 September 2013)

We investigate the thermodynamics of one-dimensional (1D) Bose gases in the strongly correlated regime. To this end, we prepare ensembles of independent 1D Bose gases in a two-dimensional optical lattice and perform high-resolution *in situ* imaging of the column-integrated density distribution. Using an inverse Abel transformation we derive effective one-dimensional line-density profiles and compare them to exact theoretical models. The high resolution allows for a direct thermometry of the trapped ensembles. The knowledge about the temperature enables us to extract thermodynamic equations of state such as the phase-space density, the entropy per particle, and the local pair-correlation function.

DOI: 10.1103/PhysRevA.88.031603 PACS number(s): 67.85.−d, 03.75.Hh, 05.30.Jp, 37.10.Jk



2. Applying the theory to cold atom experiments

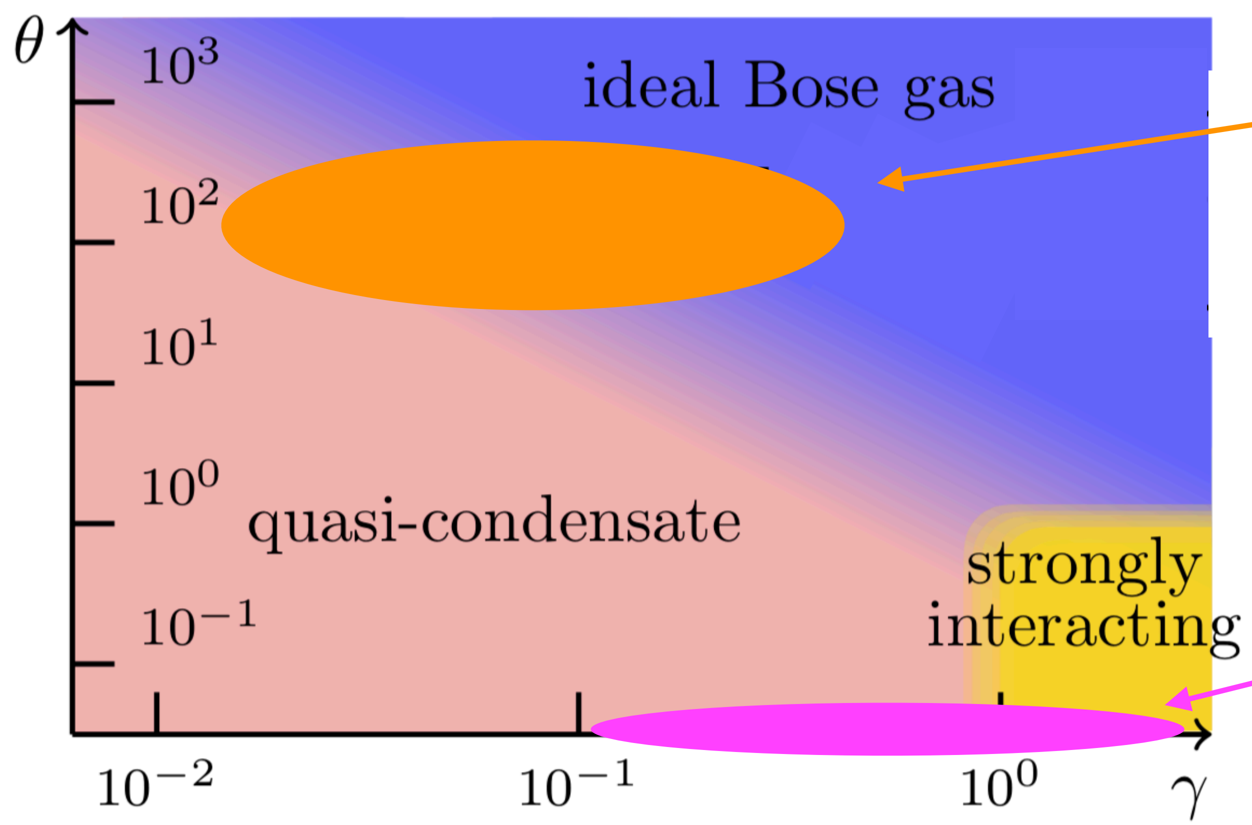
Two experiments on GHD

Regimes of the 1D Bose gas (at equilibrium):

[Petrov, Shlyapnikov, Walraven PRL 85, 2000]

dimensionless temperature

$\theta = 2\hbar^2 T/(mg^2)$



Palaiseau experiment (Bouchoule), atom chip, single 1D tube

$N \simeq 10^4$

Penn State experiment (Weiss), optical trapping, bundle of 1D tubes

$N \sim 10^1 - 10^2$

(per tube)

dimensionless repulsion strength

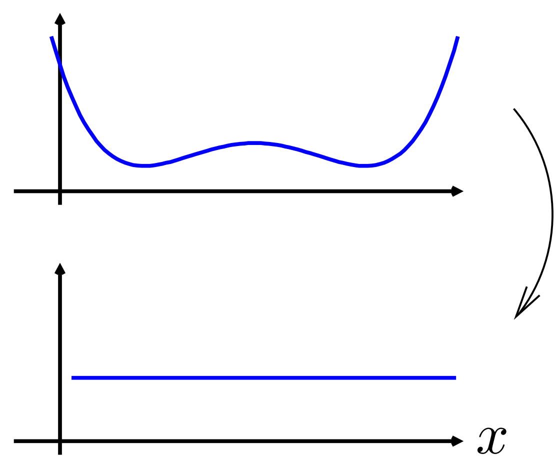
$\gamma = mg/(\hbar^2 n)$

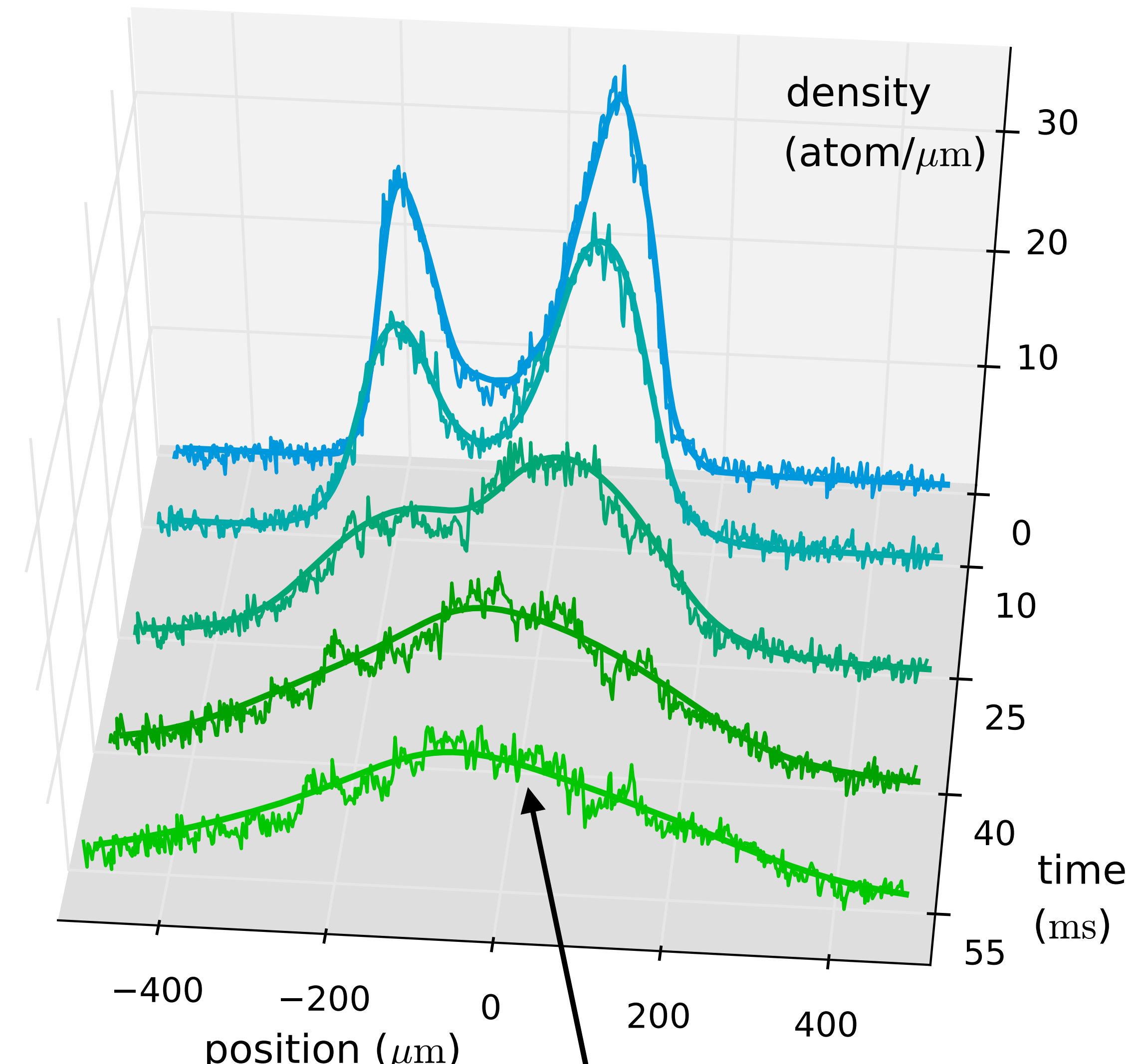
2. Applying the theory to cold atom experiments

