DYNAMICS OF SYMMETRY-RESOLVED ENTANGLEMENT MEASURES AFTER A QUENCH IN FREE-FERMIONIC SYSTEMS

“Randomness, Integrability, Universality”
May 17 , GGI Florence
Riccarda Bonsignori

In collaboration with Gilles Parez and Pasquale Calabrese

HrZZ project No. IP-2019-4-3321

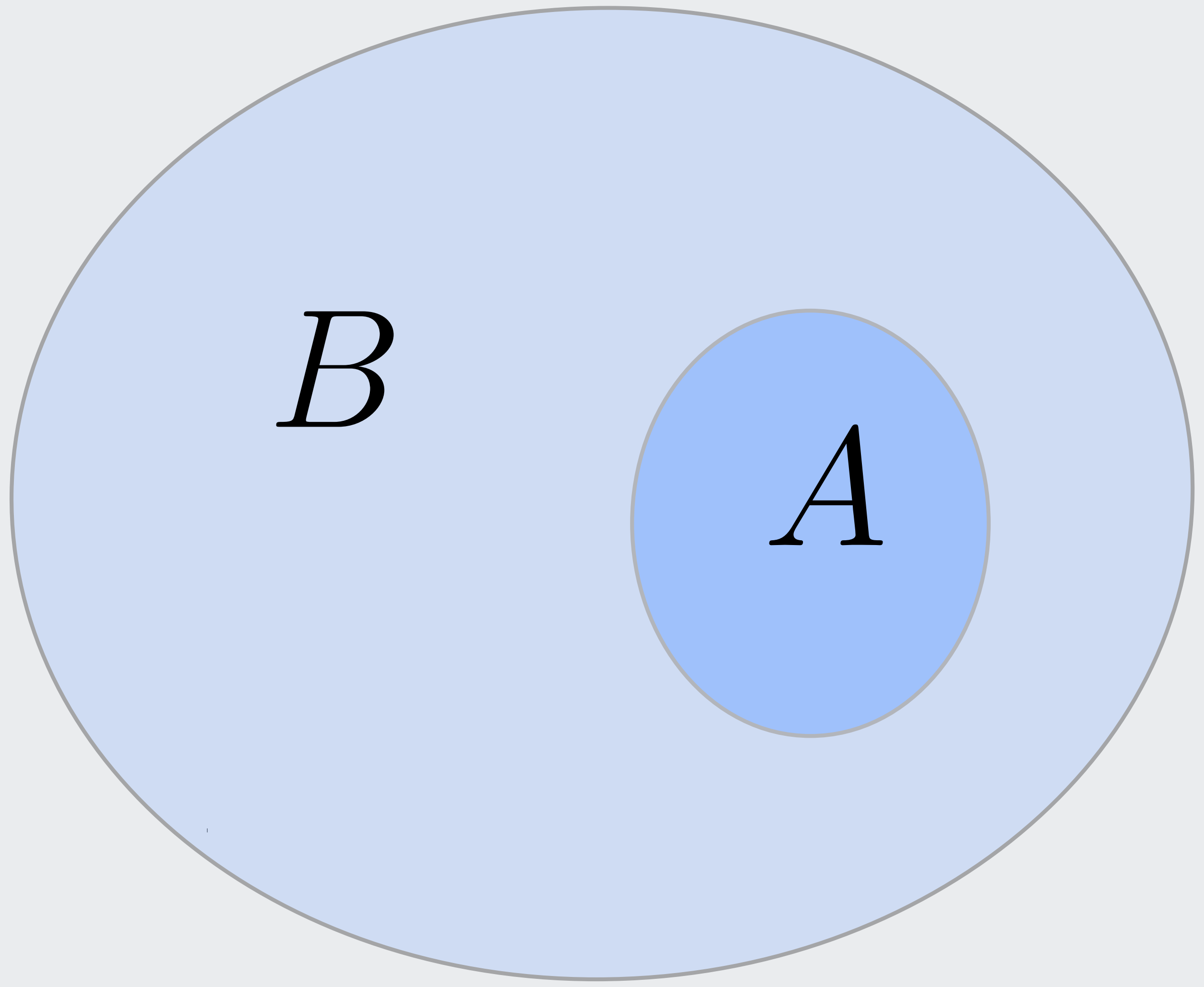
OUTLINE

- Definitions and motivations
  - Symmetry-resolved entanglement and Renyi entropies
  - Charge-imbalance-resolved entanglement negativity
- Dynamics of symmetry-resolved entanglement measures after a quench
- Conclusions

SYMMETRY RESOLVED ENTANGLEMENT ENTROPY

ENTANGLEMENT ENTROPY: DEFINITIONS

- Let us consider a bipartite quantum system $S = A \cup B$ in a pure state $\rho = |\psi\rangle\langle\psi|$
- Reduced density matrix of A: $\rho_A = \mathrm{Tr}_B \rho$



- Measures of entanglement for bipartite pure states:
  - Entanglement Entropy (EE): $S = -\mathrm{Tr}\rho_A \log \rho_A$
  - Renyi Entropies (RE): $S_n = \dfrac{1}{1-n} \log \mathrm{Tr}\rho_A^n$
- The EE is the limit $n \to 1$ of the RE.

SYMMETRY RESOLVED ENTANGLEMENT ENTROPY: DEFINITIONS

- Bipartite system with $U(1)$ internal symmetry generated by a charge $Q = Q_A + Q_B$

$[Q, \rho] = 0 \quad \Rightarrow \quad [Q_A, \rho_A] = 0 \Rightarrow \quad \rho_A$ has block diagonal structure

$$\rho_A = \oplus_q \Pi_q \rho_A = \oplus_q [p(q)\rho_A(q)], \qquad p(q) = \mathrm{Tr}(\Pi_q \rho_A)$$

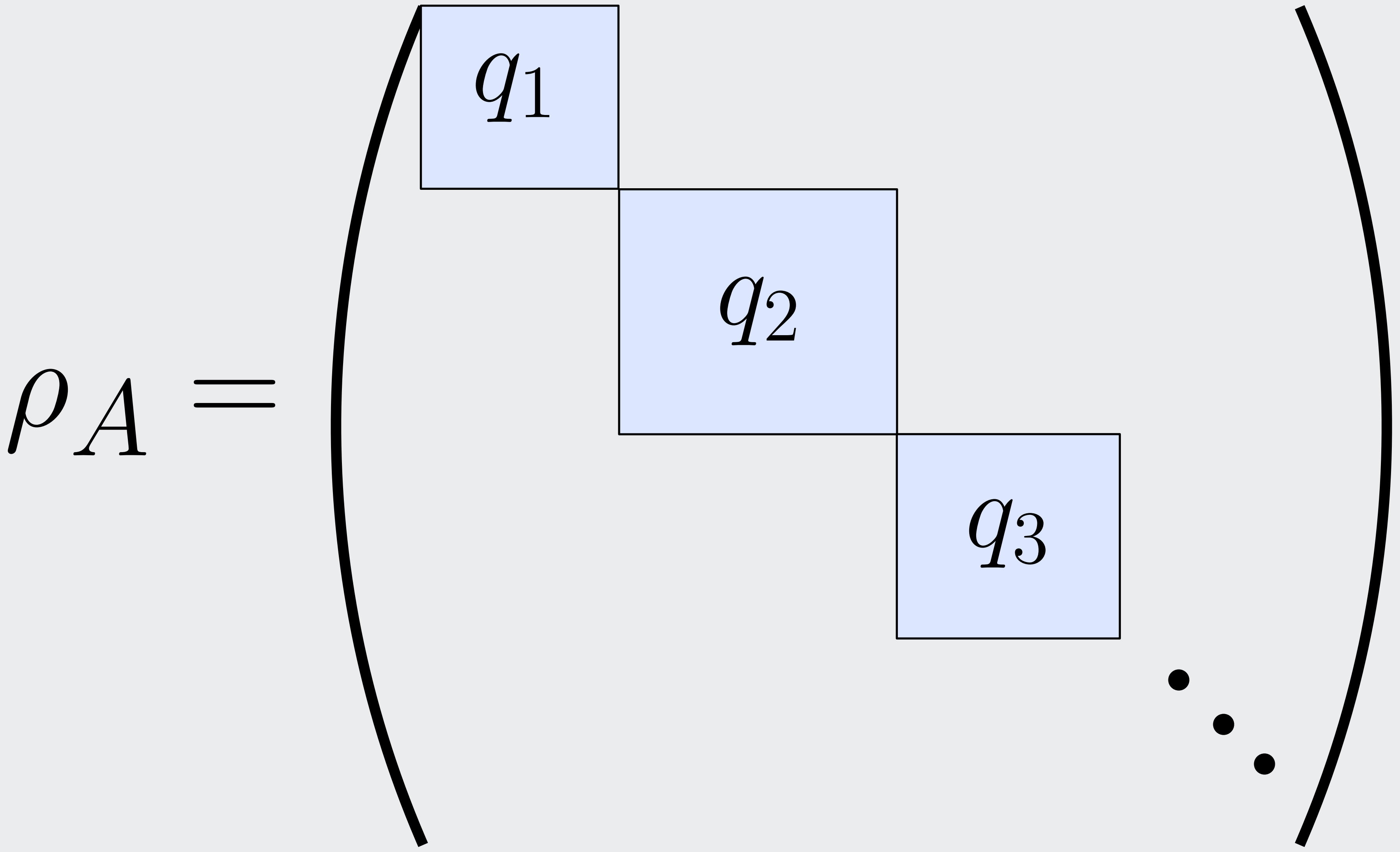
Π_q projector on the eigenspace of q

- Symmetry Resolved Entanglement Entropy

$$S(q) = -\mathrm{Tr}[\rho_A(q) \ln \rho_A(q)]$$

- Symmetry Resolved Renyi Entropies

$$S_n(q) = \frac{1}{1-n} \log \mathrm{Tr}[\rho_A(q)]^n$$

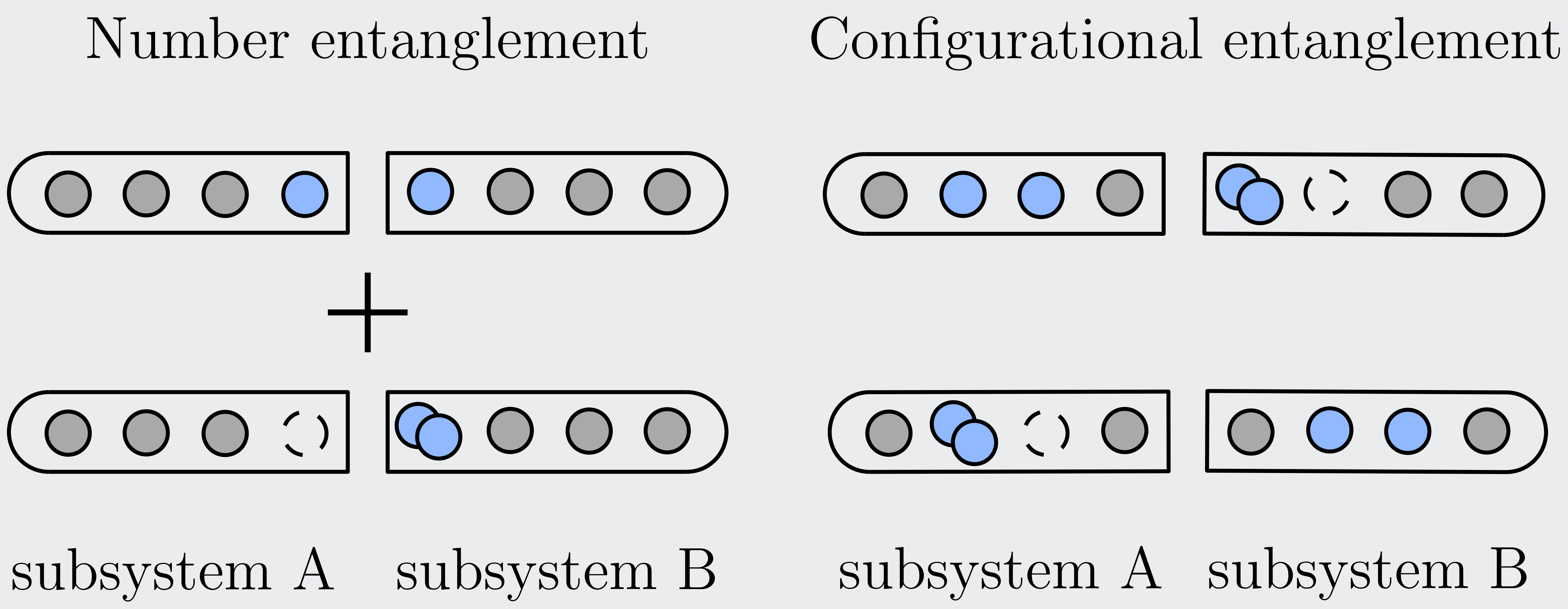


SYMMETRY DECOMPOSITION OF ENTANGLEMENT

- Decomposition of EE:

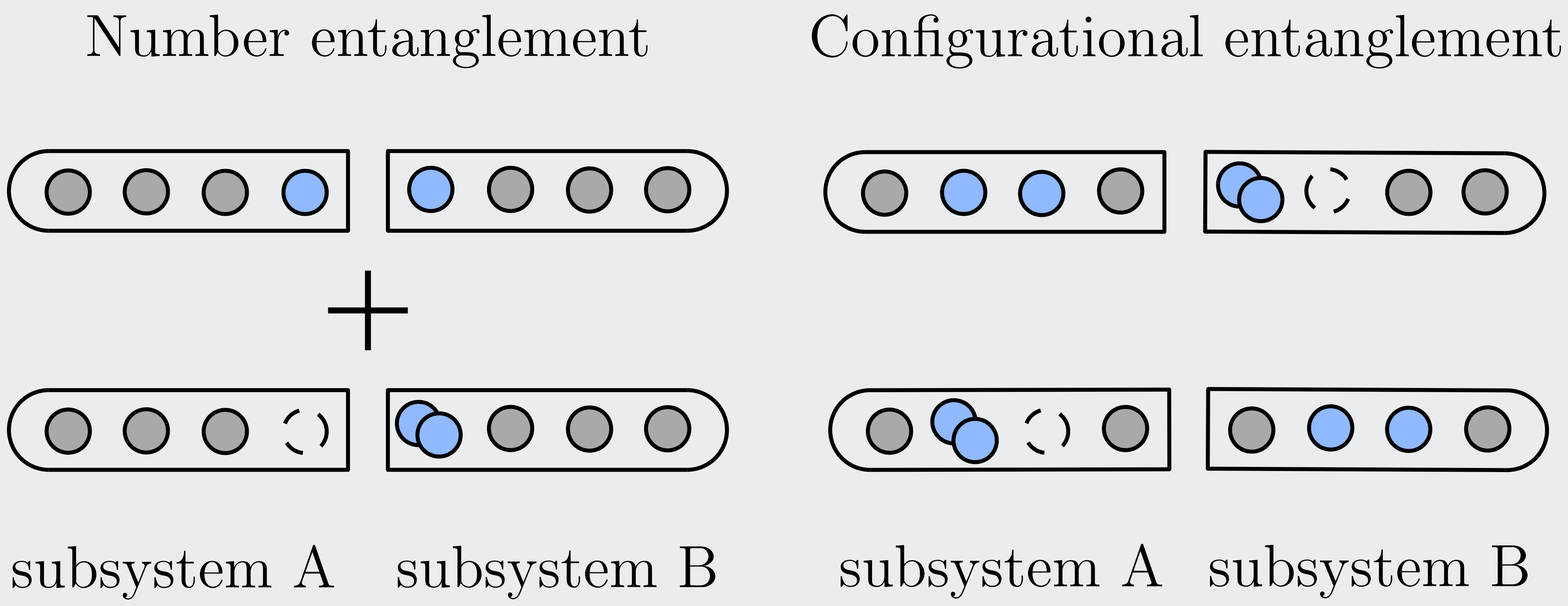
$$S = \sum_q p(q)S(q) - \sum_q p(q)\log p(q) \equiv S^c + S^n$$

