An introduction to tensor models:

from random geometry to melonic CFTs.

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GGI Lectures

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Outline

1. Matrix models: large N expansion, applications
2. Tensor models: large N expansion, random geometry
3. Melonic CFTs: SYK-like models, tensor field theories.

Reviews

Matrices: 1510.04430 Eynard, Kimura, Ribault

Random tensors:

"The tensor track" I-IV, Rivasseau 2011-2016

"Random tensors" Gurau 2016

Melonic CFTs:

1808.09434 Gurau

"The tensor track" V-VI, Rivasseau, Delporte 2018-2020

2004.08616 Benedetti

Large N expansion of matrix models

Hermitian matrices

$$Z_N(\lambda) = \int_{\mathcal{H}_N} dM \, \exp^{-N\left(\frac{1}{2}\mathrm{Tr}M^2 + \frac{\lambda}{4}\mathrm{Tr}M^4\right)}$$

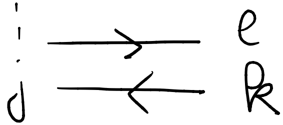
$\rightarrow \langle \mathrm{Tr}(M^{m_1}) \cdots \mathrm{Tr}(M^{m_k}) \rangle$?

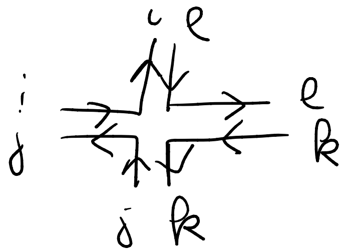
$\rightarrow$ Gaussian measure.

$$\lambda = 0 \quad \langle M_{ij} M_{kl} \rangle \equiv P_{ij,kl} = \frac{1}{Z_N(0)} \int dM \, e^{-\frac{N}{2}\mathrm{Tr}M^2} M_{ij} M_{kl}$$

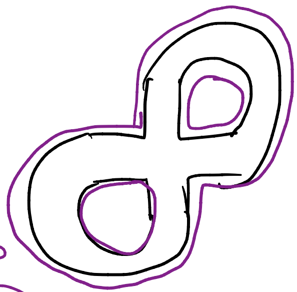
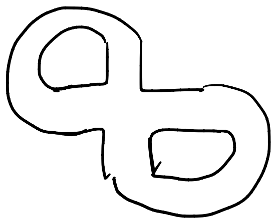
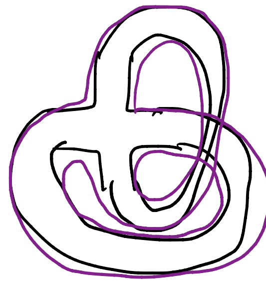
$$= \boxed{\frac{1}{N}\delta_{il}\delta_{jk}}$$

Graphical representation : ribbon graphs

$P_{ij,kl} = \frac{1}{N} \delta_{il} \delta_{jk}$ $\longrightarrow$ 

interaction $\quad N \operatorname{Tr} M^4$ $\longrightarrow$ 

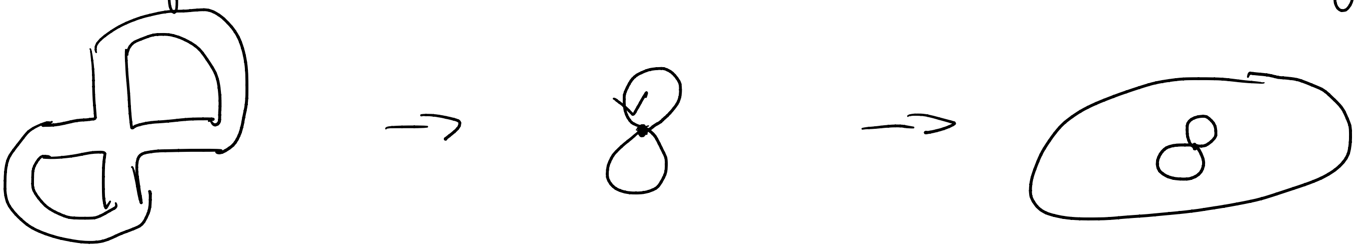
$M_{ij} M_{jk} M_{kl} M_{li}$

$\langle N \operatorname{Tr} M^4 \rangle =$  $+$  $+$ 

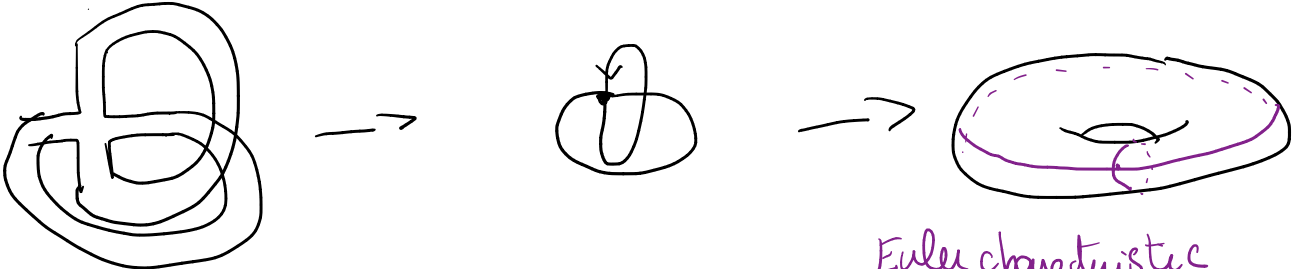
$$= \overbrace{N}^{\text{vertex}} \overbrace{\left(\frac{1}{N}\right)^2}^{2\text{ prop}} \left(N^3 + N^3 + N\right) = 2N^2 + 1$$

