Integrable deterministic dynamics with nonabelian symmetries: From KPZ mean transport of Noether charges to their anomalous fluctuations

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Menu

1. Abundant evidence on universal KPZ 2-point functions for heat-peak of Noether-charge transport in non-abelian integrable models!
2. Anomalous fluctuations (lack of scaled cumulants in FCS)!
3. Exactly solvable interacting deterministic model of FCS.

ARTICLE

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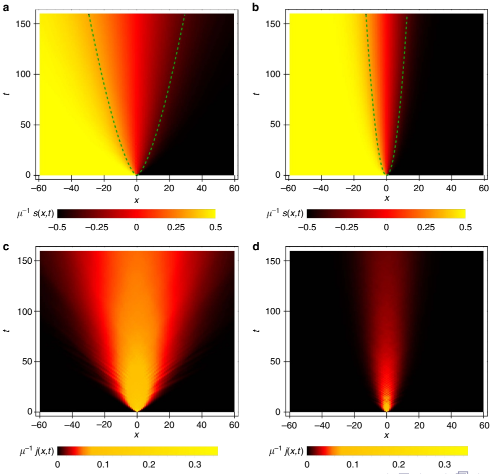
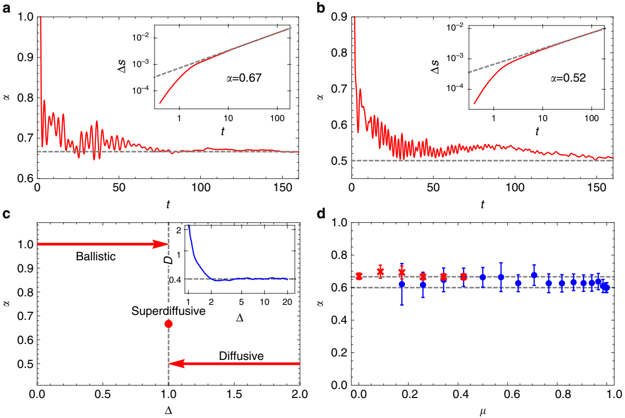
Spin diffusion from an inhomogeneous quench in an integrable system

Marko Ljubotina[1], Marko Žnidarič[1] & Tomaž Prosen[1]

$$\rho(t=0) = (\mathbb{1} + \mu\sigma^z)^{\otimes L/2} \otimes (\mathbb{1} - \mu\sigma^z)^{\otimes L/2}$$



$$\Delta s^z(t) = \int_0^t \mathrm{d}t' j(x = L/2, t') \propto t^\alpha$$



Preliminary evidence for super-diffusion with $z = 3/2$ from studies of: *boundary driven Lindblad* [M. Žnidarič, PRL **106**, 220601 (2011)], *classical limit* (lattice Landau-Lifshifz) [TP and B. Žunkovič, PRL **111**, 040602 (2013)]

PHYSICAL REVIEW LETTERS **122,** 210602 (2019)

Kardar-Parisi-Zhang Physics in the Quantum Heisenberg Magnet

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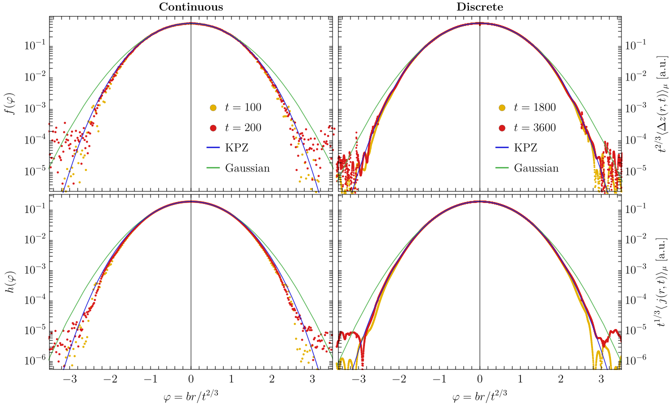
(Received 7 March 2019; published 31 May 2019)

Equilibrium spatiotemporal correlation functions are central to understanding weak nonequilibrium physics. In certain local one-dimensional classical systems with three conservation laws they show universal features. Namely, fluctuations around ballistically propagating sound modes can be described by the celebrated Kardar-Parisi-Zhang (KPZ) universality class. Can such a universality class be found also in quantum systems? By unambiguously demonstrating that the KPZ scaling function describes magnetization dynamics in the $SU(2)$ symmetric Heisenberg spin chain we show, for the first time, that this is so. We achieve that by introducing new theoretical and numerical tools, and make a puzzling observation that the conservation of energy does not seem to matter for the KPZ physics.