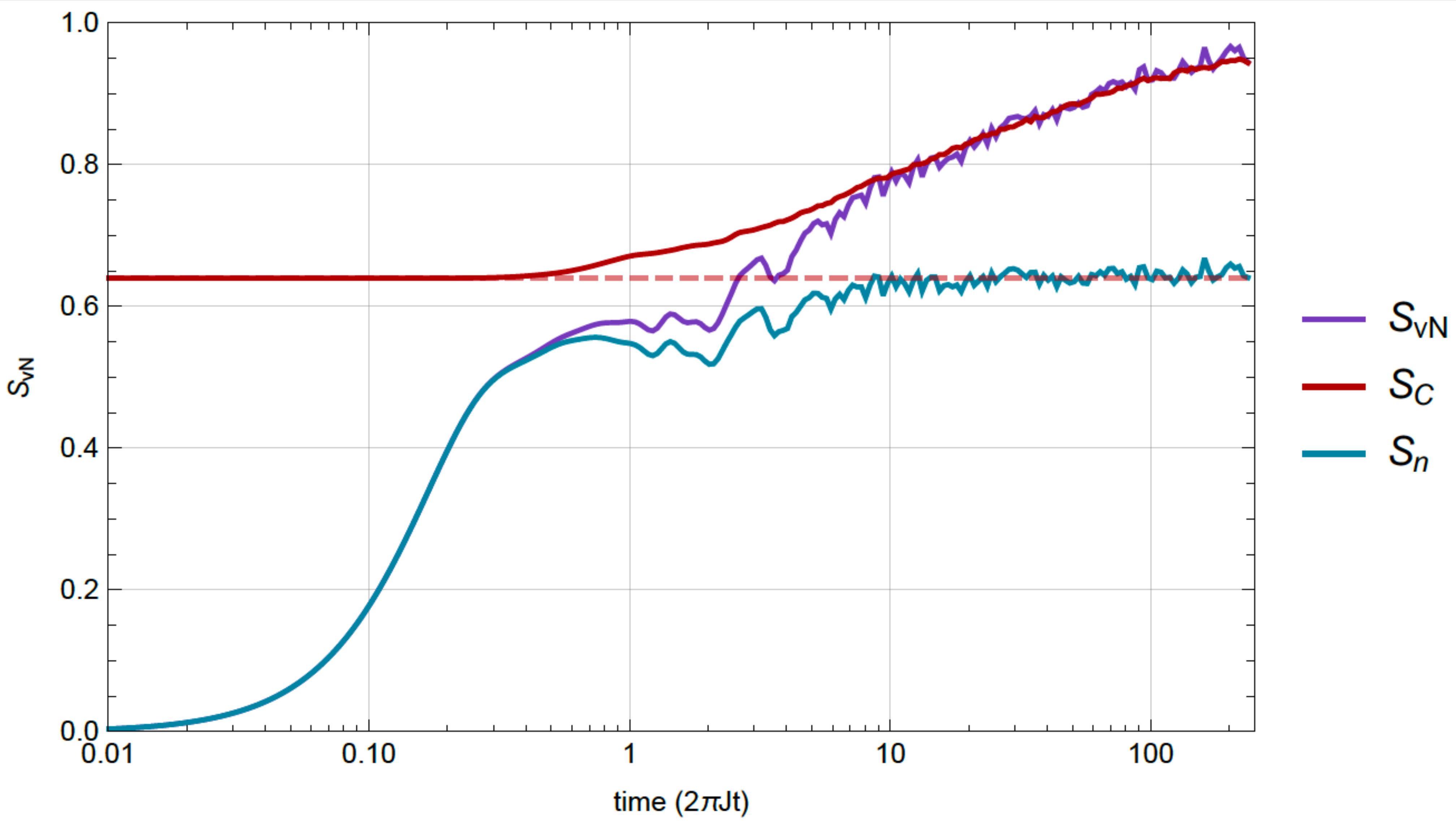
  - S^c: configurational entanglement
  - S^n : number entanglement



SYMMETRY DECOMPOSITION OF ENTANGLEMENT

A. Lukin, M. Rispoli, R. Schittko, M. E. Tai, A. M. Kaufman,S. Choi, V. Khemani, J. Leonard, and M. Greiner, Science 364, 6437 (2019).

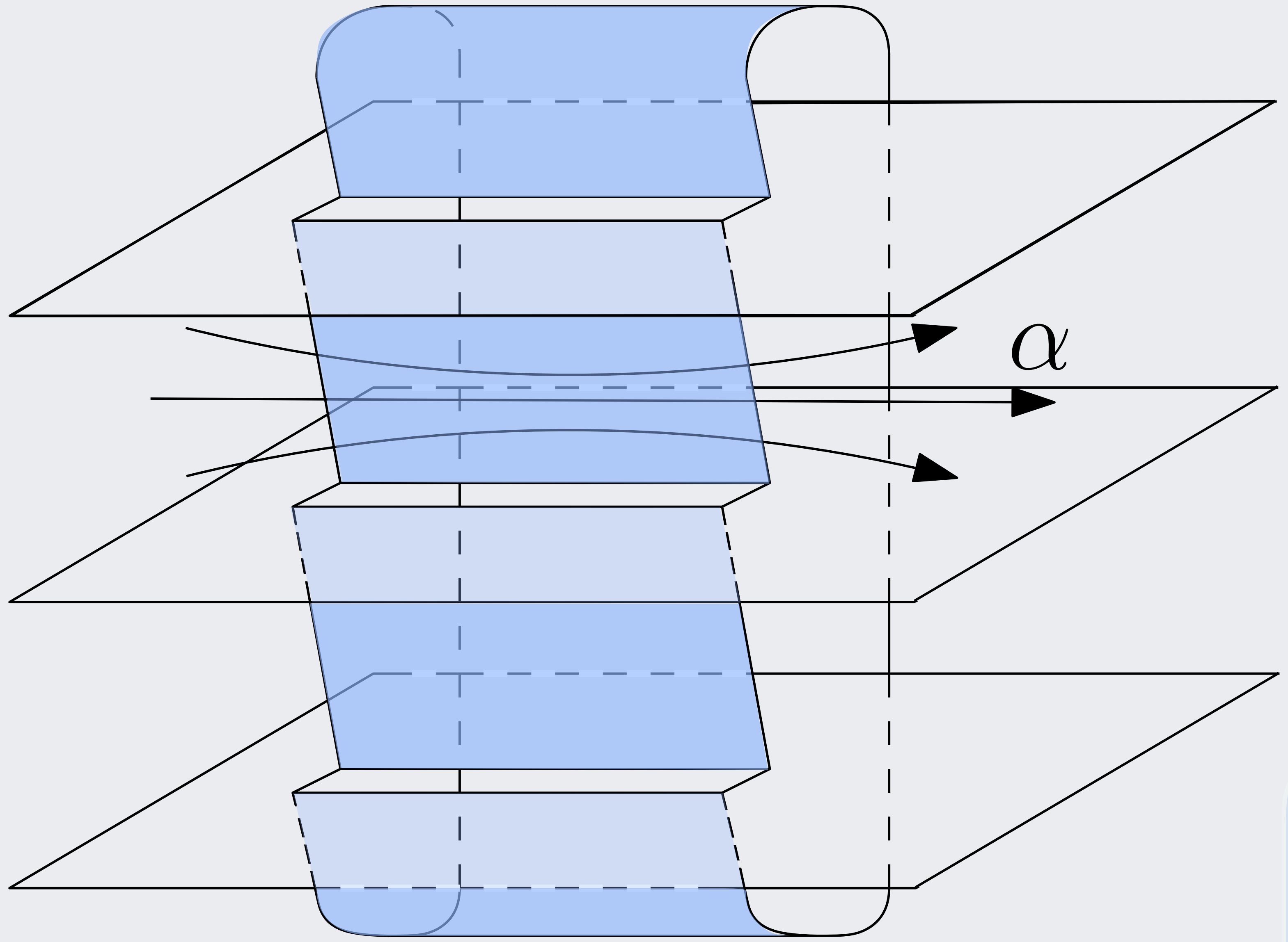




The study of the symmetry resolution of the entanglement measures is a fundamental tool for a more refined description of the entanglement content of a quantum system.

PATH INTEGRAL APPROACH

* M. Goldstein, E. Sela, *Phys.Rev.Lett.* 120 (2018) 20, 200602



- Charged moments*: $Z_n(\alpha) = \mathrm{Tr}[e^{i\alpha Q_A}\rho_A^n]$
- Fourier transform: $\mathcal{Z}_n(q) \equiv \mathrm{Tr}[\Pi_q \rho_A^n] = \int_{-\pi}^{\pi} \frac{d\alpha}{2\pi} e^{-iq\alpha} Z_n(\alpha)$
- Symmetry resolved Renyi and Entanglement Entropies:

$$S_n(q) = \frac{1}{1-n} \ln \left[\frac{\mathcal{Z}_n(q)}{\mathcal{Z}_1(q)^n} \right], \qquad S_{vN}(q) = -\partial_n \left[\frac{\mathcal{Z}_n(q)}{\mathcal{Z}_1(q)^n} \right]_{n=1}$$

- Single interval $A = [0,\ell]$, $\quad (1+1)$ D CFT:

$$Z_n(\alpha) = \langle \mathcal{T}_{n,\alpha}(\ell,0)\tilde{\mathcal{T}}_{n,\alpha}(0,0)\rangle, \qquad \Delta_{n,\alpha} = \Delta_n + \frac{\Delta_\alpha}{n}, \qquad \Delta_n = \frac{c}{24}\left(n - \frac{1}{n}\right)$$

ENTANGLEMENT EQUIPARTITION

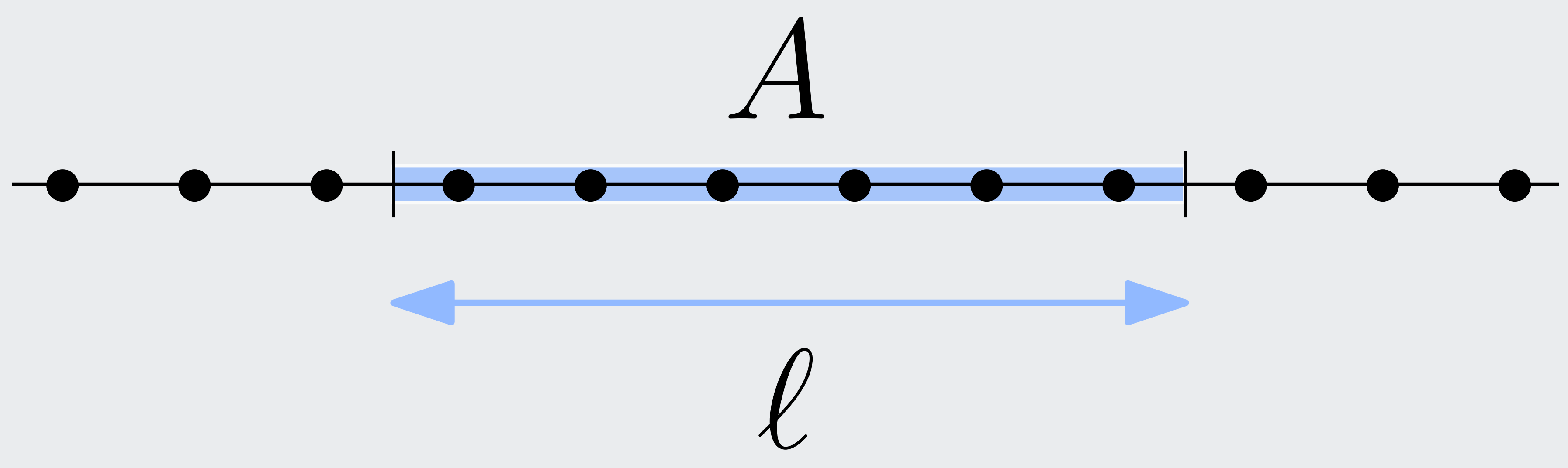
- Charged moments: $Z_n(\alpha) \sim \ell^{-\frac{c}{6}\left(n-\frac{1}{n}\right)-2\frac{\Delta_\alpha+\bar{\Delta}_\alpha}{n}}, \qquad \Delta_\alpha = \bar{\Delta}_\alpha = \frac{1}{2}\left(\frac{\alpha}{2\pi}\right)^2 K$
- Q_A-resolved moments: $\mathcal{Z}_n(q) \simeq \ell^{-\frac{c}{6}\left(n-\frac{1}{n}\right)}\sqrt{\frac{n\pi}{2K\ln\ell}}e^{\frac{n\pi^2(q-\langle Q_A\rangle)^2}{2K\ln\ell}}$

*Equipartition of entanglement**:

$$S_n(q) = S_n - \frac{1}{2}\ln\left(\frac{2K}{\pi}\ln\ell\right) + O(\ell^0), \qquad S(q) = S - \frac{1}{2}\ln\left(\frac{2K}{\pi}\ln\ell\right) + O(\ell^0)$$

* J. C. Xavier, F. C. Alcaraz, and G. Sierra, Phys. Rev. B **98** , 0401106 (2018)

SYMMETRY RESOLVED ENTANGLEMENT FOR THE XX CHAIN WITH PBC



- Free fermion chain via Jordan-Wigner transformation

$$H = -\sum_{i=-\infty}^{\infty}\left[c_i^\dagger c_{i+1} + c_{i+1}^\dagger c_i - 2h\left(c_i^\dagger c_i - \frac{1}{2}\right)\right]$$