Ribbon graphs vs Combinatorial maps

Ribbon graph → combinatorial map → embedded graph



$g = 0$



$g = 1$

Euler characteristic genus

Scaling in N: $N^{V-E+F} = N^{\chi} = \boxed{N^{2-2g}}$

Topological expansion

$$Z_N(\lambda) = \int dM \, e^{-N\left(\frac{1}{2}\mathrm{Tr}M^2 + \frac{\lambda}{4}\mathrm{Tr}M^4\right)}$$

$$= \sum_G \frac{(-\lambda)^{V(G)}}{s(G)} N^{2-2g(G)}$$

← sym. factor.

→ large N expansion

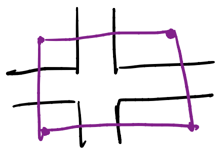
all graphs of genus g

$$\ln Z_N(\lambda) = \sum_{g\in\mathbb{N}} N^{2-2g} \mathcal{F}_g(\lambda) \qquad \sum_{\substack{G \text{ connected} \\ g(G)=g}} \frac{(-\lambda)^{V(G)}}{s(G)}$$

→ Dominated by planar graphs.

Random surfaces

- Generate quadrangulation : dual graph

 $\rightarrow$ space-time geometry.

Large N limit : planar quadrangulation

Quantum gravity in $D = 2$

Critical regimes : λ_c continuum limit

Double-scaling : large N, $\lambda \to \lambda_c$

sum over non-trivial topologies.

$\Rightarrow$ Universality : converge to Brownian sphere

$\hookrightarrow$ Random geometry behind Liouville QG

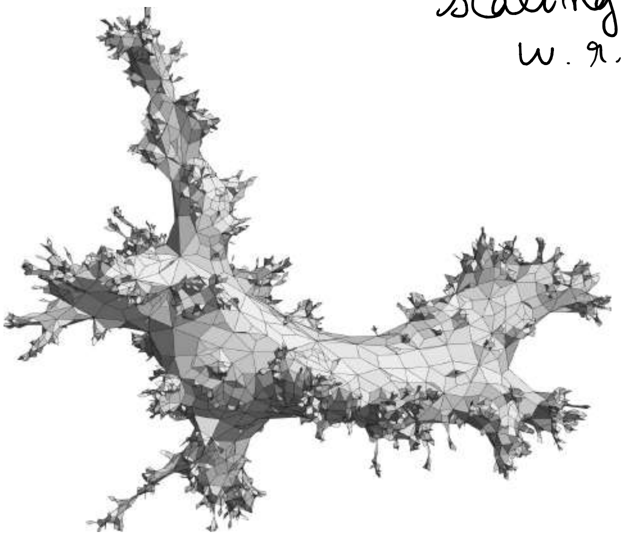
Brownian sphere

$d_{\text{spectral}} = 2$

effective dim°
diffusion process

$d_{\text{Hausdorff}} = 4$.

→ "fractalness"
scaling of the volume
w.r.t distance



Credit: T. Budd https://hef.ru.nl/~tbudd/gallery/

Other applications

Strongly coupled QFT

N: large nb of fields / sym,

↳ non-perturbatively in λ

→ Probe for holographic dualities.

Gauge ↔ Einstein

↳ 2^{nd} lecture.

What about tensors?

$$F(\lambda) = \ln \int dT \; e^{-T_{abc}T_{abc} + \frac{\lambda}{N^\alpha} T_{aeb}T_{bfc}T_{ced}T_{dfa}}$$

(Ambjørn, Durhuus Jónsson
Gross, Sasakura '90s)

→ No nice large N expansion!

→ Cannot use matrix techniques

Progress: Gurau, Rivasseau, Bonzom '10

→ more sym. $U(N)^D$: ≠ tensor fields, distinguish indices.

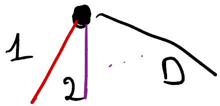
→ Universal large N expansion for any $D \geqslant 3$

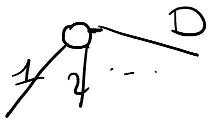
2. Complex colored tensor models

Trace invariants

D

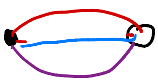
$T_{a_1 \cdots a_D}$ $\qquad$ $\bar{T}_{a_1 \cdots a_D}$

