Quantum time and a relativistic quantum spacetime

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What I'm going to talk about

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Time in quantum mechanics

a consistent formalization

based on conditional probability amplitudes

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... and a **new way to do relativistic quantum mechanics**

WHAT is time?

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... or a "coordinate"

something that "measures"
the distance between events

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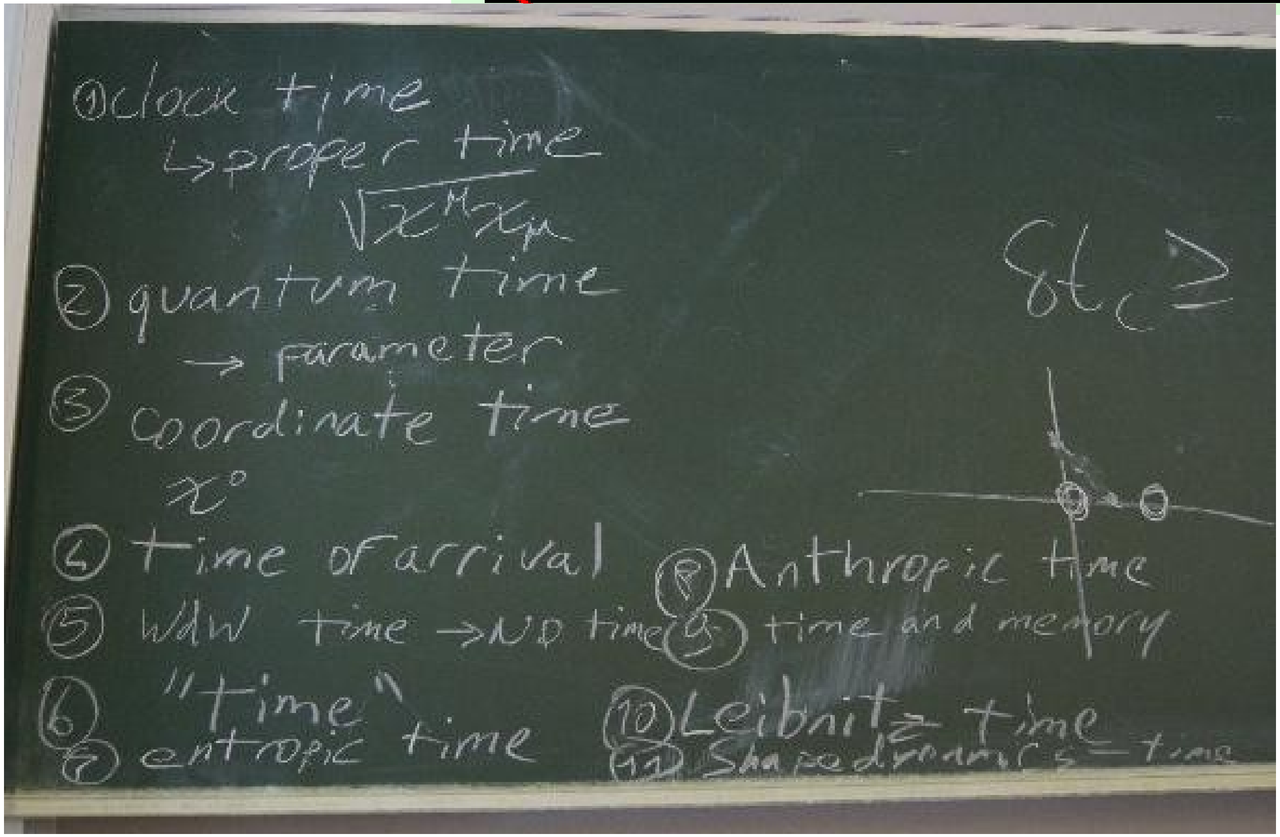
the two **main** meanings of
“time” in physics

other meanings?!

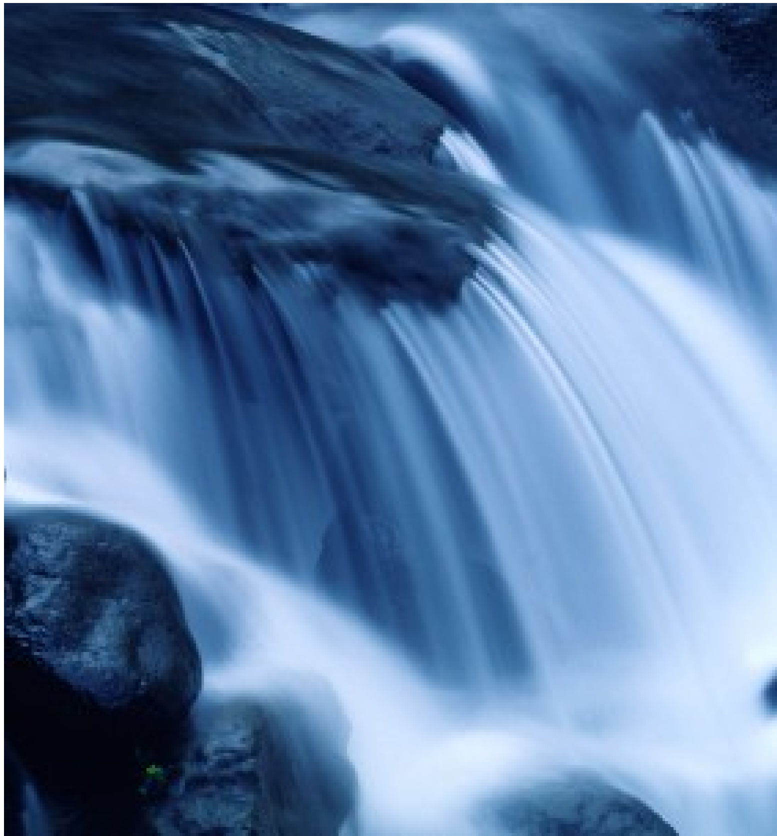
Table 2.1: Times.

Time notion	Property	Example	Form
Natural language time	memory	brain	?
Time-with-a-present	present	biology	R
Thermodynamical time	direction	thermodynamics	A
Newtonian time	unique	newtonian mechanics	M
Special relativistic time	external	special relativity	M^3
Cosmological time	spatially global	cosmological time	m
Proper time	temporally global	world line proper time	m^∞
Clock time	metric	clocks in GR	c
Parameter time	one dimensional	coordinate time	L^∞
No-time	none	quantum gravity	none



[Rovelli, “quantum gravity”]

Let's try to use our intuition?



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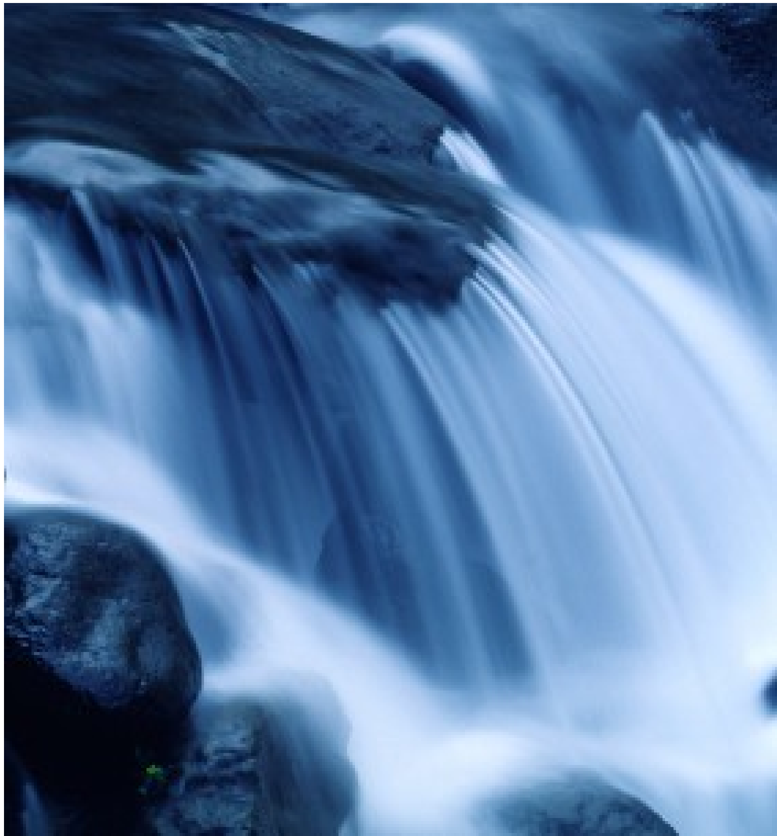

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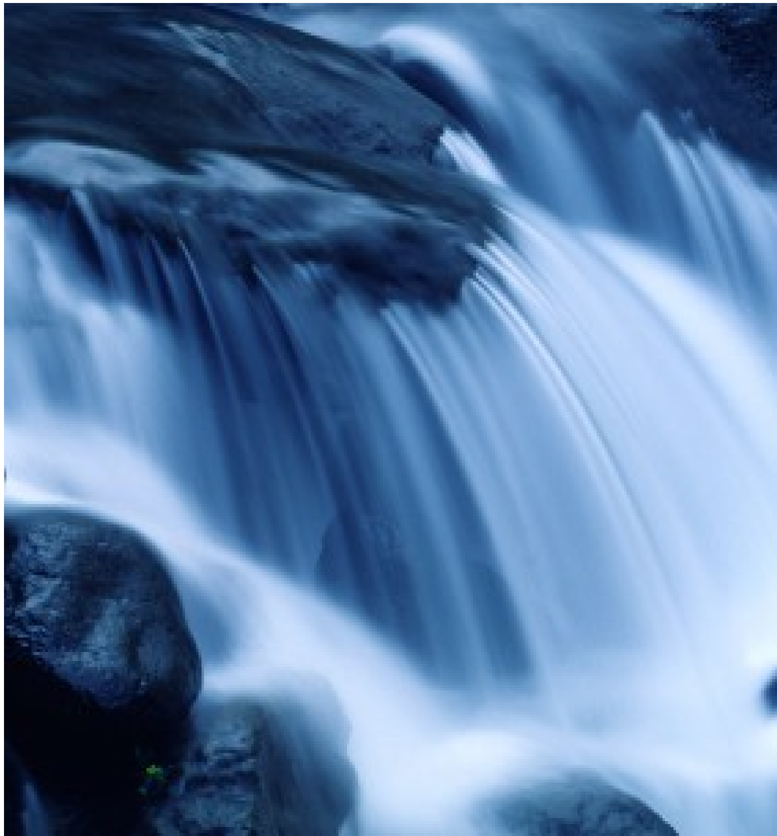

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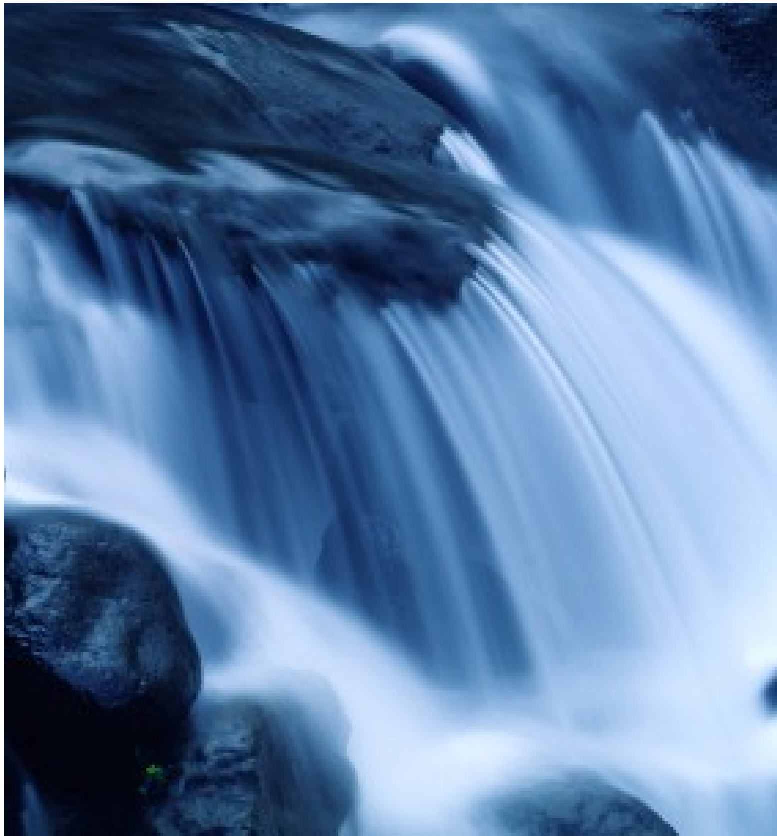

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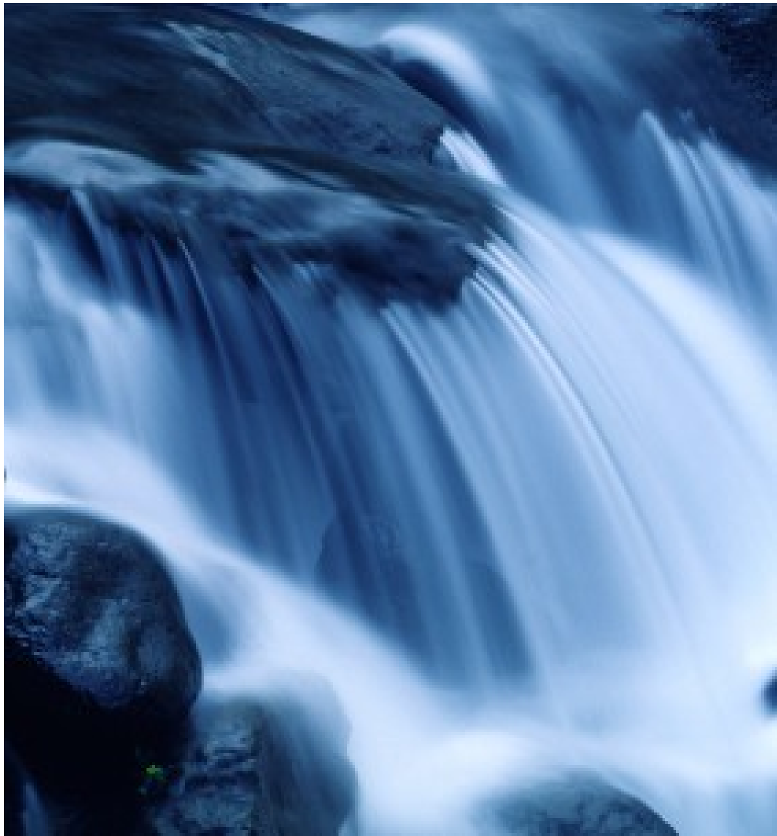

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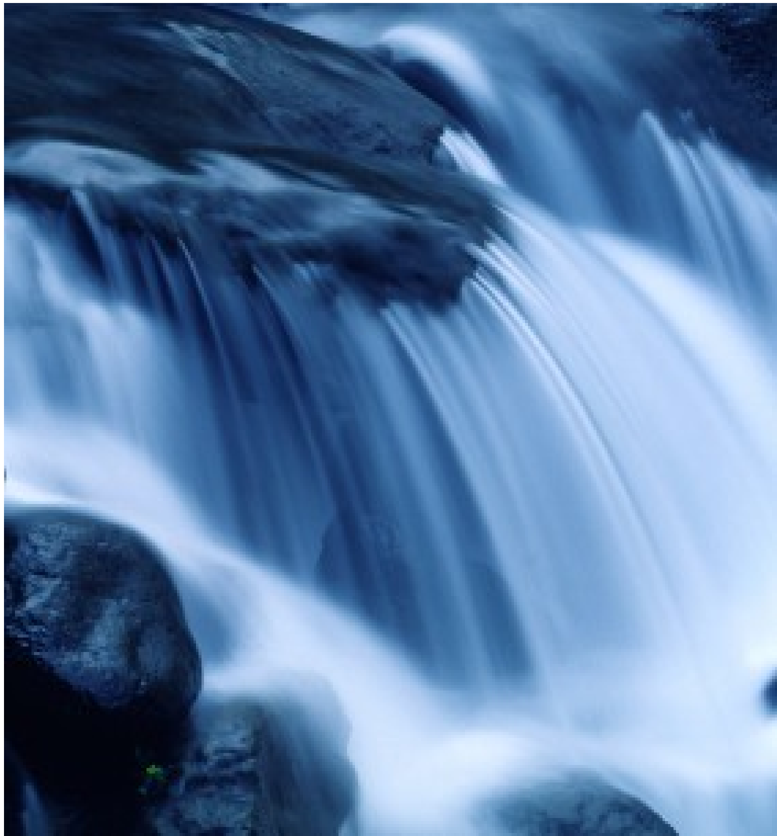

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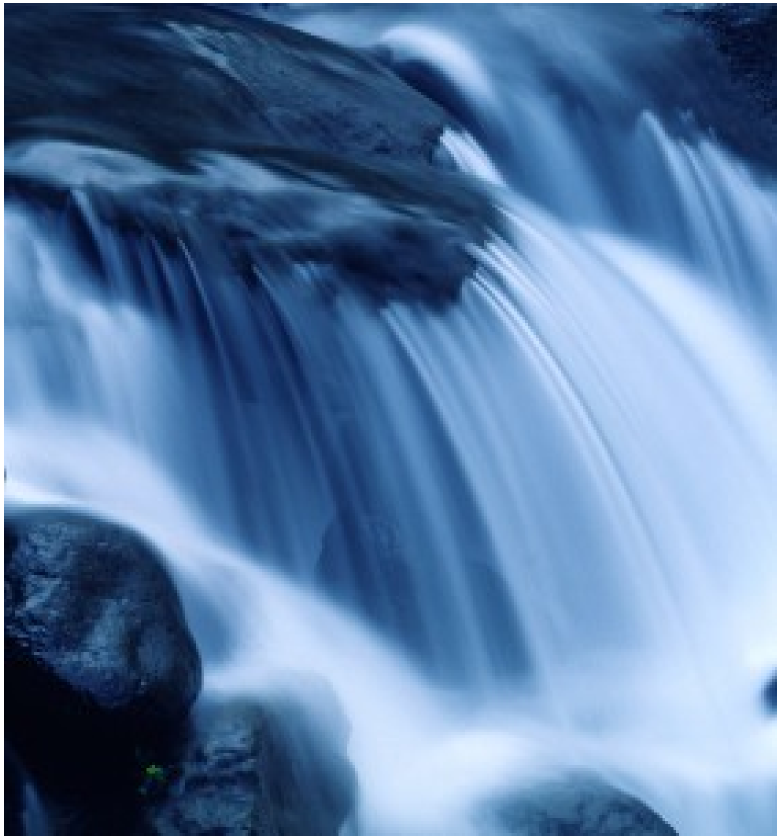