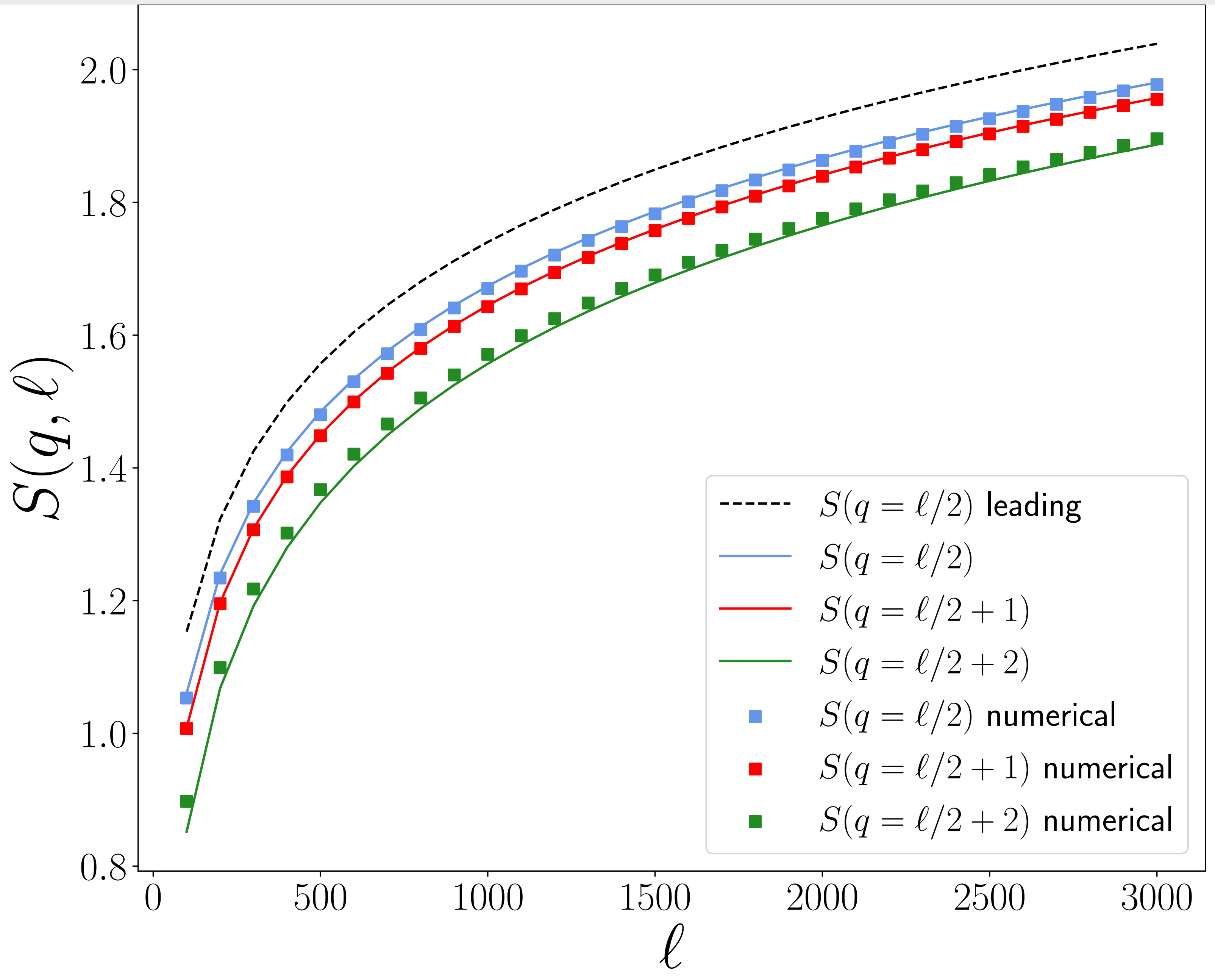
$$\mathcal{Z}_n(q) = Z_n(0)\sqrt{\frac{n\pi}{2(\ln(2\ell\,|\sin k_F|) - 2\pi^2 n\gamma_2(n))}}\,e^{-\frac{n\pi^2(q-\bar{q})^2}{2(\ln(2\ell\,|\sin k_F|) - 2\pi^2 n\gamma_2(n))}},$$

- Symmetry Resolved Entanglement Entropy:

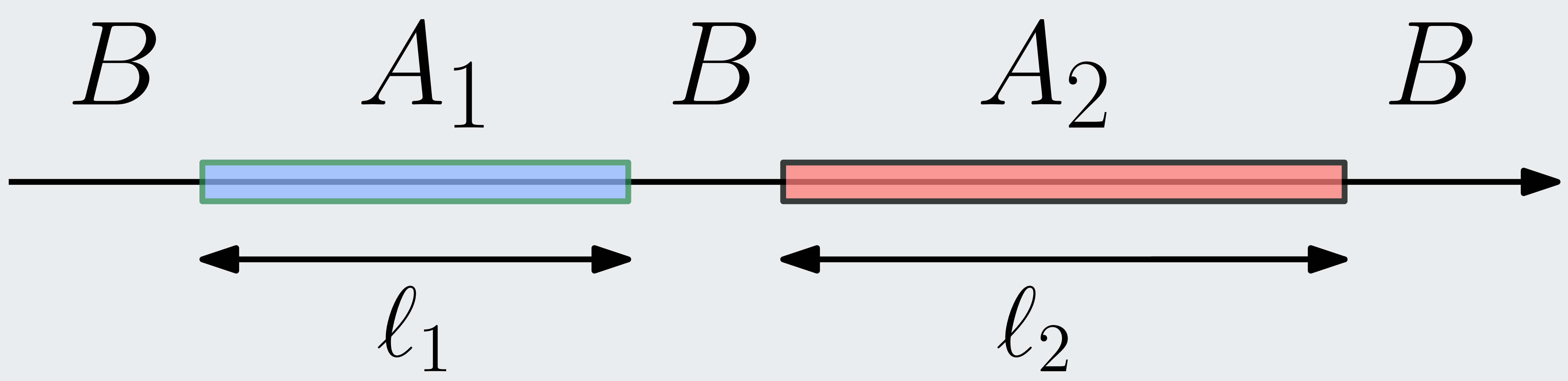
$\Delta q \equiv q - \bar{q}$

$$S(q) = S - \frac{1}{2}\ln\left(\frac{2}{\pi}\ln\delta_1(2\ell\,|\sin k_F|)\right) - \frac{1}{2} + \boxed{(q-\bar{q})}^2\,\pi^4\frac{(\gamma_2(1)+\gamma_2'(1))}{\ln^2\kappa_1(2\ell\,|\sin k_F|)} + \cdots$$

R. B., P. Ruggiero and P. Calabrese, J. Phys. A: Math. Theor. **52**, 475302 (2019).



ENTANGLEMENT NEGATIVITY



$$S = (A_1 \cup A_2) \cup B, \qquad \rho_A = \mathrm{Tr}_B \rho$$

$\{|e_i^1\rangle\}$ orthonormal basis of $\mathscr{H}_{A_1}$

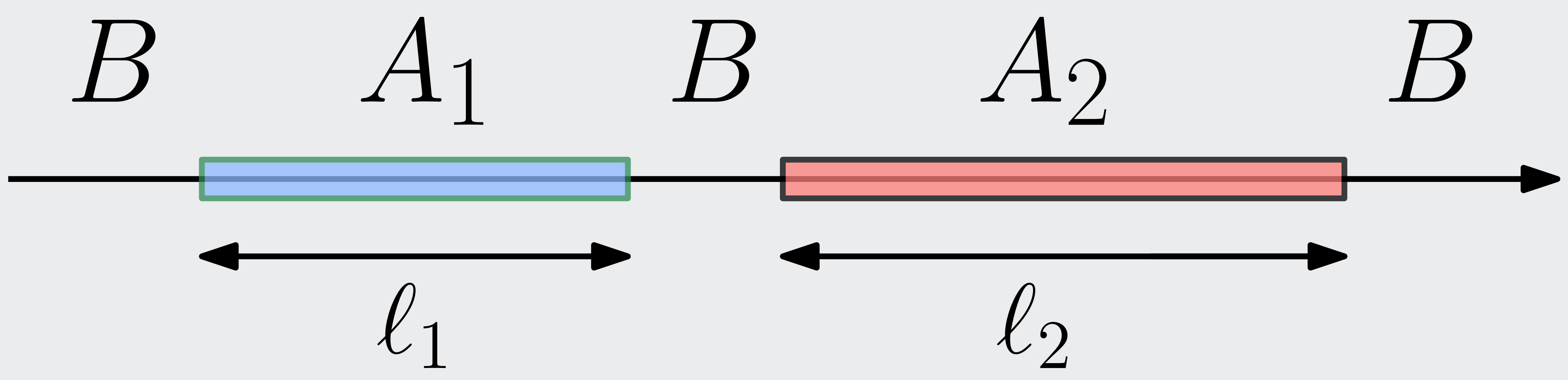
$\{|e_j^2\rangle\}$ orthonormal basis of $\mathscr{H}_{A_2}$

- Bosonic system: $$(|e_i^1, e_j^2\rangle\langle e_k^1, e_l^2|)^{T_1} \equiv |e_k^1, e_j^2\rangle\langle e_i^1, e_l^2|$$

$$\rho_A = \sum_{ijkl} \langle e_i^1, e_j^2 | \rho_A | e_k^1 e_l^2 \rangle |e_i^1, e_j^2\rangle\langle e_k^1 e_l^2| \xrightarrow{\qquad} \rho_A^{T_1} = \sum_{ijkl} \langle e_k^1, e_j^2 | \rho_A | e_i^1 e_l^2 \rangle |e_i^1, e_j^2\rangle\langle e_k^1 e_l^2|$$

- Negativity: $$\mathscr{N} = \frac{\mathrm{Tr}\rho_A^{T_1} - 1}{2}$$

FERMIONIC PARTIAL TRANSPOSE*



* H. Shapourian, K. Shiozaki, S. Ryu, Phys. Rev. B 95(16) 165101

- Fermionic system:

$$S = (A_1 \cup A_2) \cup B, \qquad \rho_A = \mathrm{Tr}_B \rho$$

Example: occupation number basis $|\{n_j\}_{j\in A_1}, \{n_j\}_{j\in A_2}\rangle = (f^\dagger_{m_1})^{n_{m_1}} \cdots (f^\dagger_{m_{l_1}})^{n_{m_{l_1}}} (f^\dagger_{m'_1})^{n_{m'_1}} \cdots (f^\dagger_{m'_{l_2}})^{n_{m'_{l_2}}} |0\rangle$

$$U_{A_1}(|\{n_j\}_{j\in A_1}, \{n_j\}_{j\in A_2}\rangle\langle\{\bar{n}_j\}_{j\in A_1}, \{\bar{n}_j\}_{j\in A_2}|)^{R_1} U^\dagger_{A_1} =$$

$$= |\{\bar{n}_j\}_{j\in A_1}, \{n_j\}_{j\in A_2}\rangle\langle\{n_j\}_{j\in A_1}, \{\bar{n}_j\}_{j\in A_2}|(-1)^{\phi(\{n_j\},\{\bar{n}_j\})}, \quad n_i, \bar{n}_j \in \{0,1\}$$

- (Fermionic)Negativity:

$$\mathcal{N} = \frac{\mathrm{Tr}\sqrt{\rho_A^{R_1}(\rho_A^{R_1})^\dagger} - 1}{2}$$

SYMMETRY DECOMPOSITION OF NEGATIVITY*

*E. Cornfeld, M. Goldstein, E.Sela ,Phys. Rev. A 98, 032302

- Single particle in one out of three boxes: $(A_1 \cup A_2) \cup B$. The system is in a pure state $|\Psi\rangle = \alpha|100\rangle + \beta|010\rangle + \gamma|001\rangle$

$$\rho_A = \begin{array}{cc} & \begin{array}{cccc} (00) & (01) & (10) & (11) \end{array} \\ & \left(\begin{array}{cc|cc} |\gamma|^2 & 0 & 0 & 0 \\ 0 & |\beta|^2 & \alpha^*\beta & 0 \\ \hline 0 & \beta^*\alpha & |\alpha|^2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right) \begin{array}{c} (00) \\ (01) \\ (10) \\ (11) \end{array} \end{array}
\qquad
\rho_A^{R_1} = \begin{array}{cc} & \begin{array}{cccc} (10) & (00) & (11) & (01) \end{array} \\ & \left(\begin{array}{cc|cc} |\alpha|^2 & 0 & 0 & 0 \\ 0 & |\gamma|^2 & i\beta^*\alpha & 0 \\ \hline 0 & i\alpha^*\beta & 0 & 0 \\ 0 & 0 & 0 & |\beta|^2 \end{array}\right) \begin{array}{c} (10) \\ (00) \\ (11) \\ (01) \end{array} \end{array}$$

- The ρ_A has block diagonal structure according to the eigenvalues $q = q_1 + q_2$.

- The $\rho_A^{R_1}$ has block diagonal structure according to the eigenvalues $\tilde{q} = q_2 - q_1$.