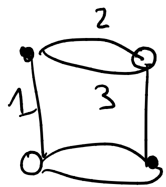


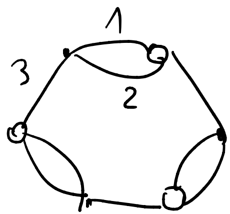


Invariants $U(N)^D$, bubble diagrams

$D = 3$







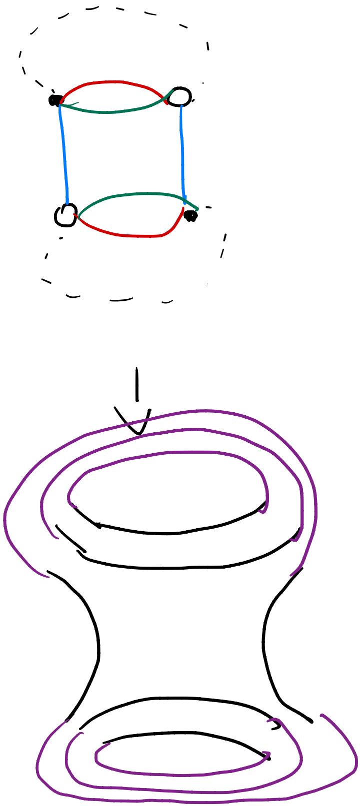
pattern of contr°

$$\tilde{F}(\lambda_B) = \ln \int dT \, e^{-N^{D-1} \bar{T} \cdot T + \sum_{B} \frac{\lambda_B}{N^{\alpha(B)}} \mathrm{Tr}_B(\bar{T}, T)}$$

fix it to have a large N expo

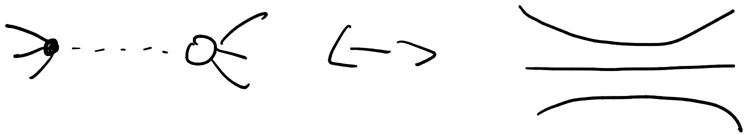
Feynman graphs

Pairings with prop. $\rightarrow$ dashed edge (color 0),



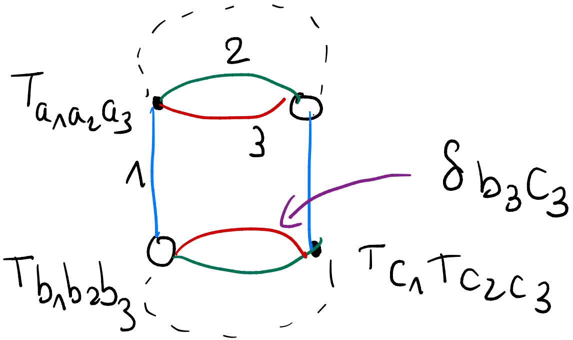
dual to stranded representation

$D=3$



(similar to ribbon graphs for matrices).

Scaling in N



Free sum for faces of color $0i$

$\hookrightarrow \delta_{b_1c_1}\delta_{b_2c_2}\delta_{b_3c_3}$

$$\propto N^{\sum_i F_{0i} - (D-1)p - \sum_{B} \omega(B)}$$

Jackets

$(D+1)$
Colored graph + cyclic permutat° σ on the colors
(except 0)

$\Rightarrow$ combinatorial map J_σ called jacket,

$D!$ jackets

↳ have well-defined notion of genus.

Gurau degree

$$\omega(G) = \frac{1}{2} \sum_{\sigma} \underbrace{g(J_\sigma)}_{\text{positive integer (orientable surface)}},$$

$$\chi = V - E + F = 2 - 2g$$

$$\vdots$$

$$\omega(G) = \frac{(D-1)!}{2}\left(D + \frac{D(D-1)\overbrace{p}^{\text{nb of prop.}}}{2} - \underbrace{F}_{\text{nb of faces}}\right)$$

Proof: Take a jacket J : $|V_J| = |V_G| = 2p$
$|E_J| = |E_G| = (D+1)p$

$$\Rightarrow F_J = 2 - 2g_J + E - V = 2 - 2g_J + (D-1)p$$

G has $D!$ jackets and each face belongs to $2(D-1)!$ jackets
(Face ij belongs to J_σ if $\sigma(i) = j$ or $\sigma(j) = i$)

$$\sum_J \rightarrow 2(D-1)!\, F = 2D! - 4\omega(G) + D!(D-1)p$$

$$\Rightarrow \boxed{\omega(G) = \frac{(D-1)!}{2}\left[D - F + \frac{D(D-1)}{2} p \right]}$$

Large N expansion

$$\mathcal{F}(\lambda_B) = \ln \int dT \, e^{N^{D-1}\left[-\bar{T}\cdot T + \sum_B \lambda_B N^{-\frac{2}{(D-2)!}\omega(B)} \mathrm{Tr}_B(\bar{T},T)\right]}$$

$$= \sum_{\omega \in \mathbb{N}} N^{D-\frac{2}{(D-1)!}\omega} \tilde{\mathcal{F}}_\omega(\lambda_B)$$

Proof:

- $D-1-\frac{2}{(D-2)!}\omega(B)$ per bubble
- $-(D-1)$ per prop.
- 1 per face of color $0i$.

$$ct \propto N^{\sum_i F_{0i} - \sum_B \left(D-1-\frac{2}{(D-2)!}\omega(B)\right) - (D-1)p}$$

$$D-1-F_{B_{\hat{i}\hat{j}}} + \frac{(D-1)(D-2)}{2} p_B$$

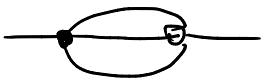
$$\propto N^{F - \frac{D(D-1)}{2} P} \propto N^{D - \frac{2}{(D-1)!} w(G)}.$$

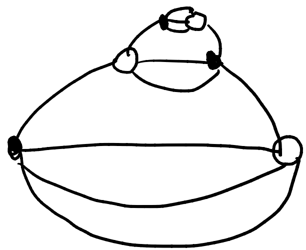
Leading order

$$N^{D - \frac{2}{(D-1)!}\omega(G)}$$ $\rightarrow$ LO : $\omega(G) = 0$

Theorem: G a $(D+1)$-colored graph $\omega(G) = 0$ iff G is melonic.