Results of the Palaiseau experiment (Bouchoule group)

[Schemmer, Bouchoule, Doyon, JD, PRL 122, 2019]

Expansion from double-well potential:

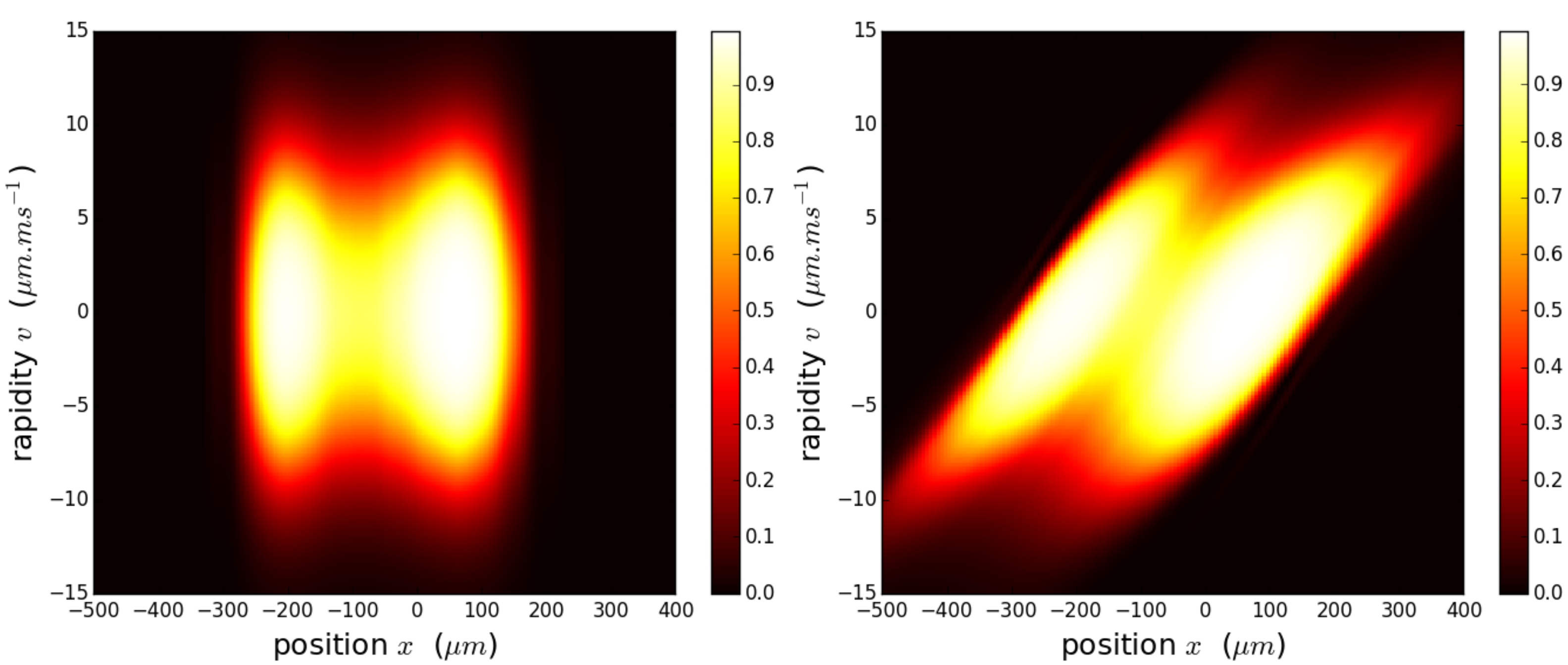




full line is theory, noisy line is experimental data

occupation of position-rapidity space simulated with GHD:

t=0 t=25 ms

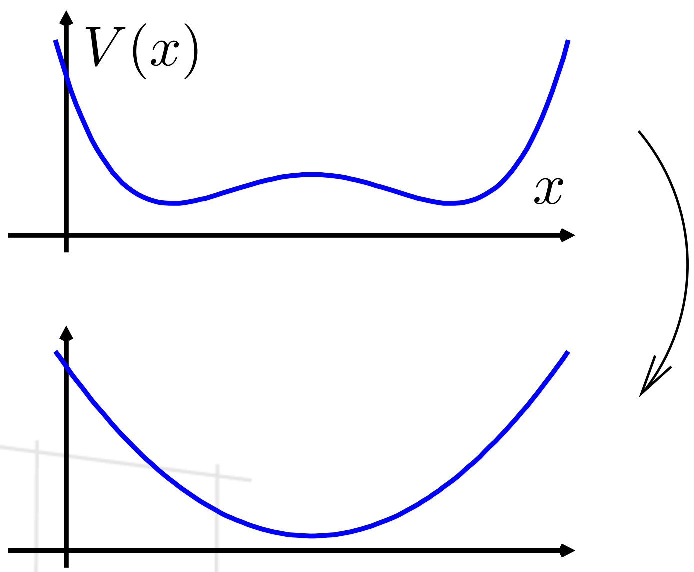


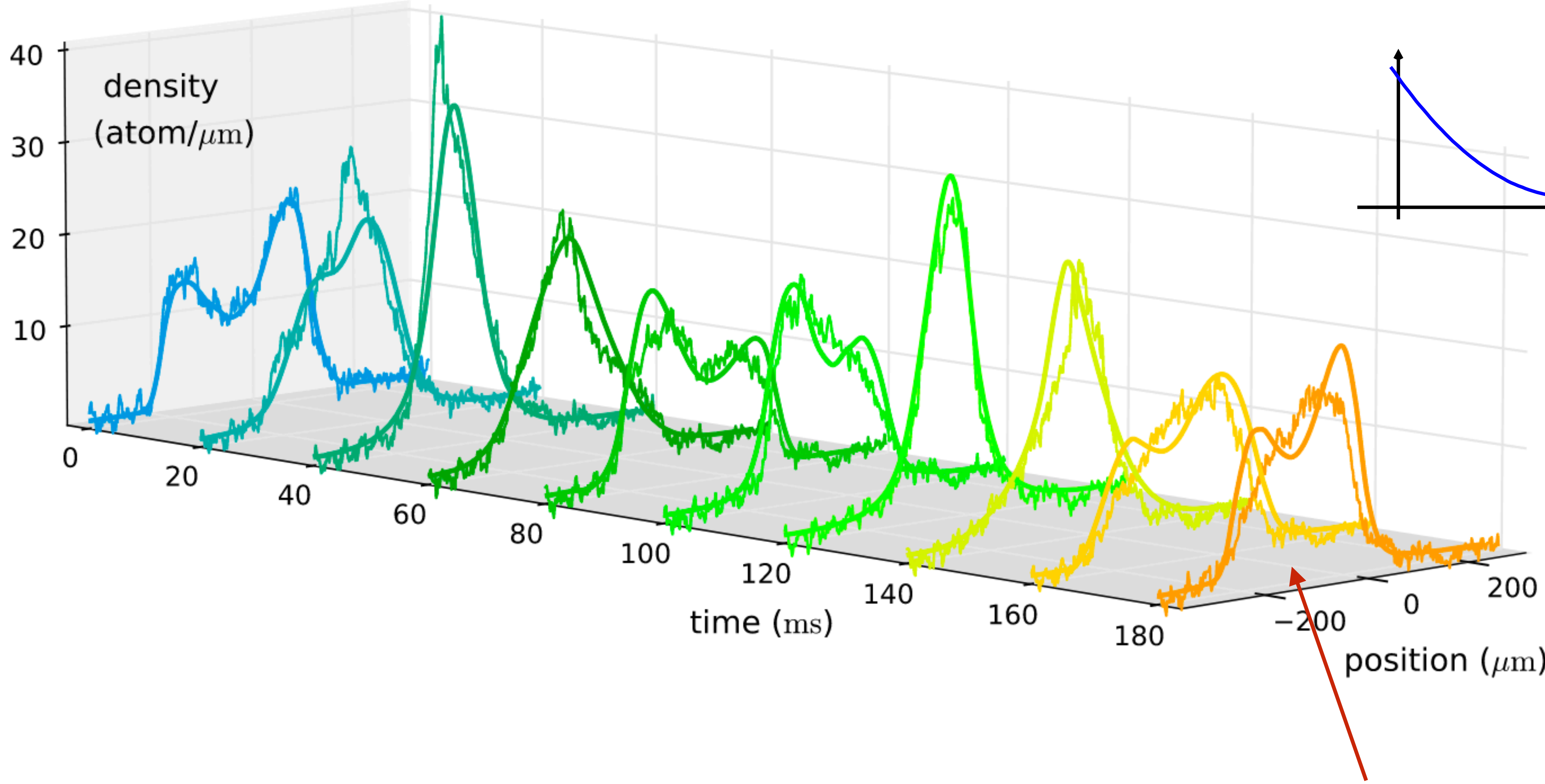
2. Applying the theory to cold atom experiments

