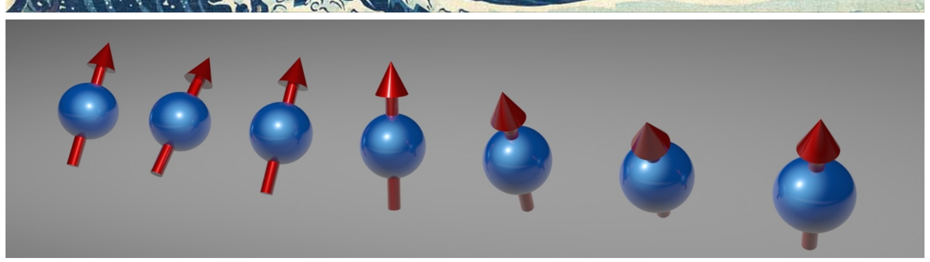
HIGH-TEMPERATURE SPIN TRANSPORT IN THE XXZ SPIN CHAIN:

DIFFUSION, KPZ DYNAMICS AND BEYOND



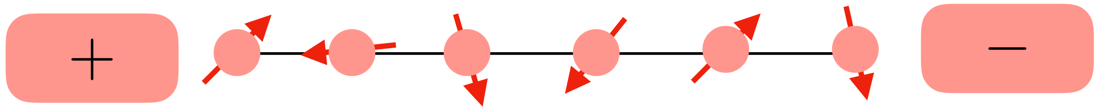
JACOPO DE NARDIS

CERGY PARIS UNIVERSITÉ

IN COLLABORATION WITH: E. ILIEVSKI, S. GOPALAKRISHNAN, R. VASSEUR, B. WARE

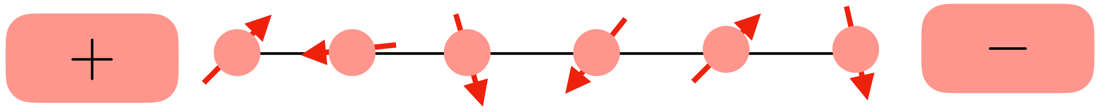
Spin transport in the XXZ chain

$$H = \sum_j \vec{S}_j \cdot \vec{S}_{j+1} + (\Delta - 1) S^z_j S^z_{j+1} + h \sum_j S^z_j$$



Spin transport in the XXZ chain

$$H = \sum_j \vec{S}_j \cdot \vec{S}_{j+1} + (\Delta - 1) S^z_j S^z_{j+1} + h \sum_j S^z_j$$

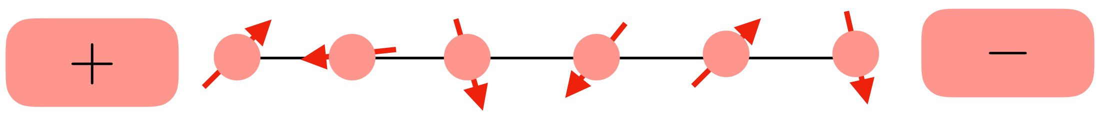


$$\sigma(\omega) = \int_0^\infty dt e^{-i\omega t} \int dx (j(x,\ t), j(0,0))$$

$$j(x) = i(S^+_x S^-_{x+1} - S^-_x S^+_{x+1})$$

Spin transport in the XXZ chain

$$H = \sum_j \vec{S}_j \cdot \vec{S}_{j+1} + (\Delta - 1) S^z_j S^z_{j+1} + h \sum_j S^z_j$$



$$\sigma(\omega) = \int_0^\infty dt e^{-i\omega t} \int dx (j(x,\ t), j(0,0))$$

$$j(x) = i(S^+_x S^-_{x+1} - S^-_x S^+_{x+1})$$

$$C^{zz}(x,t) = (S^z(x,t), S^z(0,0))$$

Ballistic

$$\sigma(\omega) \sim \delta(\omega)$$

$$\int dx\ x^2 C^{zz}(x,t) \sim t^2$$

Super (sub)-diffusive

$$\sigma(\omega) \sim \omega^{-\alpha}$$

$$\int dx\ x^2 C^{zz}(x,t) \sim t^{2/z}$$

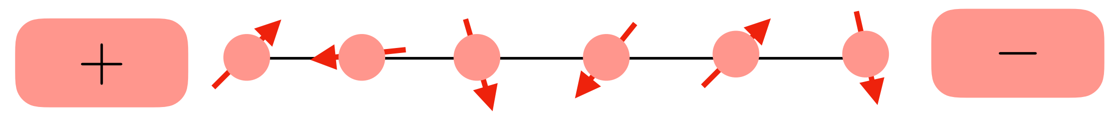
Diffusive

$$\sigma(\omega) \sim \text{const}$$

$$\int dx\ x^2 C^{zz}(x,t) \sim t$$

Spin transport in the XXZ chain

$$H = \sum_j \vec{S}_j \cdot \vec{S}_{j+1} + (\Delta - 1) S^z_j S^z_{j+1} + h \sum_j S^z_j$$

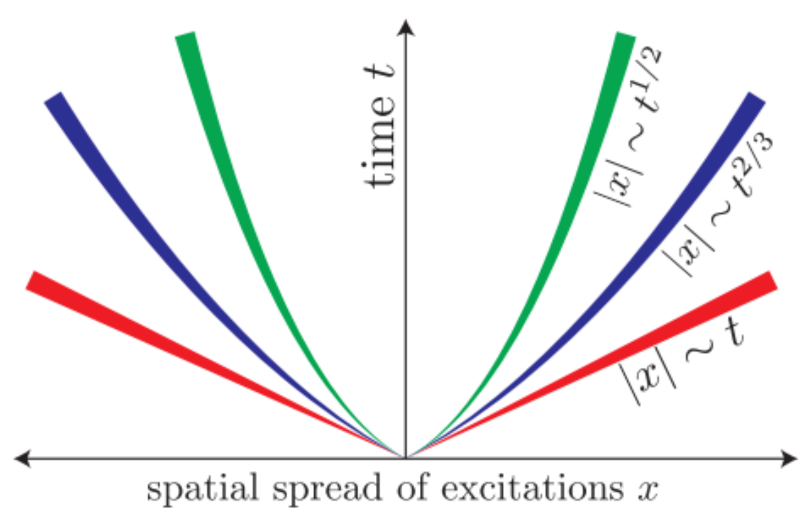


$\Delta = 0$ | $\Delta = 1$ | Δ

$\sigma(\omega) = \mathcal{D}\delta(\omega) + \dots$

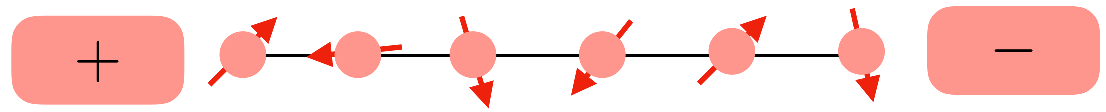
$\sigma(\omega) = K\omega^{-1/3} + \dots$

$\sigma(\omega) = \sigma_{\text{reg}}(\omega)$



KPZ dynamics at the isotropic point

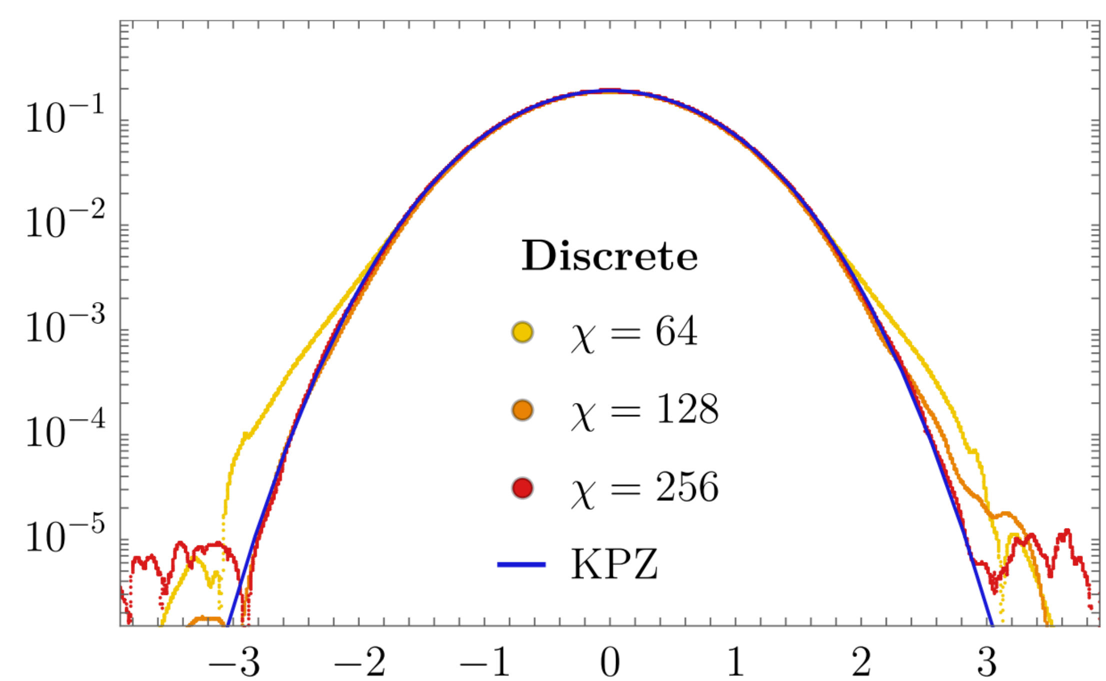
$$H = \sum_j \vec{S}_j \cdot \vec{S}_{j+1} + (\Delta - 1) S^z_j S^z_{j+1} + h \sum_j S^z_j$$



$$C^{zz}(x,t) \to \frac{\lambda}{t^{2/3}} f_{\mathrm{KPZ}}(\lambda x / t^{2/3}) \qquad \int dx x^2 C^{zz}(x,t) \sim t^{4/3}$$

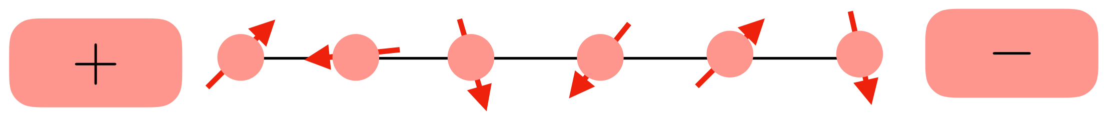
Kardar-Parisi-Zhang physics in the quantum Heisenberg magnet

Marko Ljubotina, Marko Žnidarič, and Tomaž Prosen
Physics Department, Faculty of Mathematics and Physics,
University of Ljubljana, 1000 Ljubljana, Slovenia



