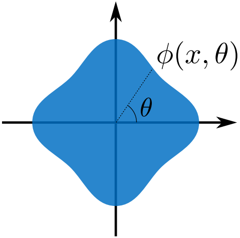
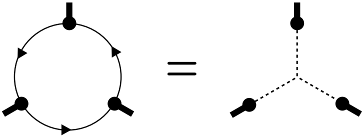
Nonlinear Bosonization of Fermi Liquids

Luca Delacrétaz

U Chicago & Unige





arXiv:**2203.05004**

Bootstrapping Nature: Nonperturbative Approaches to Critical Phenomena

October 16, 2022 @ GGI

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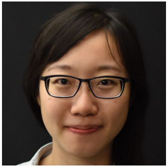
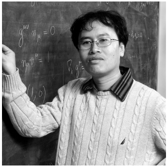


Yi-Hsien Du

Umang Mehta



Dam Thanh Son

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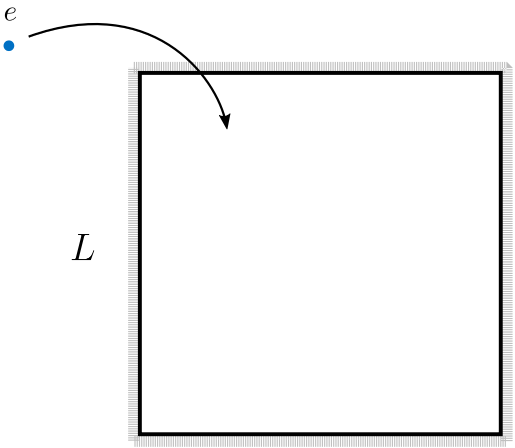
Fermi Gas



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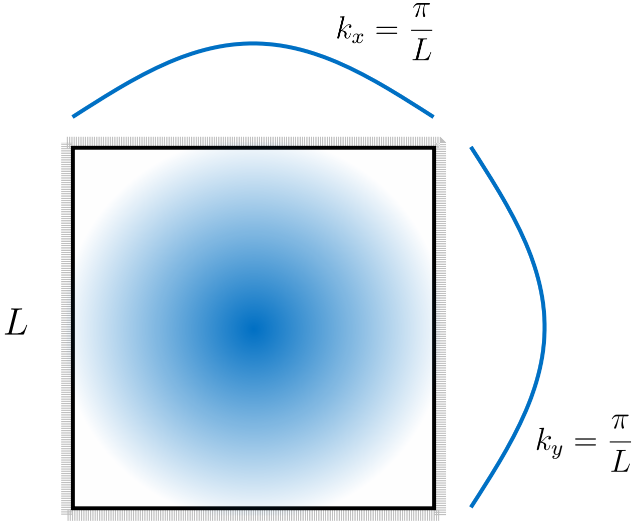

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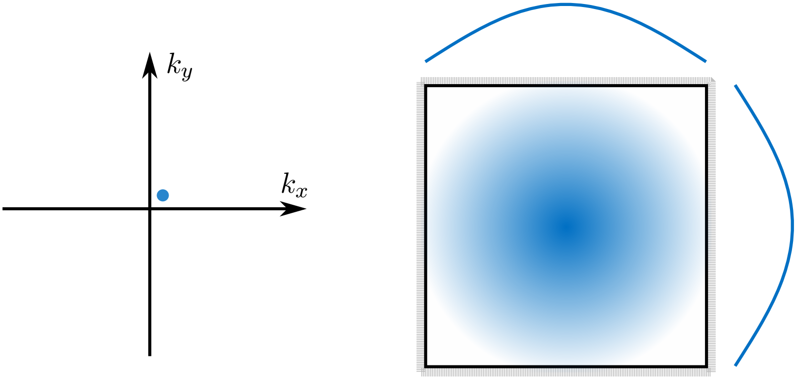

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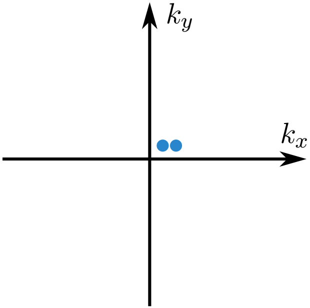