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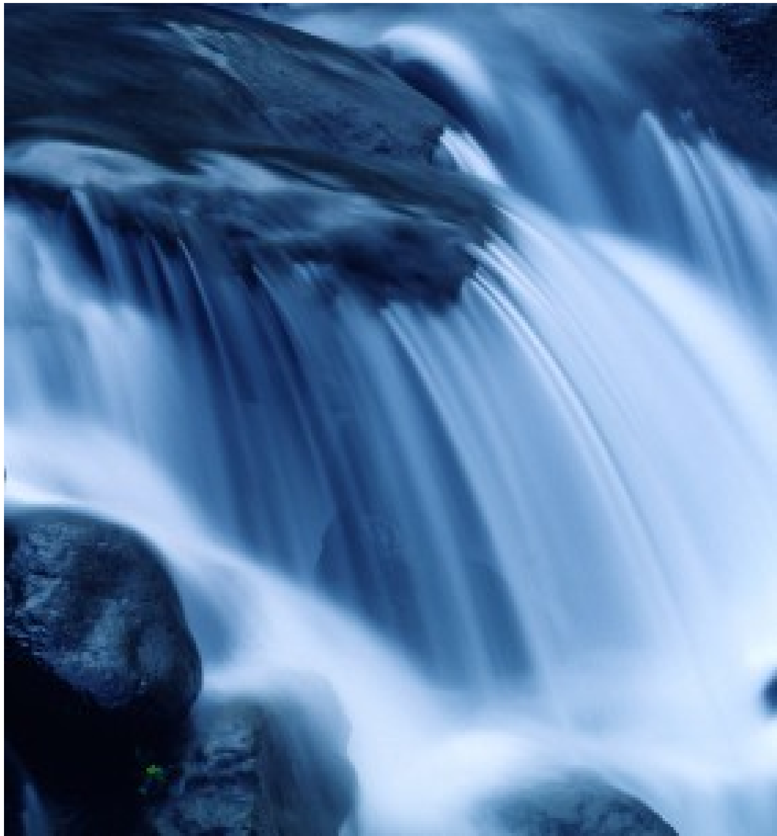

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(perhaps a little too drastic..)

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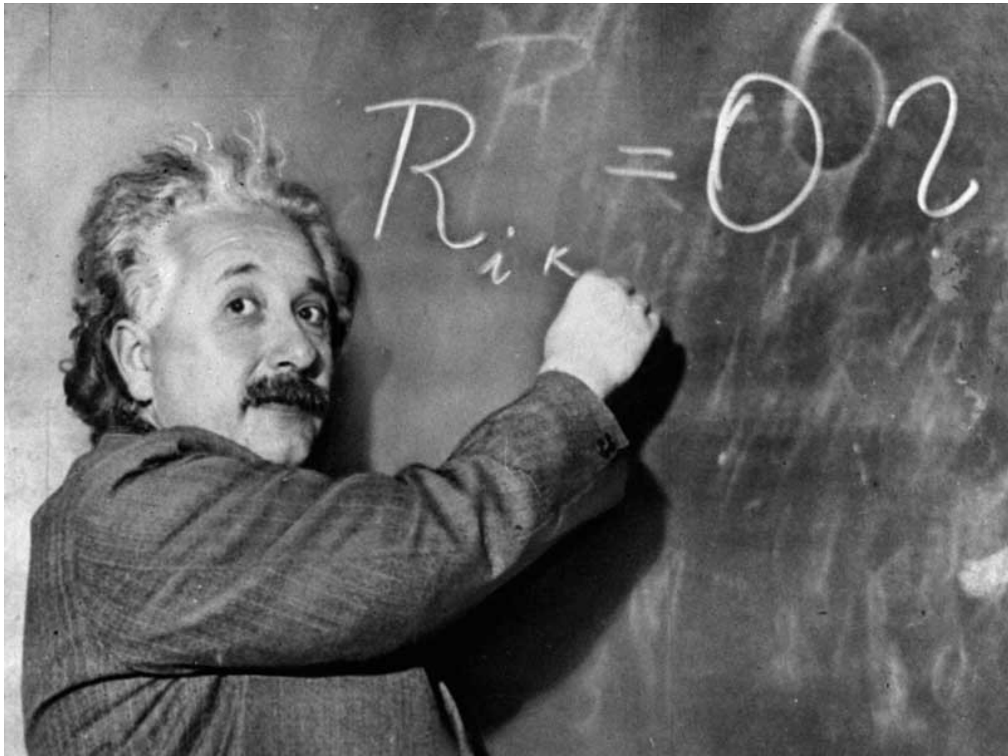
- The present "exists", the past and the future don't



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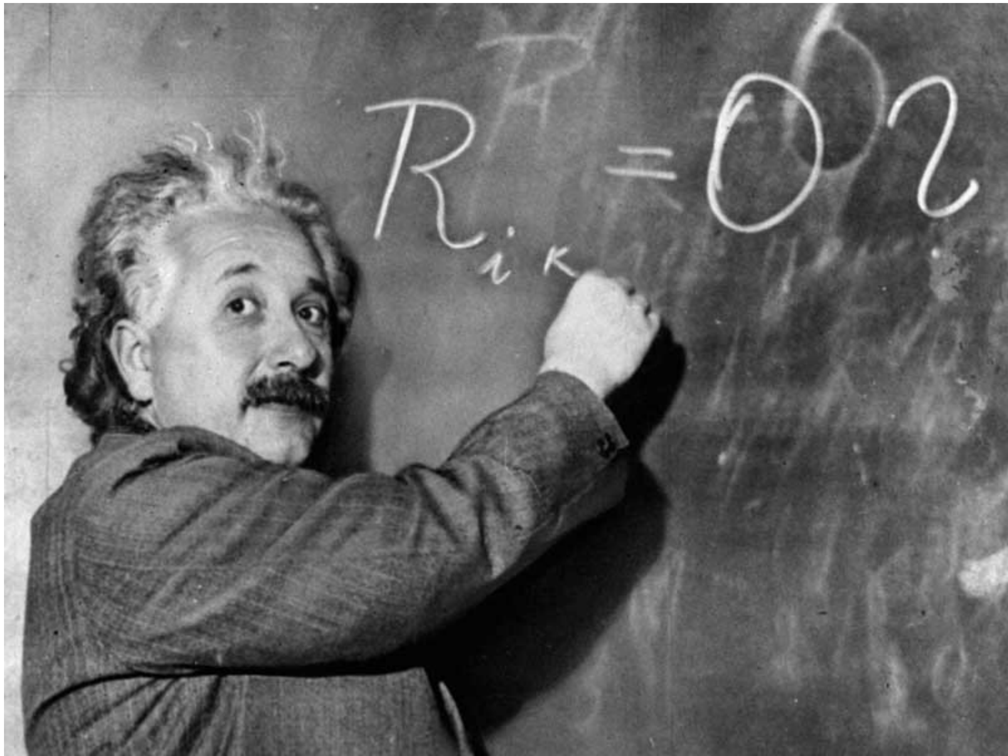

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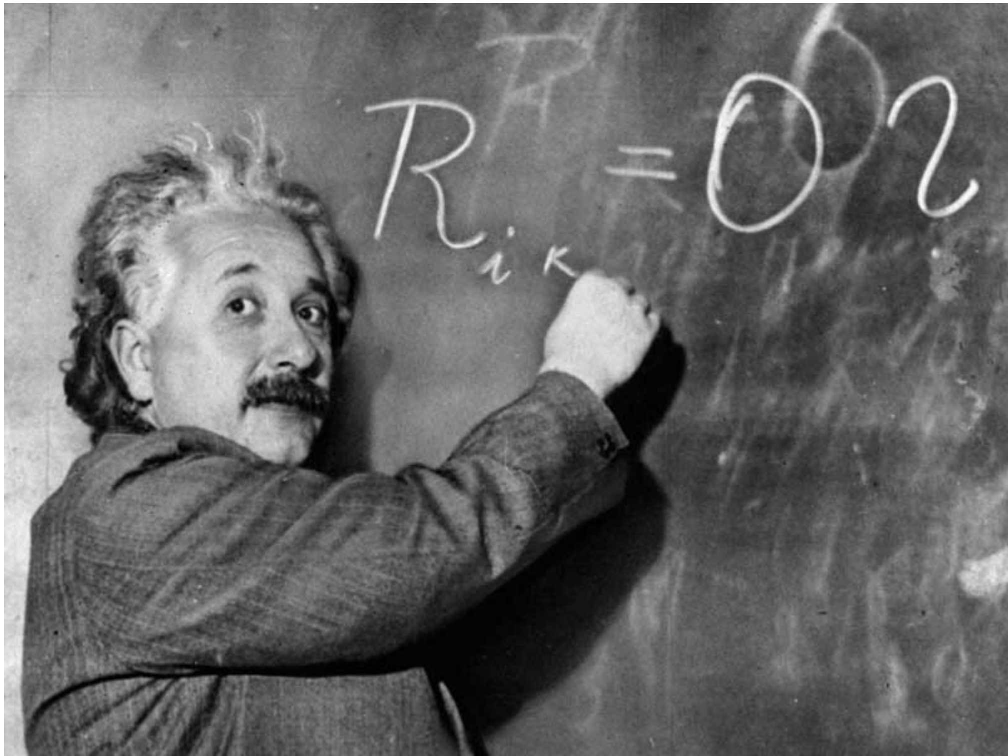

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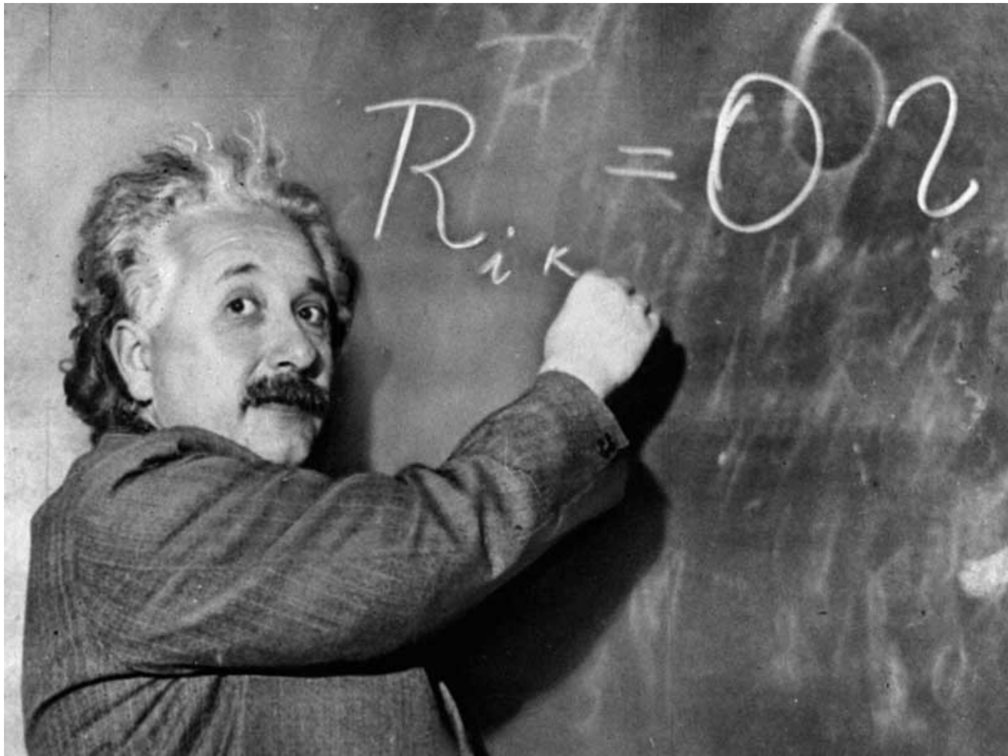
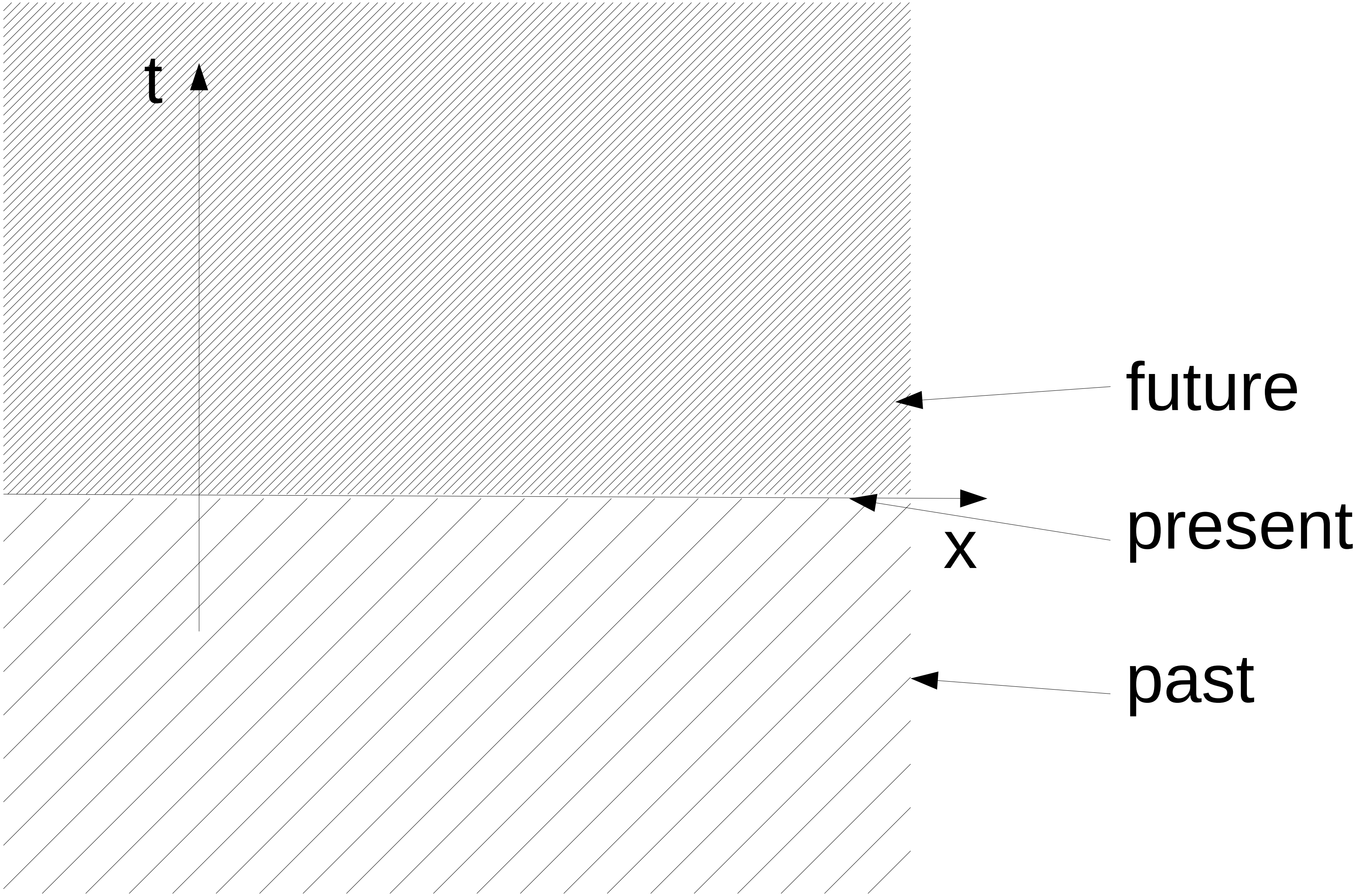
"NO"?!? why?



Relativity of simultaneity

Observers in relative motion divide spacetime in past-present-future in different ways.