SYMMETRY DECOMPOSITION OF NEGATIVITY

$[\rho_A^{R_1}, Q_2 - Q_1^{R_1}] = 0 \quad \Rightarrow \quad \rho_A^{R_1}$ has block decomposition

$$\rho_A^{R_1}(\tilde{q}) = \frac{\Pi_{\tilde{q}} \rho_A^{R_1} \Pi_{\tilde{q}}}{\mathrm{Tr}(\Pi_{\tilde{q}} \rho_A^{R_1})}, \quad \tilde{p}(\tilde{q}) = \mathrm{Tr}(\Pi_{\tilde{q}} \rho_A^{R_1})$$

$$\rho_A^{R_1} = \begin{array}{c} \begin{array}{cccc} (10) & (00) & (11) & (01) \end{array} \\ \left(\begin{array}{cc|cc} |\alpha|^2 & 0 & 0 & 0 \\ 0 & |\gamma|^2 & i\beta^*\alpha & 0 \\ \hline 0 & i\alpha^*\beta & 0 & 0 \\ 0 & 0 & 0 & |\beta|^2 \end{array} \right) \end{array} \begin{array}{c} \\ (10) \\ (00) \\ (11) \\ (01) \end{array}$$

- Charge imbalance resolved negativity:

$$\mathcal{N}(\tilde{q}) = \frac{\mathrm{Tr}\,|\rho_A^{R_1}(\tilde{q})| - 1}{2}$$

- Charge imbalance resolved logarithmic negativity:

$$\hat{\mathcal{E}}(\tilde{q}) = \log \mathrm{Tr}\,|\rho_A^{R_1}(\tilde{q})|$$

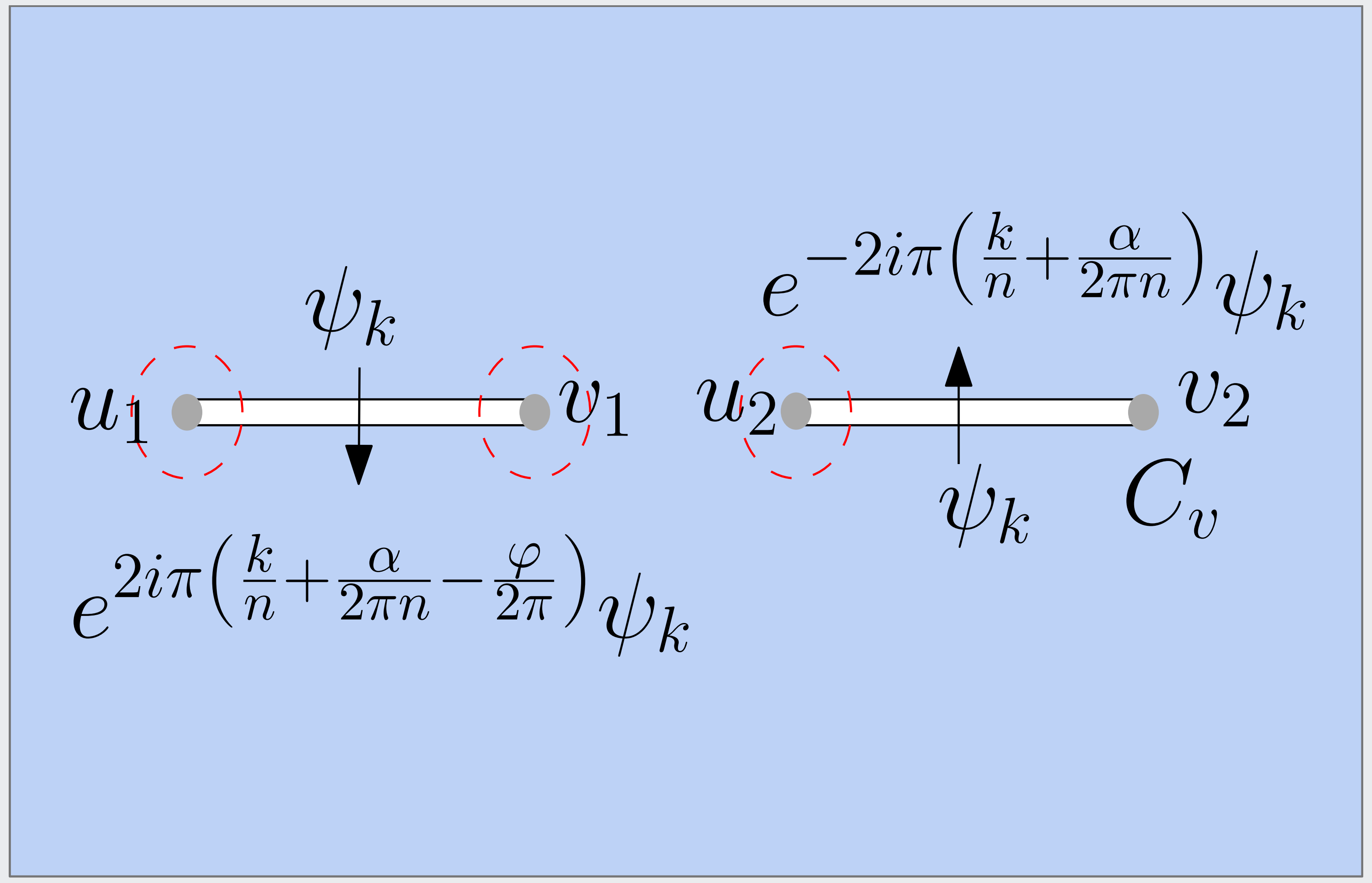
- Charge imbalance resolved Renyi negativity:

$$\hat{N}_n(\tilde{q}) = \begin{cases} \mathrm{Tr}(\rho_A^{R_1}(\tilde{q})\rho_A^{R_1}(\tilde{q})^\dagger \ldots \rho_A^{R_1}(\tilde{q})\rho_A^{R_1}(\tilde{q})^\dagger), & n \quad \text{even}, \\ \mathrm{Tr}(\rho_A^{R_1}(\tilde{q})\rho_A^{R_1}(\tilde{q})^\dagger \ldots \rho_A^{R_1}(\tilde{q})), & n \quad \text{odd}, \end{cases}$$

DEFINITION OF CHARGED MOMENTS

S. Murciano, **R. B.** and P. Calabrese, SciPost Phys. **10**, 111 (2021).

- Charged moments of the partial TR transpose: $$N_n(\alpha) \equiv \begin{cases} \mathrm{Tr}(\rho_A^{R_1}\rho_A^{R_1\dagger}\cdots\rho_A^{R_1}\rho_A^{R_1\dagger}e^{i\tilde{Q}_A\alpha}), & \text{if } n \text{ is even} \\ \mathrm{Tr}(\rho_A^{R_1}\rho_A^{R_1\dagger}\cdots\rho_A^{R_1}e^{i\tilde{Q}_A\alpha}), & \text{if } n \text{ is odd} \end{cases}$$



- Multivalued field $\Psi = (\psi_1, \cdots, \psi_n)^T$ on a single-sheet spacetime.
- Around the endpoints the field transforms according to $T_\alpha^{R_1}$ and T_α.
- Charged moments: $$N_n(\alpha) = \prod_{k=-\frac{n-1}{2}}^{\frac{n-1}{2}} Z_{R_1,k}(\alpha)$$
- Partition function in terms of vertex operators: $$Z_{R_1,k}(\alpha) = \langle \prod_{i=1}^{p} V_{k,\alpha}(u_i)V_{-k,\alpha}(v_i)\rangle$$

SYMMETRY RESOLUTION: MAIN QUANTITIES

- Charged Rényi LN: $\mathcal{E}_n(\alpha) = \log N_n(\alpha) \xrightarrow{n_e \to 1}$ Charged LN: $\mathcal{E}(\alpha)$
- Fourier transforms: $\mathcal{Z}_{R_1,n}(\tilde{q}) = \int_{-\pi}^{\pi} \frac{d\alpha}{2\pi} e^{-i\tilde{q}\alpha} N_n(\alpha), \xrightarrow{n_e \to 1} \mathcal{Z}_{R_1}(\tilde{q}) = \lim_{n_e \to 1} \mathcal{Z}_{R_1,n_e}(\tilde{q})$
- Charged probability: $N_1(\alpha) = \mathrm{Tr}(\rho_A^{R_1} e^{i\tilde{Q}_A\alpha}) \longrightarrow \tilde{p}(\tilde{q}) = \int_{-\pi}^{\pi} \frac{d\alpha}{2\pi} e^{-i\tilde{q}\alpha} N_1(\alpha)$
- Charge imbalance resolved Renyì negativity: $\hat{N}_n(\tilde{q}) = \frac{\mathcal{Z}_{R_1,n}(\tilde{q})}{\tilde{p}(\tilde{q})^n},$
- Charge imbalance resolved negativity and logarithmic negativity:

$$\mathcal{N}(q) = \frac{1}{2}\left(\frac{\mathcal{Z}_{R_1}(\tilde{q})}{\tilde{p}(\tilde{q})} - 1\right) \qquad \hat{\mathcal{E}}(q) = \log\left(\frac{\mathcal{Z}_{R_1}(\tilde{q})}{\tilde{p}(\tilde{q})}\right),$$

DYNAMICS OF SYMMETRY RESOLVED ENTANGLEMENT ENTROPY AND MUTUAL INFORMATION AFTER A QUENCH

TIME EVOLUTION

- Hamiltonian: $\mathscr{H} = \sum_{i=1}^{L} (c_i^\dagger c_{i+1} + c_{i+1}^\dagger c_i), \qquad \{c_i, c_j^\dagger\} = \delta_{ij}$
- Quench from:
  - the Néel state: $|N\rangle = \prod_{j=1}^{L/2} c_{2j}^\dagger |0\rangle$
  - Majumdar-Gosh dimer state: $|D\rangle = \prod_{j=1}^{L/2} \frac{c_{2j}^\dagger - c_{2j-1}^\dagger}{\sqrt{2}} |0\rangle$

QUENCH FROM THE NEEL STATE

G. Parez, R. Bonsignori, P. Calabrese, Phys. Rev. B **103**, L041104

- Correlation matrix:

$$[C_A(t)]_{x,x'} = \frac{\delta_{x,x'}}{2} + \frac{(-1)^{x'}}{2}\int_{-\pi}^{\pi}\frac{dk}{2\pi}e^{ik(x-x')+4it\cos k} = \frac{\delta_{x,x'}}{2} + [J_A(t)]_{x,x'}$$

- We evaluate: $\ln\left(\mathrm{Tr}[e^{i\alpha Q_A}\rho_A^n]\right) = \log Z_n(\alpha) = \sum_{m=0}^{\infty} c_{n,\alpha}(m)\mathrm{Tr}J_A^m$

- Charged moments:

$$Z_n(\alpha) = \left(\frac{\cos(\alpha/2)}{2^{n-1}}\right)^{\mathcal{J}} e^{i\ell\frac{\alpha}{2}}, \qquad \mathcal{J} = \ell - \mathrm{Tr}J_A(t)^2 = \int\frac{dk}{2\pi}\min[\ell, 2v_k t]$$

SYMMETRY RESOLVED ENTROPIES

- Symmetry resolved EE and RE:

$$S_n(q) = \mathcal{J}\log 2 + \log \mathcal{Z}_1(q)$$

For the Néel quench, the entropies do not depend on $\boldsymbol{n}$

- Using the explicit form of:

$$S_n(q) = \log \frac{\Gamma(\mathcal{J}+1)}{\Gamma\left(\frac{\mathcal{J}+2\Delta q+2}{2}\right)\Gamma\left(\frac{\mathcal{J}-2\Delta q+2}{2}\right)}$$

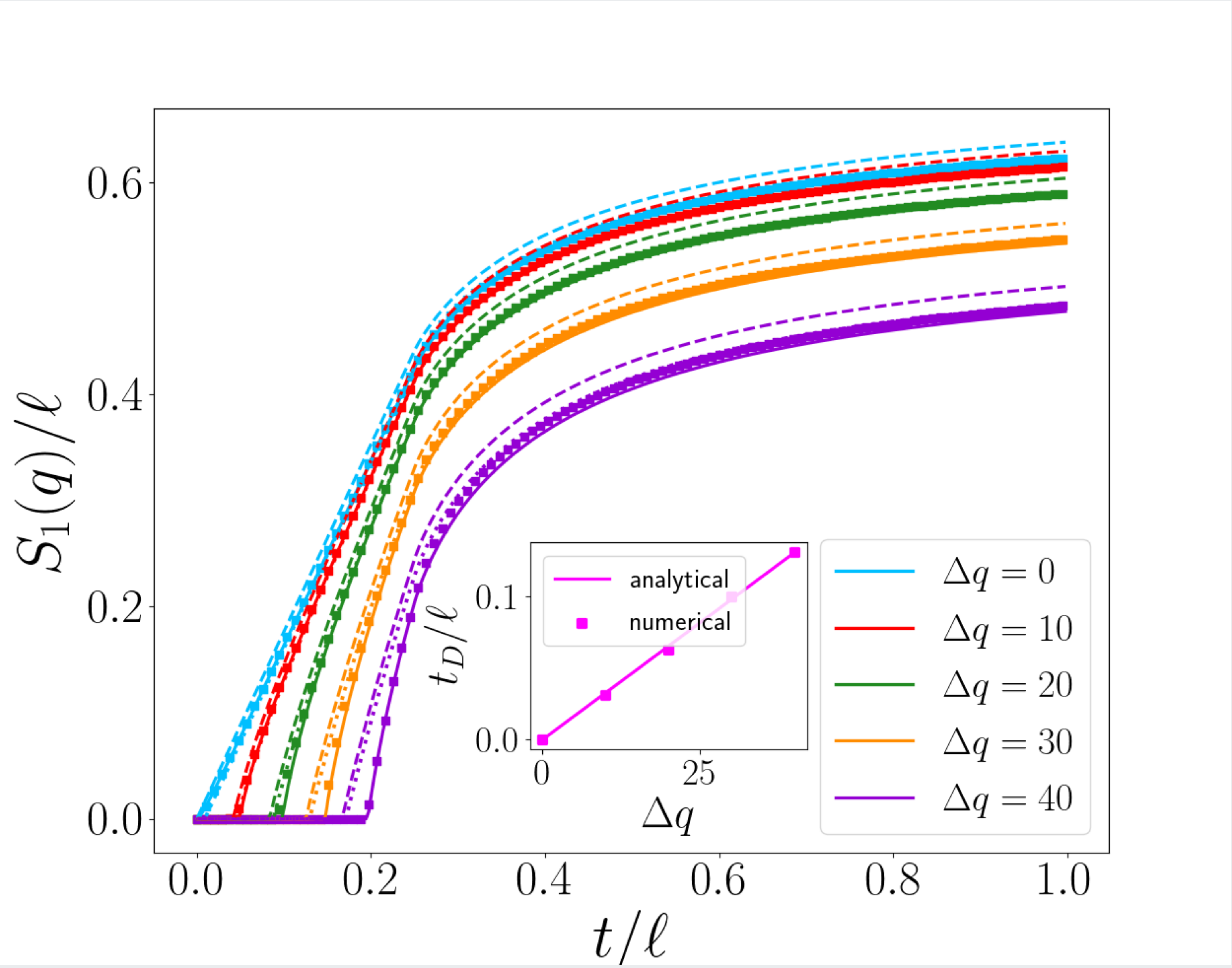
The entropies start to grow after a time delay $t_D = \pi|\Delta q|/4$