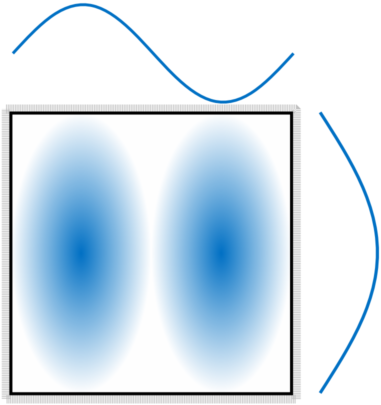

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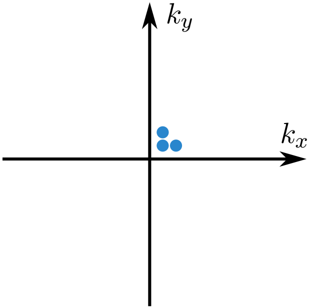

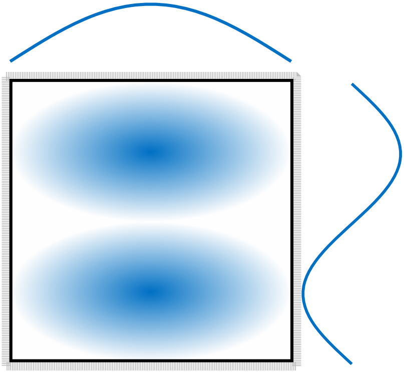

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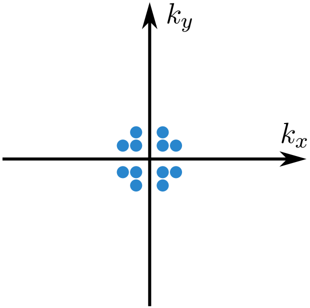

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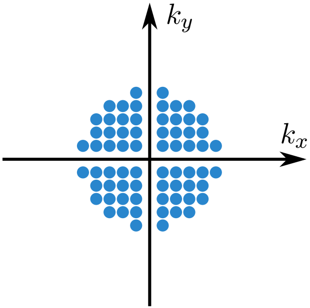

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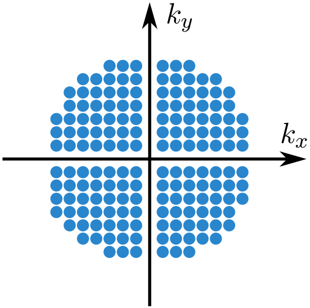
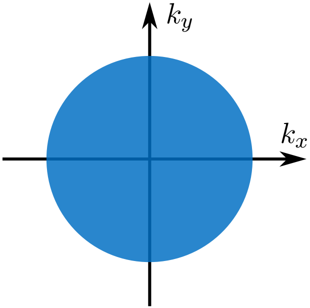
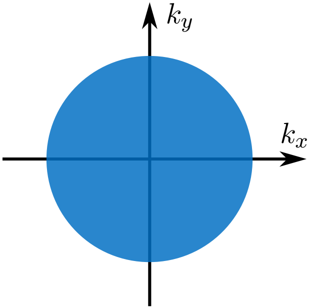
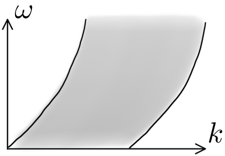
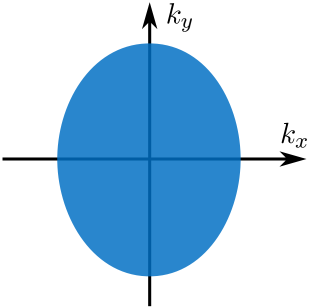



k_y
k_x



Very gapless system, compared to 'vacuum' QFT





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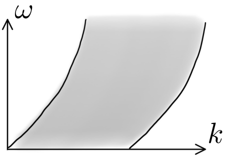

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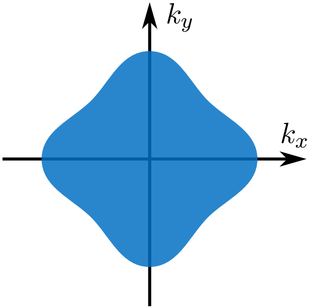