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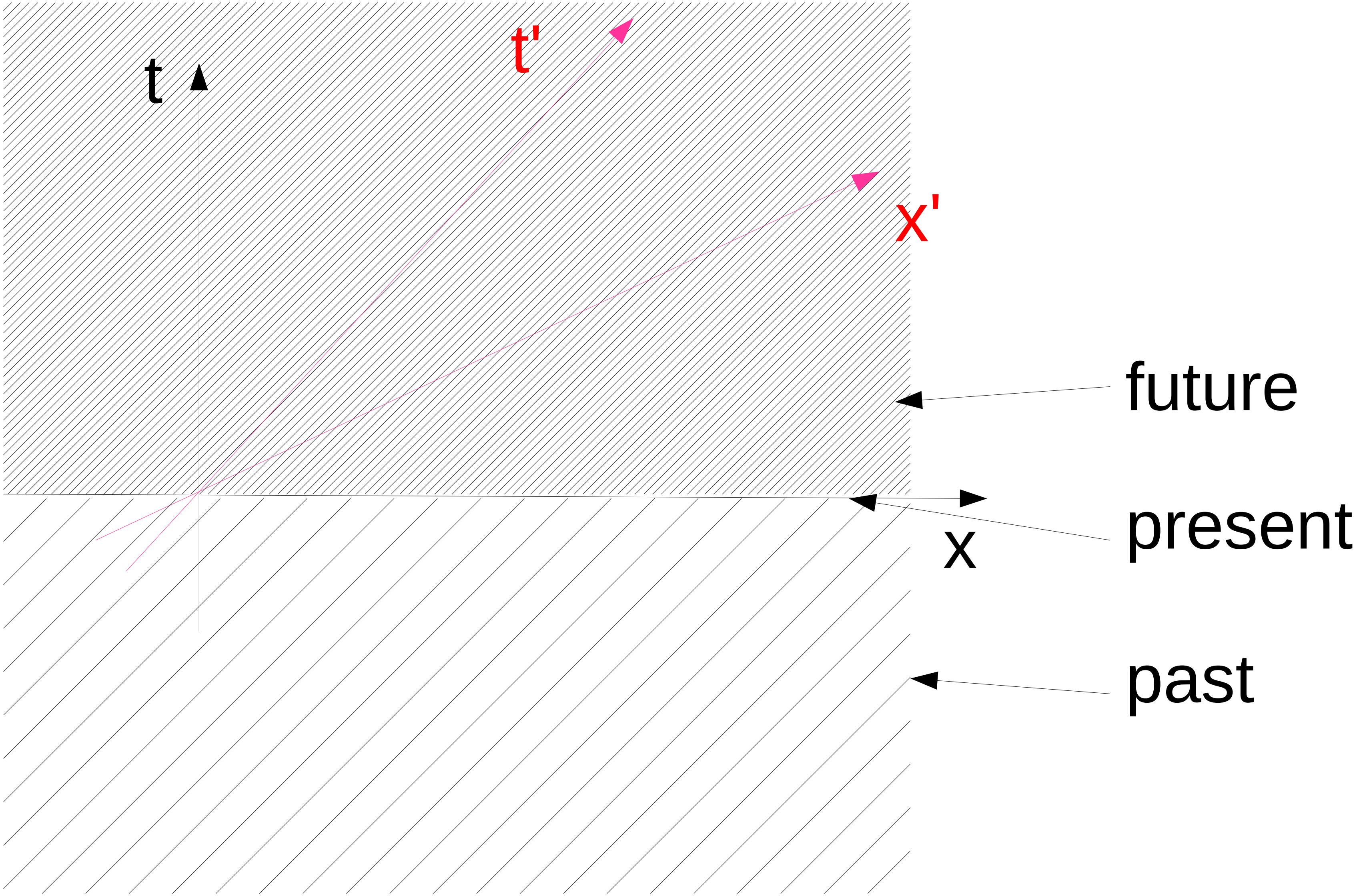

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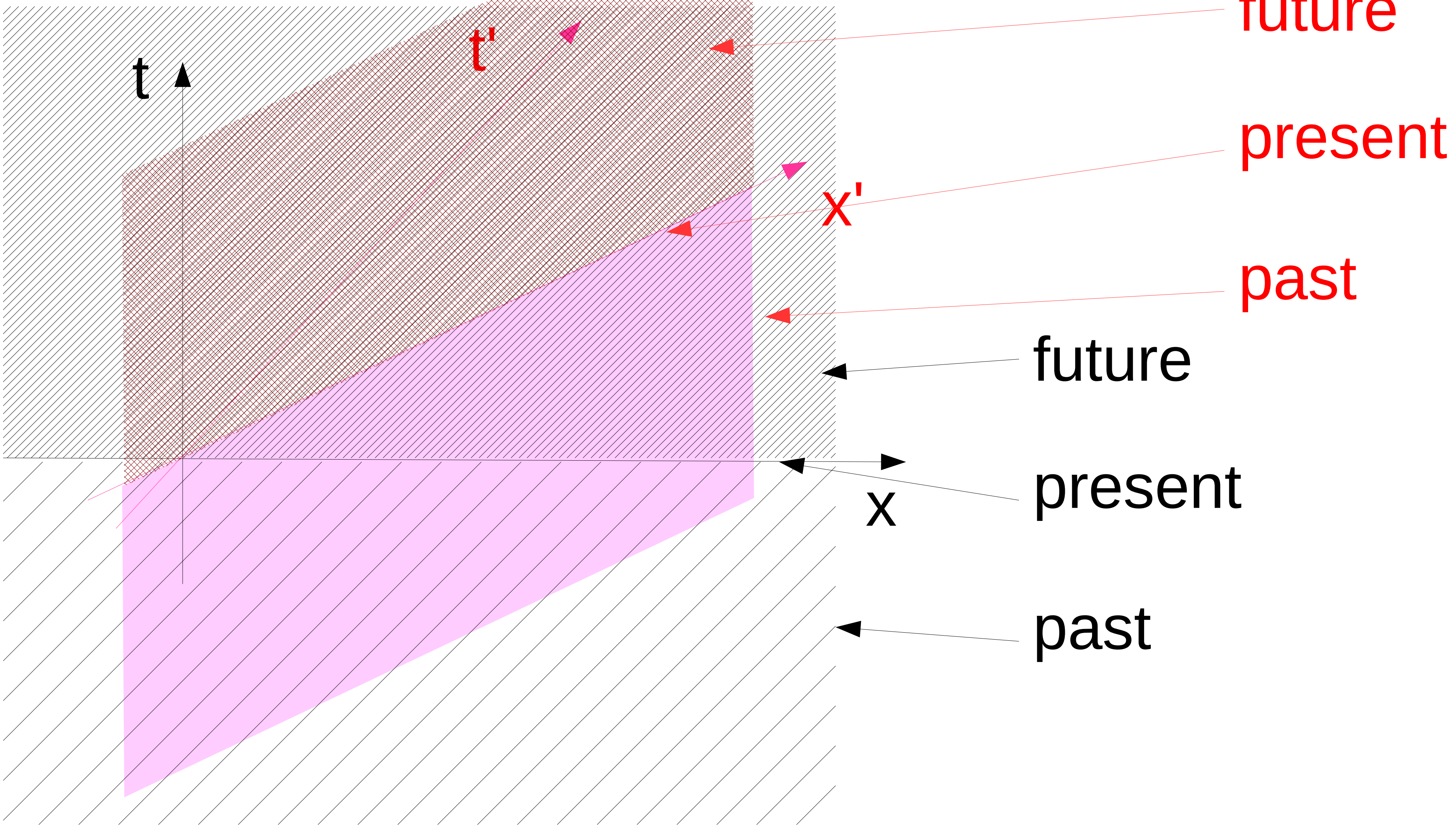

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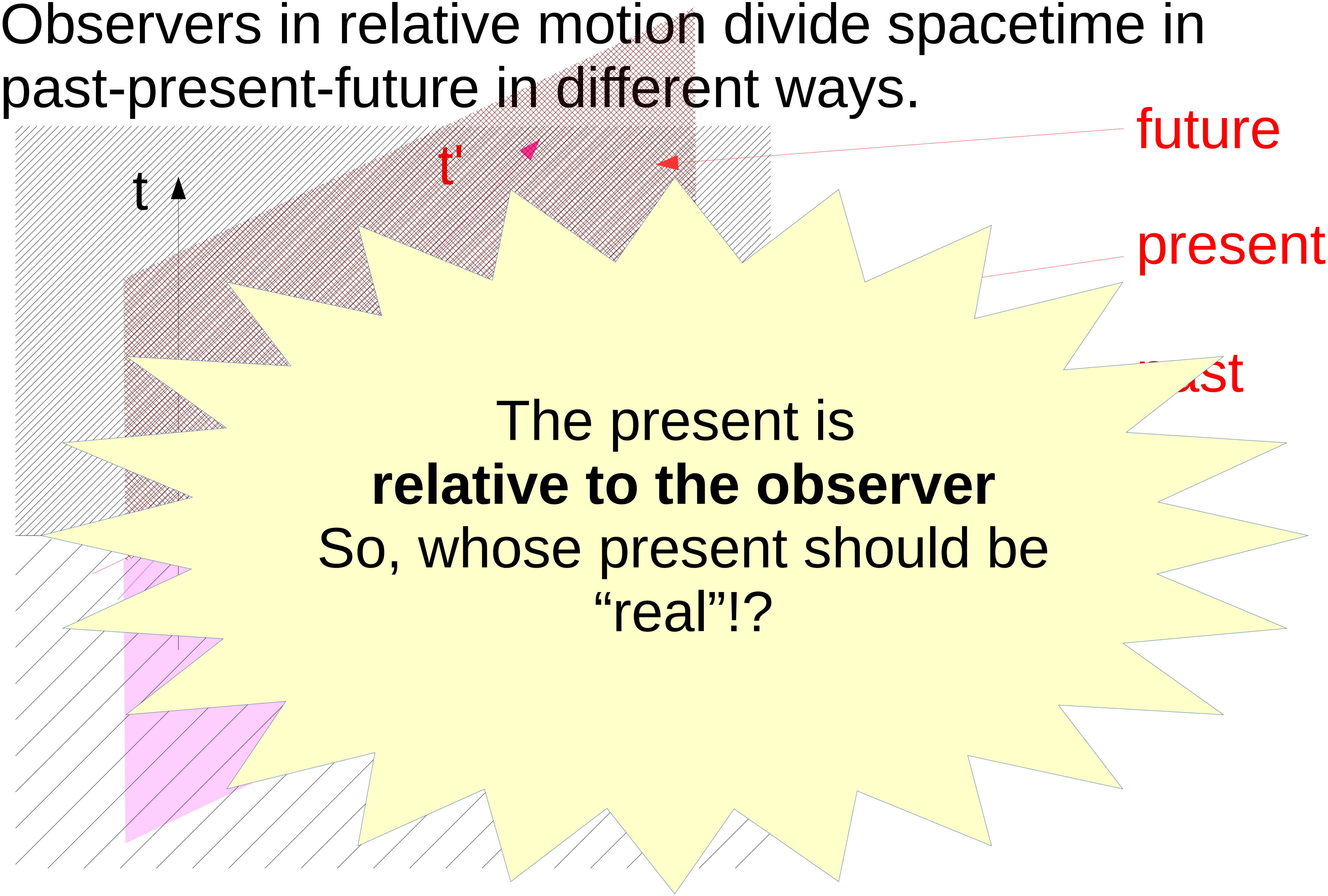

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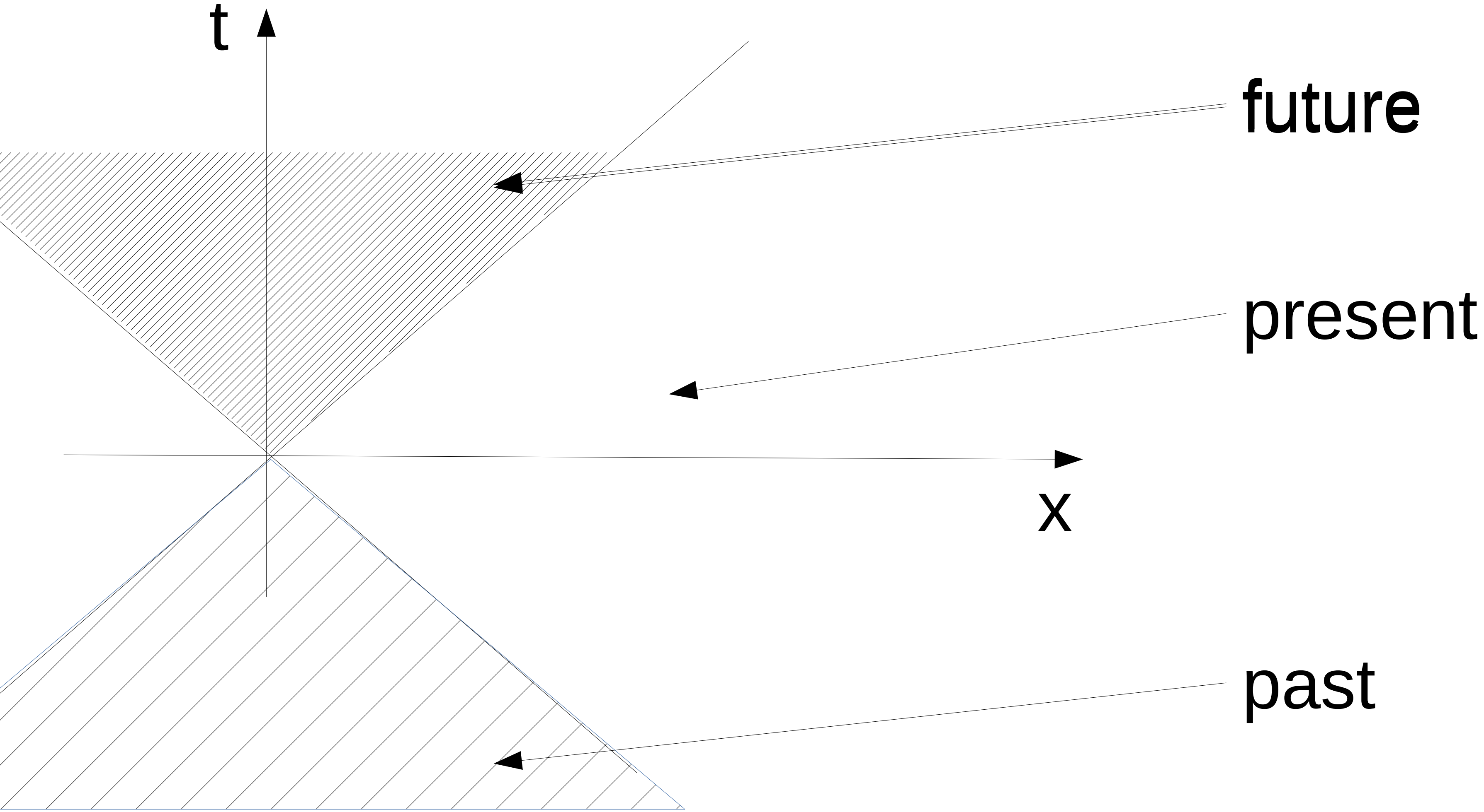

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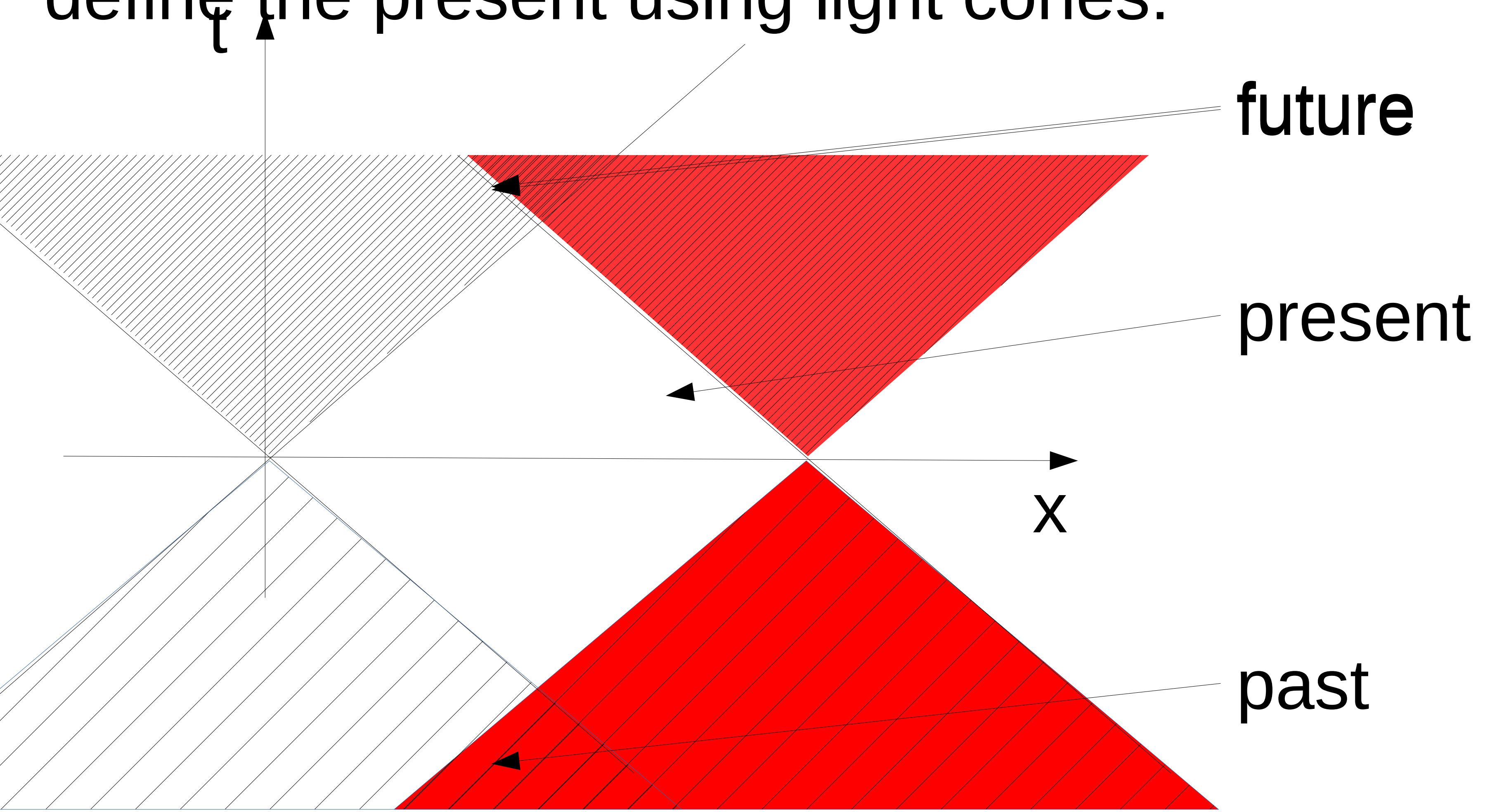

