The $O(N)$ Gross-Neveu-Yukawa Archipelago

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October 10, 2022

Prepared for Bootsrapping Nature at the Galileo Galilei Institute.
Based on [2210.02492] with Luca V. Iliesiu, Petr Kravchuk, Aike Liu, David Poland, and David Simmons-Duffin.

Introduction: The $O(N)$ Gross-Neveu-Yukawa models

$$\mathcal{L}_{O(N)} = -\frac{1}{2}(\partial\phi)^2 - i\frac{1}{2}\psi_i\partial\!\!\!/\psi_i - \frac{1}{2}m^2\phi^2 - \frac{\lambda}{4}\phi^4 - i\frac{g}{2}\phi\psi_i\psi_i$$

- 2-component Majoranas $\psi_{i=1\ldots N}$, pseudoscalar ϕ
- $O(N)$ global symmetry plus parity
- $m^2 \geq m_*^2$: $\langle\phi\rangle = 0$, ψ_i massless, parity preserved
- $m^2 < m_*^2$: $\langle\phi\rangle \neq 0$, ψ_i massive, parity broken

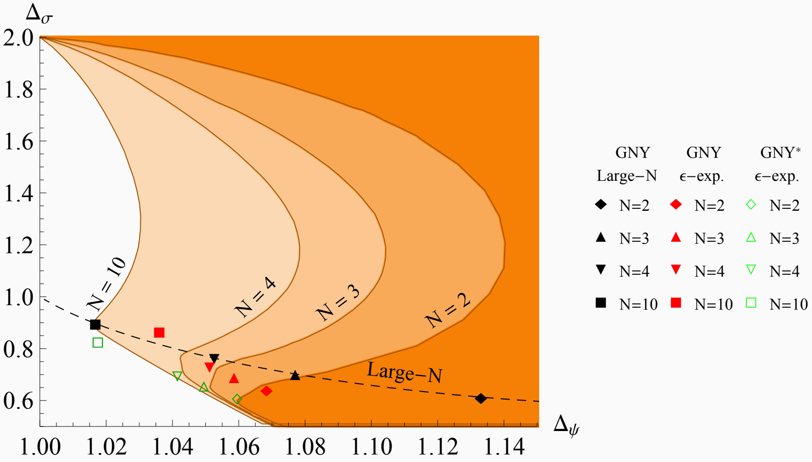
Parity breaking also means time-reversal symmetry breaking (TRSB).

Condensed matter background

- Graphene lattice ($N = 8$) [Herbut '06; Herbut, Juricic, Roy '09]
- d-wave cuprate superconductors ($N = 8$) [Vojta, Zhang, Sachdev '00]
- TRSB in topological superconductors [Grover, Sheng, Vishwanath '14]
- Experimentally realizable on the surface of He^3-B [Grover, Sheng, Vishwanath '14]
- Critical transition from Cartan symmetry class DIII to class D
- *Not* in the Chiral Ising universality class

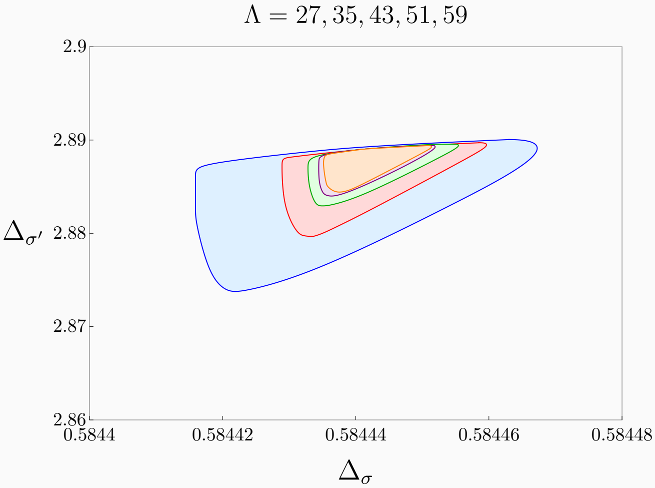
We will come back to this.

