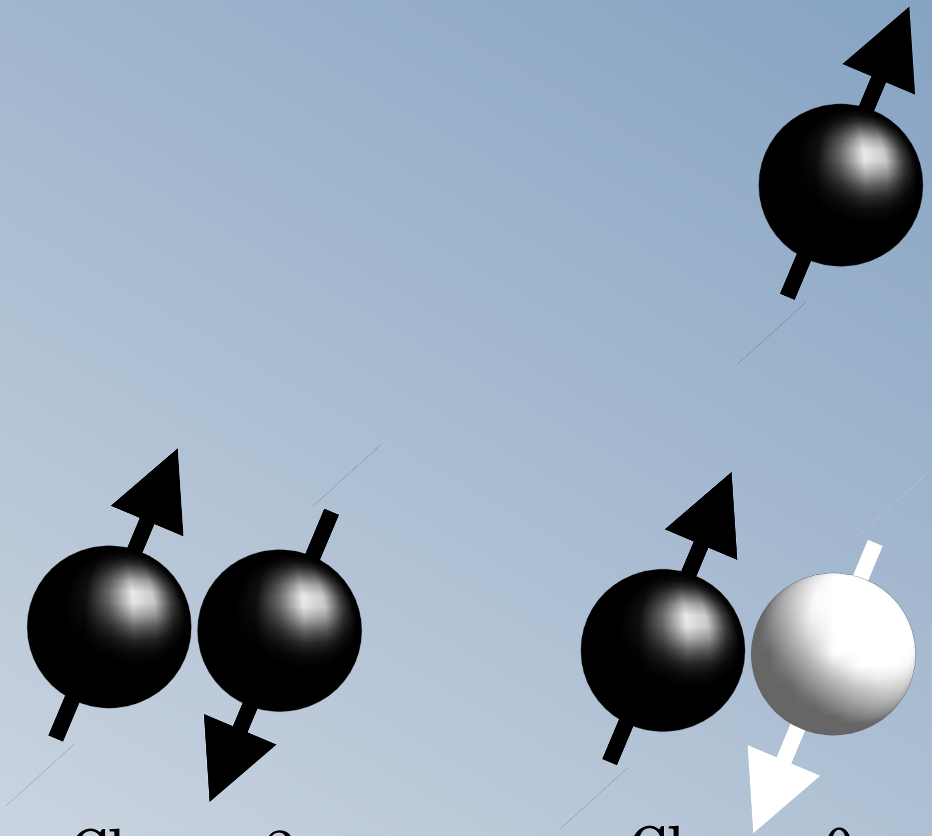
Spin-charge separation

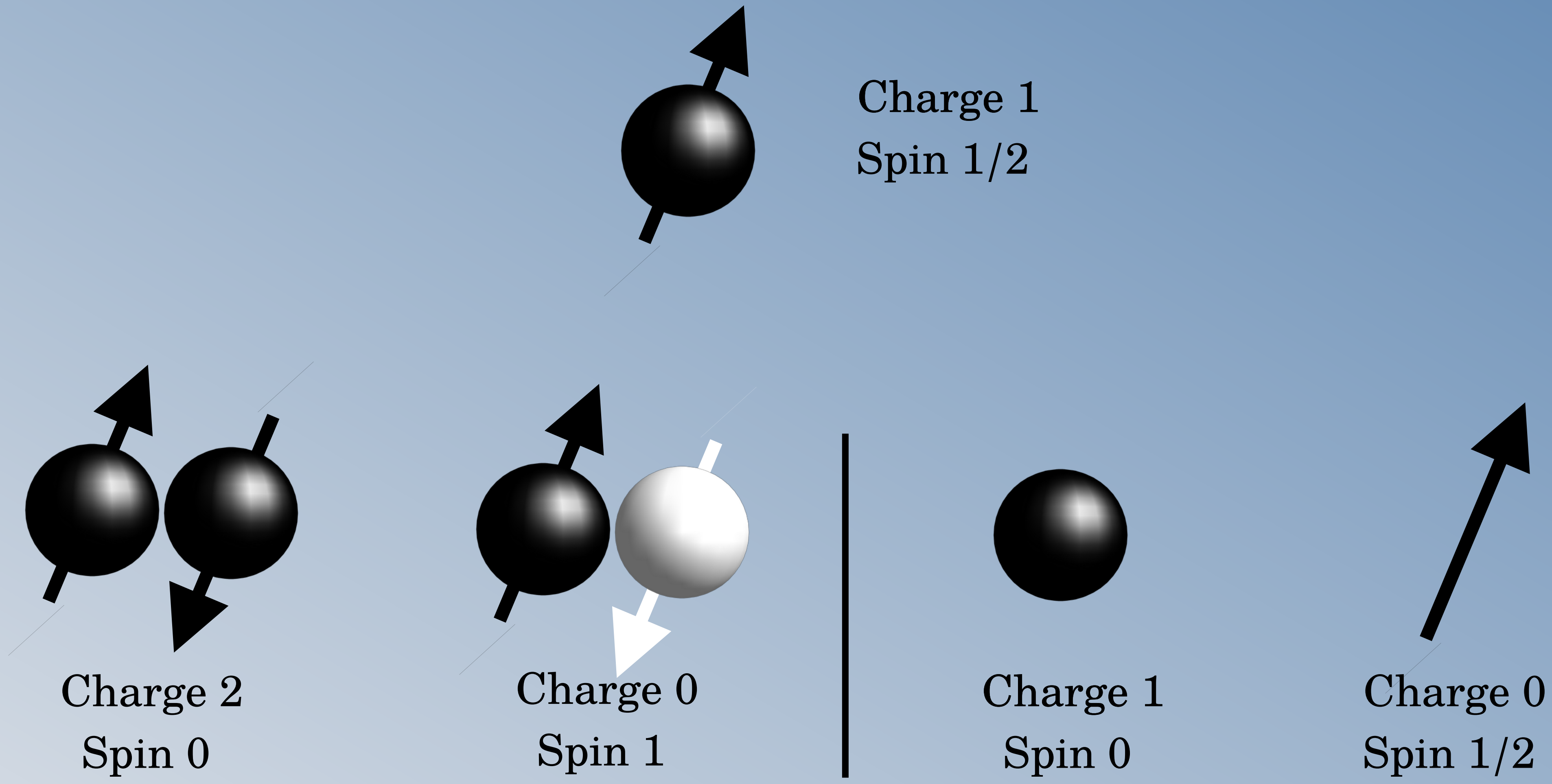


Charge 1
Spin 1/2

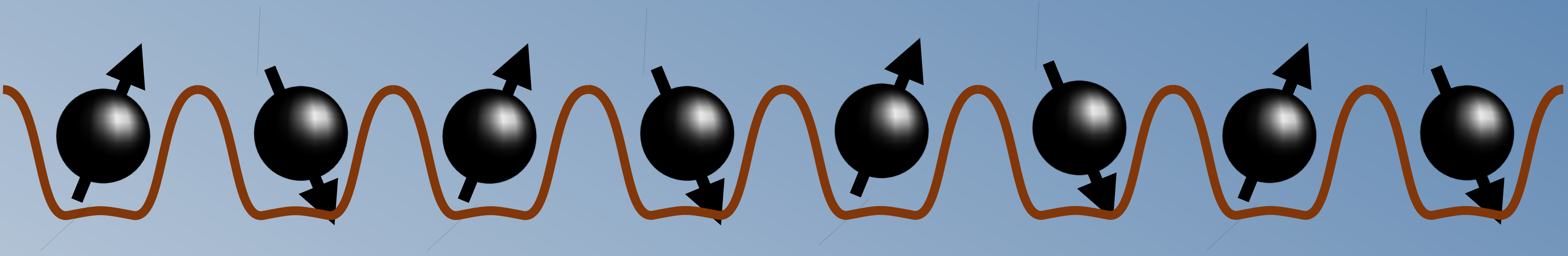
Charge 2
Spin 0

Charge 0
Spin 1

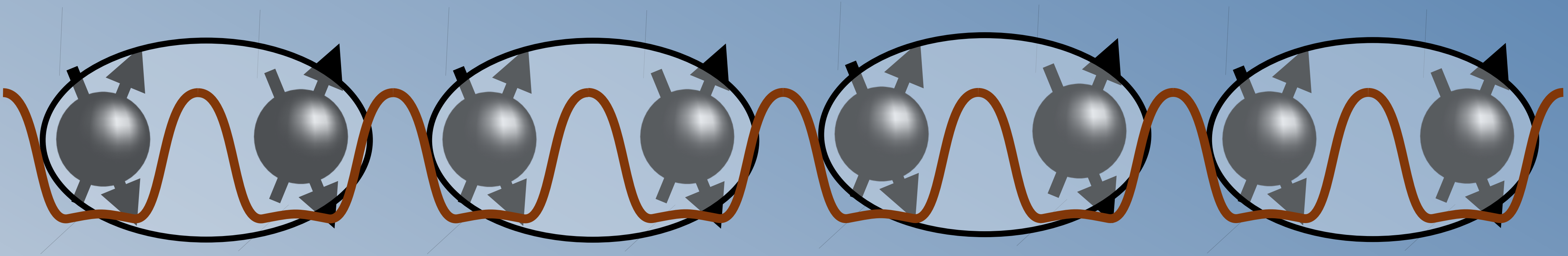
Spin-charge separation



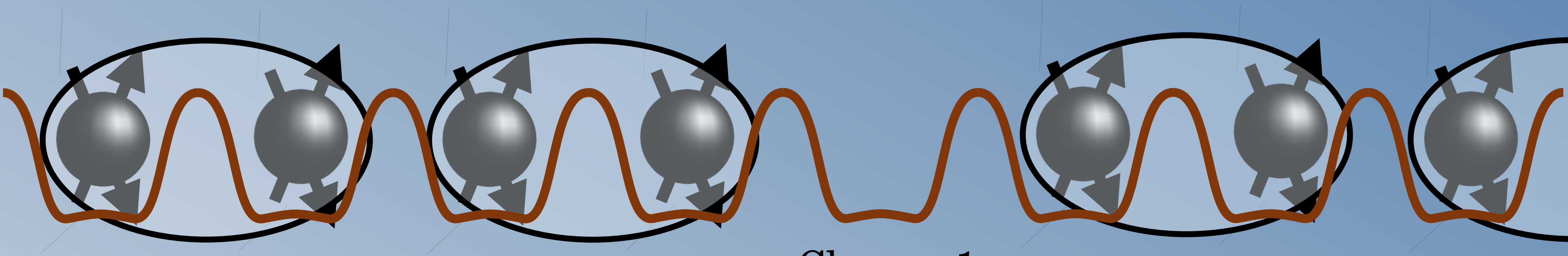
Spin-charge separation



Spin-charge separation

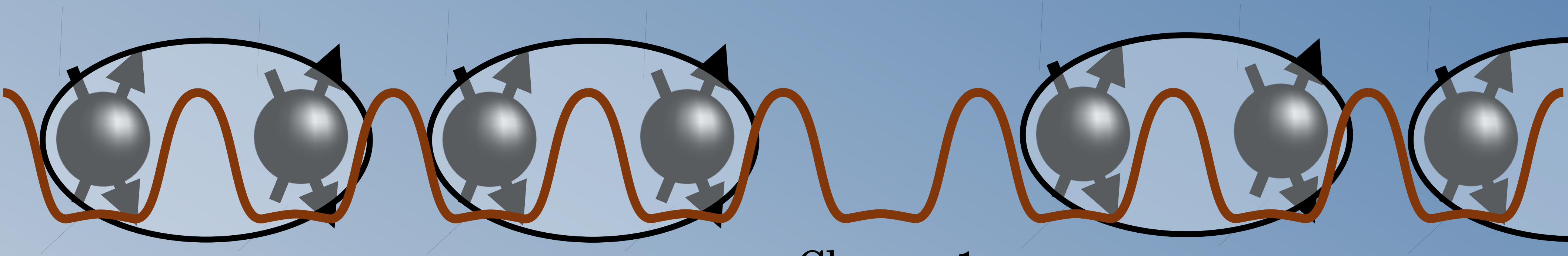


Spin-charge separation

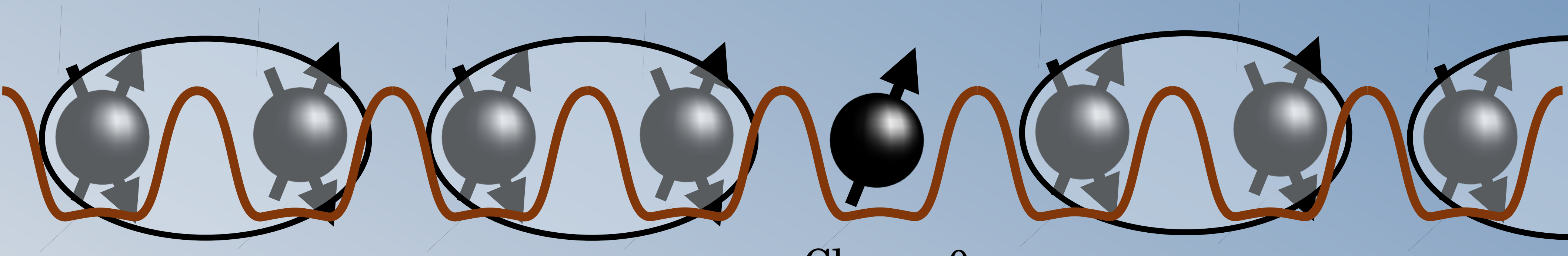


Charge -1
Spin 0

Spin-charge separation



Charge -1
Spin 0



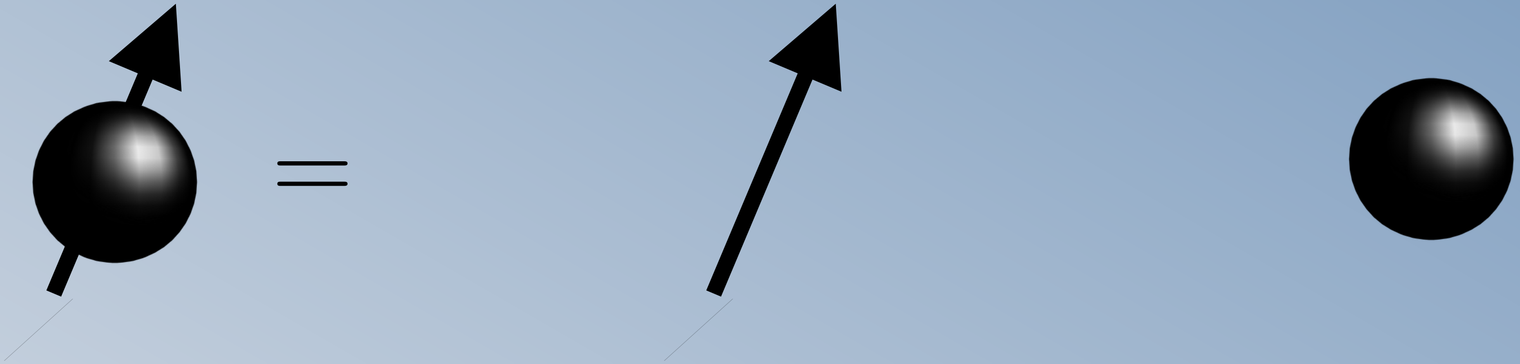
Charge 0
Spin 1/2

Spin-charge separation

$$\Psi = \Psi^s\left(\{\vec{r}_{i,\uparrow}\};\{\vec{r}_{i,\downarrow}\}\right)\Phi^c\left(\{\vec{r}_{i,\uparrow},\vec{r}_{i,\downarrow}\}\right)$$