Now even observers in the same reference disagree on what is “real”

Special relativity forces us to give the **same degree of existence to past, present and future**

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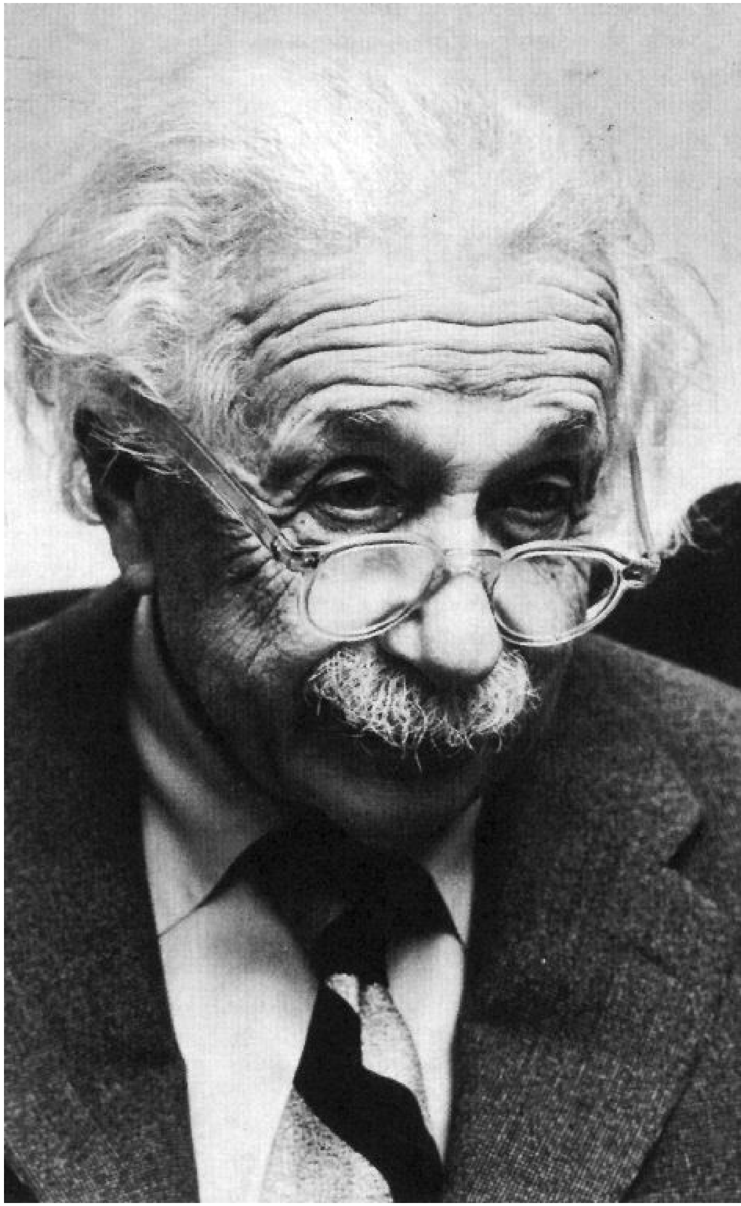
Technically:
Presentism
vs
Eternalism

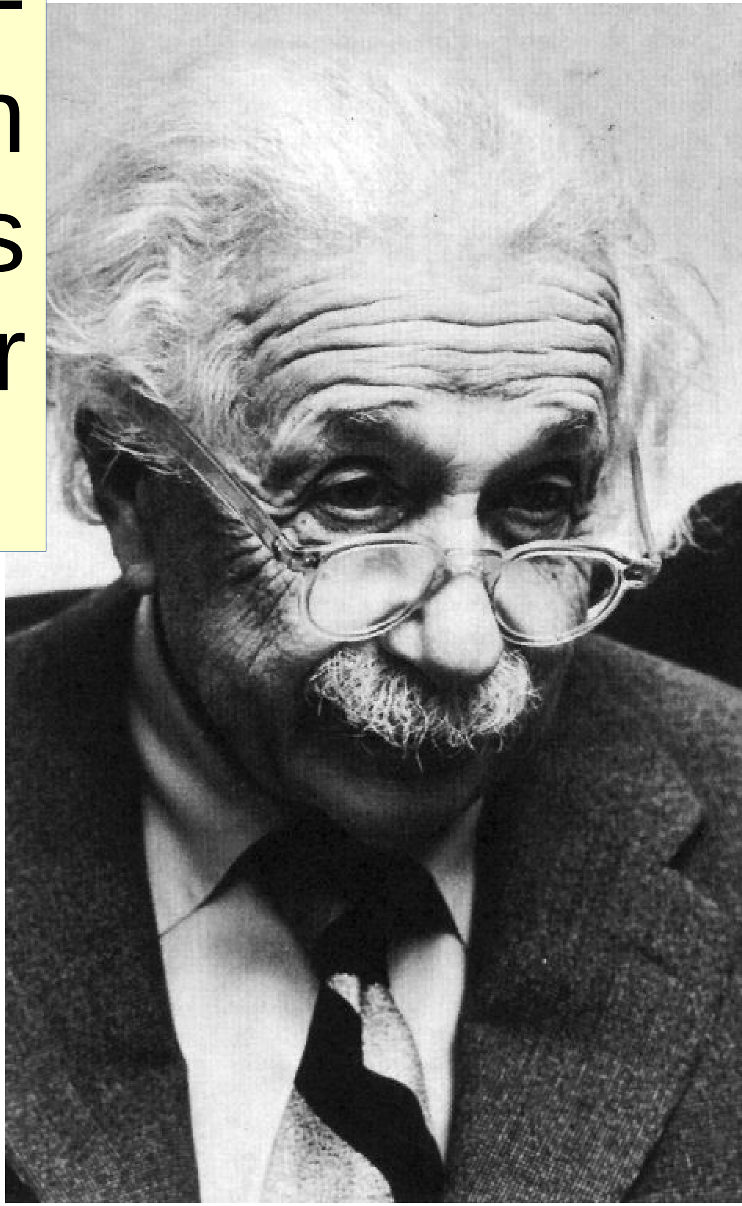
What did Einstein say about this?



What did Einstein say about this?

"For us convinced physicists the distinction between past, present, and future is only an illusion, however persistent."

Albert Einstein, 21 Maggio 1955



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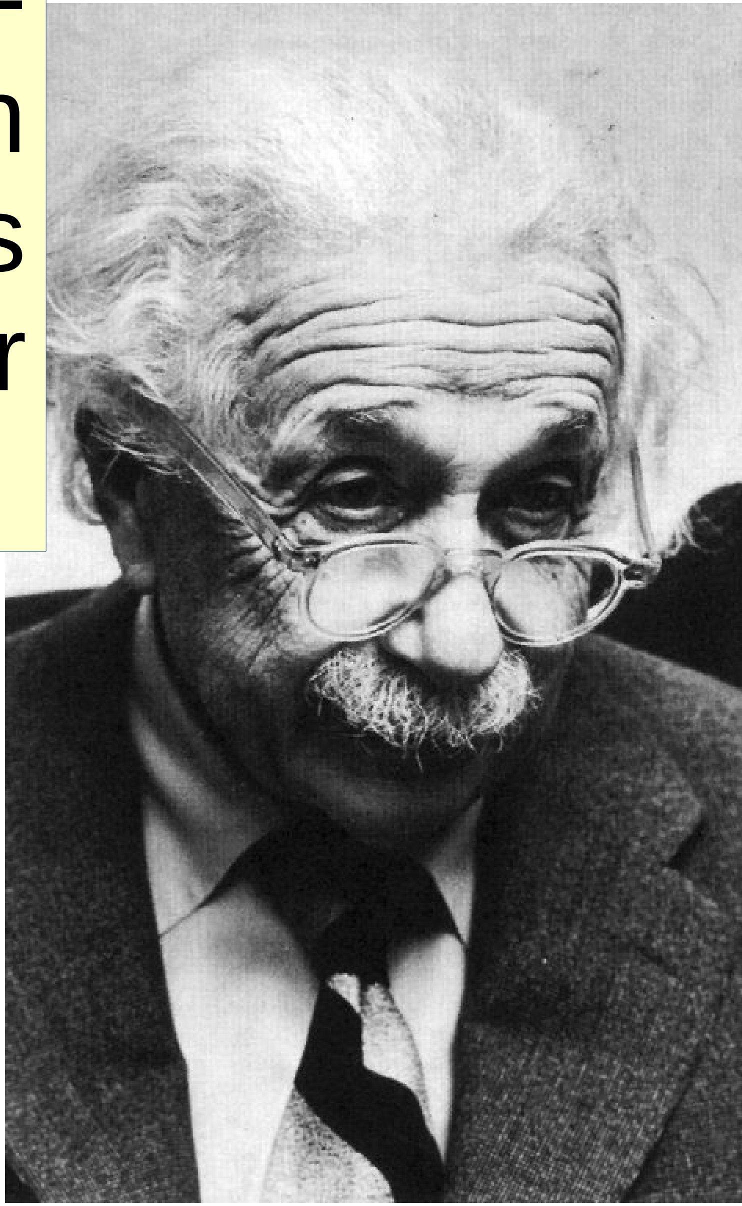
In a letter to the widow of his dear friend Michele Besso: trying to console her (or himself?) with special relativity.



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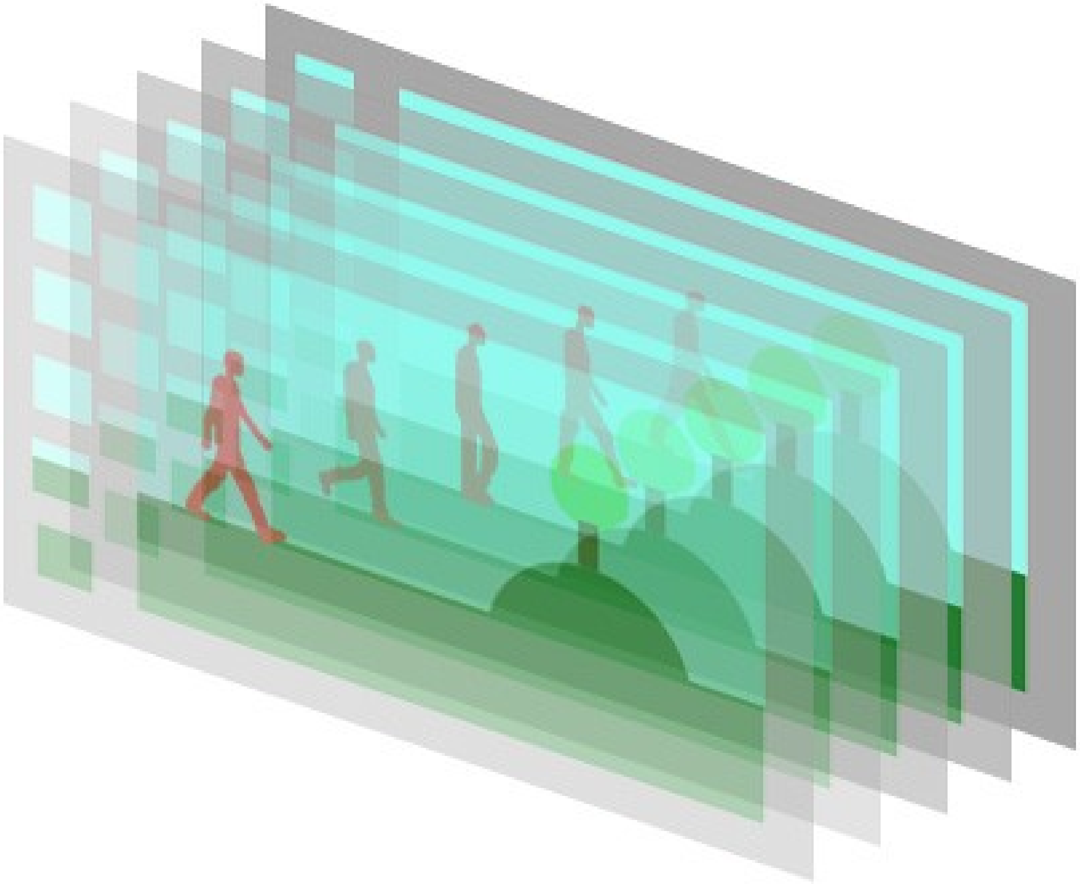
Consequence: Block universe



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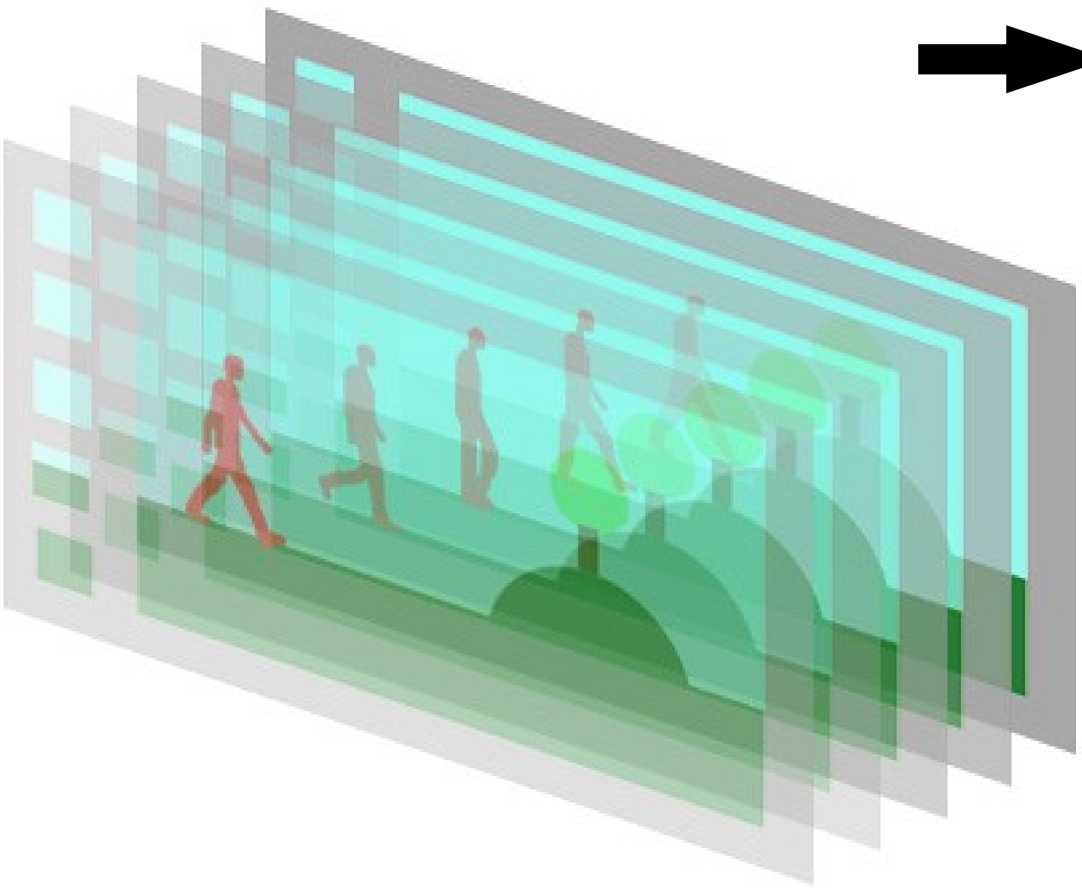

we'd like a quantum description of time that contains the BU

… can we then travel in time?!?

Palle Yourgrau, A world without time

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Travel to the future, trivial: relativistic time dilation.

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Travel to the past: possible in principle, not in practice. Closed timelike Curves (CTCs)

Palle Yourgrau, A world without time

Fate?!?!?!

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(or whatever applies to a quantum GR)

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Physical solution of an ancient religious problem!

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WHO'S RIGHT?

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Neither, but both in part...

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Time doesn't exist if nothing happens.