$D = 3$





Melons

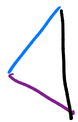
$\rightarrow$ subset of planar diagrams.

$\rightarrow$ not a topological invariant

Random geometry applications

Colored triangulations

D-colored graph $\rightarrow$ colored triangulation

 $\rightarrow$ unambiguous identification of sub-simplices and gluing.

Feynman graph $\leftrightarrow$ D+1-colored graph $\rightarrow$ triangulation of dim $\underline{D}$

$\Rightarrow$ generalization of genus expansion of matrices.

$\omega = 0$ $\rightarrow$ generate D-sphere

Leading order

-> special triangulation of the D-sphere.

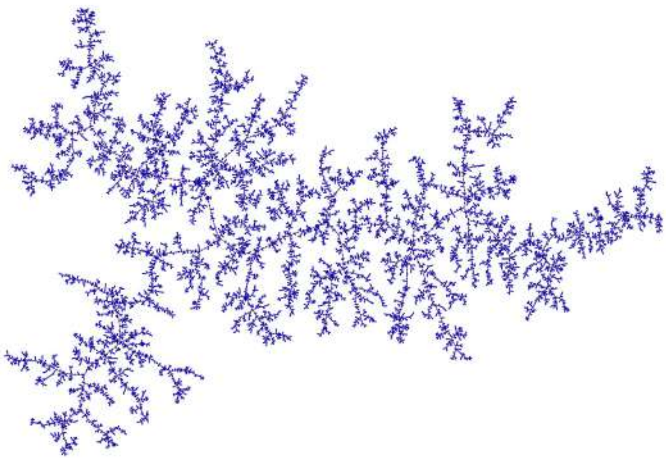
↳ tree-like combinatorial structure.

Why? For each edge can or cannot insert a melon.

-> Random trees $\xrightarrow[\text{continuous limit}]{}$ branched polymers

Branched polymers

$d_{\text{spectral}} = 4/3$ $\qquad$ $d_{\text{Hausdorff}} = 2$



Credit: I. Kortchemski https://igor-kortchemski.perso.math.cnrs.fr/images.html

Further results

- Combinatorial class° $\omega > 0$, double-scaling
- App° to Group Field theory (Oriti).
- Beyond branched polymers → open question

Summary
- TM: well-defined generalizat° of matrix models
- Melonic large N limit.
- Dominated by branched polymers.
- No new universality class so far

What about in higher dim? QM models or QFTs.

3. Melonic CFTs

→ SYK-like model

→ $O(N)^3$ bosonic tensor model

Gurau - Witten model

1610.09758

SYK: QM model of N Majorana fermions strongly interacting.

↳ large N limit dominated by melons.

↳ linked to quantum chaos, near extremal BH

↳ Pb: average over disorder.

Gaussian var.

$$J_{a_1 a_2 a_3 a_4} \psi_{a_1} \psi_{a_2} \psi_{a_3} \psi_{a_4}.$$

Witten:

$$S = \int dt \left(\frac{i}{2} \psi_{abc} \partial_t \psi_{abc} + \frac{g}{4} \psi_{abc} \psi_{ade} \psi_{fbe} \psi_{fdc} \right)$$

→ same critical behavior as SYK model but no disorder.

→ prompted numerous studies and extended to d dimension

CTKT model

Carrozza-Tanasa. Klebanov-Tarnopolsky.

CT: 1512.06718

KT: 1611.08915
1707.03866 + Giombi

Action

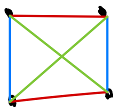
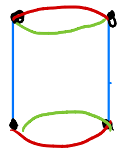
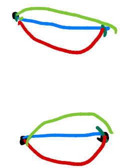
$D = 3$ $\quad O(N)^3$ sym.

bosons.

$$\varphi_{a_1 a_2 a_3} \longrightarrow \varphi'_{b_1 b_2 b_3} = O_1{}^{a_1}{}_{b_1} \, O_2{}^{a_2}{}_{b_2} \, O_3{}^{a_3}{}_{b_3} \, \varphi_{a_1 a_2 a_3}$$

$$S = \int d^d x \, \frac{1}{2} \partial_\mu \varphi_{abc} \partial^\mu \varphi_{abc} + S^{int}$$

$\to$ 3 possible bubbles.

$$S^{int} = \frac{\lambda}{4N^{3/2}}$$  $$+ \frac{\lambda_p}{4N^2}$$  $$+ \frac{\lambda_d}{4N^3}$$ 

tetrahedron

pillow

double-trace

-> int : bubble diagrams 3- colored graphs.