fermionic or bosonic

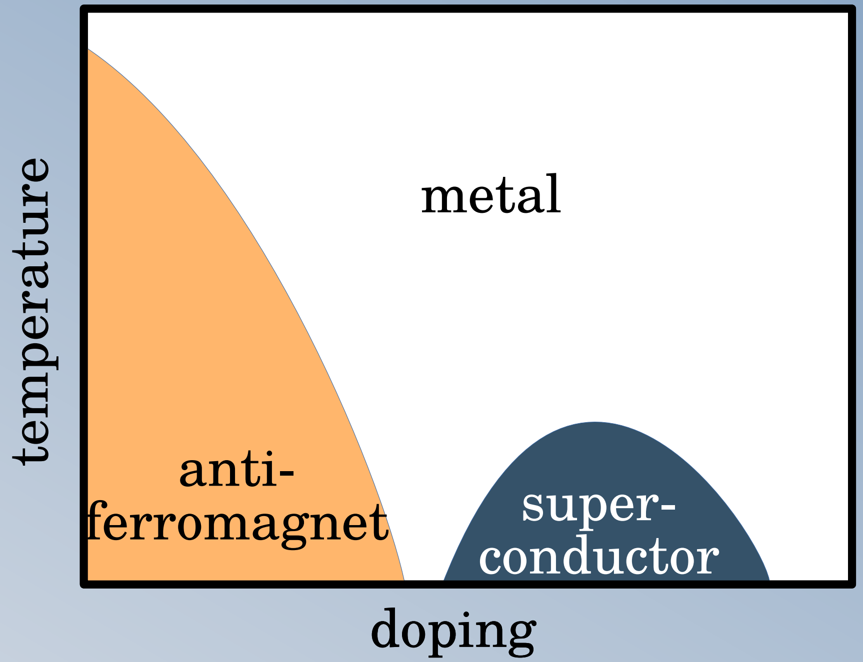
fully symmetric



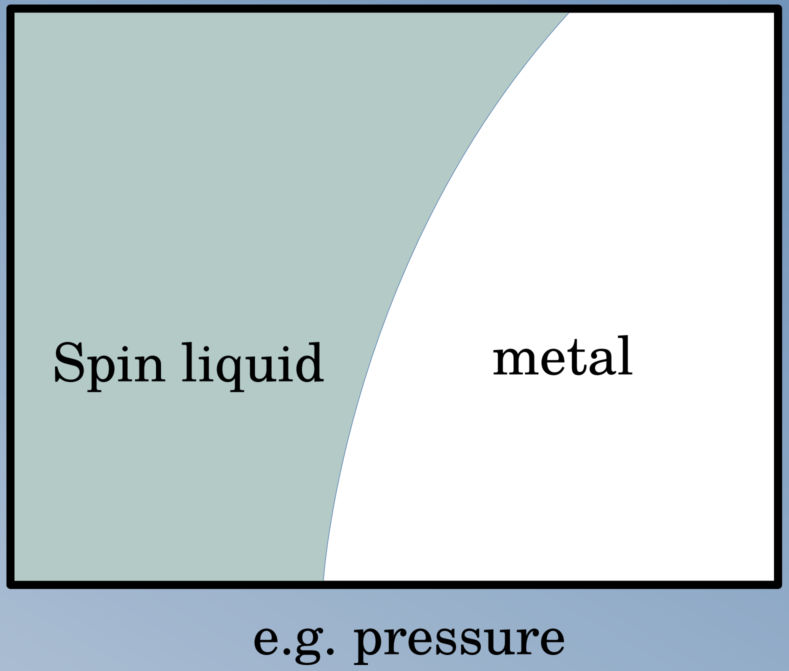
Partons: Ansätze for Ψ^s, Φ^c

Spin-charge separation

High-Tc



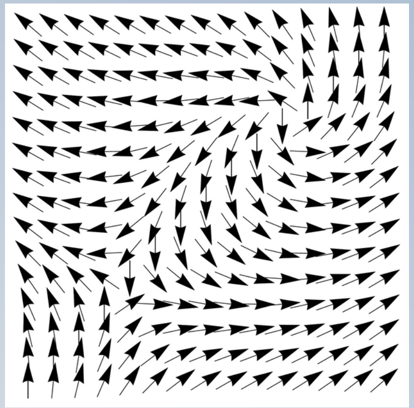
Frustrated Magnets



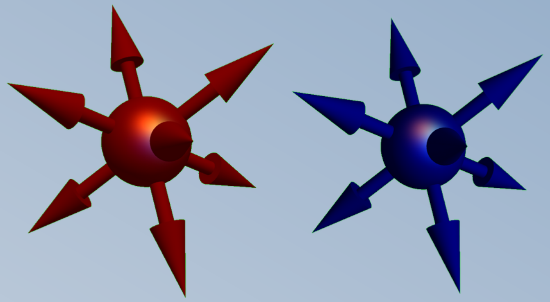
Dualities

$$\text{duality} \star \text{duality} = 1$$

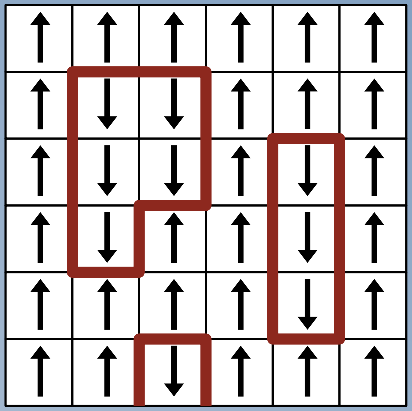
Vortices in XY model



Electric-magnetic



Kramers-Wannier



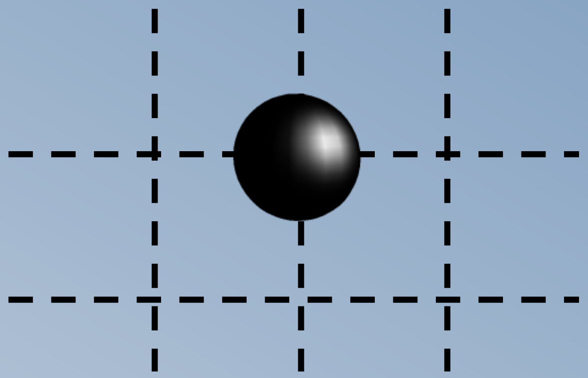
Dualities

$$\text{duality} \star X \star \text{duality} = \text{new duality}$$

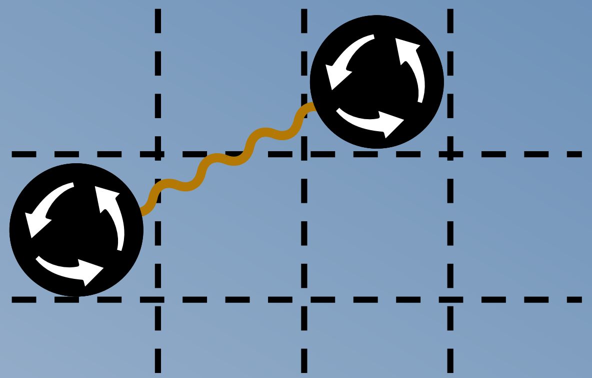
Example: Duality webs in (1+1) and (2+1) dimensions

Seiberg, Senthil, Wang, Witten (2016)
Karch, Tong (2016)
Karch, Tong, Turner (2016)