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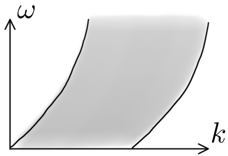

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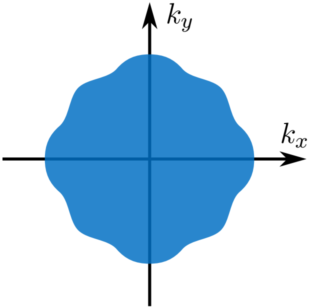

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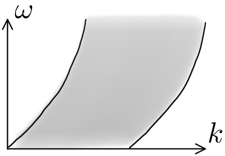

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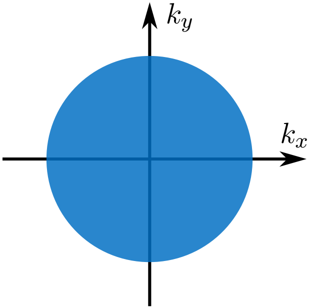

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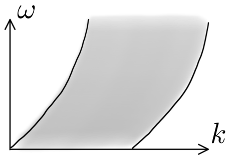
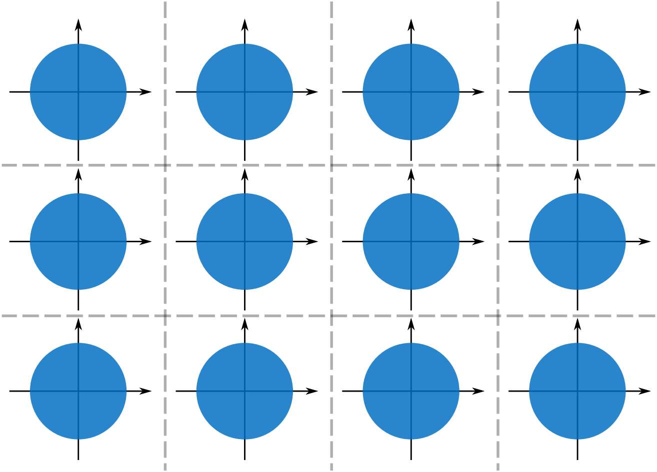


Important quantity: k_F.

Effective description for long distance physics $x \gg 1/k_F$?

Semi-classical Fermi surface

Local Fermi surface in every volume of size $\xi \gg 1/k_F$



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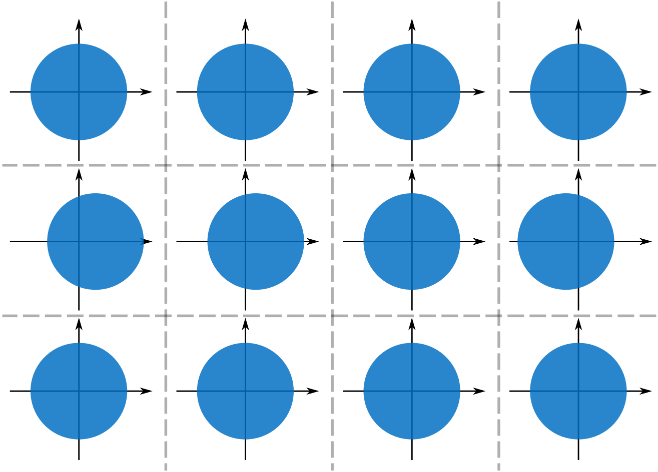

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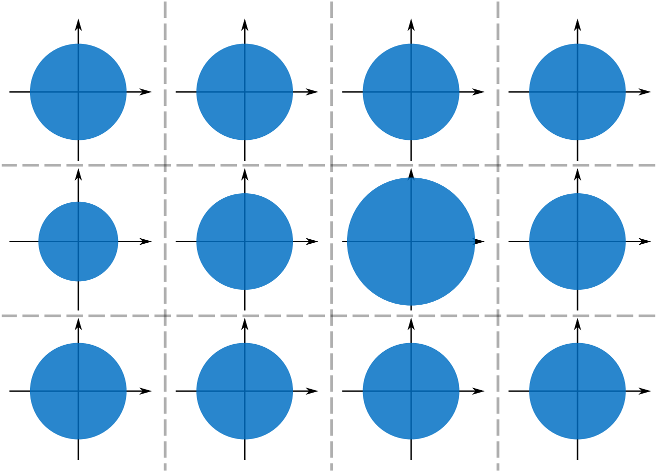

