XXX chain: spinons, bound states and form factors.

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XXZ spin chain

Defined on a one-dimensional lattice with M sites, with Hamiltonian, $H = H^{(0)} - hS_z$,

$$H^{(0)} = \sum_{m=1}^{M} \left\{ \sigma_m^x \sigma_{m+1}^x + \sigma_m^y \sigma_{m+1}^y + \Delta(\sigma_m^z \sigma_{m+1}^z - 1) \right\},$$

$$S_z = \frac{1}{2} \sum_{m=1}^{M} \sigma_m^z, \qquad [H^{(0)}, S_z] = 0.$$

$\sigma_m^{x,y,z}$ are the local spin operators (in the spin-$\frac{1}{2}$ representation) associated with each site m of the chain and $\Delta = \cos(\zeta)$, ζ real or imaginary, is the anisotropy parameter, h - external magnetic field; $h \geqslant 0$. We impose the **periodic** boundary conditions.

If $\Delta = 1$ XXX Heisenberg chain (1928), solved by H. Bethe (1931).

For $h = 0$ if $\Delta > 1$ - massive antiferromagnetic regime, $|\Delta| < 1$ - massless regime.

Form Factors

Form factors: matrix elements of **local fields**, local spin operators σ^a_m, $a = x, y, z$

$|\Psi_g\rangle$ the ground state of the model $|\Psi_e\rangle$ - an excited state

The most basic form factors

$$|\mathcal{F}_a(\Psi_e)|^2 = \frac{\langle\Psi_g|\,\sigma^a_m\,|\Psi_e\rangle\,\langle\Psi_e|\,\sigma^a_m\,|\Psi_g\rangle}{\langle\Psi_g\,|\Psi_g\rangle\,\langle\Psi_e\,|\Psi_e\rangle}$$

Then more advanced questions can be studied like matrix elements of **currents**

- Integrable QFT - F. Smirnov 1992 **bootstrap approach**
- Massive XXZ, M. Jimbo and T. Miwa 1995 q-**vertex operator** approach
- General XXZ, N.K, J.M. Maillet, V. Terras, 1999 **Algebraic Bethe ansatz** approach

Massless models: computation of the correlation functions through the form factors seems to be a strange idea (at least). However the integrable systems are very special!

- Dynamical correlation functions **at zero temperature**:

$$f_a(m,t) = \langle \sigma^a_{m+1}(t)\sigma^a_1(0)\rangle = \sum_{\Psi_e} \exp(it\Delta E_e - im\Delta p_e)\,|\mathcal{F}_a(\Psi_e)|^2$$

Turns out to be an excellent tool of **asymptotic analysis**.

- Dynamical structure factors:

$$S(k,\omega) = \int_{-\infty}^{\infty} dt \sum_{m=-\infty}^{\infty} f_a(m,t)\exp(imk - it\omega)$$

Experimentally mesurable quantity : can be computed **numerically** from the form factors (J.S. Caux et al.) and **asymptotically (edge exponents)**.

State of the art

Analytic computation of the form factors, two main approaches:

- **q-vertex operator approach:** Multiple integral representations for $h = 0,\ \Delta \geqslant 1$, infinite chain.
  - Advantages: **explicit results** for the simplest excited states
  - Disadvantages: No access to the **bound states**, difficulties to get past 4 spinons (XXX) or 2 spinons (XXZ).
- **Algebraic Bethe ansatz approach:** Determinant representations for the finite chain, all the regimes.
  - Advantages: Access to the **asymptotics**, all the regimes, possibility to treat **bound states**
  - Disadvantages: Final results always contain **Fredholm determinants**, difficulties to take the $h = 0$ limit.

The explicit results from the q-vertex operator approach were never reproduced from the ABA (with one exception: **spontaneous magnetisation**)

XXX chain: Algebraic Bethe ansatz

L.D. Faddeev, E.K. Sklyanin, L.A. Takhtajan (1979). Main object: quantum monodromy matrix:

$$T_a(\lambda) = \begin{pmatrix} A(\lambda) & B(\lambda) \\ C(\lambda) & D(\lambda) \end{pmatrix}_a.$$

- Diagonal elements $\longrightarrow$ commuting conserved charges: transfer matrix

$$\mathcal{T}(\lambda) = \mathrm{tr}_a\, T_a(\lambda) = A(\lambda) + D(\lambda), \qquad [\mathcal{T}(\lambda), \mathcal{T}(\mu)] = 0$$

- Hamiltonian:

$$H = 2i \frac{\partial}{\partial\lambda} \log \mathcal{T}(\lambda)\Big|_{\lambda=\frac{i}{2}}, \qquad [H, \mathcal{T}(\lambda)] = 0$$

- Non-diagonal elements $\longrightarrow$ creation/annihilation operators.

