Evidence for 3d Bosonization from Monopoles

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Based on: arXiv:2102.07377
and TBA with Eric Dupuis and William Witczak-Krempa

IR Duality

- IR Duality is when two quantum field theories that are different at small scales (the UV), become same CFT at large scales (the IR).
- Usually one of the CFTs is weakly coupled in the regime where the other is strongly coupled, or both are strongly coupled.
  - Classically each theory looks different, but quantum effects make them identical.
- Since at least one theory is strongly coupled, hard to check duality. All $d > 2$ cases required supersymmetry to check, e.g.
  - Original duality between 4d $\mathcal{N} = 2$ gauge theories [Seiberg '95] .
  - Generalized to dualities between 3d $\mathcal{N} = 4$ [Intrilligator, Seiberg '96] and then $\mathcal{N} = 2$ [Giveon, Kutasov '09] gauge theories.

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Duality in nature

- First experimentally relevant (i.e. non-supersymmetric) IR duality in $d > 2$ is particle/vortex duality [Peskin '78; Dasgupta, Halperin '81] :
  - QED3 with 1 complex scalar $\Leftrightarrow$ critical $O(2)$ model.
- Describes continuous transition between superfluid and Mott phase of Bose-Hubbard model at integer filling on 2d lattice.
- Compare charge q scaling dimension Δ_q from $O(2)$ lattice [Hasenbusch '20] to QED3 lattice [Kajantie et al '04, Karathik '18] :

$$O(2): \quad \Delta_0 = 1.511, \quad \Delta_{1/2} = .5191, \quad \Delta_1 = 1.236, \quad \Delta_{3/2} = 2.109$$
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Web of dualities

- All these dualities are part of web of 3d dualities [Seiberg, Senthil, Wang, Wittten '16; Karch, Tong '16] , bc given one duality can derive others by two operations:
    1. S: gauge the global symmetry of both theories.
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- Consider generalization to: $U(N_c)$ QCD3 with N scalars and CS k $\Leftrightarrow$ $SU(k)$ QCD3 with N fermions and CS $N/2 - N_c$ [Aharony '16] .
    - Many checks at leading large N_c, k and finite $N, k/N_c$ starting with correlators in [Aharony, Gur-Ari, Yacoby '12; Giombi, Minwalla, Prakash, Trivedi, Wadia, Yin '12] .
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- Break supersymmetric version of duality [Giveon, Kutasov '09] to flow to non-susy seed duality [Gur-Ari, Yacoby '15] .
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- Define monopole operators in QED3.
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- Define monopole operators in QED3.
- Describe large N, k calculation of scaling dimension to sub-leading order.
- Compare to operators in dual theories ($O(2)$ dual to $k = 0$, free fermion dual to $k = 1$), find precise match after extrapolating N, k.

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Conformal QED3

- QED3 with N complex ϕ_i and Chern-Simons level k has action:

$$\int d^3x\left[\frac{F_{\mu\nu}F^{\mu\nu}}{4e^2}+\frac{\sigma^2}{4\lambda}+|(\nabla_\mu-iA_\mu)\phi^i|^2+(\frac{1}{4}+i\sigma)|\phi^i|^2-\frac{ik}{4\pi}\epsilon^{\mu\nu\rho}A_\mu\partial_\nu A_\rho\right],$$

  - σ is real scalar Hubbard-stratonovich for ϕ^4 term.
  - k must be integer, when $k=0$ call it CP^{N-1} model.
- At large N, can show that theory flows to interacting CFT in the IR [Appelquist, Nash, Wijewardhana '88] , believed to hold at finite N except maybe $N=2$ and $k=0$.
- $e,\lambda\to\infty$ when we flow to IR, bc F^2 and σ^2 are irrelevant.
- Can construct operators from ϕ_i, σ, and A_μ in irreps of $SU(N)$ flavor symmetry, compute correlators at large N using Feynman diagrams [Halperin, Lubetsky, Ma '74; Kaul, Sachdev '08] .