Previously studied by the bootstrap [1508.00012, 1705.03484]:



Bootstrap background: $\mathcal{N} = 1$ super-Ising

Contact with $\mathcal{N} = 1$ super-Ising bootstrap [1807.04434, 1807.05702, 2201.02206]



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What are we bootstrapping?

$$\mathcal{L}_{O(N)} = -\frac{1}{2}(\partial\phi)^2 - i\frac{1}{2}\psi_i \not{\partial}\psi_i - \frac{1}{2}m^2\phi^2 - \frac{\lambda}{4}\phi^4 - i\frac{g}{2}\phi\psi_i\psi_i$$

Let's identify a list of external operators for our bootstrap setup and their spin ℓ, parity $\mathcal{P}$, and flavor rep μ:

Op.	ℓ	$\mathcal{P}$	μ
ψ_i	$\frac{1}{2}$	$+$	$\square$
$\sigma \sim \phi$	0	$-$	$\bullet$
$\epsilon \sim \phi^2$	0	$+$	$\bullet$

We will consider all 4-point correlation functions of ψ_i, σ, ϵ.

Crossing equations

Four point functions involved in the crossing equations:

$$\{\langle\psi\psi\psi\psi\rangle, \langle\epsilon\epsilon\epsilon\epsilon\rangle, \langle\sigma\sigma\sigma\sigma\rangle, \langle\psi\psi\epsilon\epsilon\rangle, \langle\psi\psi\sigma\sigma\rangle, \langle\sigma\epsilon\psi\psi\rangle, \langle\sigma\sigma\epsilon\epsilon\rangle\}.$$

For each of these equations:

$$\langle\mathcal{O}_i(x_1)\mathcal{O}_j(x_2)\mathcal{O}_k(x_3)\mathcal{O}_l(x_4)\rangle = \sum_I g^I_{ijkl}(u,v)T_{I,ijkl}$$

$$g^I_{ijkl}(u,v) = \sum_{\Delta,\rho}\sum_{a,b}\lambda^a_{ij\mathcal{O}}\lambda^b_{kl\mathcal{O}}G^{ab,I}_{ijkl,\Delta,\rho}(u,v),$$

From this point onwards, it's a straightforward process to write out what all the a, b, I are [1612.08987] and plug that all in, and then impose crossing. See our paper and code for the explicit details.

Assumptions

We used perturbative results together with bootstrap results for $\mathcal{N} = 1$ super-Ising to arrive at the following assumptions:

Channel	ℓ	$\mathcal{P}$	μ	Δ_{min}
$\sigma' \sim \phi^3$	0	$-$	$\bullet$	3[1]
$\epsilon' \sim \phi^4$	0	$+$	$\bullet$	3
$\sigma_T \sim \psi_{(i}\psi_{j)}$	0	$-$	$\square\square$	2
$\psi' \sim \psi\phi^2$	$\frac{1}{2}$	$+$	$\square$	2
$\chi \sim \psi\phi^3$	$\frac{1}{2}$	$-$	$\square$	3.5
otherwise				$\Delta_{\text{U}} + 10^{-6}$

[1] For $N = 1$ we know that $\Delta_{\sigma'} = 2.8869(25)$, so this gap assumption wasn't used for $N = 1$.

Numerics

Everything Andy talked about, plus:

- Searching over OPE coefficient ratios [1603.04436]
- Cutting surface search algorithm, and Delaunay mesh search, hotstarting [1912.03324]
- SDPB [1502.02033] with HPC parallelism [1909.09745].
- `blocks_3d` [1907.11247,2011.01959]
- A whole software stack, written in Haskell, which can be found at [gitlab.com/davidsd/fermions-3d; 2210.02492]

Searched over the space $\{\Delta_\psi, \Delta_\sigma, \Delta_\epsilon, \frac{\lambda_{\psi\psi\sigma}}{\lambda_{\sigma\sigma\epsilon}}, \frac{\lambda_{\psi\psi\epsilon}}{\lambda_{\sigma\sigma\epsilon}}, \frac{\lambda_{\epsilon\epsilon\epsilon}}{\lambda_{\sigma\sigma\epsilon}}\}$.

We had 38 crossing equations (28 for $N = 1$).

On Yale's Grace cluster and XSEDE's Expanse cluster, we used $\sim$6M CPU-hours.