-> Feynman graphs : 4- colored graphs
propagators dashed edges ----

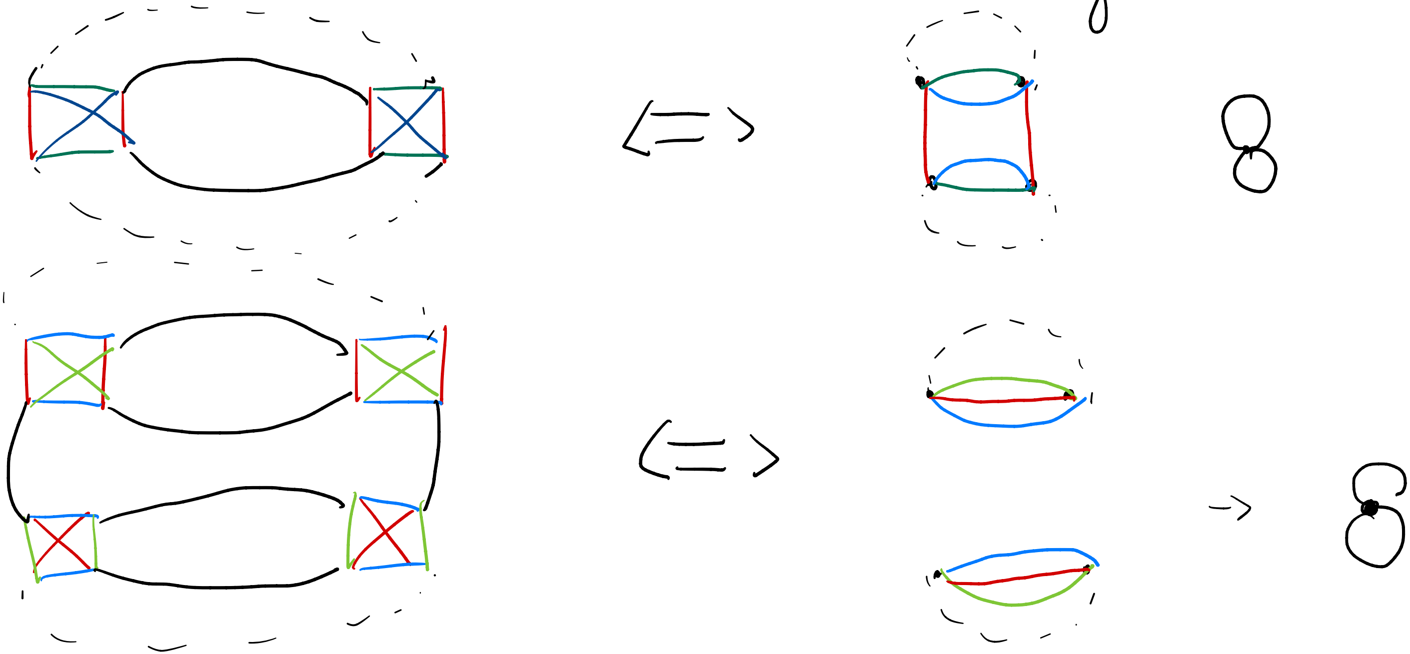
↳ needed for scaling in N

↳ shrink bubbles to a point, usual
4-valent feynman graphs -> enough to
compute integrals.

Large N expansion

One factor N per face of color 0i

Pillow and d.t : radiative corrections from tetra



$G \to \hat{G}$ with only tetrahedrons.

$$\mathcal{A}(\hat{G}) \propto N^{F_{0i}(\hat{G}) - \frac{3}{2} m_t(\hat{G})}$$

- 3 jackets by deleting color c of $\hat{G}$ $\quad \chi = 2 - g$

$$F(J^c) = E(J^c) - V(J^c) + 2 - g(J^c),$$

- 4-valent graph $\to 4V = 2E$
- each face belongs to two jackets.

$$F_{0i}(\hat{G}) = \frac{3}{2} m_t(\hat{G}) + 3 - \overbrace{\frac{1}{2} \sum_c g(J^c)}^{\text{degree} \geq 0}.$$

half-integer $\omega(\hat{G})$.

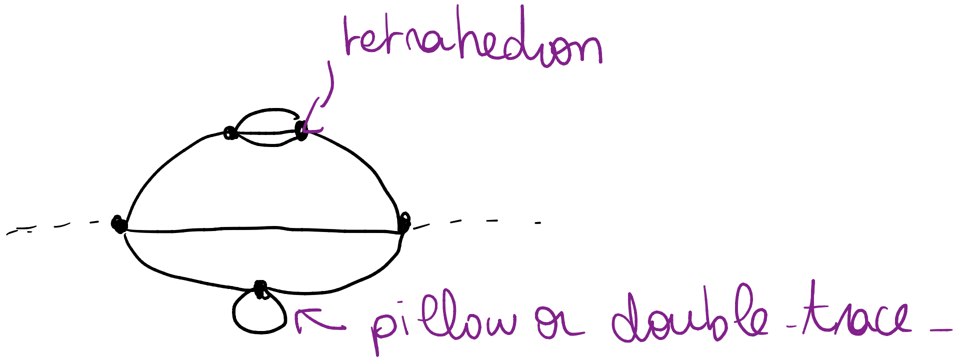
$$\Rightarrow \quad N^{3 - \omega(\hat{G})}.$$

$\omega(\hat{G}) = 0$ iff $\hat{G}$ is melonic.

Melon-tadpoles

LO : melons in terms of tetrahedral vertices

→ Back to original graph: melon-tadpole



Divergences and renormalization scheme

quartic model $\rightarrow$ same power counting as usual φ^4.

$\hookrightarrow d=4$ 2pt fct° : power div.

