Bootstrapping line defects with $O(2)$ symmetry

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arXiv:2208.11715 with E. Lauria, A. Gimenez-Grau, and P. van Vliet

Motivation

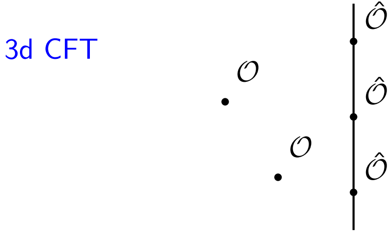
- Defects are important in both low- and high-energy physics, with or without supersymmetry.
- Impurities in condensed matter systems.
- Wilson loop in gauge theories tells us about confinement.
- Defects give access to new observables.
- 1d CFTs are nonlocal $\rightarrow$ natural interpretation as conformal line defects.

Line defects

- Conformal defects preserve a conformal subgroup of the original bulk conformal symmetry:

$$SO(4,1) \to SO(2,1) \times SO(2)\,.$$

- Bulk and defect operators:



- Only consider operators on the line $\to$ can use "ordinary" bootstrap techniques including numerical bootstrap.

Displacement

- Generically no conserved stress-energy tensor on the defect.
- Instead displacement operator D [Billò et al. (2016)]:

$$\partial_\mu T^{\mu i} = -\delta^{(q)}(\mathcal{D})\mathrm{D}^i \ .$$

- D has a protected conformal dimension

$$\Delta_{\mathrm{D}} = 2\,,$$

- and has transverse spin $s_{\mathrm{D}} = 1$.

Tilt

- The bulk CFT can have an additional global symmetry group $O(N)$ with current J^μ.
- The defect can break $O(N)$ symmetry to $O(N-1)$: no conserved current on the defect.
- Instead, the tilt t will appear [Bray et al. (1977)][Padayasi et al. (2021)]:

$$\partial_\mu J^\mu_A = \delta^{(q)}(\mathcal{D}) t_A \ , \quad A \in O(N-1) \ .$$

- t_A has a protected conformal dimension

$$\Delta_{t_A} = 1 \, ,$$

- and is a vector under $O(N-1)$.

Crossing equations for 1d CFTs

- Operators ordered on the line: $\tau_1 < \tau_2 < \tau_3 < \tau_4$.

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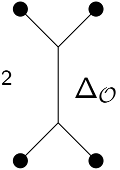
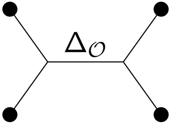
- Crossing symmetry follows from cyclicity.

$$\sum_{\mathcal{O}} (\lambda_{\mathcal{O}})^2$$



$$= \sum_{\mathcal{O}} (\lambda_{\mathcal{O}})^2$$



$$\sum_{\mathcal{O}} \lambda_{ij\mathcal{O}} \lambda_{kl\mathcal{O}} (1-\xi)^{\Delta_k+\Delta_j} g_{\Delta}^{\Delta_{ij},\Delta_{kl}}(\xi) = \sum_{\mathcal{O}} \lambda_{kj\mathcal{O}} \lambda_{il\mathcal{O}} \xi^{\Delta_i+\Delta_j} g_{\Delta}^{\Delta_{kj},\Delta_{il}}(1-\xi)$$

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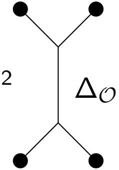
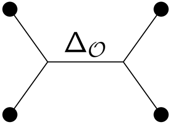
$\sum_{\mathcal{O}} (\lambda_{\mathcal{O}})^2$  $= \sum_{\mathcal{O}} (\lambda_{\mathcal{O}})^2$ 

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- No parallel spin ℓ. There is parity (and transverse spin s)

$$\mathcal{S}: \qquad \tau \to -\tau \ , \quad \mathcal{S}(\psi(\tau)) = (-1)^{S_\psi} \psi(-\tau) \ , \quad S_\psi = 0, 1 \ .$$

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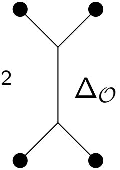
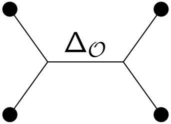
$\sum_{\mathcal{O}} (\lambda_{\mathcal{O}})^2$ 

$= \sum_{\mathcal{O}} (\lambda_{\mathcal{O}})^2$ 

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- Use numerical bootstrap to find exclusion bounds for Δ, λ.