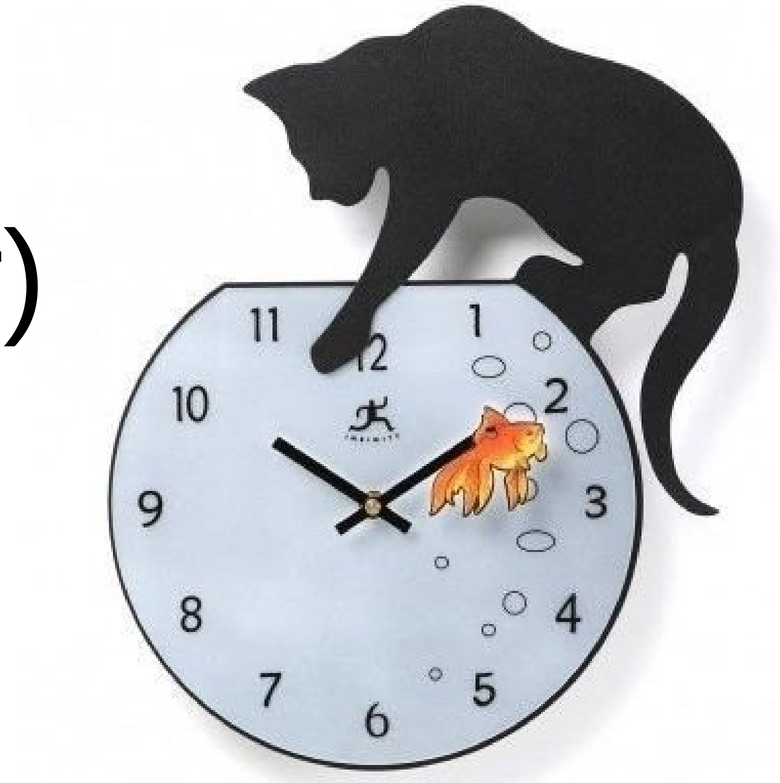
it does exist (at least as a coordinate), but it is indeed relational, if you want to give it a physical meaning

QUANTUM TIME

Start from the

NONRELATIVISTIC case

(relativity later)



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BUT... **classical systems don't exist**

in a consistent theory of quantum mechanics
(they're just a limiting situation)

define: Time is
"what is shown on a clock"

then use a **quantum** system as a clock

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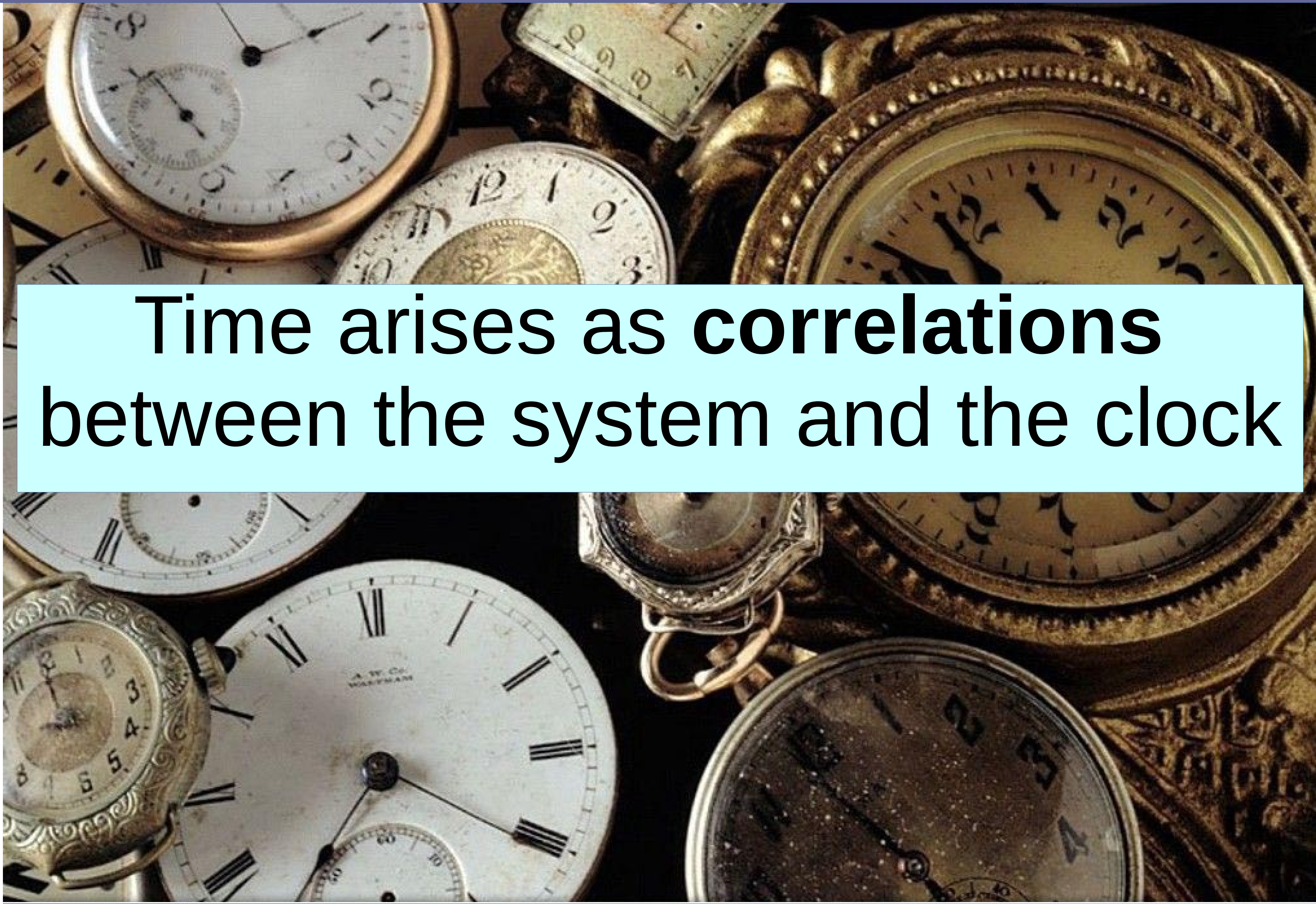
$$\mathcal{H} \equiv \mathcal{L}^2(\mathbb{R})$$

eigenbasis $\{|x\rangle\}$ — $|t\rangle$

Time and entanglement



Time arises as **correlations** between the system and the clock

The PWAK mechanism

Page and Wootters [PRD **27**,2885 (1983)]
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This means that for physical states the Hamiltonian is the generator of time translations

constraint operator:

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"position" representation=Schr eq.

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How does conventional qm fit in?

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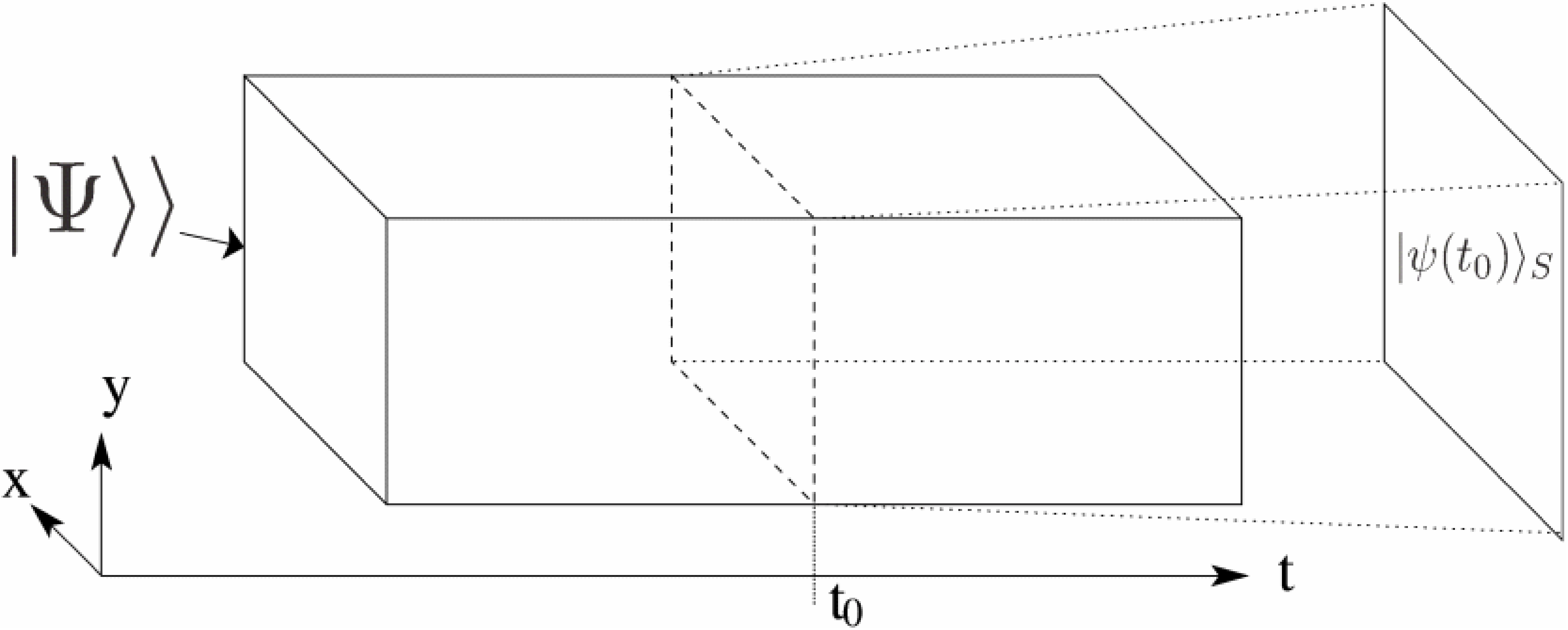
"momentum" representation=time indep. Schr eq.

what I've been saying is that

conventional qm arises in this framework through conditioning.



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is a **conditioned state**: the state *given that* the time was *t*

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Could we do with classical correlations?

$$
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Physical interpretation

The time Hilbert space is the Hilbert space of the clock that **defines** time

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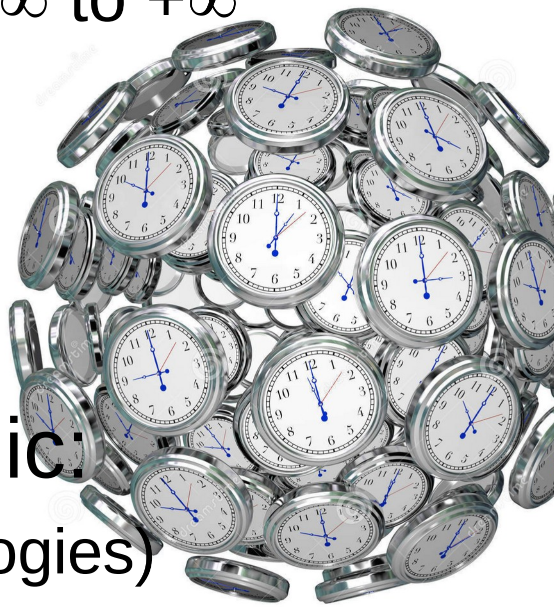

if the clock has finite energy, time is cyclic:
e.g. a spin (appropriate for certain closed cosmologies)

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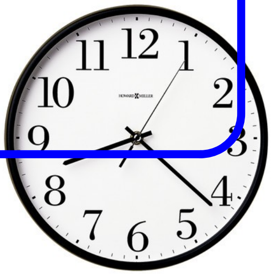
alternative:

It can be seen as an **abstract purification space**

Criticisms to time quantizations

The Pauli argument

Pauli: “time **cannot be quantized**, because a time operator that is a generator of energy translations implies that the energy is un-bounded (also from below)”



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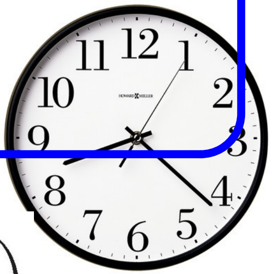

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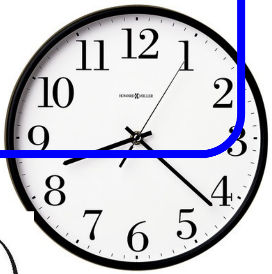

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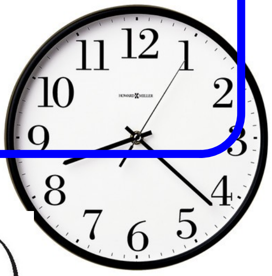

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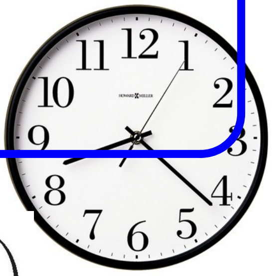

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In other words, the **Pauli argument fails** in our case because the energy-time connection is not enforced dynamically as

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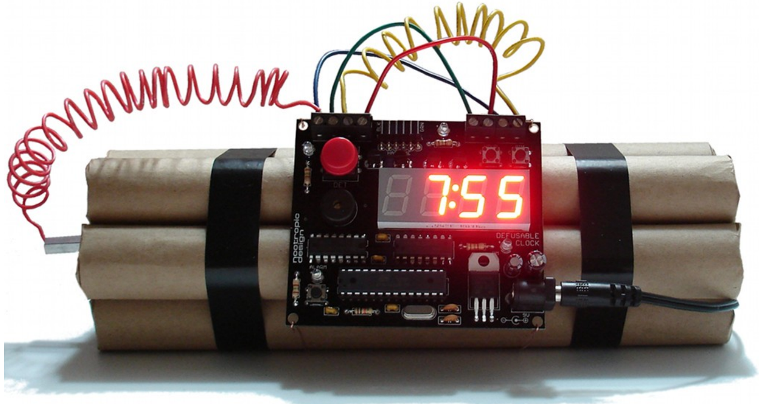

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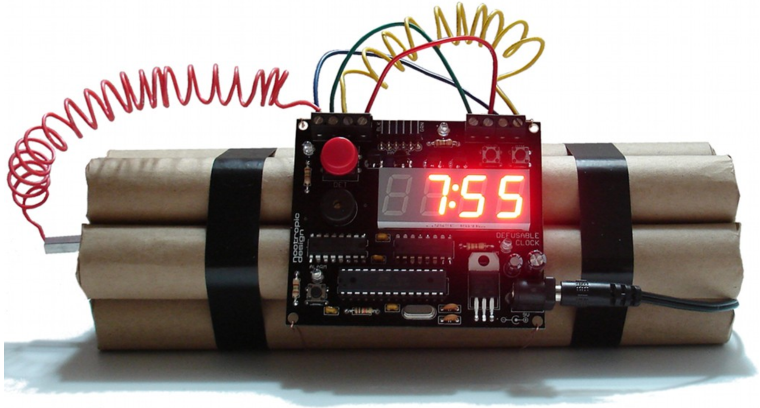

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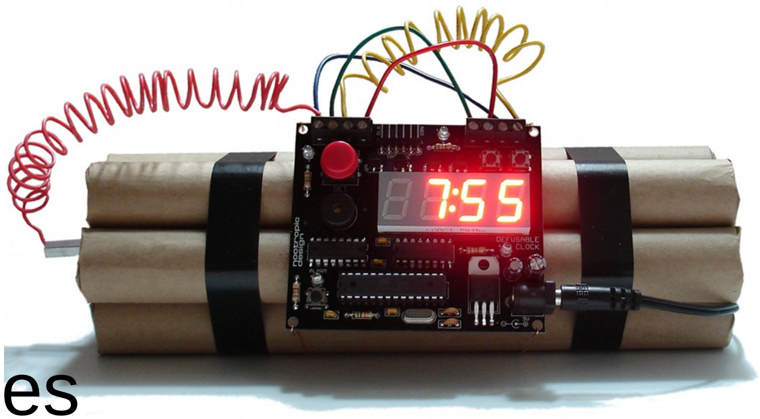
they act on different Hilbert spaces



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Quantum prob. are **conditional probabilities:**

The probability to obtain outcome $\vec{x}$ ***given*** that time is *t*

$$p(\vec{x}|\psi,t) = |\langle\vec{x}_S|\psi_S(t)\rangle|^2 = |\langle\vec{x}_H(t)|\psi_H\rangle|^2$$

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In the quantum time framework we have a ***joint*** probability that you get outcome $\vec{x}$ and that time is *t* (and then you can condition on one or the other)

The Peres argument

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- in conventional qm, time is not a dynamical variable $\Rightarrow$ no problem.

- in our case, time *is* a dynamical variable, but its translations are NOT generated by $\hat{H}_S$ (but by $\hat{\Omega}$)

The Kuchar argument against PaW

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$$|\Psi\rangle\rangle = \int dt\, |t\rangle_T \otimes |\psi(t)\rangle_S$$

$\downarrow$ time t

$$|\psi(t)\rangle$$

after a measurement of time, the state collapses to $|\psi(t)\rangle$: successive measurements give wrong statistics: **no more evolution**

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a careful formalization of **what a two-time measurement is** solves the problem!

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a careful formalization of **what a two-time measurement is** solves the problem!

The second measurement is a joint measurement on the system and on the d.o.f. that stored the outcome of the first.

The Kuchar argument against PaW

In formulas (using von Neumann's prescription for a measurement):

Measurement of observable with eigenstates $|a\rangle$ at t_0:

$$|\psi(t_0)\rangle_S |r\rangle_m \xrightarrow[t_0]{U} |\psi'\rangle_{Sm} \equiv \sum_a \psi_a |a\rangle_S |a\rangle_m$$

$$|\psi(t_0)\rangle = \sum_a \psi_a(t_0)|a\rangle$$

$$|\Psi\rangle\rangle = \int_{-\infty}^{t_0} dt |\psi(t)\rangle_S |r\rangle_m^N |t\rangle_T +$$

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$$\Rightarrow \boxed{p(a|t_0) = |\langle a|\psi(t_0)\rangle|^2 = \| {}_m\langle a|_T\langle t_0|\Psi\rangle\rangle\|^2}$$

$$= |\psi_a(t_0)|^2$$

(Born's rule)

two time measurements: same idea!!

$|a\rangle$ at t_0 and $|b\rangle$ at t_1 :

$$|\Psi\rangle\rangle = \int_{-\infty}^{t_0} dt \ldots + \int_{t_0}^{t_1} dt \sum_a \psi_a(t_0) U(t-t_0)|a\rangle_S |a\rangle_m^N |r\rangle_{m'}^N |t\rangle_T$$

$$+ \int_{t_1}^{\infty} dt \sum_{ab} \psi_a(t_0) U(t-t_1)|b\rangle_S \langle b|U(t-t_0)|a\rangle_S |a\rangle_m^N |b\rangle_{m'}^N |t\rangle_T$$

Bayes rule

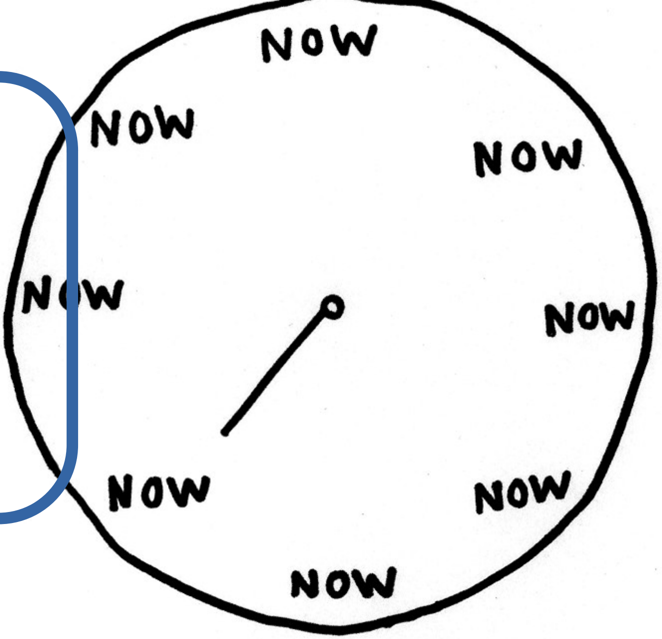
$$p(b|a,t_1) = \frac{p(b,a,t_1)}{p(a,t_1)} = \frac{\| \langle a|\langle b| \langle t_1|\Psi\rangle\rangle \|^2}{\| \langle a|\langle t_1|\Psi\rangle\rangle \|^2}$$

$$= |\langle b|U(t_1-t_0)|a\rangle|^2$$

The expected outcome!!