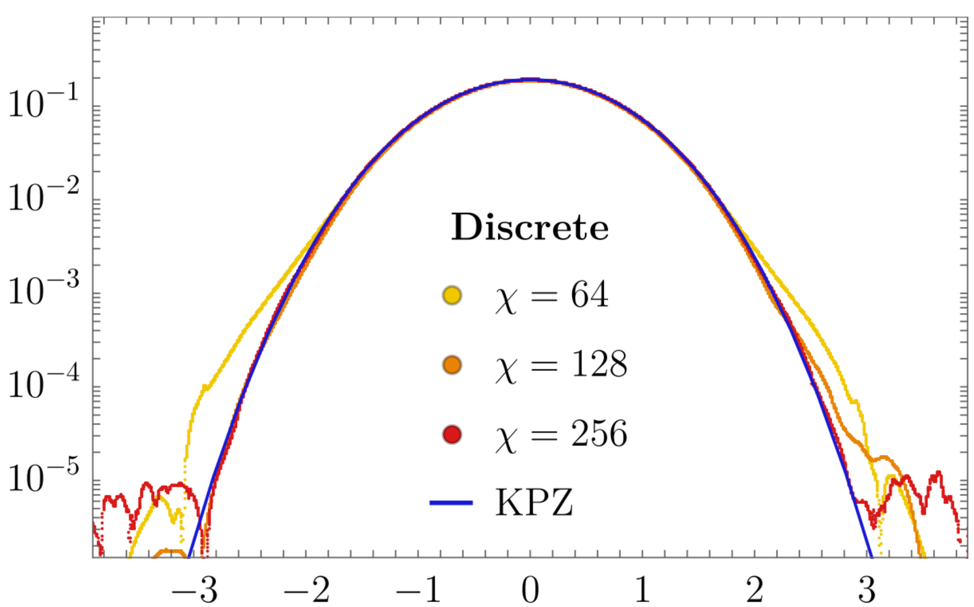
KPZ dynamics at the isotropic point

$$H = \sum_j \vec{S}_j \cdot \vec{S}_{j+1} + (\Delta - 1) S^z_j S^z_{j+1} + h \sum_j S^z_j$$



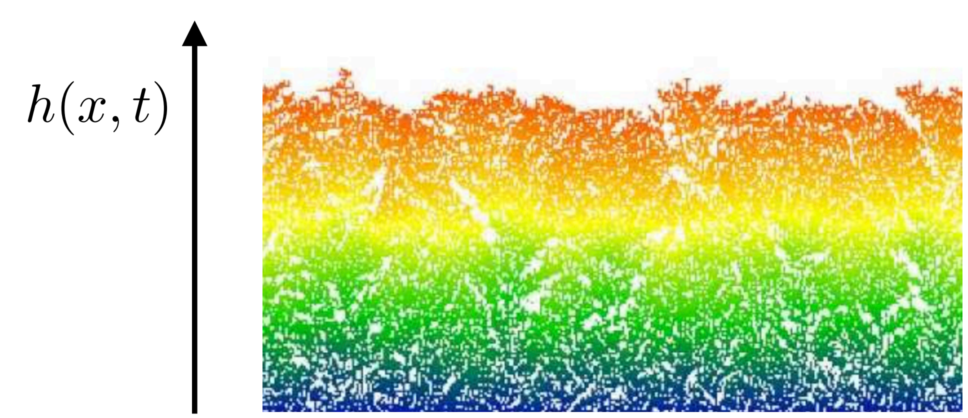
Kardar-Parisi-Zhang physics in the quantum Heisenberg magnet

Marko Ljubotina, Marko Žnidarič, and Tomaž Prosen
Physics Department, Faculty of Mathematics and Physics, University of Ljubljana, 1000 Ljubljana, Slovenia



$$\partial_t h(x,t) + \alpha(\partial_x h)^2 + D\partial_x^2 h + \eta = 0$$

$$\langle \partial_x h(x,t) \partial_x h(0,0) \rangle \to t^{-2/3} f_{\mathrm{KPZ}}(2\sqrt{2}\alpha\ x/t^{2/3}\ )$$



$$\langle S^z(x,t) S^z(0,0) \rangle \to t^{-2/3} f_{\mathrm{KPZ}}(\lambda x / t^{2/3} x)$$

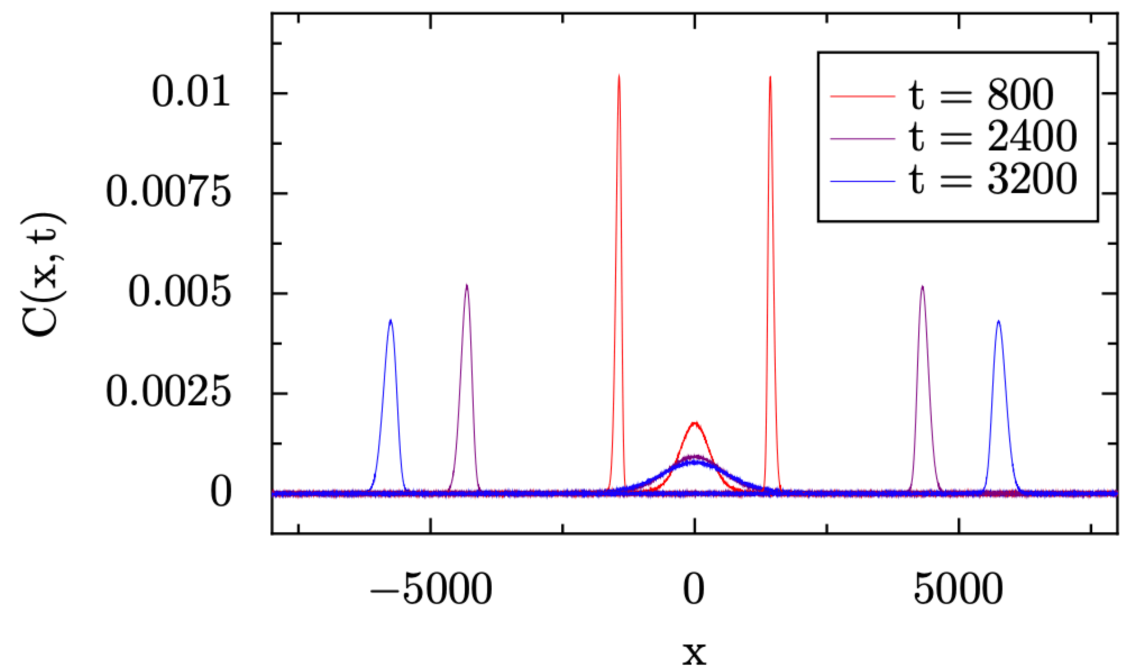
$$\partial_x h(x,t) \sim S^z(x,t)$$

Non-linear fluctuating hydrodynamics

Nonlinear Fluctuating Hydrodynamics for Anharmonic Chains

Herbert Spohn

Institute for Advanced Study, Princeton, NJ 08540,
and
Zentrum Mathematik, Physik Department, TU München,
Boltzmannstr. 3, D-85747 Garching, Germany
e-mail: spohn@tum.de



$$\partial_t q_i + \partial_x j_i(q.) = 0$$

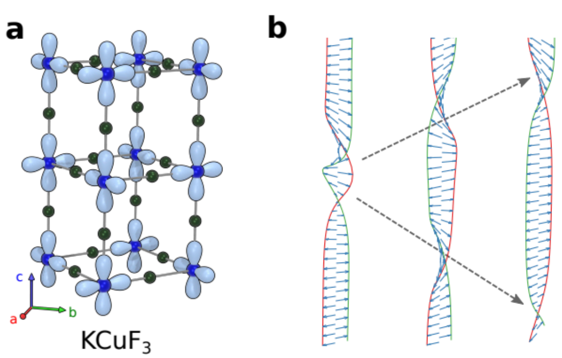
$$j_i(q.) = A_i^j \delta q_j + \frac{1}{2} G_i^{kl} \delta q_k \delta q_l$$

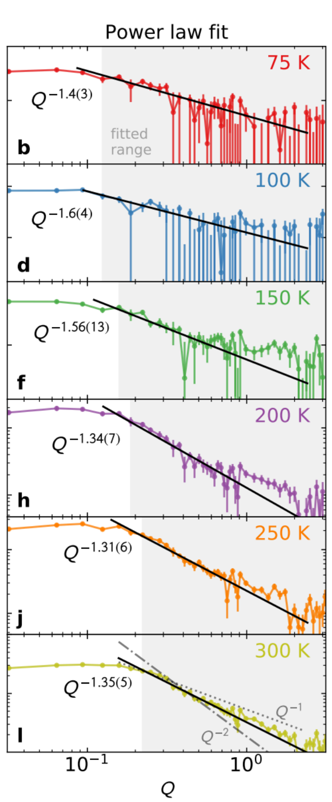
Experimental realisations

Letter | Published: 11 March 2021

Detection of Kardar–Parisi–Zhang hydrodynamics in a quantum Heisenberg spin-1/2 chain

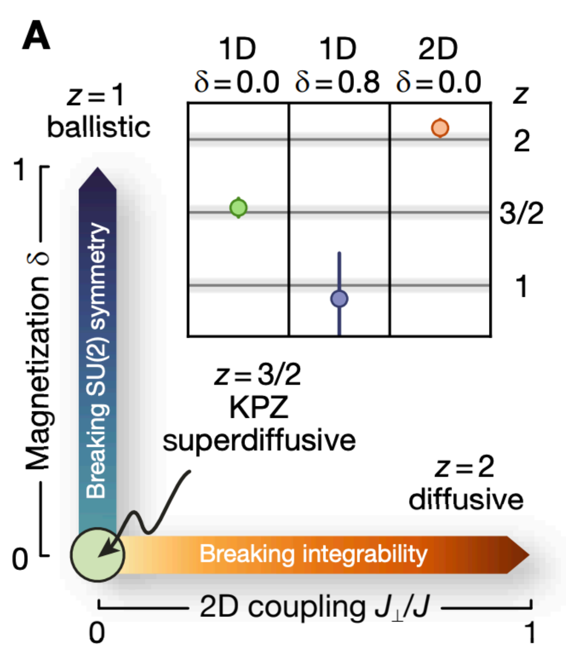
A. Scheie, N. E. Sherman, M. Dupont, S. E. Nagler, M. B. Stone, G. E. Granroth, J. E. Moore & D. A. Tennant