Bethe states

Off-shell Bethe states:

$$|\Psi(\{\lambda_1,\ldots,\lambda_N\})\rangle = B(\lambda_1)\ldots B(\lambda_N)\,|0\rangle\,, \qquad |0\rangle = |\uparrow\uparrow\ldots\uparrow\rangle$$

For any Bethe state we define **Baxter polynomial** and **exponential counting function**

$$q(\lambda)=\prod_{j=1}^{N}(\lambda-\lambda_j), \qquad \mathfrak{a}(\lambda)=\left(\frac{\lambda-\frac{i}{2}}{\lambda+\frac{i}{2}}\right)^{M}\frac{q(\lambda+i)}{q(\lambda-i)}.$$

if the **Bethe equations** are satisfied (on-shell Bethe state)

$$\mathfrak{a}(\lambda_j)+1=0, \qquad j=1,\ldots N$$

then it is an eigenstate of the **transfer matrix** and the Hamiltonian

$$\mathcal{T}(\mu)\,|\Psi(\{\lambda\})\rangle=\tau(\mu)\,|\Psi(\{\lambda\})\rangle\,, \qquad \tau(\mu)=\big(\mathfrak{a}(\mu)+1\big)\frac{q(\mu-i)}{q(\mu)}.$$

Multiplet structure

XXX chain: additional $\mathfrak{su}(2)$ symmetry:

$$[\mathcal{T}(\lambda), S_a] = 0, \qquad a = x, y, z.$$

On-shell Bethe vectors are $\mathfrak{su}(2)$ highest weight vectors

$$S_+ \left|\Psi(\{\lambda\})\right\rangle = 0, \qquad S_+ = \sum_{m=1}^{M} \sigma_m^+.$$

For XXX there are solutions of Bethe equations only if $N \leqslant \frac{M}{2}$. For $N = \frac{M}{2} - k$ they generate $2k + 1$ multiplets

$$\left|\Psi_\ell(\{\lambda\})\right\rangle = S_-^\ell \left|\Psi(\{\lambda\})\right\rangle, \quad \ell = 0, \ldots, 2k, \quad \mathcal{T}(\mu) \left|\Psi_\ell(\{\lambda\})\right\rangle = \tau(\mu) \left|\Psi_\ell(\{\lambda\})\right\rangle$$

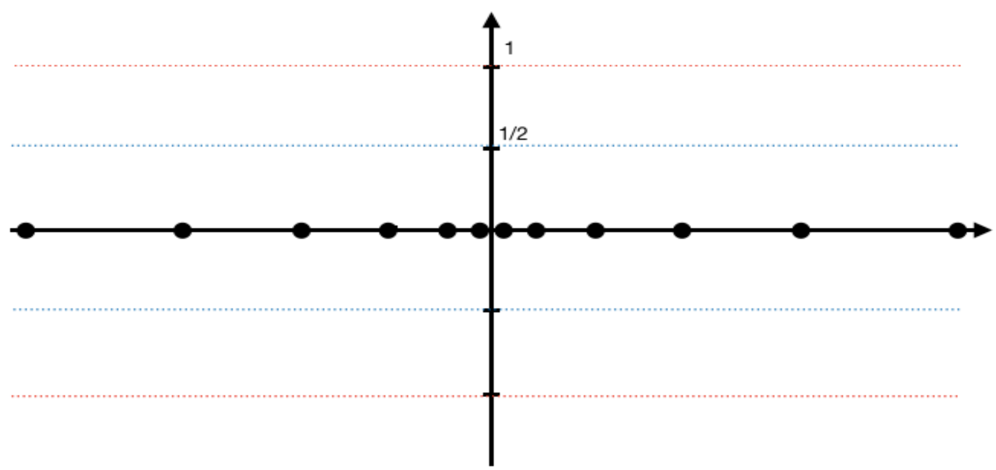
Multiplets can be seen as **Bethe states** with infinite rapidities $\lim_{\lambda\to\infty} \lambda B(\lambda) = S_-$.