Single-correlator tilt

- Start with agnostic bootstrap: no specific gap assumptions.
- Bootstrap displacement or tilt:

$$\langle t(\tau_1) t(\tau_2) \bar{t}(\tau_3) \bar{t}(\tau_4) \rangle \,.$$

- Two channels:

$$t \times \bar{t} \sim \mathbf{1} + (t\bar{t})^{\pm} + \cdots \,, \quad t \times t \sim t^2 + \cdots \,.$$

- Find maximal scaling dimension of first operator in $t \times \bar{t}$ as a function of Δ_{t^2}.

Single-correlator tilt

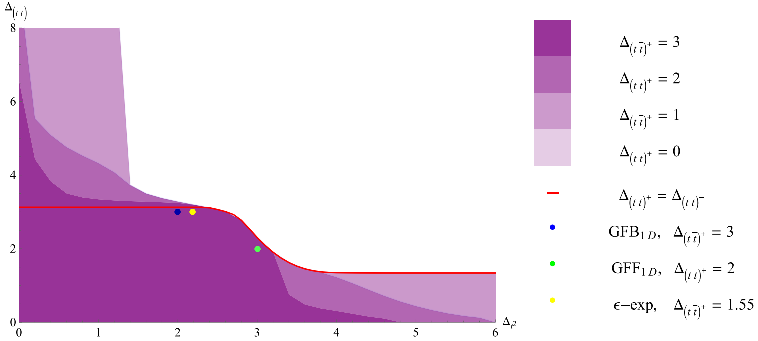


Figure: Bounds on the dimension of the $\mathcal{S}$-parity odd scalar $(t\bar{t})^-$ vs. the $\mathcal{S}$-parity even scalar $(t\bar{t})^+$ gap vs. the gap on t^2. $\Lambda = 33$.

Tilt and displacement

- Extend to multi-correlator bootstrap: t and D simultaneously.

$$\langle t(\tau_1)\bar{t}(\tau_2)t(\tau_3)\bar{t}(\tau_4)\rangle\,, \quad \langle \mathsf{D}(\tau_1)\bar{\mathsf{D}}(\tau_2)\mathsf{D}(\tau_3)\bar{\mathsf{D}}(\tau_4)\rangle$$
$$\langle t(\tau_1)\bar{t}(\tau_2)\mathsf{D}(\tau_3)\bar{\mathsf{D}}(\tau_4)\rangle\,.$$

- Access to additional channel $(t\mathsf{D})^{\pm}$.
- We still perform the agnostic bootstrap: no specific gap assumptions.

Tilt and displacement

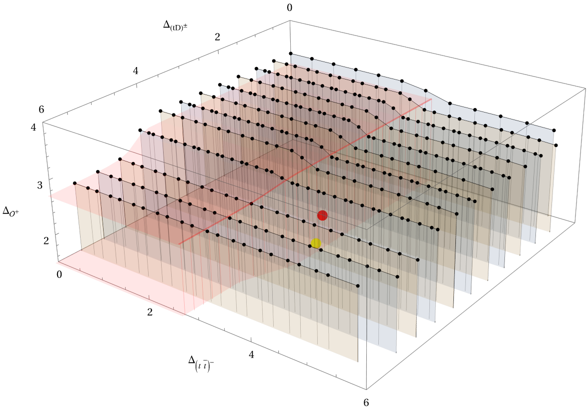
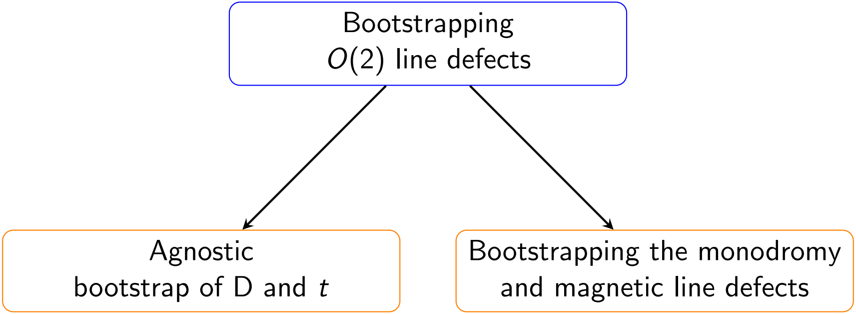