Results of the Palaiseau experiment (Bouchoule group)

[Schemmer, Bouchoule, Doyon, JD, PRL 122, 2019]

Quench from double-well to harmonic potential:





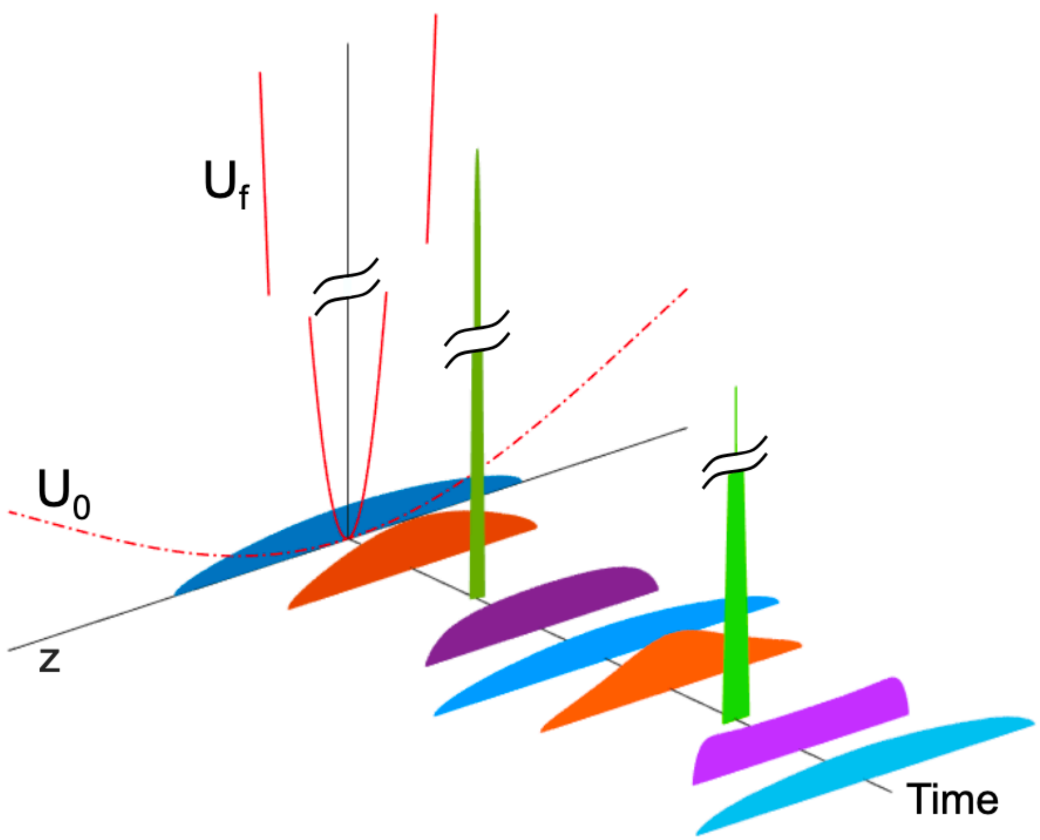
about 20% of atoms lost (not described in the theory)

2. Applying the theory to cold atom experiments

Results of the Penn State experiment (Weiss group)

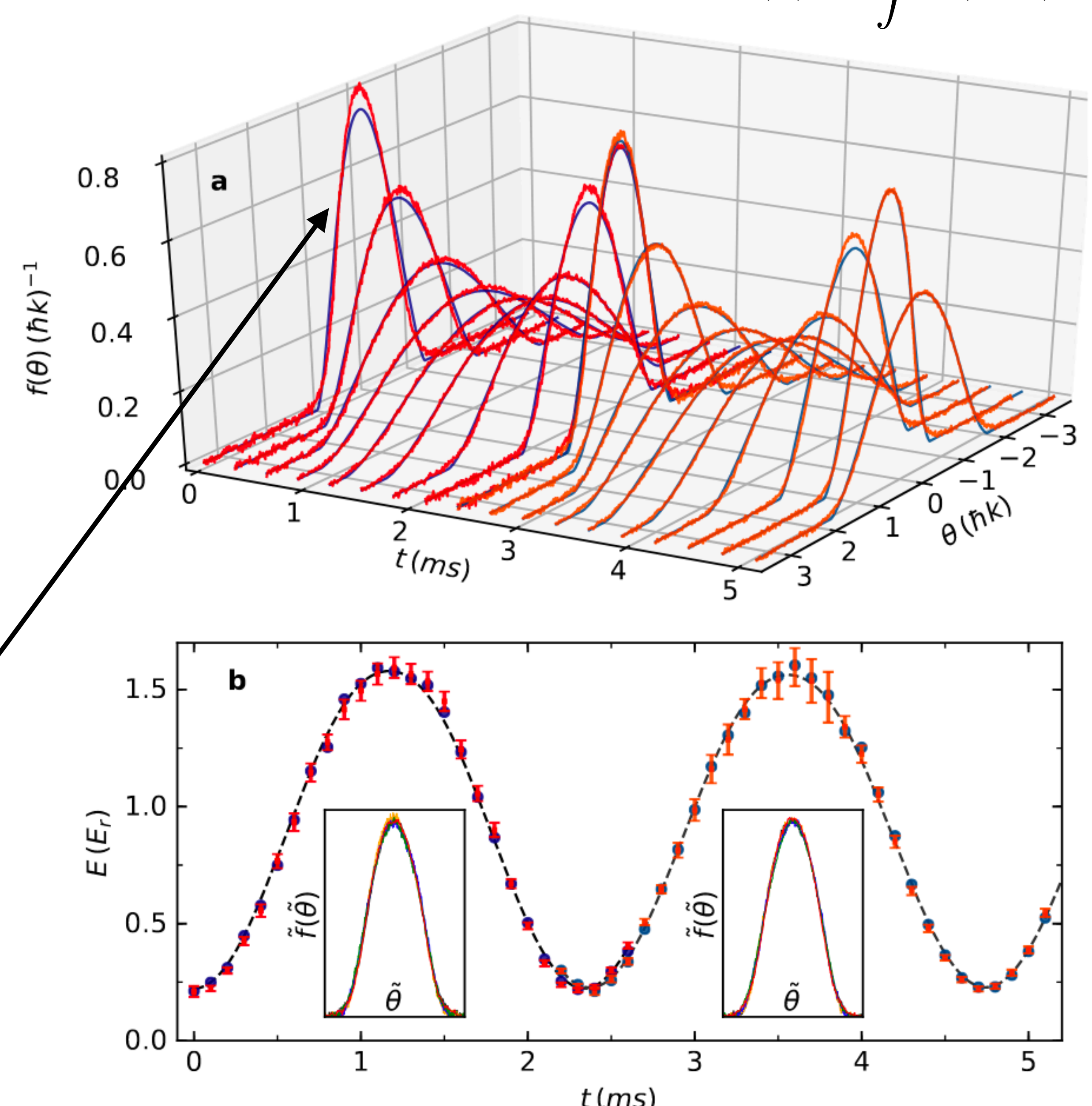
[Malvania, Zhang, Le, JD, Rigol, Weiss, arXiv:2009.06651]

Sudden increase of the depth of the 1D harmonic trap:



Results for 10-times increase of the depth:

Integrated rapidity distribution $f(\theta) = \int \rho(x,\theta)dx$



Rapidity energy $E = \int \frac{\theta^2}{2m}\rho(x,\theta)dx\,d\theta$

blue line is theory, red line is experimental data

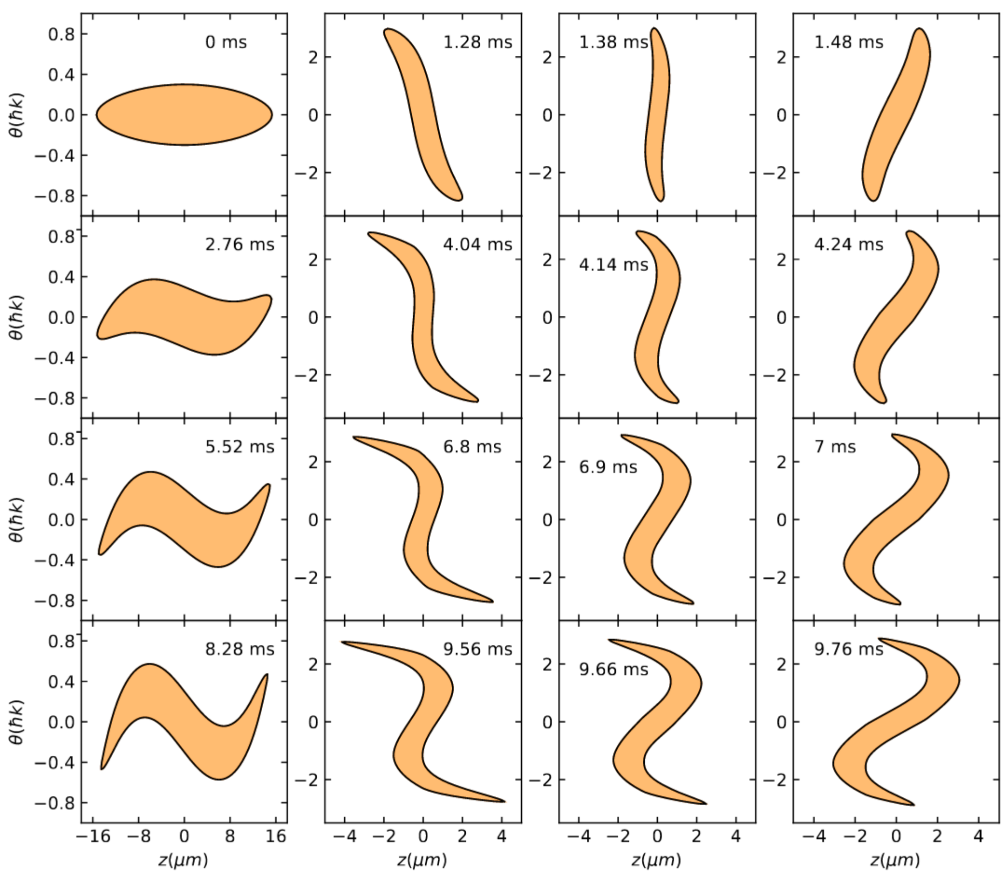
2. Applying the theory to cold atom experiments

Results of the Penn State experiment (Weiss group)

[Malvania, Zhang, Le, JD, Rigol, Weiss, arXiv:2009.06651]

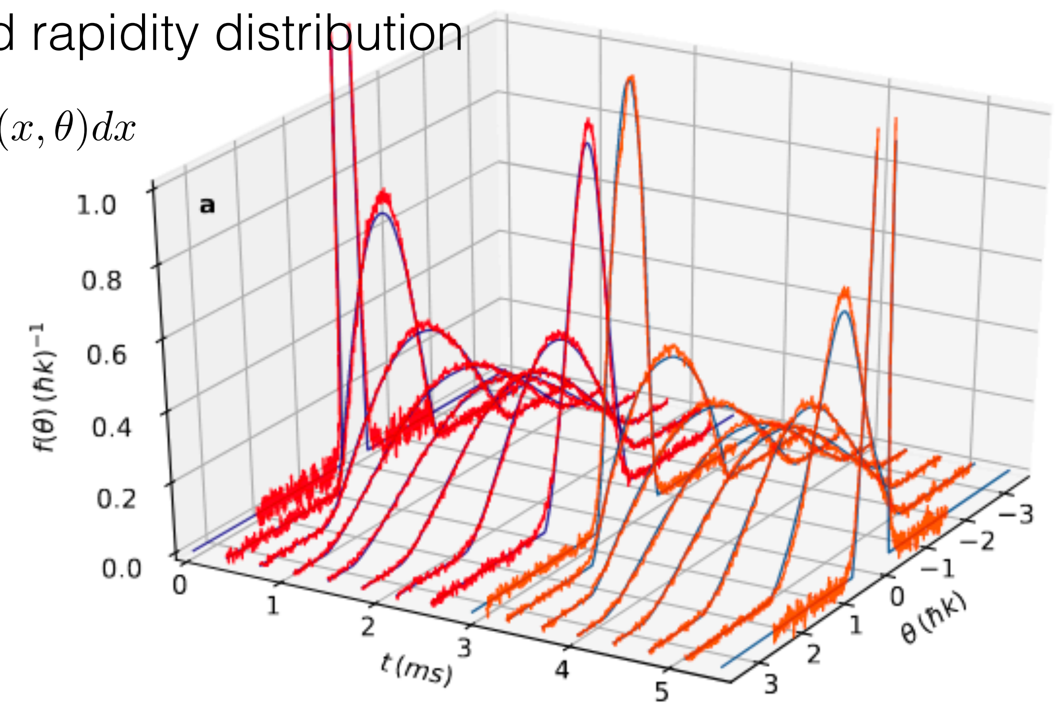
Results for 100-times increase of the depth:

occupation of position-rapidity space simulated with GHD:



Integrated rapidity distribution

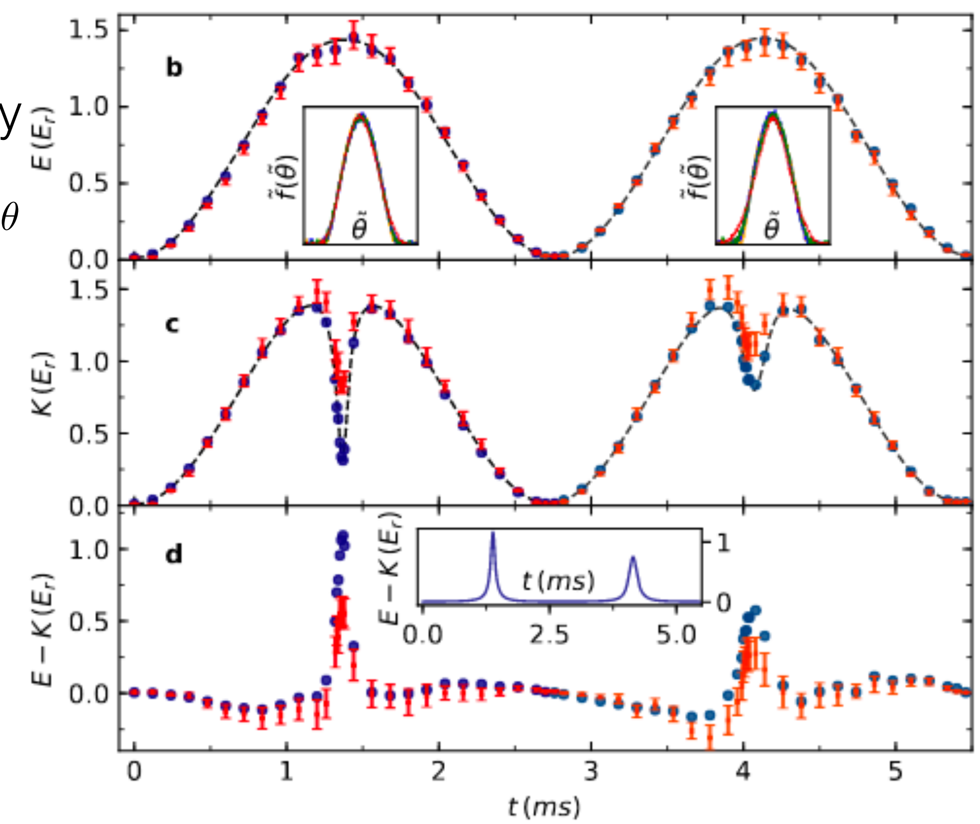
$$f(\theta) = \int \rho(x,\theta)dx$$



Rapidity energy

$$E = \int \frac{\theta^2}{2m}\rho(x,\theta)dx\, d\theta$$

Kinetic energy
(obtained from measurement of momentum dist.)



2. Applying the theory to cold atom experiments

Results of the Penn State experiment (Weiss group)

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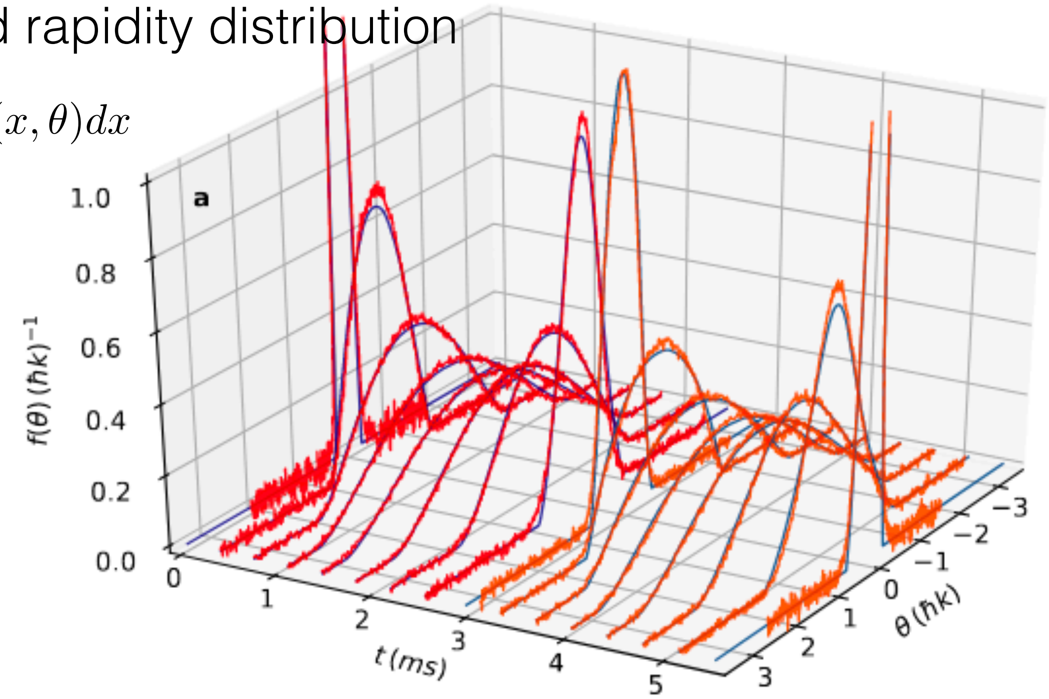
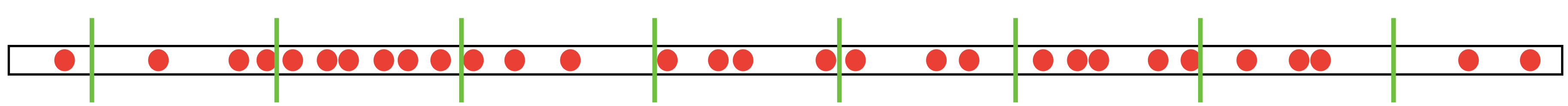


Remarkably: this is an ‘extreme’ situation where the density changes very rapidly, both in time and space. Also, the number of particles is very small: $N \simeq 11$ per tube (in average).