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Monopole operators

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- Monopole operator M_q defined as having charge q under $U(1)_T$, s.t. $\int_{S^2} F = 4\pi q$.
  - Dirac quantization condition requires $q \in \mathbb{Z}/2$.
- When $k = 0$, M_q are scalars and singlets under $SU(N)$.
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- Can compute scaling dimensions Δ_q at large N. Hard on $\mathbb{R}^3$ [Murthy, Sachdev '90] , instead use state-operator correspondence [Borokhov, Kapustin, Wu '02] .

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Δ_q from $S^2 \times \mathbb{R}$

- The state-operator correspondence relates M_q on $\mathbb{R}^3$ to state on S^2 Hilbert space with $4\pi q$ magnetic flux, s.t. Δ_q given by energy on $S^2 \times \mathbb{R}$ with $4\pi q$ flux [Borokhov, Kapustin, Wu '02] .
- Chern-Simons term contributes $2qk$ to Gauss law constraint, so need to dress vacuum with matter to make gauge invariant.
- Consider thermal free energy $F_q \equiv \frac{-\log Z}{\beta}$ on $S^2 \times S^1_\beta$ with $4\pi q$ flux, where $\beta \equiv 1/T$ is length of S^1 [SMC, Iliesiu, Mezei, Pufu '17] .
- After integrating out matter, can compute F_q from large N saddle point, s.t. holonomy of gauge field acts as chemical potential for matter fixed by saddle condition to cancel gauge charge.
- Bonus: Subleading in $1/\beta$ terms in F_q tell us degeneracy of states $\Rightarrow$ irreps of monopole operator.

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- Integrate out ϕ to get action proportional to N:

$$Z = \int dA\, d\sigma\, e^{N \mathrm{Tr} \log[\sigma + \frac{1}{4} - (\nabla_\mu - iA_\mu)^2] + N \frac{i\kappa}{4\pi} \int d^3x \epsilon^{\mu\nu\rho} A_\mu \partial_\nu A_\rho} \,.$$

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- Plug in to Z to compute leading large N free energy $NF_q^{(0)}(\alpha, \mu)$

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$$Z = \int dA\, d\sigma\, e^{N \mathrm{Tr} \log[\sigma + \frac{1}{4} - (\nabla_\mu - iA_\mu)^2] + N \frac{i\kappa}{4\pi} \int d^3x \epsilon^{\mu\nu\rho} A_\mu \partial_\nu A_\rho}\,.$$

- Consider most general saddle point $A_\mu = \mathcal{A}_\mu + \tilde{A}_\mu$ and $\sigma = \mu + i\tilde{\sigma}$ on $S^2 \times S^1_\beta$ s.t. $\int_{S^2} \mathcal{F} = 4\pi q$:

$$\mathcal{A}_\tau \equiv -i\alpha = \beta^{-1} \int_{S^1_\beta} A\,, \qquad \mathcal{F}_{\theta\phi} d\theta \wedge d\phi \equiv q \sin\theta d\theta \wedge d\phi\,,$$

- Plug in to Z to compute leading large N free energy $NF_q^{(0)}(\alpha, \mu)$

$$\begin{aligned} F_q^{(0)}(\alpha,\mu) &= \beta^{-1} \mathrm{Tr} \log \left[-(\nabla_\mu - i\mathcal{A}_\mu)^2 + \frac{1}{4} + \mu \right] - 2\kappa q \alpha\,, \\ &= \beta^{-1} \sum_{j \geq q} d_j \log[2(\cosh(\beta\lambda_j) - \cosh(\beta\alpha))] - 2\kappa q \alpha \end{aligned}$$

- λ_j, d_j are eigenvalues and degeneracies on $S^2 \times \mathbb{R}$ with $4\pi q$ flux.

