Rigorous results on a frustration-free quantum fully packed loop model

Zhao Zhang (SISSA)

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NORDITA
The Nordic Institute for Theoretical Physics

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Manuscript in preparation with Henrik S. Røising (Nordita -> Niels Bohr Institute)

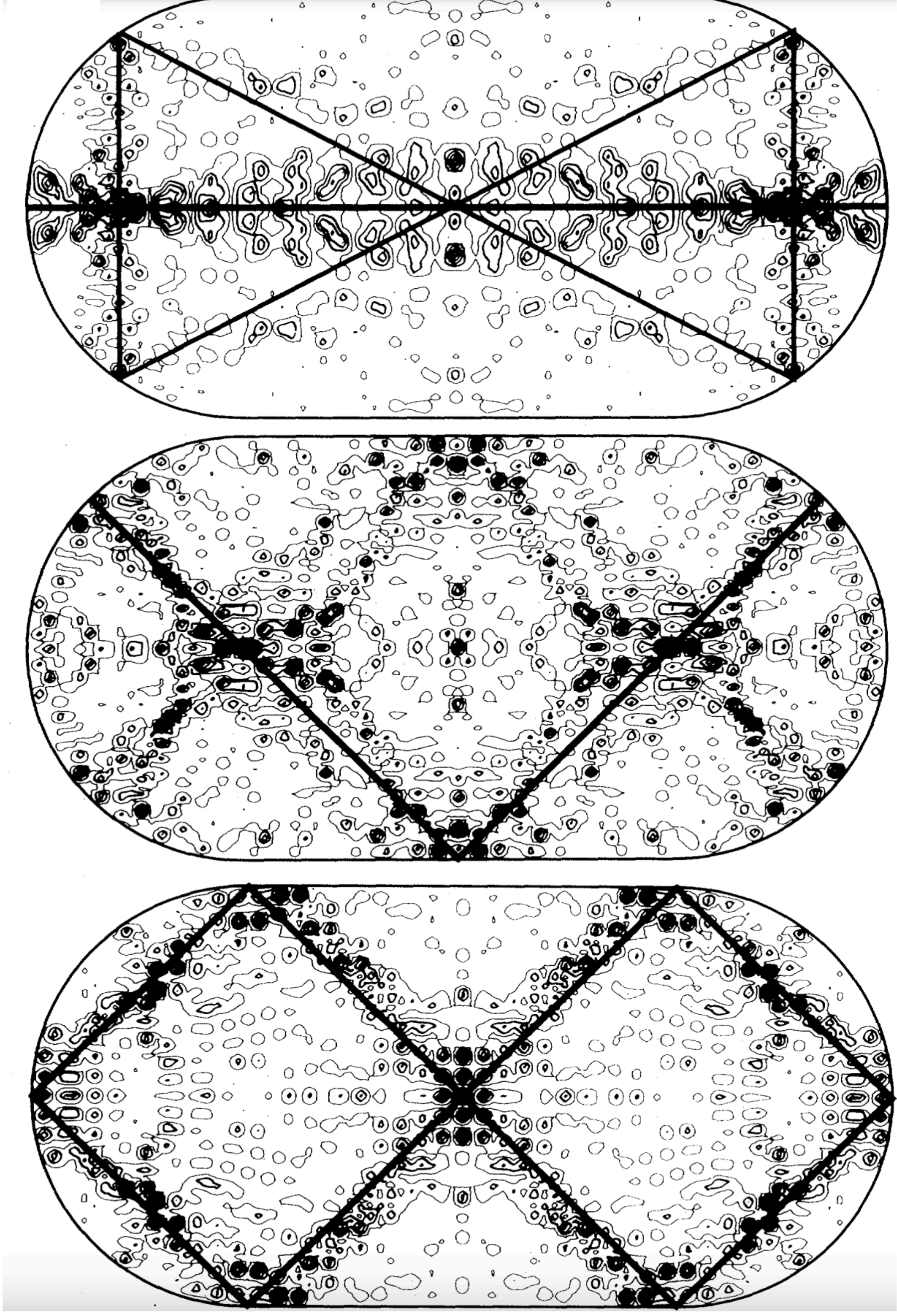
Outline

- Motivation and background
  - weak ergodicity breaking
  - entanglement entropy scaling
- Constrained Hilbert space and Hamiltonian
- Fragmentation and frustration free eigenstates
- Entanglement entropy
- Spectral gap
- Summary and Outlook

Lightning review of ergodicity and its breaking

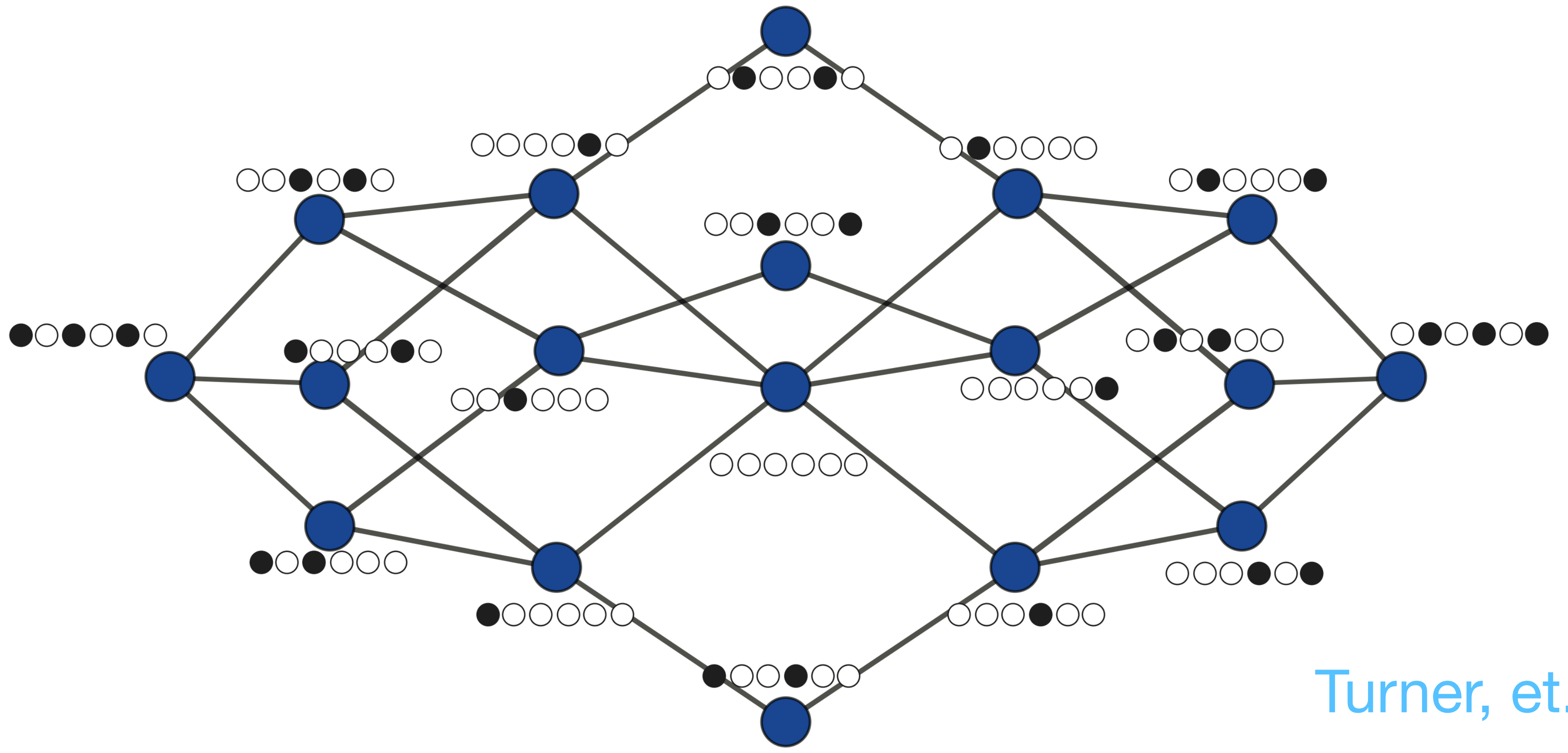
$$\lim_{t\to\infty}\rho_{\mathcal{A}}(t)=\mathrm{Tr}_{\mathcal{B}}\left(\rho^{eq}\right)\approx\rho^{eq}_{\mathcal{A}},\quad \rho^{eq}=\frac{1}{Z}e^{-\beta H}$$

Eigenstate thermalization Hypothesis



Heller, PRL, 84'

Weak ETH breaking in constrained Hilbert space



Turner, et. al., Nature Phys., 18'

PXP model
Hilbert space dim. grows as Fibonacci sequence:
2, 3, 5, 8, 13, 21, 34, 55, 89, 144, …
Asymptotically as 1.618^N

Any 2D counterparts?