Particle-vortex duality



Boson with short-range interactions

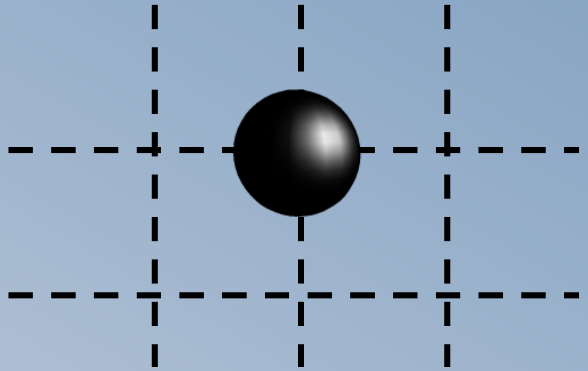


Vortices coupled to emergent photon

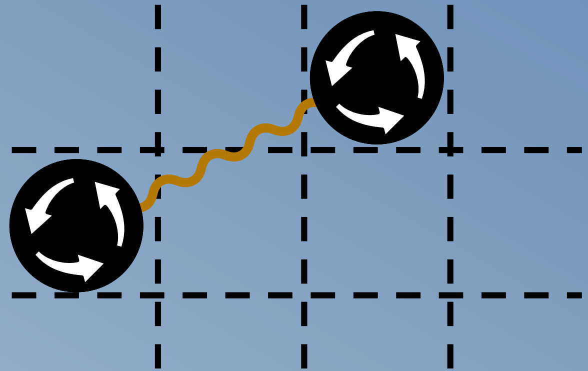
$$\mathcal{L}_{\text{Boson}} = |(\nabla - i\mathbf{A})\Psi|^2 + \ldots$$

$$\mathcal{L}_{\text{Vortex}} = |(\nabla - i\mathbf{a})\tilde{\Psi}|^2 + i\,\frac{\mathbf{A}\cdot(\nabla\times\mathbf{a})}{2\pi} + \kappa(\nabla\times\mathbf{a})^2 + \ldots$$

Particle-vortex duality



Boson with short-range interactions



Vortices coupled to emergent photon

$$\mathcal{L}_{\text{Boson}} \sim \mathbf{J}_{\text{Boson}} \cdot \mathbf{A}$$

$$\mathcal{L}_{\text{Vortex}} \sim \mathbf{J}_{\text{Vortex}} \cdot \mathbf{a} + i\frac{\mathbf{A}\cdot(\nabla \times \mathbf{a})}{2\pi}$$

Particle-vortex duality

$$\mathcal{L}_{\text{Vortex}\star\text{Vortex}} \sim \mathbf{J}_{\text{Vortex}\star\text{Vortex}} \cdot \tilde{\mathbf{a}} + i\frac{\tilde{\mathbf{a}}\cdot(\nabla\times\mathbf{a})}{2\pi} + i\frac{\mathbf{A}\cdot(\nabla\times\mathbf{a})}{2\pi}$$

$$\sim -\mathbf{J}_{\text{Vortex}\star\text{Vortex}} \cdot \mathbf{A}$$

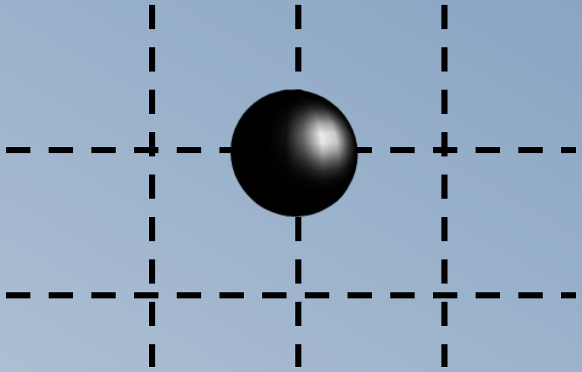
Boson with short-range interactions

Vortices coupled to emergent photon

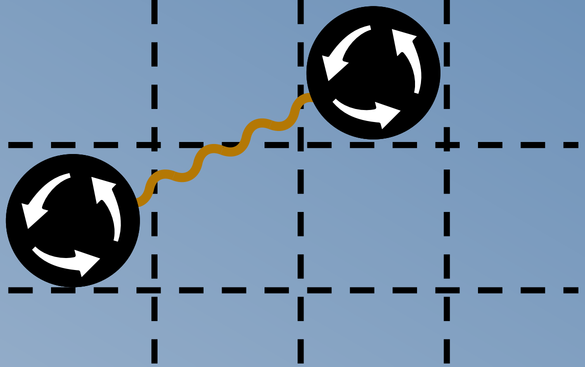
$$\mathcal{L}_{\text{Boson}} \sim \mathbf{J}_{\text{Boson}} \cdot \mathbf{A}$$

$$\mathcal{L}_{\text{Vortex}} \sim \mathbf{J}_{\text{Vortex}} \cdot \mathbf{a} + i\frac{\mathbf{A}\cdot(\nabla\times\mathbf{a})}{2\pi}$$

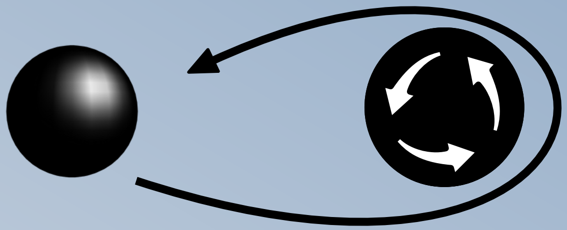
Particle-vortex duality



Boson with short-range interactions

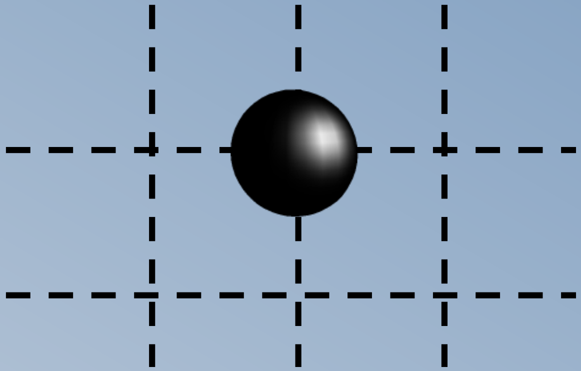


Vortices coupled to emergent photon

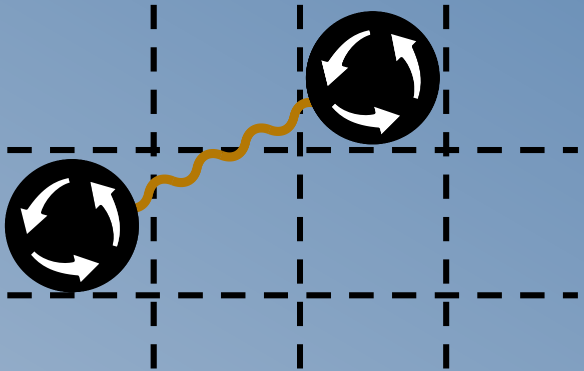


$$\exp\left[i2\pi q\tilde{n}\right]$$

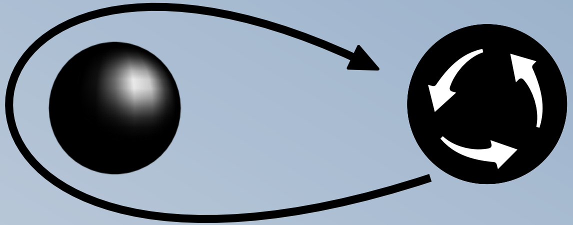
Particle-vortex duality



Boson with short-range interactions



Vortices coupled to emergent photon

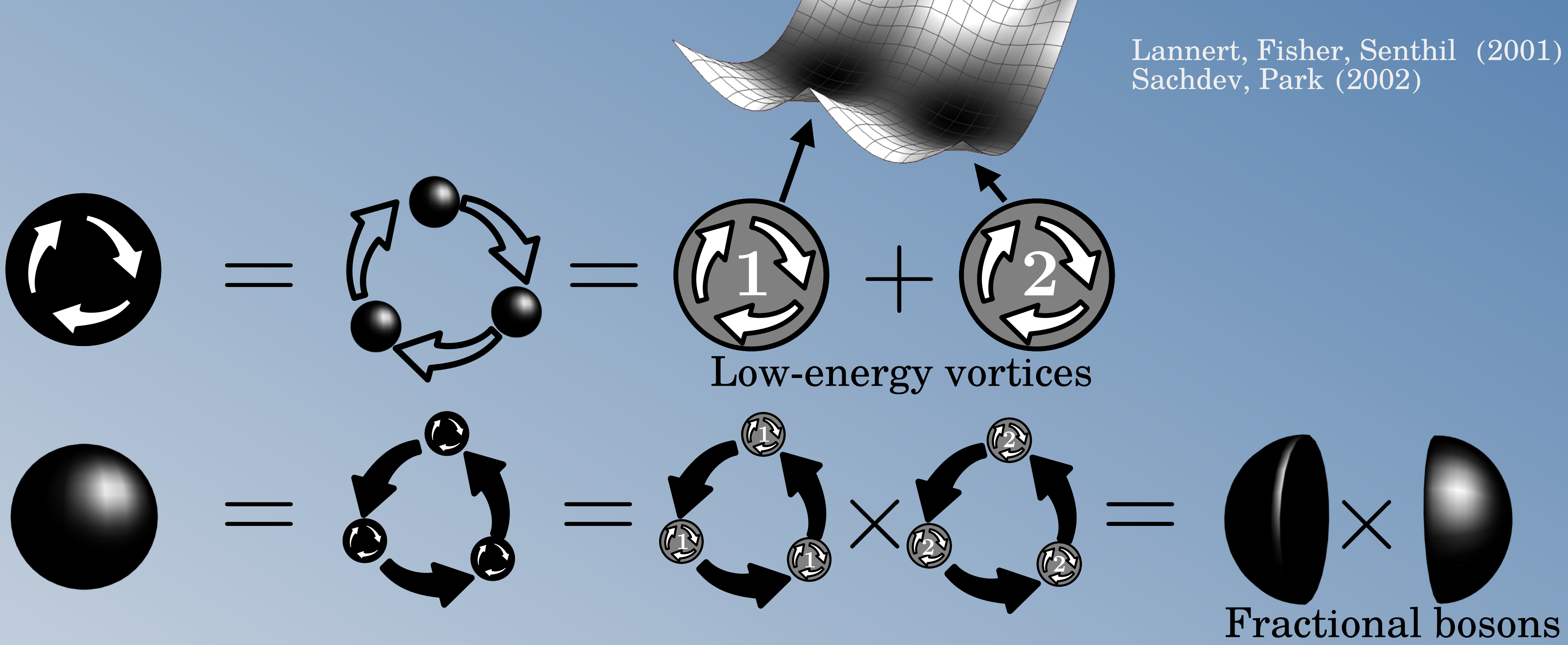


$$\exp\left[i\pi\tilde{q}\right]$$

Average boson density $\leftrightarrow$ Background magnetic flux

Half-filling per lattice site $\leftrightarrow$ π flux per dual lattice plaquette

Lannert, Fisher, Senthil (2001)
Sachdev, Park (2002)