The ground state

Ground state solution of the Bethe equations

$$\mathfrak{a}(\lambda_j) + 1 = 0, \qquad j = 1, \dots N$$

Yang and Yang 66: $N = \frac{M}{2}$ (singlet), all the roots are **real**. There is **no holes** *i.e.* all the real zeroes of $\mathfrak{a}_g(\lambda) + 1$ are Bethe roots.



The ground state density

The Bethe roots fill the real line with some density in the **thermodynamic limit**:

$$\frac{1}{M}\sum_{j=1}^{\frac{M}{2}} f(\lambda_j) = \int_{-\infty}^{\infty} f(\lambda)\rho(\lambda)d\lambda + o\left(\frac{1}{M}\right).$$

The ground state density solves the Lieb equation

$$\rho_g(\lambda) + \frac{1}{2\pi i}\int_{-\infty}^{\infty} K(\lambda-\mu)\rho_g(\mu)d\mu = \frac{1}{2\pi i}t(\lambda - i/2),$$

Where $t(\lambda) = \frac{i}{\lambda(\lambda+i)}$ and $K(\lambda) = t(\lambda) + t(-\lambda)$.

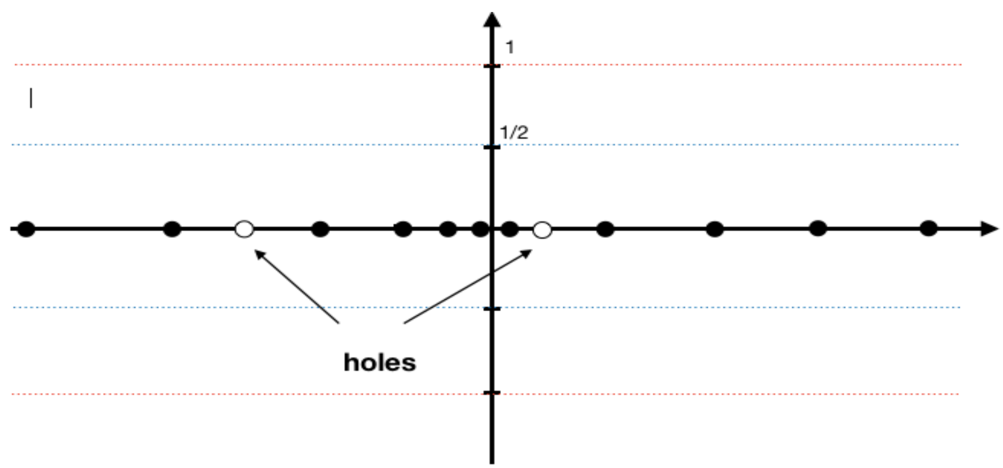
The ground state solution:

$$\rho(\lambda) = \frac{1}{2\,\cosh(\pi\lambda)}.$$

Excitations: spinons

Holes (spinons) μ_h is not a **Bethe root** but:

$$\mathfrak{a}_e(\mu_h) + 1 = 0$$



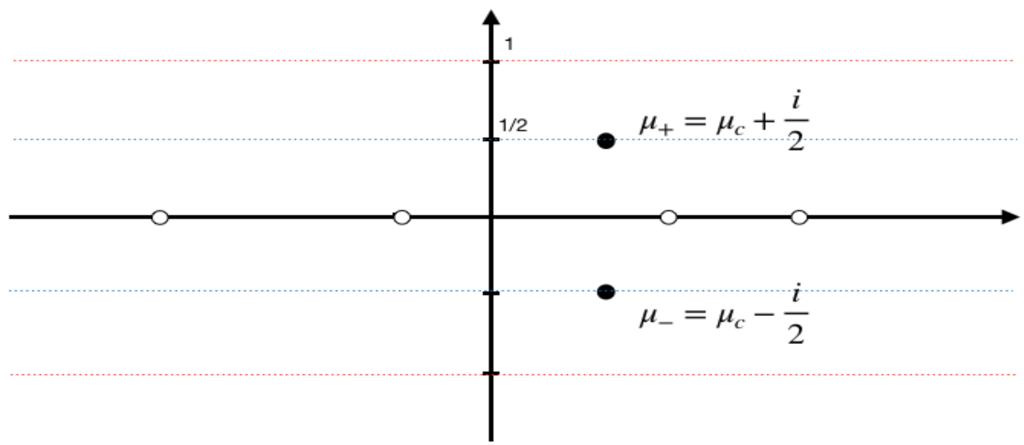
Number of holes n_h is always even, $N = \frac{M}{2} - \frac{n_h}{2}$ if all roots are real.