Ergodicity in the Quantum Dimer Model

Asked 4 years, 10 months ago Modified 3 years, 9 months ago Viewed 354 times

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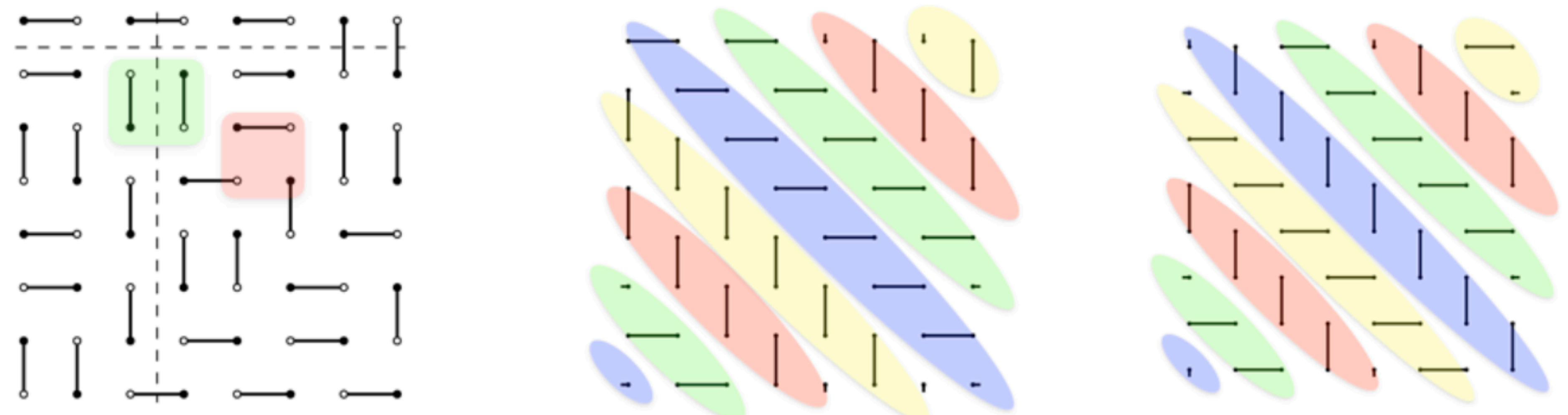
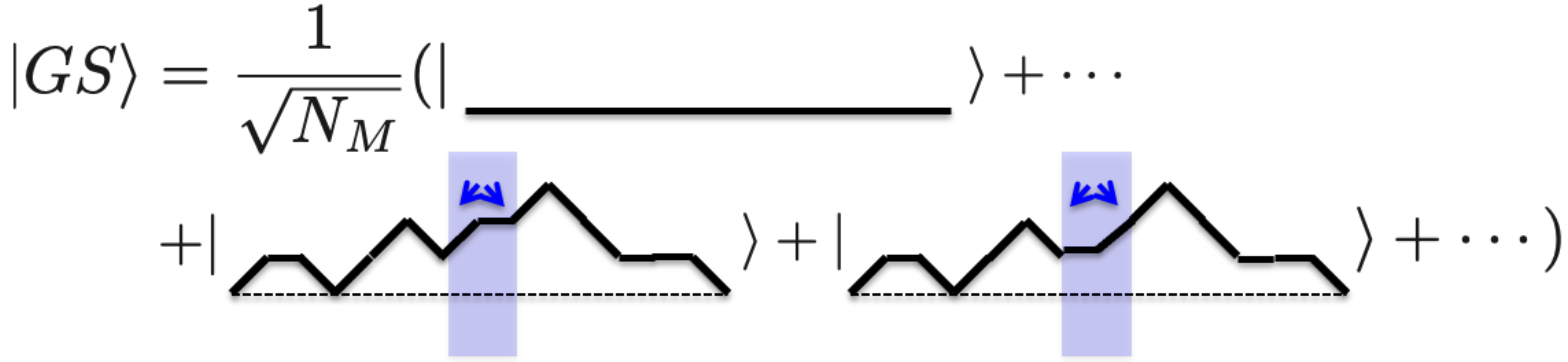
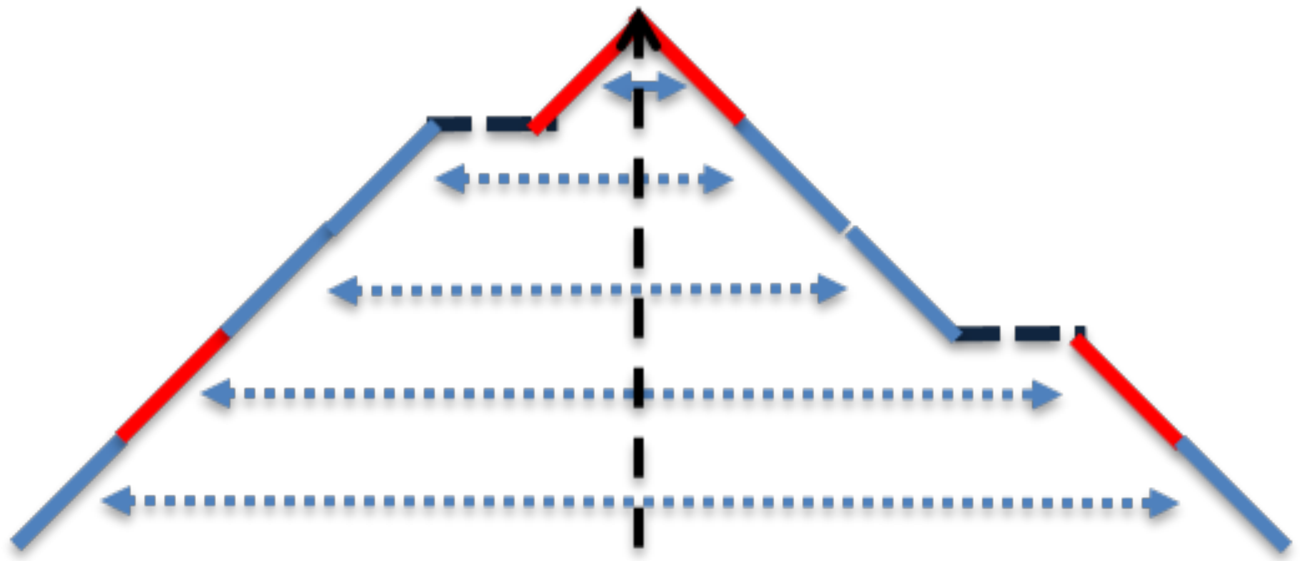
Background:

The Quantum Dimer Model is a lattice model, where each configuration is a covering of the lattice with nearest-neighbour bonds, like in the figure on the left:



Height model and entanglement





Hamiltonian as a sum of local projectors:

$$H = |\diagdown\rangle_1\langle\diagdown| + |\diagup\rangle_{2n}\langle\diagup| + \sum_{j=1}^{2n-1} |\phi\rangle_{j,j+1}\langle\phi| + |\psi\rangle_{j,j+1}\langle\psi| + |\theta\rangle_{j,j+1}\langle\theta|$$

Bravyi, et. al., PRL, 12'
Movassagh, and Shor, PNAS, 16'

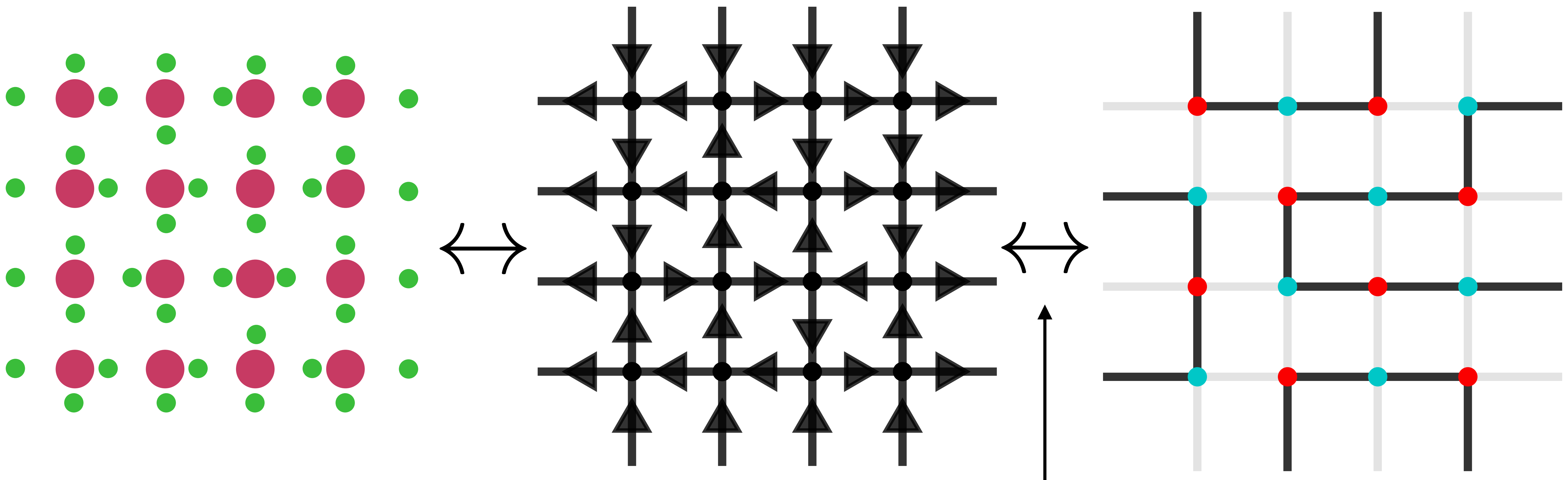
$$|\phi\rangle = \frac{1}{\sqrt{2}}(|\_\diagup\rangle - |\diagup\overline{\ \ }\rangle), |\psi\rangle = \frac{1}{\sqrt{2}}(|\overline{\ \ }\diagdown\rangle - |\diagdown\_\rangle), |\theta\rangle = \frac{1}{\sqrt{2}}(|\_\,\_\rangle - |\diagup\diagdown\rangle),$$

$$|\phi\rangle\langle\phi|(|\diagup\overline{\ \ }\rangle + |\_\diagup\rangle) = 0$$

Can this happen in 2D as well?

At least not naively.

Vertex, loop and height models



$$\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & -1 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