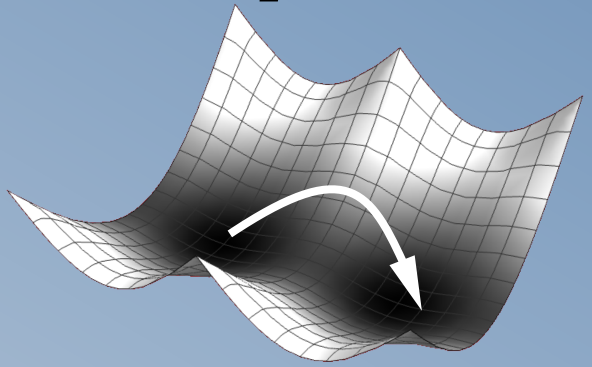


duality $\star$ low-energy limit $\star$ duality

$$\mathcal{L}_{\text{Boson}} \sim \mathbf{J}_{\text{Boson}} \cdot \mathbf{A} \quad \to \quad \sum_i \mathbf{J}_{\text{Vortex},i} \cdot \mathbf{a} + i\frac{\mathbf{A}\cdot(\nabla\times\mathbf{a})}{2\pi}$$

$$\to \quad \sum_i \mathbf{j}_i \cdot \mathbf{a}_i + \sum_i \frac{\mathbf{a}_i\cdot(\nabla\times\mathbf{a})}{2\pi} + i\frac{\mathbf{A}\cdot(\nabla\times\mathbf{a})}{2\pi}$$

$$\boxed{\mathbf{a}_1 + \mathbf{a}_2 + \mathbf{A} = 0} \quad \to \quad \mathbf{j}_1 \cdot (-\tilde{\mathbf{a}} + \tfrac{1}{2}\mathbf{A}) + \mathbf{j}_2 \cdot (\tilde{\mathbf{a}} + \tfrac{1}{2}\mathbf{A})$$



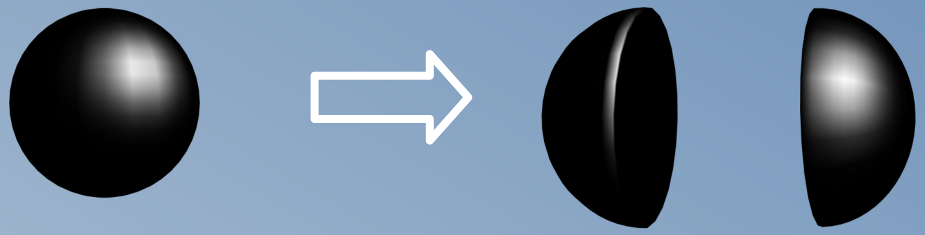
Gauge invariant quantities

$$B \sim \Psi_1\Psi_2$$

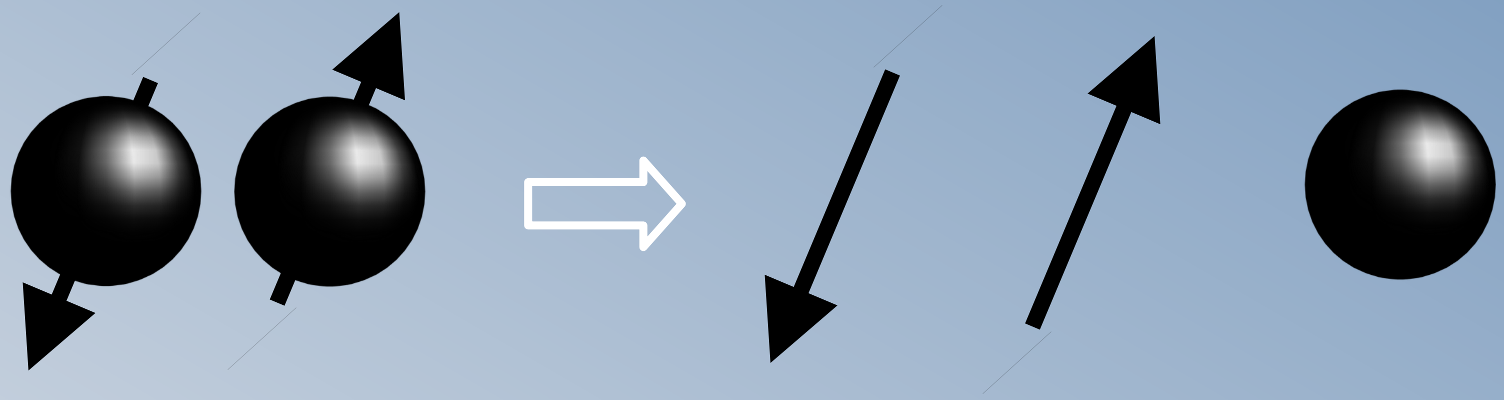
$$\vec{\nabla} \times \vec{\tilde{a}} \sim \text{'1' vortices} - \text{'2' vortices}$$

(not conserved)

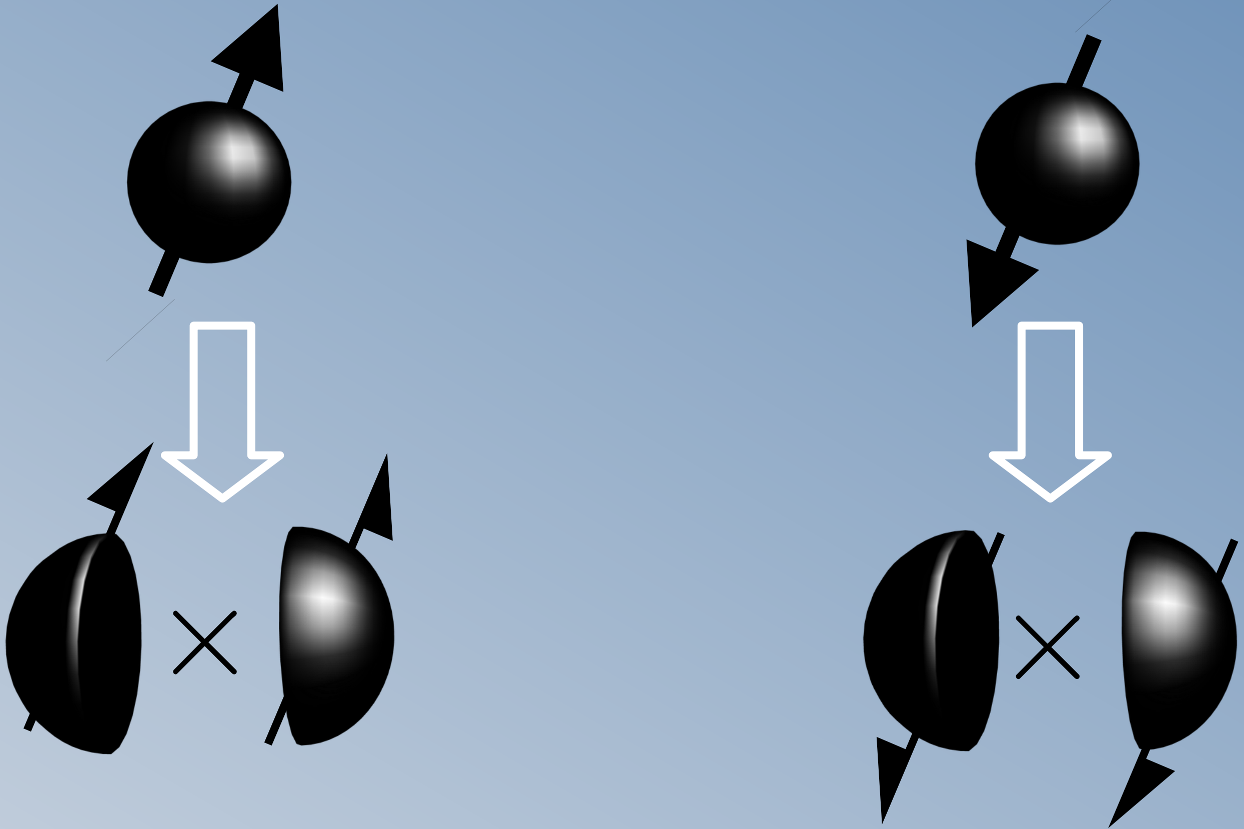
Know:



Want:



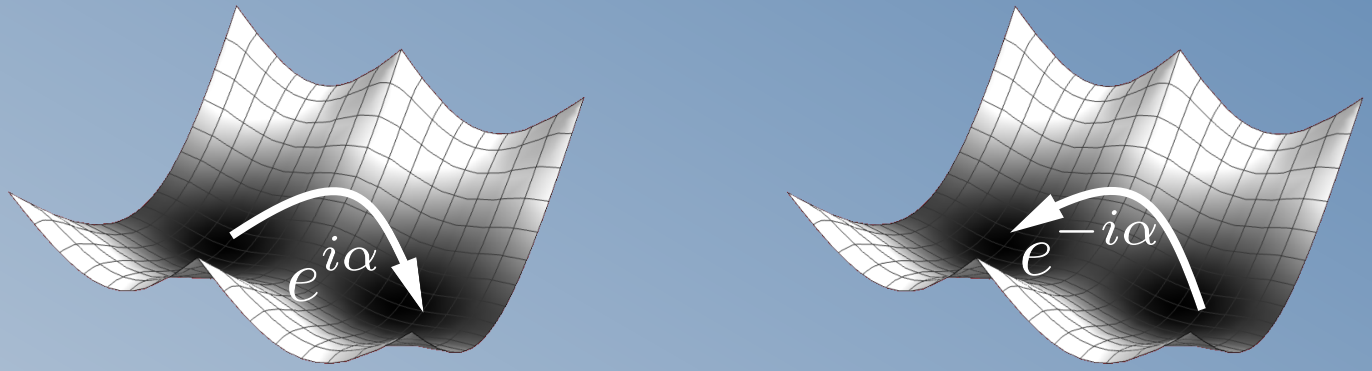
Two species of bosons $U(1) \times U(1)$

