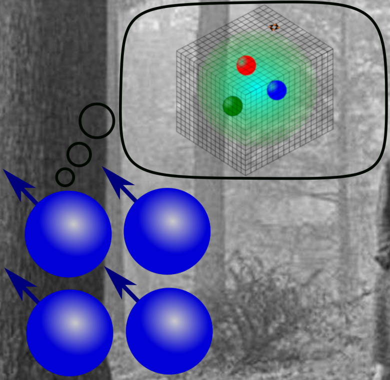
Quantum Field Theory II

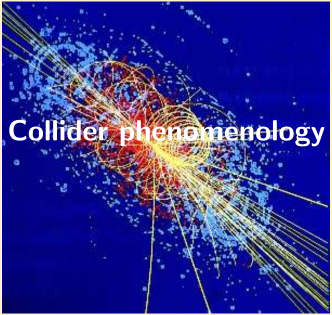
Hank Lamm

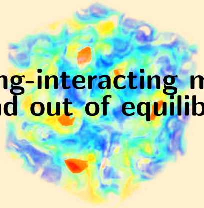


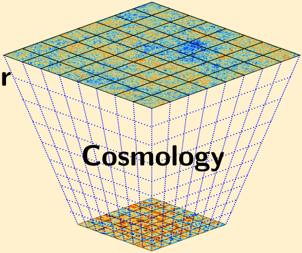
Disclaimer: Problems and solutions via my aesthetic

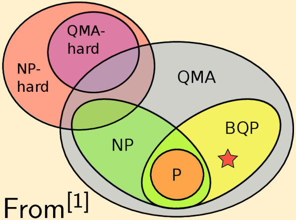
- If it **ain't** broke, don't fix it
- **Premature optimization** is the root of all evil
- **QCD** is my target

Fundamentally, HEP requires QC[2]

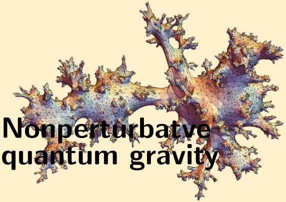








From[1]



[1] Kassal, I., J. D. Whitfield, A. Perdomo-Ortiz, M.-H. Yung, and A. Aspuru-Guzik. In: *Annual review of physical chemistry* 62 (2011).
[2] Bauer, C. W. et al. In: (Apr. 2022). arXiv: 2204.03381 [quant-ph].

Gut check!

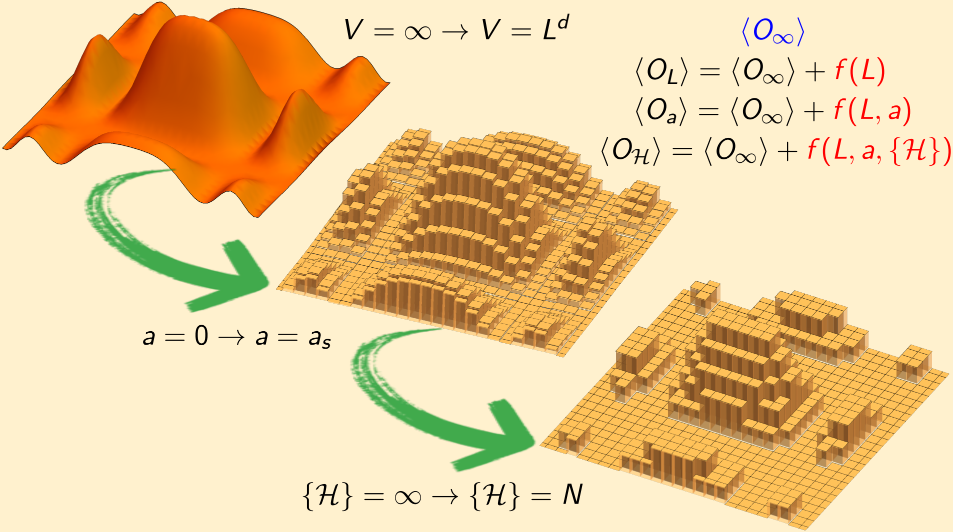
Suppose we wanted to run a circuit on a **100q** with each qubit is acting on by an entangling gate. Could we achieve 50% overall success if the gate fidelity is **95%? 99%?**

This is the **biggest** thing to remember about **current** QC

Gut check!

Suppose we wanted to run a circuit on a **100q** with each qubit is acting on by a **3q** entangling gate. Could we achieve 50% overall success if the gate fidelity is **95%? 99%?**

QFT is about infinities and how to regulate them



I'm sometimes going to talk about lattice actions

$$\langle x|e^{-iHt}|y\rangle = \int \mathcal{D}\phi e^{iS}$$

The **anisotropic Wilson** action is

$$S_{\rm W} = \frac{1}{g_t^2}\xi \sum_t \text{Tr}\, U_t + \frac{1}{g_s^2}\frac{1}{\xi}\sum_s \text{Tr}\, U_s \tag{1}$$

thru **transfer matrix**[3], $\langle i|e^{-a_0 H}|j\rangle$ derives the H_{KS}

$$H_{KS} = \frac{c}{a_s}\left[\frac{g_H^2}{2}\sum_l E_l^2 + \frac{1}{g_H^2}\sum_p \text{Tr}\, U_p\right] \tag{2}$$

...**but** this derivation requires an approximation to get E_l^2!

- H_{KS} **isn't** the Hamiltonian, but a choice with $O(a_s^2)$ errors

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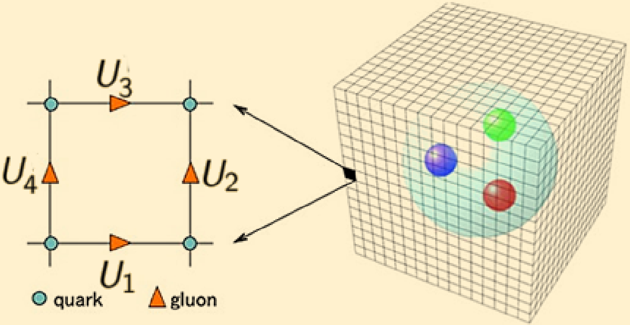
[3] Creutz, M. *Quarks, gluons and lattices*. Cambridge Monographs on Mathematical Physics. Cambridge, UK: Cambridge Univ. Press, June 1985.

LFT has been successful beyond our wildest dreams

$$S_\infty = \int d^4x \left[-\frac{1}{4} F^{\mu\nu} F_{\mu\nu} + \bar{q}(i\not{D} - m)q \right]$$

$$S_W = \sum_x [\beta \, \mathrm{Re}\, \mathrm{Tr}(1 - U_p) + S_f] \text{ with } U_p = U_1 U_2 U_3^\dagger U_4^\dagger \text{ and } U_i = e^{i a_\mu A^\mu}$$



Wick rotate $t \to i\tau$ then **sample** from e^{-S_R}

LFT can compute **most** $\langle \psi_i | \prod_n \mathcal{O}_n(\tau_n) | \psi_i \rangle = \frac{\int \mathcal{D}\phi e^{-S_R} \prod_n \mathcal{O}_n(\tau_n)}{\int \mathcal{D}\phi e^{-S_R}}$

So many choices of fermions

Nielsen-Ninomiya theorem: Assuming **locality, hermiticity, and translational symmetry**, any lattice chiral fermions have **doublers**

Ginsparg-Wilson equation: Introduce a concept of **lattice** chiral symmetry that recovered true chiral symmetry **in the continunum**

$$D\gamma_5 + \gamma_5 D = aD\gamma_5 D$$

1. *Staggered (KS) Fermions*: Spin-taste components on different lattice sites in hypercube
2. *Wilson Fermions*: add a new term to give additional mass to doublers
3. *Domain wall Fermions*: Increase dimensionality
4. *Overlap Fermions*: Use nonlocal operator to remove doublers
5. ...others

These are **categories**, which can be **improved** to remove **lattice artifacts**
Not all are formulated in **Hamiltonian** (aka for QC)...if you want a research project

So ahead of the curve, the curve becomes a sphere

(1970s) Formulate th...

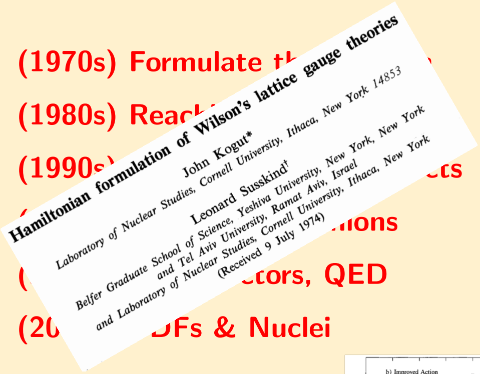
(1980s) Reac...