Figure: Bounds on the dimension of the first $\mathcal{S}$-parity even singlet O^+ as a function of the scaling dimensions $\Delta_{(tD)^{\pm}}$ and $\Delta_{(t\bar{t})^-}$. $\Lambda = 33$.

Changing gears



Monodromy line defect

- Start from $N = 2$ real scalars combined in a complex scalar Φ that satisfies

$$\Phi(r, \theta + 2\pi, \vec{x}) = e^{2\pi i v}\Phi(r, \theta, \vec{x}) \,, \quad v \sim v + 1 \,, \quad v \in [0, 1) \,.$$

- The defect modes Ψ of Φ will have fractional transverse spin $s \in \mathbb{Z} + v$ and dimensions [Söderberg 2017][Giombi et al. 2021]

$$\Delta_{\Psi_s} = 1 + |s| - \frac{\varepsilon}{2} + \frac{1}{5}\frac{v(v-1)}{|s|}\varepsilon + O(\varepsilon^2) \,.$$

- Generalization of Z_2 Ising twist defect [Gaiotto et al. 2013] [Billó et al. 2013]

Monodromy bootstrap

- D appears in the OPE of defect modes:

$$\Psi_v \times \bar{\Psi}_{v-1} \sim \mathrm{D} + \cdots .$$

- We bootstrap the defect modes of the fundamental scalar

$$\langle \Psi_v(\tau_1)\Psi_v(\tau_2)\bar{\Psi}_v(\tau_3)\bar{\Psi}_v(\tau_4)\rangle , \quad \langle \Psi_{v-1}(\tau_1)\Psi_{v-1}(\tau_2)\bar{\Psi}_{v-1}(\tau_3)\bar{\Psi}_{v-1}(\tau_4)\rangle ,$$
$$\langle \Psi_v(\tau_1)\Psi_{v-1}(\tau_2)\bar{\Psi}_{v-1}(\tau_3)\bar{\Psi}_v(\tau_4)\rangle .$$

- Only information we give is appearance of D and the external dimensions of $\Delta_{\Psi_v}, \Delta_{\Psi_{v-1}}$.

Monodromy bootstrap

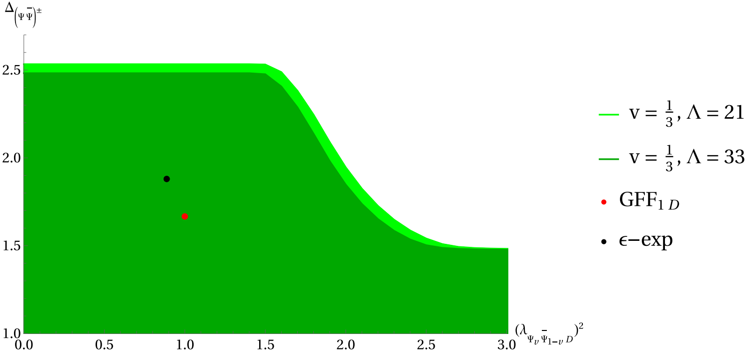


Figure: Bounds on the dimension of the first singlet in $\Psi_s \times \bar{\Psi}_s$ $\Delta_{(\Psi\bar{\Psi})^{\pm}}$ versus the OPE coefficient $(\lambda_{\Psi_v \bar{\Psi}_{1-v} D})^2$.