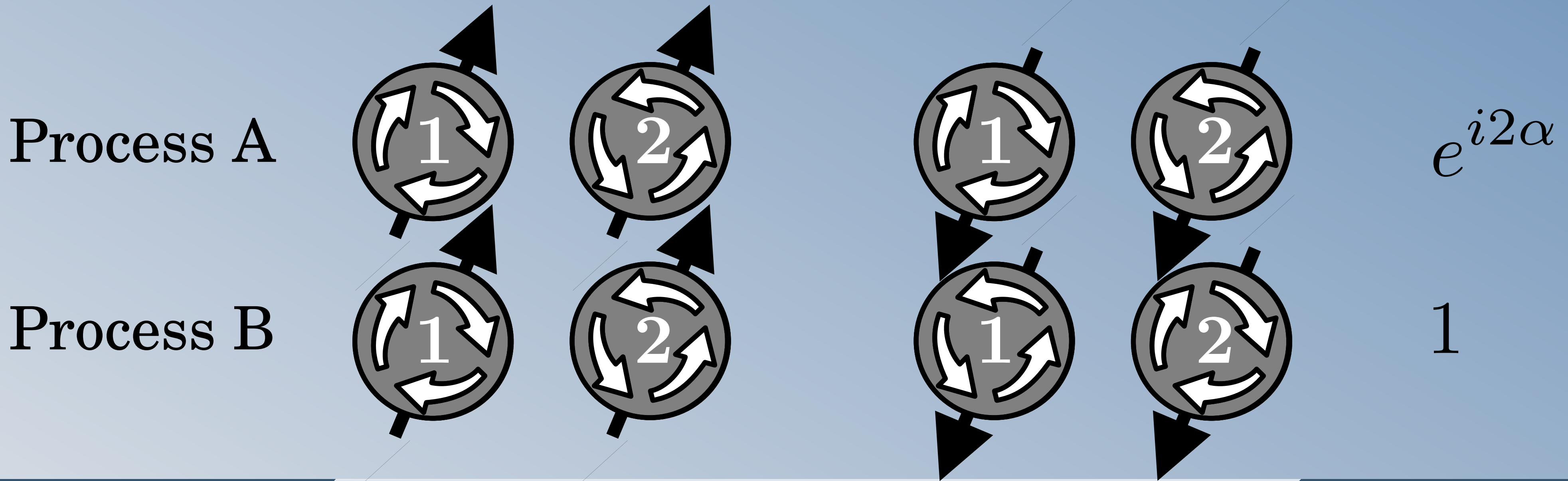


Charge 1/2
Spin 1/4

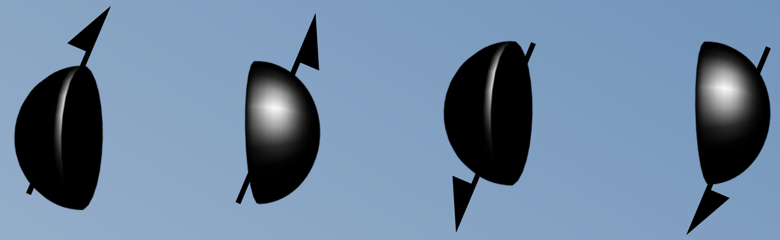
Charge 1/2
Spin -1/4

Two species of bosons $U(1) \times U(1)$

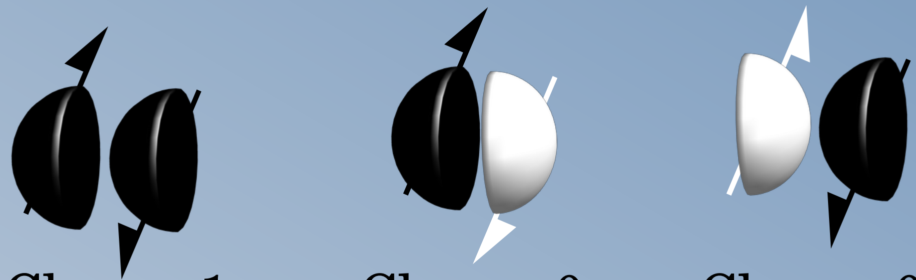




Half-charge excitations



Confined

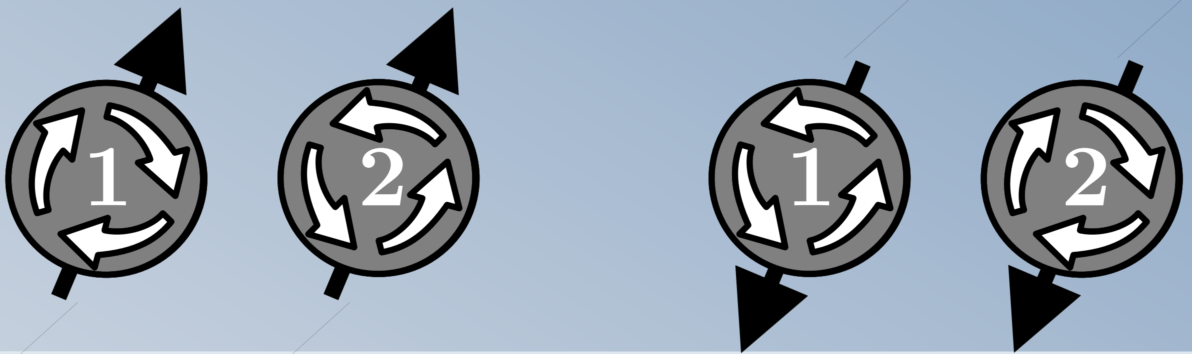


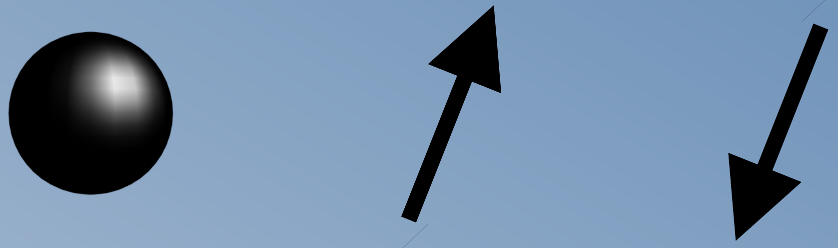
Charge 1
Spin 0

Charge 0
Spin 1/2

Charge 0
Spin -1/2

Not confined



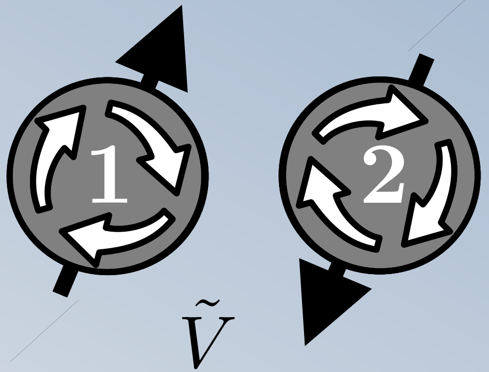


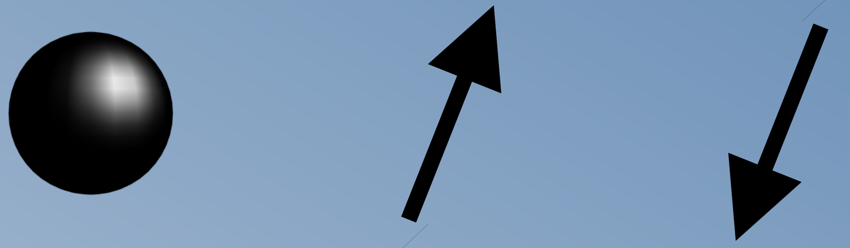
Charge 1	Charge 0	Charge 0
Spin 0	Spin 1/2	Spin -1/2

$$\mathcal{L}_{\text{Boson}} \sim \sum_\sigma \mathbf{J}_{\text{Boson},\sigma} \cdot \mathbf{A}_\sigma \quad \rightarrow \quad \sum_\sigma \mathbf{J}_{\text{Vortex},2,\sigma} \cdot \mathbf{a}_\sigma + i \sum_\sigma \frac{\mathbf{A}_\sigma \cdot (\nabla \times \mathbf{a}_\sigma)}{2\pi}$$

$$+ \sum_\sigma \tilde{\mathbf{J}}_{\text{Vortex},\sigma} \cdot [\mathbf{a}_\sigma + \mathbf{a}_{-\sigma}]$$

$$+ (\tilde{V}_\uparrow^\star \tilde{V}_\downarrow + \tilde{V}_\downarrow^\star \tilde{V}_\uparrow)$$





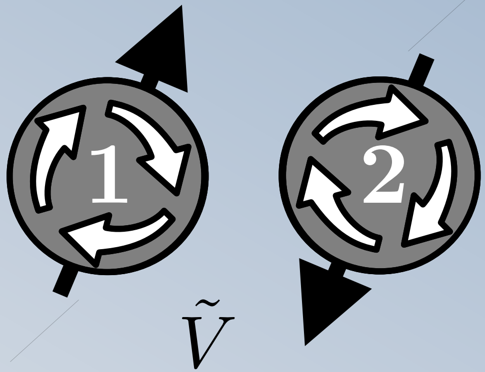
Charge 1
Spin 0

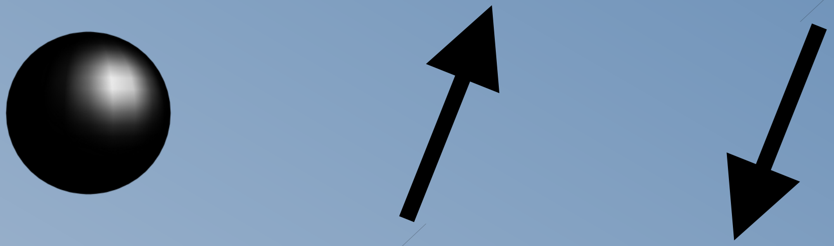
Charge 0
Spin 1/2

Charge 0
Spin -1/2

$$\mathcal{L}_{\text{Boson}} \sim \sum_\sigma \mathbf{J}_{\text{Boson},\sigma} \cdot \mathbf{A}_\sigma \quad \rightarrow \quad \sum_\sigma \mathbf{J}_{\text{Vortex},2,\sigma} \cdot \mathbf{a}_\sigma + i \sum_\sigma \frac{\mathbf{A}_\sigma \cdot (\nabla \times \mathbf{a}_\sigma)}{2\pi}$$

$$+ \tilde{\mathbf{J}}_{\text{Vortex}} \cdot [\mathbf{a}_\sigma + \mathbf{a}_{-\sigma}]$$





$$\mathcal{L}_{\text{Boson}} \sim \sum_\sigma \mathbf{J}_{\text{Boson},\sigma} \cdot \mathbf{A}_\sigma \quad \rightarrow \quad \sum_\sigma \mathbf{J}_{\text{Vortex},2,\sigma} \cdot \mathbf{a}_\sigma + i \sum_\sigma \frac{\mathbf{A}_\sigma \cdot (\nabla \times \mathbf{a}_\sigma)}{2\pi}$$

$$+ \tilde{\mathbf{J}}_{\text{Vortex}} \cdot [\mathbf{a}_\sigma + \mathbf{a}_{-\sigma}]$$

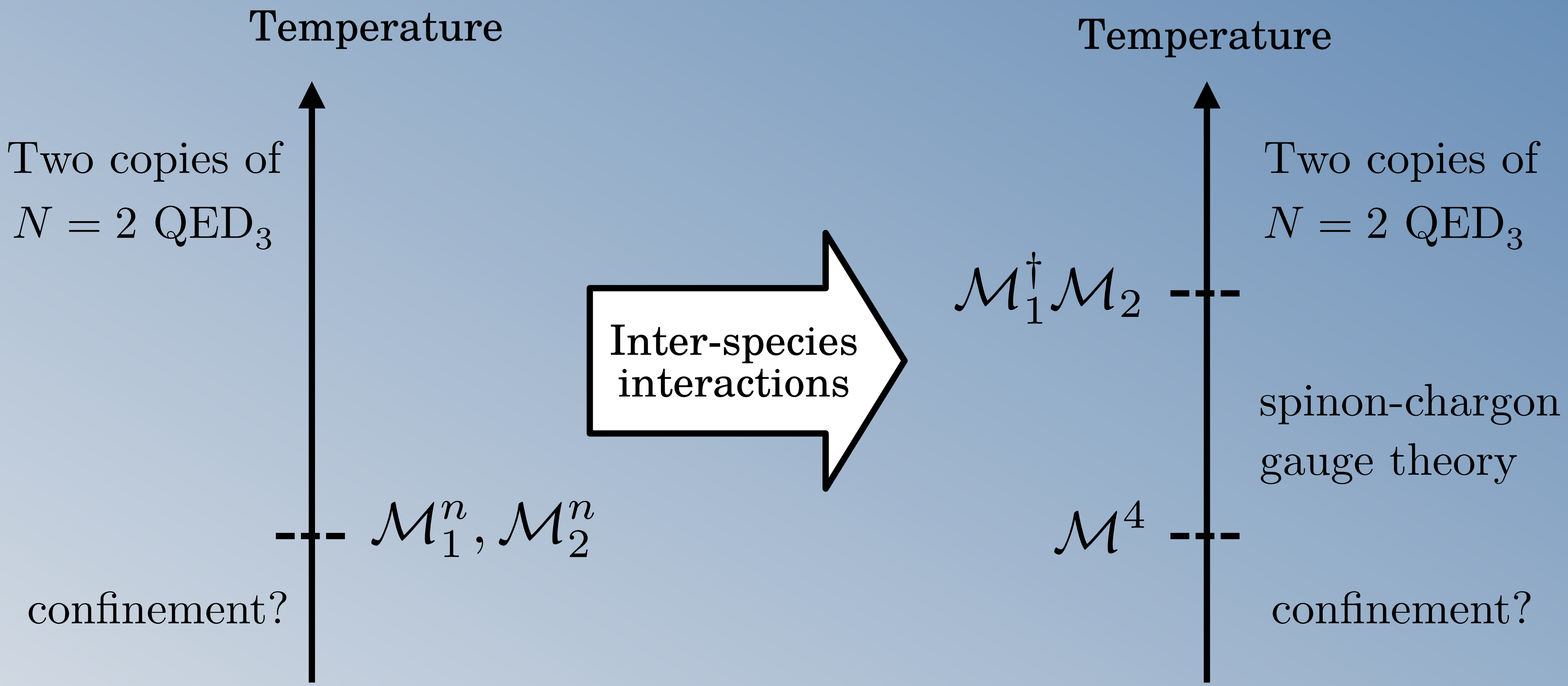
$$\rightarrow \quad \sum_\sigma \mathbf{J}_{\text{spinon},\sigma} \cdot (\mathbf{A}_\sigma - \mathbf{a}) + \mathbf{J}_{\text{chargon}} \cdot \mathbf{a}$$