So the hydrodynamic picture of a continuous medium made of ‘fluid cells’ that have time to locally relax to a Generalized Gibbs Ensemble is **challenged**.



This suggests that **‘GHD’ might not be a hydrodynamic theory after all** (cf. Tonks-Girardeau case). **More theory work is needed there.**

$$H = \int dx \left(\frac{\hbar^2}{2} (\partial_x \psi^\dagger)(\partial_x \psi) + \frac{g}{2} \psi^{\dagger 2} \psi^2 + V(x) \psi^\dagger \psi \right)$$

full **quantum** many-body problem

classical evolution equation (GHD)

$$\partial_t \rho + \partial_x \left(v^{\rm eff}_{[\rho]} \, \rho \right) \; = \; \frac{\partial_x V}{m} \partial_v \rho$$

$$v^{\rm eff}_{[\rho]}(v) = v + \int dw \, \Delta(v-w) \, \rho(w) \, \left(v^{\rm eff}_{[\rho]}(w) - v^{\rm eff}_{[\rho]}(v) \right)$$

- **Q1**: where does the quantumness of the microscopic model enter the large-scale hydrodynamic description?
- **Q2**: are there quantum effects that are lost in that description?

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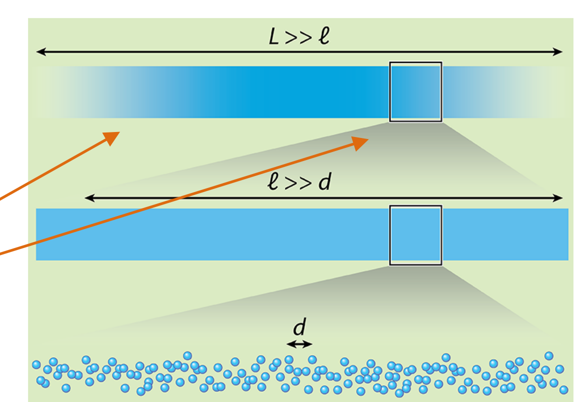
- **Q1**: where does the quantumness of the microscopic model enter the large-scale hydrodynamic description?
    - **A1**: it enters through the Wigner time delay $\Delta(v-w)$
- **Q2**: are there quantum effects that are lost in that description?
    - **A2**: yes: entanglement and correlations between fluid cells at equal time

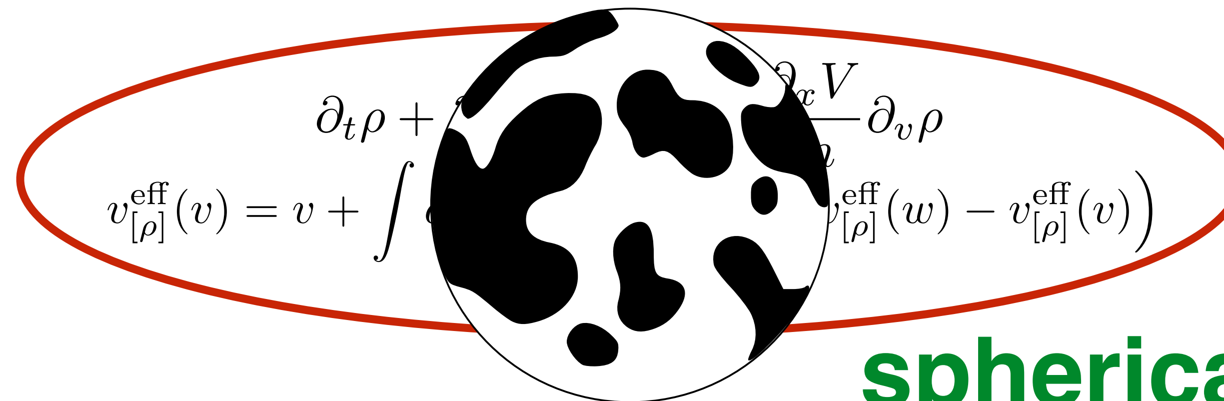
in classical hydrodynamics, no correlations between different fluid cells at equal time. Yet, in the original quantum system, such correlations are present



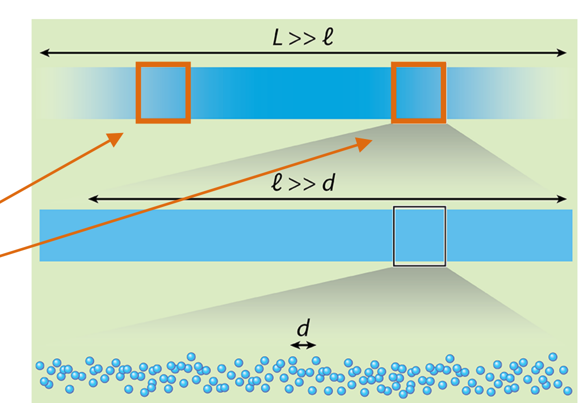
real beast

full **quantum** many-body problem

classical evolution equation (GHD)



spherical cow



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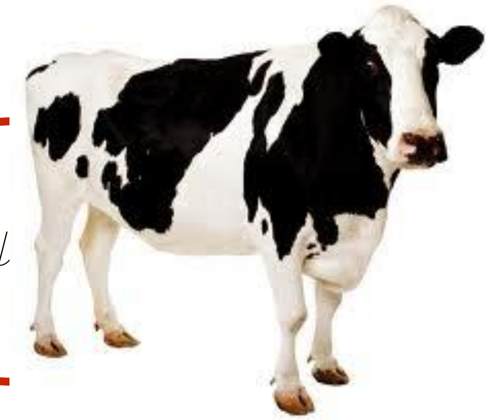
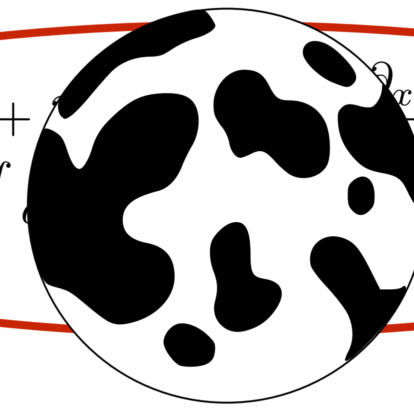


$H = \int dx \left(\frac{\hbar^2}{2} (\partial_x \psi \quad \ldots \quad (x) \psi^\dagger \psi \right)$

full **quantum** many-body problem

classical evolution equation (kinetic equation for quasi-particles)

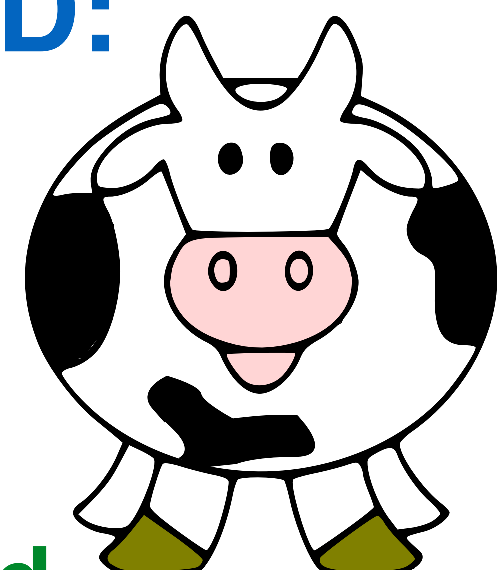


$\partial_t \rho + \ldots \frac{\partial_x V}{\ldots} \partial_v \rho$

$v^{\rm eff}_{[\rho]}(v) = v + \int \ldots \left(v^{\rm eff}_{[\rho]}(w) - v^{\rm eff}_{[\rho]}(v) \right)$