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Saddle point values

- Holonomy α and μ are constants determined from saddle point equations:

$$\frac{\partial F_q^{(0)}(\alpha,\mu)}{\partial\alpha}\Big|_{\alpha,\mu} = \frac{\partial F_q^{(0)}(\alpha,\mu)}{\partial\mu}\Big|_{\alpha,\mu} = 0\,.$$

- For α we find up to $O(e^{-\beta})$ a unique saddle that gives real F_q:

$$\alpha(\kappa) = -\operatorname{sgn}(\kappa)\left(\lambda_q + \beta^{-1}\log\frac{\xi}{1+\xi}\right)\,,\qquad \xi \equiv \frac{2q|\kappa|}{d_q}\,.$$

- For μ, given by solution to equation (regularize with zeta functions):

$$\sum_{j\geq q}\frac{d_j}{\lambda_j(\mu)} + \frac{\xi d_q}{\lambda_q(\mu)} = 0\,,\qquad \lambda_j \equiv \sqrt{(j+1/2)^2 - q^2 + \mu}\,,\qquad d_j \equiv 2j+1\,.$$

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Leading order free energy

- Plug α, μ into $F_q^{(0)}$ to get final answer

$$F_q = NF_q^{(0)} + F_q^{(1)} + \dots, \qquad F_q^{(0)} = \Delta_q^{(0)} - \frac{1}{\beta} S_q^{(0)} + O(e^{-\beta}).$$

- The energy $N\Delta_q^{(0)}$ is the monopole scaling dimension by state-operator correspondence, and entropy $S_q^{(0)}$ is log# of operators with $\Delta_q^{(0)}$ at large N:

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Sub-leading calculation

- Subleading $F_q^{(1)}$ from 2nd order in fluctuations around saddle:

$$\exp(-\beta F_q^{(1)}) = \int D\tilde{A}D\tilde{\sigma}\,\exp\Bigg[-\frac{N}{2}\int d^3x d^3x'\,\Big(\tilde{A}_\mu(x)K_q^{\mu\nu}(x,x')\tilde{A}_\nu(x') + \tilde{\sigma}(x)K_q^{\sigma\sigma}(x,x')\tilde{\sigma}(x') + 2\tilde{\sigma}(x)K_q^{\sigma\nu}(x,x')\tilde{A}_\nu(x')\Big)\Bigg],$$

  - Consider ratio $\frac{\exp(-\beta F_q^{(1)})}{\exp(-\beta F_0^{(1)})}$ to cancel divergent gauge modes.

- The kernels $K_q(x,x')$ are written terms of the Green's function:

$$\left[-(\nabla_\mu - i\mathcal{A}_\mu)^2 + \frac{1}{4} + \mu\right] G_q(x,x') = \delta(x-x')\,.$$

  - Computed for general q in [SMC, Iliesiu, Mezei, Pufu '17] in terms of infinite sum of monopole spherical harmonics.
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Microcanonical interpretation

- We would like to explain our results using an oscillator construction, which is only valid in the UV at $e^2 N \to 0$.

- Not obvious that such a construction should remain valid for the IR CFT at $e^2 N \to \infty$. Evidence for conjecture from thermal results.

- For scalar QED_3, expand ϕ_I in modes on Lorentzian $S^2 \times \mathbb{R}$:

	energy	spin	gauge charge	$SU(N)$ irrep	degenera
$a^{i,\dagger}_{jm}$	λ_j	j	$+1$	$\mathbf{N}$	Nd_j
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M_{bare}	$N\sum_j d_j\lambda_j$	0	$2qN\kappa$	$\mathbf{1}$	1

  - M_{bare} is vacuum in presence of $4\pi q$ flux.
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Microcanonical interpretation: Leading order

Recall:
$$N\Delta_q^{(0)} = N\Big[\sum_{j\geq q} d_j\lambda_j + \xi d_q\lambda_q\Big]\,,$$
$$NS_q^{(0)} = N[-d_q\,(\xi\log\xi - (1+\xi)\log[1+\xi])]\,.$$