Chargons and spinons couple to emergent gauge field

$$\vec{\nabla} \times \vec{a} \sim \text{‘}1_\uparrow\text{’ vortices} - \text{‘}2_\uparrow\text{’ vortices}$$
$$+ \text{‘}1_\downarrow\text{’ vortices} - \text{‘}2_\downarrow\text{’ vortices}$$

(not conserved)

• Mott transition of two boson species at half-filling



Gauge theory of spinons and chargons from duality

- Generalizations: Fermions and $N > 2$
- Exact duality on wire arrays
- Parent Hamiltonians for phases and transitions
- $\mathbb{Z}_2$ analogue in 1+1 dimensions

Generalizations: Fermions and $N > 2$

- Flux attachment: Spin-charge separation for fermions

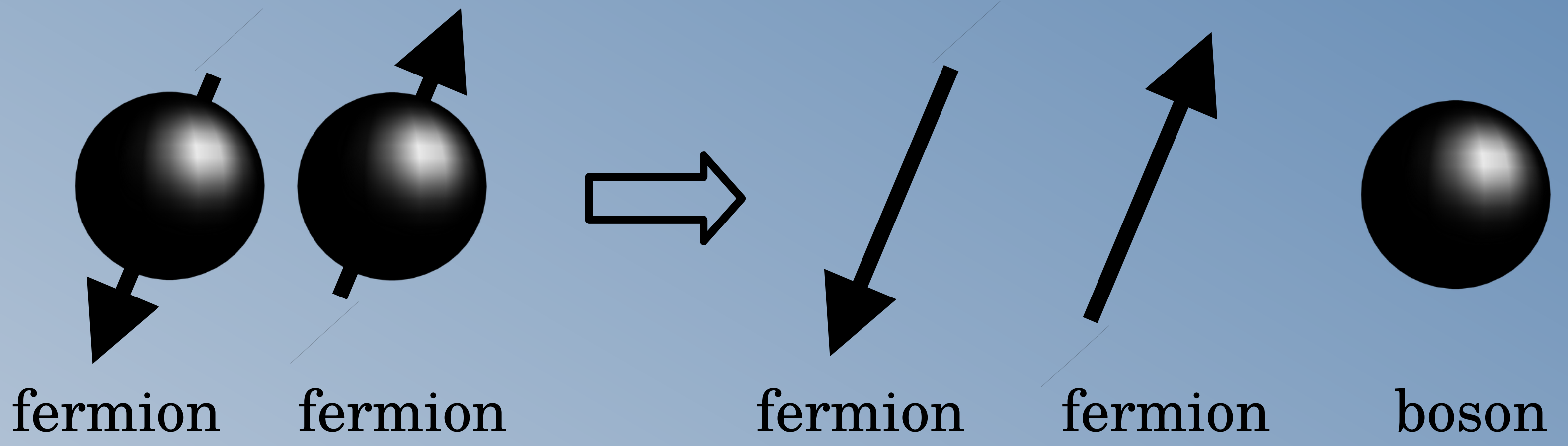
$$\sum_\sigma \mathbf{J}_{\text{Fermion},\sigma} \cdot \mathbf{A}_\sigma$$

$$\to \sum_\sigma \mathbf{J}_{\text{Boson},\sigma} \cdot [\mathbf{A}_\sigma + \mathbf{a}_\sigma^{\text{CS}}] + \text{CS}[\mathbf{a}_\sigma^{\text{CS}}]$$

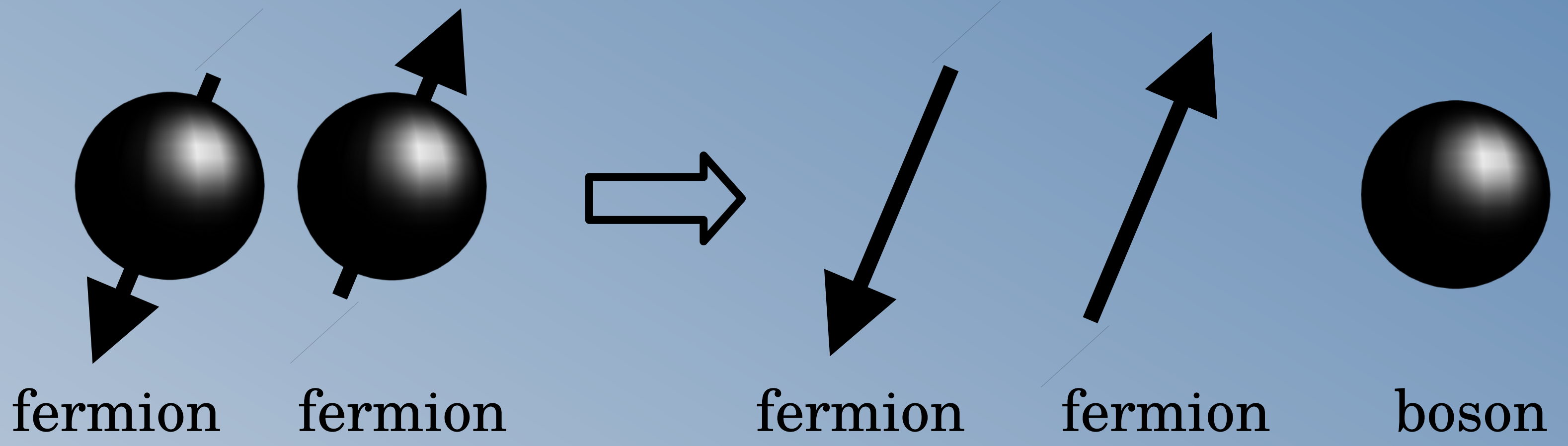
$$\to \sum_\sigma \mathbf{J}_{\text{spinon},\sigma} \cdot (\mathbf{A}_\sigma + \mathbf{a}_\sigma^{\text{CS}} - \mathbf{a}) + \mathbf{J}_{\text{chargon}} \cdot \mathbf{a} + \text{CS}[\mathbf{a}_\sigma^{\text{CS}}]$$

$$\to \sum_\sigma \mathbf{J}_{\text{fermionic spinon},\sigma} \cdot (\mathbf{A}_\sigma - \mathbf{a}) + \mathbf{J}_{\text{chargon}} \cdot \mathbf{a}$$

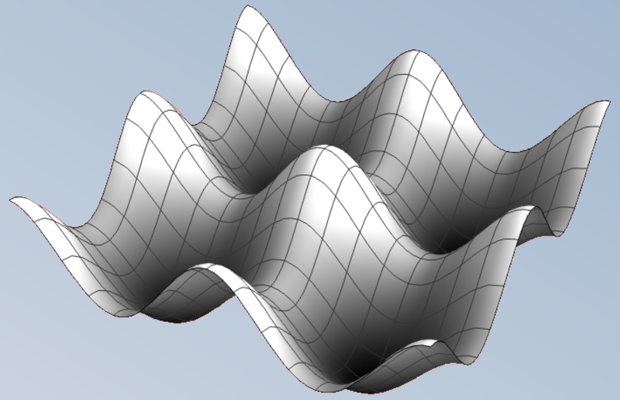
- Flux attachment: Spin-charge separation for fermions

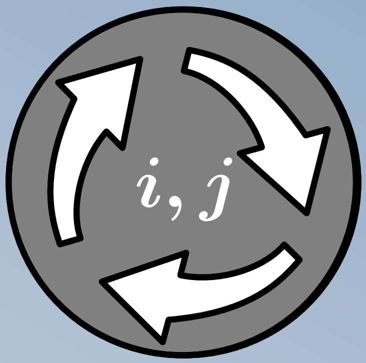


- Flux attachment: Spin-charge separation for fermions



- N bosons species at filling $\frac{1}{N}$





Exact duality on wire arrays

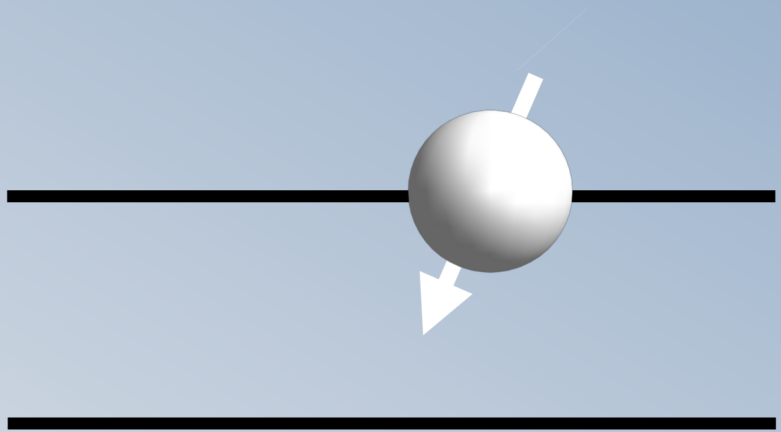
Derivations of (2+1) dualities in wire models

DFM, Alicea, Motrunich (2016, 2017)
Leviatan, DFM (2020, 2022)

Exact duality on wire arrays

$$\text{Bosons} \sim e^{i\varphi}$$

$$\mathcal{L} \sim (\partial_{x,\tau}\varphi - a_{x,\tau})^2 + \cos[\Delta_y \varphi - a_y] + \dots$$