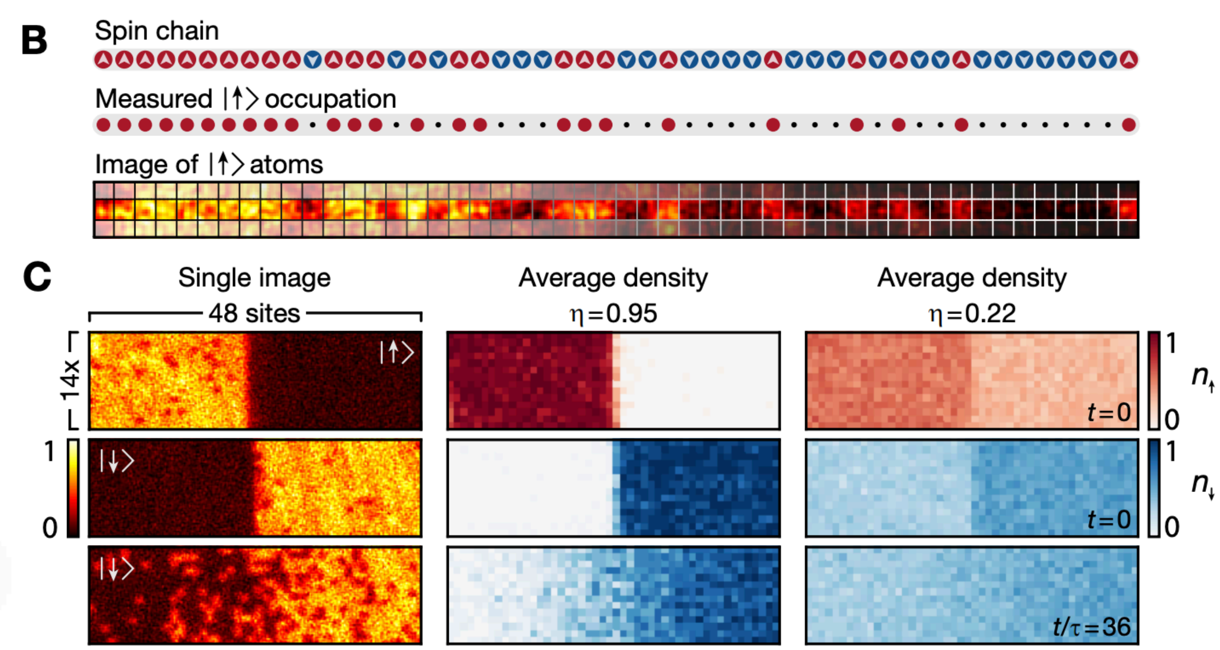




Quantum gas microscopy of Kardar-Parisi-Zhang superdiffusion

David Wei,[1,2] Antonio Rubio-Abadal,[1,2,*] Bingtian Ye,[3] Francisco Machado,[3,4] Jack Kemp,[3] Kritsana Srakaew,[1,2] Simon Hollerith,[1,2] Jun Rui,[1,2,†] Sarang Gopalakrishnan,[5,6] Norman Y. Yao,[3,4] Immanuel Bloch,[1,2,7] and Johannes Zeiher[1,2]





Hydrodynamic (thermodynamic) description

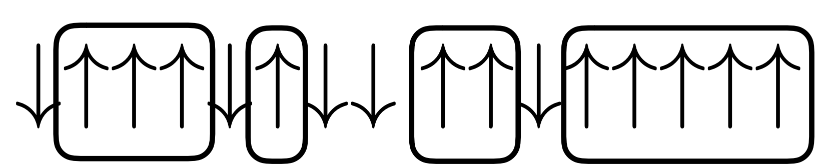
$$H = \sum_j \vec{S}_j \cdot \vec{S}_{j+1} + (\Delta - 1) S^z_j S^z_{j+1} + h \sum_j S^z_j$$

$$| \downarrow\downarrow\downarrow \ldots \downarrow\downarrow \rangle = |\text{vac}\rangle$$

$\boxed{\uparrow}$ $\quad m_1 = 1$

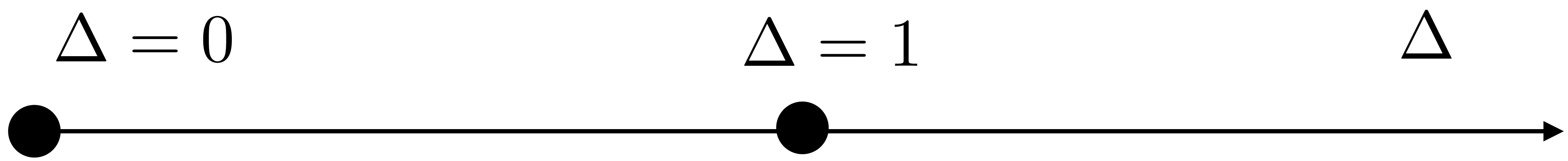
$\boxed{\uparrow \ldots \uparrow}$ $\quad m_2 = 2$

$\boxed{\uparrow \ldots \uparrow \ldots \uparrow}$ $\quad m_3 = 3$



$$\rho_s(\theta) \qquad v^{\text{eff}}_s(\theta) \qquad m^{\text{eff}}_s(\theta)$$

The ballistic regime



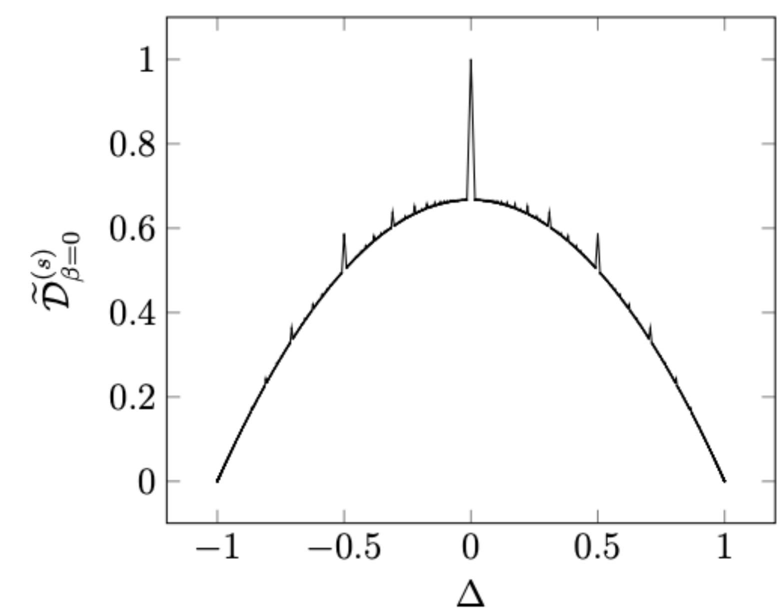
$$\sigma(\omega) = \mathcal{D}\delta(\omega) + \sigma_{\text{reg}}(\omega)$$

$$\int dx (j(x,t), j(0,0))$$

$$\to \sum_{k,l} \int dx (j(x,t), Q_k) C^{-1}_{k,l} (Q_l, j(0,0)) = BC^{-1}B$$

Prosen, 2011

Ilievski, De Nardis, 2017

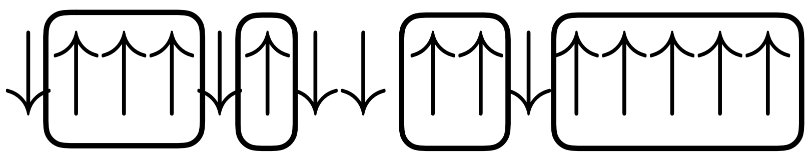


$$\mathcal{D} = \sum_s \int dk(\theta) n_s(\theta)(1 - n_s(\theta))(v^{\text{eff}}_s(\theta) m^{\text{dr}}_s(\theta))^2 \sim \sum_s \rho_s (v^{\text{eff}} m^{\text{dr}}_s)^2$$

Zotos, 1999

De Nardis, Doyon, Medenjak, Panfil, 2021

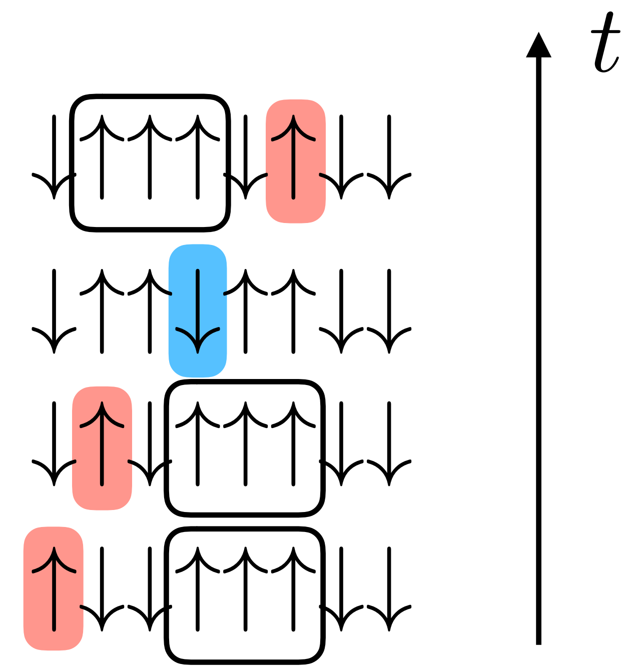
The regime Delta >= 1



$$m_s = s$$

$$\rho_s \quad v_s^{\mathrm{eff}} \qquad m_s^{\mathrm{dr}} \sim \langle S^z \rangle g_s$$

$\Delta = 1$	$\Delta > 1$
$v_s \sim s^{-1}$	$v_s \sim e^{-cs}$
$\rho_s \sim s^{-3}$	$\rho_s \sim s^{-3}$



Screening of magnetisation

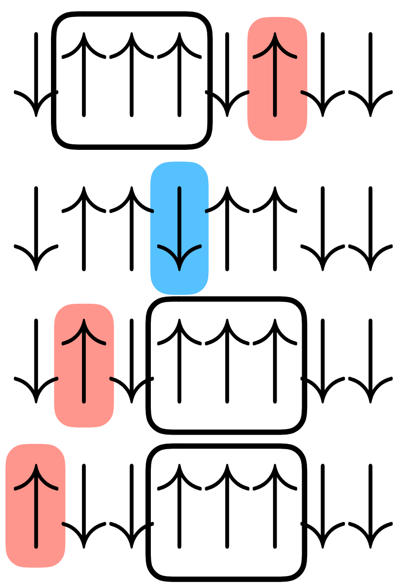
$$\int dx (j(x,t), j(0,0)) \simeq (j, Q_k)[C^{-1}]_{kl}(Q_l(t), j) = (j, Q_k)[C^{-1}]_{kl} e^{-t/\tau_l}(Q_l, j)$$

$$\sigma(\omega) \simeq \sum_s \int dt e^{-i\omega t} \rho_s (v_s^{\text{eff}} m_s)^2 e^{-t/\tau_s}$$

$$\rho_s \sim s^{-3}$$

$$m_s = s$$

$$\tau_s \sim \frac{1}{v_s} \frac{1}{\rho_{s'>s}}$$



De Nardis, Gopalakrishnan, Vasseur, Ware, 2021

Screening of magnetisation

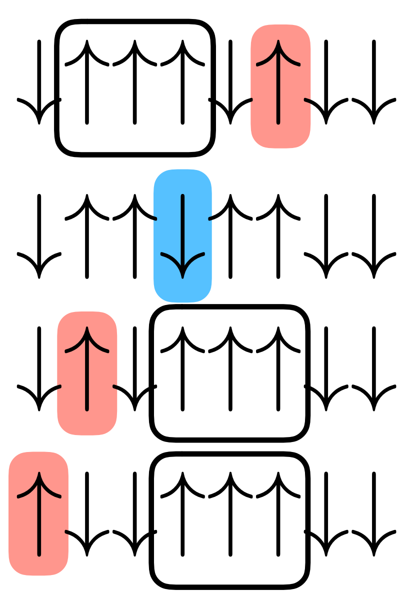
$$\int dx (j(x,t), j(0,0)) \simeq (j, Q_k)[C^{-1}]_{kl}(Q_l(t), j) = (j, Q_k)[C^{-1}]_{kl} e^{-t/\tau_l}(Q_l, j)$$

$$\sigma(\omega) \simeq \sum_s \int dt e^{-i\omega t} \rho_s (v_s^{\text{eff}} m_s)^2 e^{-t/\tau_s}$$

$$\rho_s \sim s^{-3}$$

$$m_s = s$$

$$\tau_s \sim \frac{1}{v_s} \frac{1}{\rho_{s'>s}}$$



$\Delta = 1$	$\Delta > 1$
$v_s \sim s^{-1}$	$v_s \sim e^{-cs}$
$\tau_s \sim s^3$	$\tau_s \sim s^3$
$\sigma(\omega) \sim \omega^{-1/3}$	$\sigma(\omega) \sim \omega^0$