Kardar-Parisi-Zhang equation:
random surface growth and scaling:
Non-Equilibrium

$$C(r,t) = \langle [h(r,t) - h(0,0) - t\langle \partial_t h\rangle]^2\rangle$$

$$g(\varphi) = \lim_{t\to\infty} \frac{C\left(\left(2\lambda^2 t^2 \Gamma \nu^{-1}\right)^{-1/3}\varphi, t\right)}{\left(\frac{1}{2}\lambda t \Gamma^2 \nu^{-2}\right)^{2/3}}$$

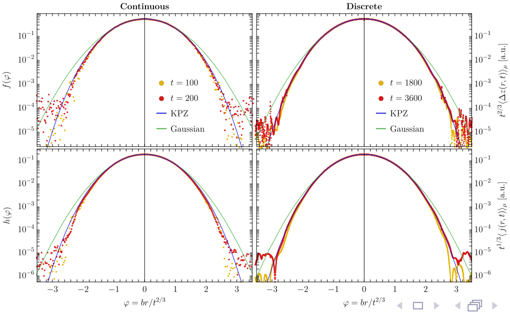
$$\partial_t h = \frac{1}{2}\lambda\,(\partial_r h)^2 + \nu \partial_r^2 h + \sqrt{\Gamma}\zeta$$

$$f(\varphi) = \frac{1}{4} g''(\varphi) \sim \partial_r^2 C(r,t)\,.$$

Equilibrium high-T spin dynamics in Heisenberg model ($z \equiv s^z$):

$$\langle z(0,0) z(r,t)\rangle = \lim_{\mu\to\infty} \tfrac{1}{2\mu}(\langle z(r-1,t)\rangle - \langle z(r,t)\rangle) \qquad \Rightarrow \qquad \partial_r h \leftrightarrow z$$

$$\partial_r z(r,t) = \frac{a\mu}{t^{2/3}} f\left(\frac{br}{t^{2/3}}\right) \qquad j(r,t) = \frac{2a\mu}{3b^2 t^{1/3}} h\left(\frac{br}{t^{2/3}}\right)$$



From 2019, several works appeared discussing further evidence and/or possible mechanisms for KPZ physics in Heisenberg spin chain and related models:

A. Das, M. Kulkarni, H. Spohn and A. Dhar, PRE **100**, 042116 (2019),

S. Gopalakrishnan and R.Vasseur, PRL **122**, 127202 (2019),

J. De Nardis, M. Medenjak, C. Karrasch and E. Ilievski, PRL **123**, 186601 (2019),

M. Dupont, J. E. Moore, PRB **101**, 121106 (2020),

V. B. Bulchandani, Phys. Rev. B **101**, 041411 (2020),

F. Weiner, P. Schmitteckert, S. Bera and F. Evers, PRB **101**, 045115 (2020),

Ž. Krajnik, T. Prosen, JSP **179**, 110 (2020),

E. Ilievski, J. De Nardis, S. Gopalakrishnan, R. Vasseur, B. Ware, PRX **11**, 031023 (2021),

...

PHYSICAL REVIEW LETTERS **122,** 127202 (2019)

Kinetic Theory of Spin Diffusion and Superdiffusion in *XXZ* Spin Chains

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[1]*Department of Physics and Astronomy, CUNY College of Staten Island, Staten Island, New York 10314; Physics Program and Initiative for the Theoretical Sciences, The Graduate Center, CUNY, New York, New York 10016, USA*
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(Received 11 December 2018; published 26 March 2019)

We address the nature of spin transport in the integrable *XXZ* spin chain, focusing on the isotropic Heisenberg limit. We calculate the diffusion constant using a kinetic picture based on generalized hydrodynamics combined with Gaussian fluctuations: we find that it diverges, and show that a self-consistent treatment of this divergence gives superdiffusion, with an effective time-dependent diffusion constant that scales as $D(t) \sim t^{1/3}$. This exponent had previously been observed in large-scale numerical simulations, but had not been theoretically explained. We briefly discuss *XXZ* models with easy-axis anisotropy $\Delta > 1$. Our method gives closed-form expressions for the diffusion constant D in the infinite-temperature limit for all $\Delta > 1$. We find that D saturates at large anisotropy, and diverges as the Heisenberg limit is approached, as $D \sim (\Delta - 1)^{-1/2}$.

PHYSICAL REVIEW X **11,** 031023 (2021)

Superuniversality of Superdiffusion

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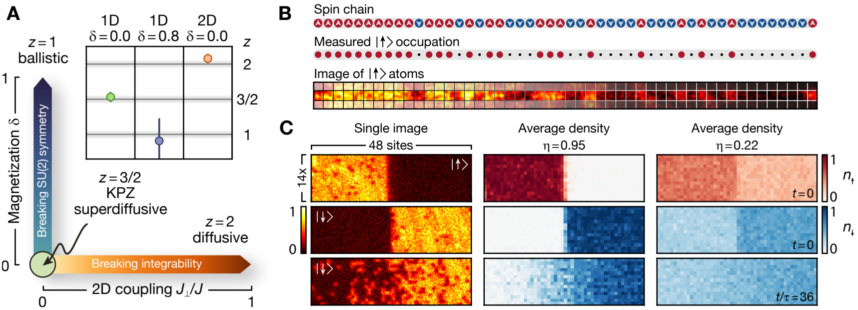
(Received 24 October 2020; revised 4 May 2021; accepted 19 May 2021; published 28 July 2021)