(Born's rule)

⇒ Kuchar's objection is defeated!



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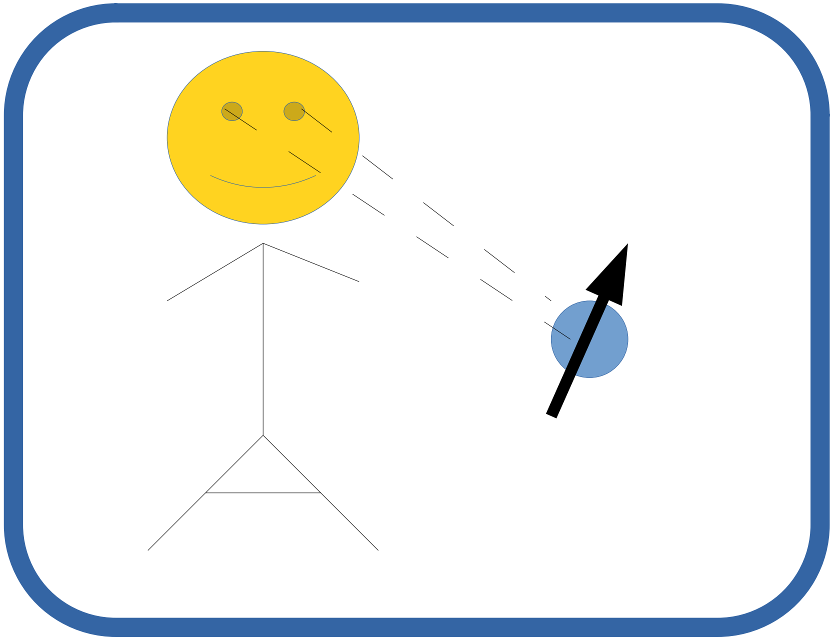

Alice's lab

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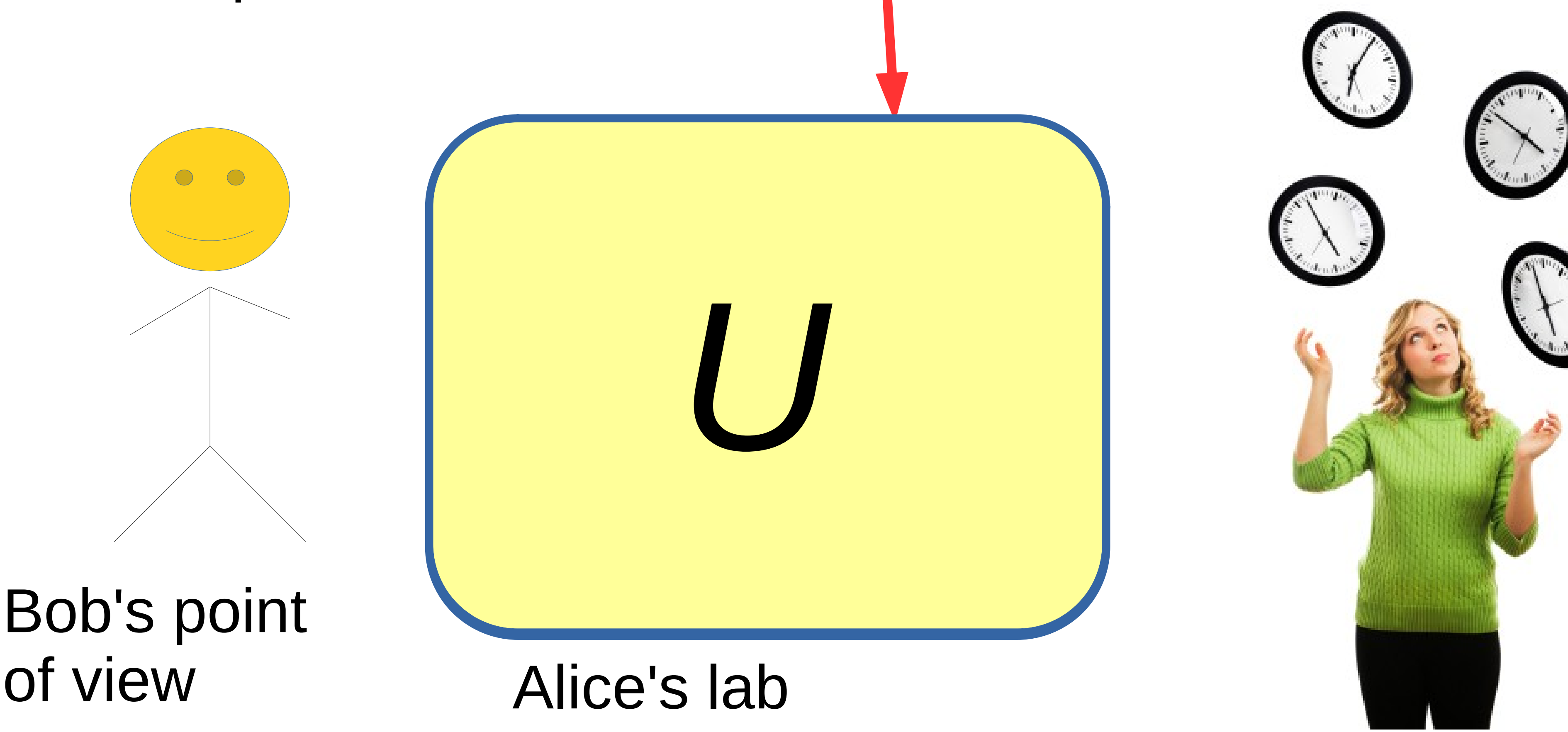

Motivations

same treatment of time and space in qm

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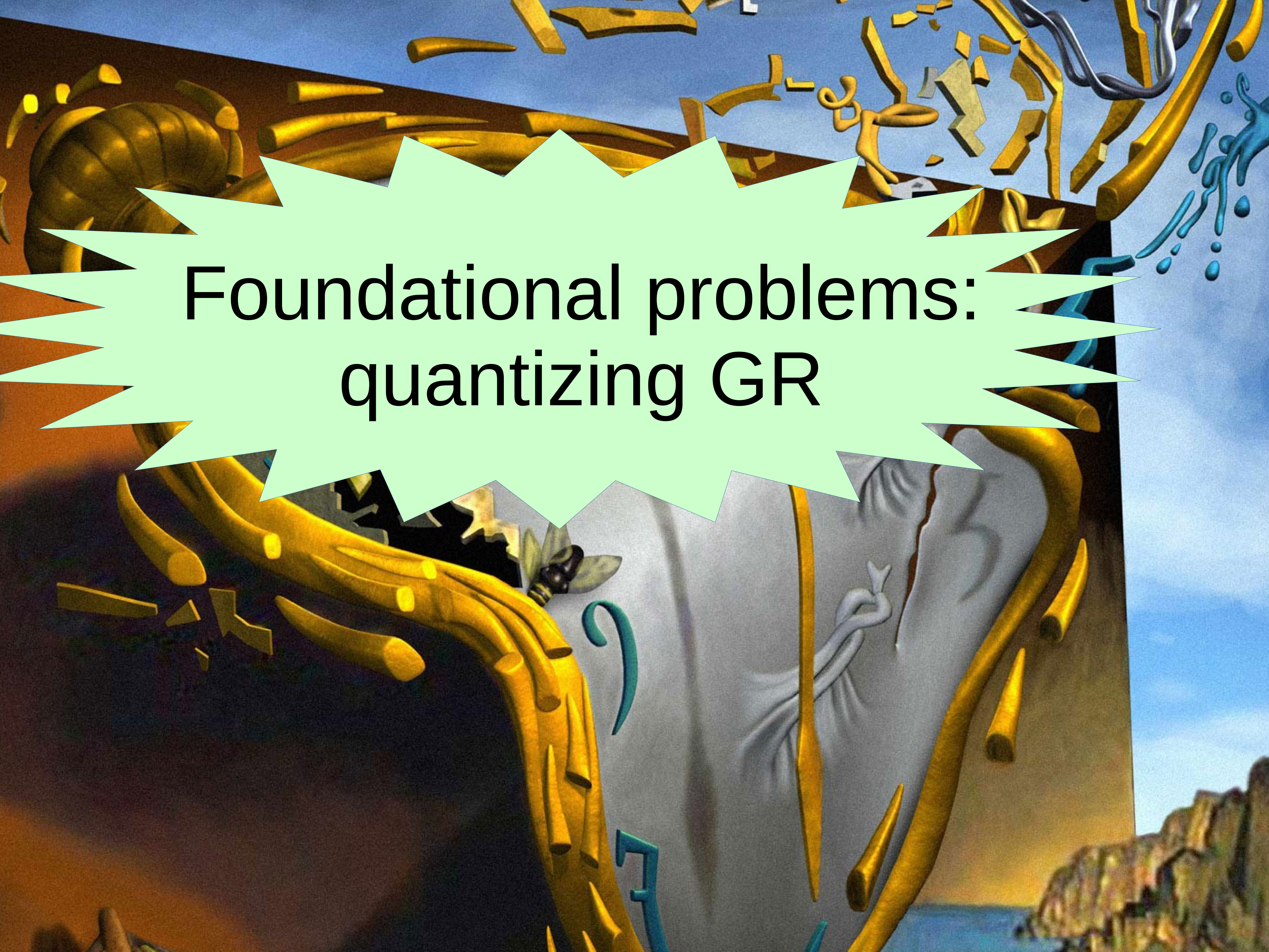
OUR FRAMEWORK permits the QUANTIZATION OF EVENTS



Foundational problems: quantizing GR

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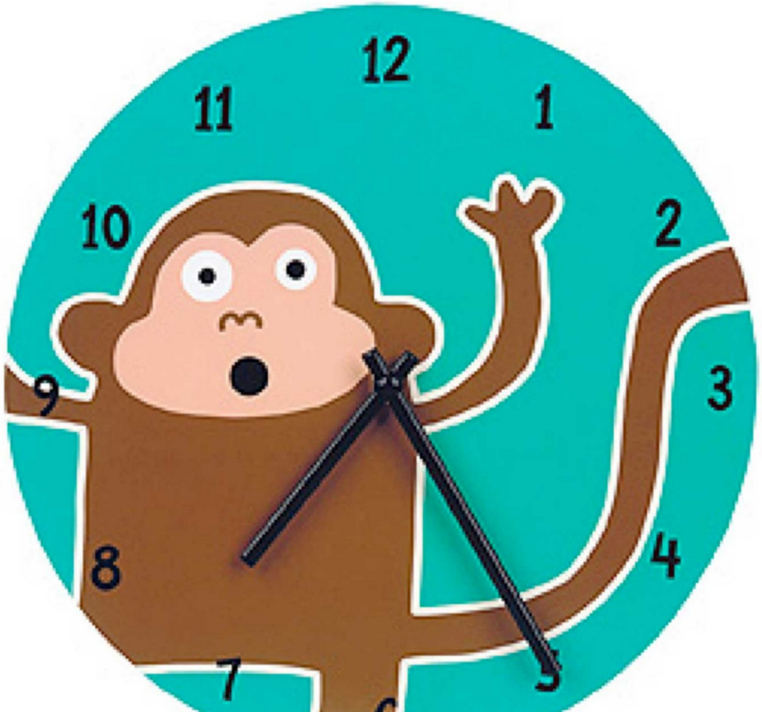
- GR ⟶ events



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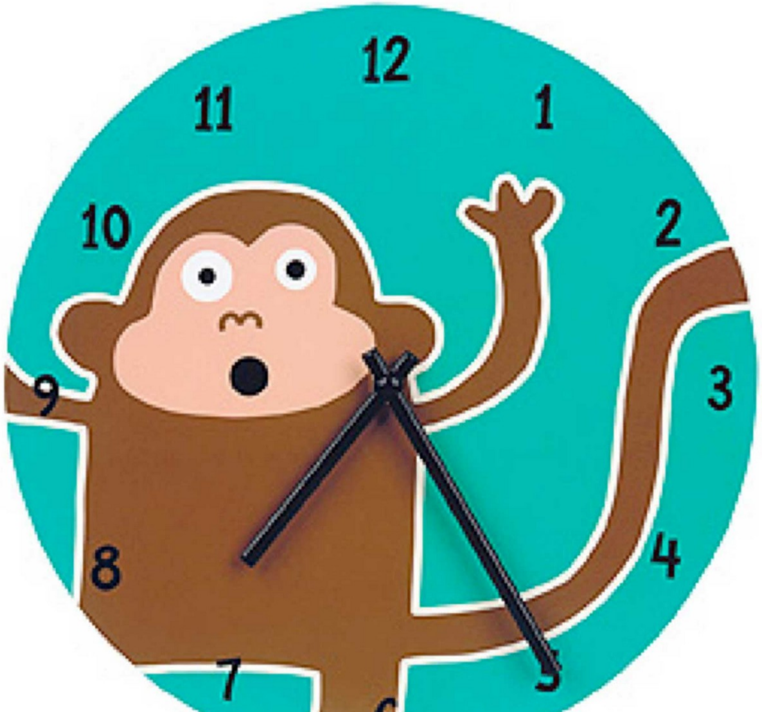

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What do YOU think?!

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- General relativistic theory of QM

Systems are more fundamental: GR is made of quantum fields

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Events are more fundamental:

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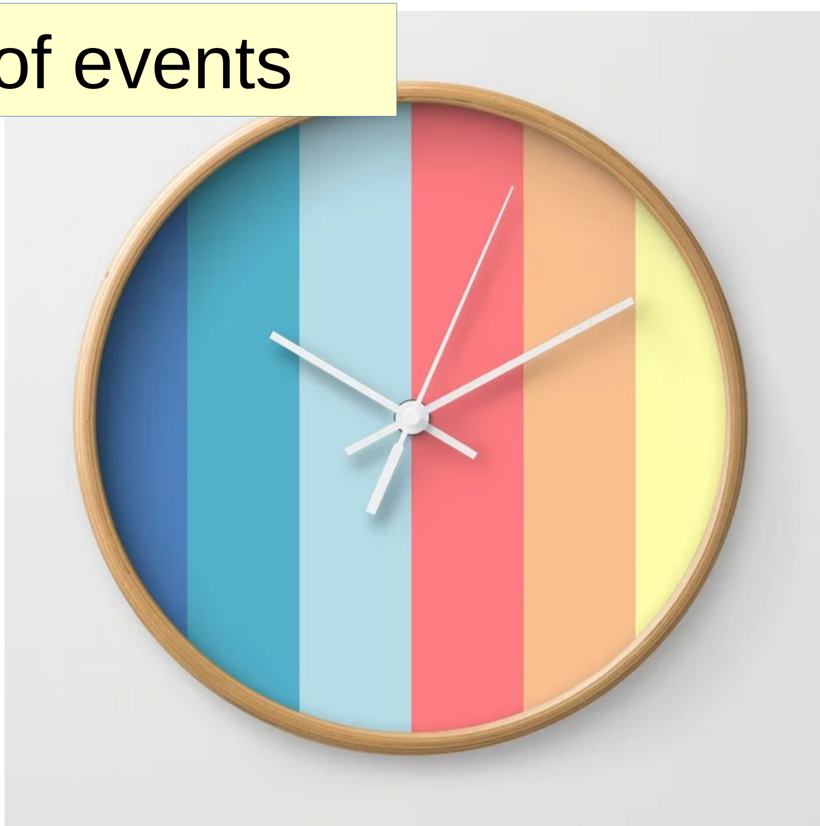

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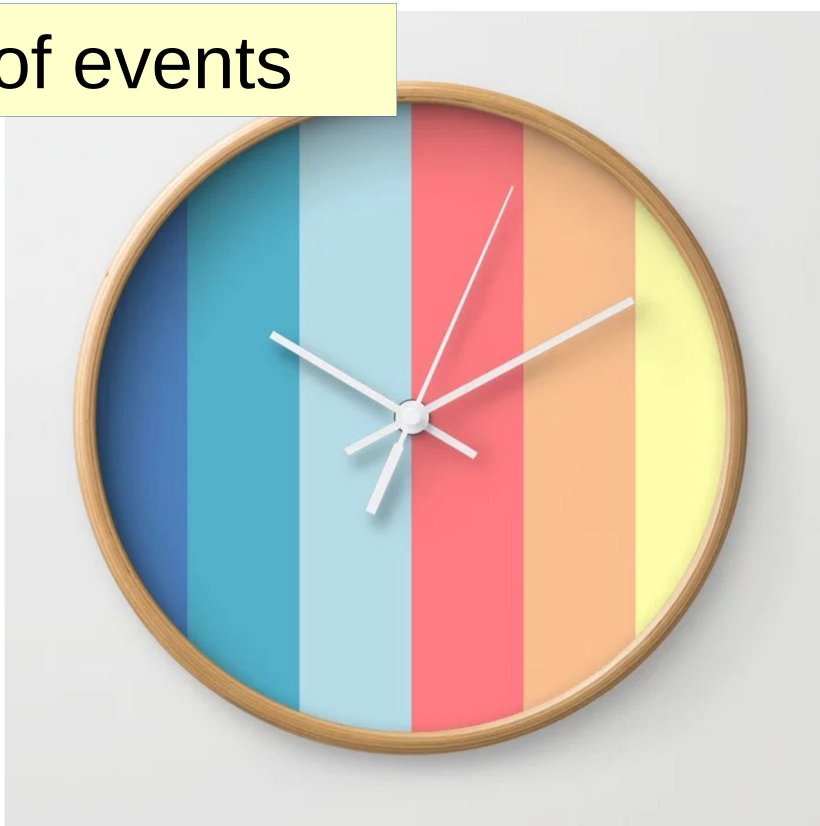