Complex roots: strings

Complex roots: if μ_+ is one of the Bethe roots then $\mu_- = \overline{\mu}_+$ is also a root (bound state). For a finite chain with large M the simplest configuration: **2-string**:

$$\mu_+ = \mu_c + \frac{i}{2} - i\delta, \quad \mu_- = \mu_c - \frac{i}{2} + i\delta,$$

Where $\mu_c \in \mathbb{R}$ - string center and $\delta = O(M^{-\infty})$ - string deviation.

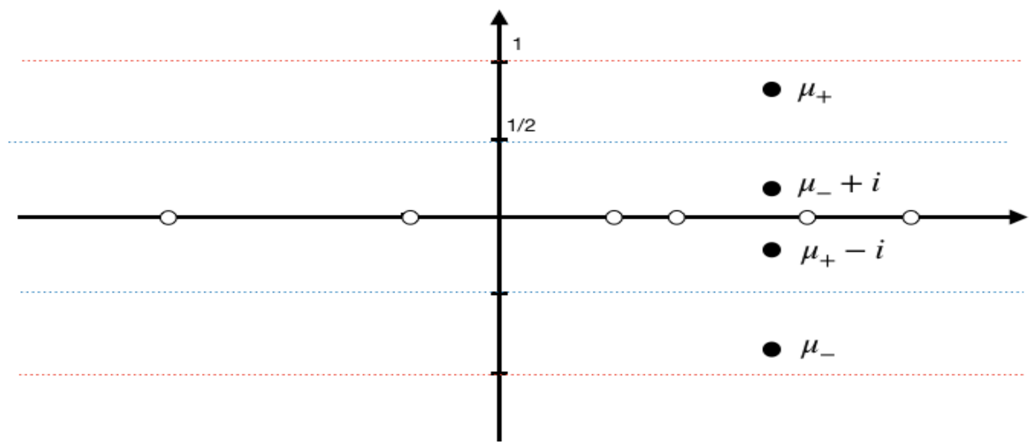


Complex roots: close pairs and quartets

Destri, Lowenstein 1982; Babelon, de Vega, Viallet 1983.

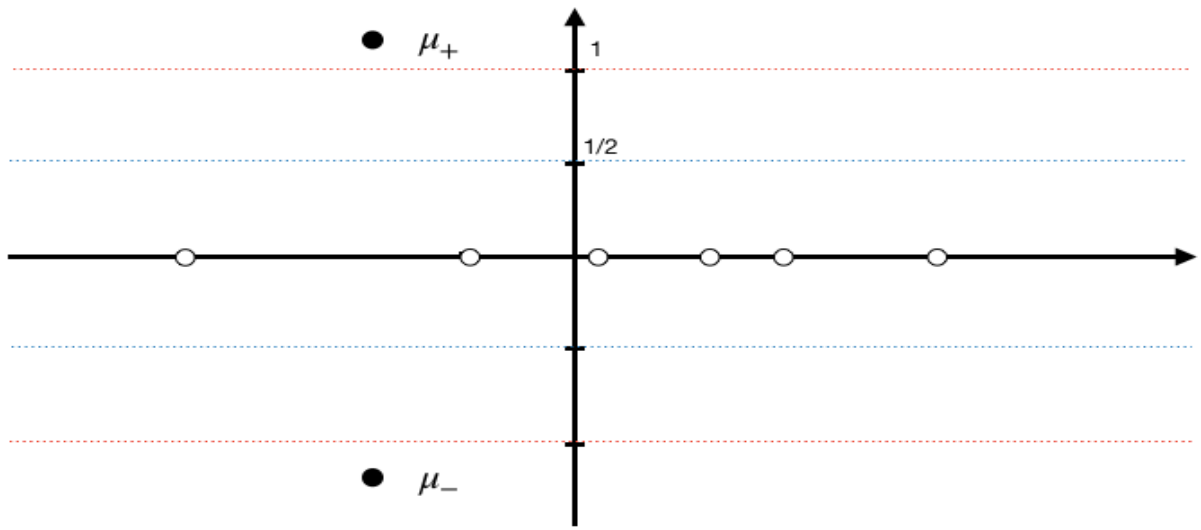
Close pair: $\mu_+, \mu_-, \quad 0 < \Im(\mu_+) < 1$. Note: **2-string** is a close pair. Otherwise close pairs form quartets:

$$\mu_+, \quad \mu_- + i - i\delta, \quad \mu_+ - i + i\delta, \quad \mu_-.$$



Complex roots: wide pairs

Wide pair: $\mu_+, \mu_-, \quad \Im(\mu_+) > 1$.