De Nardis, Gopalakrishnan, Vasseur, Ware, 2021

Screening of magnetisation

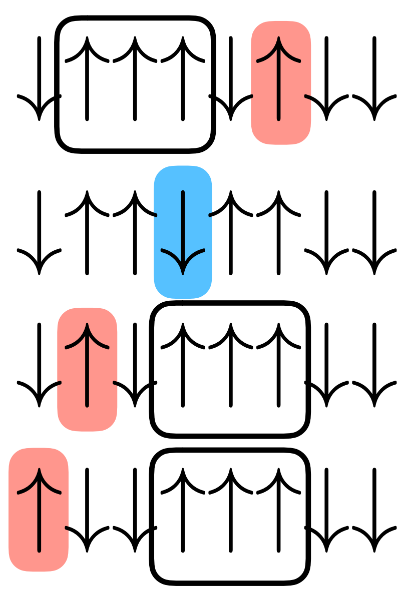
$$\int dx (j(x,t), j(0,0)) \simeq (j, Q_k)[C^{-1}]_{kl}(Q_l(t), j) = (j, Q_k)[C^{-1}]_{kl} e^{-t/\tau_l}(Q_l, j)$$

$$\sigma(\omega) \simeq \sum_s \int dt e^{-i\omega t} \rho_s (v_s^{\rm eff} m_s)^2 e^{-t/\tau_s}$$

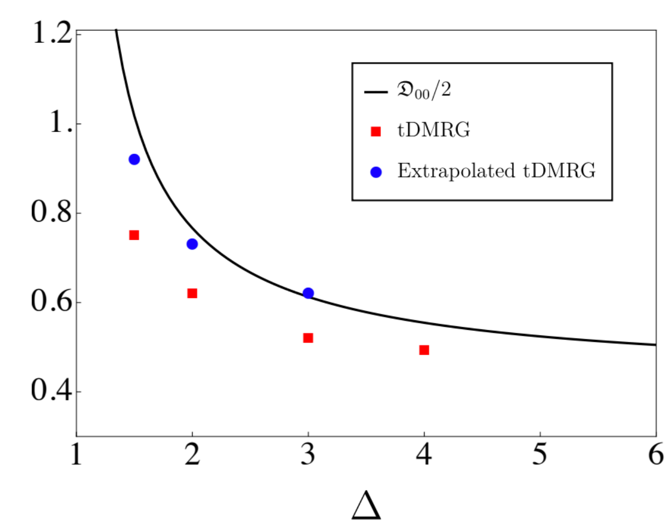
$$\rho_s \sim s^{-3}$$

$$m_s = s$$

$$\tau_s \sim \frac{1}{v_s} \frac{1}{\rho_{s'>s}}$$



$\Delta = 1$	$\Delta > 1$
$v_s \sim s^{-1}$	$v_s \sim e^{-cs}$
$\tau_s \sim s^3$	$\tau_s \sim s^3$
$\sigma(\omega) \sim \omega^{-1/3}$	$\sigma(\omega) \sim \omega^0$



$$D = \sum_s \rho_s (1 - n_s) |v_s^{\rm eff}| \lim_{\langle S^z \rangle \to 0} (m^{\rm dr} / \langle S^z \rangle)^2$$

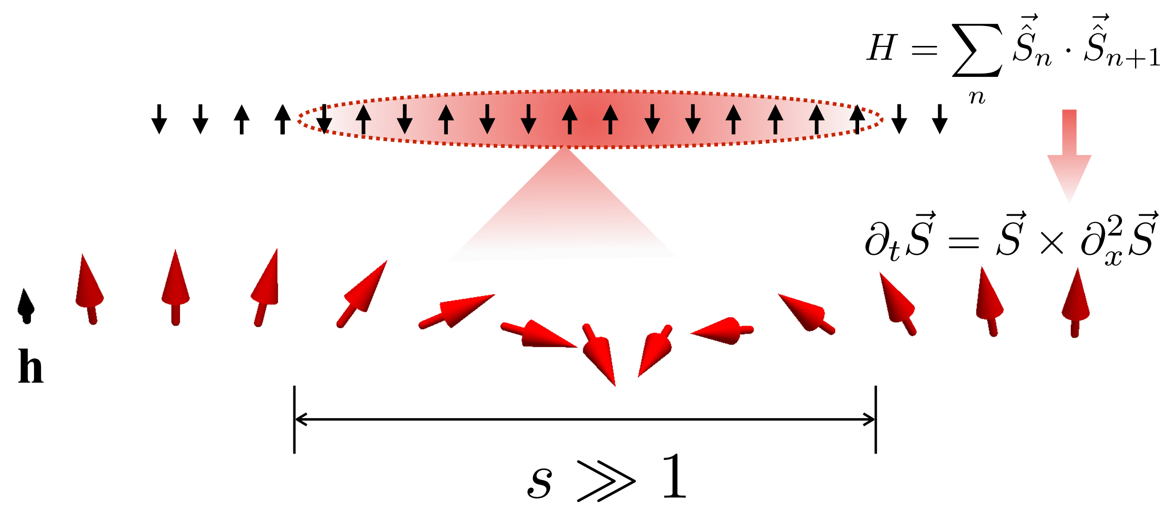
De Nardis, Bernard, Doyon, 2019

Large quasiparticles and solitons gases

$$D = \sum_s \rho_s (1 - n_s) |v_s^{\rm eff}| \lim_{\langle S^z \rangle \to 0} (m^{\rm dr} / \langle S^z \rangle)^2 \;\; = \sum_s \frac{1}{s^3} \frac{1}{s} s^4 \sim s_{\rm max}$$

Large quasiparticles and solitons gases

$$D = \sum_s \rho_s (1 - n_s) |v_s^{\rm eff}| \lim_{\langle S^z \rangle \to 0} (m^{\rm dr} / \langle S^z \rangle)^2 \;\; = \sum_s \frac{1}{s^3} \frac{1}{s} s^4 \sim s_{\rm max}$$



De Nardis, Gopalakrishnan, Ilievski, Vasseur, 2020

$$T_{s,s'}(\theta) \to T^{\rm giant}_{\xi,\xi'}(u) = \frac{1}{\pi} \log \frac{4u^2 + (\xi + \xi')^2}{4u^2 + (\xi - \xi')^2}$$

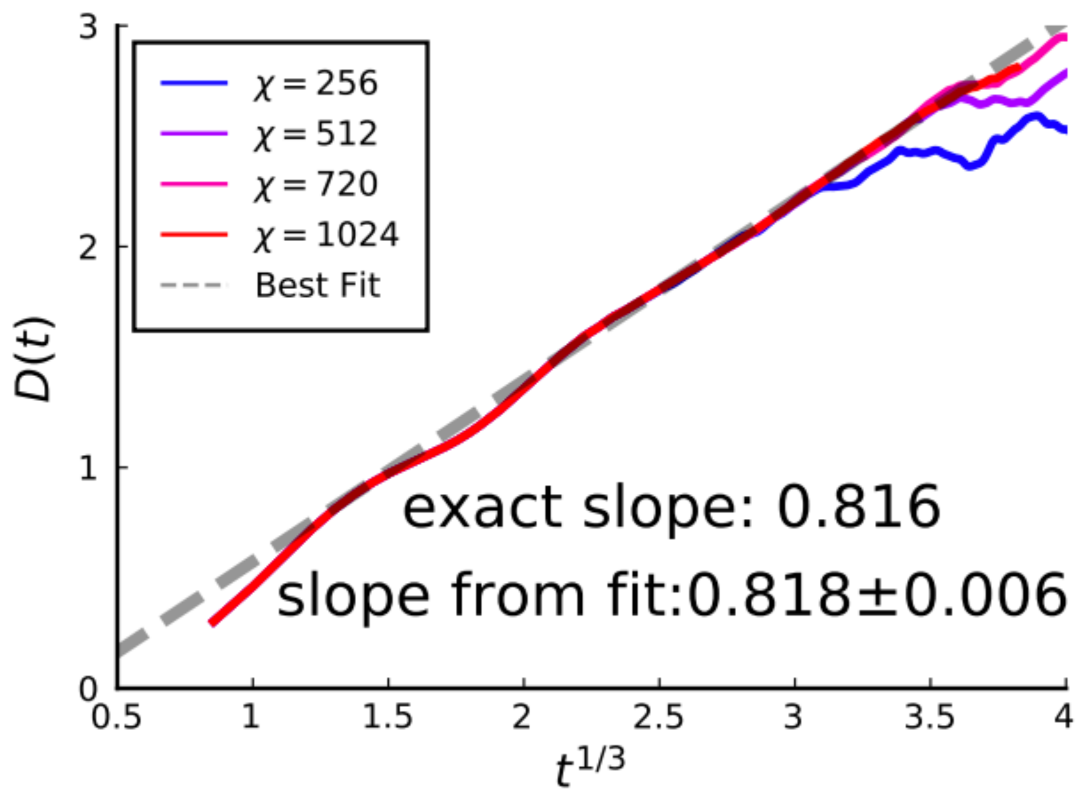
$$D(t) = 2^{5/3} D^{4/3}_{\rm classical} \chi^{2/3} \; t^{1/3} + \dots$$

Large quasiparticles and solitons gases

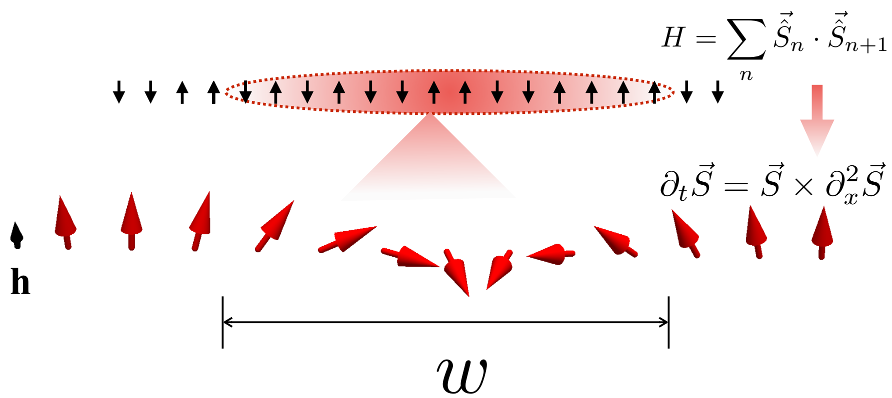
$$D(t) = 2^{5/3} D_{\text{classical}}^{4/3} \chi^{2/3}\ t^{1/3} + \ldots$$

$$T_{s,s'}(\theta) \to T^{\text{giant}}_{\xi,\xi'}(u) = \frac{1}{\pi} \log \frac{4u^2 + (\xi+\xi')^2}{4u^2 + (\xi-\xi')^2}$$

$$D_{\text{classical}} = \int_0^\infty d\xi \int_{-\infty}^{\infty} du D(\xi, u) = 5\pi/27$$