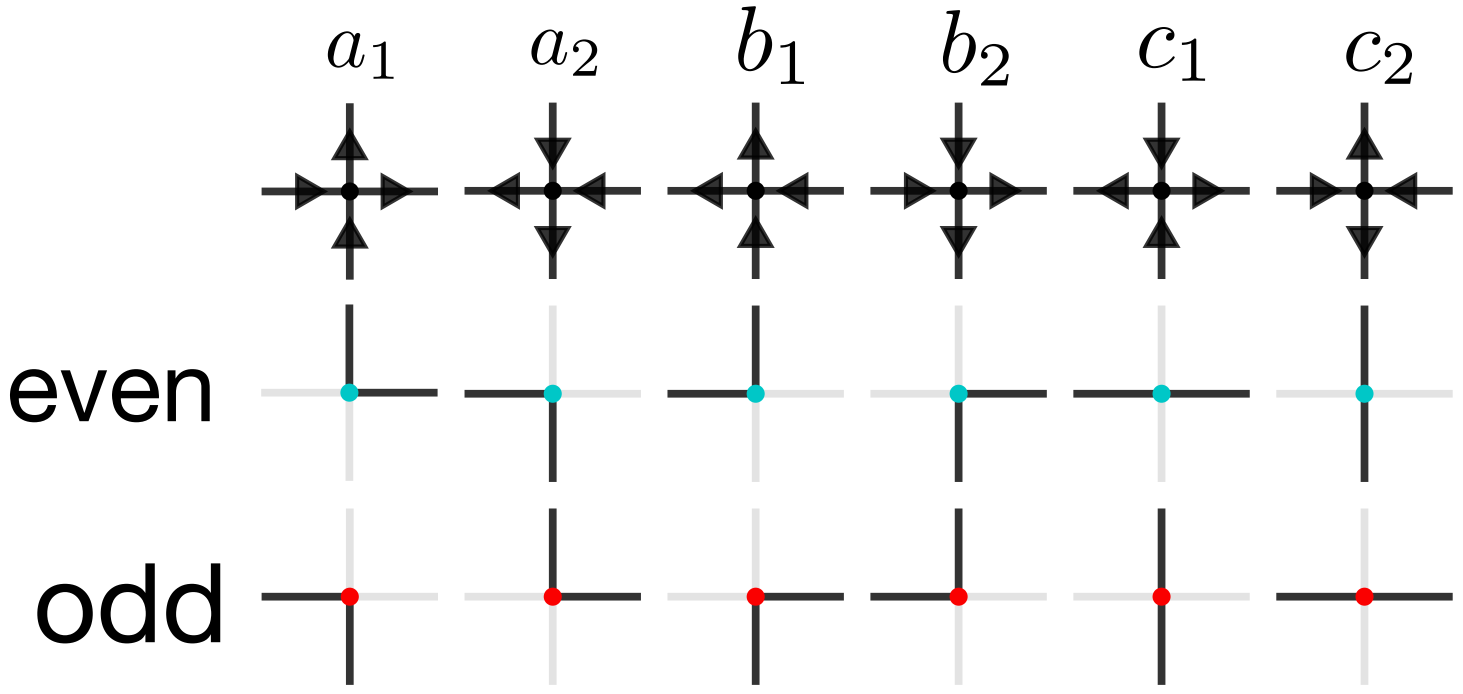
square ice ↔ six vertex ↔ FPL ↔ ASM

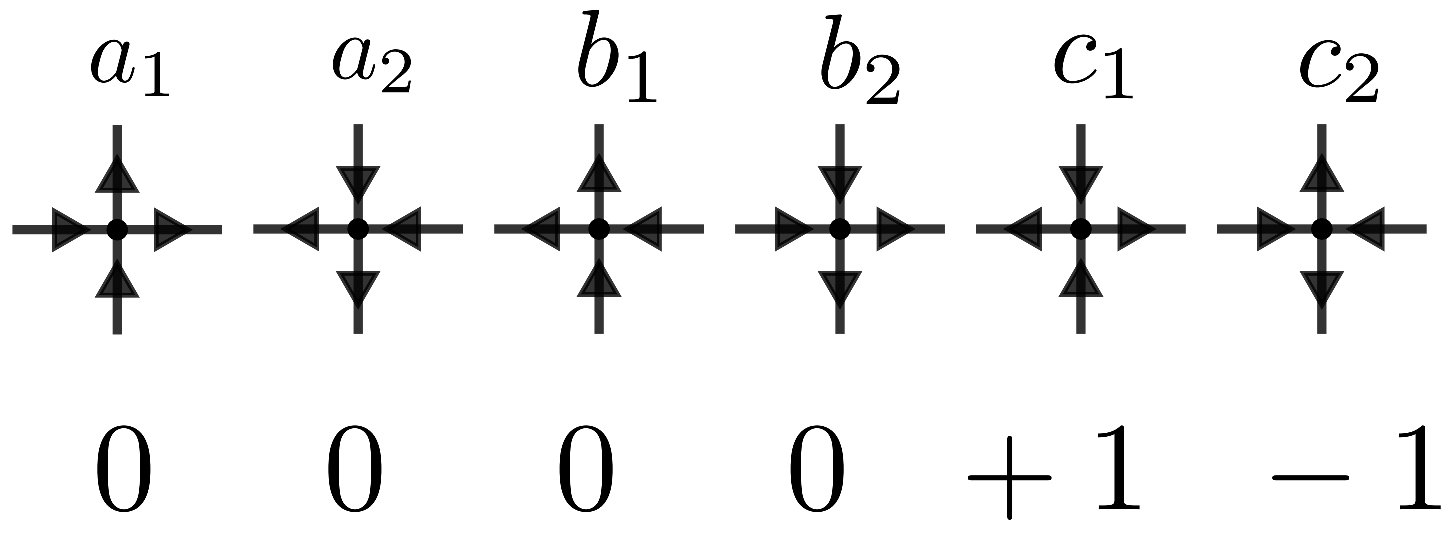
a=b=c

Domain wall boundary

Hilbert space grows as:
2, 7, 42, 429, 7436, 218348, …
Asymptotically as 1.299^{N^2}

$$A(N) = \prod_{n=0}^{N-1} \frac{(3n+1)!}{(N+n)!}$$

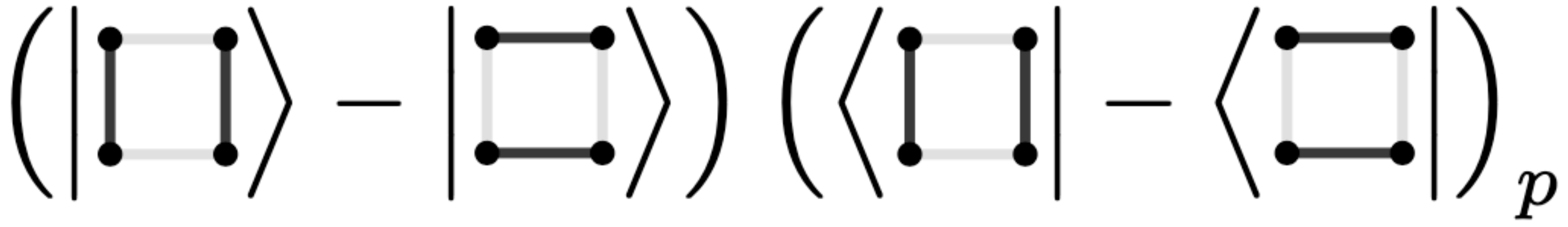




How to make them quantum (frustration free)?

By introducing off-diagonal terms in Hamiltonian

$$H = \sum_{p \in \mathrm{bulk}} P_p + H_\partial,$$

$P_p =$ 

$$H_0 = V \sum_{i,j=1}^{N} \left(S_x(i,j) + S_y(i,j) + S_y(i,j+1) + S_x(i+1,j)\right)^2, \; V \to \infty$$

$$H_\partial = H_\partial^u + H_\partial^d + H_\partial^l + H_\partial^r + 2N,$$

$$H_\partial^y = \sum_{x=1}^{N} (-1)^x \left|\downarrow\right\rangle \left\langle\downarrow\right|_{x,N},$$

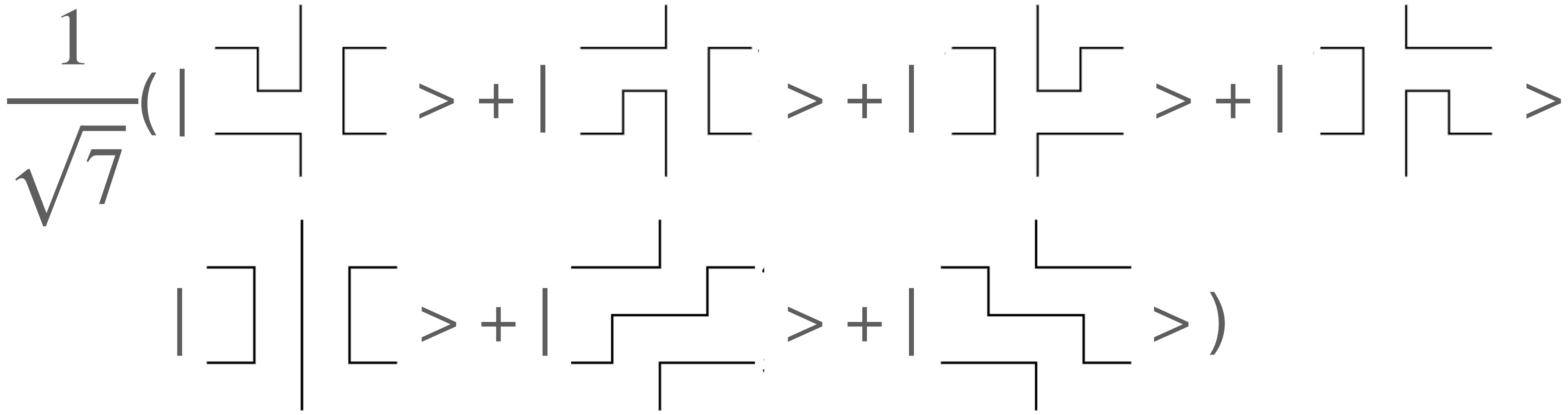
$$H_\partial^d = \sum_{x=1}^{N} (-1)^{x+1} \left|\uparrow\right\rangle \left\langle\uparrow\right|_{x,1}$$

$$H_\partial^l = \sum_{y=1}^{N} (-1)^y \left|\rightarrow\right\rangle \left\langle\rightarrow\right|_{1,y},$$

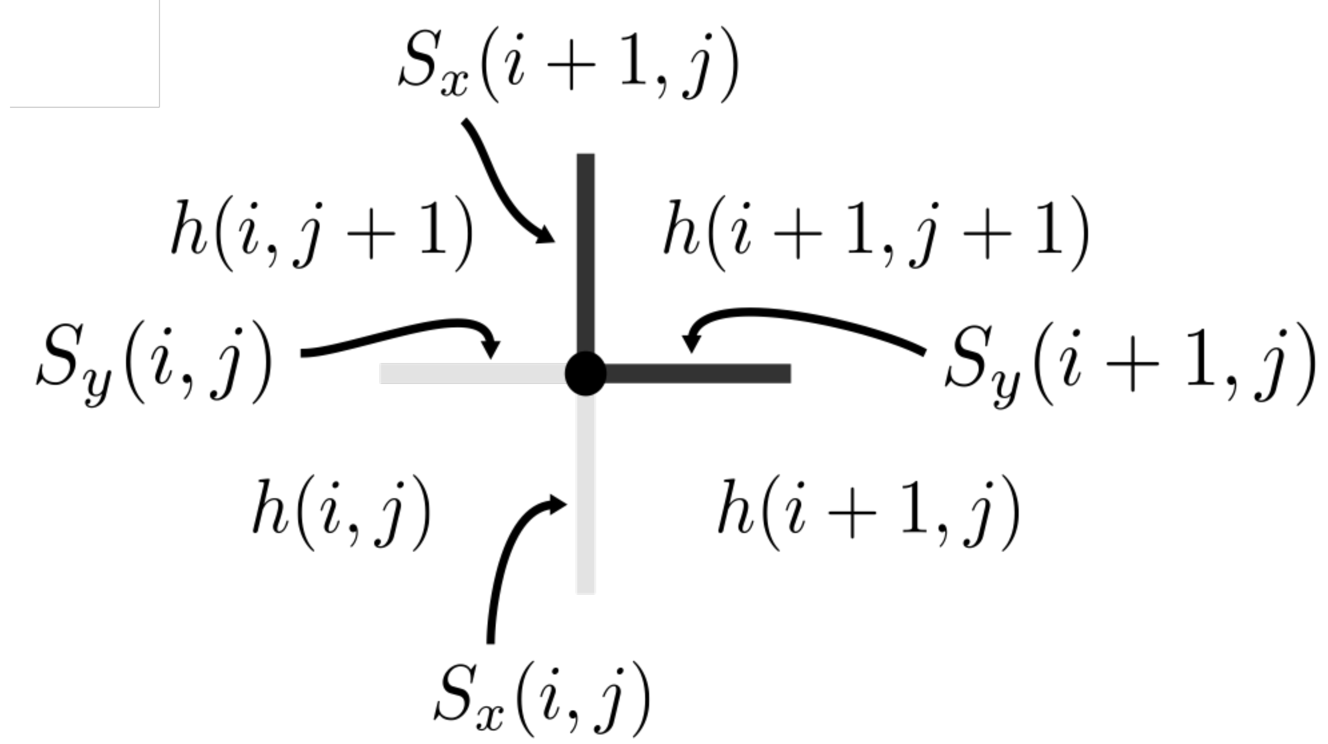
$$H_\partial^r = \sum_{y=1}^{N} (-1)^{y+1} \left|\leftarrow\right\rangle \left\langle\leftarrow\right|_{N,y}.$$