- First term in $N\Delta_q^{(0)}$ is Casimir energy of M_{bare}, second are $N\xi d_q$ lowest energy λ_q modes needed to cancel gauge charge of M_{bare}.
- These $N\xi d_q$ modes each in the fundamental of $SU(N)$, together form many degenerate $SU(N)\times SU(2)_{\text{rot}}$ irreps. E.g.:

$$d_q = 2: \qquad \bigoplus_{\ell=0}^{N\xi}(\mathbf{R}_\ell, \mathbf{2\ell+1})\,, \qquad \mathbf{R}_\ell \equiv$$

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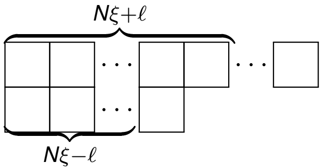

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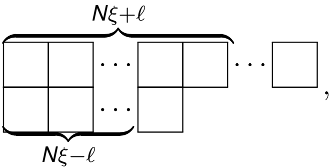

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- Conjecture continuous density of states $C(E - E_0)^\alpha$ at large N:

$$F = -\frac{\log \int dE \mathcal{D}(E) e^{-\beta E}}{\beta} = E_0 + (\alpha+1)\frac{\log\beta}{\beta} - \frac{\log(C\Gamma(\alpha+1))}{\beta} + O(\beta^{-2})$$

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$$\mathcal{D}(E) \approx \frac{e^{NS_1^{(0)}}}{N^{1/2}\pi\xi^2(1+\xi)^2 C_{1/2,1}^{3/2}}(E - N\Delta_{1/2}^{(0)} - \Delta_{1/2}^{(1)})^{1/2}$$

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$$\mathcal{D}(E) \approx \frac{(2\ell+1)\dim\mathbf{R}_\ell}{\Delta E_\ell} \approx \frac{2e^{NS_1^{(0)}}}{N^{1/2}\pi\xi^2(1-\xi)^2}y^2 e^{-\frac{1}{\xi(1-\xi)}y^2}\frac{dy}{dE}.$$

- Solve to get energy splitting $E \approx N\Delta_{1/2}^{(0)} - \Delta_{1/2}^{(1)} + C_{1/2,1}y^2$.

Microcanonical interpretation: Subleading order

- Conjecture continuous density of states $C(E-E_0)^\alpha$ at large N:

$$F=-\frac{\log\int dE\mathcal{D}(E)e^{-\beta E}}{\beta}=E_0+(\alpha+1)\frac{\log\beta}{\beta}-\frac{\log(C\Gamma(\alpha+1))}{\beta}+O(\beta^{-2})$$

- Compare to β^{-1} terms from F_q (set $d_q=2$ for simplicity) we find

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Comparison of Δ_q to duality: particle/vortex

- Scalar QED3 with $N = 1$ and $k = 0 \Leftrightarrow$ critical $O(2)$ Wilson Fisher.
- $M_q \Leftrightarrow$ lowest dimension operator made of $2q$ complex bosons ϕ:
  - $M_{1/2} \Leftrightarrow \phi$, and $M_1 \Leftrightarrow \phi\phi$, and $M_{3/2} \Leftrightarrow \phi\phi\phi$.
- All these operators are unique scalars, so no degeneracy breaking terms in monopole calculation.
- $O(2)$ operators computed for $q \leq 2$ at high precision from numerical bootstrap [SMC, Landry, Liu, Poland, DSD, Su, Vichi '20; Liu, Meltzer, Poland, DSD '20] .
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q	$\Delta^{(0)}_{q,0}$	$\Delta^{(1)}_{q,0}$	$N = 1$	$O(2)$	Error (%)
$1/2$	0.12459	0.38147	0.50609	0.519130434	2.5
1	0.31110	0.87452	1.1856	1.23648971	4.1
$3/2$	0.54407	1.4646	2.0087	2.1086(3)	4.7
2	0.81579	2.1388	2.9546	3.11535(73)	5.2
$5/2$	1.1214	2.8879	4.0093	4.265(6)	5.8
3	1.4575	3.7053	5.1628	5.509(7)	6.3
$7/2$	1.8217	4.5857	6.4074	6.841(8)	6.3
4	2.2118	5.5249	7.7367	8.278(9)	6.5
$9/2$	2.6263	6.5194	9.1458	9.796(9)	6.6
5	3.0638	7.5665	10.630	11.399(10)	6.7