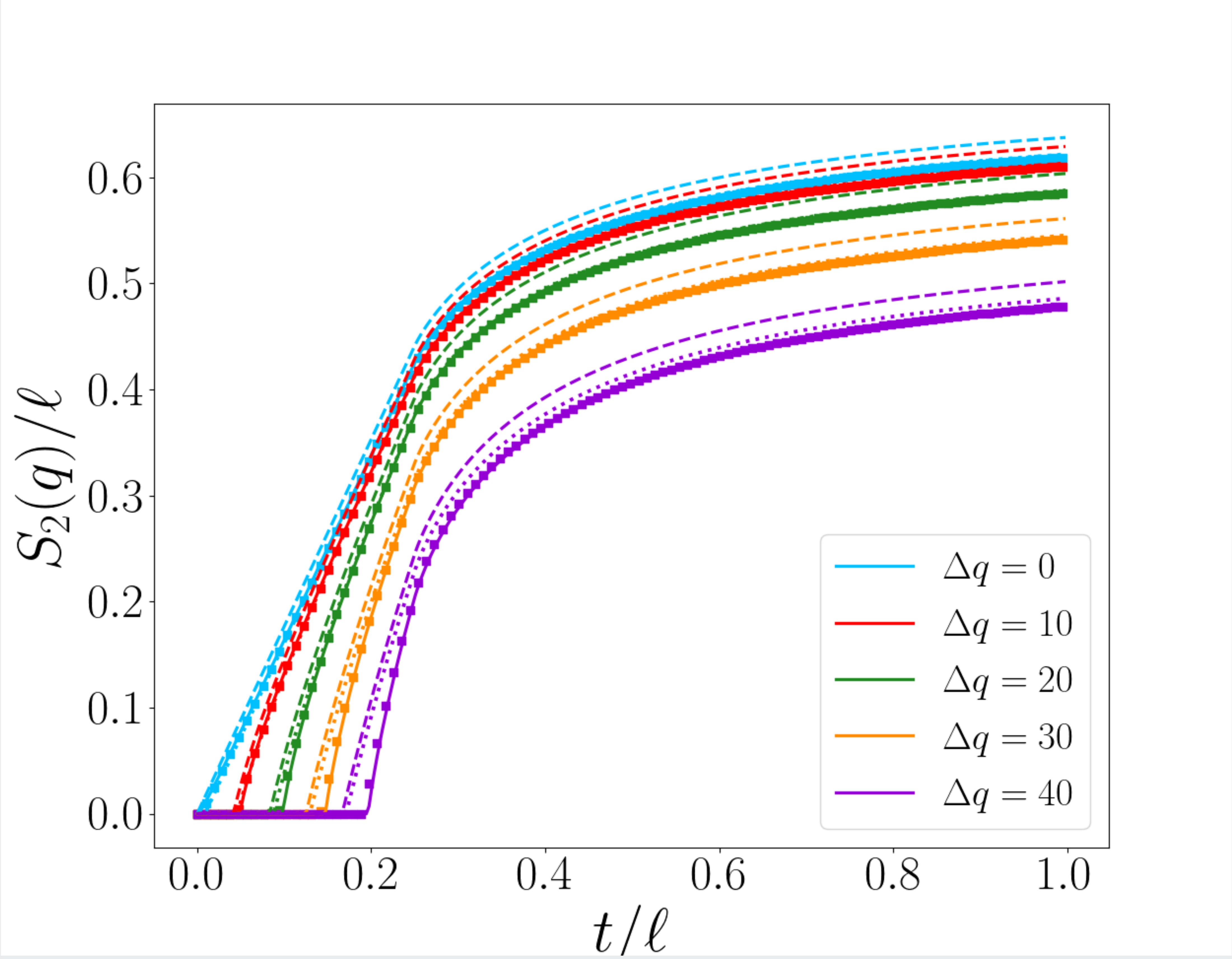
- For large ℓ and small $|\Delta q| \ll \mathcal{J}$:

$$S_n(q) = \mathcal{J}\left(\log 2 - 2\left(\frac{|\Delta q|}{\mathcal{J}}\right)^2\right)$$

For small $|\Delta q|$ *there is effective equipartition of entanglement*

Time evolution of the symmetry-resolved entanglement and Renyi 2 entropies, after a quench from the Néel state: Analytical predictions vs numerical results





G. Parez, R. Bonsignori, P. Calabrese, Phys. Rev. B **103**, L041104

CFT RESULTS*

*P. Calabrese and J. Cardy, J. Stat. Mech. (2005) P04010

- Charged moments:

$$\log Z_n^{Neel/Dimer}(\alpha) = \log Z_n^{Neel/Dimer}(0) - \alpha^2 \mathcal{J}_{0/n}$$

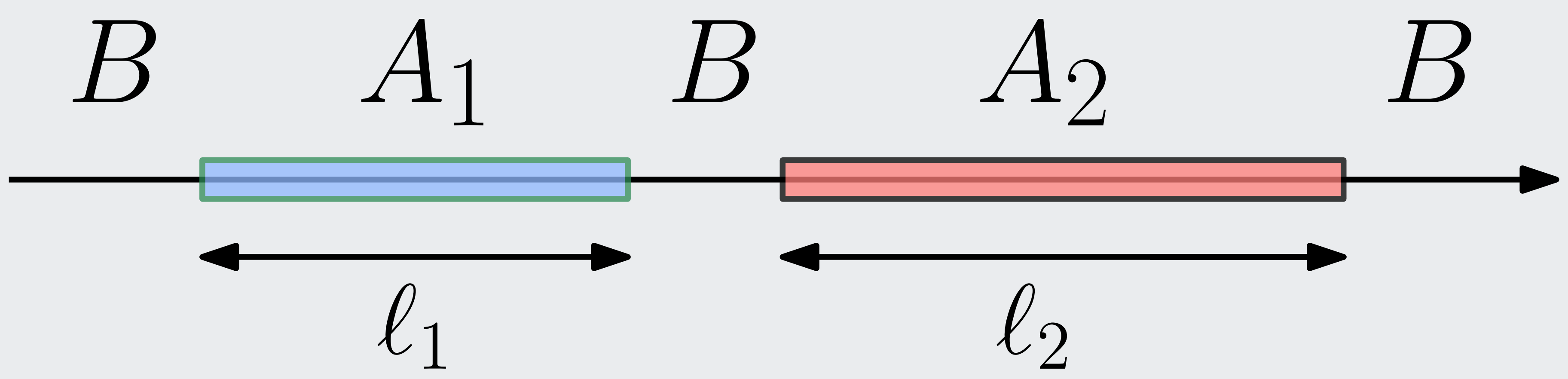
$$\log Z_n^{CFT}(\alpha) = \log Z_n^{CFT}(0) - \frac{K\alpha^2}{4\pi n}\frac{\min[2vt, \ell]}{\tau_0}$$

- Observations for the CFT case:
  - Equipartition of entanglement
  - Absence of delay time

CFT captures only the universal properties of the charged moments

SYMMETRY RESOLVED MUTUAL INFORMATION

G. Parez, R. Bonsignori, P. Calabrese, J. Stat. Mech. **2021**, 093102 (2021).41104



- Mutual Information:

$$I^{A_1:A_2} = S^{A_1} + S^{A_2} - S^{A_1\cup A_2}$$

- Symmetry Resolved Mutual Information:

$$I_1^{A_1:A_2}(q) = \sum_{q_1=0}^{q} p(q_1, q-q_1)\Big(S_1^{A_1}(q_1) + S_1^{A_2}(q-q_1)\Big) - S_1^{A_1\cup A_2}(q)$$

$$p(q_1, q-q_1) = \frac{\boxed{\mathcal{Z}_1^{A_1:A_2}(q_1, q-q_1)}}{\mathcal{Z}_1^{A_1\cup A_2}(q)}$$

Double Fourier transform of

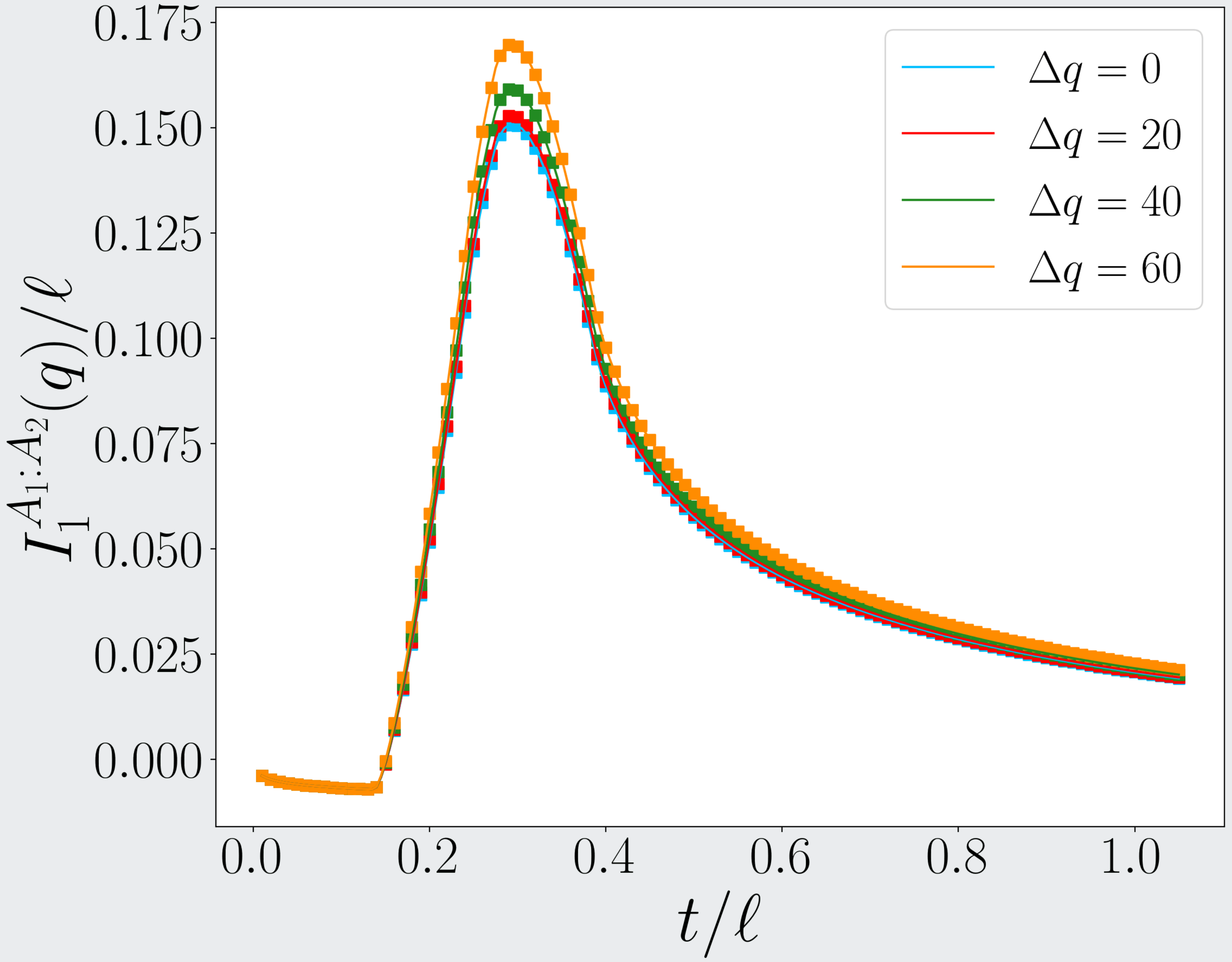
$$Z_1^{A_1:A_2}(\alpha,\beta) = \mathrm{Tr}[\rho_A e^{i\alpha Q_{A_1}+i\beta Q_{A_2}}]$$

- Total Mutual Information:

$$I_1^{A_1:A_2} = \sum_q p(q) I_1^{A_1:A_2}(q) + S^{A_1,n} + S^{A_2,n} - S^{A_1\cup A_2,n} = \sum_q p(q) I_1^{A_1:A_2}(q) + I^{A_1:A_2,n}$$

Time evolution of the symmetry-resolved Mutual Information after a quench from the Néel state: Analytical predictions vs numerical results

G. Parez, R. Bonsignori, P. Calabrese, J. Stat. Mech. **2021**, 093102 (2021).41104

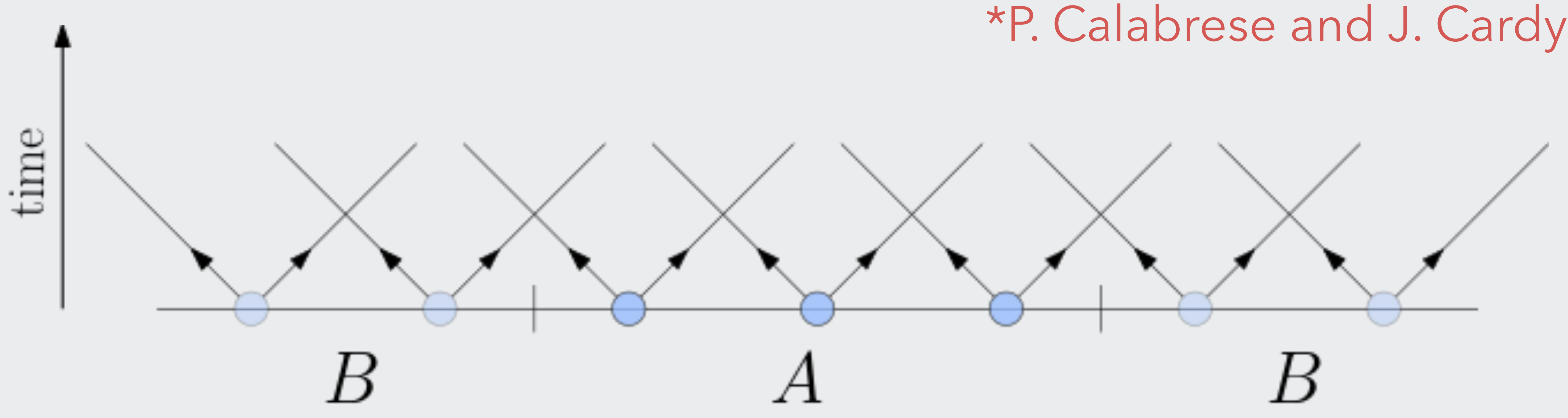


$$I_1^{A_1:A_2}(q) = (\mathcal{J}_{A_1} + \mathcal{J}_{A_2} - \mathcal{J}_d)\log 2 - \frac{1}{2}\Big(\log\frac{\mathcal{J}_{A_1}\mathcal{J}_{A_2}\pi}{2\mathcal{J}_d}\Big) - \frac{4\mathcal{J}_{A_1}\mathcal{J}_{A_2} - \mathcal{J}_m^2}{8\mathcal{J}_d}\Big(\frac{1}{\mathcal{J}_{A_1}} + \frac{1}{\mathcal{J}_{A_2}}\Big)$$
$$-2\Delta q^2\Big\{\Big(\frac{-\mathcal{J}_{A_1} + \mathcal{J}_{A_2} - \mathcal{J}_d}{2\mathcal{J}_d}\Big)^2\frac{1}{\mathcal{J}_{A_1}} + \Big(\frac{\mathcal{J}_{A_1} - \mathcal{J}_{A_2} - \mathcal{J}_d}{2\mathcal{J}_d}\Big)^2\frac{1}{\mathcal{J}_{A_2}} - \frac{1}{\mathcal{J}_d}\Big\}.$$

- At the leading order there is equipartition of the symmetry resolved Mutual Information.