Large quasiparticles as Goldstone modes

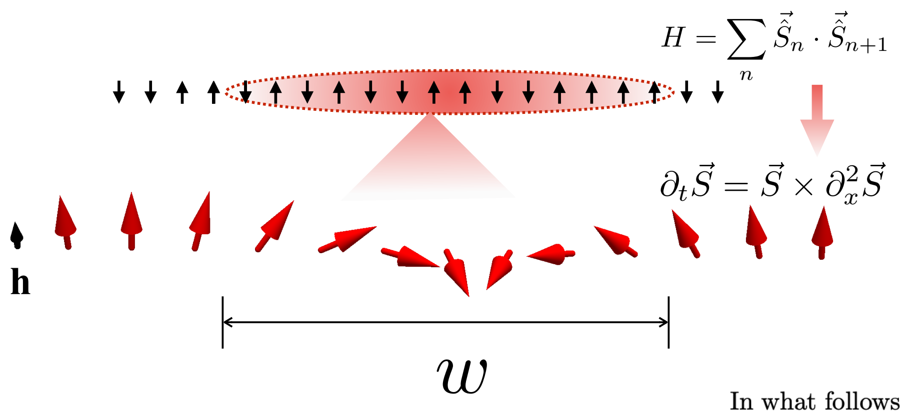


$$k \sim 1/w$$

$$\varepsilon \sim 1/w^2$$

$$v \sim 1/w$$

Large quasiparticles as Goldstone modes



$$k \sim 1/w$$
$$\varepsilon \sim 1/w^2$$
$$v \sim 1/w$$

Superuniversality of superdiffusion

Enej Ilievski,[1] Jacopo De Nardis,[2] Sarang Gopalakrishnan,[3] Romain Vasseur,[4] and Brayden Ware[4]

[1] *Faculty for Mathematics and Physics, University of Ljubljana, Jadranska ulica 19, 1000 Ljubljana, Slovenia*
[2] *Department of Physics and Astronomy, University of Ghent, Krijgslaan 281, 9000 Gent, Belgium*
[3] *Department of Physics and Astronomy, CUNY College of Staten Island, Staten Island, NY 10314; Physics Program and Initiative for the Theoretical Sciences, The Graduate Center, CUNY, New York, NY 10016, USA*
[4] *Department of Physics, University of Massachusetts, Amherst, Massachusetts 01003, USA*

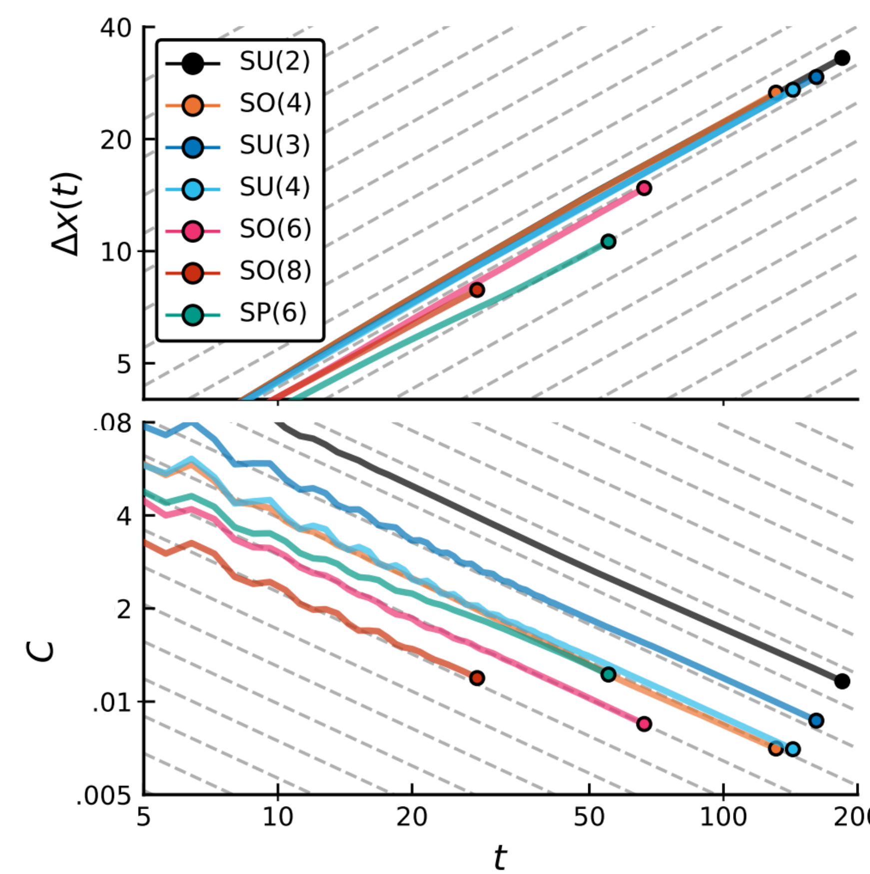
In what follows we will establish $z = 3/2$ within a universal algebraic description of the thermodynamic dressing equations and link them to the underlying symmetries structures and representation theory of quantum groups (Yangians). We find, remarkably, that the Fermi functions assume universal algebraic scaling at large-s,

$$\boxed{n_{a,s} \sim \frac{1}{s^2}}, \qquad (29)$$

which comes out as a direct corollary of fusion identities amongst the quantum character associated to the Yangian symmetry [72–76]. Similarly, we find that the total state densities and the dressed velocities of giant magnons (when multiplied by regular function and integrate over the rapidity domain, cf. Sec. IV G) decay as

$$\boxed{\rho^{\rm tot}_{a,s} = \frac{\rho_{a,s}}{n_{a,s}} \sim \frac{1}{s}} \quad \text{and} \quad \boxed{v^{\rm eff}_{a,s} \sim \frac{1}{s}}, \qquad (30)$$

respectively, for all flavors $a = 1, \ldots, r$. Most remarkably,



KPZ fluctuations?

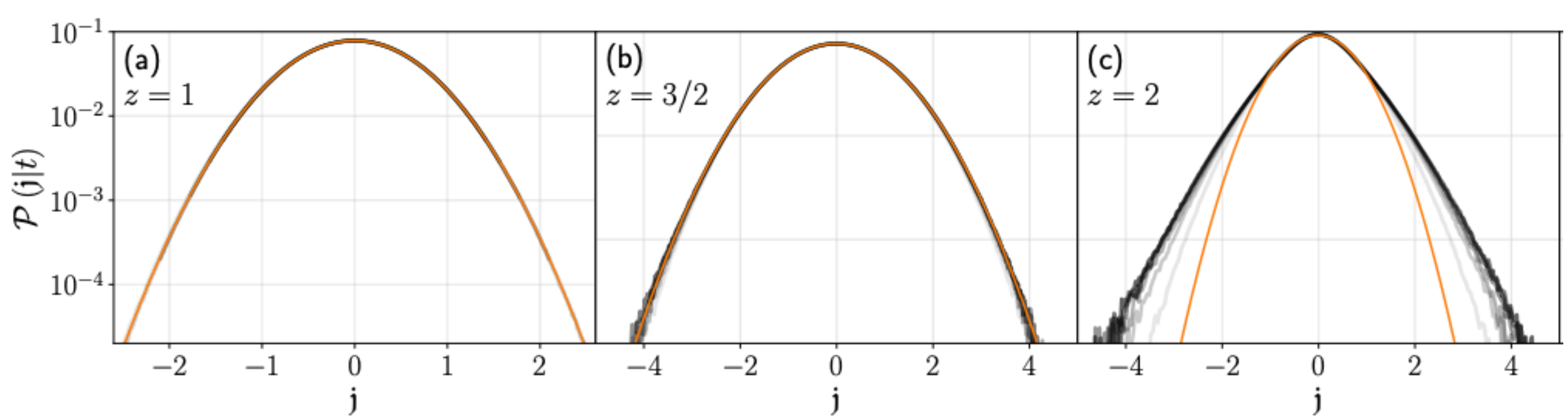
$$\langle S^z(x,t)S^z(0,0)\rangle \to t^{-2/3} f_{\mathrm{KPZ}}(\lambda x/t^{2/3}x)$$

$$\partial_x h(x,t) \sim S^z(x,t)$$

$$\int_0^t dt' j(0,t') \sim h(0,t)$$

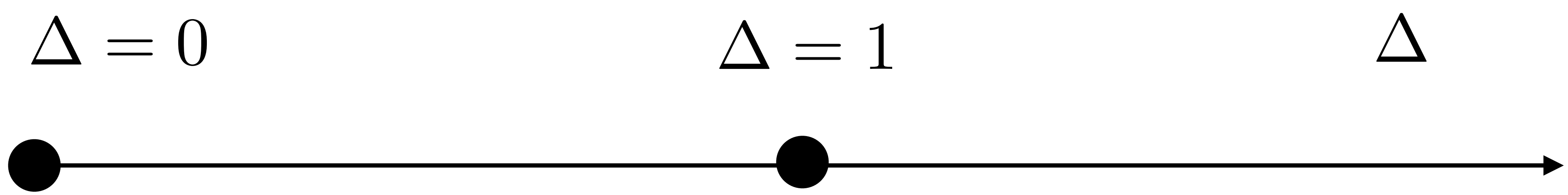
Absence of Normal Fluctuations in an Integrable Magnet

Žiga Krajnik,[1] Enej Ilievski,[1] and Tomaž Prosen[1]