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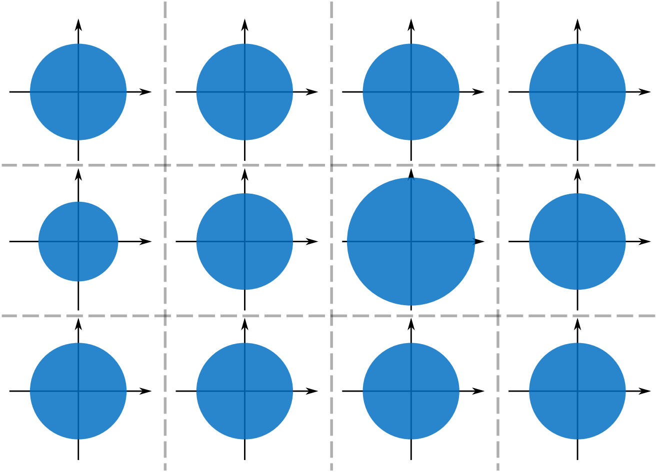
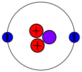


Described by kinetic equation

$$\partial_t f(t, x, p) + \vec{v}(p) \cdot \nabla_x f(t, x, p) = 0$$

Successes of FL theory

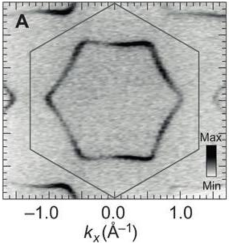
^{3}He



- Specific heat $c_V \propto T$ (compare to CFT $c_V \sim T^d$)
- Observation of collective excitations (e.g. zero sound)
- Measurement of Landau parameters
- Viscosity, sound attenuation, equilibration time $\tau \sim E_F/T^2$

Metals

- Visualize Fermi surfaces with ARPES
- Magnetic oscillations, Hall coefficient, $\sigma_\perp(q) \sim k_F/q$
- $\rho_{\rm dc} \sim T^2$



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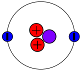
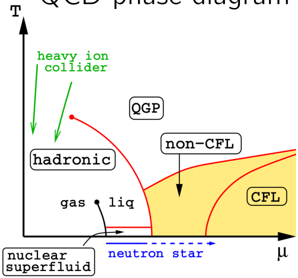
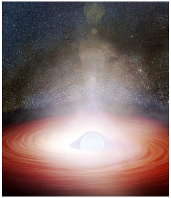


QCD phase diagram



Neutron stars, White dwarfs



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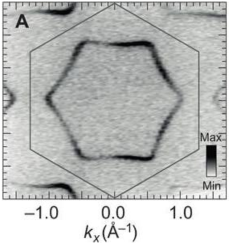
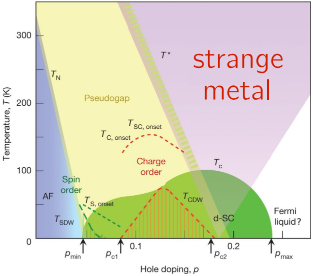


Beyond FL

Experiment

High-T_c superconductors



[Keimer Kivelson Norman Uchida Zaanen '15]

Fermi surface, but thermodynamics and transport not FL-like

Theory

S_{FL} + gapless boson

NFL fixed point

Hertz '76, ... Nayak Wilczek '94, Altshuler Ioffe Millis '94,

Kim Furusaki Wen Lee '94, ... Lee '09, Metlitski Sachdev '10,

... Esterlis Guo Patel Sachdev '21, ...