4pt fct° : log div

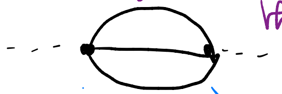
$\geqslant 6$ pt : UV conv.

Dimensional regularization: $d = 4 - \varepsilon$

Subtract° at zero momentum, mass regulator μ -

SDE

$$G(p)^{-1} = \overbrace{C(p)^{-1}}^{p^2} - \Sigma(p)$$

$\Sigma(p) = \lambda^2$  $- \lambda_p$  $- \lambda_d$ 

decorated by the full 2pt. fct.

→ in the large N limit → scale $p^{d/2}$

UV: $G(p)^{-1} \sim p^2$ → free scaling

IR: $G(p)^{-1} \sim p^{d/2}$ → anomalous scaling

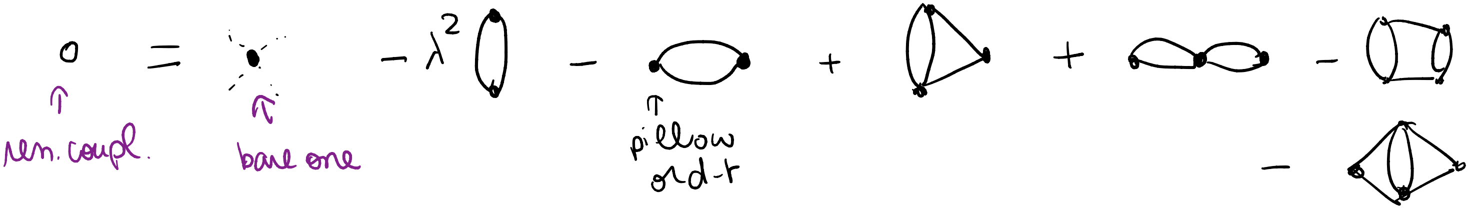
Neglect $C(p)^{-1}$ in the IR, solve SDE $G(p) = \dfrac{Z}{p^{d/2}}$ wave function renormalizat°

$$\Rightarrow \quad Z = \left(\frac{d/4 \; \Gamma(3d/4)\,(4\pi)^d}{\lambda^2 \; \Gamma(1-d/4)} \right)^{1/4}$$

4-point function

Tetrahedron: at large N $\Rightarrow$ no radiative corrections.

Pillow, double-trace: bare expansion.



Reparametrization: $\lambda_1 = \dfrac{\lambda_p}{3}$ and $\lambda_2 = \lambda_d + \lambda_p$

$\hookrightarrow$ orthogonal.

Beta functions

$$g_1 = \mu^{-\varepsilon} Z^2 \Gamma^{(4,1)}(0,0,0,0) \,, \quad g_2 = \mu^{-\varepsilon} Z^2 \Gamma^{(4,2)}(0,0,0,0)$$

$$\beta_{g_i} = \mu \partial_\mu g_i$$

$$\Rightarrow \quad \beta_t = -\varepsilon g + 2g^3$$

$$\beta_1 = -\varepsilon g_1 + 2g^2 + 2g_1^2 - 2g^2 g_1$$

$$\beta_2 = -\varepsilon g_2 + 6g^2 + 2g_2^2 - 10g^2 g_2$$

Fixed points

- Trivial sol°
- Non-vanishing tetrahedron: $g^* = \sqrt{\frac{\varepsilon}{2}}$

$$\Rightarrow \quad g_1^* = \pm i\sqrt{\frac{\varepsilon}{2}}$$

$$g_2^* = \pm i\sqrt{\frac{3\varepsilon}{2}}$$

→ purely imaginary g_1, g_2 → pb because of reflect° positivity

→ dimension of $\mathcal{O} = \varphi_{abc}\varphi_{abc}$

$$\Delta_{\mathcal{O}} = \frac{1}{2}\left(2(d-2) + 2g_2^*\right) = 2 \pm i\sqrt{6\varepsilon} + \mathcal{O}(\varepsilon)$$

→ unstable, non-unitary CFT.

Long-range model

Based on:

1903.03578

1909.07767

2002.07652

2011.11276

2111.11792

Action

$$S = \frac{1}{2}\int d^d x \; \varphi_{abc} (-\Delta)^{\zeta} \varphi_{abc} + S^{int}$$

same as for CTKT model.

-> long-range model.

$0 < \zeta < 1$ ← preserve reflect° positivity

↑ well-defined thermodynamic limit

-> Vast array of app°, LR Ising

0907.0323 Campa, Dauxois Ruffo.

Canonical dim° of the field $\Delta_\varphi = \frac{d-2\zeta}{2}$

↳ quartic int° : irrelevant $\zeta < d/4$ -> mean-field behavior

relevant $\zeta > d/4$

marginal $\boxed{\zeta = d/4}$ -> $C(p) = \frac{1}{p^{d/2}}$ IR scaling

Power counting

for $\gamma = \frac{d}{4}$ $\quad d < 4$

$\rightarrow$ 2pt: power div, 4pt: log div, $\geqslant$ 6pt: UV convergent.

Dim reg: Fix d, $\gamma = \frac{d+\varepsilon}{4}$

Subtraction at zero momentum, regulator μ.