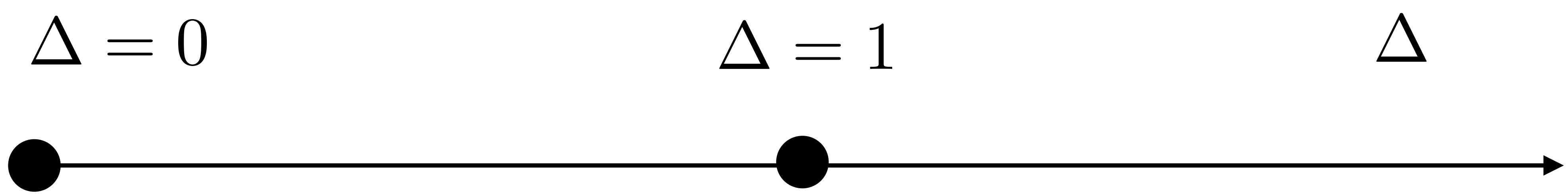
Beyond integrability

$$H = H_{\mathrm{XXZ}} + V$$



Beyond integrability

$$H = H_{\mathrm{XXZ}} + V$$



$$\sigma(\omega) = \mathcal{D}\delta(\omega) + \ldots$$

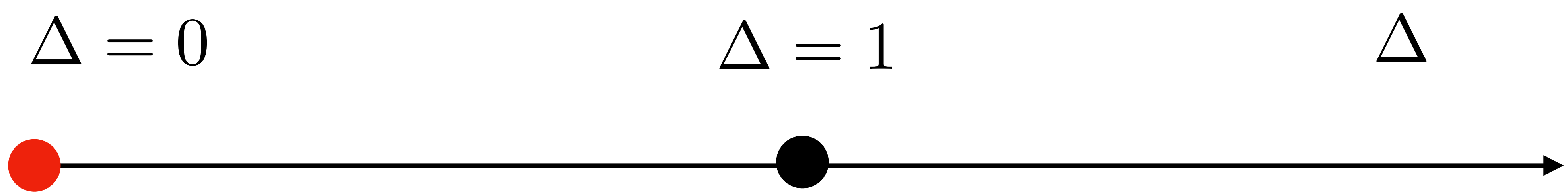
$$\to \frac{\gamma}{\gamma^2 + \omega^2}$$

$$\sigma(\omega) \simeq \sum_s \int dt e^{-i\omega t} \rho_s (v_s^{\mathrm{eff}} m_s)^2 e^{-t/\gamma}$$

Friedman, Gopalakrishnan, Vasseur, 2020

Beyond integrability

$$H = \sum_j \vec{S}_j \cdot \vec{S}_{j+1} + (\Delta - 1) S^z_j S^z_{j+1} + h \sum_j S^z_j \; + V$$

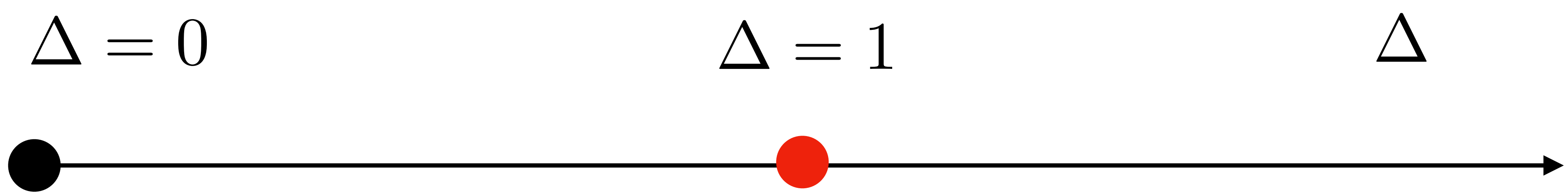


$$H = \sum_i S^x_i S^x_{i+1} + S^y_i S^y_{i+1} + \sqrt{\gamma}\eta_i(t) S^z_i$$

$$\partial_t n_k = -\gamma n_k + \gamma/2$$

$$D = \frac{1}{\gamma}$$

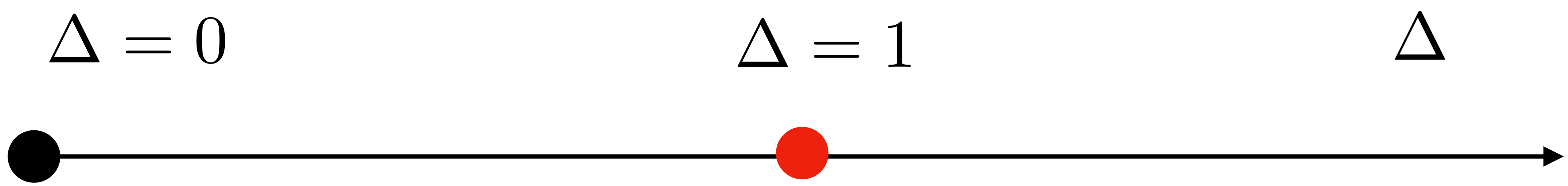
Beyond integrability: Heisenberg point



$$\sigma(\omega) \simeq \sum_s \int dt e^{-i\omega t} \rho_s (v_s^{\rm eff} m_s)^2 e^{-t/\tau_s} e^{-t/\tau_s^V}$$

De Nardis, Gopalakrishnan, Vasseur, Ware, 2021

Beyond integrability: Heisenberg point



$$SU(2) \qquad H = H_{\mathrm{XXX}} + \sqrt{\gamma} \sum_j \eta_j(t) h_{j,j+1}$$

$$U(1) \qquad H = H_{\mathrm{XXX}} + \sqrt{\gamma} \sum_j \eta_j(t) S_j^z$$

$$\sigma(\omega) \simeq \sum_s \int dt e^{-i\omega t} \rho_s (v_s^{\mathrm{eff}} m_s)^2 e^{-t/\tau_s} e^{-t/\tau_s^V}$$

$$SU(2) \qquad \tau_s^V \sim s^2 \qquad \sigma(\omega) \sim \log \omega$$

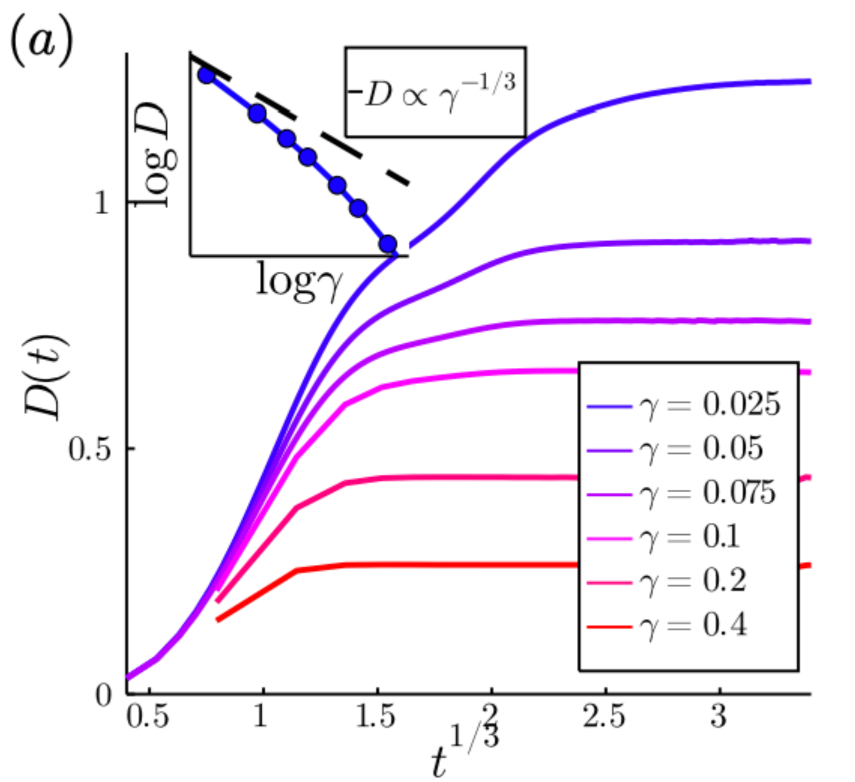
$$U(1) \qquad \tau_s^V \sim s^0 \qquad \sigma(\omega) \sim \omega^0$$

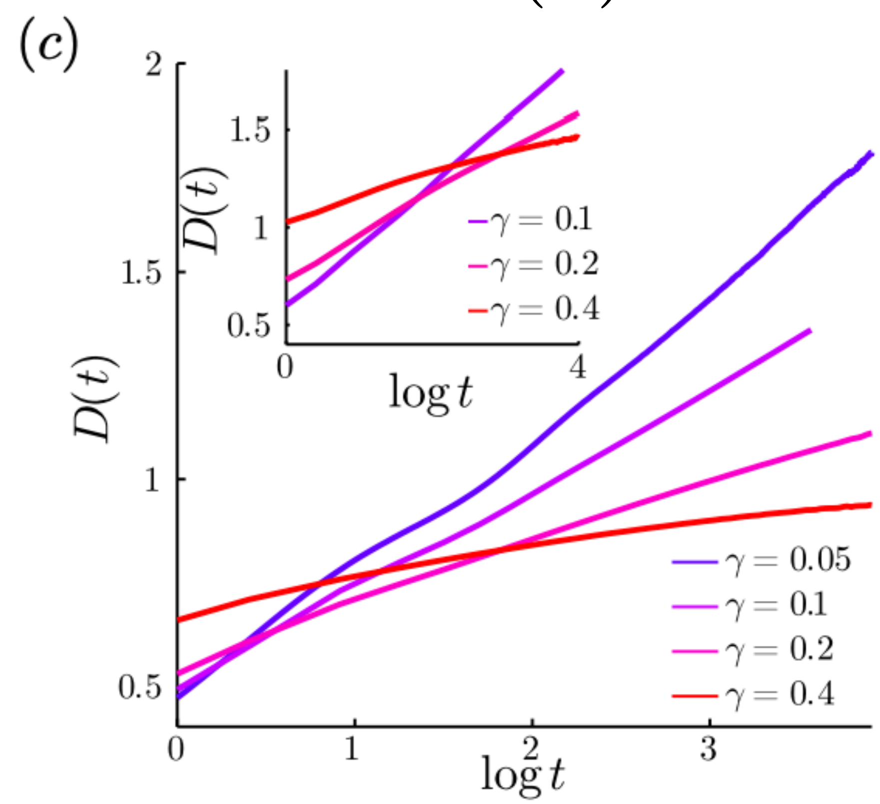
De Nardis, Gopalakrishnan, Vasseur, Ware, 2021