NFL is hard

Interacting CFT	NRCFT ($z = 2$)	Lifschitz (z, θ)	NFL
Perturbative constructions:	Perturbative constructions:	Perturbative constructions:	Perturbative constructions:
$4-\epsilon$, $2+\epsilon$, N	similar	free theories, ..?	$d-\epsilon$: nonlocal, break symmetries, UV/IR mixing N: subtleties $[\lim_{\omega\to 0}, \lim_{N\to\infty}] \neq 0$
Non-perturbative methods:	Non-perturbative methods:	Non-perturbative methods:	Non-perturbative methods:
OPE, operator-state, analytic/numerical bootstrap, dualities, SUSY, holography	OPE, operator-state, holography (bottom-up)	holography (bottom-up)	

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NRCFT $(z=2)$

Perturbative constructions:

similar

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Lifschitz (z,θ)

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NFL

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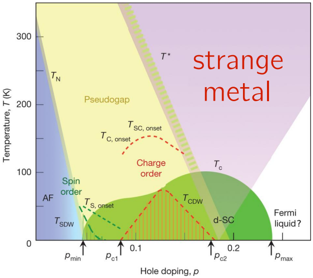
High-T_c superconductors



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P.W. Anderson, 1989

"Know the enemy!"

[Keimer Kivelson Norman Uchida Zaanen '15]

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What is an Effective Field Theory (EFT)?

"A microscopics-insouciant description of a system based on general principles"

An EFT for FL

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E.g.: spontaneous symmetry breaking

Pions in QCD

$$U \in \frac{SU(2)\times SU(2)}{SU(2)}$$

$$S = \int \mathrm{Tr}\left(\partial_\mu U^\dagger \partial^\mu U\right)$$

Spin waves in Ferromagnet

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[Callan Coleman Wess Zumino '70]

An EFT for FL

What is S_{FL} ? EFT for Fermi liquids

Shankar '91, Polchinski '92

$$S = \int dt \int d^d p \, \psi_p^\dagger (\partial_t + \epsilon(p)) \psi_p + \int_{p_1 p_2 p_3} V(p_1, p_2, p_3, p_4) \psi_1^\dagger \psi_2^\dagger \psi_3 \psi_4 + \cdots$$

Landau parameters and BCS couplings as only marginal terms

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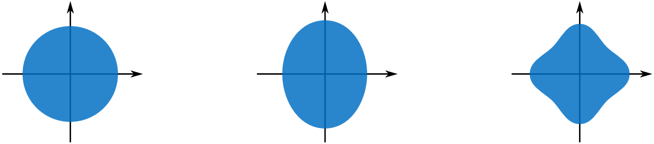
Our strategy: write an EFT directly for the shape, or $f(x, p, t)$



Nonlinear completion of multidimensional bosonization

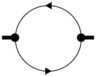
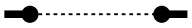
Why bosonization?

In 1+1d, bosonization can solve interacting fermion problems

Even for free fermions, bosonization is useful

$$S = \int dt dx \, i\bar{\psi}\gamma^{\mu}\partial_{\mu}\psi \qquad \leftrightarrow \qquad S = \frac{1}{4\pi}\int dt dx \, (\partial_{\mu}\phi)^2$$

$$\rho = \psi^{\dagger}\psi \qquad \leftrightarrow \qquad \rho = \frac{1}{2\pi}\partial_x\phi$$

$\langle\rho\rho\rangle(\omega, q) =$  $\leftrightarrow$ $\langle\rho\rho\rangle(\omega, q) =$ 

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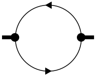
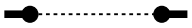
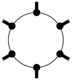
$$\langle \rho\rho \rangle(\omega, q) =$$  $$\leftrightarrow \qquad \langle \rho\rho \rangle(\omega, q) =$$ 

$$\langle \rho\rho \cdots \rho \rangle =$$  $= ???$

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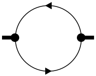
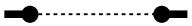
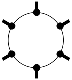
Even for free fermions, bosonization is useful

$$S = \int dtdx\, i\bar{\psi}\gamma^{\mu}\partial_{\mu}\psi \qquad \leftrightarrow \qquad S = \frac{1}{4\pi}\int dtdx\,(\partial_{\mu}\phi)^2$$

$$\rho = \psi^{\dagger}\psi \qquad \leftrightarrow \qquad \rho = \frac{1}{2\pi}\partial_x\phi$$

$\langle\rho\rho\rangle(\omega, q) =$  $\leftrightarrow$ $\langle\rho\rho\rangle(\omega, q) =$ 

$\langle\rho\rho\cdots\rho\rangle =$  $= ???$ $\leftrightarrow$ $\langle\rho\rho\cdots\rho\rangle = 0$!