(1990s) ...cts

(... ...ions

(... ...ctors, QED

(20... ...DFs & Nuclei


Hamiltonian formulation of Wilson's lattice gauge theories

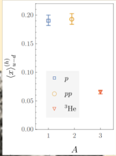
John Kogut*

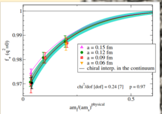
Laboratory of Nuclear Studies, Cornell University, Ithaca, New York 14853

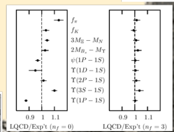
Leonard Susskind†

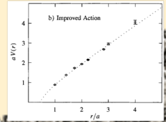
Belfer Graduate School of Science, Yeshiva University, New York, New York and Tel Aviv University, Ramat Aviv, Israel and Laboratory of Nuclear Studies, Cornell University, Ithaca, New York

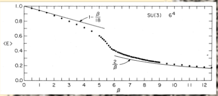
(Received 9 July 1974)











Confinement of quarks*

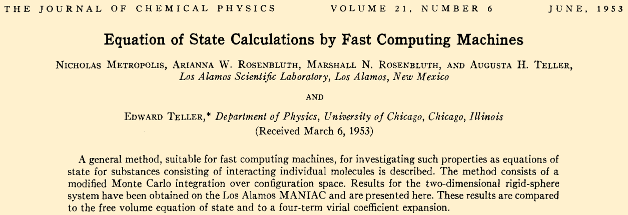
Kenneth G. Wilson

Laboratory of Nuclear Studies, Cornell University, Ithaca, New York 14850

(Received 12 June 1974)

What about Monte Carlos?

When the state space gets too big, to evaulate $\int \mathrm{d}x\, p(x)$, randomly sample values according to $p(x)$



THE JOURNAL OF CHEMICAL PHYSICS VOLUME 21, NUMBER 6 JUNE, 1953

Equation of State Calculations by Fast Computing Machines

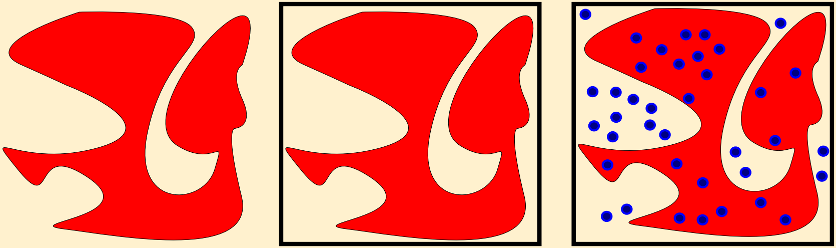
NICHOLAS METROPOLIS, ARIANNA W. ROSENBLUTH, MARSHALL N. ROSENBLUTH, AND AUGUSTA H. TELLER,
Los Alamos Scientific Laboratory, Los Alamos, New Mexico

AND

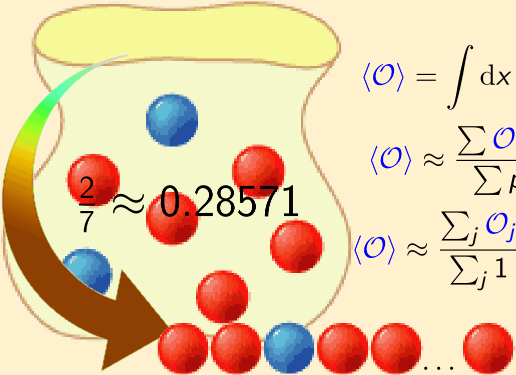
EDWARD TELLER,* *Department of Physics, University of Chicago, Chicago, Illinois*

(Received March 6, 1953)

A general method, suitable for fast computing machines, for investigating such properties as equations of state for substances consisting of interacting individual molecules is described. The method consists of a modified Monte Carlo integration over configuration space. Results for the two-dimensional rigid-sphere system have been obtained on the Los Alamos MANIAC and are presented here. These results are compared to the free volume equation of state and to a four-term virial coefficient expansion.



Monte Carlo methods present a practical solution...



$\frac{2}{7} \approx 0.28571$

$$\langle \mathcal{O} \rangle = \int \mathrm{d}x\, \mathcal{O} p(x)$$

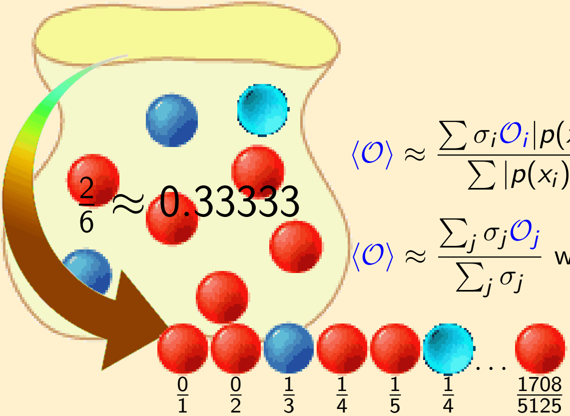
$$\langle \mathcal{O} \rangle \approx \frac{\sum \mathcal{O}_i p(x_i)}{\sum p(x_i)}$$

$$\langle \mathcal{O} \rangle \approx \frac{\sum_j \mathcal{O}_j}{\sum_j 1} \text{ where } j \text{ sampled with } p(x_j)$$

$\frac{0}{1}$ $\frac{0}{2}$ $\frac{1}{3}$ $\frac{1}{4}$ $\frac{1}{5}$ $\ldots$ $\frac{1708}{5979} \approx 0.28566$

- As $N \to \infty$, $\langle \mathcal{O} \rangle \to \mathcal{O}_{\mathrm{exact}}$.
- Computable uncertainty which decreases as N grows!
- ...but what if $p(x_i) \neq [0, 1]$ (e.g. e^{-S} is not real)

Monte Carlo methods present a practical solution...



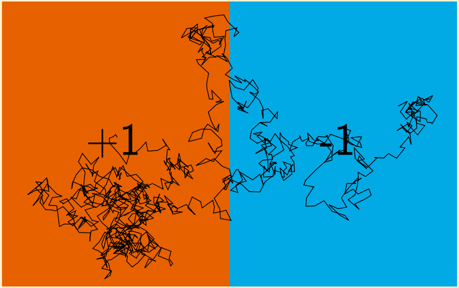
$$\langle\mathcal{O}\rangle \approx \frac{\sum \sigma_i \mathcal{O}_i |p(x_i)|}{\sum |p(x_i)|}\frac{\sum |p(x_i)|}{\sum \sigma_i |p(x_i)|}$$

$$\langle\mathcal{O}\rangle \approx \frac{\sum_j \sigma_j \mathcal{O}_j}{\sum_j \sigma_j} \text{ where } j \text{ sampled with } |p(x_j)|$$

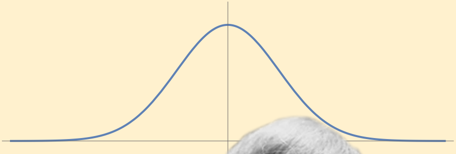
- Reweighting: assign probabilities $|p(x_i)|$ and make the relative sign, σ_i part of the observable
- ...but what happens when the cancellations are strong?

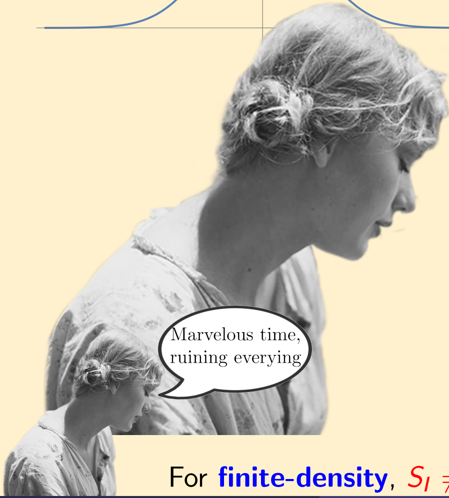
...but struggle with sign problems

- Sign problem: *when stochastic sampling requires precise cancellations of positive and negative contributions, which is generically* **exponentially** *bad in particle number or volume*
- $\int_{-1}^{1} \mathrm{d}x \int_{-1}^{1} \mathrm{d}y \, [\Theta(-x) - \Theta(x)] = 0$



Sign(al-to-noise) problems stymie HEP





$S_I = 0,\ \langle\sigma\rangle_{S_R} = 1$

$$\langle\mathcal{O}\rangle = \frac{\int \mathcal{D}\phi\ e^{-iS_I}\mathcal{O}e^{-S_R}}{\int \mathcal{D}\phi\ e^{-S_R}} \frac{\int \mathcal{D}\phi\ e^{-S_R}}{\int \mathcal{D}\phi\ e^{-S_R}e^{-iS_I}}$$

$$= \frac{\left\langle \mathcal{O}e^{-iS_I}\right\rangle_{S_R}}{\langle\sigma\rangle_{S_R}}$$

$S_I \neq 0,\ \langle\sigma\rangle_{S_R} \leq 1$

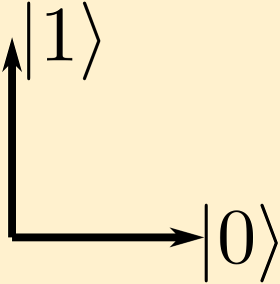
For **finite-density**, $S_I \neq 0$! For **dynamics**, $S_R = 0$!

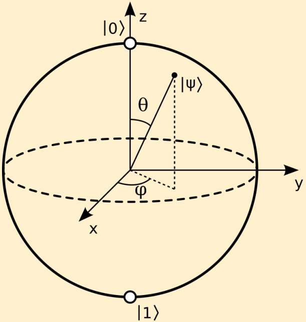
Stated succinctly...