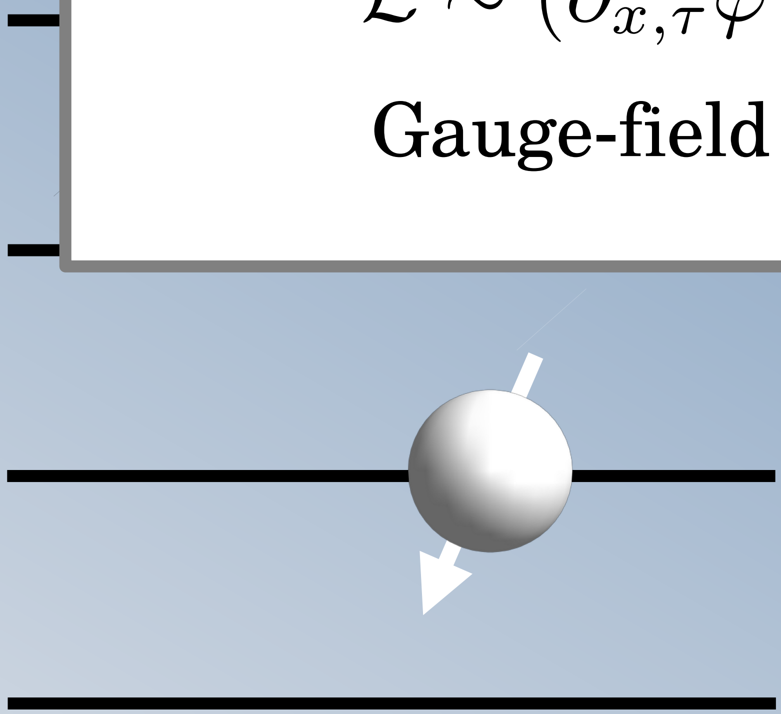


Exact duality on wire arrays

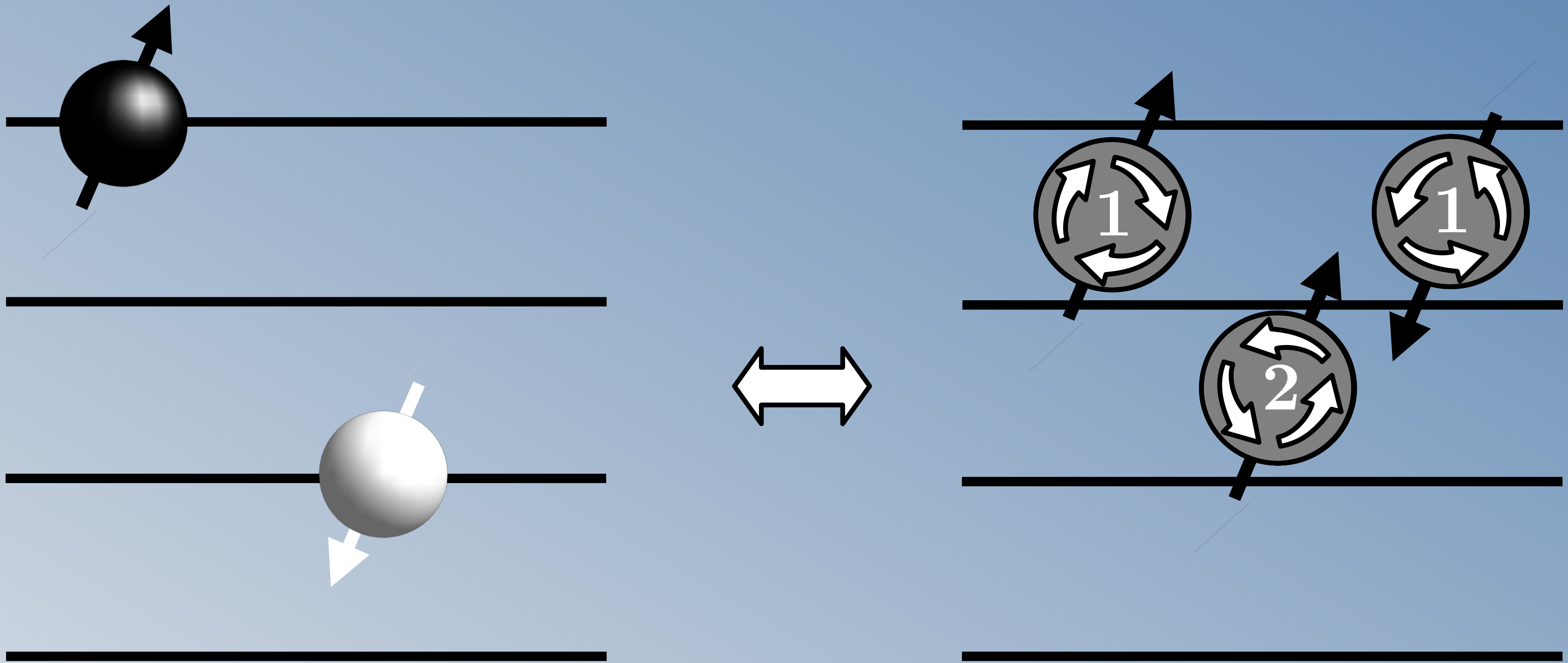
Bosons $\sim e^{i\varphi}$

$$\mathcal{L} \sim (\partial_{x,\tau}\varphi - a_{x,\tau})^2 + \cos[\Delta_y \varphi] + \ldots$$