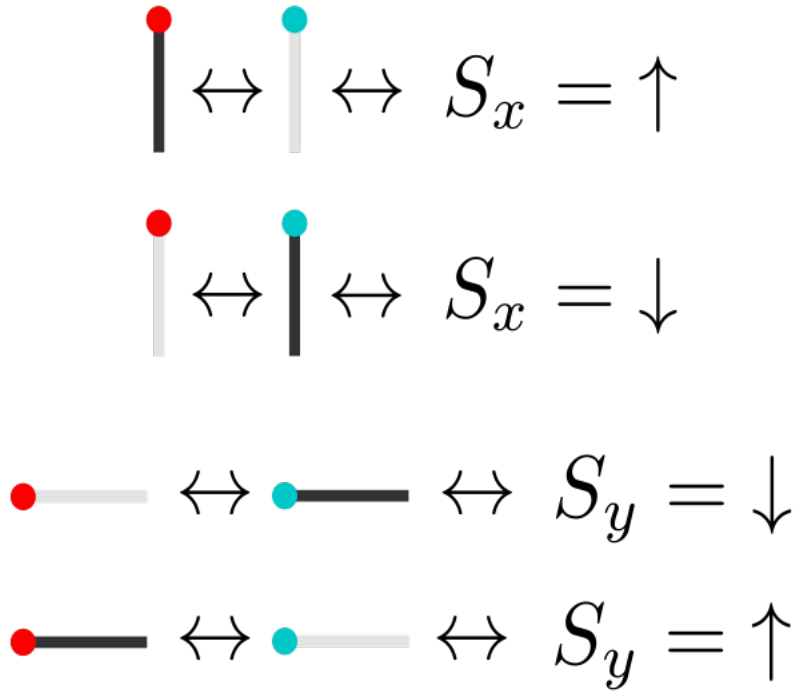
Ergodicity within fixed boundary configuration

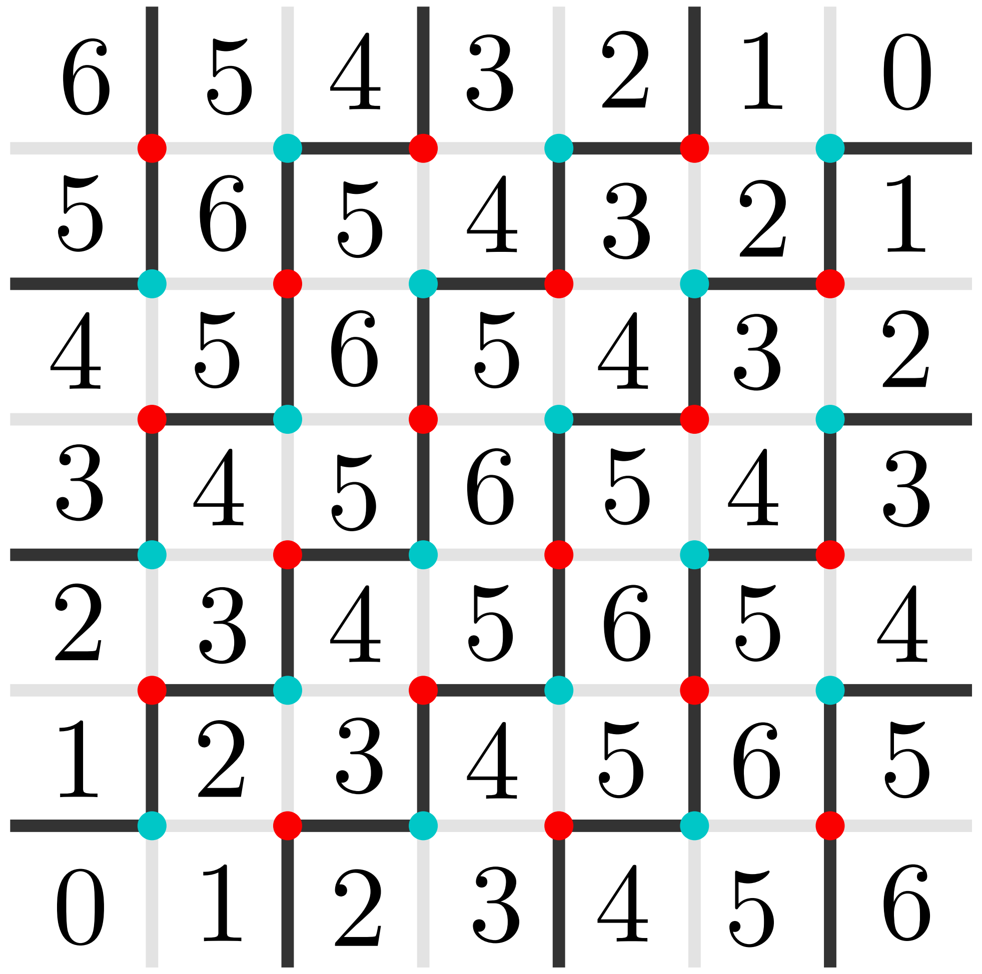
$$\Longrightarrow \; |\mathrm{GS}\rangle = \frac{1}{\sqrt{A(N)}} \sum_{\mathcal{F} \in \mathrm{FPL\ with\ DWBC}} |\mathcal{F}\rangle$$