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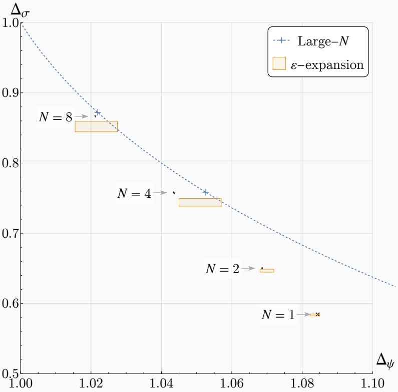
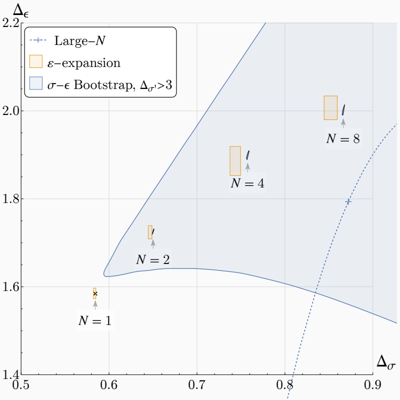
Results: Archipelago

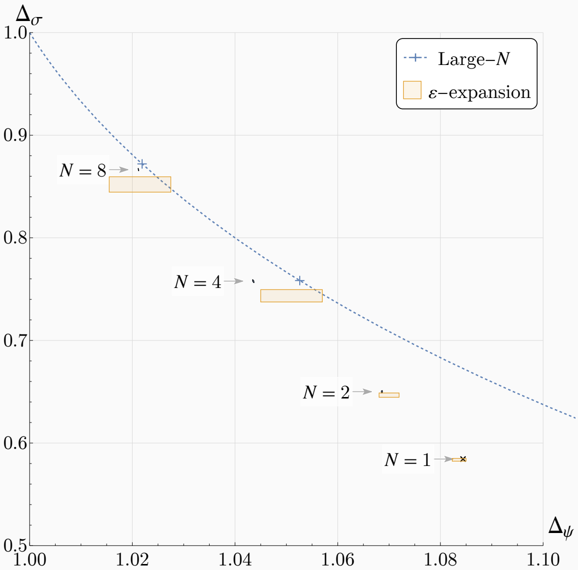




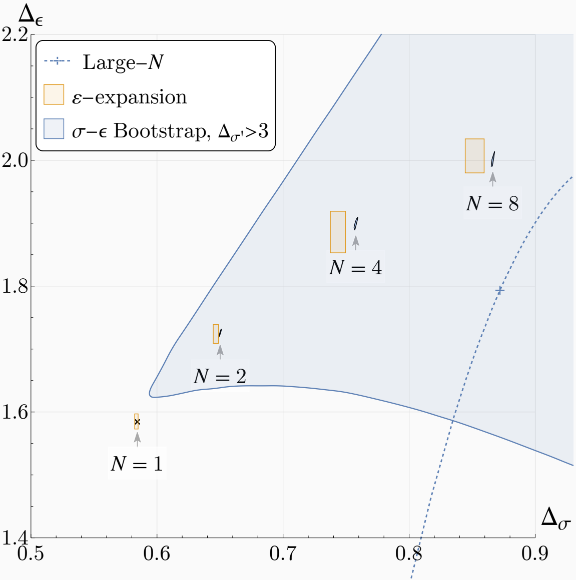
ε-exp [1806.04977], Large-N [hep-th/9306107, 1607.05316];

σ-ϵ bootstrap [1807.05702]