Beyond integrability: Heisenberg point

$$D(t) = \frac{1}{2}\partial_t \sum_x x^2 \langle S_x(t) S_0(0) \rangle$$

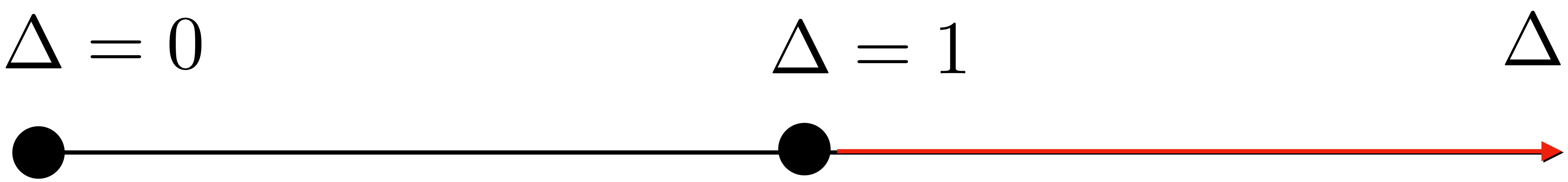
$U(1)$ $SU(2)$

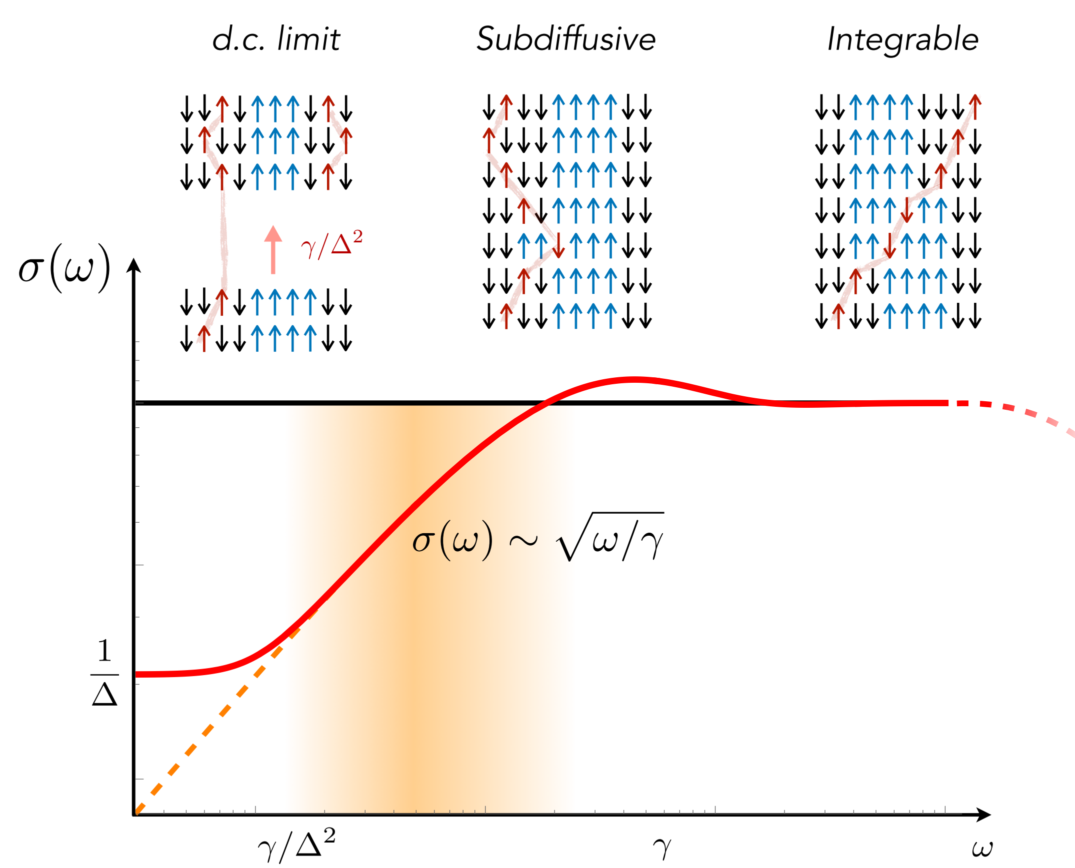




Beyond integrability: large Delta regime

$$H = H_{\mathrm{XXZ}} + \sqrt{\gamma} \sum_j \eta_j(t) S_j^z$$



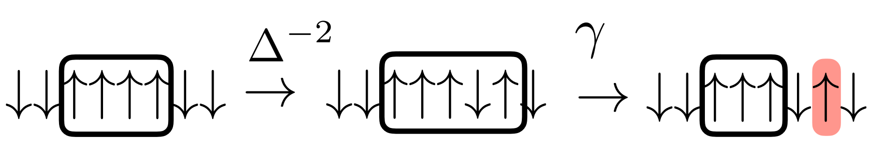


Sub-diffusive dynamics

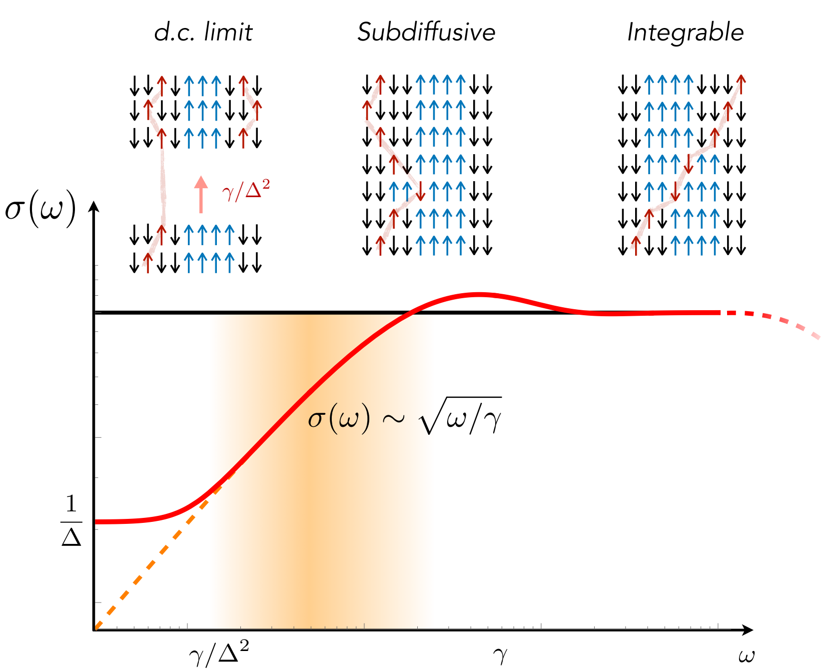
$$D = \sum_s \rho_s (1 - n_s) |v_s^{\mathrm{eff}}| \lim_{\langle S^z \rangle \to 0} (m^{\mathrm{dr}} / \langle S^z \rangle)^2$$

$$t_1^* = 1/\gamma \qquad v^{\mathrm{dr}} \sim \frac{1}{\sqrt{t}} \qquad D \sim t^{-1/2}$$