Excited states

We denote n_h - number of **holes** (even), n_s - number of **2-strings**, n_q - number of **quartets**, n_w - number of **wide pairs**. Total number of Bethe roots

$$N = \frac{M}{2} - \frac{n_h}{2} + n_s + 2n_q + 2n_w, \quad N \leqslant \frac{M}{2}$$

. Positions of holes $\mu_{h_a} \in \mathbb{R}$, $a = 1, \ldots n_h$ (arbitrary in the thermodynamic limit). The position of complex roots are defined from the position of holes from the **higher level Bethe equations** Destri, Lowenstein 1982; Babelon, de Vega, Viallet 1983.

$$\prod_{a=1}^{n_h} \frac{z_j - \mu_{h_a} - \frac{i}{2}}{z_j - \mu_{h_a} + \frac{i}{2}} \prod_{k=1}^{n_c} \frac{z_j - z_k + i}{z_j - z_k - i} + 1 = 0, \quad j = 1, \ldots, n_c.$$

here $n_c = n_s + 2n_q + 2n_w$,

$z_j = \mu_c$ for a **2 string**

$z_j = \mu_+ - \frac{i}{2}, \quad z_{j+1} = \mu_- + \frac{i}{2}$ for a **quartet** and a **wide pair**.

Energy and momentum

The complex roots don't influence **energy and momentum**, they depend only on hole positions (spinon rapidities).

$$\Delta E \equiv E_e - E_g = \sum_{a=1}^{n_h} \varepsilon(\mu_{h_a}), \qquad \varepsilon(\mu) = \frac{\pi}{2\cosh \pi\mu},$$

$$\Delta P \equiv P_e - P_g = \sum_{a=1}^{n_h} p(\mu_{h_a}), \qquad p(\mu) = \frac{\pi}{2} - \arctan(\sinh \pi\mu).$$

With fixed **spinon rapidities** 2^{n_h}-fold degeneracy.

Example: **two-spinon sector**, 4-fold degenerate:

- singlet with two holes μ_{h_1}, μ_{h_2} and one 2-string $\mu_c + \frac{i}{2}, \mu_c - \frac{i}{2}$, higher level Bethe equations have only one solution: $\mu_c = \frac{1}{2}(\mu_{h_1} + \mu_{h_2})$
- triplet with two holes μ_{h_1}, μ_{h_2} and no complex roots

Form factors and multiplet structure

We want to compute:

$$|\mathcal{F}_z|^2 = \frac{\langle \Psi_e \,|\, \sigma_m^z \,|\, \Psi_g \rangle \, \langle \Psi_g \,|\, \sigma_m^z \,|\, \Psi_e \rangle}{\langle \Psi_g \,|\, \Psi_g \rangle \, \langle \Psi_e \,|\, \Psi_e \rangle},$$

for XXX no difference between x, y or z form factors.

It is easy to see from $\sigma_m^z = [S^+, \sigma_m^-]$ that there are non-trivial form factors **only** for the **triplet** states ($N = \frac{M}{2} - 1$) and

$$\begin{aligned}
\left\langle \Psi_{e_1} \,\middle|\, \sigma_m^z \,\middle|\, \Psi_g \right\rangle &= -2 \left\langle \Psi_{e_0} \,\middle|\, \sigma_m^+ \,\middle|\, \Psi_g \right\rangle, \\
\left\langle \Psi_g \,\middle|\, \sigma_m^z \,\middle|\, \Psi_{e_1} \right\rangle &= \left\langle \Psi_g \,\middle|\, \sigma_m^+ \,\middle|\, \Psi_{e_2} \right\rangle, \\
\left\langle \Psi_{e_1} \,\middle|\, \Psi_{e_1} \right\rangle &= 2 \left\langle \Psi_{e_0} \,\middle|\, \Psi_{e_0} \right\rangle.
\end{aligned}$$