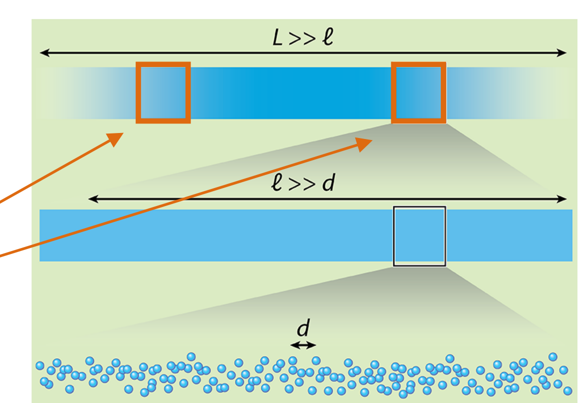
spherical cow

beyond GHD:



quantum fluctuating GHD theory?

improved spherical cow

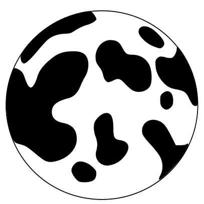


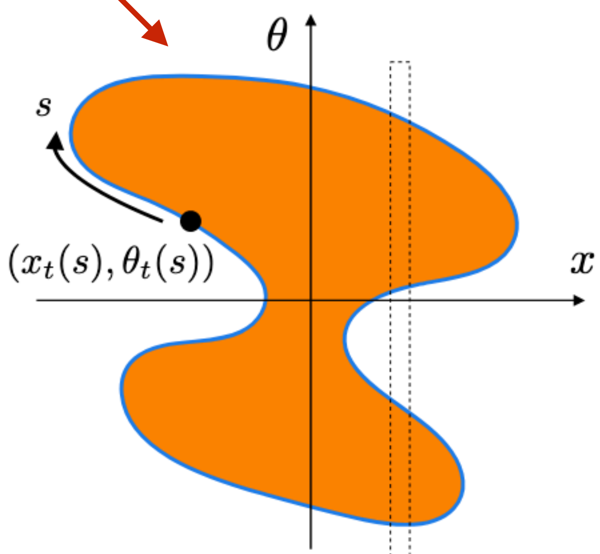
in classical hydrodynamics, no correlations between different fluid cells at equal time. Yet, in the original quantum system, such correlations are present

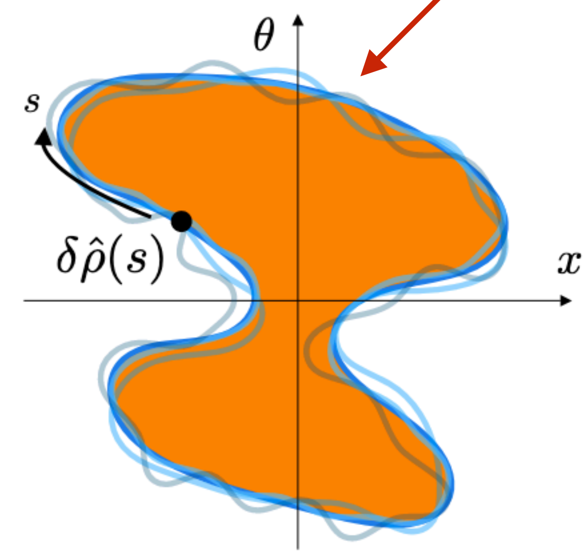
Quantum fluctuations around GHD

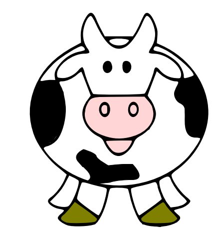
‘standard’ GHD predicts time-evolution of region of phase-space occupied by quasi-particles

the idea is to allow this region to fluctuate in phase space









- technically, this is done by defining an operator $\delta\hat{\rho}(s)$ which measures the small fluctuation of the contour encircling the region of phase-space occupied by quasi-particles

- the operator satisfies the commutation relations of a chiral boson: $[\delta\hat{\rho}(s), \delta\hat{\rho}(s')] = \frac{1}{2\pi i}\delta'(s-s')$

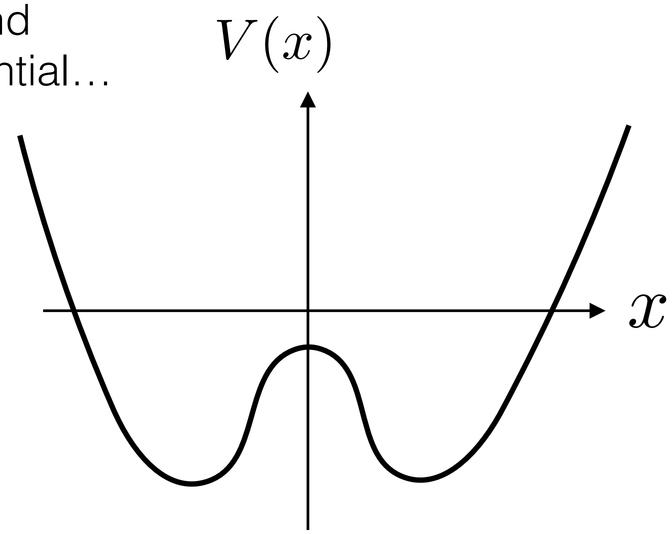
- the Hamiltonian that governs the time evolution is quadratic in $\delta\hat{\rho}(s)$ so we are dealing with a free boson theory (as in Luttinger liquid theory).

- the difficulty is to compute the propagator of the bosonic field. This is done numerically.

Examples of results obtained with this approach

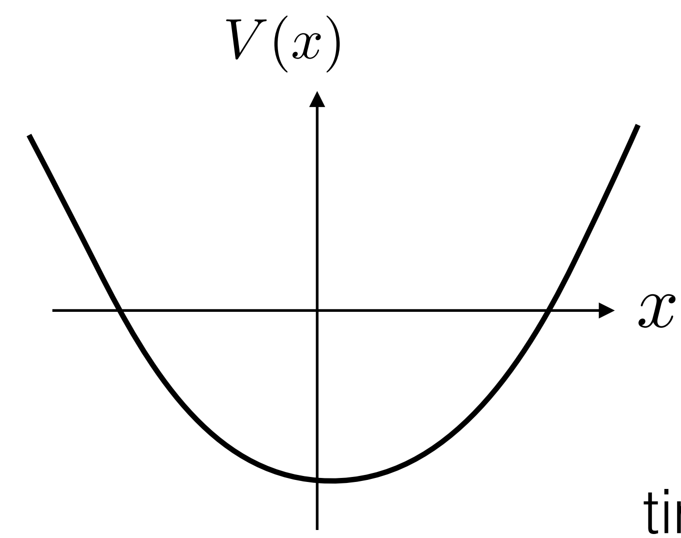
quench from the ground state in a double-well potential…