2-point function

decorated by full 2pt. fct°

$G(p)^{-1} = p^{2\varsigma} - \lambda^2$  $+ \lambda_p$  $+ \lambda_d$ 

0 in dim reg.

$\varsigma = d/4$

have same scaling

$$G(p) = \frac{Z}{p^{d/2}} \Rightarrow \frac{1}{Z^4} - \frac{1}{Z^3} = \frac{\lambda^2}{(4\pi)^d} \frac{\Gamma(1-d/4)}{\frac{d}{4}\,\Gamma(3d/4)}$$

$d<4$ finite

$\Rightarrow$ No wave function renormalization.

(finite rescaling).

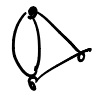
Beta functions

↳ no term from wave fct° renormalization.

$$\beta_t = 0$$

$$\beta_1 = \qquad + 2(g_1^2 + g^2) + 4\alpha_{s,0}\, g_1 g^2$$

$$\beta_2 = \qquad + 2(g_2^2 + 3g^2) + 12\alpha_{s,0}\, g_2 g^2$$

$\alpha_{s,0} \rightarrow$ 

$$= 2\psi(d/4) - \psi(d/2) - \psi(1).$$

→ g free parameter. $\quad g = Z^2 \lambda$

$$g_c^{-2} = \Gamma(1 - d/4)\left[(4\pi)^d\,(d/4)\,\Gamma\left(\frac{3d}{4}\right)\right]^{-1} \Rightarrow Z = 1 - \frac{g^2}{g_c^2}$$

λ real: $\lambda(g)$ invertible to $g(\lambda)$ for any λ, $\quad Z \xrightarrow[\lambda \to \infty]{} 0$

λ purely imaginary: $\lambda(g)$ invertible up to $|g| < 3^{-1/2} g_c$. $\quad Z$ bounded.

Fixed points

$$g_1^* = \pm\sqrt{-g^2} - \alpha_{S,0}\, g^2 + \mathcal{O}(g^3)$$

$$g_2^* = \pm\sqrt{3}\sqrt{-g^2} - 3g^2\alpha_{S,0} + \mathcal{O}(g^3).$$

$\rightarrow$ g real : g_1^*, g_2^* imaginary.

g purely imaginary : g_1^*, g_2^* real.

Four lines of fixed points.

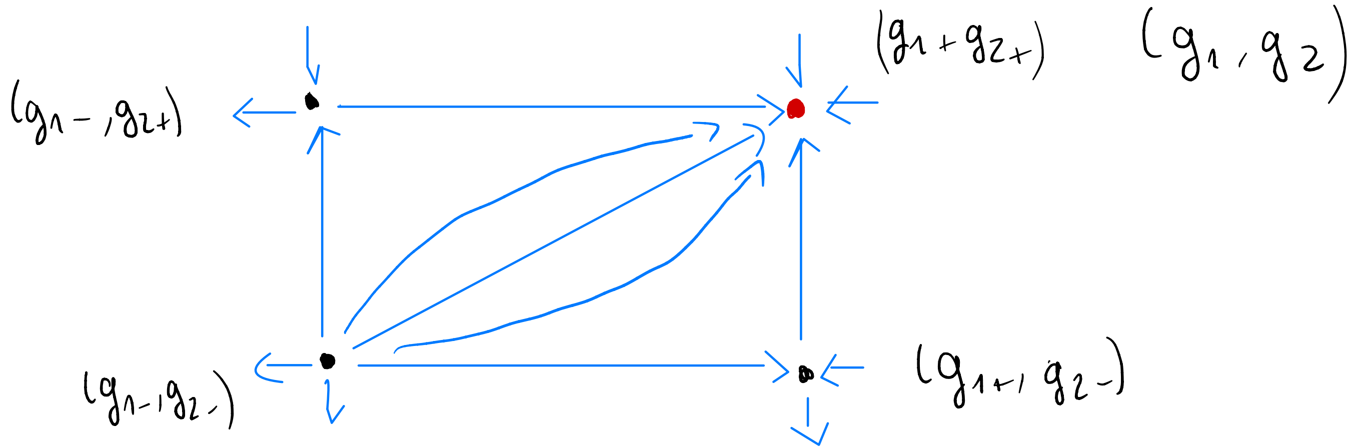
Critical exponents

→ triangular stability matrix

$$\partial\beta_1(g^*) = \mp 4\sqrt{-g^2} + O(g^3)$$
$$\partial\beta_2(g^*) = \pm 4\sqrt{-3g^2} + O(g^3)$$

} real if g purely imaginary.

→ one IR attractive stable FP.

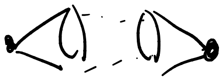


Non-perturbative result

Idea: chain 1V irreducible pieces.

3 types: pure ladders  $U(g) = g^2 + g^4 + \dots$

caps  $S(g) = \lambda_1 + \lambda_1 g^2 + \dots$

double-caps  $T(g) = \lambda_1^2 + \lambda_1^2 g^2 + \dots$

$$g_1 = -U(g) + \lambda_1 \frac{(1 + S(g))^2}{1 + \lambda_1 T(g)}$$ $\to$ exact

$$\Rightarrow \quad \beta_{g_1} = \underline{\beta_0^g} - 2\underline{\beta_1^g}\, g_1 + \underline{\beta_2^g}\, g_1^2$$ | quadratic in g_1^2

series in g^2.