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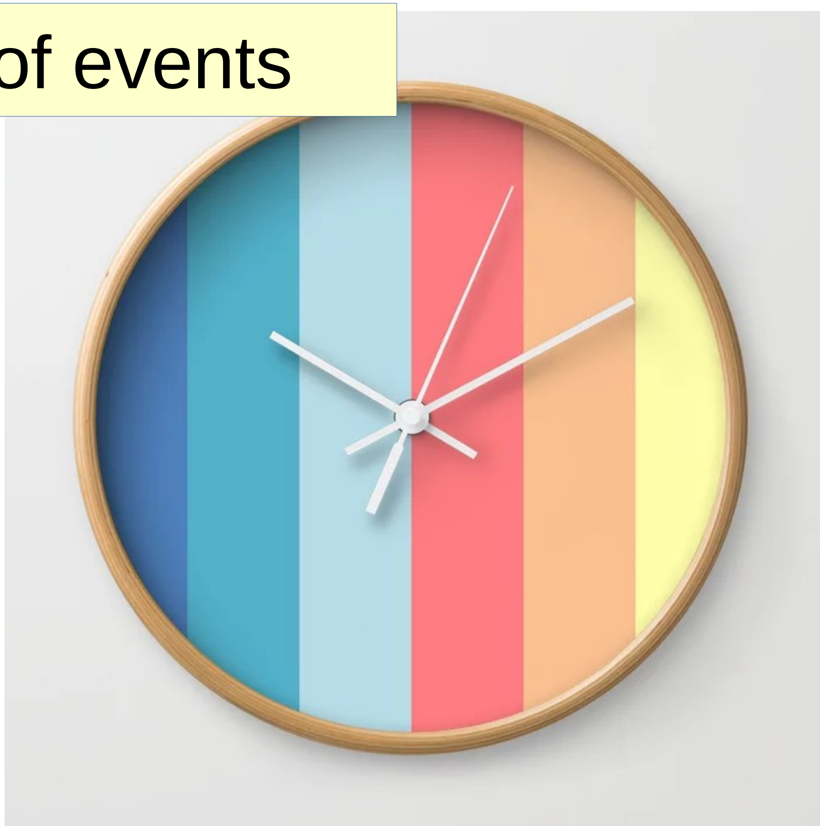

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event=what happens to a quantum system

•Quantum theory of GR

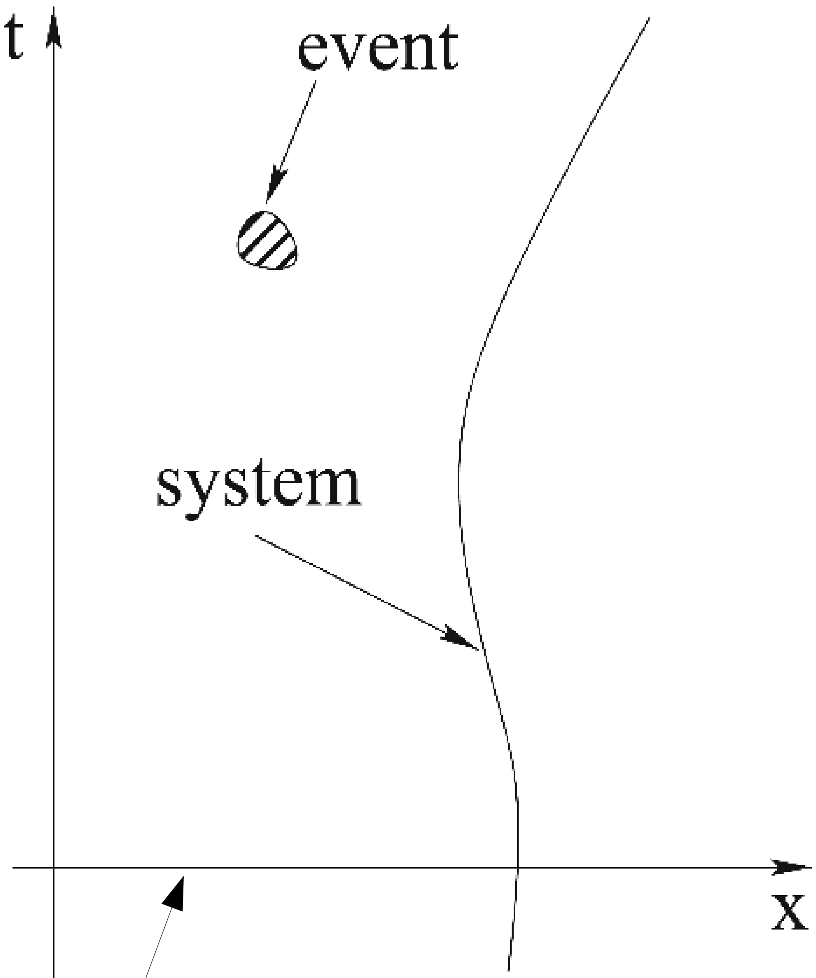
Events are more fundamental:

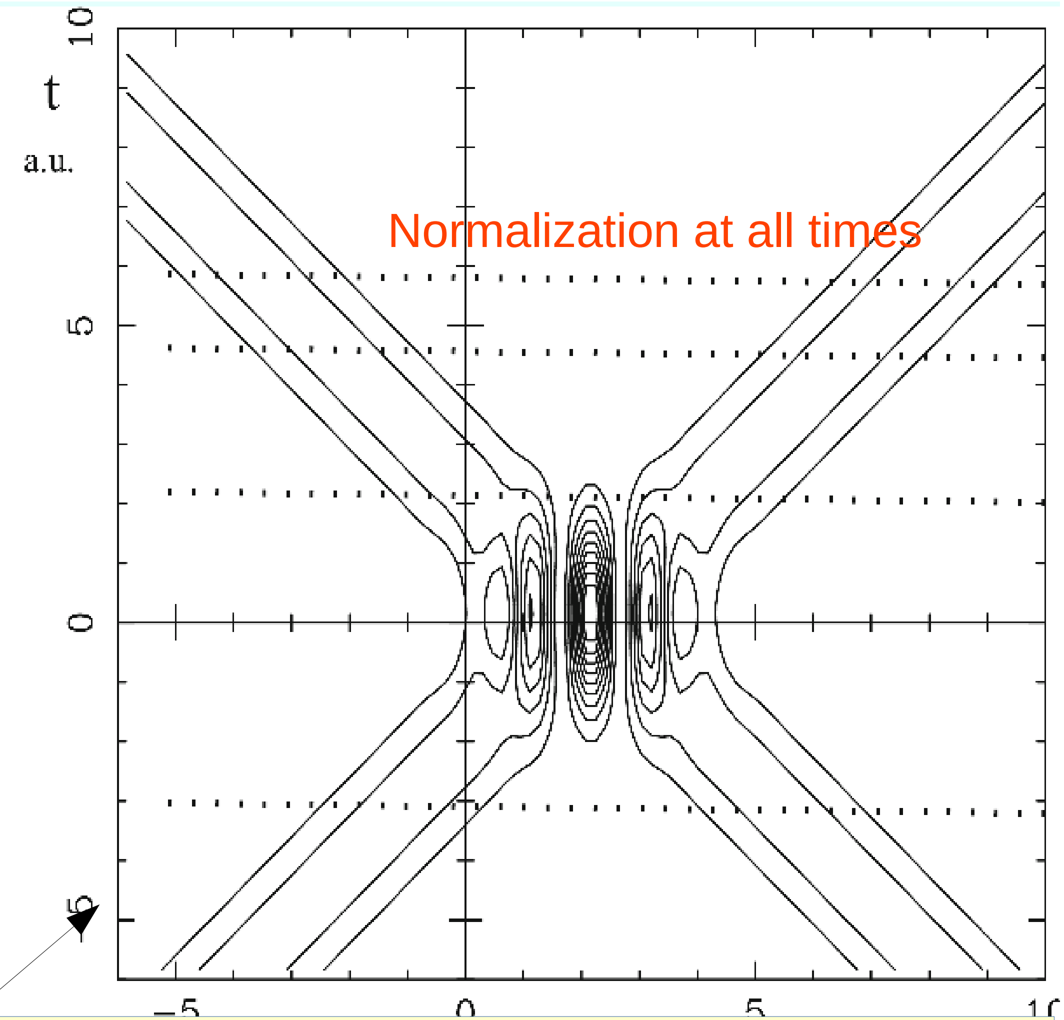
Quantum system=succession of events

Quantum gravity approaches up to now?

Explore the alternative!!

Current QM not able to deal with q events





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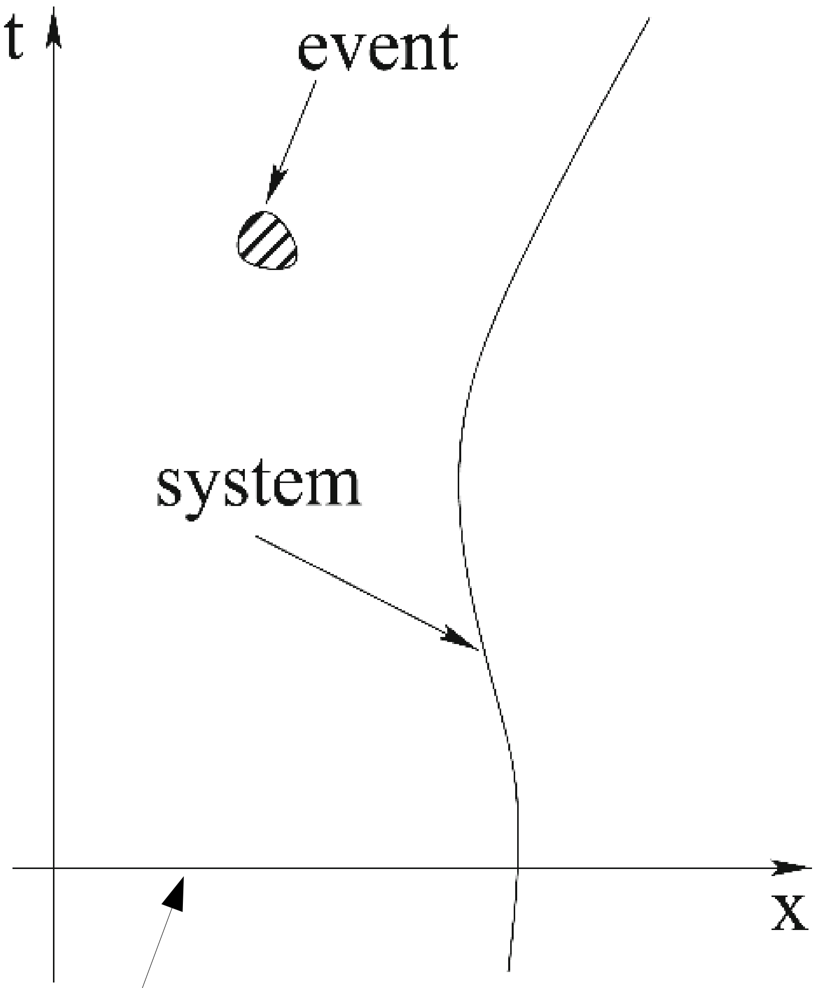

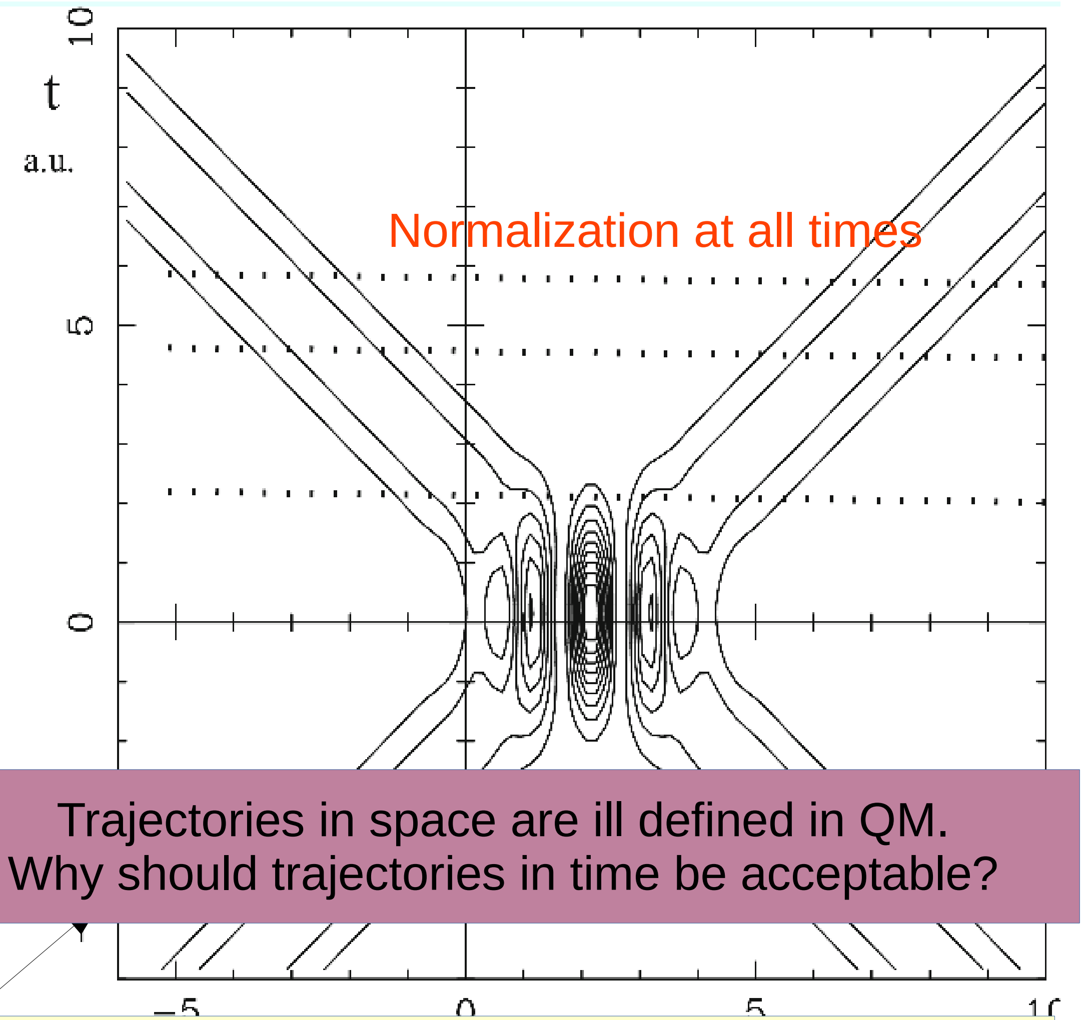

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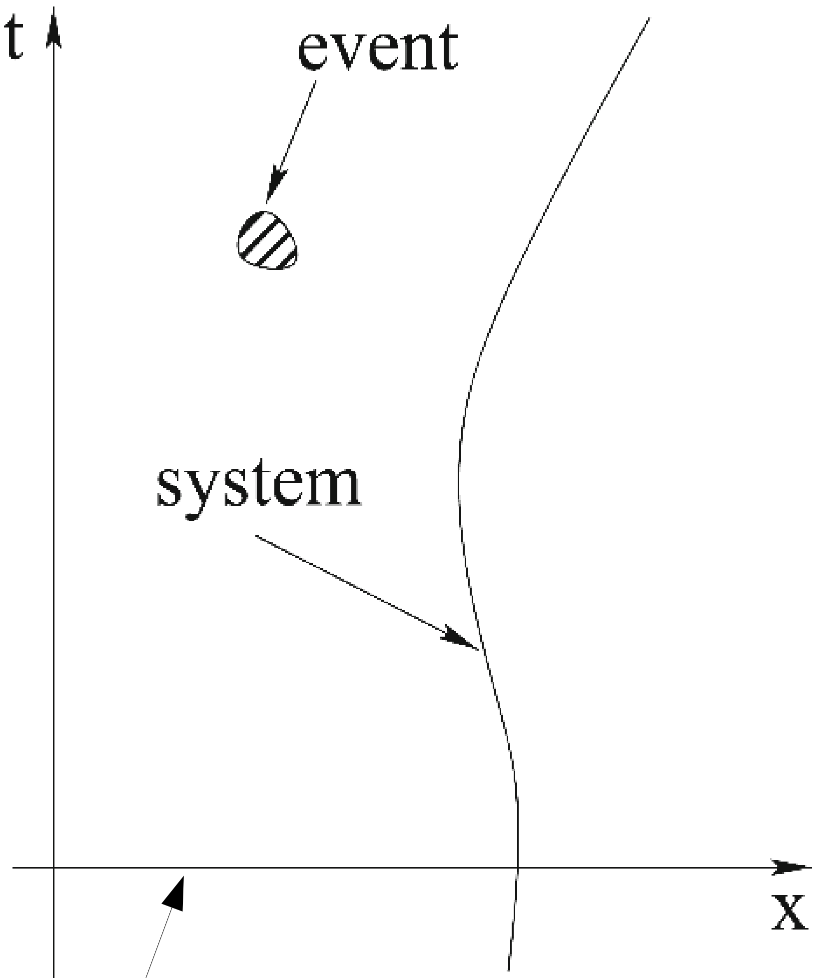