$z = 3/2$: universal prediction based on kinematics of **giant quasiparticles** within generalised hydrodynamics framework

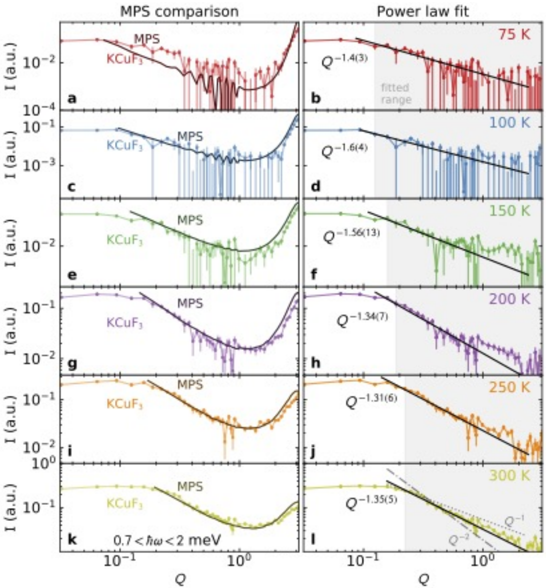
Experiments (cold atoms):

D. Wei, A. Rubio-Abadal, B. Ye, F. Machado, J. Kemp, K. Srakaew, S. Hollerith, J. Rui, S. Gopalakrishnan, N. Yao, I. Bloch, J. Zeiher, arXiv:2107.00038



Experiments (solid state, neutron scattering):

A. Scheie, N. E. Sherman, M. Dupont, S. E. Nagler, M. B. Stone, G. E. Granroth, J. E. Moore, D. A. Tennant, Nature Phys. (2021)



SciPost Phys. 9, 038 (2020)

Integrable matrix models in discrete space-time

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Abstract

We introduce a class of integrable dynamical systems of interacting classical matrix-valued fields propagating on a discrete space-time lattice, realized as many-body circuits built from elementary symplectic two-body maps. The models provide an efficient integrable Trotterization of non-relativistic σ-models with complex Grassmannian manifolds as target spaces, including, as special cases, the higher-rank analogues of the Landau–Lifshitz field theory on complex projective spaces. As an application, we study transport of Noether charges in canonical local equilibrium states. We find a clear signature of superdiffusive behavior in the Kardar–Parisi–Zhang universality class, irrespectively of the chosen underlying global unitary symmetry group and the quotient structure of the compact phase space, providing a strong indication of superuniversal physics.

Two-matrix map

Consider a pair of complex matrices $M_1, M_2 \in \mathrm{End}(\mathbb{C}^N)$, a formal parameter τ, and define a map over $\mathrm{End}(\mathbb{C}^N) \times \mathrm{End}(\mathbb{C}^N)$:

$$(M_1', M_2') = \Phi_\tau(M_1, M_2)$$

via

$$\begin{aligned} M_1' &= (M_1 + M_2 + \mathrm{i}\tau\mathbb{1})\, M_2\, (M_1 + M_2 + \mathrm{i}\tau\mathbb{1})^{-1}, \\ M_2' &= (M_1 + M_2 + \mathrm{i}\tau\mathbb{1})\, M_1\, (M_1 + M_2 + \mathrm{i}\tau\mathbb{1})^{-1}. \end{aligned}$$

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The map has several cute properties:

1. $\Phi_\tau^{-1} = \Phi_{-\tau}$
2. $M_j^2 = \mathbb{1} \Leftrightarrow (M_j')^2 = \mathbb{1}$, $j = 1, 2$
3. M_j Hermitian $\Leftrightarrow$ M_j' Hermitian, $j = 1, 2$, for $\tau \in \mathbb{R}$
4. $M_1 + M_2 = M_1' + M_2'$

Phase-space

A convenient local phase space for the map is the *Complex Grassmannian*

$$\mathcal{M}_1 \equiv \mathrm{Gr}_{\mathbb{C}}(k,N) := \left\{ M \in GL(N;\mathbb{C}); M^\dagger = M, M^2 = \mathbb{1}, \mathrm{Tr}\, M = N - 2k \right\}.$$

which can be naturally parametrised via

$$M = g\,\Sigma^{(k,N)}\,g^\dagger, \quad g \in SU(N)$$

where:

$$\Sigma^{(k,N)} = \mathrm{diag}(\underbrace{-1,-1,\ldots,-1}_{k},\underbrace{1,\ldots,1}_{N-k}).$$

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Hence:

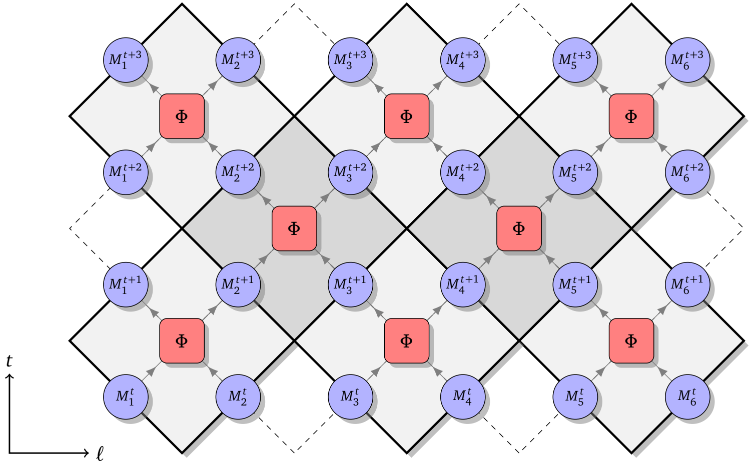
$$\mathcal{M}_1 \equiv \mathrm{Gr}_{\mathbb{C}}(k,N) \simeq \frac{U(N)}{U(k)\times U(N-k)} \cong \frac{SU(N)}{S(U(k)\times U(N-k))}.$$

$$\dim \mathcal{M}_1 = 2(N-k)k.$$

Many-body matrix dynamical system

Space-time discrete dynamics $x \in \mathbb{Z}_L$, $t \in \mathbb{Z}$:

$$(M_{2\ell-1}^{2t+2}, M_{2\ell}^{2t+2}) = \Phi_\tau(M_{2\ell-1}^{2t+1}, M_{2\ell}^{2t+1}), \qquad (M_{2\ell}^{2t+1}, M_{2\ell+1}^{2t+1}) = \Phi_\tau(M_{2\ell}^{2t}, M_{2\ell+1}^{2t}).$$



Many-body phase-space $\mathcal{M}_L = \mathcal{M}_1^{\times L}$ and one time-step many-body map:

$$\Phi_\tau^{\rm full} : \mathcal{M}_L \to \mathcal{M}_L$$

defined via brick-work functional circuit:

$$\Phi_\tau^{\rm full} = \Phi_\tau^{\rm even} \circ \Phi_\tau^{\rm odd},$$

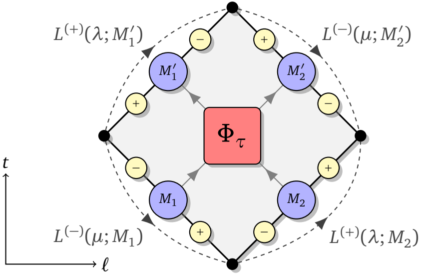
where

$$\Phi_\tau^{(j)} = \underbrace{I \otimes \cdots \otimes I}_{j-1} \otimes \Phi_\tau \otimes \underbrace{I \otimes \cdots \otimes I}_{L-j-1},$$

$$\Phi_\tau^{\rm odd} = \prod_{\ell=1}^{L/2} \Phi_\tau^{(2\ell-1)}, \qquad \Phi_\tau^{\rm even} = \prod_{\ell=1}^{L/2} \Phi_\tau^{(2\ell)}.$$

Minimal set of assumptions:

1. Symmetric parallel transport $L_+(\lambda, M) \equiv L_-(\lambda, M) =: L(\lambda, M)$
2. Linearity of Lax operator $L(\lambda, M) = \lambda \mathbb{1} + M$.
3. Nonlinear constraint $M^2 = \mathbb{1}$.



Assumptions (1-3) imply that a discrete zero-curvature condition

$$F^{1/2} L^{(+)}(\lambda; M_2) L^{(-)}(\mu; M_1) F^{-1/2} = F^{-1/2} L^{(-)}(\mu; M_2') L^{(+)}(\lambda; M_1') F^{1/2}$$

is equivalent to our two-matrix map, generalised by a *twist* $F \in GL(N)$:

$$M_1' = \mathrm{Ad}_{FS_\tau}(M_2), \qquad M_2' = \mathrm{Ad}_{FS_\tau}(M_1), \qquad S_\tau \equiv M_1 + M_2 + \mathrm{i}\tau \, \mathbb{1}.$$

Poisson structure and invariant measure

1. There is a natural Poisson bracket on $\mathcal{M}_1$:

$$\{M \overset{\otimes}{,} M\} = -\frac{\mathrm{i}}{2}\Big[\Pi, M \otimes \mathbb{1}_N - \mathbb{1}_N \otimes M\Big], \qquad \Pi(a \otimes b) \equiv b \otimes a,$$

extending to $\mathcal{M}_L$ by $\{M_\ell \overset{\otimes}{,} M_{\ell'}\} = \delta_{\ell,\ell'}\{M_\ell \overset{\otimes}{,} M_\ell\}$.

2. There is a natural maximum entropy measure over $\mathcal{M}_1$, and over $\mathcal{M}_L$ by extension, which in suitable affine coordinates $z_j, \bar{z}_j$ reads:

$$\mathrm{d}\Omega(M) = \frac{1}{\mathcal{V}}\prod_{j=1}^{n} \mathrm{d}z_j \mathrm{d}\bar{z}_j, \quad n = k(N-k), \quad \dim \mathcal{M}_1 = 2n.$$

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The following can be straightforwardly shown:

1. The matrix map Φ_τ is symplectic over $\mathcal{M}_2$, hence Φ^{full} defines symplectic dynamics over $\mathcal{M}_L$, i.e. the map preserves the above Poisson bracket.