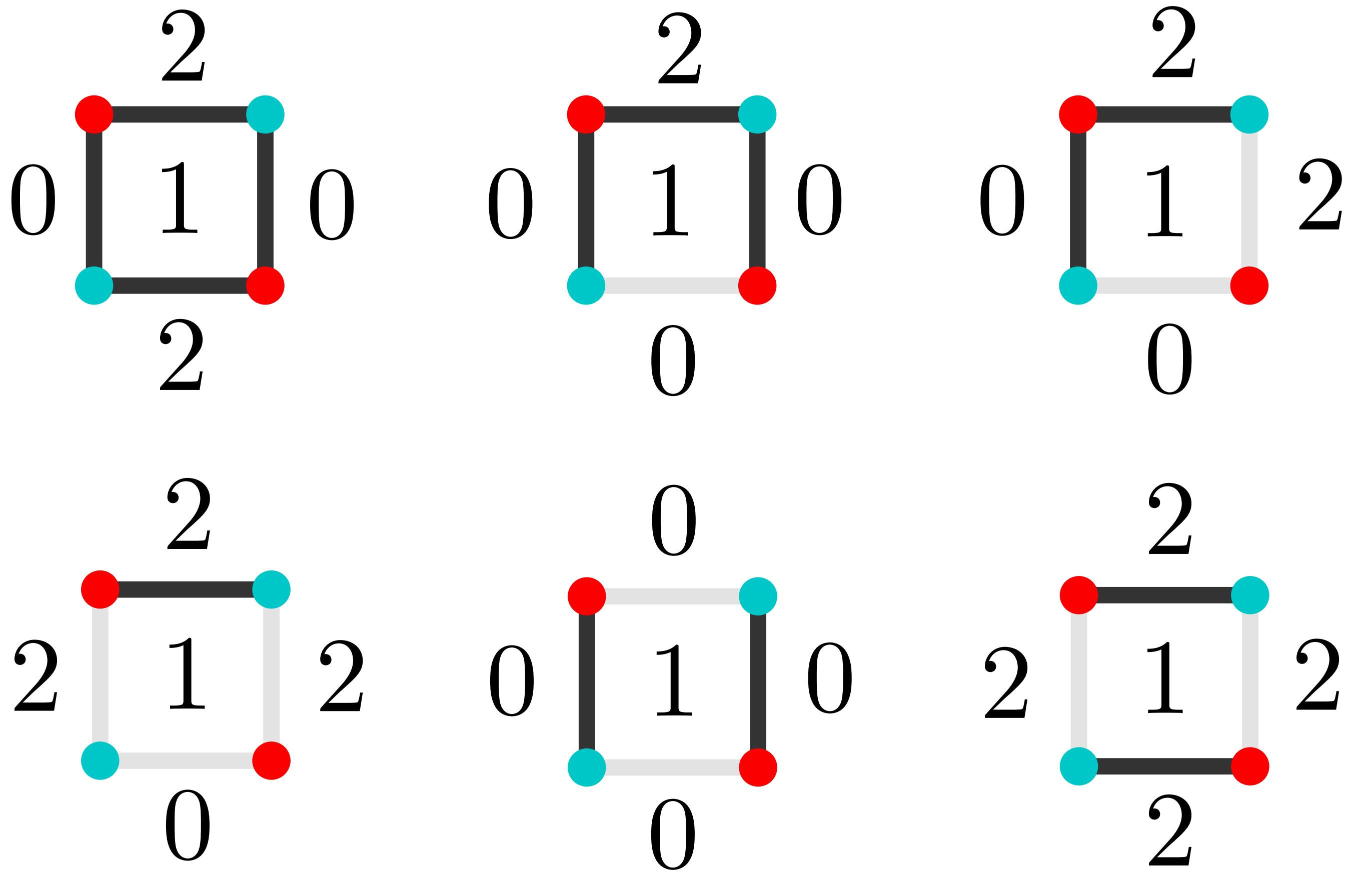


Dual height representation









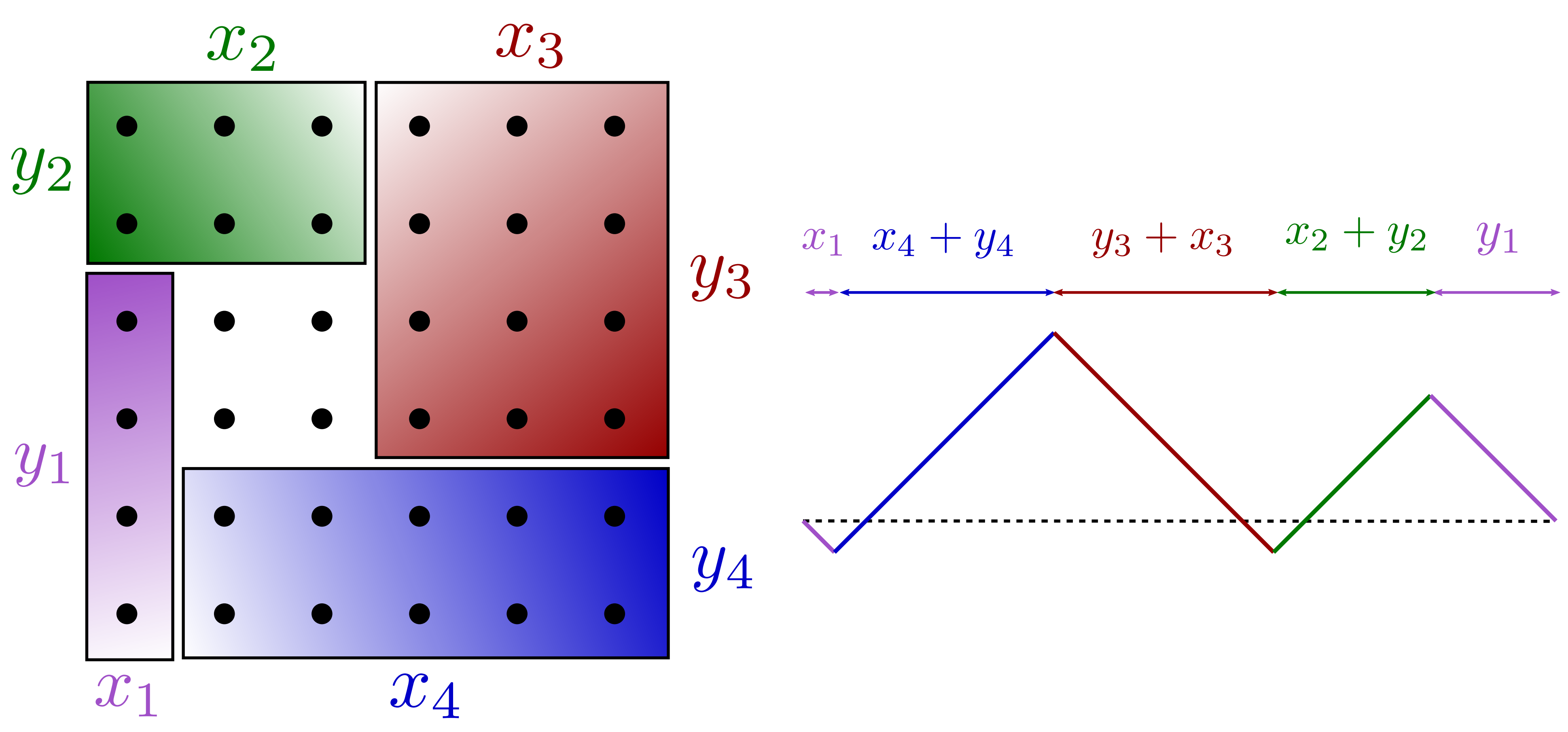
$$\vec{S}(i,j) = \vec{\nabla} h(i,j) \implies \vec{\nabla} \times \vec{S}(i,j) = \vec{0}$$

$$H_0^* = V \sum_{\langle p,q \rangle} \left((h_p - h_q)^2 - 1 \right)^2, \; V \gg 1$$

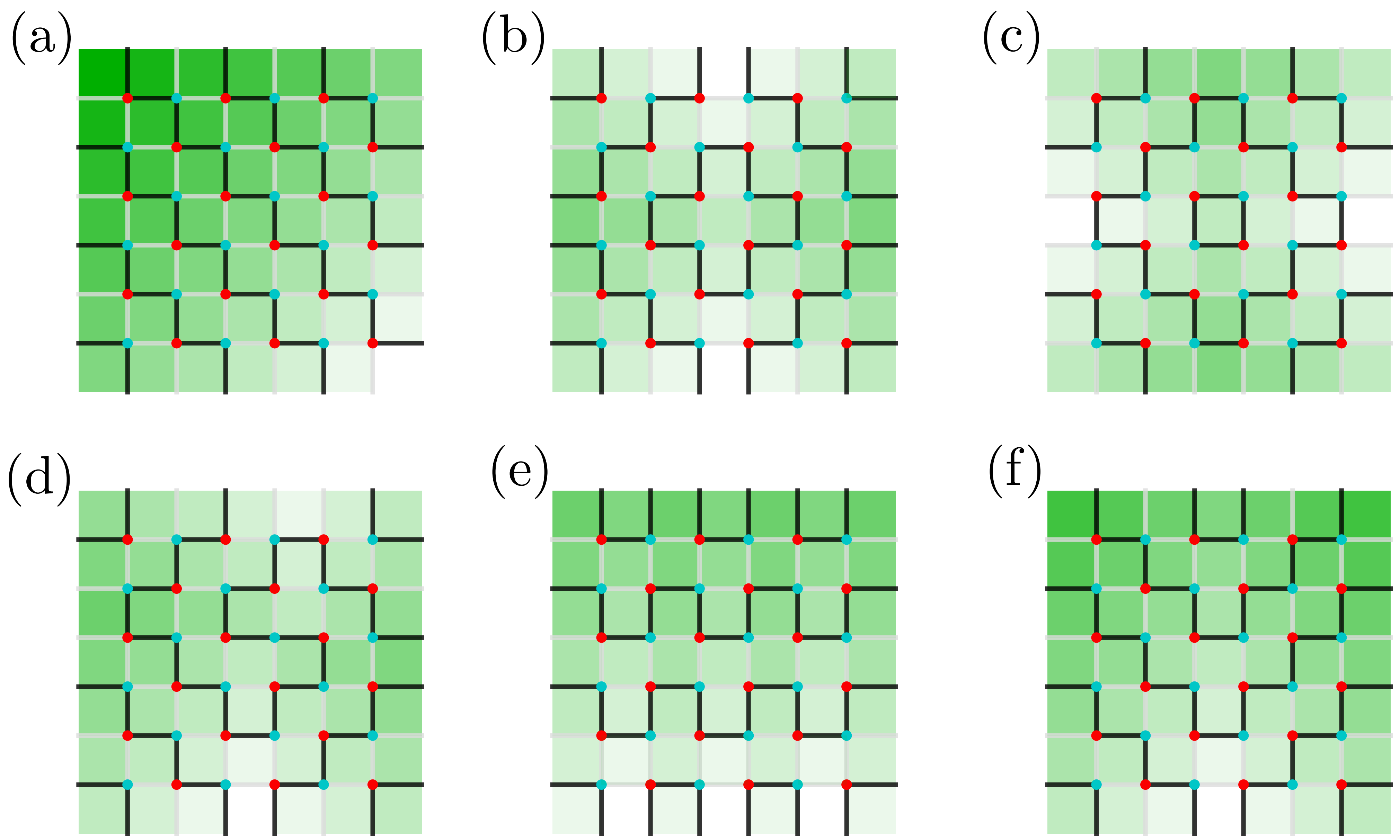
$$H^* = \sum_{p \in \text{bulk}} \left(\Pi_p^> + \Pi_p^< - \Pi_p^> h_p^+ \Pi_p^< - \Pi_p^< h_p^- \Pi_p^> \right) + H_\partial^*$$

$$H_\partial^* = h(1,1) - h(1,N+1) - h(N+1,1) + h(N+1,N+1) + 2N.$$

Hilbert space fragmentation

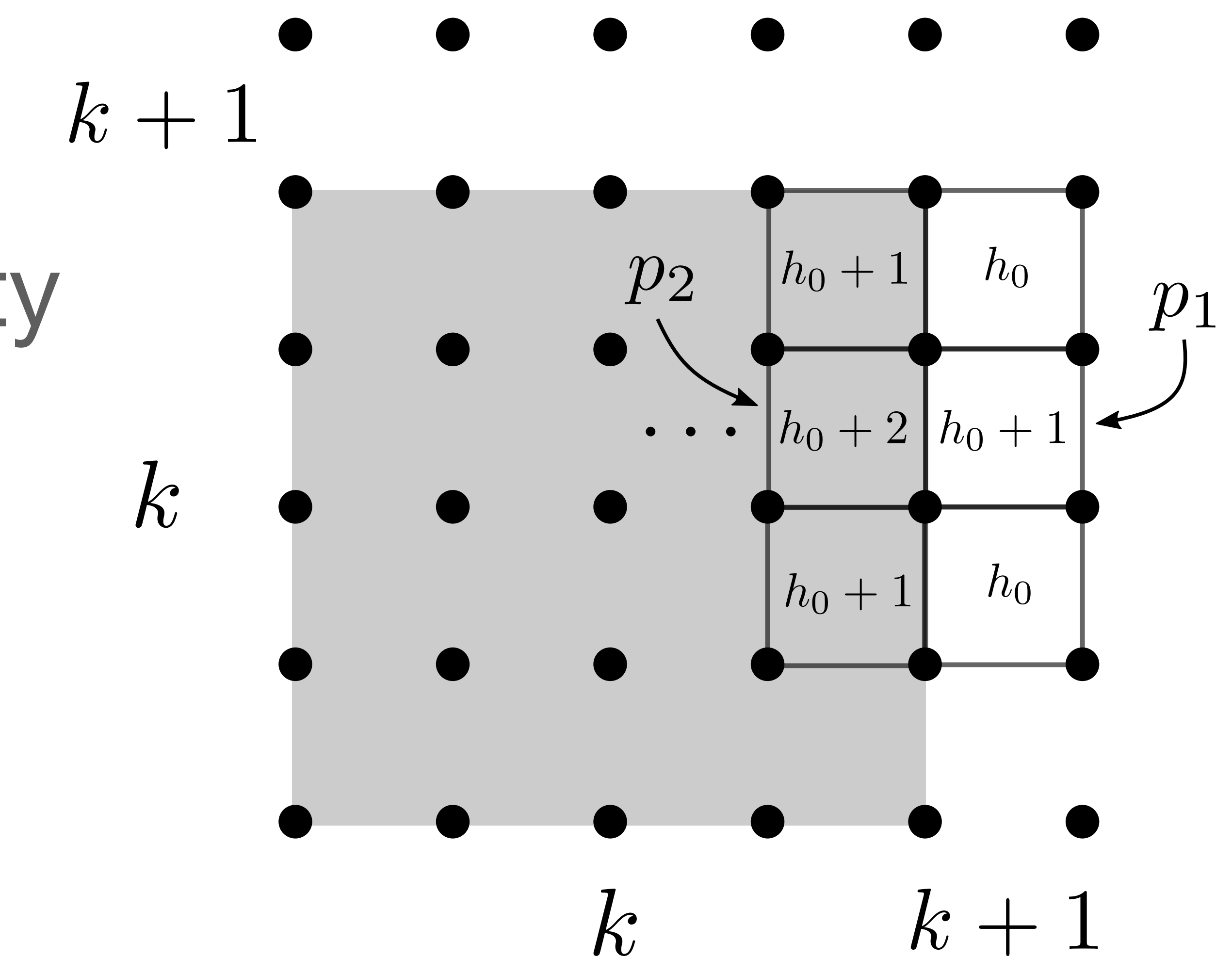


Product state eigenstates:



Inductive proof of ergodicity within Krylov subspaces

$$|2n, \mathcal{S}_n\rangle \propto \sum_{\mathcal{F} \in \text{FPL with } \mathcal{S}_n} |\mathcal{F}\rangle$$

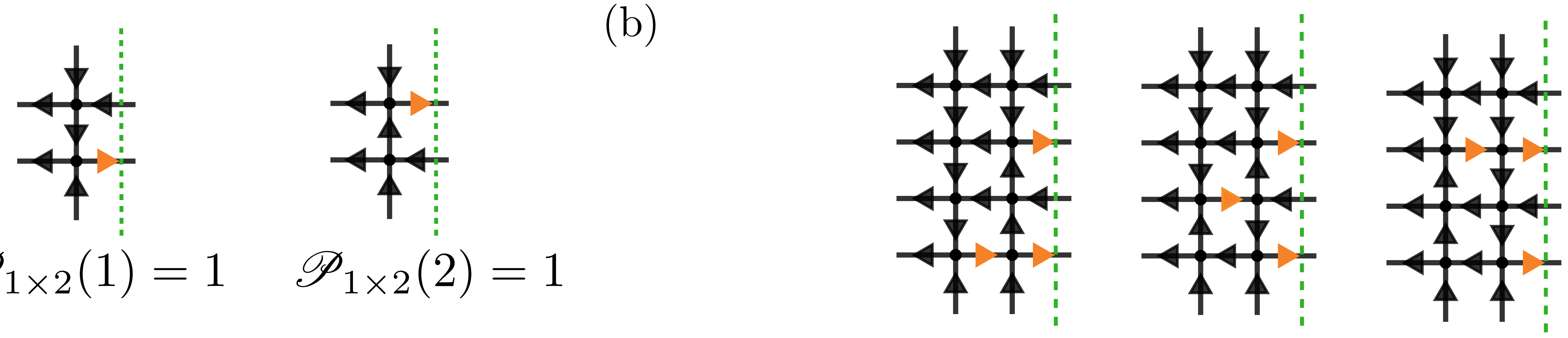


Entanglement entropy

$$|\mathrm{GS}\rangle = \sum_{\{\vec{m}\}} \sqrt{p_{\vec{m}}}\, |\mathcal{P}_{L,\vec{m}}(N/2)\rangle \otimes |\mathcal{P}_{R,\vec{m}}(N/2)\rangle$$

$$p_{\vec{m}} = \frac{|\mathcal{P}_{L,\vec{m}}(N/2)||\mathcal{P}_{R,\vec{m}}(N/2)|}{A(N)} \qquad S = -\sum_{\{\vec{m}\}} p_{\vec{m}} \log p_{\vec{m}}$$

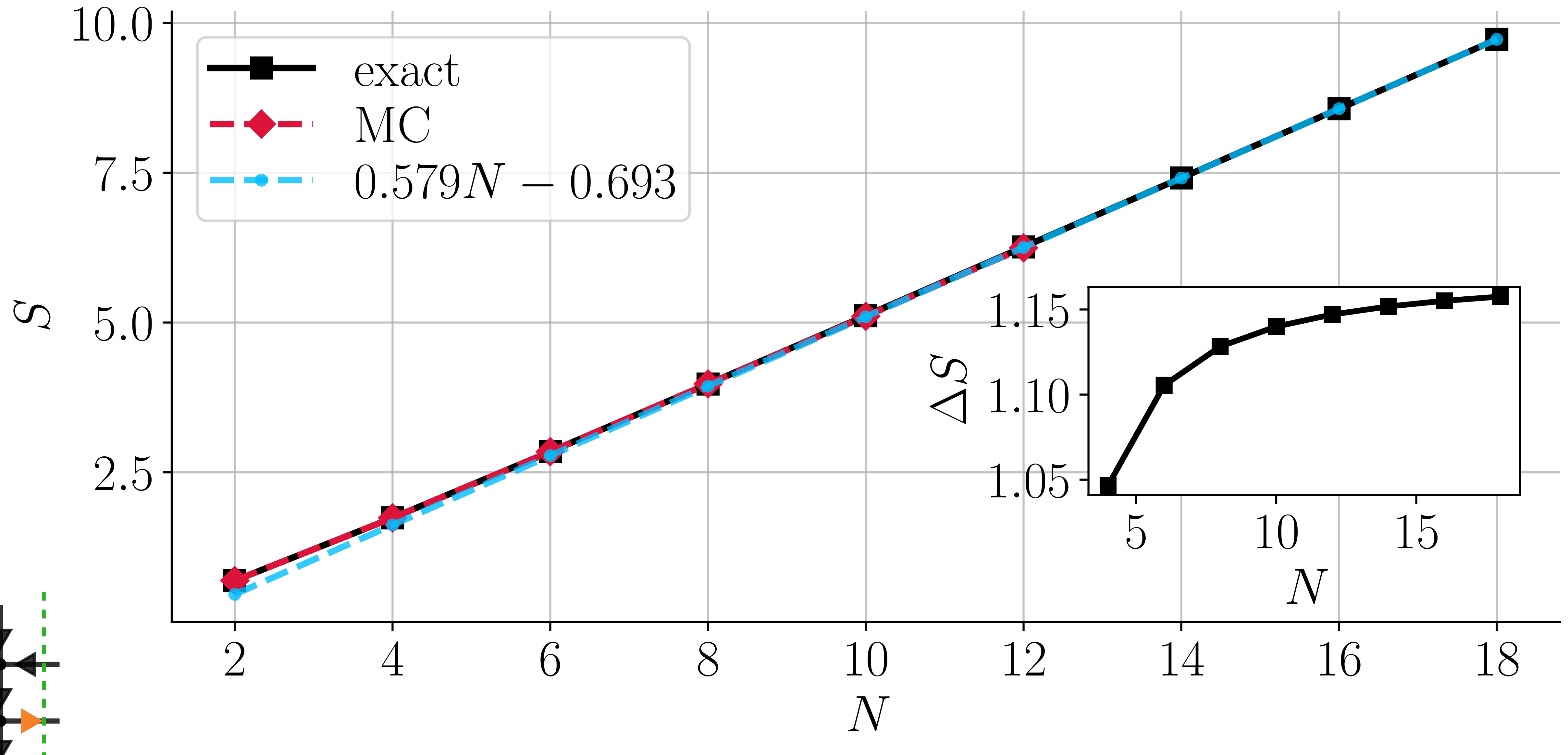
(a) (b)

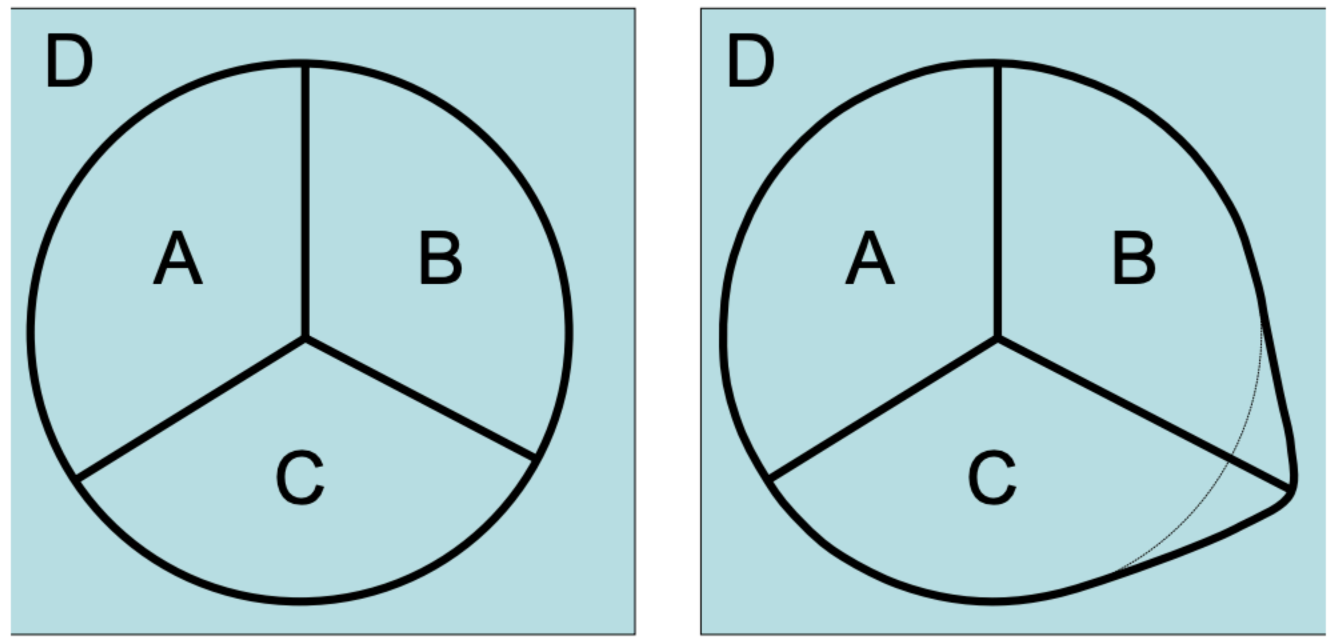


$\mathscr{P}_{1\times 2}(1) = 1$ $\quad \mathscr{P}_{1\times 2}(2) = 1$

$$\mathscr{P}_{2\times 4}(1,3) = \mathscr{P}_{1\times 4}(1) + \mathscr{P}_{1\times 4}(2) + \mathscr{P}_{1\times 4}(3)$$

$$\mathscr{P}_{(\frac{N}{2}+1)\times(N+2)}(\vec{x}) = \sum_{\substack{x_1 \le y_1 \le x_2 \le y_2 \le \ldots \le y_{\frac{N}{2}} \le x_{\frac{N}{2}+1} \\ y_1 \ne y_2 \ne \ldots \ne y_{\frac{N}{2}}}} \mathscr{P}_{\frac{N}{2}\times(N+2)}(\vec{y})$$