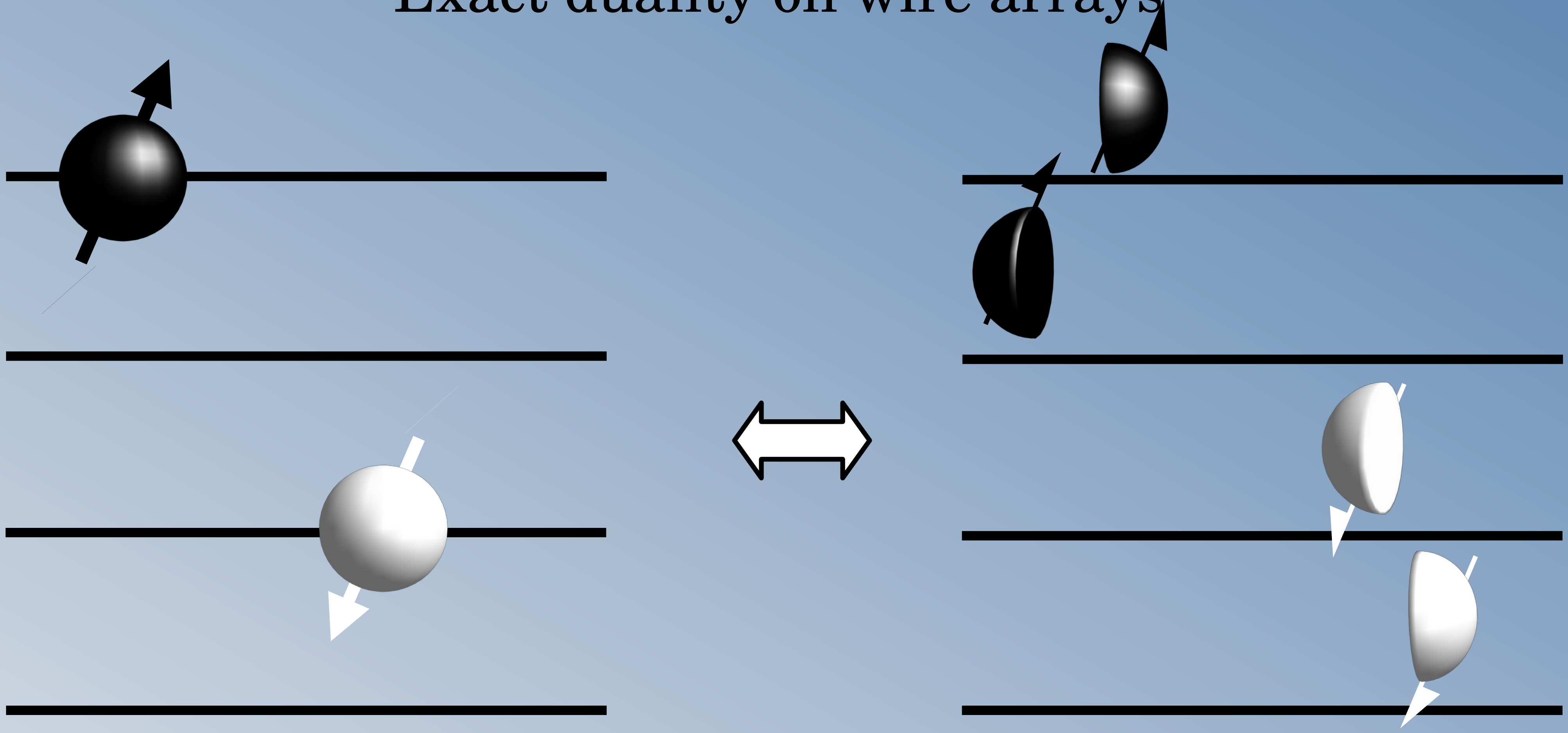
Gauge-field only in Gaussian action



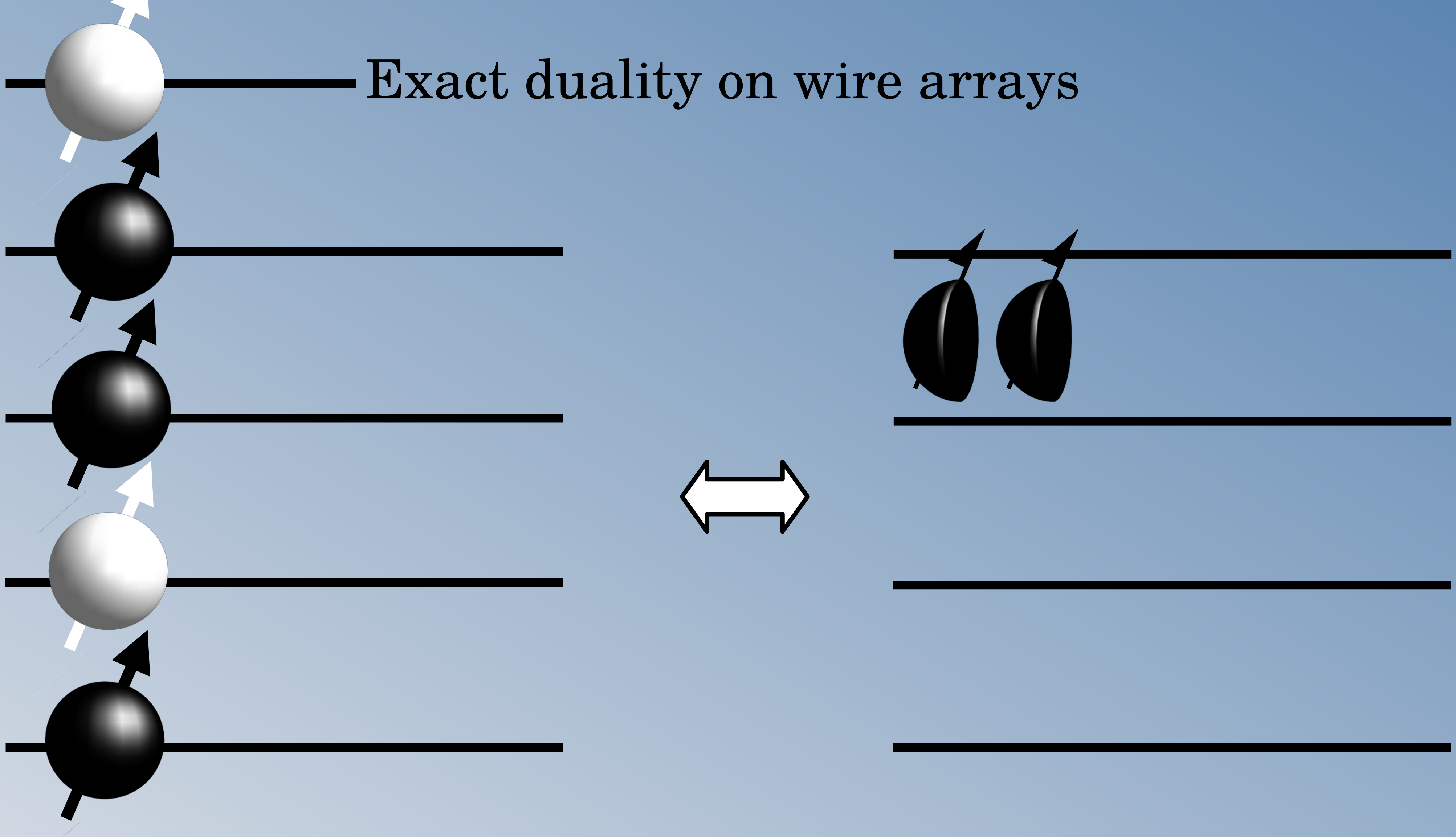
Exact duality on wire arrays



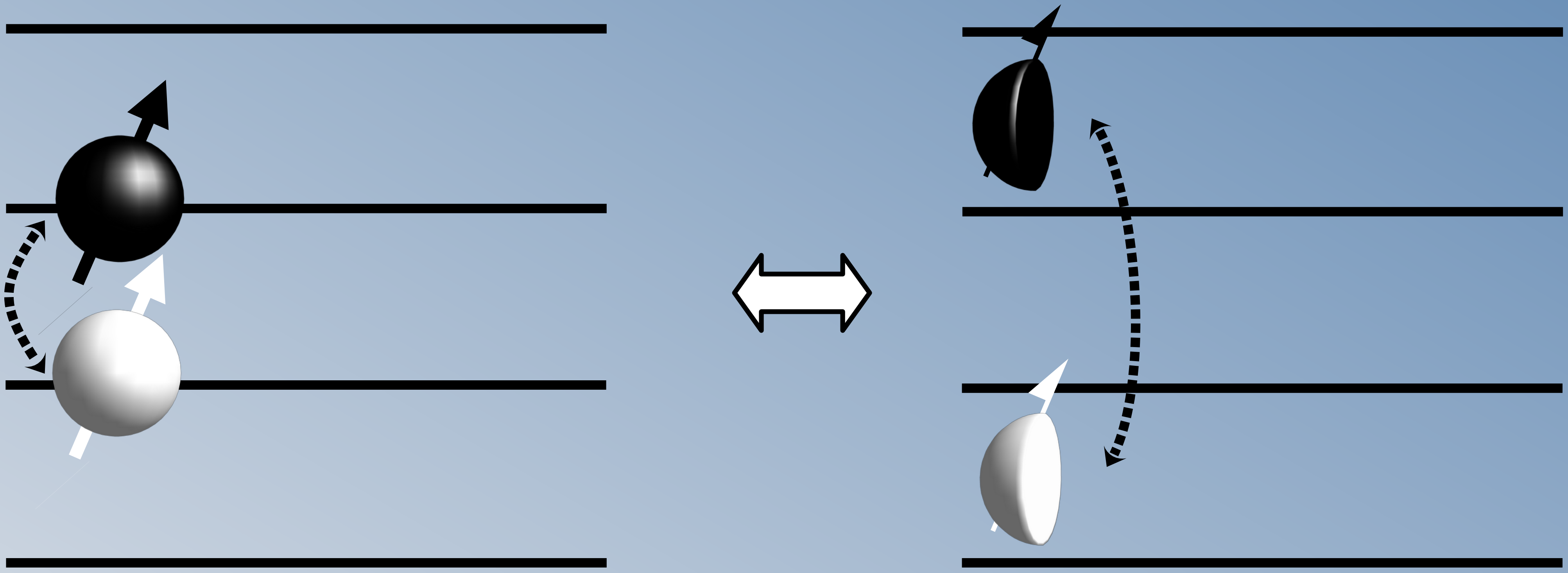
Exact duality on wire arrays



Exact duality on wire arrays



Exact duality on wire arrays



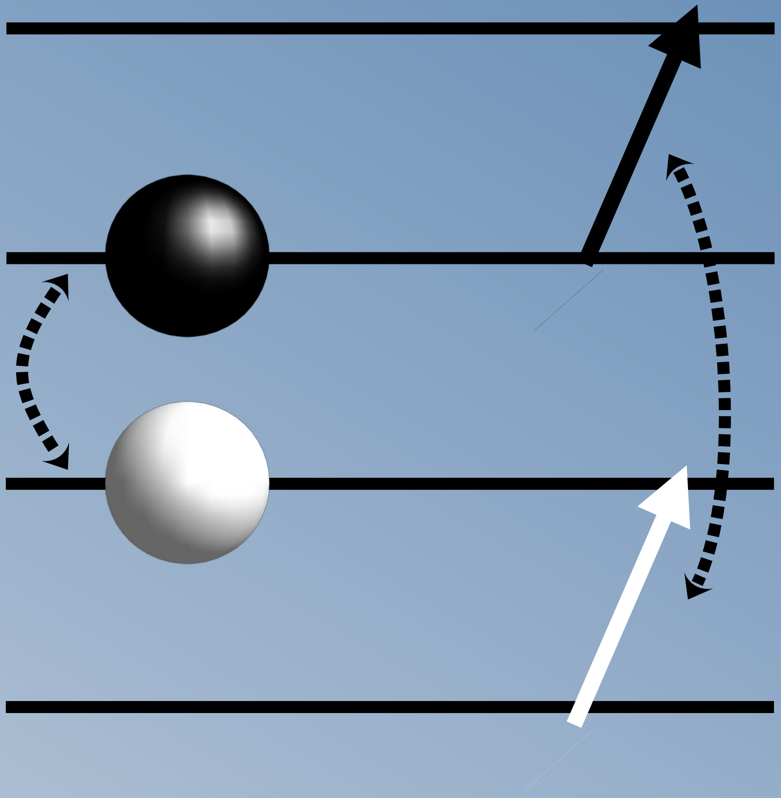
Exact duality on wire arrays

Chargon creation: non-local

Chargon hopping: local

Spinon creation: non-local

Spinon hopping: local



Exact duality on wire arrays

$$\text{Bosons} \sim e^{i\varphi}$$

$$\mathcal{L} \sim (\partial_{x,\tau}\varphi - a_{x,\tau})^2 + \cos[\Delta_y \varphi] + \dots$$

Gauge-field only in Gaussian action

$$\int \mathcal{D}a e^{-\mathcal{S}[a,\psi^c,\psi^s_\sigma]} \propto e^{-\mathcal{S}^{\text{non-local}}[\psi^c,\psi^s_\sigma]} = e^{-\mathcal{S}^{\text{local}}[b_\sigma]}$$

Parent Hamiltonians

Wire constructions for topological phases

Kane, Mukhopadhyay, Lubensky (2002) Nersesyan, Tsvelik (2003)
Lu, Vishwanath (2012) Vazifeh (2013) Oreg, Sela, Stern (2014)
Teo and Kane (2014) Klinovaja, Loss (2014) Meng et al. (2014)
Klinovaja, Tserkovnyak (2014) Sagi, Oreg (2014) Meng, Sela (2014)
Mong et al. (2014) Seroussi, Berg, Oreg (2014) Vaezi (2014) Neupert, et al. (2014)
and many more

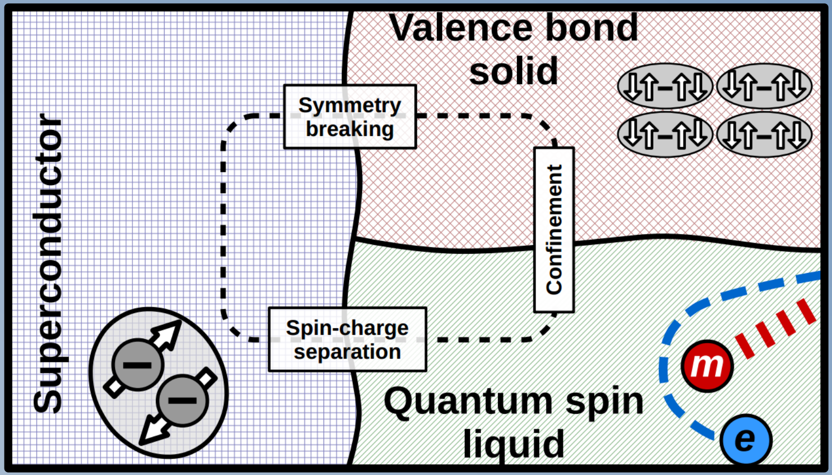
Parent Hamiltonians for phases and transitions

$$\underbrace{H_{\text{Chargon}}, H_{\text{Spinon}}}_{\text{simple}} \Longrightarrow H_{\text{Microscopic}}$$

fix H_{Spinon}

transition in H_{Chargon} $\Longrightarrow$ **Mott transition without closing spin gap**

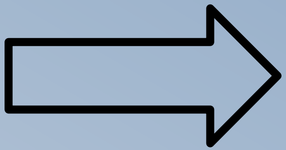
Parent Hamiltonians for phases and transitions



Leviatan, DFM (2022)

fix H_{Spinon}

transition in H_{Chargon}



Mott transition without closing spin gap

$\mathbb{Z}_2$ analogue in 1+1 dimensions

Duality webs in (1+1) and (2+1) dimensions

Seiberg, Senthil, Wang, Witten (2016)
Karch, Tong (2016)
Karch, Tong, Turner (2016)

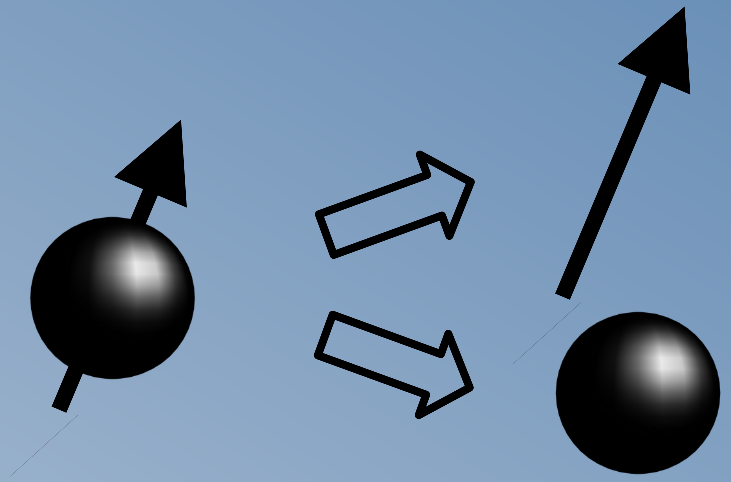
Deconfined criticality on (1+1) dimensions

Jiang, Motrunich (2019)

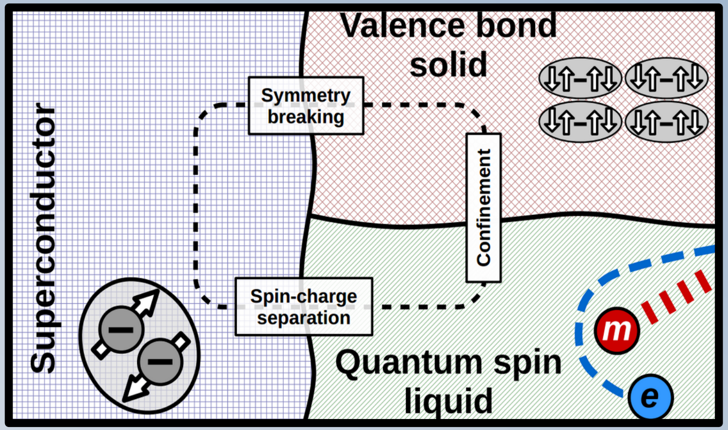
2+1 dimensions		1+1 dimensions
• Conserved bosons at half-filling	$\Longleftrightarrow$	Spins with $\mathbb{Z}_2 \times \mathbb{Z}_2$ symmetry
• U(1) gauge fields	$\Longleftrightarrow$	$\mathbb{Z}_2$ gauge fields
• Spinful bosons	$\Longleftrightarrow$	Spin ladder

Conclusions

Spin-charge separation



Duality



Microscopic models

(1+1) vs. (2+1) dimensions