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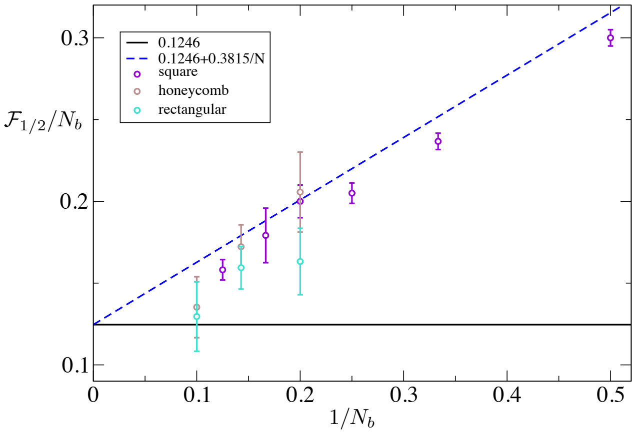
Comparison to lattice for $N > 1$ and $k = 0$



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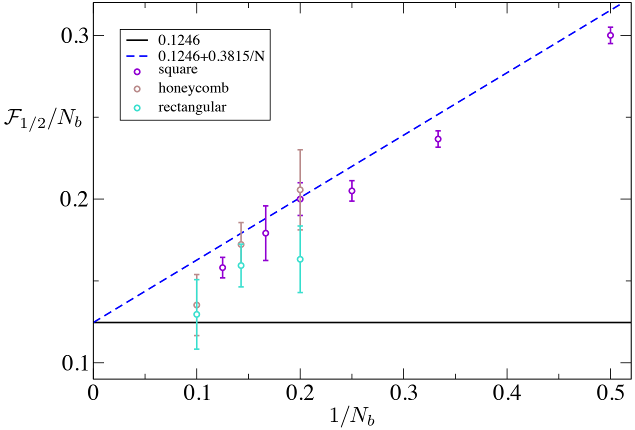

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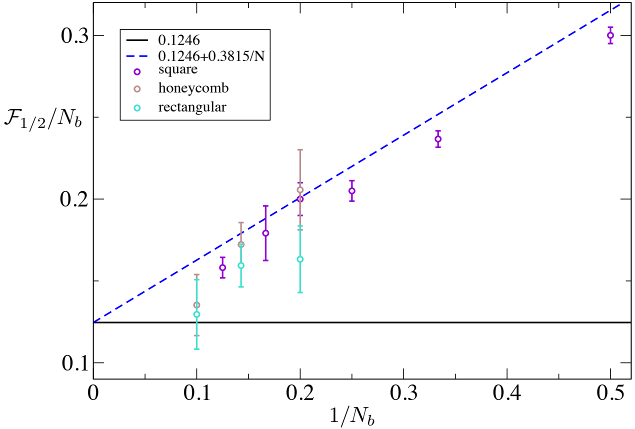

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Comparison of Δ_q to duality: 3d bosonization

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- For higher q need to dress with derivatives bc of antisymmetry, so degenerate operators with same q and dimension, e.g. for $q = 2$:
  1. $\epsilon^{\alpha\beta}\psi_\alpha\psi_\beta\epsilon^{\gamma\delta}\partial_\mu\psi_\gamma\partial_\nu\psi_{\gamma\delta}$ has $\Delta = 6$ and spin 2.
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Operators in free fermion theory