[Dzyaloshinskii Larkin '74]

Why bosonization?

Bosonization also helps study deformations of original problem

Example: Prove that Schwinger model (1+1d QED) is gapped

$$S = \int \bar{\psi}\gamma^{\mu}(\partial_{\mu} + e a_{\mu})\psi + (da)^2$$

Relevant interaction!

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Relevant interaction!

Bosonized description directly gives $m_{\text{photon}} = e$.

Why bosonization?

In higher dimensions, or for $\epsilon(p) \neq p$, still *approximate* loop cancellations

Kopietz Hermisson Schönhammer '95, Metzner Castellani Di Castro '97

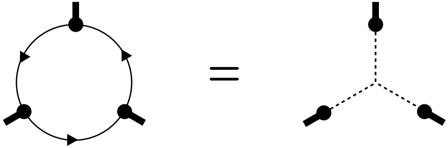
These make perturbative analyses of non-Fermi liquids difficult

Metlitski Sachdev '10, Holder Metzner '15

Why bosonization?

These near cancellations are transparent in the bosonic picture

The 'small' remainder comes from 'weak' interactions



LVD Du Mehta Son '22

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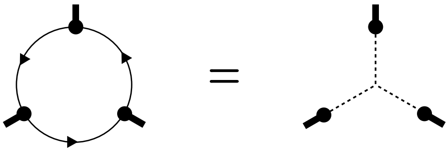
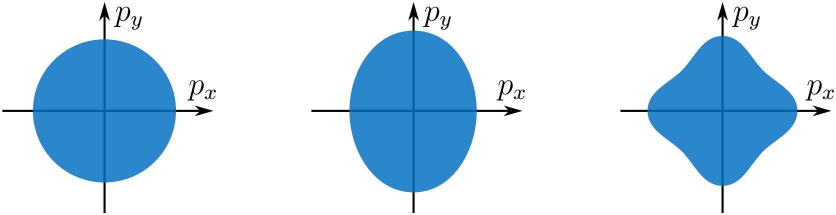


LVD Du Mehta Son ’22

Two sources of nonlinearities:

- $\epsilon(p) \neq p$
- Geometry of the Fermi surface

Our strategy: action for the *shape* of the Fermi surface



$$f(t,x,p) = \begin{cases} 1 & \text{if } p \in \\ 0 & \text{if } p \notin \end{cases}$$


We would like an action S_{FL} for $f(t,x,p)$

LVD Du Mehta Son '22

For free fermions with single particle $H = \epsilon(p) + V(x)$, the equation of motion should be the Boltzmann equation

$$\partial_t f + \nabla_p H \cdot \nabla_x f - \nabla_x H \cdot \nabla_p f = 0$$

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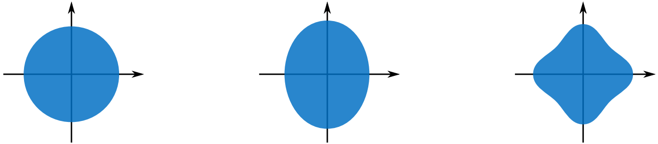
$$\partial_t f + \nabla_p H \cdot \nabla_x f - \nabla_x H \cdot \nabla_p f = 0$$

From $f_0(x, p) = \Theta(p_F - p)$, not every $f(x, p)$ can be reached

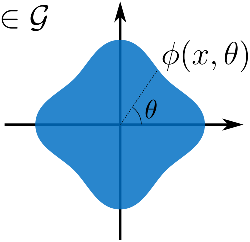


$$\partial_t f = [H, f] \qquad \Rightarrow \qquad f = e^{Ht} f_0 e^{-Ht}$$

allowed f's $\quad = \quad$ orbit of f_0 under $\mathcal{G}$

We can parametrize the degree of freedom as

$$f = U f_0 U^{-1}\,, \qquad \text{with} \;\; U = e^{\phi} \in \mathcal{G}$$



with $\phi = \phi(x, p)$

But: $\quad e^{\phi} \sim e^{\phi} e^{\alpha} \quad \Rightarrow \quad U = e^{\phi} \in \mathcal{G}/\mathcal{H}$