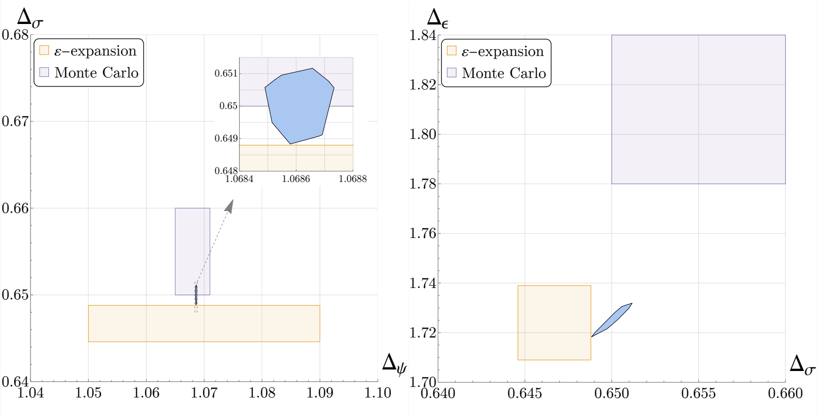
Results: Archipelago



Results: Archipelago

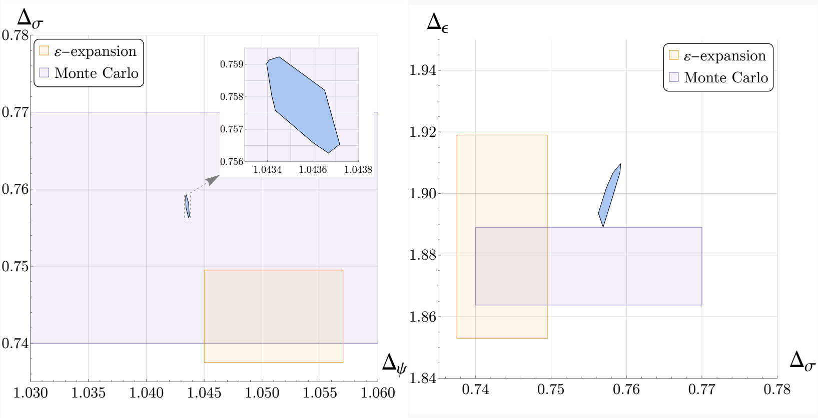


Results: N=2



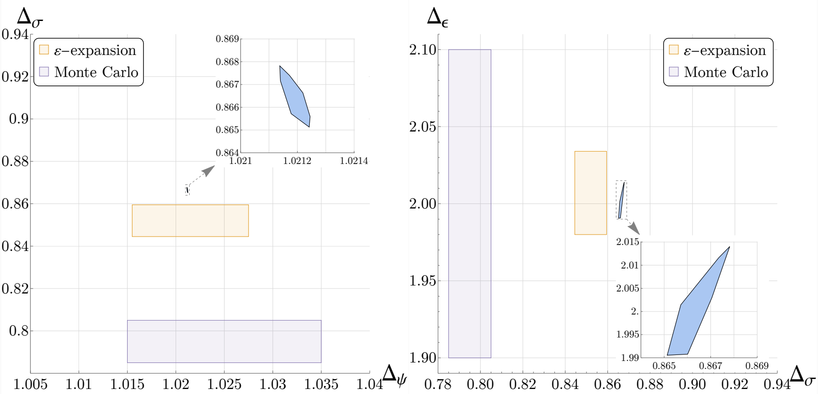
ε-expansion [1806.04977]; Monte Carlo [2112.09209]

Results: N=4



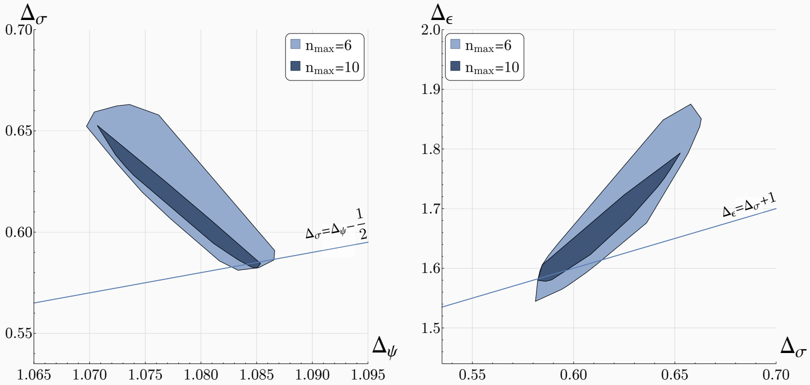
ε-expansion [1806.04977]; Monte Carlo* [1912.12823]

Results: N=8



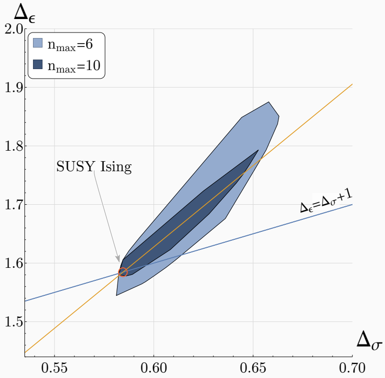
ϵ-expansion [1806.04977]; Monte Carlo* [1910.07430]

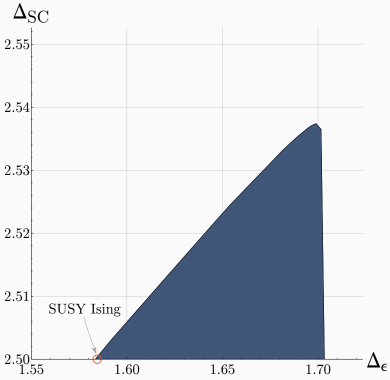
Results: N=1



$\Delta_\sigma = 0.58444(8)$ along the SUSY constraint line.