- We can determine spectrum of free fermion theory by looking at free energy on $S^2 \times \mathbb{R}$ in presence of background $U(1)$ flux q.
- Fermionic modes of spin $j = 1/2, 3/2, \ldots$ have eigenvalue $\lambda_j = j + 1/2$, charge 1/2, and $2j + 1$ in each energy shell.
- Operators with charge q that correspond to states of n filled energy shells are unique scalars have charge and dimension:

$$q = \sum_{j=1/2}^{n-1/2} (2j+1) = n(n+1)/2\,, \qquad \text{e.g.} \quad q = 1, 3, 6, 10, \ldots$$

$$\Delta = \sum_{j=1/2}^{n-1/2} (2j+1)\lambda_j = \frac{2}{3} q\sqrt{1+8q}\,, \quad \text{e.g.} \quad \Delta = 2, 10, 28, 60, \ldots$$

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$3/2$	4.5873	-1.052	3.535	4	15
2	6.9380	-1.534	5.404	6	9.9
$5/2$	9.5904	-2.070	7.52	8	6.0
3	12.514	-2.655	9.859	10	1.4
6	34.727	-7.032	27.70	28	1.1
10	74.141	-14.71	59.43	60	0.95
15	135.67	-26.63	109.04	110	0.87
21	224.23	-43.75	180.5	182	0.82

- Purple are unique scalar operators (i.e. filled energy shells)
- Find match for unique scalars, that improves with q.

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- Purple are unique scalar operators (i.e. filled energy shells)
- Find match for unique scalars, that improves with q.

Evidence for 3d bosonization from monopoles

q	$\Delta_{q,1}^{(0)}$	$\Delta_{q,1}^{(1)}$	$N=1$	Fermion	Error (%)
$1/2$	1	-0.2789	0.7211	1	28
1	2.5833	-0.6312	1.952	2	2.4
$3/2$	4.5873	-1.052	3.535	4	15
2	6.9380	-1.534	5.404	6	9.9
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Match for other q

- Operators in free fermion theory that NOT unique scalars do not match our monopole calculation (tho mismatch shrinks with q).
- This could be because of the degeneracy breaking term in the large N calculation, that we have not taken into account.
- If we take $\Delta_q^{\text{free}} = \frac{2}{3}q\sqrt{1+8q}$ of unique scalars in free fermion theory, which only valid for $q = 1, 3, 6, \ldots$, and analytically continue to general q then we get precise match now for all q:

$$\Delta_{1/2}^{\text{ferm}} = .7454, \ \Delta_{3/2}^{\text{ferm}} = 3.606, \ \Delta_{2}^{\text{ferm}} = 5.498,$$
$$\Delta_{1/2}^{\text{mono}} = .7211, \ \Delta_{3/2}^{\text{mono}} = 3.535, \ \Delta_{2}^{\text{mono}} = 5.404,$$

- Suggests that large N calculation might correspond to effective large q theory, which only applies to unique scalars but is analytic in q [Komargodski, Mezei, Pal, Raviv-Moshe '21] .

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Conclusion

- Computed scaling dimensions of monopoles in QED3 with N scalars and CS k at large N, k and fixed $\kappa \equiv k/N$ to sub-leading order.
  - Generalized previous results for $\kappa = 0$.
- Extrapolating to $N = 1$ and $\kappa = 0$ matches operators in critical $O(2)$ model, first check of particle-vortex duality for charged operators.
- Extrapolating to $N = 1$ and $\kappa = 1$ matches operators in free fermion theory, first dynamical check of 3d bosonization!
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Future directions

- Improve large N calculation of monopoles for non-unique scalars to get match to free fermion theory.
  - Check how higher orders in $1/N$ contribute.
- Derive analytic proof of 3d bosonization at large charge, hinted by our answer
- Generalize to other 3d gauge theories at large N, k and fixed $\kappa \equiv k/N$, e.g.:
  - QED3 with N fermions, use to check duality between QED3 with $N = 1$ fermion and $k = 1/2$, and critical $O(2)$ model.
  - $\mathcal{N} = 1$ SQED, check dualities in that case [Benini, Benvenuti '18] .
  - QCD3 with general finite rank gauge group ($\kappa = 0$ already done in [Dyer, Mezei, Pufu '15]), check other dualities e.g. [Aharony, Benini, Hsin, Seiberg '17] .

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