$|\psi\rangle$ is a **complex-valued** probability amplitude

What do I gain with a quantum computer?[4]

$$\langle\psi_i|\prod_n \mathcal{O}_n(t_n)|\psi_i\rangle = \langle\psi_i|e^{iHt_0}\mathcal{O}_0 e^{iH\delta t}\mathcal{O}_1 \ldots e^{-iHT}|\psi_i\rangle$$





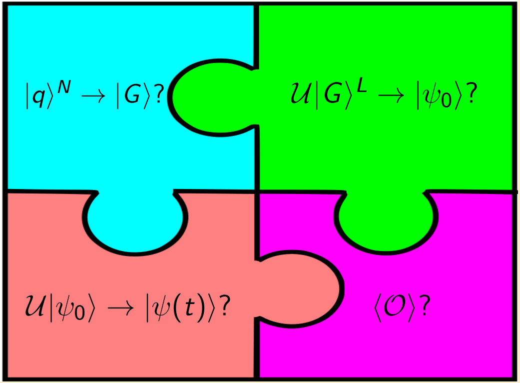
QC can **efficently represent** superpositions and entanglement

Digital QC provide entangled qubits and gates, **not** field theories.

[4] Feynman, R. P. In: *Int.J.Theor.Phys.* 21 (1982).

What “champagne problems” need to be solved?

- **Encoding**: How are fields represented as registers?
- **Initalize**: How can registers be set to a state?
- **Propagate**: How can gates evolve states?
- **Evaluate**: How can observables be computed?



- **Mitigate**: Can LFT-specific QEC/QEM be cheaply designed?

Fermions on quantum computers

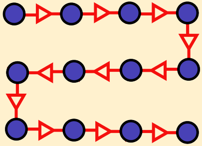
Most quantum computers are built from **bosonic** degrees of freedom

This is a **problem**... since fermions **anticommute**, $\{\psi_a, \psi_b\} = \delta_{ab}$

Fermionic states are fully antisymmetric $\implies$ **nontrivial map** to qudits

Most common...but there are others

- **Jordan-Wigner**: $a_j = -(\otimes_{k=1}^{j-1} Z_k) \otimes \sigma_j \implies$ Good in 1+1d, but...



- **Bravyi-Kitaev**: Uses parity get $\mathcal{O}(\log(m))$ gates

Can be application limited e.g. some work poorly for LGT

What about **QEC+fermion encodings**?[5] ...if you want a research project

[5] Landahl, A. J. and B. C. A. Morrison. In: (Oct. 2021). arXiv: 2110.10280 [quant-ph].

How do I digitize a gluon?

All things considered...

Exploring Digitizations of Quantum Fields for Quantum Devices

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Hersh Singh,[5,6,§] Jesse Stryker,[4,6,¶] and Judah Unmuth-Yockey[7]
[1] *University of Iowa, Iowa City, Iowa, 52242***
[2] *Department of Physics, Boston University, 590 Commonwealth Avenue, Boston, MA 02215, USA*
[3] *Fermi National Accelerator Laboratory, Batavia, Illinois, 60510, USA*
[4] *Maryland Center for Fundamental Physics and Department of Physics, University of Maryland, College Park, MD 20742, USA*
[5] *Department of Physics, Box 90305, Duke University, Durham, North Carolina 27708, USA*
[6] *Institute for Nuclear Theory, University of Washington, Seattle, WA 98195, USA*
[7] *Syracuse University, Syracuse NY*[††]

In this LOI we undertake to enumerate promising digitization schemes for quantum fields that could allow near-term calculations on quantum devices. Further we discuss the outstanding questions that must be resolved in evaluating their potential, providing potential benchmarking on the way to practical quantum advantage in high energy physics.

Lots of choices for digitizing gauge bosons[6]:

- Some combination of: Hamiltonian, basis, and **truncation**
- I am going to focus on **discrete subgroups**

What qualities make a GOOD scheme?

- What **quantum resources** are required to get physical point?
- What symmetries are being **broken** in digitization?
- Can the scheme be simulated **classically**?

[6] Gustafson, E. et al. In: *Snowmass 2021 LOI* TF10-97 (2020).

Example of digitization:

Start from **Kogut-Susskind** Hamiltonian (a **lattice-reg'd** version of H):

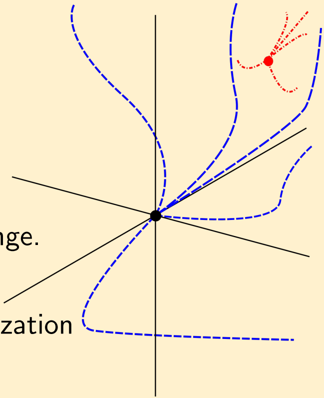
$$H_{KS} = \frac{c}{a_s}\left[\frac{g_H^2}{2}\sum_l E_l^2 + \frac{1}{g_H^2}\sum_p \mathrm{Tr}\, U_p\right]$$

Notice there are two **natural** basis: E_l-basis & $U-$basis
Truncate the basis, e.g. $E_l \leq E_{\max}$ but now you aren't using H_{KS}

$$H_{\rm trunc} = \frac{c}{a_s}\left[\frac{g_H^2}{2}\sum_l E_l^2 + \frac{1}{g_H^2}\sum_p \mathrm{Tr}\, U_p\right] + \mathcal{O}_{\rm trunc}$$

$\mathcal{O}_{\rm trunc}$ may break symmetries, unitarity – and could be **relevant** operator – and will be affected by **noise**

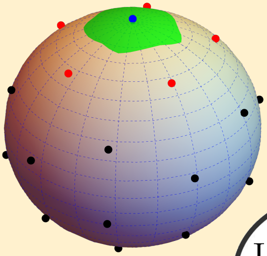
This is not a triviality!



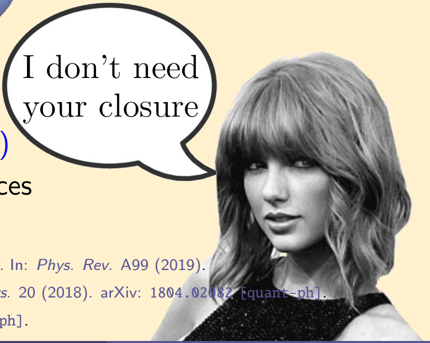
- This **defines** your EFT
- Qubit costs scale as **function of** a_s
- **Continuum theory** approximated can change.
- **Mixing** of matrix elements under renormalization
- O_{trunc} is **not** necessarily obtained from replacement e.g. $U \to U + \delta$ but can be **lowest** dimension operator which breaks symmetry

Discrete subgroups allow plug-and-play[7][8][9]

Replace $G \to H$ in $e^{-S}, e^{-i\mathcal{H}}$



- $SU(3) \to \mathbb{V}$ reduces qubits by $O(10^2)$
- I **believe** endgame will be **3x3** matrices

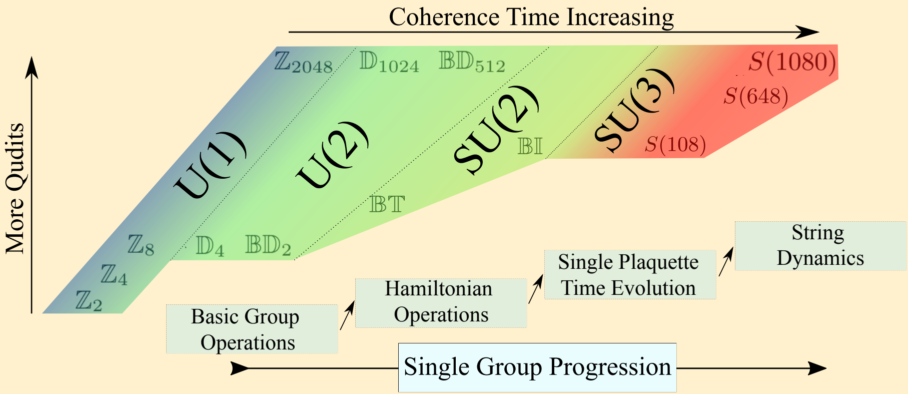


[7] Bhanot, G. In: *Phys. Lett.* 108B (1982), Hackett, D. C. et al. In: *Phys. Rev.* A99 (2019).

[8] Bender, J., E. Zohar, A. Farace, and J. I. Cirac. In: *New J. Phys.* 20 (2018). arXiv: 1804.02082 [quant-ph].

[9] Haase, J. F. et al. In: (June 2020). arXiv: 2006.14160 [quant-ph].

Approximating Continuous Gauge Groups

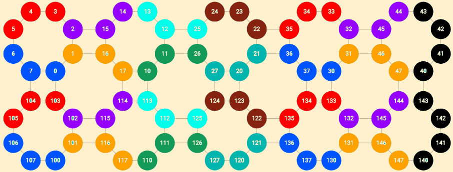


For **any** finite group, we can map elements g_i to **integers** i.