Form factors and inverse problem

Quantum inverse problem: local operators in terms of the monodromy matrix elements N.K., J.M. Maillet and V. Terras 1999:

$$\begin{aligned}
\sigma_m^z &= \mathcal{T}^{m-1}\left(\tfrac{i}{2}\right)\left\{A\left(\tfrac{i}{2}\right) - D\left(\tfrac{i}{2}\right)\right\}\mathcal{T}^{-m}\left(\tfrac{i}{2}\right),\\
\sigma_m^- &= \mathcal{T}^{m-1}\left(\tfrac{i}{2}\right) B\left(\tfrac{i}{2}\right)\mathcal{T}^{-m}\left(\tfrac{i}{2}\right),\\
\sigma_m^+ &= \mathcal{T}^{m-1}\left(\tfrac{i}{2}\right) C\left(\tfrac{i}{2}\right)\mathcal{T}^{-m}\left(\tfrac{i}{2}\right).
\end{aligned}$$

Due to the commutation relations of the monodromy matrix elements everything expressed in terms of **scalar products** of off-shell and on-shell **multiplet** Bethe states.

$$|\mathcal{F}_z|^2 = -\frac{\tau_e(\frac{i}{2})}{\tau_g(\frac{i}{2})}\frac{\left\langle \Psi_{e_0} \,\middle|\, C(\frac{i}{2}) \,\middle|\, \Psi_g \right\rangle \left\langle \Psi_g \,\middle|\, C(\frac{i}{2}) \,\middle|\, \Psi_{e_2} \right\rangle}{\left\langle \Psi_g \,\middle|\, \Psi_g \right\rangle \left\langle \Psi_{e_0} \,\middle|\, \Psi_{e_0} \right\rangle},$$

Scalar products and norms

N. Slavnov, 1989: $\{\lambda_1, \ldots \lambda_N\}$ - solution of Bethe equations, $\{\mu_1, \ldots \mu_N\}$ - generic set of parameters.

$$\langle \Psi(\{\mu\}) \,|\Psi(\{\lambda\})\rangle = \frac{\prod_{k=1}^{N} q(\mu_k - i)}{\prod_{j>k}(\lambda_j - \lambda_k)(\mu_k - \mu_j)} \det_N \mathcal{M}(\{\lambda\}|\{\mu\}),$$

$$\mathcal{M}_{j,k}(\{\lambda\}|\{\mu\}) = \mathfrak{a}(\mu_k)t(\mu_k - \lambda_j) - t(\lambda_j - \mu_k), \quad t(\lambda) = \frac{i}{\lambda(\lambda + i)}.$$

Norms of the on-shell Bethe states are given by the Gaudin formula

$$\langle \Psi(\{\lambda\}) \,|\Psi(\{\lambda\})\rangle = (-1)^N \frac{\prod_{j=1}^{N} q(\lambda_j - i)}{\prod_{j\neq k}(\lambda_j - \lambda_k)} \det \mathcal{N}(\{\lambda\}),$$

$$\mathcal{N}_{j,k}(\{\lambda\}) = \mathfrak{a}'(\lambda_j)\delta_{j,k} - K(\lambda_j - \lambda_k), \quad K(\lambda) = t(\lambda) + t(-\lambda).$$

Slavnov formula can be adapted for the **multiplet states** Foda, Wheeler 2012.

Finite chain determinant representation for the factors form factors corresponding to arbitrary triplet excited state:

$$|\mathcal{F}_z|^2 = -2 \prod_{j=1}^{\frac{M}{2}-1} \frac{q_g(\mu_j - i)}{q_e(\mu_j - i)} \prod_{k=1}^{\frac{M}{2}} \frac{q_e(\lambda_k - i)}{q_g(\lambda_k - i)}$$

$$\times \frac{\det_{\frac{M}{2}} \mathcal{M}(\{\lambda\}|\{\mu_1 \dots \mu_{\frac{M}{2}-1}, \frac{i}{2}\}) \det_{\frac{M}{2}+1} \mathcal{M}^{(2)}(\{\mu\}|\{\lambda_1, \dots \lambda_{\frac{M}{2}}, \frac{i}{2}\})}{\det_{\frac{M}{2}} \mathcal{N}(\{\lambda\}) \det_{\frac{M}{2}-1} \mathcal{N}(\{\mu\})}.$$

Here $\mathcal{M}^{(2)}$ - Foda Wheeler variant of the Slavnov matrix for triplets (two extra rows).