2. The measure $\Omega^{\times L}$ is invariant under the map Φ^{full}.

Monodromy matrix and Transfer map

Claim: Dynamical system $(\Phi^{\mathrm{full}}, \mathcal{M}_L, \Omega^{\times L})$ is completely integrable.

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Proof: Define the *monodromy matrix* $\mathbb{M}_\tau : \mathbb{C} \times \mathcal{M}_L \to \mathrm{End}(\mathbb{C}^N)$:

$$\mathbb{M}_\tau(\lambda|\{M_\ell\}) = L(\lambda; M_L)\, L(\lambda+\tau; M_{L-1}) \cdots L(\lambda; M_2)\, L(\lambda+\tau; M_1).$$

and consequently, the *transfer map* $T_\tau : \mathbb{C} \times \mathcal{M}_L \to \mathbb{C}$:

$$T_\tau(\lambda|\{M_\ell\}) = \mathrm{Tr}\, \mathbb{M}_\tau\big(\lambda|\{M_\ell\}\big).$$

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Directly telescoping the discrete zero curvature condition, and using definition of the Poisson bracket, we have:

$$T_\tau(\lambda) \circ \Phi^{\text{full}}_\tau = T_\tau(\lambda),$$

$$\Big\{T_\tau(\lambda|\{M_\ell\}), T_\tau(\lambda'|\{M_\ell\})\Big\} = 0, \qquad \forall \quad \lambda, \lambda' \in \mathbb{C}.$$

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$T_\tau(\lambda)$ generates an extensive set of independent constants of motions. For rank-1 Grassmannians ($k=1$),

$$({\rm d}/{\rm d}\lambda)^m \log T_\tau(\lambda)|_{\lambda \in \{\pm {\rm i}, \pm {\rm i} - \tau\}}$$

are local conservation laws.

Writing $F = \exp(-\mathrm{i}\tau B/2)$ and letting the limit $\tau \to 0$, we obtain

$$\frac{\mathrm{d}M_\ell}{\mathrm{d}t} = \{M_\ell, H_{\mathrm{lattice}}\} = -\mathrm{i}\big[M_\ell, (M_{\ell-1} + M_\ell)^{-1} + (M_\ell + M_{\ell+1})^{-1} + B\big],$$

with

$$H_{\mathrm{lattice}} = \sum_{\ell=1}^{L} \left(\mathrm{Tr}\big(M_\ell B\big) - \mathrm{Re}\,\mathrm{Tr}\log(M_\ell + M_{\ell+1})\right).$$

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Further, introducing a *matrix field* $M_\ell(t) \to M(x = \ell\Delta, t)$ with $B \to (\Delta^2/2)B$ and letting the lattice spacing $\Delta \to 0$, we obtain a Hamiltonian field theory:

$$\partial_t M = \{M(x,t), \mathcal{H}\} = \frac{1}{2\mathrm{i}}\big[M, \partial_x^2 M\big] + \mathrm{i}[B, M]$$

with

$$\mathcal{H} = \int \mathrm{d}x \left[\frac{1}{4}\mathrm{Tr}\big(\partial_x M\big)^2 + \mathrm{Tr}(M\,B)\right].$$

Generalised (Grassmannian/SU(N)) (lattice) Landau-Lifshitz models!

[Krajnik & P, J. Stat. Phys. 2020]

$$M_\ell = \vec{S}_\ell \cdot \vec{\sigma}, \quad \vec{S}_\ell \cdot \vec{S}_\ell = 1,$$

$$\Phi_\tau(\vec{S}_1, \vec{S}_2) = \frac{1}{\tau^2 + \varrho^2} \left(\varrho^2 \, \vec{S}_1 + \tau^2 \, \vec{S}_2 + \tau \, \vec{S}_1 \times \vec{S}_2, \varrho^2 \, \vec{S}_2 + \tau^2 \, \vec{S}_1 + \tau \, \vec{S}_2 \times \vec{S}_1 \right),$$

where $\varrho^2 \equiv (1 + \vec{S}_1 \cdot \vec{S}_2)$. In the continuous time limit $\tau \to 0$

$$H_{\text{lattice}} = H_{\text{LLL}} = -\sum_{\ell=1}^{L} \log(1 + \vec{S}_\ell \cdot \vec{S}_{\ell+1}).$$

Define charge density $q^a_\ell(t) \equiv \mathrm{Tr}(X^a\, M^t_\ell)$, and the corresponding correlator

$$C_{q^a}(\ell,t) = \langle q^a_\ell(t)\, q^a_0(0)\rangle - \langle q^a_\ell(0)\rangle\langle q^a_0(0)\rangle$$

where $\langle\bullet\rangle$ defines expectation w.r.t. invariant (maximum entropy) state Ω.