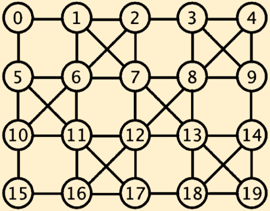
Then **encode** g_i into qubits via the **bit-string** of the integer

For example: $|g_{23}\rangle = |23\rangle = |10111\rangle$

What might a register look like?[10]

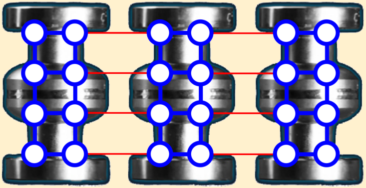


Multi-Qudits per $|g\rangle$



1 Qudit per $|g\rangle$

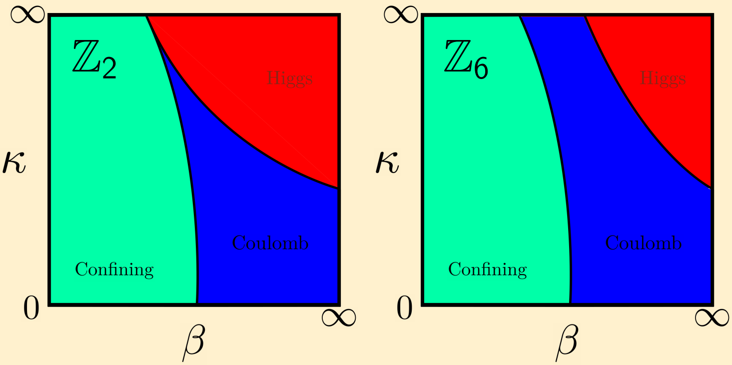
Multi-$|g\rangle$ per Qudit



[10] Ciavarella, A., N. Klco, and M. J. Savage. In: (Jan. 2021). arXiv: 2101.10227 [quant-ph].

Discrete groups can't reach continuum[11][12][13]

$$S = \beta \sum \mathrm{Re}\,\mathrm{Tr}\, U_P + \kappa \sum \phi \cdot D\{U_P\} \cdot \phi^\dagger + c.c.$$

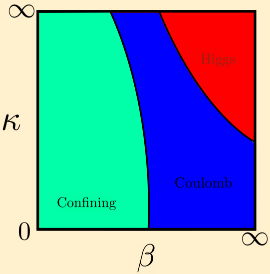


Integrating over ϕ leads to S_{eff} with new irreps of G

[11] Fradkin, E. H. and S. H. Shenker. In: *Phys. Rev. D* 19 (1979).
[12] Horn, D., M. Weinstein, and S. Yankielowicz. In: *Phys. Rev. D* 19 (1979).
[13] Labastida, J. M. F., E. Sanchez-Velasco, R. E. Shrock, and P. Wills. In: *Phys. Rev. D* 34 (1986).

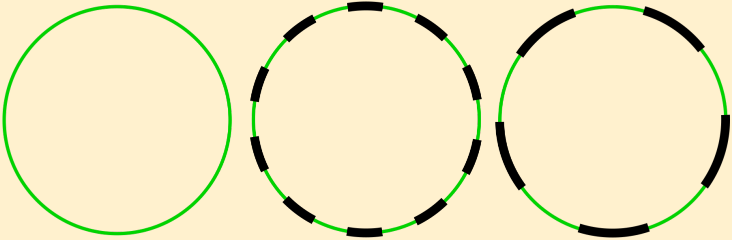
So, discrete groups are continuous groups+Higgs

- Starting from G **coupled to** ϕ
- The rep of ϕ **determines** the breaking $G \to H$
- **Higher** rep (larger H) $\to$ **smaller** a_f
- **Dislike this?** note that $SO(4)$ is **never** recovered for $O(1/a)$ states
- On-going work to understand how Higgs couples to **Nonabelian** G[14]



[14] Das, S. and A. Hook. In: *JHEP* 10 (2020). arXiv: 2006.10767 [hep-ph].

So how can we predict a_f?[15]



$$\beta_{f,U(1)} = \frac{\log(1+\sqrt{2})}{1-\cos\left(\frac{2\pi}{N}\right)} \approx \kappa_2 N^2, \text{ which extends to } \beta_{f,SU(N_c)} \approx \kappa N^{\frac{N_c^2-1}{2}}$$

But whereas $\mathbb{Z}_N$ can be **taken to ∞**, **limited** number for $SU(N_c)$

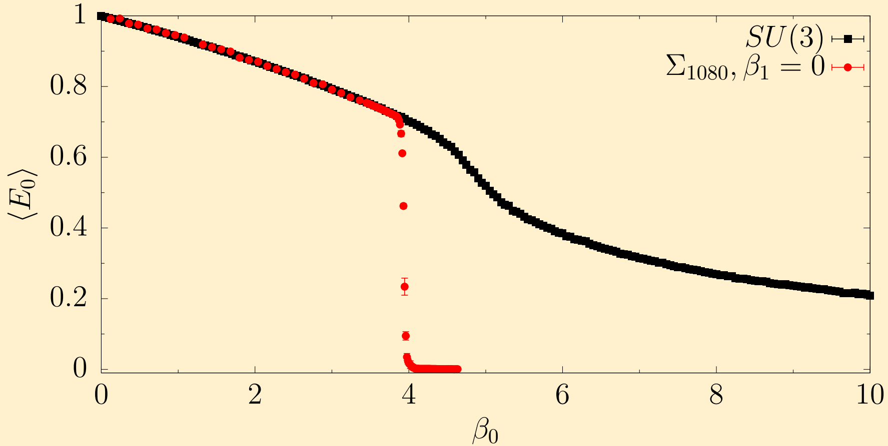
$$\beta \propto \frac{1}{\log(a)} \implies a_f \propto e^{-\beta_f}$$

So the important question is $a_s > a_f$?

[15] Petcher, D. and D. H. Weingarten. In: *Phys. Rev.* D22 (1980), Hartung, T., T. Jakobs, K. Jansen, J. Ostmeyer, and C. Urbach. In: (Jan. 2022). arXiv: 2201.09625 [hep-lat].

What do we know from Wilson Action?

- $U(1) \to \mathbb{Z}_N$, $N > 4$
- $SU(2) \to \mathbb{BO}, \mathbb{BI}$
- $SU(3) \to \mathbb{V}$ has $\beta_f = 3.935(5) < \beta_s \approx 6$
- One **1152** qubit $SU(3)$ link vs $\sim 4^3$ lattice of **11** qubits for $\mathbb{V}$ link



But why use the Wilson action?

The Wilson action is inadequate for many issues

$$S_W = \beta \,\mathrm{Re}\,\mathrm{Tr}[1 - U_p] \approx -\frac{1}{4} F_{\mu\nu} F_{\mu\nu} + \frac{1}{12} a^2 D_\mu F_{\mu\nu} D_\mu F_{\mu\nu}$$

...which can be treated with **Symanzik improvement**[16]

$$\begin{aligned} S_{LW} =& \beta \,\mathrm{Re}\,\mathrm{Tr}[1 - U_p] + \beta_2 \,\mathrm{Re}\,\mathrm{Tr}[1 - U_{rt}] + \beta_3 \,\mathrm{Re}\,\mathrm{Tr}[1 - U_{par}] \\ & \approx -\frac{1}{4} F_{\mu\nu} F_{\mu\nu} + O(a^4) \end{aligned}$$

but you could also add **local** terms proportional to other **irreps**...e.g.[17]

$$S_M = \beta \,\mathrm{Re}\,\mathrm{Tr}[1 - U_p] + \beta_a \,\mathrm{Re}\,\mathrm{Tr}[U_p]\,\mathrm{Tr}[U_p^\dagger] \tag{3}$$

---

[16] Symanzik, K. In: *Communications in Mathematical Physics* 18 (1970).

[17] Bhanot, G. In: *Phys. Lett.* 108B (1982), Fukugita, M., T. Kaneko, and M. Kobayashi. In: *Nucl. Phys. B* 215 (1983), Hasenbusch, M. and S. Necco. In: *JHEP* 08 (2004). arXiv: hep-lat/0405012 [hep-lat].

'Same' physics at $\beta_W \equiv f(\beta_f, \beta_a)$ have diff. errors[18]

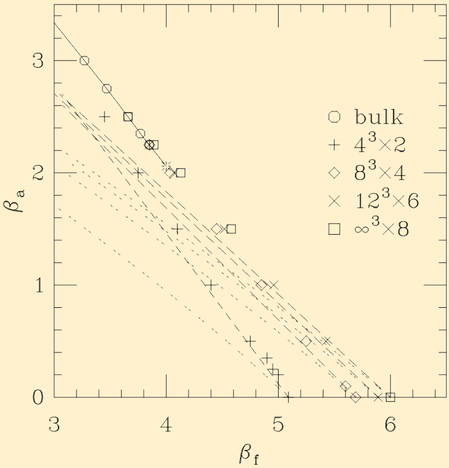
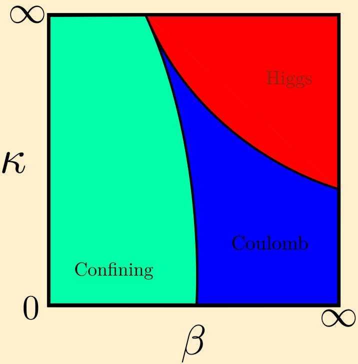


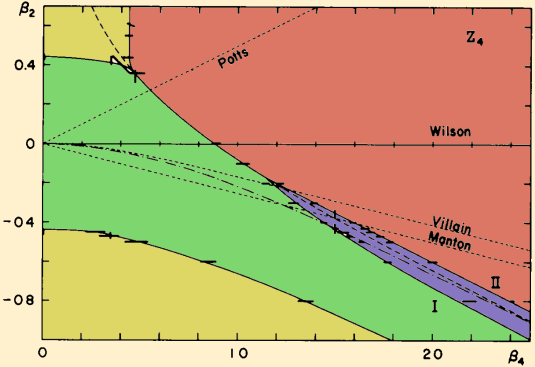
Figure 6: Lines of constant physics as predicted by perturbation theory (dotted lines) and tadpole improved perturbation theory (dashed lines) together with the deconfinement transitions for $N_t = 2$, 4, 6, and 8.