Can use this to choose $\phi = \phi(x, \theta)$

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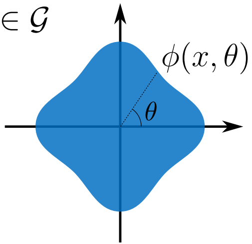

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Can use this to choose $\phi = \phi(x,\theta)$

There is a WZW term (Kirillov-Kostant-Souriau form)

$$S_{\mathrm{WZW}} = \int dt\; \mathrm{Tr}\left(f_0 U^{-1} \partial_t U\right)$$

Kirillov '78, Khveshchenko '94

Otherwise, any action that is invariant under $\mathcal{H}$ is allowed

$$S = S_{\mathrm{WZW}} - \int dt dx dp\; f(x,p,t)\,(\epsilon(p) + V(x)) + \cdots$$

In the spirit of EFT, one should include all possible terms

$$
\begin{aligned}
S[f] = S_{\mathrm{WZW}} &- \int_{txp} f(x,p,t)\,\epsilon(p) \\
&+ \int_{txp_1p_2} f(x,p_1,t)f(x,p_2,t)\,F^{(2)}(p_1,p_2) \\
&+ \int_{txp_1p_2p_3} f(x,p_1,t)f(x,p_2,t)f(x,p_3,t)F^{(3)}(p_1,p_2,p_3) \\
&+ \cdots
\end{aligned}
$$

Fluctuations

$$S \simeq -\frac{p_F}{2} \int dt d^2x d\theta \; \hat{n}(\theta) \cdot \nabla\phi \left[\dot{\phi} + v_F \left(\hat{n}(\theta) \cdot \nabla\phi \right) \right]$$

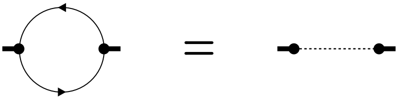
(recover Haldane '92, Castro Neto Fradkin '93, Houghton Kwon Marston '94)

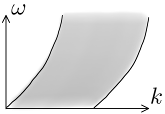
The density operator is

$$\rho(t, x) \simeq p_F \int d\theta \; \hat{n}(\theta) \cdot \nabla\phi(t, x, \theta)$$

'chiral boson on the Fermi surface'

$$\langle \rho\rho \rangle(\omega, q) = p_F \int d\theta \frac{\hat{n}(\theta) \cdot q}{\omega - v_F \hat{n}(\theta) \cdot q} = \frac{p_F}{v_F} \left[1 - \frac{|\omega|}{\sqrt{\omega^2 - v_F^2 q^2}} \right]$$



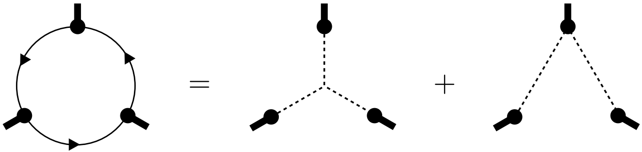


Nonlinear response

$$S_{\mathrm{WZW}} = -p_F \int_{tx\theta} \dot{\phi}\hat{n}\cdot\nabla\phi + \frac{2}{3}\frac{1}{p_F}\hat{n}\cdot\nabla\phi\left((\partial_\theta\hat{n})\cdot\nabla\phi\partial_\theta\dot{\phi}\right) + \cdots$$

$$S_H = -p_F \int_{tx\theta} \epsilon'(\hat{n}\cdot\nabla\phi)^2 + \frac{1}{3}\frac{1}{p_F}\left(\epsilon' + \epsilon'' p_F\right)\left(\hat{n}\cdot\nabla\phi\right)^3 + \cdots$$

$$\rho = p_F \int_\theta \hat{n}\cdot\nabla\phi + \frac{1}{2}\frac{1}{p_F}(\partial_\theta\hat{n})\cdot\nabla\left(\partial_\theta\phi\hat{n}\cdot\nabla\phi\right) + \cdots$$