QUASIPARTICLE PICTURE*

*P. Calabrese and J. Cardy, J. Stat. Mech. (2005) P04010



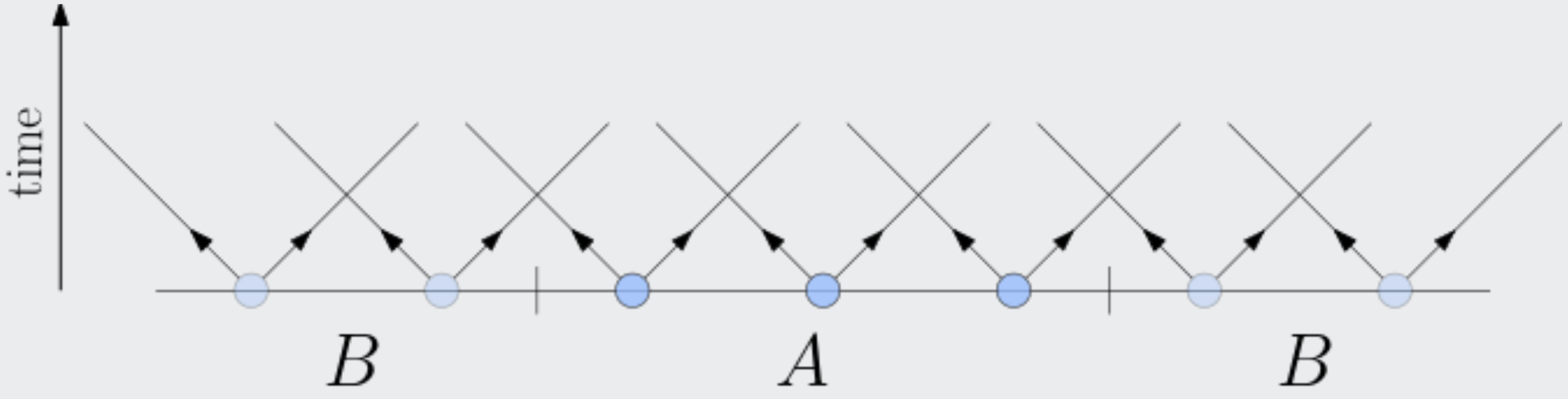
- The entanglement between a subsystem A and its complement B is proportional to the number of entangled pairs of quasiparticles shared between A and B

$$S(t) = 2t \int_{2v_k t<\ell} \frac{dk}{2\pi}\, v_k s(k) + \ell \int_{2v_k t>\ell} \frac{dk}{2\pi}\, s(k) = \int \frac{dk}{2\pi} s(k) \min[2v_k t, \ell]$$

- For free-fermion models:

$$s(k) = \frac{1}{2\pi}(-n_k \log n_k - (1-n_k)\log(1-n_k))$$

QUASIPARTICLE PICTURE



- The expression obtained for the charged moments has the form:

$$\log Z_n(\alpha) = i\langle Q_A\rangle\alpha + \int \frac{dk}{2\pi} f_{n,\alpha}(k)\min[2v_k t, \ell]$$

- The result confirms the existence of a delay time t_D, that in the quasiparticle picture can be seen as the time needed to change the charge by an amount $|\Delta q|$ within the subsystem A.

DYNAMICS OF CHARGE-IMBALANCE-RESOLVED ENTANGLEMENT NEGATIVITY AFTER A QUENCH

DISJOINT INTERVALS

H. Shapourian, K. Shiozaki, S. Ryu, Phys. Rev. B 95(16) 165101

G. Parez, R. Bonsignori, P. Calabrese, *arXiv*:2202.05309

$$J_{A_1 \cup A_2} = \begin{pmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{pmatrix}, \qquad J_{\pm} = \begin{pmatrix} -J_{11} & \pm i J_{12} \\ \pm i J_{21} & J_{22} \end{pmatrix} \qquad J_{\mathrm{x}} = (\mathbb{I} + J_{+}J_{-})^{-1} \cdot (J_{+} + J_{-}),$$

- Charged Renyi logarithmic negativity:

$$\log N_{n_e}(\alpha) = -i\frac{\ell\alpha}{2} + \mathrm{Tr}\log\Big[\Big(\frac{\mathbb{I}+J_{\mathrm{x}}}{2}\Big)^{\frac{n_e}{2}} e^{i\alpha} + \Big(\frac{\mathbb{I}-J_{\mathrm{x}}}{2}\Big)^{\frac{n_e}{2}}\Big] + \frac{n_e}{2}\mathrm{Tr}\log\Big[\Big(\frac{\mathbb{I}+J}{2}\Big)^2 + \Big(\frac{\mathbb{I}-J}{2}\Big)^2\Big]$$

- Charged logarithmic negativity:

$$\mathscr{E}(\alpha) = -i\frac{\ell\alpha}{2} + \mathrm{Tr}\log\Big[\Big(\frac{\mathbb{I}+J_{\mathrm{x}}}{2}\Big)^{\frac{1}{2}} e^{i\alpha} + \Big(\frac{\mathbb{I}-J_{\mathrm{x}}}{2}\Big)^{\frac{1}{2}}\Big] + \frac{1}{2}\mathrm{Tr}\log\Big[\Big(\frac{\mathbb{I}+J}{2}\Big)^2 + \Big(\frac{\mathbb{I}-J}{2}\Big)^2\Big]$$

- Charged probability: $\log N_1(\alpha) = -i\frac{\ell\alpha}{2} + \mathrm{Tr}\log\Big[\frac{\mathbb{I}+J_{+}}{2}e^{i\alpha} + \frac{\mathbb{I}-J_{+}}{2}\Big]$

CHARGED PROBABILITY

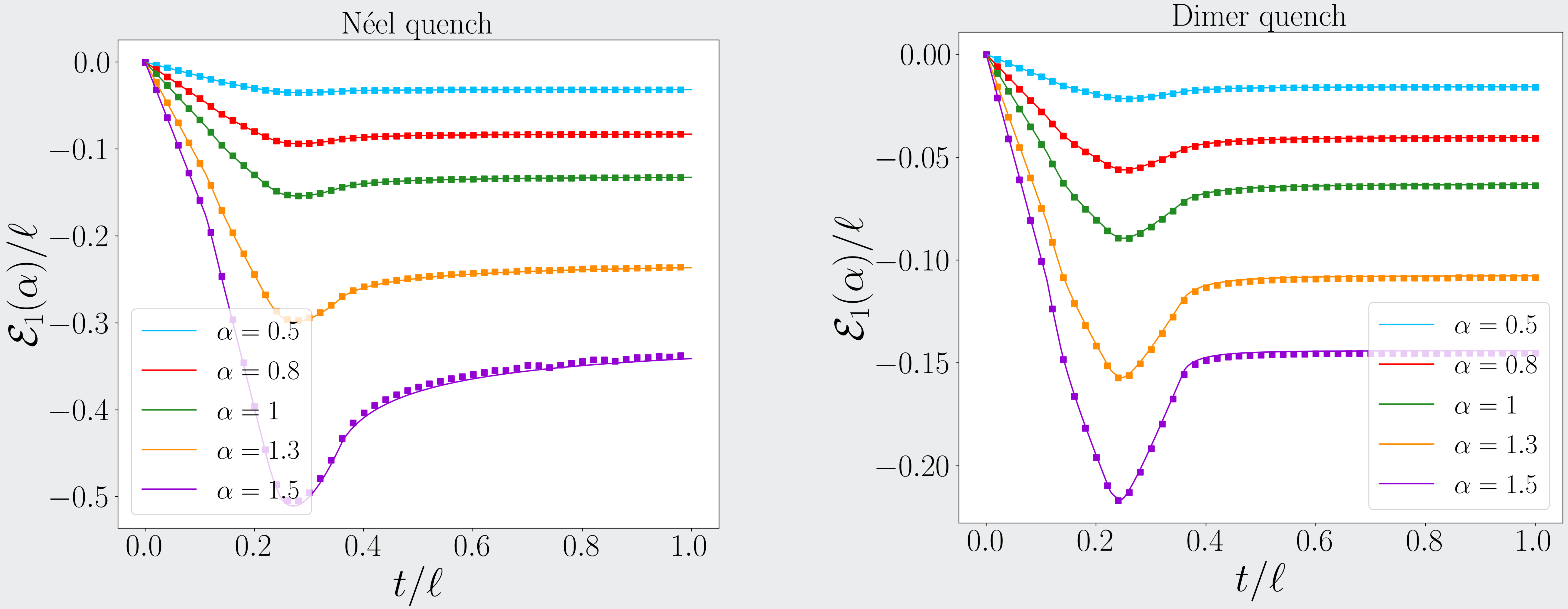
G. Parez, R. Bonsignori, P. Calabrese, *arXiv*:2202.05309

- Charged probability: $\mathscr{E}_1(\alpha) = \int \frac{dk}{2\pi} \mathrm{Re}[h_{1,\alpha}(x_k)](\min(\ell_1, 2v_k t) + \min(\ell_2, 2v_k t)) +$

$$- \int \frac{dk}{2\pi} \mathrm{Re}[h_{1,\alpha}(x_k) - \frac{1}{2} h_{1,2\alpha}(x_k)](\max(d, 2v_k t) + \max(d+\ell, 2v_k t) - \max(d+\ell_1, 2v_k t) - \max(d+\ell_2, 2v_k t))$$

where $h_{n,\alpha}(x) = \log\left[\left(\frac{1+x}{2}\right)^n e^{i\alpha} + \left(\frac{1-x}{2}\right)^n\right]$

$$\begin{cases} x_k^{Neel} = 0 \\ x_k^{Dimer} = \cos k \end{cases}$$



CHARGED RENYI LOGARITHMIC NEGATIVITIES

- Charged Renyi logarithmic negativities:

$$\mathscr{E}_n(\alpha) = \int \frac{dk}{2\pi} \mathrm{Re}[h_{n,\alpha}(x_k)](\min(\ell_1, 2v_k t) + \min(\ell_2, 2v_k t)) +$$

$$- \int \frac{dk}{2\pi} \mathrm{Re}[h_{n,\alpha}(x_k) - h^{(2)}_{n,\alpha}(x_k)](\max(d, 2v_k t) + \max(d+\ell, 2v_k t) - \max(d+\ell_1, 2v_k t) - \max(d+\ell_2, 2v_k t))$$