$$\mathcal{M}^{(\ell)}_{j,k}(\{\lambda\}|\{\mu\}) = \mathfrak{a}(\mu_k)(\mu_k + i)^{j-N-1} - \mu_k^{j-N-1}, \qquad \text{for } j > N.$$

Computation of determinants

The main idea is extremely simple: we compute the following matrices

$$F_g = \mathcal{N}^{-1}(\{\lambda\})\mathcal{M}\left(\{\lambda\}|\{\mu_1 \dots \mu_{\frac{M}{2}-1}, \frac{i}{2}\}\right),$$
$$F_e = \mathcal{N}^{(2)^{-1}}(\{\mu\})\mathcal{M}^{(2)}\left(\{\mu\}|\{\lambda_1, \dots \lambda_{\frac{M}{2}}, \frac{i}{2}\}\right),$$

For the first (ground state) matrix: system of linear equations

$$\mathfrak{a}_g'(\lambda_j)F_{g_{j,k}} - \sum_{a=1}^{\frac{M}{2}} K(\lambda_j - \lambda_a)F_{g_{a,k}} = \mathfrak{a}_g(\mu_k)t(\mu_k - \lambda_j) - t(\lambda_j - \mu_k).$$

We set

$$\mathfrak{a}_g'(\lambda_j)F_{g_{j,k}} = G_g(\lambda_j; \mu_k)$$

Linear equations $\longrightarrow$ Contour integral equation for $G_g(\lambda;\mu)$

$$G_g(\lambda;\mu_k) - \frac{1}{2\pi i}\oint\limits_{\Gamma} d\nu\, K(\lambda-\nu)\frac{G_g(\nu;\mu_k)}{1+\mathfrak{a}_g(\nu)} = (\mathfrak{a}_g(\mu_k)+1)t(\mu_k-\lambda),$$

We set

$$G_g(\lambda;\mu) = \big(1+\mathfrak{a}_g(\mu)\big)\rho_g(\lambda;\mu)$$

.

Thermodynamic limit $\longrightarrow$ Integral equation

$$\rho_g(\lambda;\mu) - \frac{1}{2\pi i}\int\limits_{\mathbb{R}+i\epsilon} d\nu\, K(\lambda-\nu)\rho_g(\nu;\mu) = t(\mu-\lambda).$$

Lieb equation for the density of Bethe roots!

Solution:

$$F_{g\,j,k} = \frac{\mathfrak{a}_g(\mu_k)+1}{\mathfrak{a}'_g(\lambda_j)}\frac{\pi}{\sinh\pi(\mu_k-\lambda_j)}$$

F_e is slightly more complicated (holes contributions, Foda-Wheeler rows) but also has this basic Cauchy structure.

Example without complex roots:

$$F_{e_{j,k}} = \frac{\mathfrak{a}_e(\lambda_k)+1}{\mathfrak{a}'_e(\mu_j)} \left(\frac{\pi}{\sinh \pi(\lambda_k - \mu_j)} - 2\pi i \sum_{a=1}^{n_h} \frac{\rho_h(\mu_j - \mu_{h_a})}{\mathfrak{a}'_e(\mu_{h_a})} \frac{\pi}{\sinh \pi(\lambda_k - \mu_{h_a})} \right), \qquad j \leqslant \frac{M}{2} - 1$$

$$F_{e_{\frac{M}{2},k}} = \mathfrak{a}_e(\lambda_k) - 1, \qquad F_{e_{\frac{M}{2}+1,k}} = \mathfrak{a}_e(\lambda_k)(\lambda_k + i) - \lambda_k.$$

Composed only of **Cauchy columns**! Very similar structure with the complex roots.

Structure of results

After this step typically the form factor is written as a product of two **Cauchy** determinants and determinants of a $n_h \times n_h$ matrix and a $n_c \times n_c$ matrix.

$$|\mathcal{F}_z|^2 = -2 \prod_{j=1}^{M/2} \frac{\tau_e(\lambda_j)}{\tau_g(\lambda_j)} \prod_{k=1}^{M/2-1} \frac{\tau_g(\mu_k)}{\tau_e(\mu_k)} \det_{\frac{M}{2}} \mathcal{C}_g \det_{\frac{M}{2}+1} \mathcal{C}_e$$

$$\times \frac{\prod_{j=1}^{M/2} \prod_{k=1}^{M/2-1} (\lambda_j - \mu_k)^2}{\prod_{j\neq k}^{M/2} (\lambda_j - \lambda_k) \prod_{j\neq k}^{M/2-1} (\mu_j - \mu_k)} \det_{n_c} \mathcal{Q}_g \det_{n_h} \mathcal{Q}_e$$