Results: N=1 Supercurrent





$\Delta_{SC} < 2.5003219$ at the SUSY Ising point.

Results: Table with critical exponents

	Δ_ψ	Δ_σ	Δ_ϵ	η_ψ	η_ϕ	ν^{-1}
N = 2						
$n_{\max} = 18, \Delta_{\sigma'} > 2.5$	1.0672(**25**)	0.657(**13**)	1.74(**4**)	0.134(**5**)	0.313(**25**)	1.26(**4**)
$n_{\max} = 18, \Delta_{\sigma'} > 3$	1.06861(**12**)	0.6500(**12**)	1.725(**7**)	0.13722(**24**)	0.3000(**23**)	1.275(**7**)
ϵ-exp w/DREG$_3$	1.07(2)	0.6467(21)	1.724(15)	0.1400(39)	0.2934(42)	1.276(15)
Monte Carlo	1.068(3)	0.655(5)	1.81(3)	0.136(5)	0.31(1)	1.19(3)
N = 4						
$n_{\max} = 18, \Delta_{\sigma'} > 3$	1.04356(**16**)	0.7578(**15**)	1.899(**10**)	0.08712(**32**)	0.5155(**30**)	1.101(**10**)
ϵ-exp w/DREG$_3$	1.051(6)	0.744(6)	1.886(33)	0.102(12)	0.487(12)	1.114(33)
Monte Carlo*	—	0.755(15)	1.876(13)	—	0.51(3)	1.124(13)
N = 8						
$n_{\max} = 18, \Delta_{\sigma'} > 3$	1.02119(**5**)	0.8665(**13**)	2.002(**12**)	0.04238(**11**)	0.7329(**27**)	0.998(**12**)
ϵ-exp w/DREG$_3$	1.022(6)	0.852(8)	2.007(27)	0.043(12)	0.704(15)	0.993(27)
Monte Carlo*	1.025(10)	0.79(1)	2.0(1)	0.05(2)	0.59(2)	1.0(1)

*s indicate studies conducted on the chiral Ising universality class.

Results: Table with OPE coefficient ratios

	Δ_ψ	Δ_σ	Δ_ϵ	$\lambda_{\psi\psi\sigma}/\lambda_{\sigma\sigma\epsilon}$	$\lambda_{\psi\psi\epsilon}/\lambda_{\sigma\sigma\epsilon}$	$\lambda_{\epsilon\epsilon\epsilon}/\lambda_{\sigma\sigma\epsilon}$
N = 2						
$n_{\max} = 18, \Delta_{\sigma'} > 2.5$	1.0672(**25**)	0.657(**13**)	1.74(**4**)	0.5071(15)	0.2347(35)	1.636(17)
$n_{\max} = 14, \Delta_{\sigma'} > 3$	1.06860(**16**)	0.6498(**14**)	1.724(**8**)	0.5087(10)	0.2392(6)	1.629(13)
$n_{\max} = 18, \Delta_{\sigma'} > 3$	1.06861(**12**)	0.6500(**12**)	1.725(**7**)	—	—	—
N = 4						
$n_{\max} = 18, \Delta_{\sigma'} > 3$	1.04356(**16**)	0.7578(**15**)	1.899(**10**)	0.4386(6)	0.15530(19)	1.682(18)
N = 8						
$n_{\max} = 18, \Delta_{\sigma'} > 3$	1.02119(**5**)	0.8665(**13**)	2.002(**12**)	0.3322(8)	0.08082(12)	1.71(4)

For $N = 2$ $\Delta_{\sigma'} > 3$ at $n_{\max} = 18$ we didn't have enough statistics to feel comfortable reporting the estamimates of the OPE coefficient ratios.