[18] Blum, T. et al. In: *Nucl. Phys.* B442 (1995). arXiv: hep-lat/9412038 [hep-lat].

Modifed actions can lower truncation needed[19]

$$f(z) = \beta_0 + \tfrac{1}{2}\beta_4(z + z^{-1}) + \beta_2 z^2 \, .$$

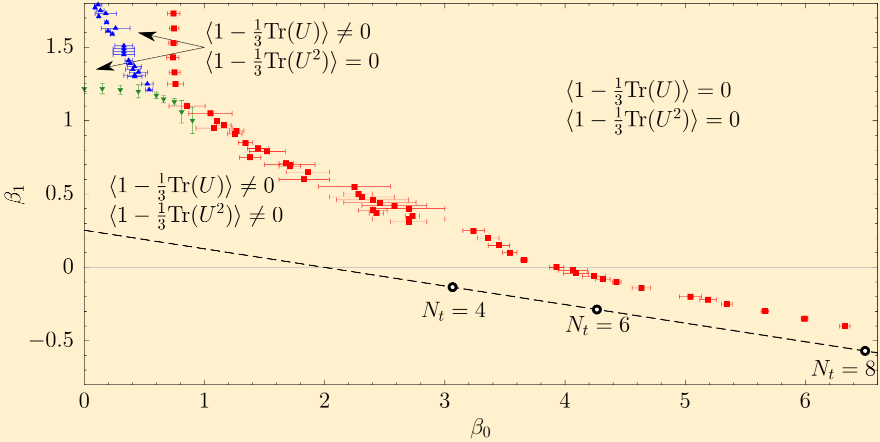




[19] Fukugita, M., T. Kaneko, and M. Kobayashi. In: *Nucl. Phys. B* 215 (1983).

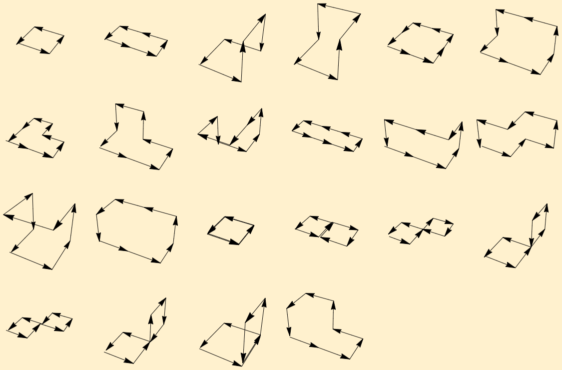
Can modified actions help $S(1080)$?

Define a trajectory to study continuum limit



Classical “State Prepartion” with operator basis

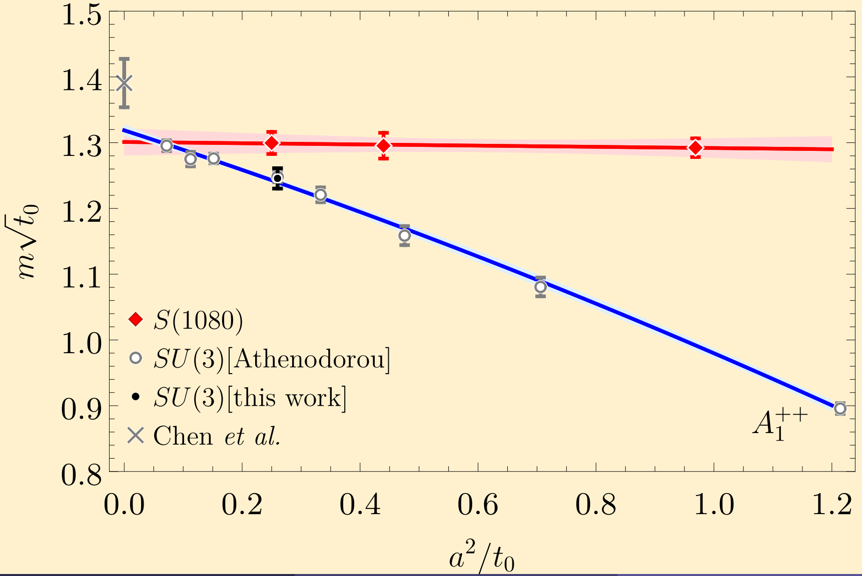
$$\langle C_{ij}(\tau)\rangle = \langle\beta|\mathcal{O}_i(0)\mathcal{O}_j^\dagger(\tau)|\beta\rangle = \sum_k c_{ijk}e^{-E_k\tau}$$



10,016 independent operators from $p = 0$ operators across 20 symmetry sectors with $n_{\text{smear}} = 2, 4, 6, 8$ levels of *stout-smearing*[20].

[20] Morningstar, C. and M. J. Peardon. In: *Phys. Rev.* D69 (2004). arXiv: hep-lat/0311018 [hep-lat].

Seems to work for glueballs[21]



Low-lying glueball masses are consistent with $SU(3)$

irrep	$S(1080)$	$SU(3)^{[22]}$	$SU(3)^{[23]}$
A_1^{++}	1.301(20)	1.319(8)	1.391(37)
A_1^{-+}	2.090(31)	2.049(17)	2.089(20)
E^{++}	1.899(21)	1.902(7)	1.946(17)

$S(1080)$ reproduces $SU(3)$ at **10**$\times$ higher energy than $T_c\sqrt{t_0} \approx 0.25$

$S(1080)$ good until **at least** $\mathcal{O}(10^5)$ qubit devices

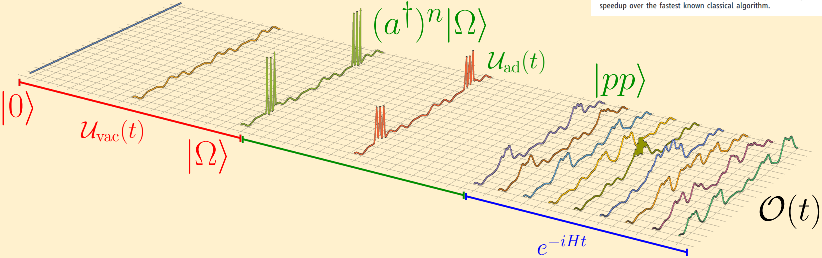
[22] Athenodorou, A. and M. Teper. In: *JHEP* 11 (2020). arXiv: 2007.06422 [hep-lat].
[23] Chen, Y. et al. In: *Phys. Rev.* D73 (2006). arXiv: hep-lat/0510074 [hep-lat].

What might a galactic algorithm look like?

Quantum Algorithms for Quantum Field Theories

Stephen P. Jordan,[1*] Keith S. M. Lee,[2] John Preskill[3]

Quantum field theory reconciles quantum mechanics and special relativity, and plays a central role in many areas of physics. We developed a quantum algorithm to compute relativistic scattering probabilities in a massive quantum field theory with quartic self-interactions (ϕ^4 theory) in spacetime of four and fewer dimensions. Its run time is polynomial in the number of particles, their energy, and the desired precision, and applies at both weak and strong coupling. In the strong-coupling and high-precision regimes, our quantum algorithm achieves exponential speedup over the fastest known classical algorithm.

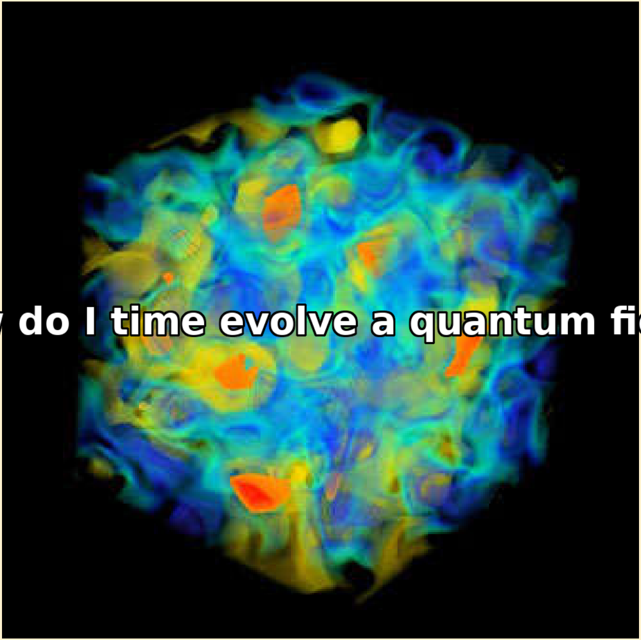


Vacuum Prep+**Adiabatic evolution**+**Trotterization**+Measurements[24]

Example: $|\langle pp|U(t)|\pi\pi\pi\pi\rangle|^2$ needs $\mathcal{O}(10^8)$ **logical qubits**

$\approx \left(\frac{4\text{ fm}}{0.05\text{ fm}}\right)^3 \times (3\text{ links} \times 11\text{ qubits} + 3\text{ colors} \times 2\text{ flavors} \times 2\text{ spins} \times 1\text{ qubit})$

[24] Jordan, S. P., K. S. M. Lee, and J. Preskill. In: *Science* 336 (2012). arXiv: 1111.3633 [quant-ph].