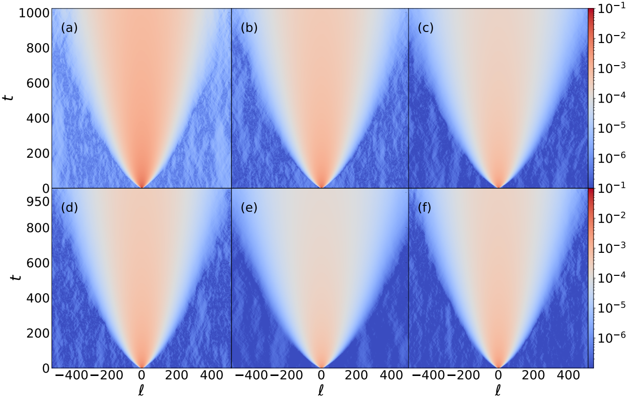

Figure 5: Space-time profiles of the charge autocorrelation function $C_{\mathbb{q}}(\ell,t)$ (shown the absolute value in logarithmic scale) for various local variables $M \in \mathcal{M}_1 = \mathrm{Gr}_{\mathbb{C}}(k,N)$: (a) $(k,N) = (1,2)$, (b) $(k,N) = (1,3)$, (c) $(k,N) = (1,4)$, (d) $(k,N) = (2,4)$, (e) $(k,N) = (1,5)$ and (f) $(k,N) = (2,5)$. The data shown for parameters $\tau = 1$, $N_s = 10^5$ and $L = 2^{10}$.

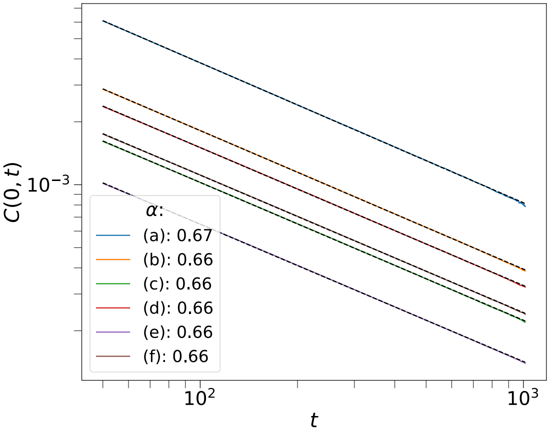

Figure 6: Algebraic dynamical exponents $\alpha = 1/z$ characterizing the asymptotic decay of correlators $C_{\mathbb{q}}(0,t) \sim |t|^{-\alpha}$ (for the corresponding datasets shown in Fig. 5) obtained by least square fit.

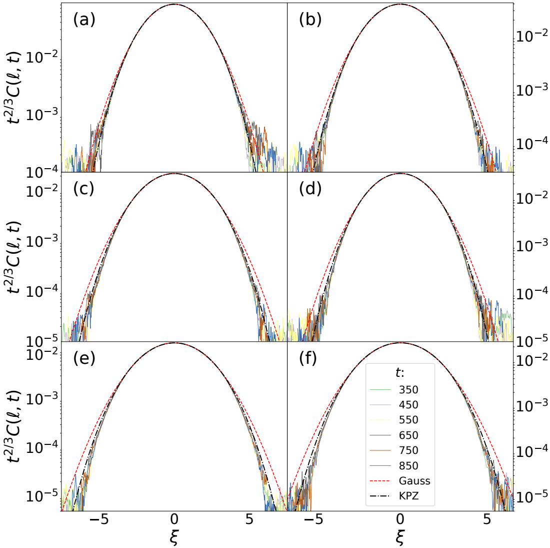

Figure 7: Convergence to the stationary cross sections of the scaled dynamical structure factors $\widetilde{C}_{\mathbb{q}}(\xi, t)$, fitted with the KPZ universal function $g_{\rm PS}$ (black dashed curve), for the corresponding datasets shown in Fig. 5. In comparison, the red dashed lines display the best fit with a Gaussian profile (red dashed curve), showing systematic deviations in the tails.

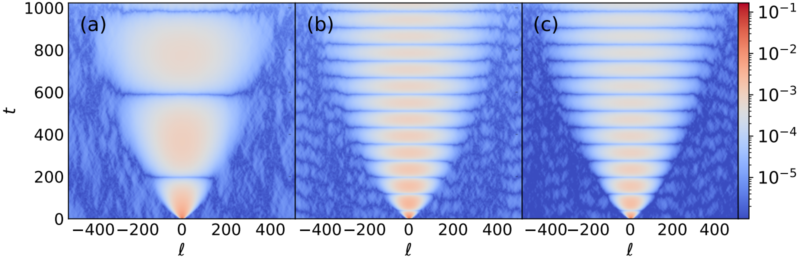

Figure 9: Effect of an applied magnetic field $F_\tau = \exp(-i\tau(h/2)\sum_a n_a X^a)$ (cf. Eq. (115)) on the correlation function of the Noether charges perpendicular to the polarization direction $\mathbf{n}$, shown for (a) $N=2$, $h=10^{-3}$, (b) $N=2$, $h=10^{-2}$, (c) $N=3$, $h=10^{-2}$ (with parameters $N_s=10^3$, and $L=2^{10}$).