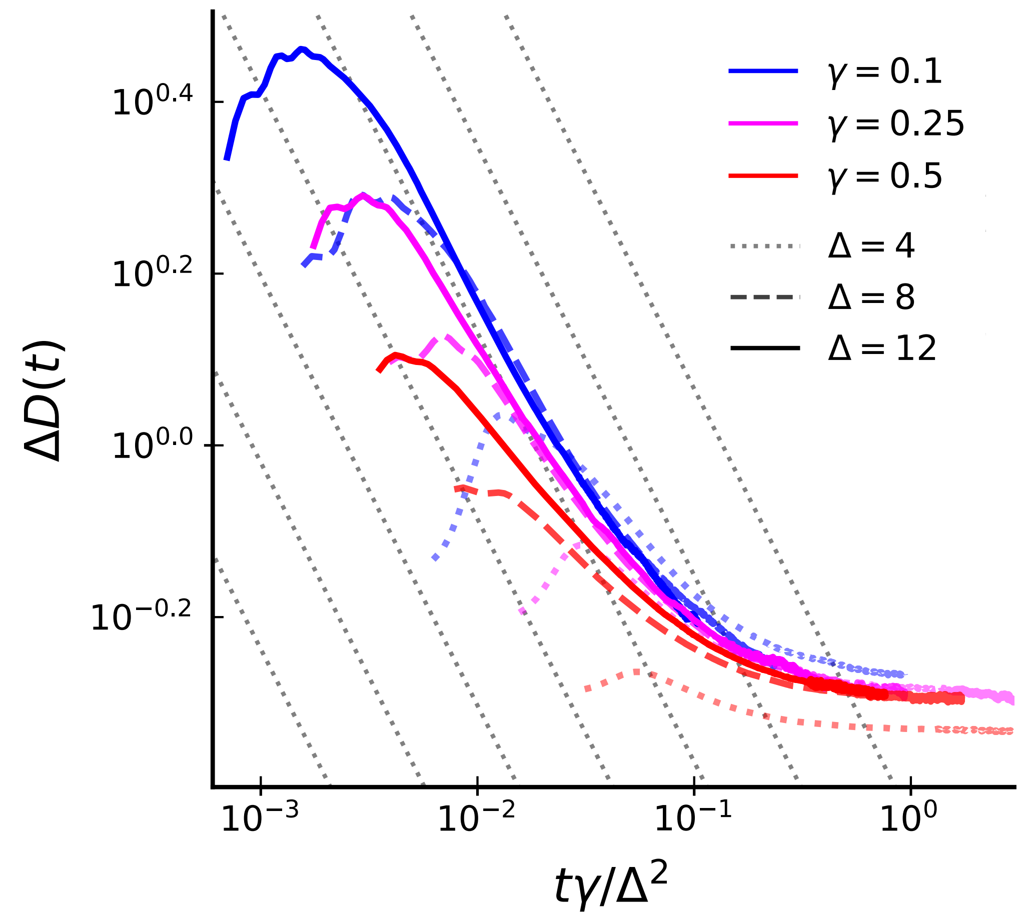
$$t_2^* = \Delta^2/\gamma$$



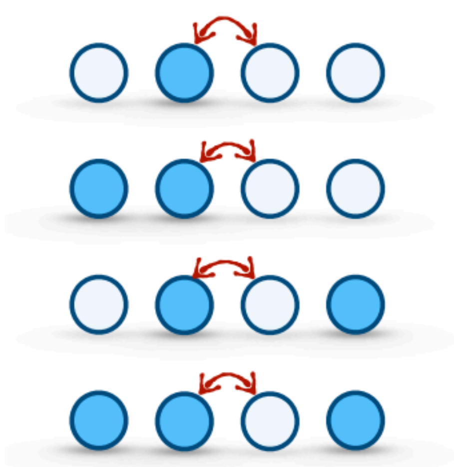
$$D \sim 1/\Delta$$



$$D(t) = \frac{1}{2} \partial_t \sum_x x^2 \langle S_x(t) S_0(0) \rangle$$

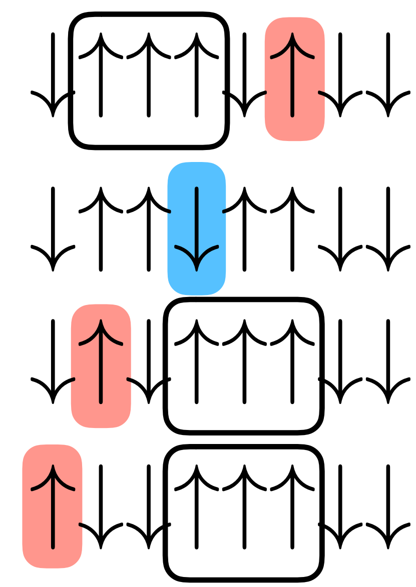


Beyond integrability: large Delta regime

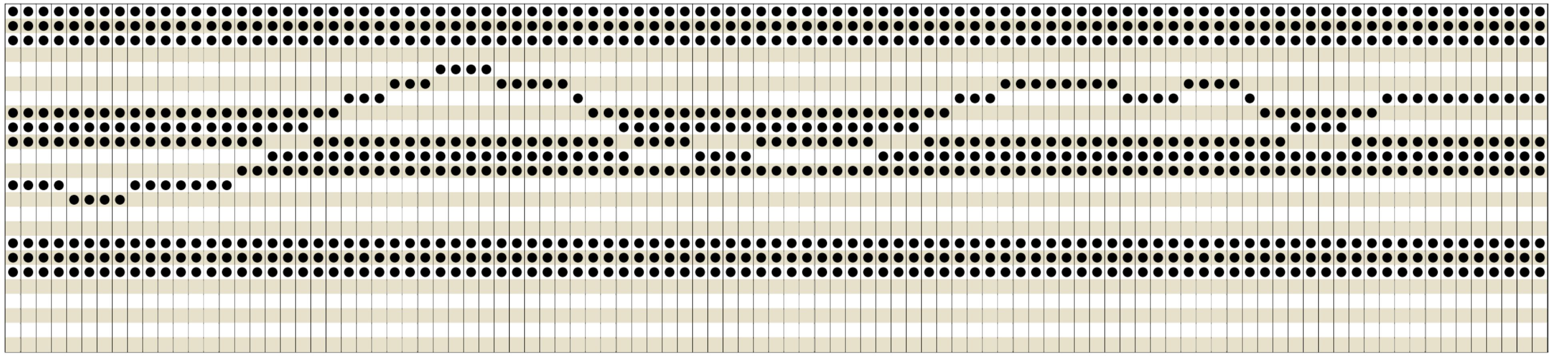


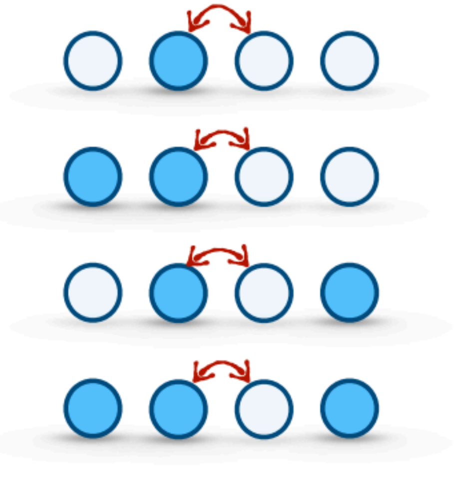
PXYP	U(1) East	XNOR	Fredkin	GLT
✗	✗	✓	✓	✗
✗	✗	✗	✗	✓
✗	✓	✗	✓	✓
✓	✓	✓	✓	✓

Singh, Ware, Vasseur, Friedman, 2021



XNOR gates





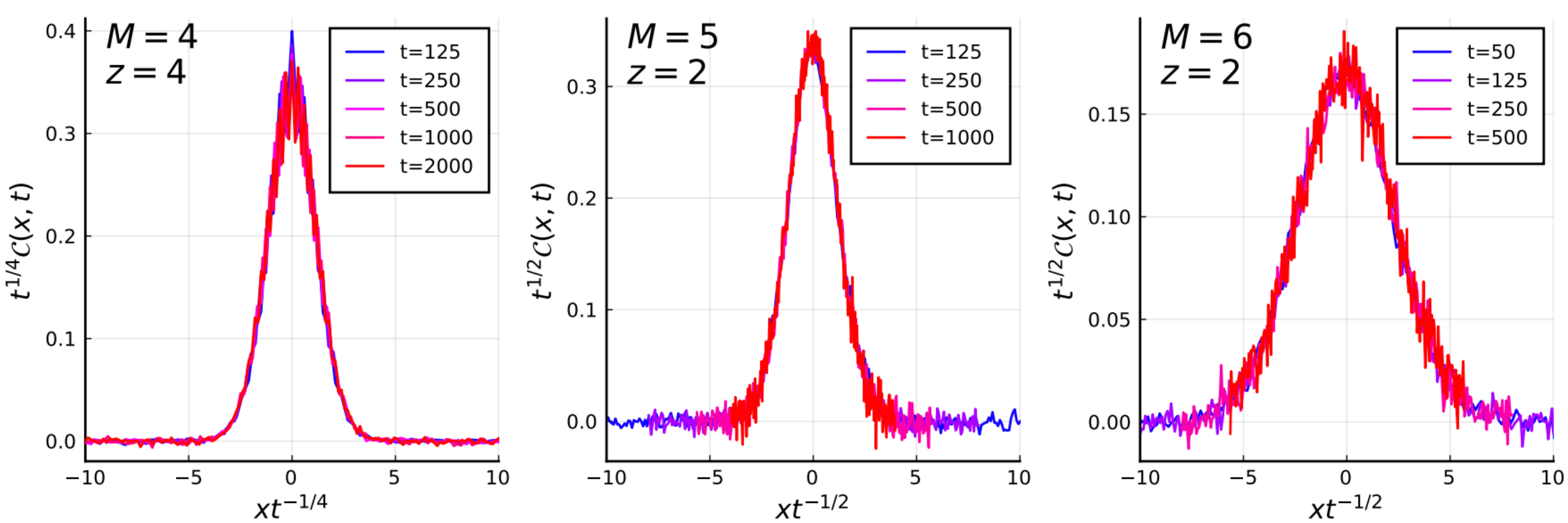
	PXYP	U(1) East	XNOR	Fredkin	GLT
	✗	✗	✓	✓	✗
	✗	✗	✗	✗	✓
	✗	✓	✗	✓	✓
	✓	✓	✓	✓	✓

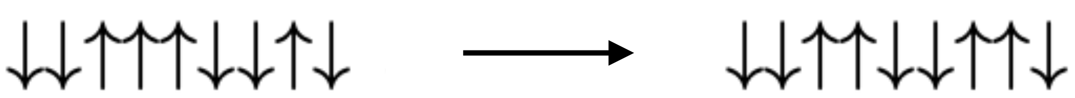
Singh, Ware, Vasseur, Friedman, 2021

Importance of magnon conservation

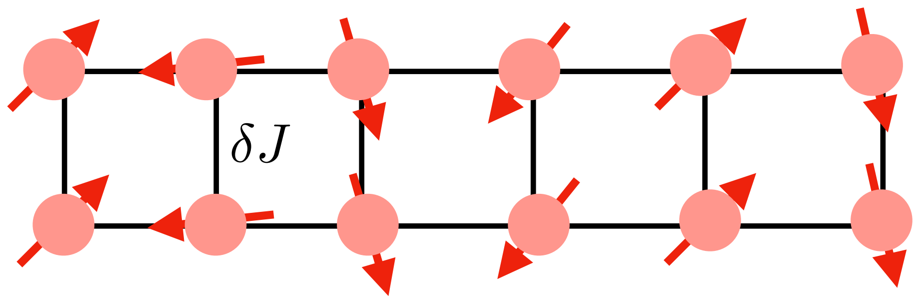
$$H = \sum_j \vec{S}_j \cdot \vec{S}_{j+1} + (\Delta - 1) S_j^z S_{j+1}^z + \sqrt{\gamma}\eta_j(t) S_j^z$$

$t_2^* \neq \exp \Delta$ *Abanin, De Roeck, Ho, Huveneers, 2017*





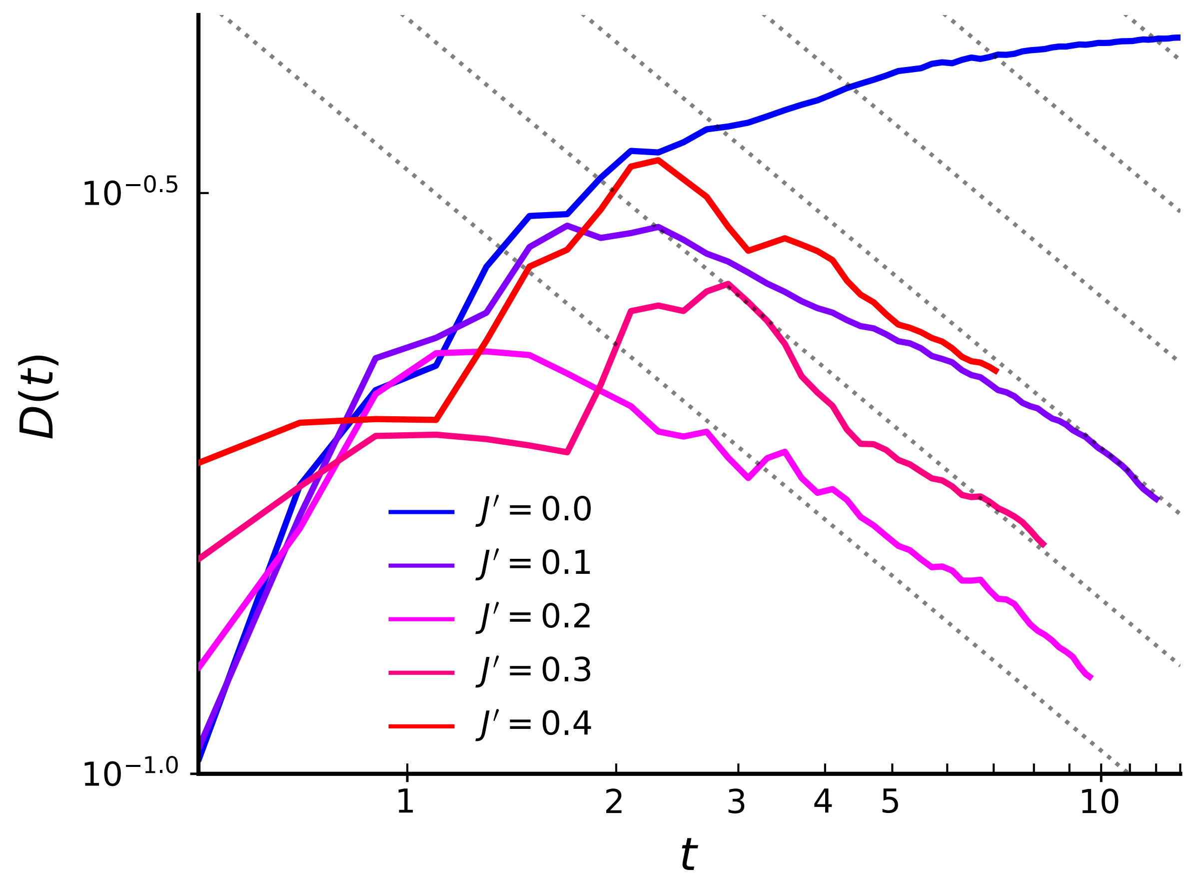
Adding noise to spin: sub-diffusion



$$t_1^* \sim 1/(\delta J)^2$$

$$t_2^* \sim \Delta^3/(\delta J)^2$$

$$D(t) \to \sim \Delta^{-3/2}$$



$$\downarrow\boxed{\uparrow\uparrow}\downarrow\boxed{\uparrow\uparrow}\downarrow\downarrow \;\to\; \downarrow\boxed{\uparrow\uparrow\uparrow}\downarrow\uparrow\downarrow\downarrow$$

$$\Gamma_{2+2\to 3+1} \sim |P|^2 g(R) \sim \left|\frac{\delta J}{\Delta}\right|^2 \frac{1}{\Delta}$$

Conclusions