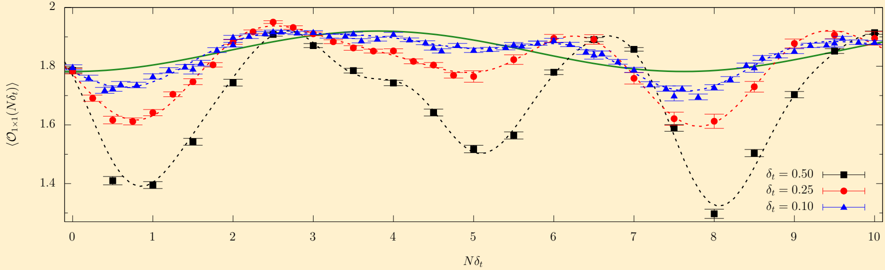


How do I time evolve a quantum field?

What is trotterization?

$$\mathcal{U}(t) = e^{-iHt} \approx \left(e^{-i\delta t \frac{H_V}{2}} e^{-i\delta t H_K} e^{-i\delta t \frac{H_V}{2}} \right)^{\frac{t}{\delta t}}$$

$$\approx \exp\left\{ -it\left(H_K + H_V + \frac{\delta t^2}{24}(2[H_K,[H_K,H_V]] - [H_V,[H_V,H_K]]) \right) \right\}$$



- δt is bare $c(a, a_t)$ **not** physical a_t
- Introduces **higher dimension operators**
- Eigenstates **mix** at $a_t \neq 0 \rightarrow$ **quantum smearing?**...if you want a research project

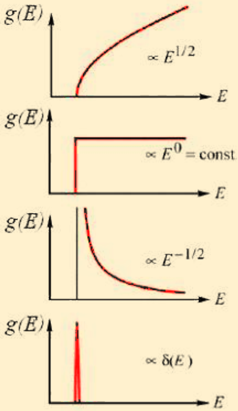
UV states could really be a problem

3-D (bulk) $\quad g(E) = \frac{1}{2\pi^2}\left(\frac{2m^*}{\hbar^2}\right)^{\frac{3}{2}}\sqrt{E_g - E}$

2-D (slab) $\quad g(E) = \frac{m^*}{\pi\hbar^2}\sigma(E_g - E)$

1-D (wire) $\quad g(E) = \frac{m^*}{\pi\hbar}\sqrt{\frac{m^*}{2(E_g - E)}}$

0-D (dot) $\quad g(E) = 2\delta(E_g - E)$

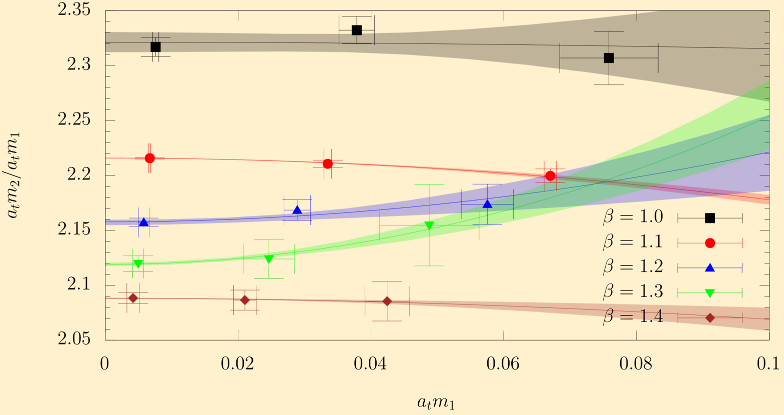


Unlike Euclidean, they **don't** naturally dissipate

Your digitization affects **trotter-mixing into UV** & must be investigated

Reduced mixing $\implies$ larger a_t $\implies$ **shallower circuits**

Approaching the continuum[25]

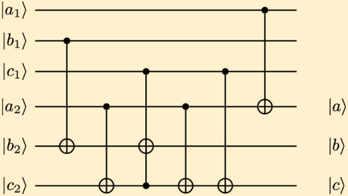


- *Hamiltonian* limit: $a_t \to 0$ (unnecessarily expensive)
- *Continuum* limit: $a_t, a \to 0$ (the one that I want)
- Fix $\xi = a/a_t$ to **efficiently** get QFT

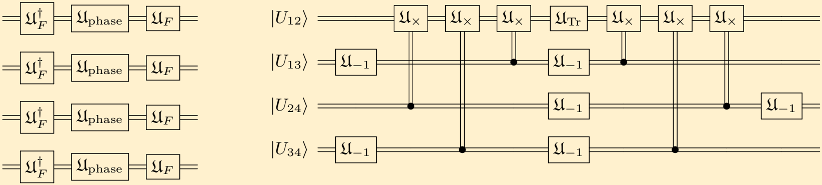
[25] Carena, M., **H. Lamm**, Y.-Y. Li, and W. Liu. In: *Phys. Rev. D* 104 (2021). arXiv: 2107.01166 [hep-lat].

How do we build $U_K = e^{iH_K}$ **and** $U_V = e^{iH_V}$?

- Inversion gate: $\mathfrak{U}_{-1}\left|g\right\rangle = \left|g^{-1}\right\rangle$

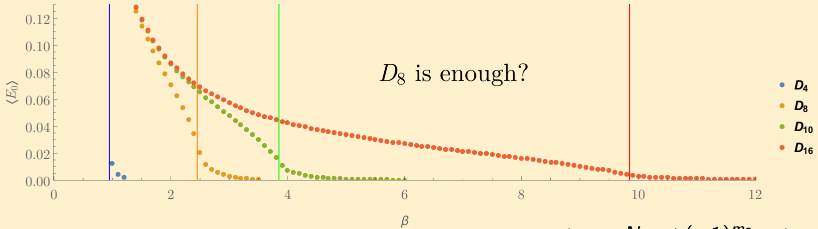


- Multiplication gate: $\mathfrak{U}_\times \left|g\right\rangle \left|h\right\rangle = \left|g\right\rangle \left|gh\right\rangle$
- Trace gate $\mathfrak{U}_{\mathrm{Tr}}(\theta)\left|g\right\rangle = e^{i\theta\,\mathrm{Re\,Tr}\,g}\left|g\right\rangle$
- Fourier Transform gate: $\mathfrak{U}_F \sum_{g\in G} f(g)\left|g\right\rangle = \sum_{\rho\in\hat{G}} \hat{f}(\rho)_{ij}\left|\rho, i, j\right\rangle$

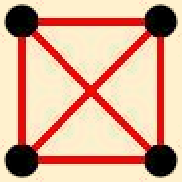


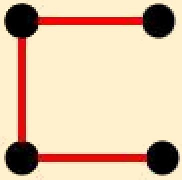
[26] **Lamm, H.**, S. Lawrence, and Y. Yamauchi. In: *Phys. Rev.* D100 (2019). arXiv: 1903.08807 [hep-lat].

Small steps with D_{2N} for quantum leaps



D_{2N} multiply: $h_1 \times h_2 = v^{m_1} u^{n_1} v^{m_2} u^{n_2} = v^{m_1+m_2} u^{Nm_2+(-1)^{m_2} n_1+n_2}$





All-to-all connectivity

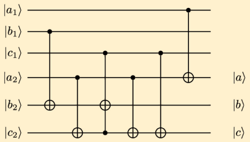
$N_{CNOT} = 21N_q - 31$

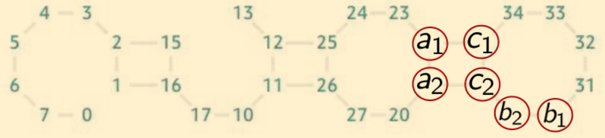
Linear connectivity

$N_{CNOT} \leq 126N_q^2 - 438N_q + 372$

Ancilla qubits: $N_q - 1$

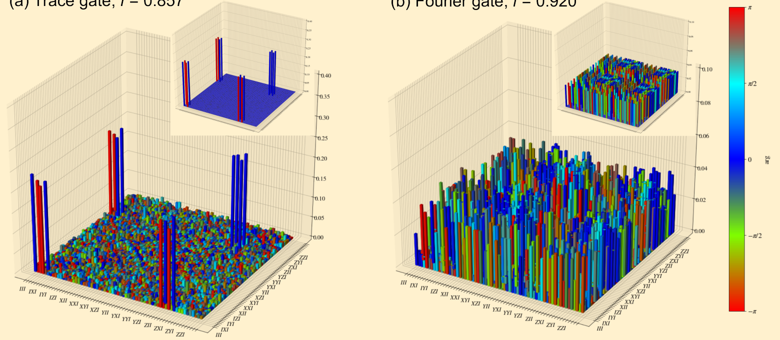
Primitive gates on Rigetti[27]





Primitive gates for D_4 have $\geq$ 80% fidelity – **CCPHASE** critical!

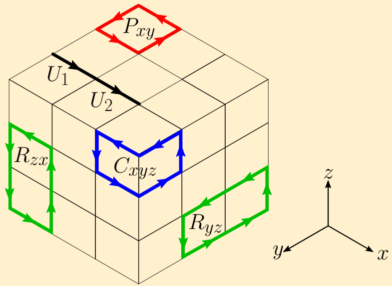
(a) Trace gate, f = 0.857 (b) Fourier gate, f = 0.920



[27] Alam, M. S., S. Hadfield, **H. Lamm**, and A. C. Y. Li. In: (Aug. 2021). arXiv: 2108.13305 [quant-ph].