PHYSICAL REVIEW LETTERS **128,** 090604 (2022)

Absence of Normal Fluctuations in an Integrable Magnet

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Faculty for Mathematics and Physics, University of Ljubljana, Jadranska ulica 19, 1000 Ljubljana, Slovenia

We consider integrable space-time discretization of $\mathrm{SU}(N=2)$ spins and its anisotropic deformation `[Krajnik,Ilievski,P,Pasquier,SciPost Phys.('21)]`:

- Consider unbiased, infinite temperature *equilibrium* state
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- Variance fixes the equilibrium dynamical exponent z, $\langle [\mathfrak{J}(t)]^2 \rangle^c \sim t^{1/z}$
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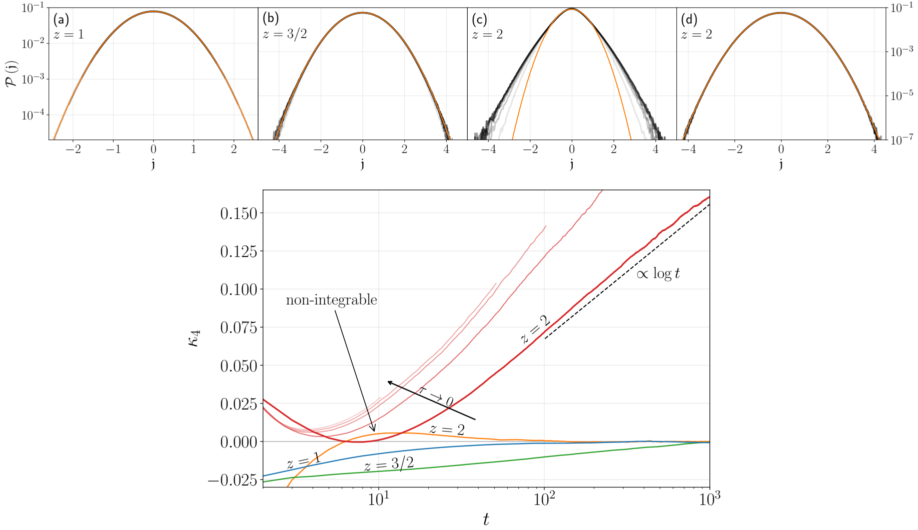
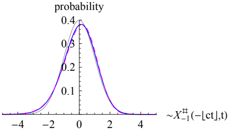
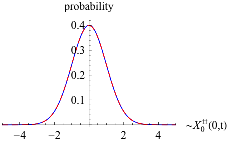

(a)
$z = 1$
(b)
$z = 3/2$
(c)
$z = 2$
(d)
$z = 2$
$\mathcal{P}(\mathrm{j})$
j
κ_4
t
non-integrable
$\tau \to 0$
$z = 2$
$z = 2$
$z = 1$
$z = 3/2$
$\propto \log t$

For comparison:

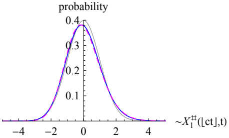
Previously observed Baik-Rains (KPZ) fluctuations for sound peaks in non-integrable classical anharmonic chains
[`Mendl and Spohn, JSTAT (2015) P03007`]



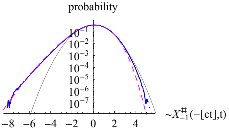
(a) left sound peak



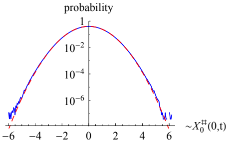
(b) heat peak



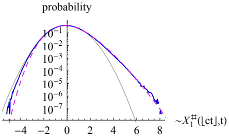
(c) right sound peak



(d) left sound peak



(e) heat peak



(f) right sound peak

Divergence of scaled cumulants

- Large deviation principle: $\mathbb{P}\left(t^{-1/z}\mathfrak{J}(t)=j\right) \simeq e^{-t^{1/z} I(j)}$
- Legendre–Fenchel transform of rate function $I(j) = \max_\lambda[\lambda\, j - F(\lambda)]$ yields *scaled cumulant generating function*

$$F(\lambda) = \lim_{t\to\infty} t^{-1/z} \log\left\langle e^{\lambda \mathfrak{J}(t)}\right\rangle.$$

However, in general:

$$s_n(t) = t^{-1/z}\left\langle[\mathfrak{J}(t)]^n\right\rangle^c, \quad \lim_{t\to\infty} s_n(t) \neq \partial_\lambda^n F(\lambda).$$