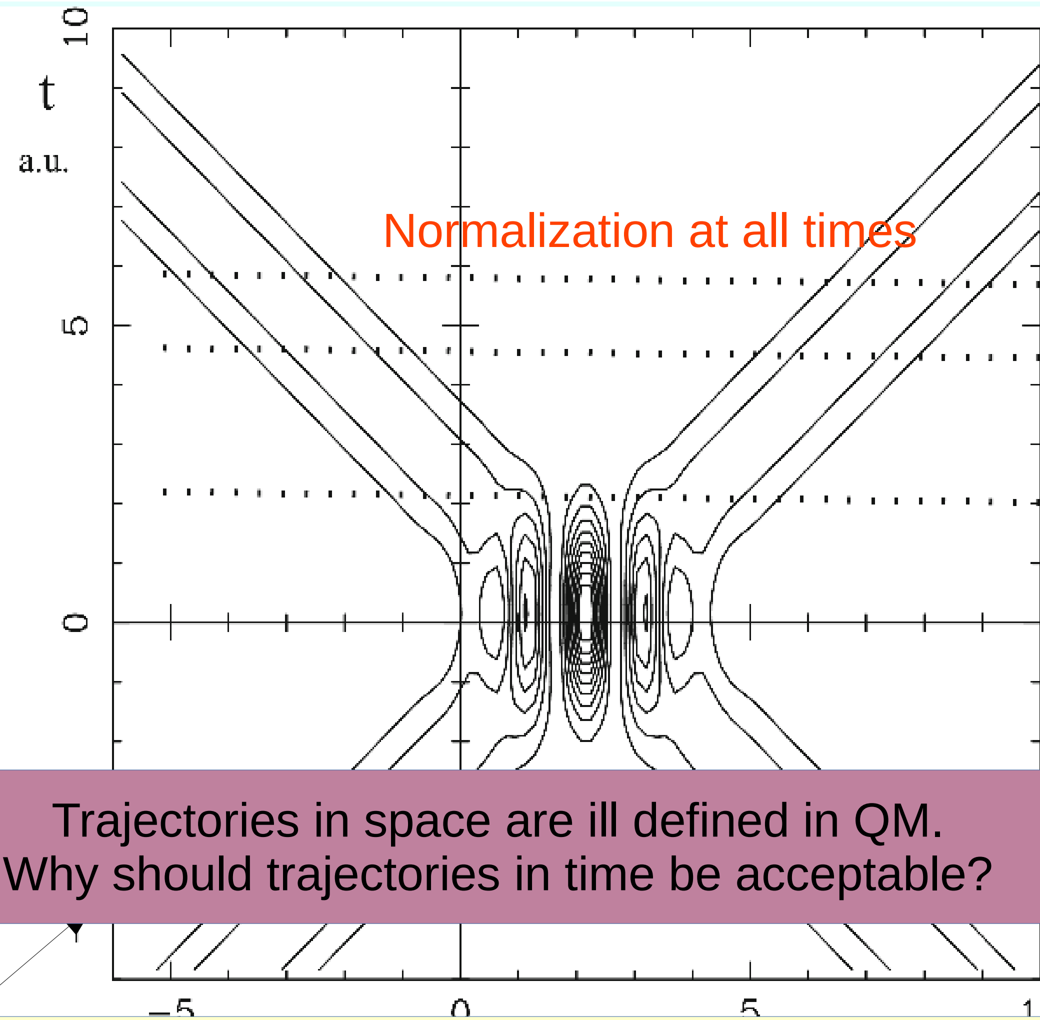

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Need: Hilbert space for events (and its composition rule!)

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$$|\psi(t)\rangle$$ States (Schroedinger picture)

$$X(t)$$ Observables (Heis. picture)

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$$\Downarrow$$

CANNOT be relativistically covariant

(covariance="formulas look the same in all reference frames")

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2) Use a state that is invariant for Lorentz transforms, e.g the vacuum $|0\rangle$

Our approach: Geometric Event-Based QM

GEB

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GEB

quantum events → **fundamental**

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GEB

quantum events → **fundamental**

quantum systems → derived: a **quantum state for a succession of events** in q spacetime

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Poincare' algebra:

$$\begin{aligned} [M^{\mu\nu}, P^\rho] &= -i(\eta^{\mu\rho} P^\nu - \eta^{\nu\rho} P^\mu), \\ [M^{\mu\nu}, M^{\rho\sigma}] &= i(\eta^{\nu\rho} M^{\mu\sigma} - \eta^{\mu\rho} M^{\nu\sigma} \\ &\quad -\eta^{\mu\sigma} M^{\rho\nu} + \eta^{\nu\sigma} M^{\rho\mu}) \end{aligned}$$

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GEB

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$$p(\vec{x}|\psi,t) = |\langle\vec{x}_S|\psi_S(t)\rangle|^2 = |\langle\vec{x}_H(t)|\psi_H\rangle|^2$$

Probability that the particle is at position $\vec{x}$

given that the time is t!!!

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UNCONDITIONED probability that the event is in spacetime position $\overline{x}$

QM probabilities are NOT covariant
GEB probabilities ARE covariant

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$\Delta X^\mu \Delta P^\mu \geqslant 1/2,$ $\mu = 0, 1, 2, 3$ UR!!!

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Cannot localize an event in time unless it has an energy spread

LORENTZ TRANSFORMS IN GEB

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Just a **unitary transformation on the GEB state** (Wigner's prescription on how to describe symmetries of a theory)

$$|\Phi'\rangle = U_\Lambda |\Phi\rangle$$

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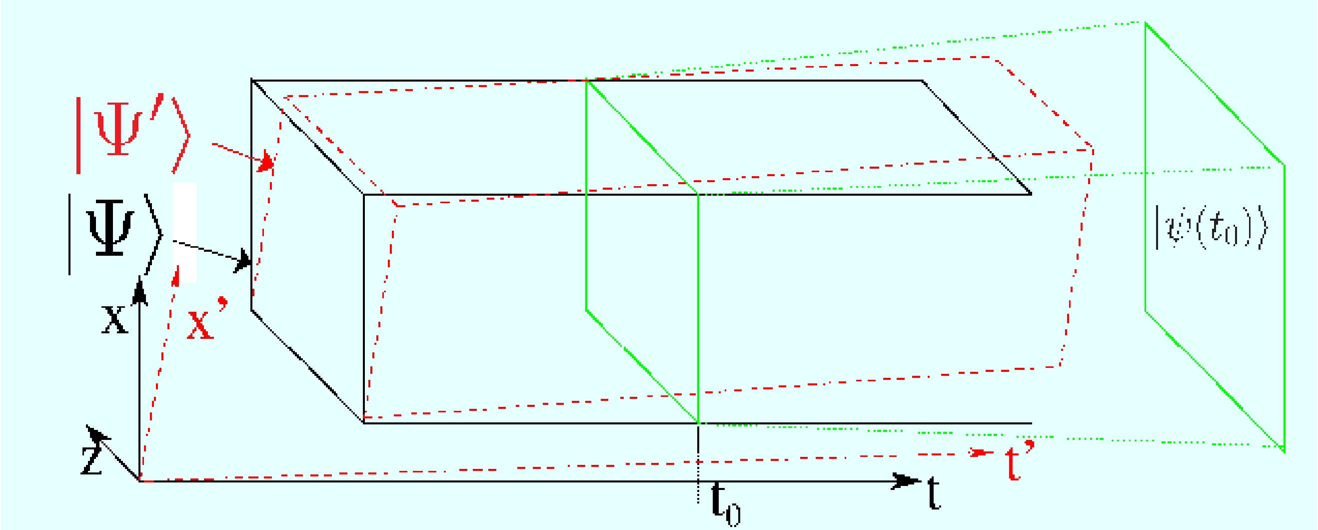

LORENTZ TRANSFORMS IN QFT

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… or you can take a shortcut through GEB

LORENTZ TRANSFORMS IN QFT

Nightmare
from the
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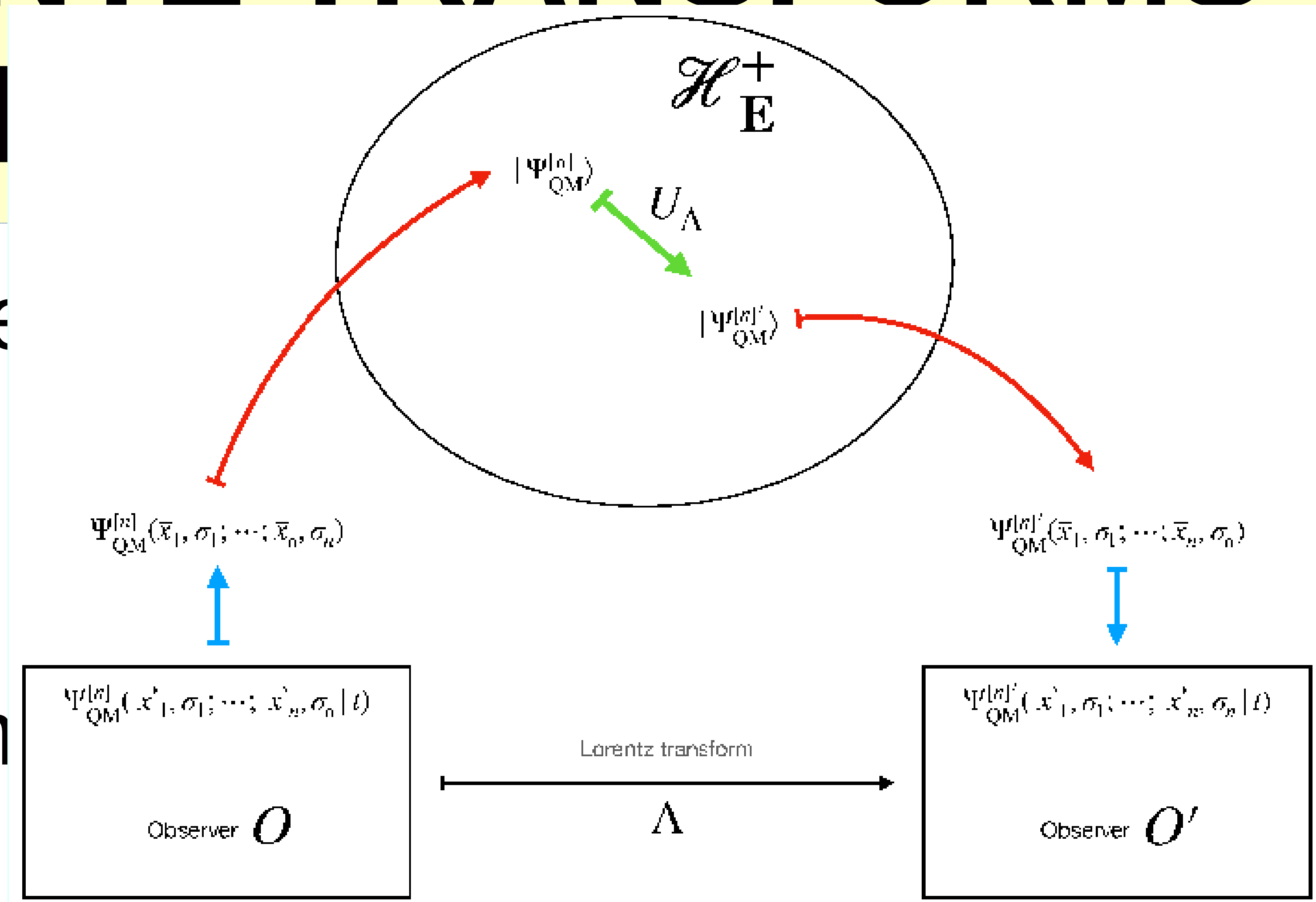

… GEB

Easier than requantizing everything: a good first motivation for GEB

Multiple events: tensor products!

(if fixed number of events n)

$$|\Phi^{[n]}\rangle = \sum_{\sigma_1,\cdots,\sigma_n} \int d^4x_1 \cdots d^4x_n \; \Phi^{[n]}(\bar{x}_1, \sigma_1; \cdots ; \bar{x}_n, \sigma_n) \left|\bar{x}_1, \sigma_1; \cdots ; \bar{x}_n, \sigma_n\right\rangle$$

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Fock space

(otherwise)

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Creation operators: create an event at position x_1

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EACH EVENT WITH ITS OWN TIME!!!!

(cfr Dirac's multiparticle-multitime)

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Commutators:

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Bosonic events → Bosons

Fermionic events → Fermions

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4D vacuum

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Different state from the 3D vacuum of QFT
($|0\rangle_3$ =no particles at time t=0 in the Heis pic)

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$$|0\rangle_3 = \text{foliate}(a^\dagger_{\overline{p}=0}|0\rangle_4)$$

Event state of **zero 4-momentum**: ground state of the field

QFT FROM GEB

Use **constraints** (as in the quantum time P&W, WdW, etc.)!

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$$\Psi_{\rm QM}(\vec{x},\sigma|t)$$

← (prob amplitude of finding particle in x given that the time is t)

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$$\Psi_{\rm QM}(\overline{x},\sigma) := \Psi_{\rm QM}(\vec{x},\sigma|t)$$

$$|\Psi_{\rm QM}\rangle \stackrel{!}{:=} \sum_{\sigma}\int d^4x\; \Psi_{\rm QM}(\overline{x},\sigma)\;|\overline{x},\sigma\rangle$$

(GEB state describing the whole dynamics of the **particle as a state of a sequence of events**)

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3) Write it as an eigenstate of a constraint op. →

$$K|\Psi_{\rm QM}\rangle = 0$$

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Same procedure works for more complex QFT systems e.g.

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Similarly for the Dirac eq. constraint.

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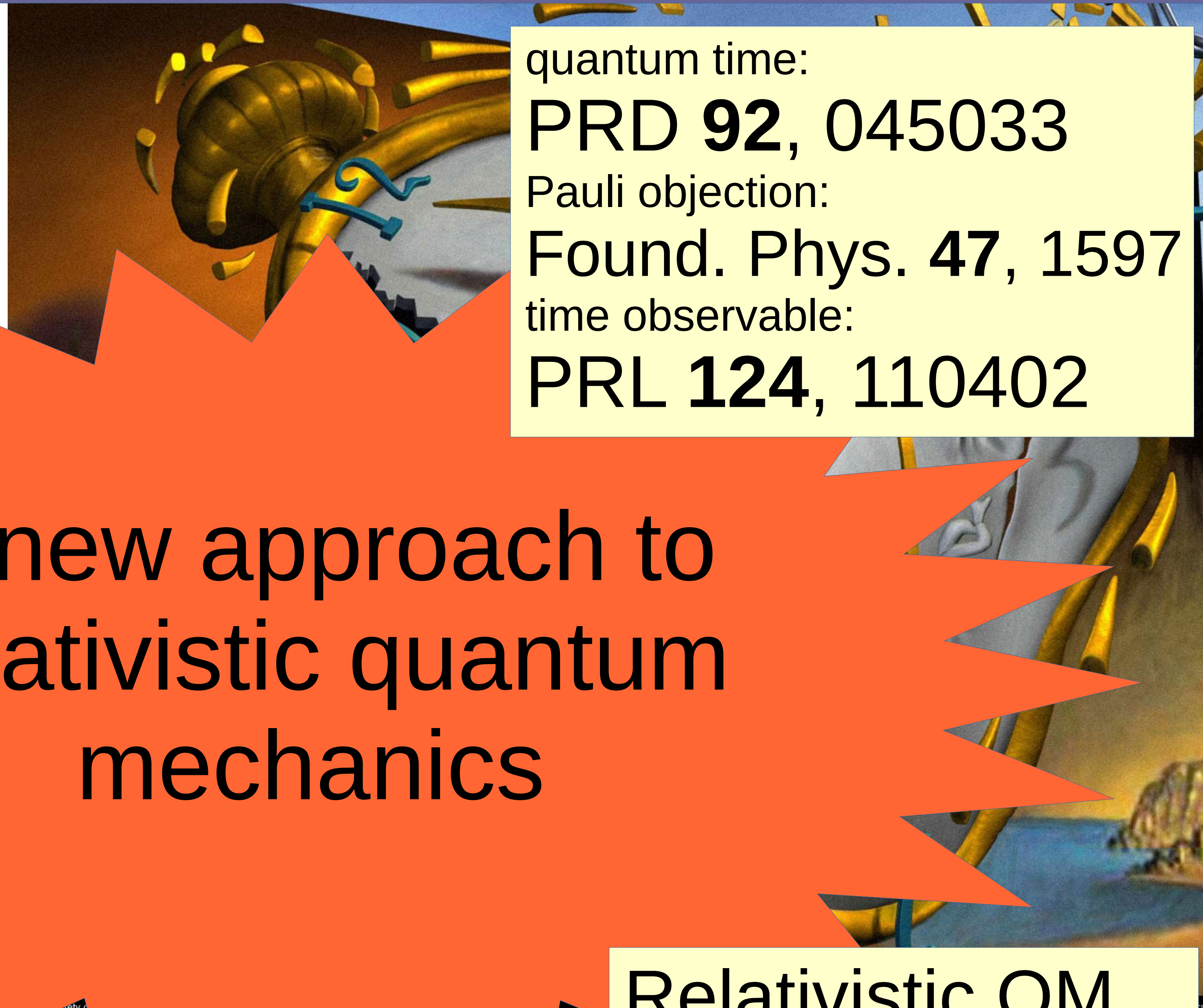
- QFT from GEB: KG and Dirac

Take home message



quantum time:

PRD **92**, 045033

Pauli objection:

Found. Phys. **47**, 1597

time observable:

PRL **124**, 110402

A new approach to relativistic quantum mechanics

Relativistic QM
arXiv:2206.08359

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