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The $O(N/2)^2 \rtimes \mathbb{Z}_2$ GNY Model

$$\mathcal{L}_{O(N/2)^2\rtimes\mathbb{Z}_2} = -\frac{1}{2}(\partial\phi)^2 - \frac{1}{2}m^2\phi^2 - \frac{\lambda}{4}\phi^4 \\ - i\frac{1}{2}\psi_i^A \partial\!\!\!/ \psi_i^A - i\frac{g}{2}\phi(\psi_i^L\psi_i^L - \psi_i^R\psi_i^R)$$

- 2-component Majoranas $\psi_{i=1\ldots N/2}^{A=L,R}$, scalar ϕ
- $O(N/2)^2$ symmetry plus a "chiral" symmetry that exchanges them, hence $O(N/2)^2 \rtimes \mathbb{Z}_2$, plus parity
- $m^2 \geq m_*^2$: $\langle\phi\rangle = 0$, ψ_i massless, "chirality" preserved
- $m^2 < m_*^2$: $\langle\phi\rangle \neq 0$, ψ_i massive, "chirality" broken

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Conclusions

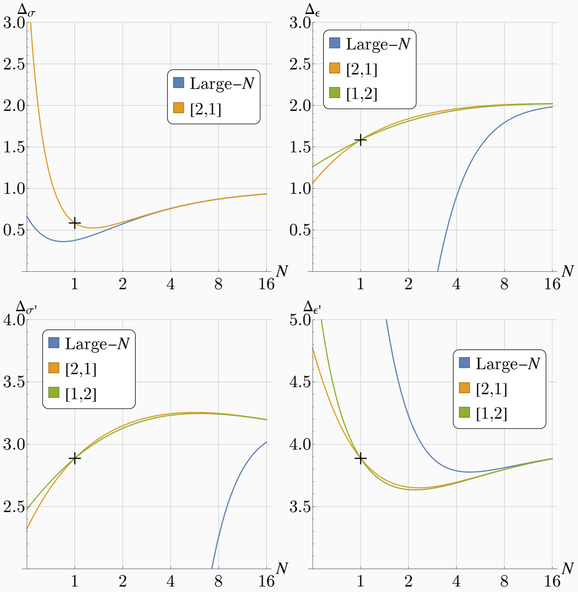
- New archipelago of numerical bootstrap results improves upon existing results
- Distinction between $O(N)$ vs $O(N/2)^2 \rtimes \mathbb{Z}_2$ GNY
- Agreement with emergent supersymmetry for $N = 1$
- We now have a highly sophisticated software stack

Future work

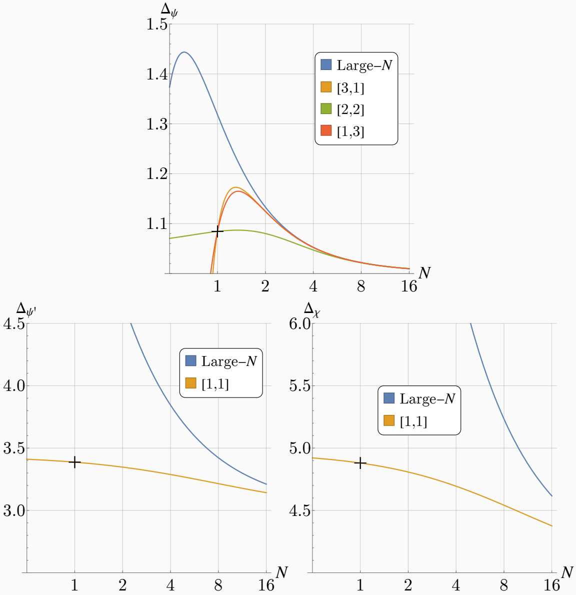
- Can we use the navigator [2104.09518] to strengthen our gap assumptions?
- Can we numerically resolve the difference between $O(N)$ and $O(N/2)^2 \rtimes \mathbb{Z}_2$?
- What else can we investigate?
- What tools can we develop next?

Thank you!