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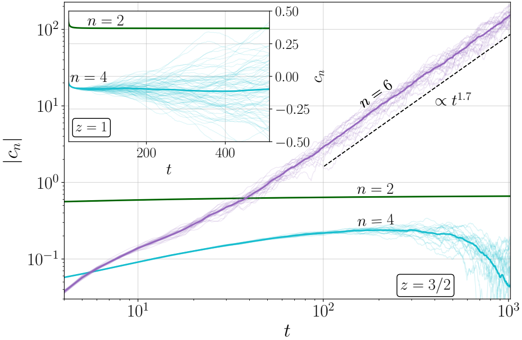
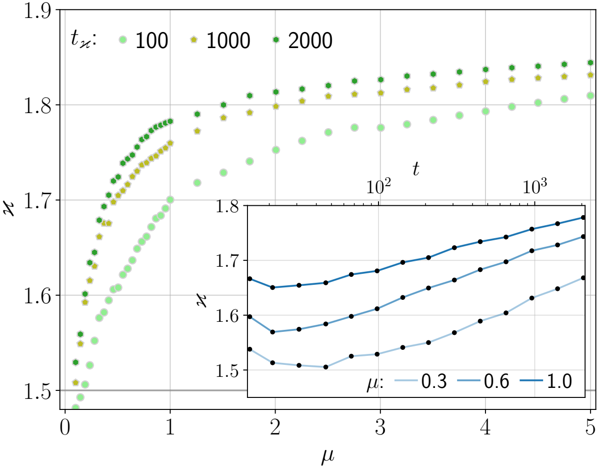


Bi-partitioning protocol at finite magnetization bias μ,

$$\rho(t=0) \sim \exp\left(-\mu \sum_{x<0} S^3_x + \mu \sum_{x>0} S^3_x\right)$$

$$\langle \mathfrak{J}(t) \rangle \sim t^{1/\varkappa}$$



Exact Anomalous Current Fluctuations in a Deterministic Interacting Model

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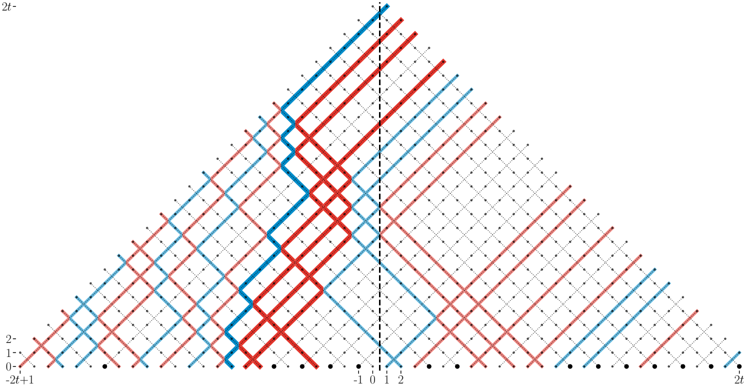
[4]Institut de Physique Théorique, Université Paris Saclay, CEA, CNRS UMR 3681, 91191 Gif-sur-Yvette, France

[PRL **128**, 160601 (2022)]

$$(\emptyset,\emptyset)\leftrightarrow(\emptyset,\emptyset),\qquad(\emptyset,q)\leftrightarrow(q,\emptyset),\qquad(q,q')\leftrightarrow(q,q'),\qquad\text{for}\quad q,q'\in\{+,-\}.$$



$$\mathbb{P}(\{q_\ell\})=\prod_\ell p(q_\ell),\qquad p(\pm)=\rho\frac{1\pm b}{2},\qquad p(\emptyset)=\bar{\rho}=1-\rho$$

Exact finite-time Moment Generating Function ($\mu_\pm \equiv \cosh\lambda \mp b\sinh\lambda$):

$$G(\lambda|t) = \rho^{2t}\sum_{l=0}^{t}\sum_{r=0}^{t}\binom{t}{l}\binom{t}{r}\nu^{l+r}\mu_-^{|\Lambda_-|}\mu_+^{|\Lambda_+|}, \quad |\Lambda_\pm| = \frac{|l-r|\pm(l-r)}{2}$$

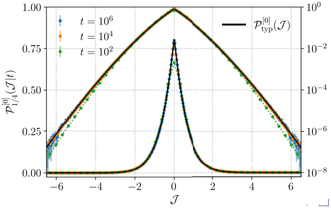
Divergence of (scaled) cumulants, $z^{[b\neq 0]} = 1$, $z^{[b=0]} = 2$:

$$c_2^{[b]}(t) \sim t, \quad c_{2n>2}^{[b]}(t) \sim t^{n-1/2} \qquad \text{and} \qquad c_{2n}^{[0]}(t) \sim t^{n/2}.$$

Exact PDFs of typical fluctuations:

$$\mathcal{P}_{\rm typ}^{[b]}(j) = \frac{1}{\sqrt{2\pi\sigma^2}}\exp\left[-\frac{j^2}{2\sigma^2}\right],$$

$$\mathcal{P}_{\rm typ}^{[0]}(j) = \frac{1}{\sqrt{2}\pi\varDelta}\int_{\mathbb{R}} {\rm d}u \exp\left[-\left(\frac{u^2}{2\varDelta}\right)^2 - \frac{j^2}{2u^2}\right].$$



- Conjecture (superuniversality of superdiffusion): *all 1+1D integrable models with non-abelian global symmetries exhibit superdiffusion of Noether charges of KPZ type with $z = 3/2$ in equilibrium states with unbroken symmetry.*
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