Magnetic line defect

- The action of the localized magnetic field line defect is given by:

$$S = \int d^d x \left(\frac{1}{2}(\partial_\mu \phi_a)^2 + \frac{\lambda_0}{4!}(\phi_a^2)^2 \right) + h_0 \int_{-\infty}^{\infty} d\tau \, \phi_1(x(\tau)) \, ,$$

- Defect breaks bulk $O(3)_F$ symmetry to defect $O(2)_F$ symmetry.
- Breaking introduces an $O(2)$ vector $t_{\hat{a}}$ and a scalar ϕ_1 with dimension [Cuomo et al. 2022]

$$\Delta_{\phi_1} = 1 + \varepsilon - \frac{184}{121}\varepsilon^2 + O(\varepsilon^3) \xrightarrow{\text{Padé}} 1.55 \, .$$

- Displacement is given by a transverse derivative:

$$\mathrm{D} \propto \nabla \phi_1 \, .$$

Magnetic line bootstrap

- We bootstrap the tilt and the fundamental scalar:

$$\langle t(\tau_1)\bar{t}(\tau_2)t(\tau_3)\bar{t}(\tau_4)\rangle\,, \quad \langle \phi_1(\tau_1)\phi_1(\tau_2)\phi_1(\tau_3)\phi_1(\tau_4)\rangle\,,$$
$$\langle t(\tau_1)\bar{t}(\tau_2)\phi_1(\tau_3)\phi_1(\tau_4)\rangle\,.$$

- Special feature: externals appear in OPE

$$\phi_1 \times \phi_1 \sim \mathbf{1} + \phi_1 + s_- + \cdots\,, \quad (t \times \bar{t})^+ \sim \mathbf{1} + \phi_1 + s_- + \cdots\,,$$
$$(t \times \bar{t})^- \sim A + \cdots\,, \quad t \times t \sim T + \dots\,, \quad t \times \phi_1 \sim t + V + \cdots\,,.$$

- Reminiscent of Ising model island!

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- Reminiscent of Ising model island!
- Gap assumptions from ε-expansion results at $O(\varepsilon)$:

$$\Delta_{s_-} = 2.36\,, \qquad \Delta_A = 3\,, \qquad \Delta_T = 2.18\,, \qquad \Delta_V = 3.18\,,$$

Magnetic line bootstrap

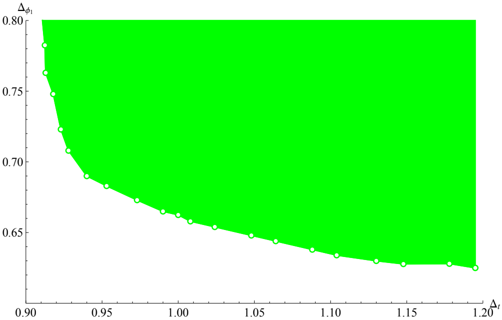


Figure: Bounds on the scaling dimensions Δ_{ϕ_1} and Δ_t for $\Lambda = 21$.

- Close to rediscovering the tilt!

Magnetic line bootstrap

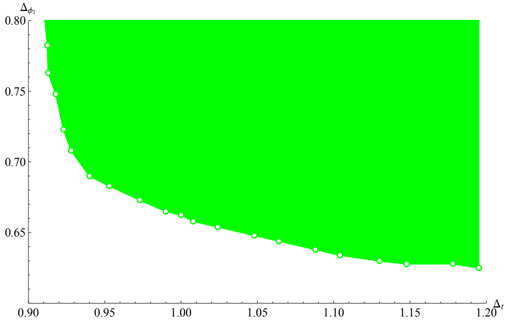


Figure: Bounds on the scaling dimensions Δ_{ϕ_1} and Δ_t for $\Lambda = 21$.