Two-sided Padé Approximants of Scalar Δs



Two-sided Padé Approximants of Fermion Δs



We define the three-point conformal structures,

	$\mathcal{O} \in (l, P, \mu)$	$\langle \mathcal{O}_a \mathcal{O}_b \mathcal{O}_c \rangle = \{\text{structure}_1, \text{structure}_2, \cdots\}$
$\sigma \times \sigma$ $\epsilon \times \epsilon$	$(l \in 2\mathbb{Z}, \text{even}, \bullet)$	$\langle \sigma\sigma\mathcal{O} \rangle, \langle \epsilon\epsilon\mathcal{O} \rangle = \{\vert 0, l\rangle\}$
$\sigma \times \epsilon$	$(l \in 2\mathbb{Z}, \text{odd}, \bullet)$	$\langle \sigma\epsilon\mathcal{O} \rangle = \{\vert 0, l\rangle\}$ $\langle \epsilon\sigma\mathcal{O} \rangle = \{(-1)^l \vert 0, l\rangle\}$
$\sigma \times \psi$	$(l \in \mathbb{Z} + \frac{1}{2}, \text{even}, \square)$	$\langle \sigma\psi^i\mathcal{O}^j \rangle = \{\delta^{ij}(-1)^{l+\frac{1}{2}} \vert \frac{1}{2}, l + \frac{1}{2}\rangle\}$ $\langle \psi^i\sigma\mathcal{O}^j \rangle = \{\delta^{ij} \vert \frac{1}{2}, l - \frac{1}{2}\rangle\}$
	$(l \in \mathbb{Z} + \frac{1}{2}, \text{odd}, \square)$	$\langle \sigma\psi^i\mathcal{O}^j \rangle = \{\delta^{ij}(-1)^{l-\frac{1}{2}} \vert \frac{1}{2}, l - \frac{1}{2}\rangle\}$ $\langle \psi^i\sigma\mathcal{O}^j \rangle = \{\delta^{ij} \vert \frac{1}{2}, j + \frac{1}{2}\rangle\}$
$\epsilon \times \psi$	$(l \in \mathbb{Z} + \frac{1}{2}, \text{even}, \square)$	$\langle \epsilon\psi^i\mathcal{O}^j \rangle = \{\delta^{ij}(-1)^{l-\frac{1}{2}} \vert \frac{1}{2}, l - \frac{1}{2}\rangle\}$ $\langle \psi^i\epsilon\mathcal{O}^j \rangle = \{\delta^{ij} \vert \frac{1}{2}, l + \frac{1}{2}\rangle\}$
	$(l \in 2\mathbb{Z} + \frac{1}{2}, \text{odd}, \square)$	$\langle \epsilon\psi^i\mathcal{O}^j \rangle = \{\delta^{ij}(-1)^{l+\frac{1}{2}} \vert \frac{1}{2}, l + \frac{1}{2}\rangle\}$ $\langle \psi^i\epsilon\mathcal{O}^j \rangle = \{\delta^{ij} \vert \frac{1}{2}, l - \frac{1}{2}\rangle\}$

$\psi \times \psi$	$(l \in 2\mathbb{Z}, \text{ even}, \mu \in \{\bullet, \square\square\})$	$\langle \psi^i\psi^j\mathcal{O}^a \rangle = \{T^{ija}_\mu \vert 0, l\rangle, T^{ija}_\mu \vert 1, l\rangle\}$
	$(l \in 2\mathbb{Z} + 1, \text{ even}, \mu = \boxminus)$	$\langle \psi^i\psi^j\mathcal{O}^a \rangle = \{T^{ija}_\mu \vert 0, l\rangle, T^{ija}_\mu \vert 1, l\rangle\}$
	$(l \in 2\mathbb{Z}, \text{ odd}, \mu \in \{\bullet, \square\square\}),$	$\langle \psi^i\psi^j\mathcal{O}^a \rangle = \{T^{ija}_\mu (\sqrt{l+1} \vert 1, l+1\rangle - \sqrt{l} \vert 1, l-1\rangle)\}$
	$(l \in 2\mathbb{Z} + 1, \text{ odd}, \mu \in \{\bullet, \square\square\})$	$\langle \psi^i\psi^j\mathcal{O}^a \rangle = \{T^{ija}_\mu (\sqrt{l} \vert 1, l+1\rangle + \sqrt{l+1} \vert 1, l-1\rangle)\}$
	$(l \in (2\mathbb{Z})_{\geq 2}, \text{ odd}, \mu = \boxminus)$	$\langle \psi^i\psi^j\mathcal{O}^a \rangle = \{T^{ija}_\mu (\sqrt{l} \vert 1, l+1\rangle + \sqrt{l+1} \vert 1, l-1\rangle)\}$
	$(l \in 2\mathbb{Z} + 1, \text{ odd}, \mu = \boxminus)$	$\langle \psi^i\psi^j\mathcal{O}^a \rangle = \{T^{ija}_\mu (\sqrt{l+1} \vert 1, l+1\rangle - \sqrt{l} \vert 1, l-1\rangle)\}$