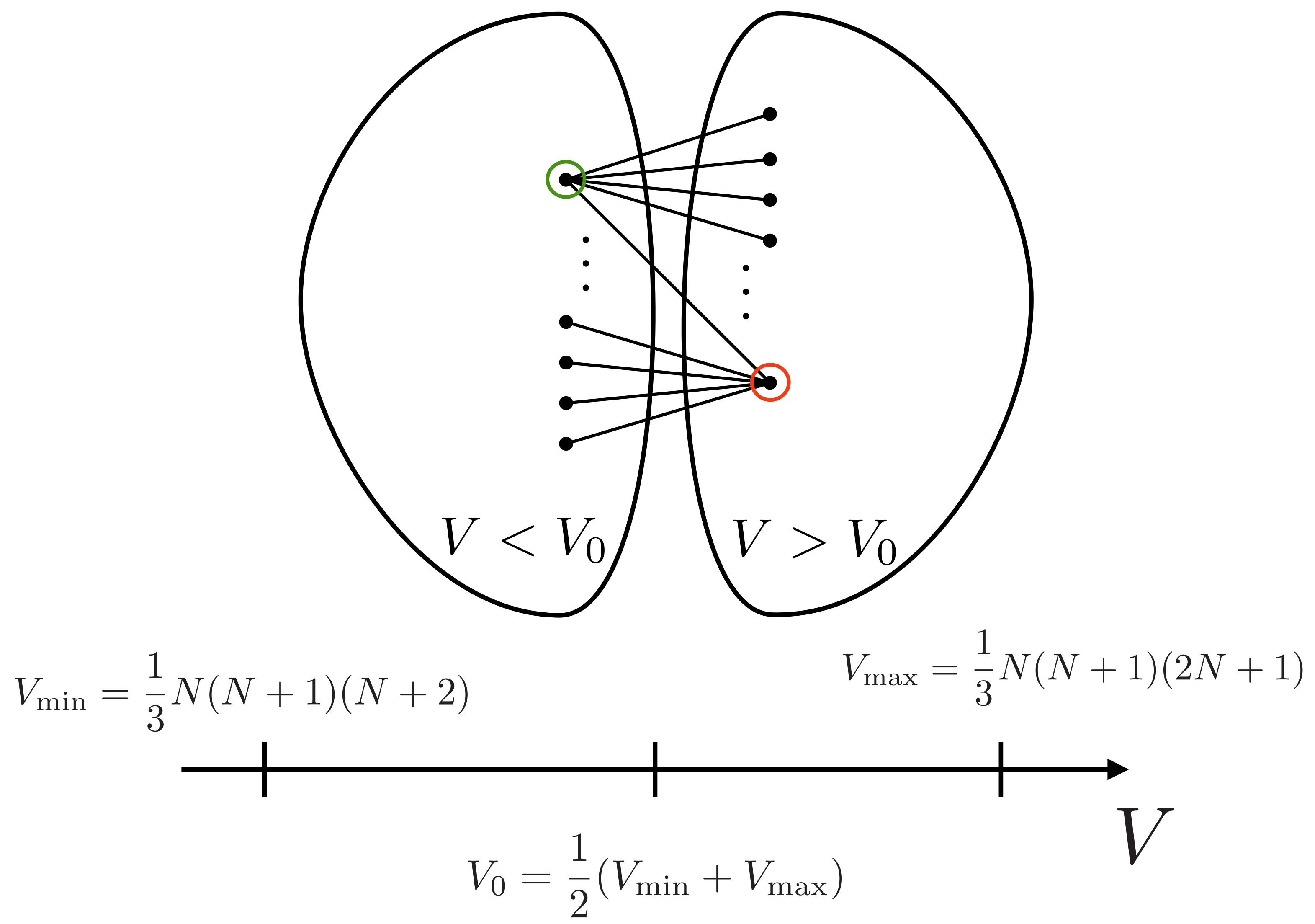




$$S_{\mathrm{topo}} \equiv S_A + S_B + S_C - S_{AB} - S_{BC} - S_{AC} + S_{ABC}$$

bulk gapless excitation

Natural ordering of configurations by volume

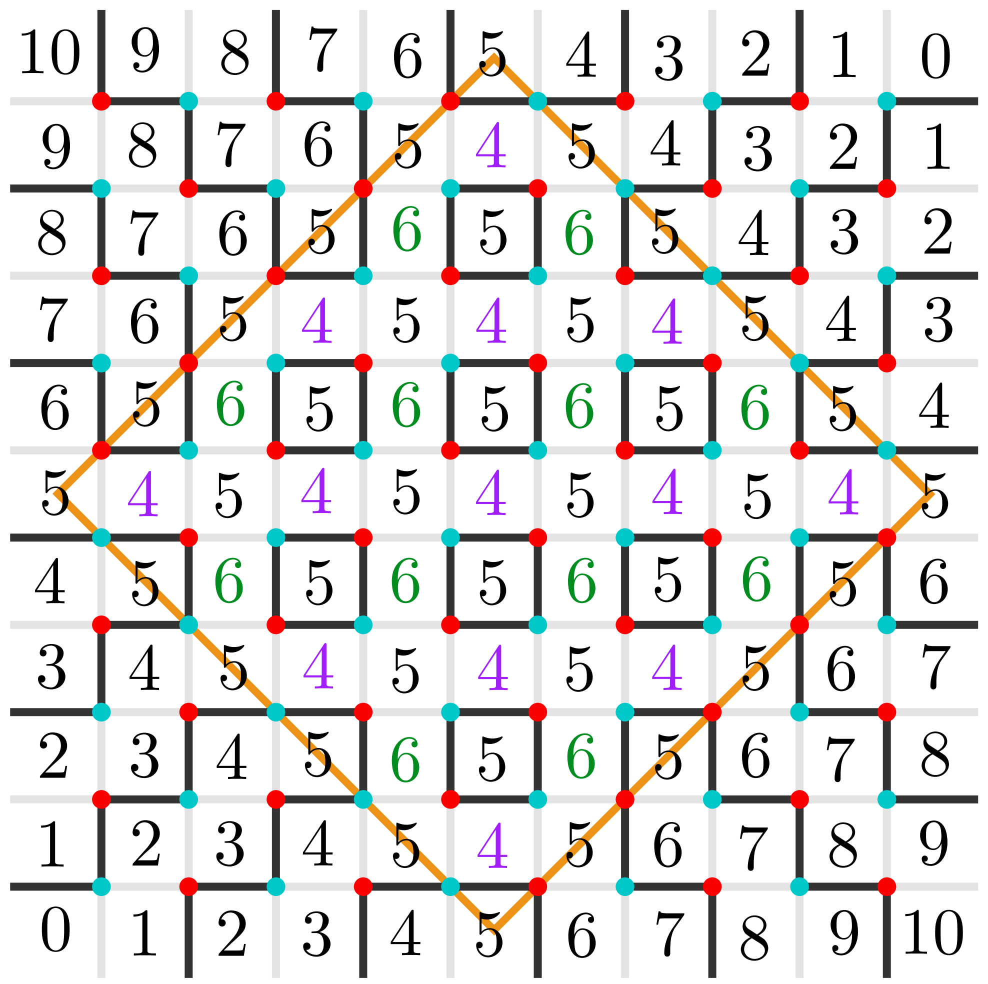


$$|\pi\rangle = \sum_{\mathcal{F}\in \text{FPL with DWBC1}} \text{sgn}(V(\mathcal{F}) - V_0)\, |\mathcal{F}\rangle$$

$$\langle \text{GS}|\pi\rangle = 0 \qquad \langle \pi|\pi\rangle = A(N)$$

$$M = \frac{N^2}{4}$$ mobile plaquettes $$\begin{cases} \frac{M+1}{2} \to \frac{N}{2} - 1, \\ \frac{M-1}{2} \to \frac{N}{2} + 1. \end{cases}$$

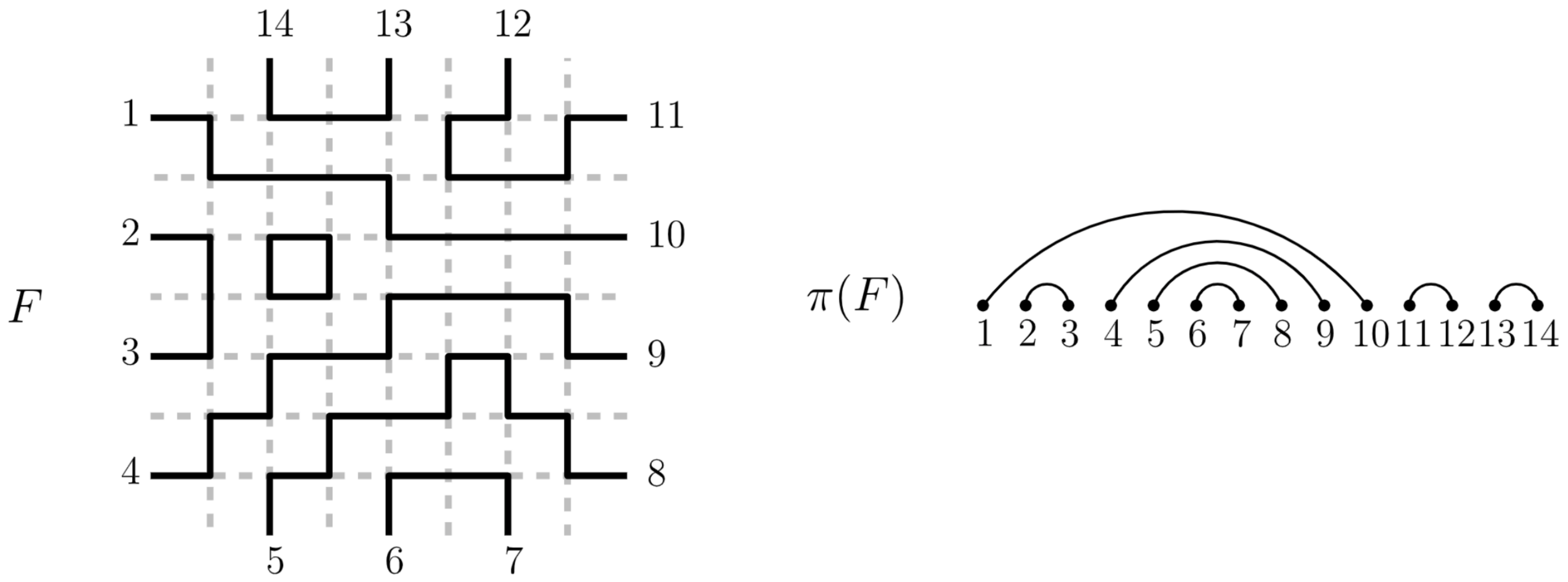
total number of such configurations on the boundary $$\sum_{f=0}^{\frac{M-1}{2}} \binom{\frac{M-1}{2}}{f} \binom{\frac{M+1}{2}}{f} \equiv \binom{M}{\frac{M+1}{2}}$$