- Close to rediscovering the tilt!
- Set $\Delta_t = 1$ and bootstrap

$$(\lambda_{\phi_1\phi_1\phi_1})^2 + (\lambda_{tt\phi_1})^2 \,, \quad \tan\theta = \frac{\lambda_{\phi_1\phi_1\phi_1}}{\lambda_{tt\phi_1}} \,.$$

Magnetic line bootstrap

- We found a series of cusps. Improves with higher number of derivatives?

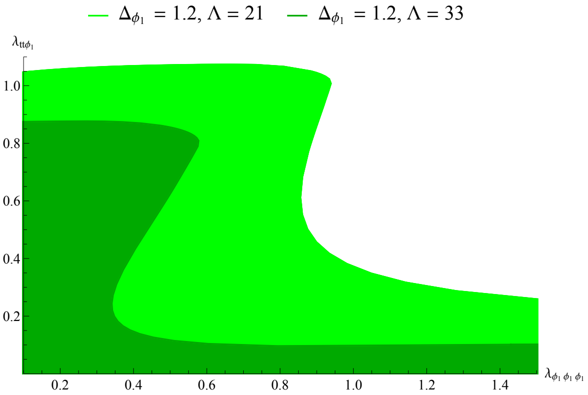
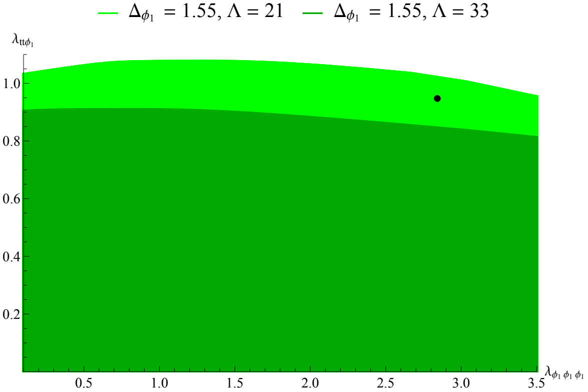


Figure: Upper bounds on the OPE coefficient $\lambda_{\phi_1\phi_1\phi_1}$ as a function of $\lambda_{tt\phi_1}$.

Magnetic line bootstrap

- Predicted value of $\Delta_{\phi_1} = 1.55$, falls outside of numerical bounds.



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Conclusions

- Focus on canonical operators: D and t.
- Agnostic searches already show interesting behaviour.
- Without rigorous gap assumptions hard to isolate solutions.
- Gap assumptions inspired by ε-expansion show very interesting behavior, but too strong.

Future directions

- Monodromy defect breaks global symmetry.
- Generalize to supersymmetric setup:

$$OSp(2|4) \to SU(1,1|1) \times U(1)\,.$$

- Appearance of tilt, displacement $\to$ work in progress.

Thank you!

Backup

The numerical bootstrap

- Functionals to rule out possible solutions

$$\lambda^2_{\mathcal{O}'}\alpha\Big(F_{\Delta_{\mathcal{O}'}}\Big) = -\alpha\Big(F_0\Big) - \sum_{\mathcal{O}} \lambda^2_{\mathcal{O}}\alpha\Big(F_{\Delta_{\mathcal{O}}}\Big)\,.$$

- Upper bound if you can find α s.t.

$$\alpha\Big(F_{\Delta_{\mathcal{O}'}}\Big) = 1\,, \quad \alpha\Big(F_{\Delta_{\mathcal{O}}}\Big) \geq 0\,.$$

- Allowed and disallowed solutions for conformal dimensions.
- Upper and lower bounds for OPE coefficients.