,

$\mathcal{C}_{jk} = \frac{\pi}{\sinh \pi(\mu_k - \lambda_j)}$ - **Cauchy determinant** (can be computed)

Thermodynamic limit : 2-spinon case

Two-spinon form factor: no complex roots, the only "extra" matrix

$$\mathcal{Q}_e = \begin{pmatrix} \frac{1}{\mathfrak{a}'_e(\mu_{h_1})} & \frac{1}{\mathfrak{a}'_e(\mu_{h_2})} \\ \frac{\mu_{h_1}+\frac{i}{2}}{\mathfrak{a}'_e(\mu_{h_1})} & \frac{\mu_{h_2}+\frac{i}{2}}{\mathfrak{a}'_e(\mu_{h_2})} \end{pmatrix}, \qquad \det \mathcal{Q}_e = \frac{\mu_{h_1}-\mu_{h_2}}{\pi^2 M^2} \prod_{a=1}^{2} \cosh \pi \mu_{h_a}$$

With expected scaling $\frac{1}{M^2}$. Final result for the scaled form factor:

$$|\mathcal{Y}(\mu_{h_1}-\mu_{h_2})|^2 = \lim_{M\to\infty} M^2 |\mathcal{F}_z|^2 = \frac{2}{G^4\left(\frac{1}{2}\right)} \left| \frac{G\left(\frac{\mu_{h_1}-\mu_{h_2}}{2i}\right) G\left(1+\frac{\mu_{h_1}-\mu_{h_2}}{2i}\right)}{G\left(\frac{1}{2}+\frac{\mu_{h_1}-\mu_{h_2}}{2i}\right) G\left(\frac{3}{2}+\frac{\mu_{h_1}-\mu_{h_2}}{2i}\right)} \right|^2 .$$

Where $G(z)$ iz the Barnes G-function (related to the double Γ-function).

$$G(z+1) = \Gamma(z)G(z), \qquad G(1) = 1.$$

Relation with q-vertex operator approach

Using integral representations for $\log G(z)$ we obtain

$$|\mathcal{Y}(\mu_{h_1} - \mu_{h_2})|^2 = 2e^{-I(\mu_{h_1} - \mu_{h_2})},$$

$$I(\mu_{h_1} - \mu_{h_2}) = \int_0^\infty \frac{dt}{t} e^t \frac{\cos\left(2(\mu_{h_1} - \mu_{h_2})t\right)\cosh(2t) - 1}{\cosh(t)\sinh(2t)}.$$

This reproduces the result for the two-spinon form factor obtained in the q-vertex operator framework from the M. Jimbo and T. Miwa multiple integral formulas by A. H. Bougourzi, M. Couture and M. Kacir

Starting from **four-spinon** case the things become more interesting as we use a different basis with respect to the Jimbo-Miwa approach.

General case

General excited state with n_h **spinons** and $n_c = n_s + 2n_q + 2n_w$ **complex roots**

General representation: pre-factor (Cauchy determinants) and finite size determinant

$$\lim_{M\to\infty} M^{n_h} |\mathcal{F}_z|^2 = \prod_{j>k} |\mathcal{Y}(\mu_{h_j} - \mu_{h_k})|^2 \, \mathcal{R}(\mu_h, \mu_c) \det_{n_c} \widehat{\mathcal{Q}_g} \det_{n_h} \widehat{\mathcal{Q}_e}.$$

Here $\mathcal{R}(\mu_h, \mu_c)$ - rational function of holes and complex roots
$\widehat{\mathcal{Q}_g}$ and $\widehat{\mathcal{Q}_e}$ - finite matrices also written in terms of positions of **holes** and **complex roots**. Includes also a **Higher level Gaudin matrix**

Simpler form for finite matrices: work in progress.

Interesting interplay with Jimbo-Miwa-Smirnov fermionic approach.

Conclusion and outlook

Advantages of the new approach:

- Explicit results, no Fredholm determinants.
- We know how to deal with bound states
- Possibility to apply in a systematic way for all the regimes of the XXZ chain

Open problems: out-of-equilibrium systems, overlaps instead of form factors

- Second densification (holes, complex roots distributed with some density)
- Can we apply this method far from the **ground state**?
- Macroscopic changes in the system (**quenches**).