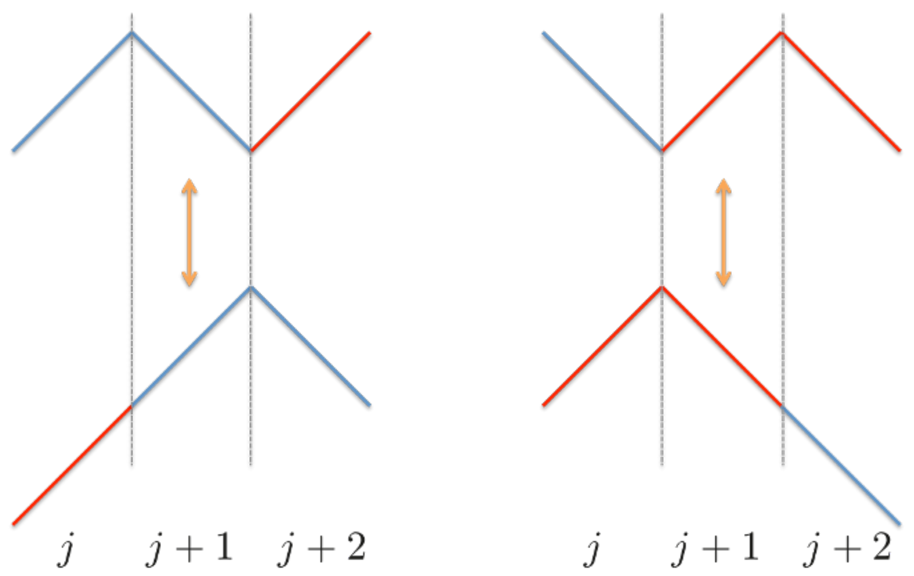


$$\langle \pi|\, H\, |\pi\rangle = \sum_{\mathcal{F}', \mathcal{F}' = V_0 \pm 1} \langle \mathcal{F}|\, H\, |\mathcal{F}'\rangle$$
$$= \frac{M+1}{2} \binom{M}{\frac{M+1}{2}}.$$

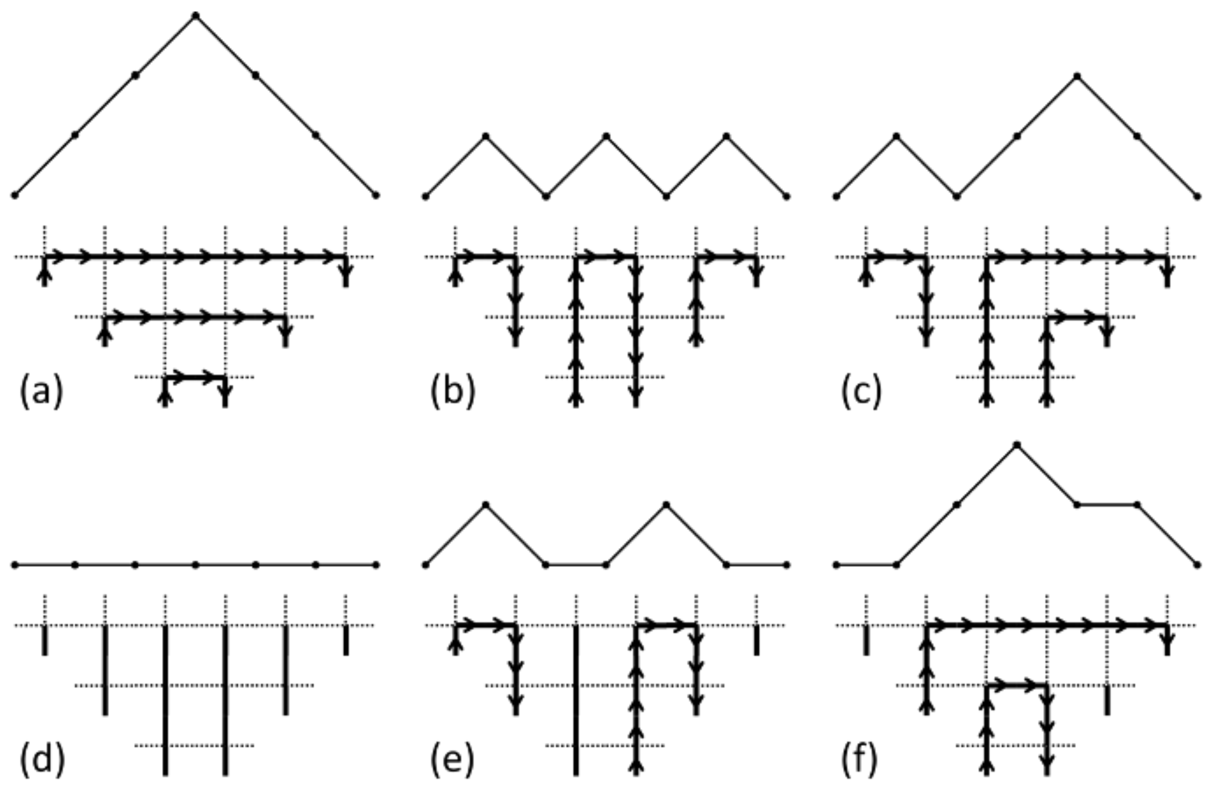
$$\lim_{N\to\infty} \frac{\langle \pi|\, H\, |\pi\rangle}{\langle \pi|\pi\rangle} \propto N \left(\frac{\sqrt[4]{2}}{3\sqrt{3}/4} \right)^{N^2} \to 0$$

Link pattern, Wieland gyration and holography dual

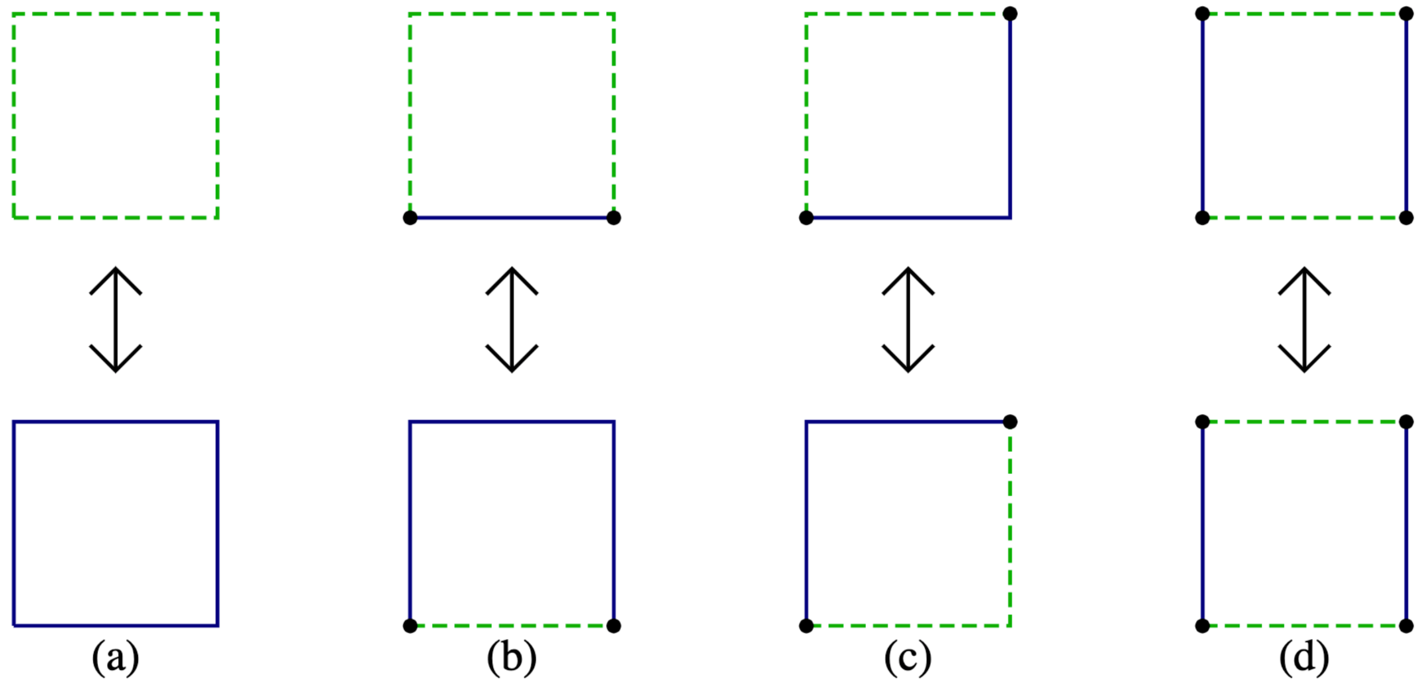




Dell'Anna, et. al., PRB, 16'
Salberger, and Korepin, Rev. Math. Phys., 17'

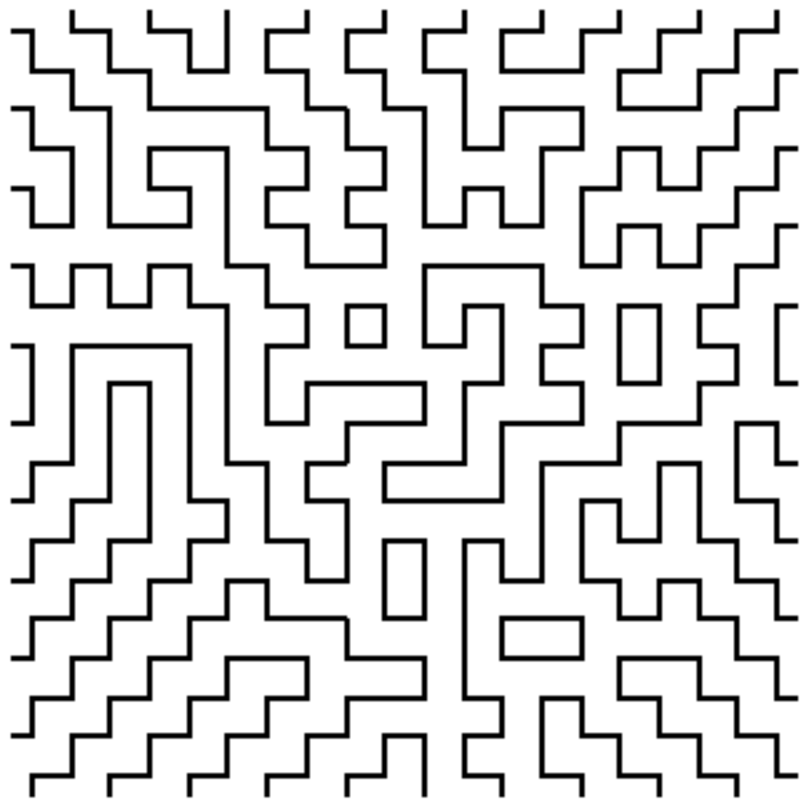


Alexander, et. al., PRB, 19'



Wieland, Elec. J Comb., 00'

$$[H_{\text{bulk}} + H'_{\partial}, \mathcal{G}] = 0$$



Chiral edge mode?

Lemm and Mozgunov, J. Math. Phys., 19'

Open questions

- Quantum many-body scar dynamics? Turner, et. al., Nature Phys., 18’
- What if there is kinetic term on boundary or periodic boundary condition?
- Add color degree of freedom to increase entanglement? Movassagh, and Shor, PNAS, 16’
- Current gapless proof doesn’t work for q-deformed model, could the be a phase transition at $q=1$?

  Zhang, Ahmadain, Klich, PNAS, 17’
- Other lattices and constraints (coming soon)
- Spin Hamiltonian on vertices (coming soon) Cano, and Fendley, PRL, 12’