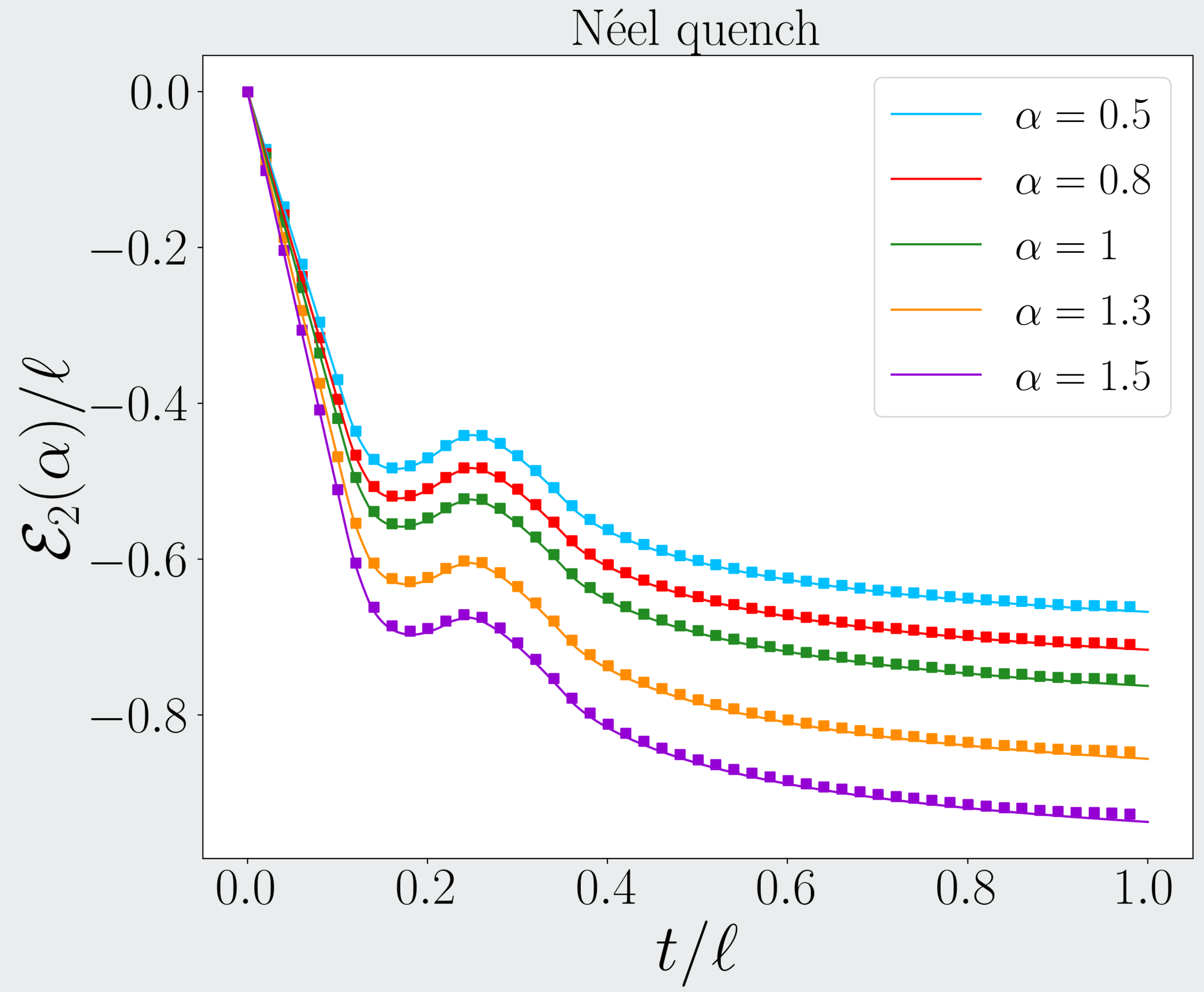
where

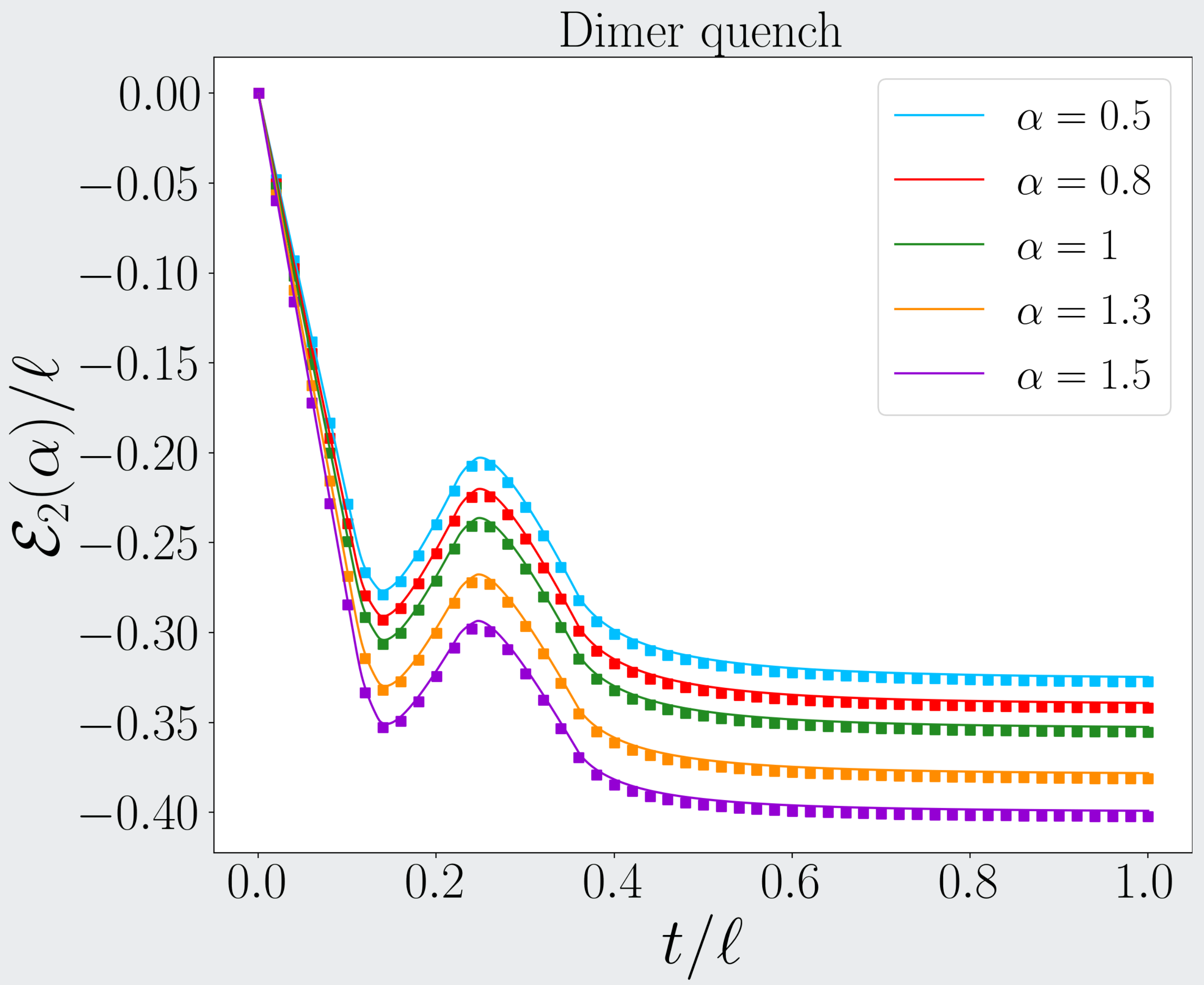
$$h^{(2)}_{n,\alpha}(x_k) = \begin{cases} \frac{1}{2} h_{n,2\alpha}(x_k), & \text{odd } n, \\ h_{\frac{n}{2},\alpha}(x_k), & \text{even } n\,. \end{cases}$$

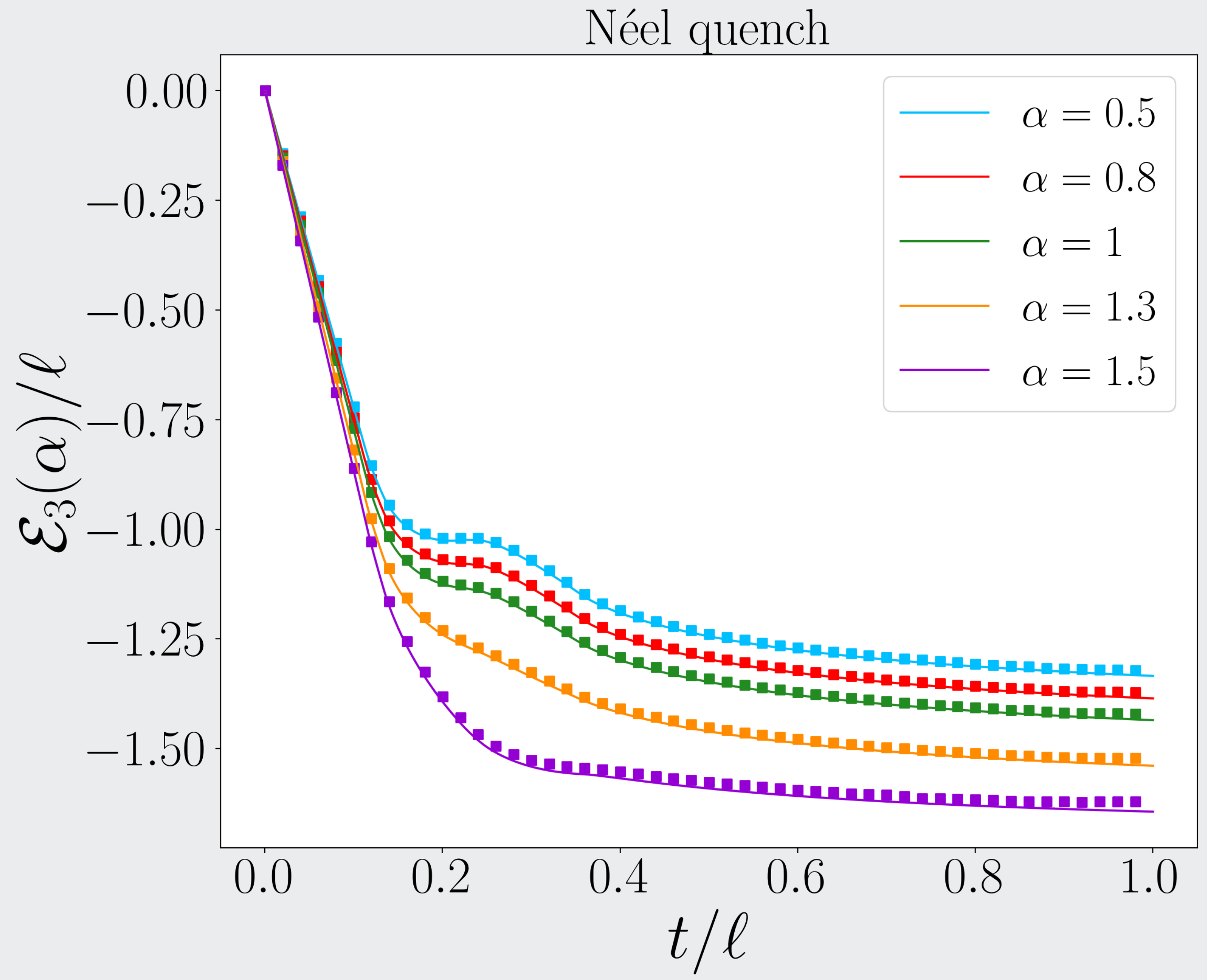
- For $n_{odd} \to 1$ → charged probability $\mathscr{E}_1(\alpha)$

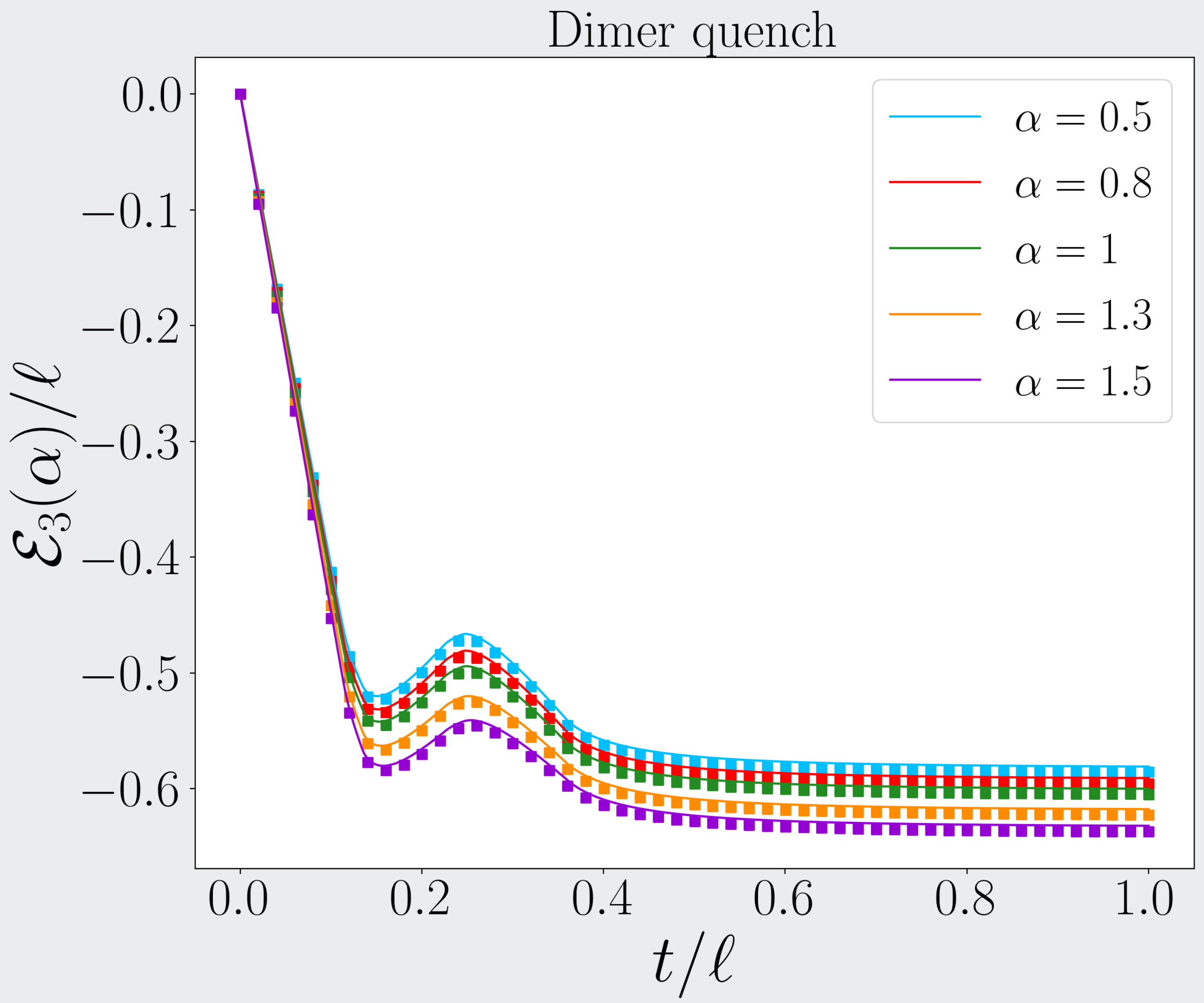
- For $n_{even} \to 1$ → charged logarithmic negativity $\mathscr{E}(\alpha)$