Kogut-Susskind[28] is not only Hamiltonian[29]

$$H_{\rm co} = \frac{1}{2}\int \mathrm{d}^d x\, \mathrm{Tr}\left[\mathbf{E}^2(\mathbf{x}) + \mathbf{B}^2(\mathbf{x})\right]$$



$$H_{KS} = K_{KS} + V_{KS} + \mathcal{O}(a^2)$$

Including additional terms reduces discretization effects

$$H_I = K_I + V_I + \mathcal{O}(a^4)$$
$$V_I = \beta_{V0} V_{KS} + \beta_{V1} V_{\rm rect} + \beta_{V2} V_{\rm bent}$$
$$K_I = \beta_{K0} K_{KS} + \beta_{K1} K_{2L}$$

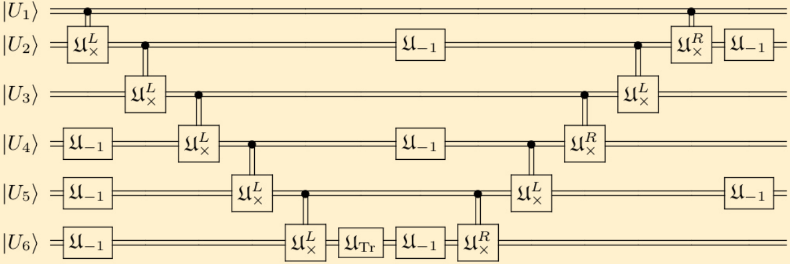
$\gtrsim 2^d$ fewer qubits without increasing gate cost

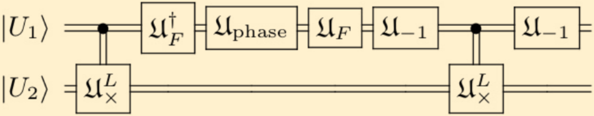
[28] Kogut, J. and L. Susskind. In: *Phys. Rev. D* 11 (2 1975).

[29] Carena, M., **H. Lamm**, Y.-Y. Li, and W. Liu. In: (Mar. 2022). arXiv: 2203.02823 [hep-lat].

Reducing resources with improved Hamiltonians[30]

Larger $a_s \implies$ **fewer** qubits for fixed discretization error

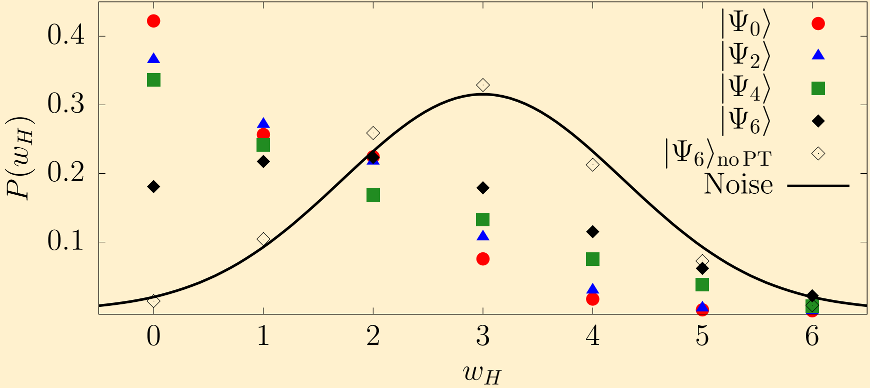




[30] Carena, M., **H. Lamm**, Y.-Y. Li, and W. Liu. In: (Mar. 2022). arXiv: 2203.02823 [hep-lat].

Can we implement this Hamiltonian today?

Quantum Fidelity for $\mathcal{U}_{V_{\rm rect}}$ for $\mathbb{Z}_2$ is $\lesssim 55\%$ on `ibm_perth`

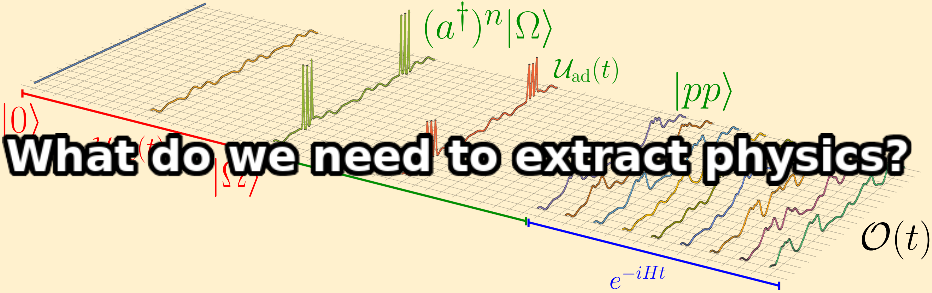


$P(w_H)$ is **probability** of measuring a state with w_H 1's in it

e.g. $|001010\rangle$ has $w_H = 2$

Noiseless results would be $P(w_H = 0) = 1$

What do we need to extract physics?



How do I compute $\langle\Psi|\prod_n \mathcal{O}(t_n)|\Psi\rangle$?[31]

Want to measure $\langle O(t)\rangle$? **Measure** the qubits, or **phase estimation**

Acting on a quantum state $|\Psi\rangle$ with the first Hermitian $\mathcal{O}(t_0)$ leads to...

WAVEFUNCTION COLLAPSE

So what is to be done?

Perturb $H \to H + \epsilon\mathcal{O}\delta(t)$, and take derivatives:

$$\langle\Psi|\mathcal{O}(t)\mathcal{O}(0)|\Psi\rangle = \frac{\partial}{\partial\epsilon_t}\frac{\partial}{\partial\epsilon_0}\langle\Psi|e^{-iHt}e^{-i\mathcal{O}\epsilon_t}e^{iHt}e^{i\mathcal{O}\epsilon_0}|\Psi\rangle$$

[31] Pedernales, J. S., R. Di Candia, I. L. Egusquiza, J. Casanova, and E. Solano. In: *Phys. Rev. Lett.* 113 (2 2014).

Deriving Lattice Hamiltonian Operators[32]

$$\eta = \frac{V}{T}\int_0^\infty \mathrm{d}t \langle T_{12}(t) T_{12}(0)\rangle$$

We construct a lattice Hamiltonian version of $T_{\mu\nu}$ that depends on $F_{\mu\nu}$

TABLE I. Gauge-invariant lattice operators in the Hamiltonian formalism in $3+1d$ dimensions: naive operators with $O(a)$ errors and improved operators with errors that are $O(a^2)$. Components of the energy-momentum tensor $T_{\mu\nu}$ are constructed as linear combinations of these operators according to Eq. (8). The plaquette $\hat{P}$ and clover $\hat{C}$ are defined in Eq. (10) and Eq. (15), respectively. Spatial indices are $i \neq j \neq k$.

Operator	$O(a)$	$O(a^2)$
$\mathrm{Tr} F_{0i}F_{0i}(n)$	$\frac{g_s^2}{a^4}\mathrm{Tr}\left[\pi_{n,i}^2\right]$	$\sum_{x=0,1}\frac{g_s^2}{2a^4}\mathrm{Tr}\left[\pi^2_{n-x\hat{i},i}\right]$
$\mathrm{Tr} F_{0i}F_{0j}(n)$	$\frac{g_s^2}{a^4}\mathrm{Tr}\left[\pi_{n,i}\pi_{n,j}\right]$	$\frac{g_s^2}{4a^4}\Big(\mathrm{Tr}\left[\hat{\pi}_{n,i}\hat{\pi}_{n,j}\right] + \mathrm{Tr}\left[\hat{\pi}_{n,i}\hat{U}^\dagger_{n-\hat{j},j}\hat{\pi}_{n-\hat{j},j}\hat{U}_{n-\hat{j},j}\right] + \mathrm{Tr}\left[\hat{U}^\dagger_{n-\hat{i},i}\hat{\pi}_{n-\hat{i}}\hat{U}_{n-\hat{i},i}\hat{\pi}_{n,j}\right]$ $+\mathrm{Tr}\left[\hat{U}^\dagger_{n-\hat{i},i}\hat{\pi}_{n-\hat{i},i}\hat{U}_{n-\hat{i},i}\hat{U}^\dagger_{n-\hat{j},j}\hat{\pi}_{n-\hat{j},j}\hat{U}_{n-\hat{j},j}\right]\Big)$
$\mathrm{Tr} F_{0j}F_{ij}(n)$	$-\frac{1}{a^4}\mathrm{Tr}\left[\hat{\pi}_{n,j}\mathrm{Im}\hat{P}_{ij}(n)\right]$	$-\frac{1}{2a^4}\Big(\mathrm{Tr}\left[\hat{\pi}_{n,j}\mathrm{Im}\hat{C}_{ij}(n)\right] + \mathrm{Tr}\left[\hat{U}^\dagger_{n-\hat{j},j}\hat{\pi}_{n-\hat{j},j}\hat{U}_{n-\hat{j},j}\mathrm{Im}\hat{C}_{ij}(n)\right]\Big)$
$\mathrm{Tr} F_{ij}F_{ij}(n)$	$\frac{2}{g_s^2 a^4}\mathrm{ReTr}\left[1-\hat{P}_{ij}(n)\right]$	$\sum_{x=0,1}\sum_{y=0,1}\frac{1}{2g_s^2a^4}\mathrm{ReTr}\left[1-\hat{P}_{ij}(n-x\hat{i}-y\hat{j})\right]$
$\mathrm{Tr} F_{ij}F_{kj}(n)$	$\mathrm{Tr}[\hat{F}^N_{ij}(n)\hat{F}^N_{kj}(n)]$	$\mathrm{Tr}[\hat{F}^C_{ij}(n)\hat{F}^C_{kj}(n)]$

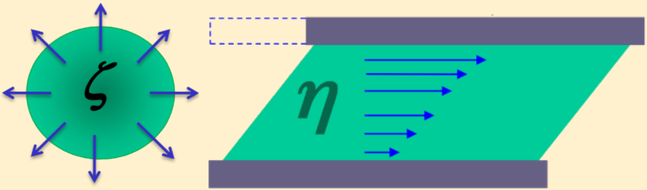
[32] Cohen, T. D., **H. Lamm**, S. Lawrence, and Y. Yamauchi. In: (Apr. 2021). arXiv: 2104.02024 [hep-lat].