Single-correlator tilt

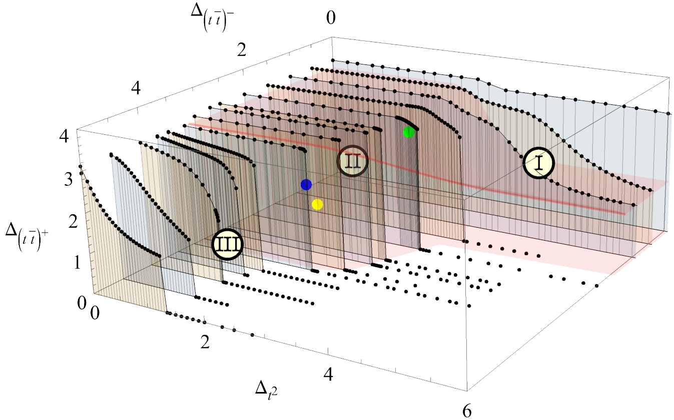

Figure: Bounds on the maximal gap on the dimension of the $\mathcal{S}$-parity even scalar $(t\bar{t})^+$ vs. the $\mathcal{S}$-parity odd scalar gap $(t\bar{t})^-$ vs. the gap on the leading charged operator t^2. $\Lambda = 33, P = 53$.

Single-correlator tilt

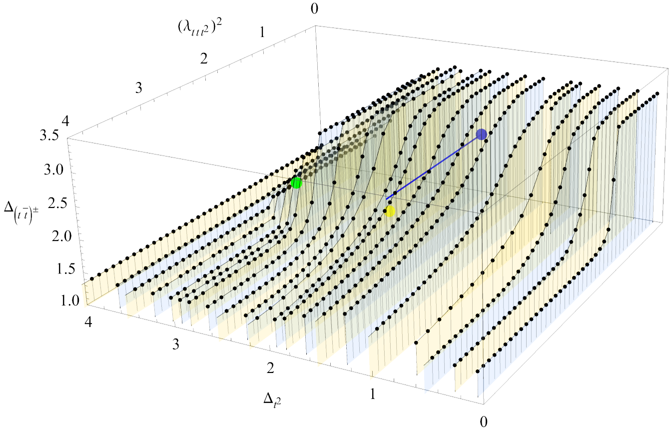

Figure: Bounds on the dimension of the first singlet in the $t \times \bar{t}$ OPE as a function of the gap on the dimension Δ_{t^2} and the OPE coefficient $(\lambda_{ttt^2})^2$ of the first operator charged under $O(2)_F$ in the $t \times t$ OPE.
$\Lambda = 49, P = 69$.

Tilt and displacement

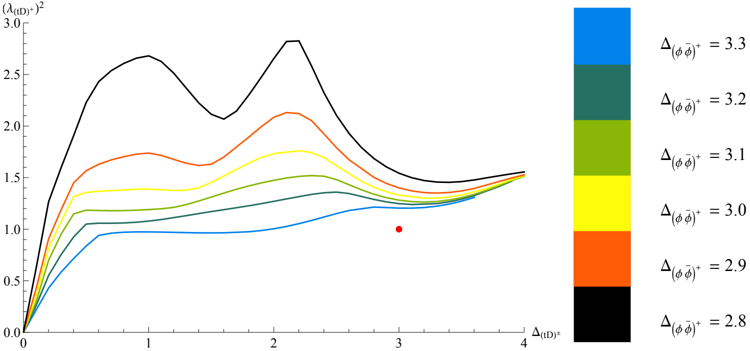

Figure: Bounds on $(\lambda_{tD(tD)^+})^2$ as a function of the scaling dimension of $\Delta_{(tD)^+}$ and of the scaling dimension of the first parity-even singlet $\Delta_{(\phi\bar{\phi})^+}$.

Monodromy line defect

- The scaling dimensions of Ψ_s are given by [Giombi et al. 2021]:

$$\Delta_{\Psi_s} = 1 + |s| - \frac{\varepsilon}{2} + \frac{1}{5}\frac{v(v-1)}{|s|}\varepsilon + O(\varepsilon^2)\,.$$