CHARGED RENYI LOGARITHMIC NEGATIVITIES

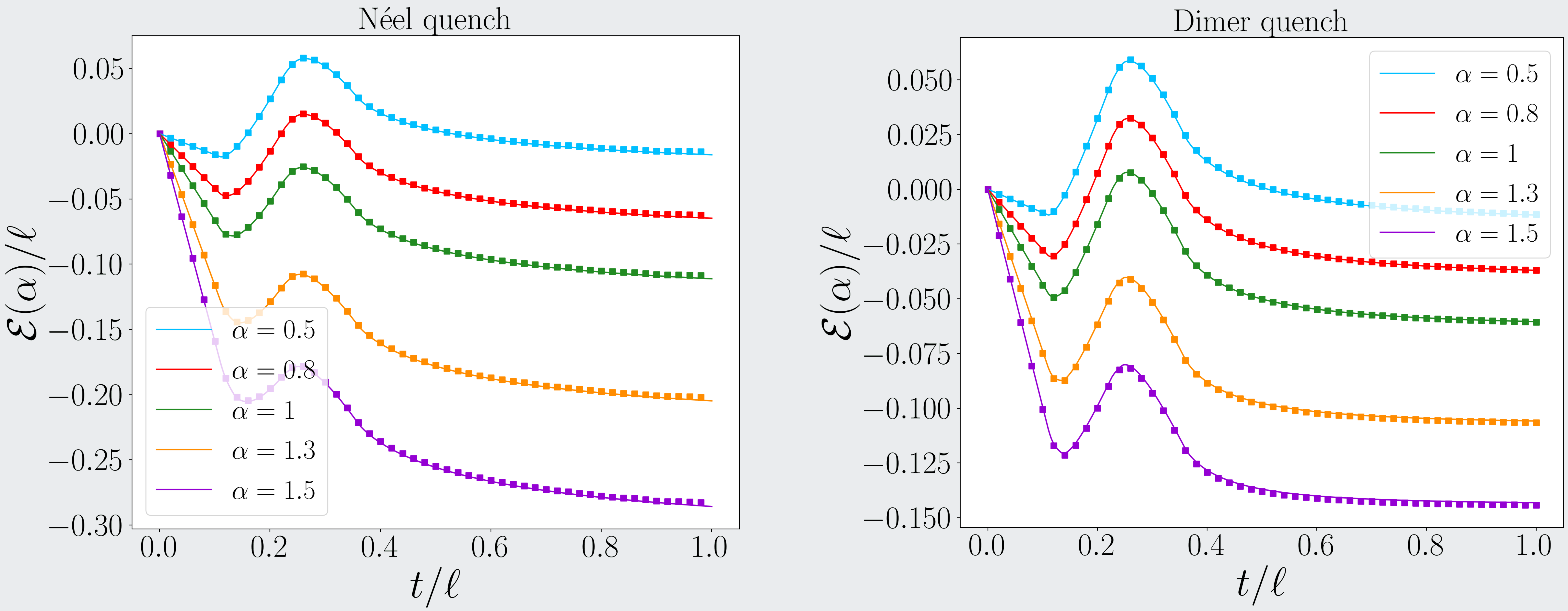

Néel quench


Dimer quench


Néel quench

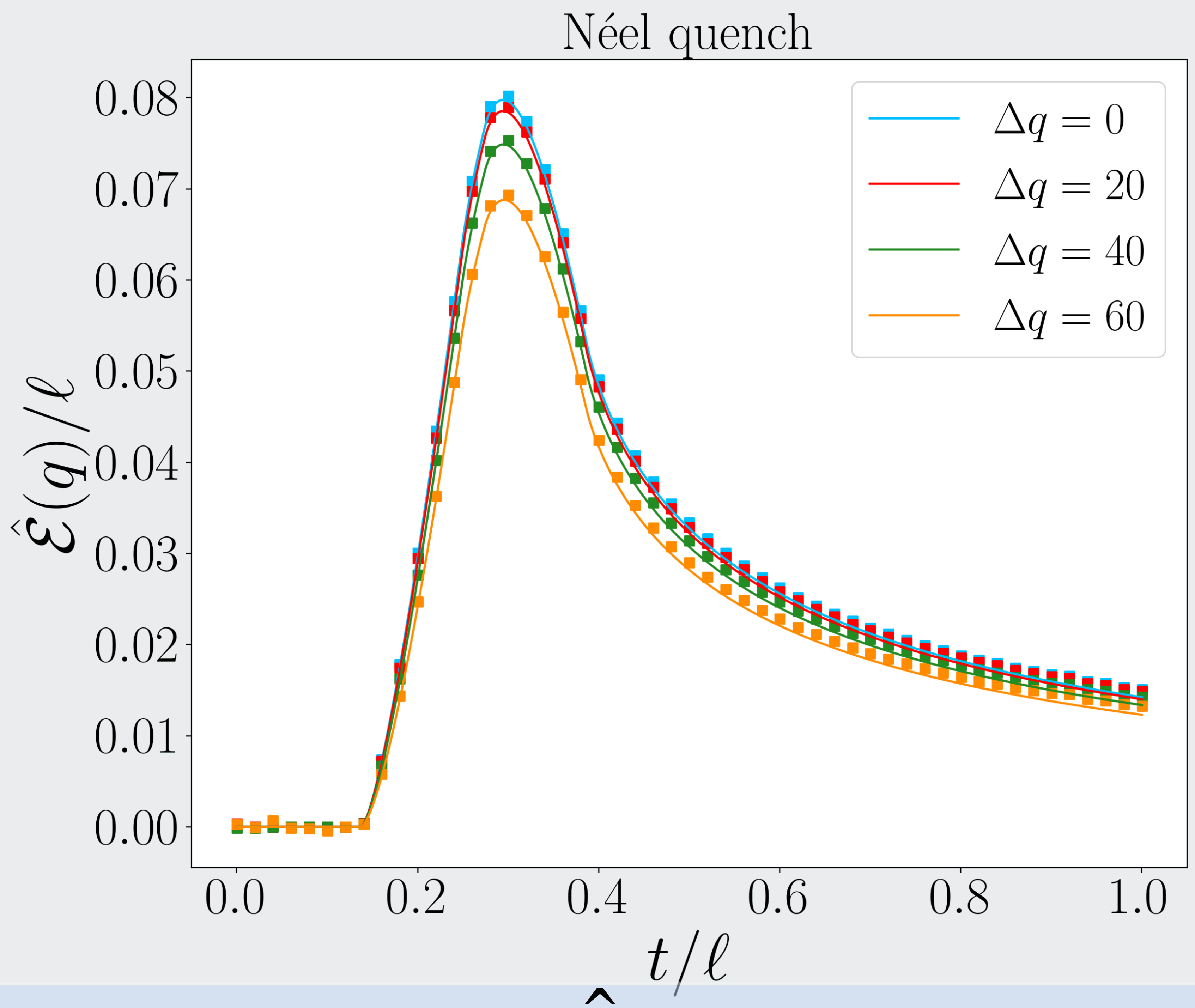

Dimer quench

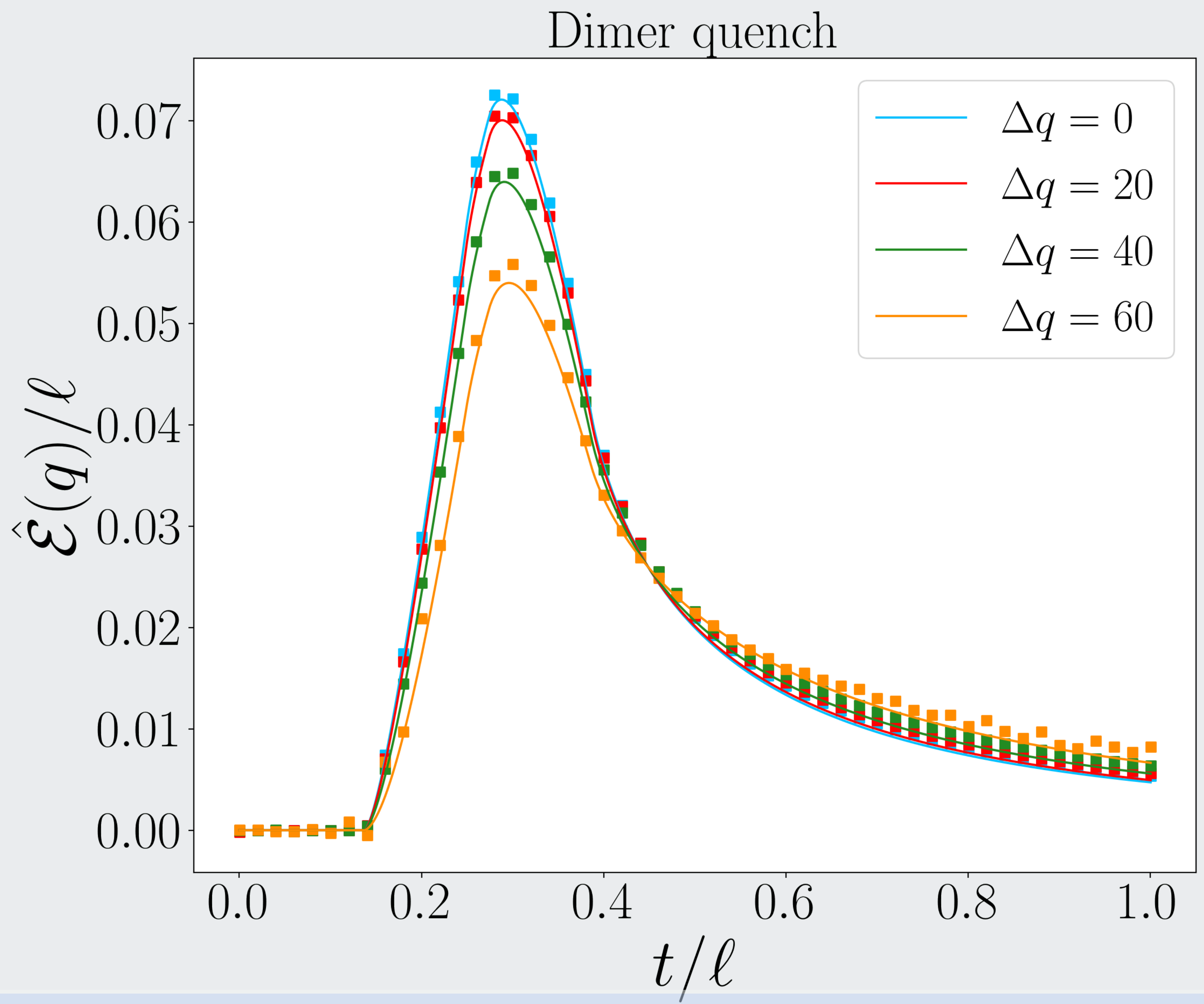
CHARGED LOGARITHMIC NEGATIVITY



CHARGE-IMBALANCE-RESOLVED LOGARITHMIC NEGATIVITY

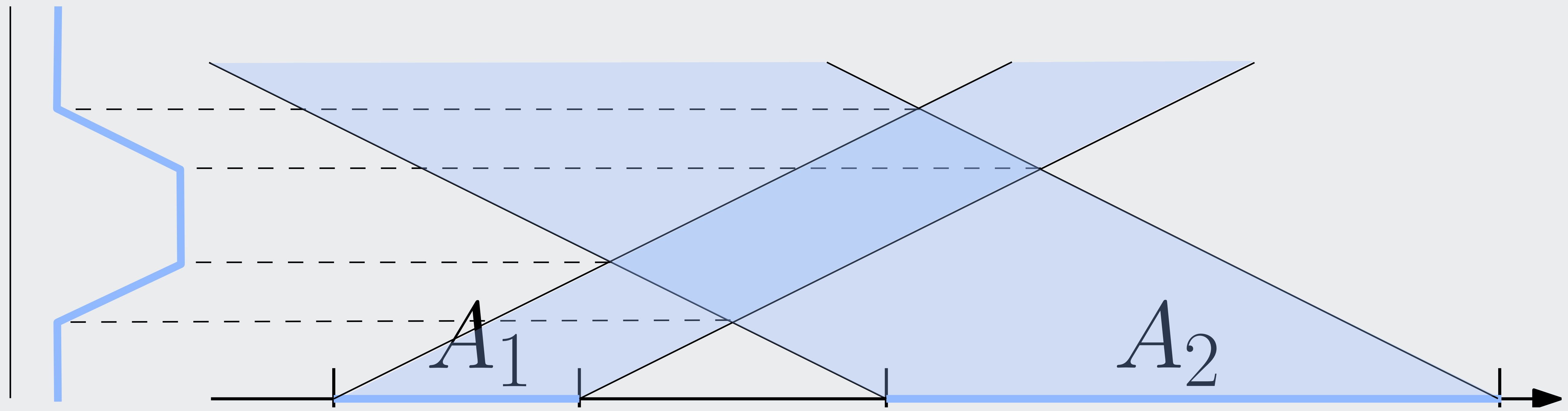
$$\hat{\mathcal{E}}(q) = \mathcal{E}(0) - 2\Delta q^2 \left(\frac{1}{\mathcal{J}^{(1)}_{A_1,A_2} - \mathcal{J}^{(1)}_{m} + \mathcal{J}^{(1/2)}_{m}} - \frac{1}{\mathcal{J}^{(1)}_{A_1,A_2} + \mathcal{J}^{(1)}_{m}} \right) + \frac{1}{2} \log \left(\frac{\mathcal{J}^{(1)}_{A_1,A_2} + \mathcal{J}^{(1)}_{m}}{\mathcal{J}^{(1)}_{A_1,A_2} - \mathcal{J}^{(1)}_{m} + \mathcal{J}^{(1/2)}_{m}} \right)$$





Equipartition of $\hat{\mathcal{E}}(q)$ *for early and large times, broken at order* $\Delta q^2/\ell$ *for intermediate times*

QUASIPARTICLE DYNAMICS FOR LOGARITHMIC NEGATIVITY



- The Quasiparticle prediction for the logarithmic negativity is*

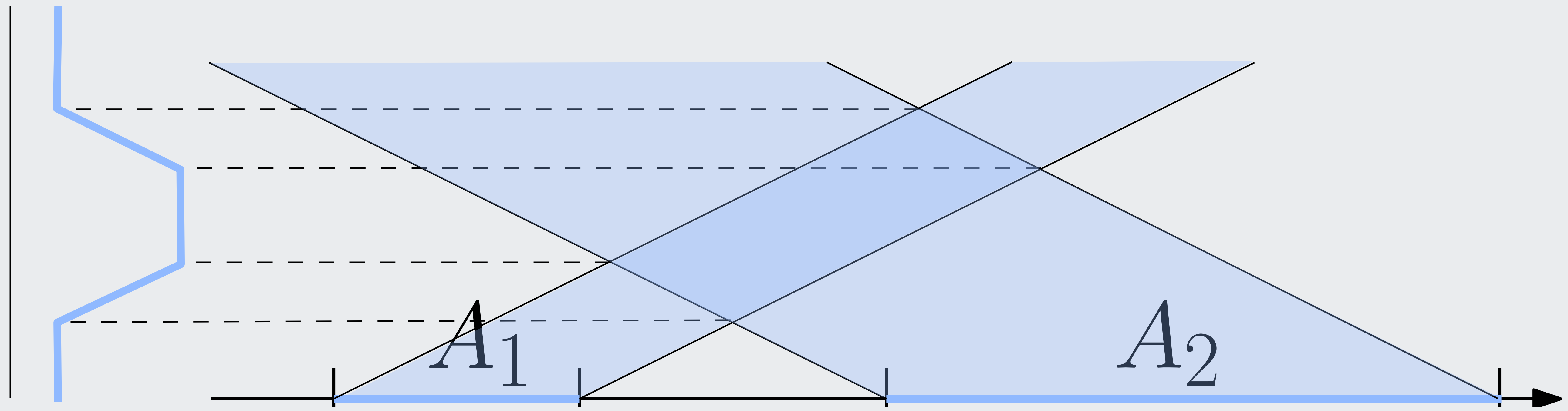
$$\mathscr{E} = \int dk\ \epsilon(k)(\max(d,2v_k t) + \max(d+\ell,2v_k t) - \max(d+\ell_1,2v_k t) - \max(d+\ell_2,2v_k t))$$

- For free-fermion models:

$$\epsilon(k) = h_{1/2,0}(2n_k - 1), \qquad n_k = \begin{cases} \frac{1}{2}, & \text{Neel quench,} \\ \frac{(1+\cos k)}{2}, & \text{Dimer quench} \end{cases}$$

*V. Alba, P. Calabrese, PNAS 114, 7947 (2017).

QUASIPARTICLE DYNAMICS FOR $\mathscr{E}_n$



$$\mathscr{E}_n = \int \frac{dk}{2\pi} \epsilon_n(k)(\min(\ell_1, 2v_k t) + \min(\ell_2, 2v_k t)) +$$

$$-\int \frac{dk}{2\pi}(\epsilon_n(k) - \epsilon_n^{(2)}(k))(\max(d, 2v_k t) + \max(d+\ell, 2v_k t) - \max(d+\ell_1, 2v_k t) - \max(d+\ell_2, 2v_k t))$$

$$\epsilon_n^{(2)}(k) = \begin{cases} \frac{1}{2}\epsilon_n(k), & \text{odd } n, \\ \epsilon_{\frac{n}{2}}(k), & \text{even } n \end{cases}$$

*S. Murciano, V. Alba, P. Calabrese, arXiv:2110.14589.

QUASIPARTICLE DYNAMICS FOR $\mathscr{E}_n(\alpha)$

$$\mathscr{E}_n(\alpha) = \int \frac{dk}{2\pi} \epsilon_{n,\alpha}(k)(\min(\ell_1, 2v_k t) + \min(\ell_2, 2v_k t) +$$

$$- \int \frac{dk}{2\pi} (\epsilon_{n,\alpha}(k) - \epsilon^{(2)}_{n,\alpha}(k))(\max(d, 2v_k t) + \max(d+\ell, 2v_k t) - \max(d+\ell_1, 2v_k t) - \max(d+\ell_2, 2v_k t))$$

$$\epsilon^{(2)}_{n,\alpha}(k) = \begin{cases} \frac{1}{2}\epsilon_{n,2\alpha}(k), & \text{odd } n, \\ \epsilon_{\frac{n}{2},\alpha}(k), & \text{even } n\,. \end{cases}$$

- *Our results for can be understood in the framework of the quasiparticle picture for the entanglement dynamics*
- *The conjecture is expected to hold for a large variety of integrable models*

CONCLUSIONS

- *The study of the symmetry resolution of the entanglement measures gives a deeper understanding of the entanglement dynamics of many-body quantum systems.*

- Dynamics of symmetry-resolved entanglement entropy and mutual information
  - Existence of delay time t_D
  - Equipartition for small $|\Delta q|$
- Dynamics of charge-imbalance-resolved negativity
  - Equipartition with violations of order $\Delta q^2/\ell$ at intermediate times
- Within the quasiparticle picture, we conjectured a general formula for the charged entropy and charged Renyi logarithmic negativities that is expected to hold for arbitrary integrable models and predicts:

THANK YOU!