What will it take for practical quantum advantage?

$$N_{qudits} \propto N_{dof} \times \left[\frac{L}{a}\right]^d \;\&$$

$$N_{gates} \propto N_{\mathcal{U}}(N_{dof}[L/a]^d) \times \left[\frac{T}{a_t}\right]$$

- **Hadron scattering**: $L, T = O(10)$ **fm,** $a, a_t = O(0.1)$ **fm**[33]
- **Transport coefficients**: $L, T = O(1)$ **fm,** $a, a_t = O(1)$ **fm**[34]
- $\mathcal{U}_{\eta\mathrm{circ}} \sim$ **Thermal** state prep + **Quench** + Trotterization



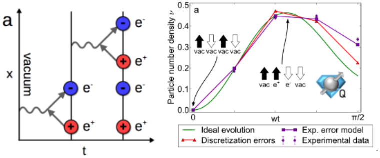
[33] Jordan, S. P., K. S. M. Lee, and J. Preskill. In: *Science* 336 (2012). arXiv: 1111.3633 [quant-ph].

[34] Cohen, T. D., **H. Lamm**, S. Lawrence, and Y. Yamauchi. In: (Apr. 2021). arXiv: 2104.02024 [hep-lat].

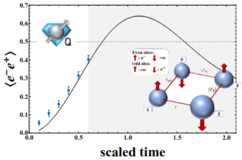
Dynamics in the Schwinger Model - Abelian Gauge Theory

— 1+1 dim QED —

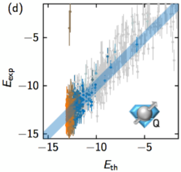
Innesbruck

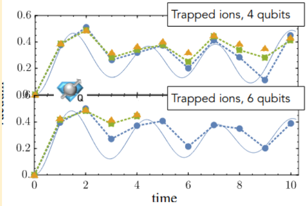


ORNL-Washington-Basque

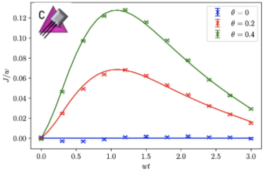


Innesbruck

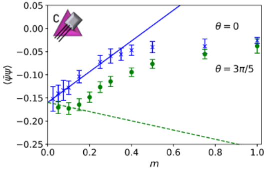




Maryland



Kharzeev-Kikuchi



Chakraborty, Honda, Izubuchi, Tomiya

As we walk in, $1+1d$ gauge could be trouble

- Nondynamic gauge field $\Longrightarrow$ **removable**
- Dramatic optimizations **may not generalize**
- Solvable[35] $\Longrightarrow$ **no** quantum advantage
- Often **"superrenormalizable"/conformal** $\Longrightarrow$ lots of simplifications

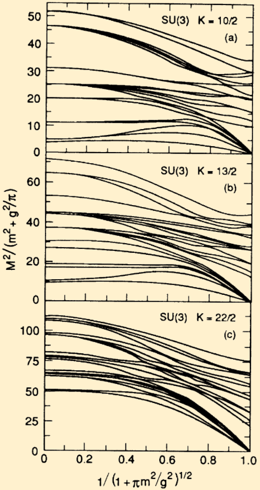
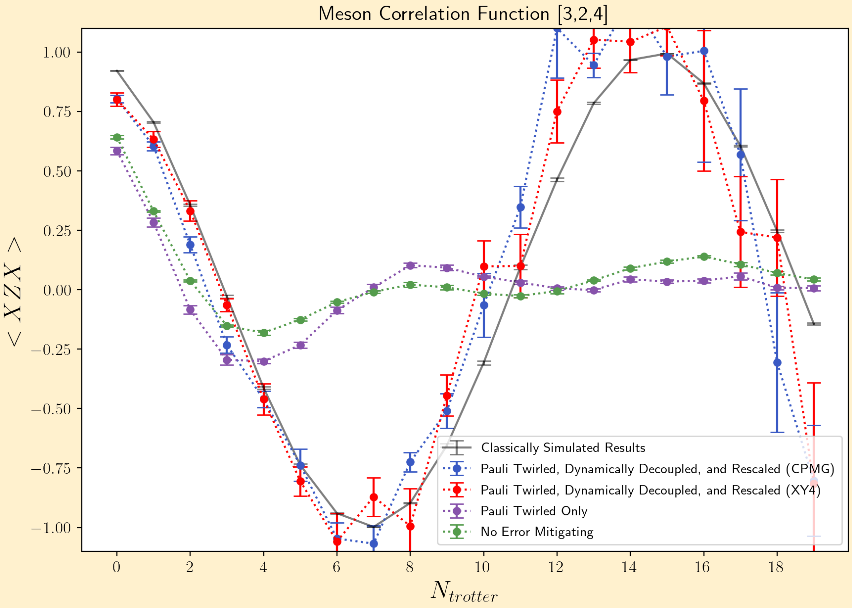


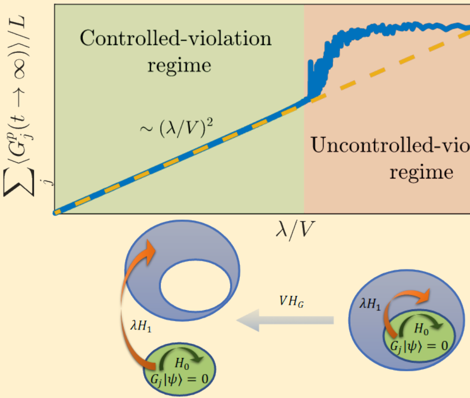
FIG. 2. Spectra for $N=3$, baryon number $B=0$, 1, and 2 as a function of g/m; K fixed.

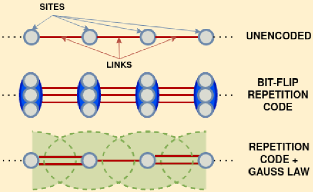
[35] Hornbostel, K., S. J. Brodsky, and H. C. Pauli. In: *Phys. Rev. D* 41 (1990).

Error mitigation crucial to question of NISQ QA



Specialized Error Mitigation and Correction[36][37]





[36] Halimeh, J. C. and P. Hauke. In: *Phys. Rev. Lett.* 125 (2020). arXiv: 2001.00024 [cond-mat.quant-gas].

[37] Rajput, A., A. Roggero, and N. Wiebe. In: (Dec. 2021). arXiv: 2112.05186 [quant-ph].

Today's estimate: $\mathcal{O}(10^8)$ q & $\mathcal{O}(10^{55})$ T-gates[38]

"...99.998% of the gate counts stem from **QFOPs**...The SU(3) *HI collision* problem is...>3 **yrs** of runtime on an **exa-scale** quantum supercomputer."

- *pp* scattering on $(L/a)^d = 100^3$ lattice
  - Observables dictate L/a, T/a_t, $d \implies$ **fewer** qubits
- **Kogut-Susskind** Hamiltonian
  - Improved Hamiltonians will increase $a \implies$ **fewer** qubits
- Truncate to $\Lambda = 10$ in the electric field values **(24q)**
  - Better truncations allow **fewer** qubits per link near continuum
- **Trotterization** $\mathcal{U}(T)$ with **loose** error bound $\epsilon_{Trotter}$
  - Other methods: variational, QDRIFT, qubitization ...
- **Decomposing specific** unitaries into gates introduces $\epsilon_{synthesis}$
  - Different **platforms**: Analog, Digital, CV, Qudits
- $\epsilon \equiv \epsilon_{Trotter} + \epsilon_{synthesis} = 10^{-8}$
  - Current theoretical errors can be $\mathcal{O}(1)$

Cracking RSA and Quantum Chemistry need $\mathcal{O}(10^7)$ q & $\mathcal{O}(10^{20})$!

[38] Kan, A. and Y. Nam. In: *arXiv preprint arXiv:2107.12769* (2021).

It's time to go

- Devices are expected to rapidly scale
  - Theorists should be engaged **early**
  - **Toy models** simulations in $\lesssim$5 years
- Investigate desirable properties
  - **Entanglement in QG? Viscosity?, Cosmology?**
- Must improve over **expensive** algorithms
  - e.g. Consider theory errors, tighter bound on trotterization, reduce QFOPs
- Need to develop workforce with **new skills**