Scaling manifest! $\langle\rho(\omega_1, q_1)\cdots\rho(\omega_n, q_n)\rangle \sim 1$

Beyond the anomaly

Nonlinear completion of the 'emergent LU(1) anomaly'

Else Thorngren Senthil '20

$$\partial_\mu j^\mu(t, \vec{x}, \theta) = \frac{\kappa}{8\pi} dA \wedge dA \,, \qquad \text{with} \quad \vec{A}(\theta) = \vec{k}_F(\theta)$$

Two issues:

- $\vec{k}_F$ is dynamical (and related to j^0)
- θ is in momentum space

What this equation really is, is a Ward identity in phase space

$$0 = \nabla_I j^I(t, \vec{x}, \vec{p}) = \partial_I j^I + \{A_I, j^I\} \,.$$

(Recover LU(1) anomaly upon linearization) LVD Du Mehta Son '22

Couple the bosonized theory to a gapless boson

$$S = -\frac{p_F}{2} \int_{tx\theta} \hat{n}(\theta) \cdot \nabla\phi \left[\dot{\phi} + v_F \left(\hat{n}(\theta) \cdot \nabla\phi\right)\right] + \lambda \int_{tx\theta} \Phi \hat{n}(\theta) \cdot \nabla\phi + \int_{tx} (\nabla\Phi)^2$$

Find $z = 3$ at tree-level

$$\langle\Phi\Phi\rangle(\omega, q) = \frac{1}{q^2 + \lambda^2 \frac{|\omega|}{\sqrt{\omega^2 - v_F^2 q^2}}}$$

Lawler Barci Fernández Fradkin Oxman '06, Chubukov Khveshchenko '06

Also produces specific heat $c_V \sim T^{2/3}$

Beyond Fermi Liquids

We know the structure of nonlinearities: they are relevant, $d_c = 3$.

Can generalize $\Phi\nabla^{1+\epsilon}\Phi$ to make $d_c = 2$ Nayak Wilczek '94

Revamp existing approaches, with an improved perturbative expansion

New perturbative approach: $d = 3 - \epsilon$.

One target: $z \neq 3$? Sachdev Metlitski '10, Holder Metzner '15

Non-perturbative statements from our Ward identity (similar to Shi Else Goldman Senthil '22)

More...

- Which phases of matter can arise from CFT + μ ? Sachdev '12
  $\rightsquigarrow$ spectrum of CFT large charge operators
  Hellerman Orlando Reffert Watanabe '15, Monin Pirtskhalava Rattazzi Seibold '16, Cuomo '19, Komargodkski Mezei Pal Raviv-Moshe '21
  $\rightsquigarrow$ UV/IR constraints as in relativistic EFTs?
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- Spinful Fermi surfaces, BCS, $2k_F$ physics

- Future directions: Fermi Surface + X

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Thanks!

extra slides

HDL

Alternatively, obtain $\langle \rho\rho\cdots\rangle$ by solving kinetic equation in the presence of a source $A_0(t,x)$

$$(\partial_t + v(p)\cdot\nabla_x + E\cdot\nabla_p)\, f(t,x,p) = 0\,, \qquad f = \Theta(p_F(t,x,\theta) - p)$$

Used in Manuel '95 to compute HDLs in QCD

(see also Blaizot Iancu '93, Kelly Liu Lucchesi Manuel '94 for HTLs)

HDL

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Scaling of n-point functions different in QCD:

The algebra is extended to $\mathfrak{g}\otimes u(N)$

$$[F,G]^c = \{F^0, G^c\} - \{G^0, F^c\} + f_{ab}{}^c F^a G^b$$

$\rightsquigarrow$ lower derivative terms in the EFT: $S_{\rm WZW} \sim \int f_{abc}\nabla\phi^a\dot{\phi}^b\phi^c$

$\rightsquigarrow$ different scaling $\langle \rho^{a_1}\cdots\rho^{a_n}\rangle \sim \frac{1}{q^{n-2}}$

Braaten Pisarski '92, Frenkel Taylor '92