- The displacement D appears in the OPE $\Psi_v \times \bar{\Psi}_{v-1}$.
- Other results we need to compare to the numerics [Giombi et al. 2021]:

$$|\lambda_{\Psi_v \bar{\Psi}_{v-1} \mathrm{D}}|^2 = 1 + \frac{1}{10}\left(2H_{1-v} + 2H_v - 3\right)\,,$$

$$\Psi_v \times \bar{\Psi}_v \sim \mathbf{1} + \mathcal{O}_0 + \ldots, \quad \Psi_{v-1} \times \bar{\Psi}_{v-1} \sim \mathbf{1} + \mathcal{O}_0 + \ldots.$$

$$\Delta_{\mathcal{O}_0} = \frac{4}{5(1+2v)} + \frac{2(v-1)}{5}\,.$$

Monodromy bootstrap

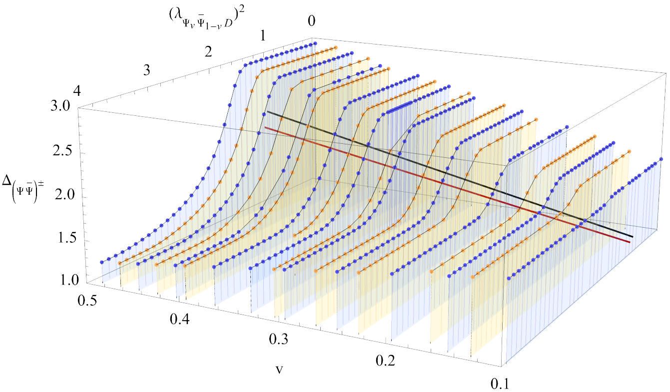

Figure: Bounds on the dimension of the first singlet in the $\Psi_s \times \bar{\Psi}_s$ OPE $\Delta_{(\Psi\bar{\Psi})^{\pm}}$ versus the OPE coefficient of the displacement operator and the monodromy v. $\Lambda = 21, P = 41$.

Magnetic line defect

- We can compute the scaling dimensions of leading operators [Cuomo et al. 2022]:

$$\Delta_{\phi_1} = 1 + \varepsilon - \frac{3N^2 + 49N + 194}{2(N+8)^2}\varepsilon^2 + O(\varepsilon^3) \xrightarrow{\text{Padé}} 1.55\,,$$

$$\Delta_{s_\pm} = 2 + \varepsilon\,\frac{3N + 20 \pm \sqrt{N^2 + 40N + 320}}{2(N+8)} + O(\varepsilon^2)\,,$$

$$\Delta_A = 3 + O(\varepsilon^2)\,, \qquad \Delta_T = 2 + \frac{2\varepsilon}{N+8} + O(\varepsilon^2)\,,$$

$$\Delta_V = 2 + \varepsilon\,\frac{N+10}{N+8} + O(\varepsilon^2)\,.$$

- To compare with numerics, we also need the OPE coefficients

$$\lambda_{\phi_1\phi_1\phi_1} = \frac{3\pi\varepsilon}{\sqrt{N+8}} + O(\varepsilon^2)\,, \qquad \lambda_{tt\phi_1} = \frac{\pi\varepsilon}{\sqrt{N+8}} + O(\varepsilon^2)\,.$$

Magnetic line bootstrap

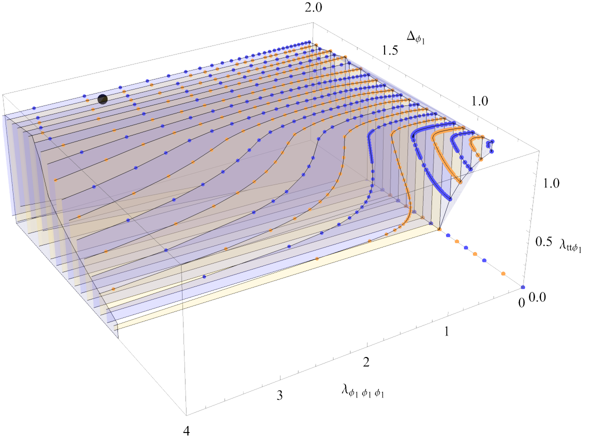

Figure: Bounds on the OPE coefficients $\lambda_{\phi_1 t\bar{t}}$ and $\lambda_{\phi_1\phi_1\phi_1}$ as a function of the gap Δ_{ϕ_1} for the $O(3)$-breaking magnetic line defect. $\Lambda = 21, P = 41$.