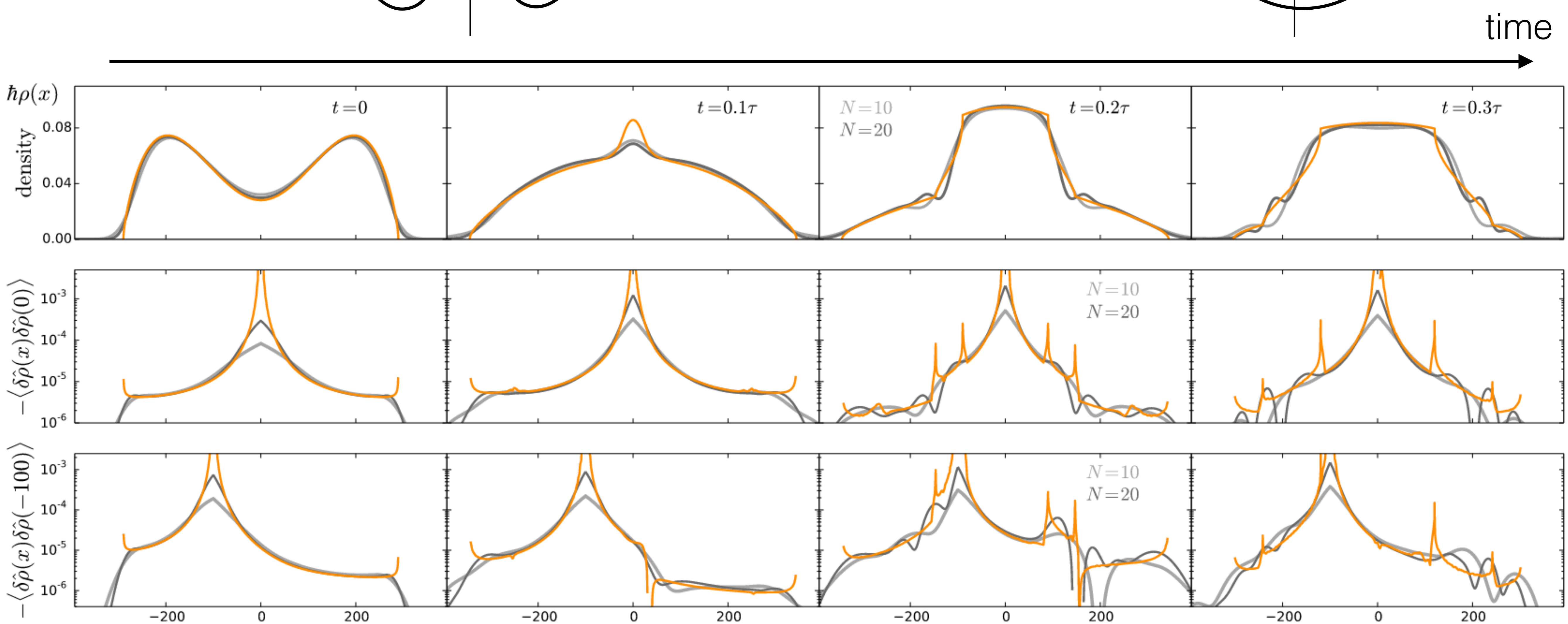
$t = 0$



…to harmonic potential

$t > 0$





from **[Ruggiero, Doyon, Calabrese, JD, 2020]**

Examples of results obtained with this approach

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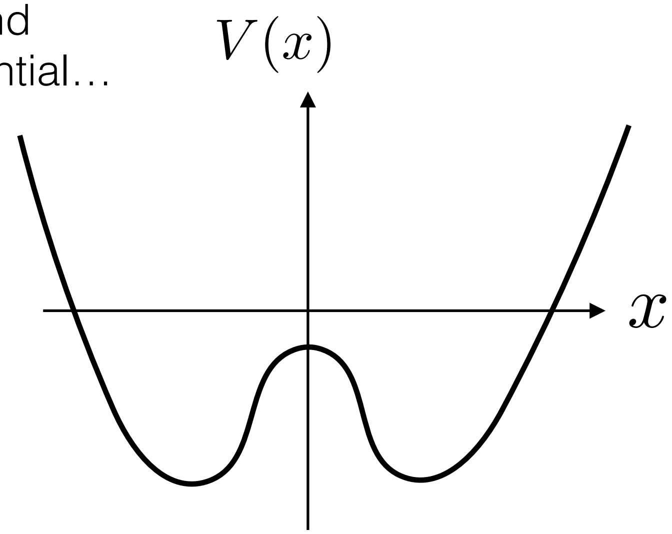

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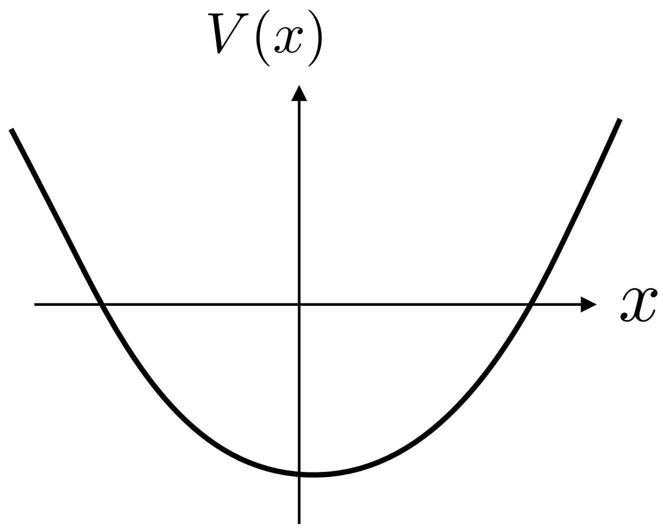

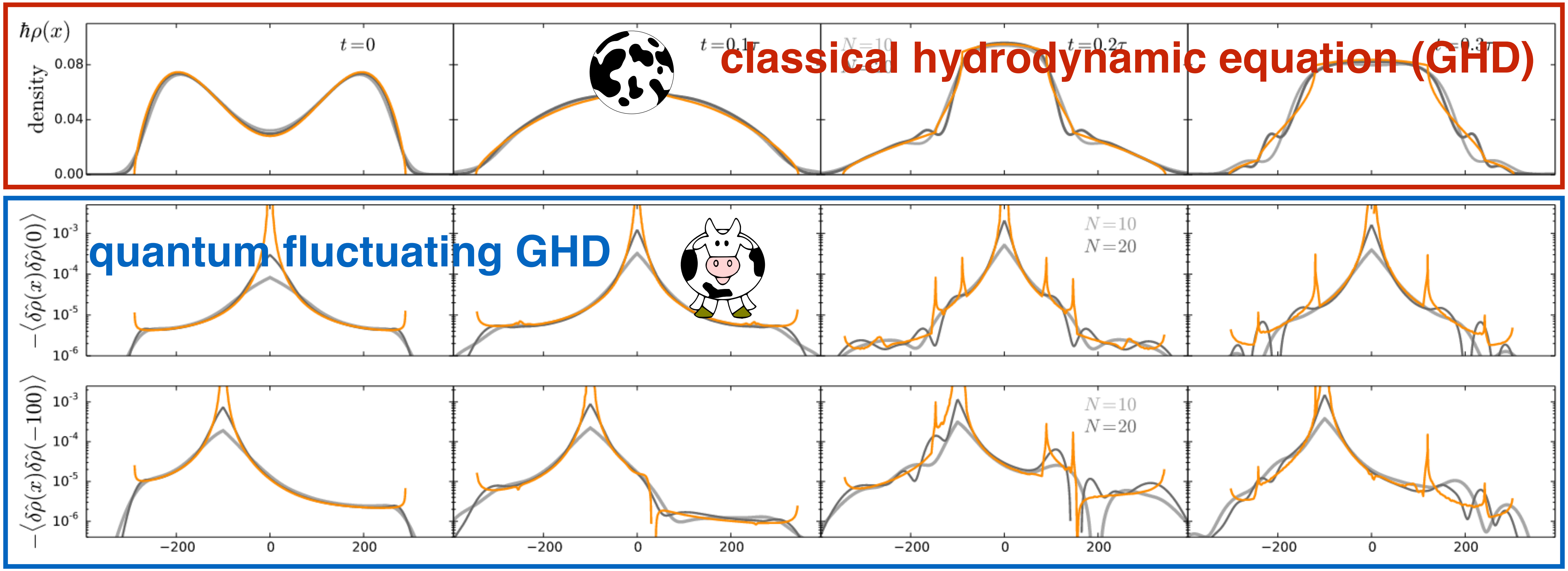

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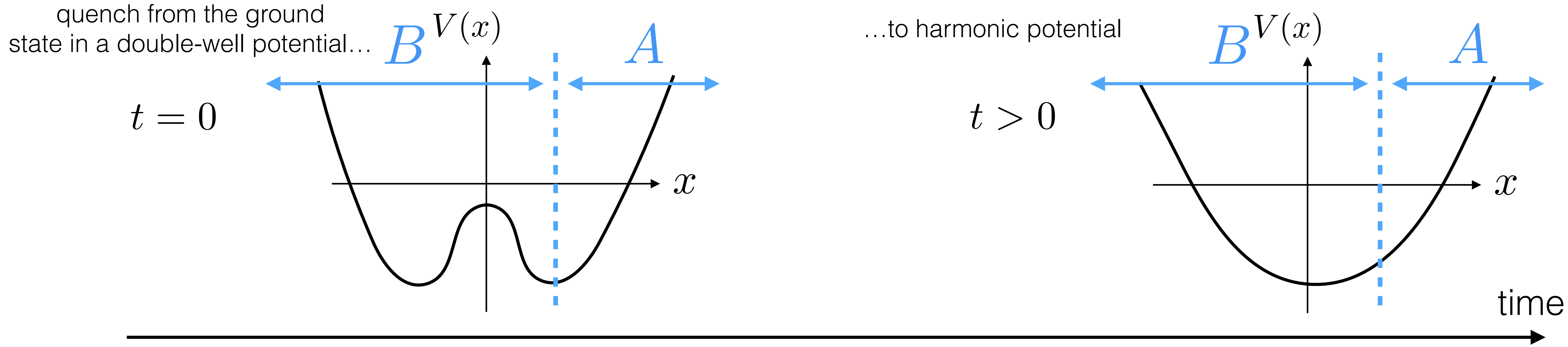
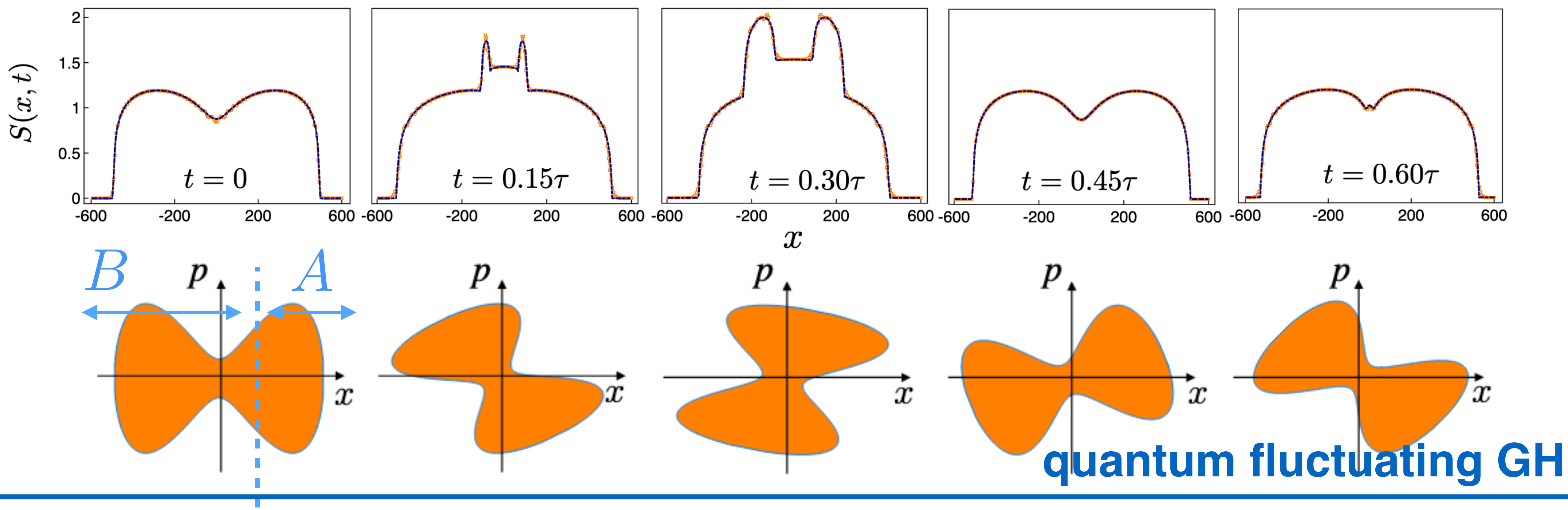