Four-point Structures and Crossing Equations

$$\langle \psi^i \psi^j \psi^k \psi^l \rangle = \sum_{I,a} t_I \, Q_a^{ijkl} g_{\psi\psi\psi\psi}^{I,a}(u,v),$$

Flavor structures,

$$Q^+ = \delta^{ij}\delta^{kl} + \delta^{il}\delta^{jk}, \quad Q^3 = \delta^{ik}\delta^{jl}, \quad Q^- = \delta^{ij}\delta^{kl} - \delta^{il}\delta^{jk},$$

Conformal structures with nice symmetry properties,

$$[q_1 q_2 q_3 q_4].$$

Based on how the structures transform under (13) permutation, we obtain the factor M_J^I in the crossing equations.

$$g_{ijkl}^I(u,v) \pm \sum_J M_J^I g_{kjil}^J(v,u) = 0.$$

Four-point Structures and Crossing Equations

The fully mixed system of $\{\psi, \epsilon, \sigma\}$ produces 38 crossing equations.

We will search for allowed regions in the 6d space of parameters

$$\{\Delta_\psi, \Delta_\epsilon, \Delta_\sigma, \frac{\lambda_{\psi\psi\sigma}}{\lambda_{\epsilon\epsilon\epsilon}}, \frac{\lambda_{\psi\psi\epsilon}}{\lambda_{\epsilon\epsilon\epsilon}}, \frac{\lambda_{\sigma\sigma\epsilon}}{\lambda_{\epsilon\epsilon\epsilon}}\}$$

Gap Assumptions

Operator	Parity	$O(N)$ rep.	Δ at large N.	Δ in ϵ-exp.
ψ_i	$+$	V	$1+\frac{4}{3\pi^2 N}+\frac{896}{27\pi^4 N^2}+\dots$	$\frac{3}{2}-\frac{N+5}{2(N+6)}\epsilon+\dots$
$\sigma=\phi$	$-$	S	$1-\frac{32}{3\pi^2 N}+\frac{32(304-27\pi^2)}{27\pi^4 N^2}+\dots$	$1-\frac{3}{N+6}\epsilon+\dots$
$\epsilon=\phi^2$	$+$	S	$2+\frac{32}{3\pi^2 N}-\frac{64(632+27\pi^2)}{27\pi^4 N^2}+\dots$	$2+\frac{\sqrt{N^2+132N+36}-N-30}{6(N+6)}\epsilon+\dots$
$\psi_i'=\phi^2\psi_i$	$+$	V	$3+\frac{100}{3\pi^2 N}+\dots$	-
$\chi_i=\phi^3\psi_i$	$-$	V	$4+\frac{292}{3\pi^2 N}+\dots$	-
$\sigma'=\phi^3$	$-$	S	$3+\frac{64}{\pi^2 N}-\frac{128(770-9\pi^2)}{9\pi^4 N^2}+\dots$	$3+\frac{\sqrt{N^2+132N+36}-N-30}{6(N+6)}\epsilon+\dots$
$\epsilon'=\phi^4$	$+$	S	$4+\frac{448}{3\pi^2 N}-\frac{256(3520-81\pi^2)}{27\pi^4 N^2}+\dots$	-
$\sigma^T=\psi_{(i}\psi_{j)}$	$-$	T	$2+\frac{32}{3\pi^2 N}+\frac{4096}{27\pi^4 N^2}+\dots$	-

$$\left(\tfrac{1}{2},\text{odd},\square\right):\quad \partial\!\!\!/\psi_i = ig\phi\psi_i \quad\Rightarrow\quad \phi^3\psi_i$$

$$\Delta_{\psi'}>2,\Delta_{\chi}>3.5,\Delta_{\epsilon'}>3,\Delta_{\sigma'}>3,\Delta_{\sigma^T}>2$$