$\beta_0{}^g$, $\beta_1{}^g$ $\beta_2{}^g$ $\rightarrow$ finite at all orders, sum is also finite
(multi-scale analysis, taylor sub° operator).

$\Rightarrow$ Lines of FP : non-perturbative.
↳ stable FP $\rightarrow$ strongly-coupled FP.

Unitarity?

$$\varphi_1(x_1)\,\varphi_2(x_2) = \sum_{\mathcal{O}\ \text{primary}} \overbrace{C_{\mathcal{O}}}^{\text{OPE coeff.}} \underset{\mathcal{O}}{\bullet\!\!-\!\!-\!\!\bullet}$$

4pt function CFT

$$\langle\varphi\varphi\varphi\varphi\rangle = \langle\varphi\varphi\rangle\langle\varphi\varphi\rangle + \sum_J \int_{\frac{d}{2}-i\infty}^{d/2+i\infty} \frac{dh}{2\pi i}\,\frac{1}{1-\underbrace{k(h,J)}_{\text{eigenvalues of the 4pt kernel.}}}\,\overbrace{\mu(h,J)}^{\text{measure}}\,\overbrace{G^{\Delta_\varphi}_{h,J}(x_i)}^{\text{conformal block}}$$

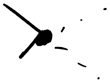
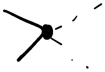
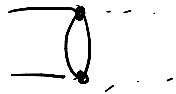
Deform the contour and pick up poles.

$\rightarrow$ only contributing poles $h_{m,J}$ s.t. $k(h_{m,J}, J) = 1$

$\hookrightarrow$ dim° of bilinears

$\hookrightarrow$ residue at these poles are $C_{m,J}^2$: square of the OPE coeff.

Kernel is known in the large N limit.

$K = -\lambda_p$  $-\lambda_d$  $+3\lambda^2$ 

↳ diagonalized by the 3pt functions.

→ Dim° of bilinears:

$$h_{0\pm} = \frac{d}{2} \pm \alpha_0 \sqrt{-g^2} + O(g^3)$$

$$h_n = \frac{d}{2} + 2n + \alpha_n g^2 + O(g^3)$$

→ g purely imaginary: real dimensions.

→ Real OPE coeff.

Further work

NLO ? $O(N^3)$,

SR ?

- small N
- vector-like large N : $\overset{\to\infty}{O(N_1)} \times \overset{\text{fixed}}{O(N_2)} \times O(N_3)$ } no FP with non zero tetrahedral coupling.
- matrix like :

Short-range model: NLO -> critical exp acquire a real part
-> true stable FP.

Long-range model: NLO -> critical exp get complex correct°
-> breaking unitarity at NLO.

F-theorem

→ generalization in $d = \underline{3}$ of the c-theorem, a-theorem.
$(d=2)$ $(d=4)$

→ sphere free energy decrease along the RG flow.

→ $(\Delta)^{\varsigma}$ on the sphere?

→ is satisfied for the long-range $O(N)^3$ model.

↳ perturbatively as a reversal of sign of a coupling cst.

⇒ More hints of unitarity at large N.

Other models

Model	Sym.	d	FP	Stable	Unitary	NLO
CTKT [1]	$O(N)^3$	$4-\epsilon$	Complex	×	×	stable FP
BGHS [2]	$O(N)^3$	$d<4$	Real	✓	✓	Non-unitary
Prismatic [3]	$O(N)^3$	$3-\epsilon$	Real	✓	✓	✓
Sextic [4]	$U(N)^3$	$3-\epsilon$	Real	✓	×	?
Sextic [4]	$U(N)^3$	$d<3$	Real	✓	?	?
Rank 5 [4]	$O(N)^5$	$3-\epsilon$	Trivial	-	-	-

[1][Carrozza, Tanasa, Klebanov, Tarnopolsky, Giombi, ...] [2][Benedetti, Gurau, SH, Suzuki]
[3][Giombi, Klebanov, Popov, Prakash, Tarnopolsky] [4][Benedetti, Delporte, SH, Sinha]

Sextic int°

$U(N)^3$ → SR : two stable FP → NLO still exist.
→ LR : line of FP → no precursor

$O(N)^5$ → SR/LR : gaussian FP → rank imp. parameter.

→ supersymmetric models (Popov, Lettera-Vichi)

→ Fermionic models (Prakash, Simha).

→ Tensorial Gross-Neveu (Benedetti, Carrozza, Gurau, Sfondrini, Delporte),

→ Multi-matrix (Ferrari, Rivasseau, Tanasa, Toriumi, Valette),

Conclusion

Tensor models for strongly-coupled QFTs:

- → Third family of large N theories, both rich and tractable.
- → Can reproduce SYK-like physics without disorder.
- → New family of CFTs "melonic CFTs" that can be studied analytically.
  - ↳ LR $O(N)^3$, unitary at large N.
- → Allows us to test properties of QFT in rigorous set ups → F-theorem.

Open questions:

- Higher order int° : can we find similar FP?
- Diff. sym. : rank 3, J, |sym. traceless tensors
  |anti-sym
  - ↳ what about QFT based on this model?
- What about the AdS dual?
  - ↳ Mello-Koch.
  - ↳ Hard question : factorial growth of the invariant