entanglement entropy of subsystem A (compared with numerics in the hard core limit)

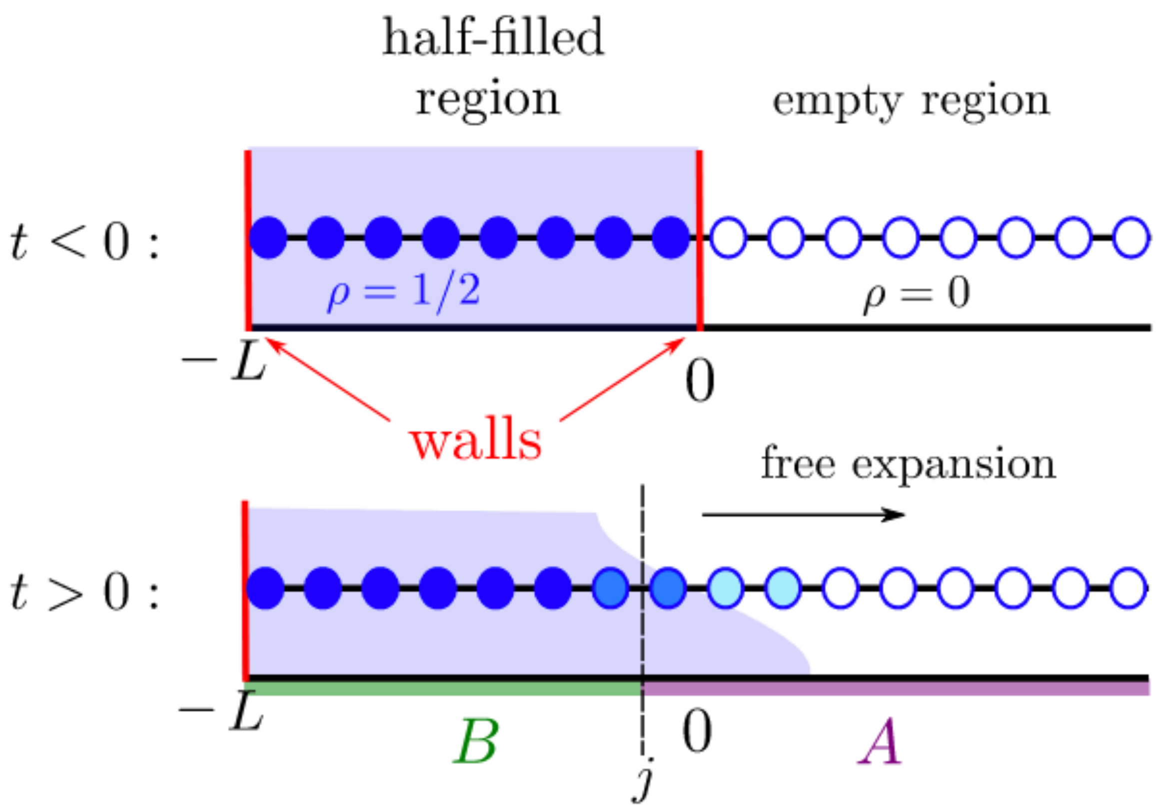


quantum fluctuating GHD

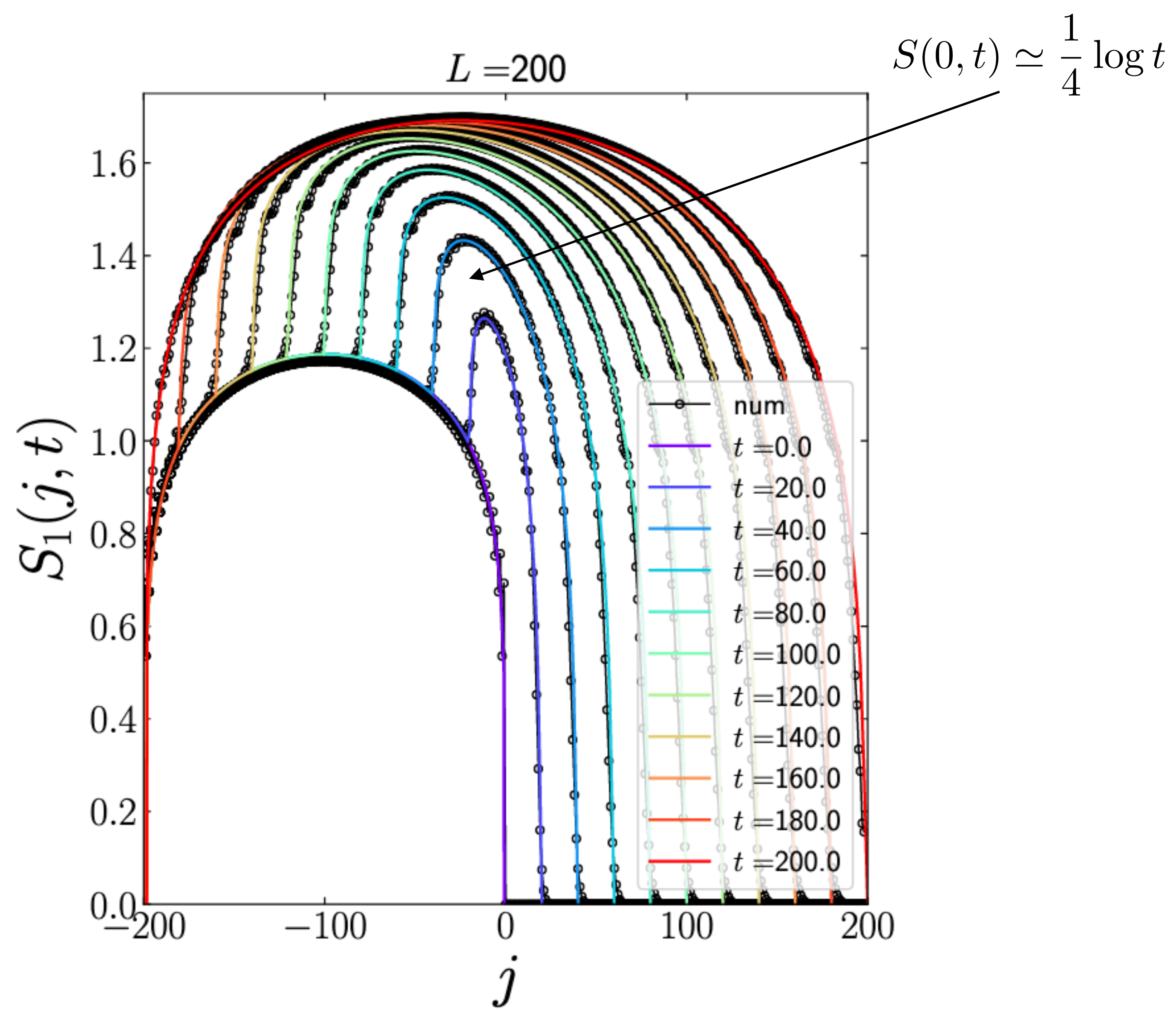
from **[Ruggiero, Doyon, Calabrese, JD, 2022]**

Examples of results obtained with this approach

quench from ground state of half-filled lattice gas on the left, empty system on the right



entanglement entropy profiles from ‘quantum GHD’, compared to numerics (non-interacting fermions)



[Scopa, Krajenbrink, Calabrese, JD 2021]. See also **[Eisler 2021], [Scopa, Calabrese, JD 2022]**

(probably way over time by now…)

Thank you!