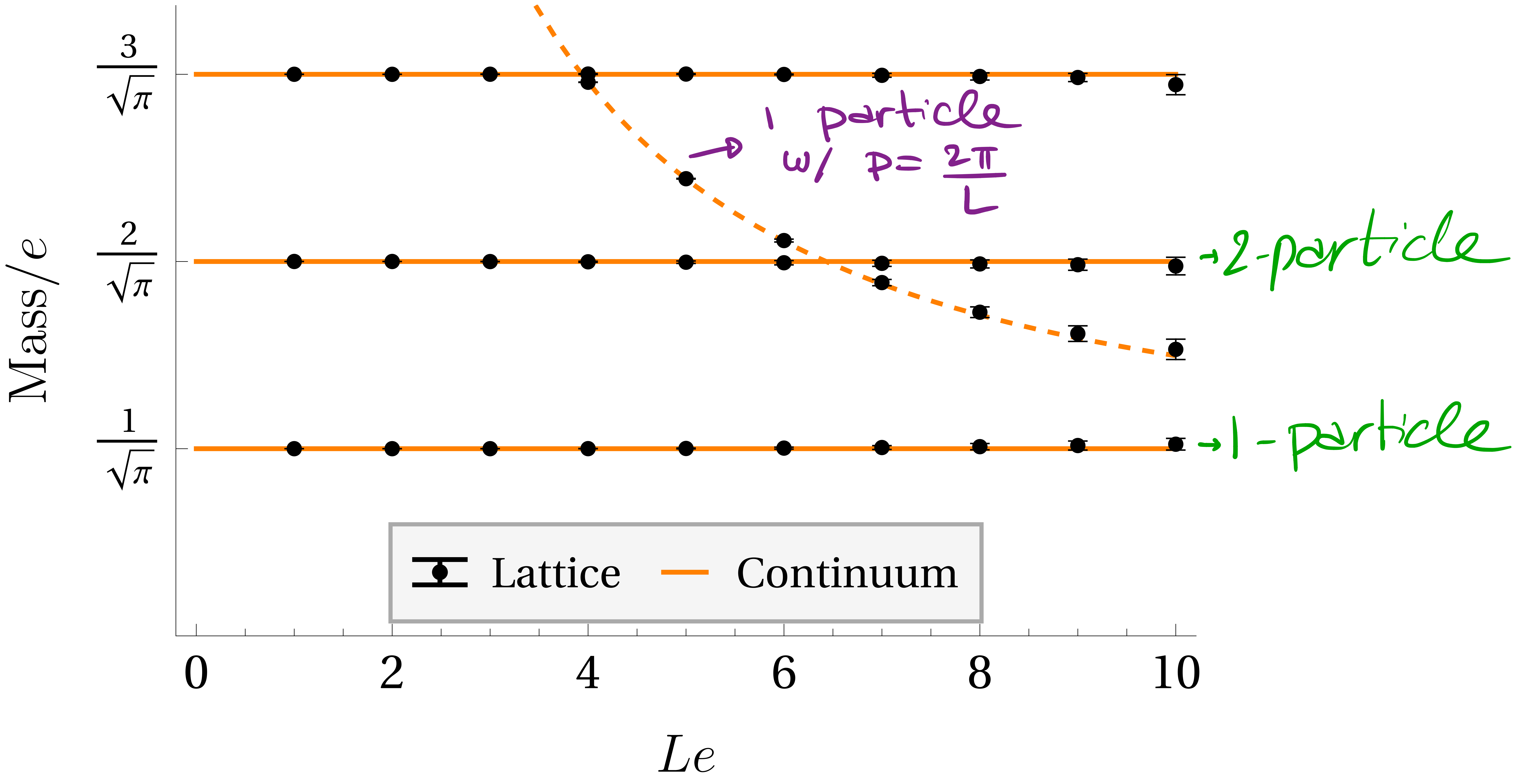
PBC : finite $N \to \infty$ number of states

Truncate : $\mathcal{E} = \frac{1}{N} \sum_n L_n \in [-\mathcal{E}_{MAX}, \mathcal{E}_{MAX}]$

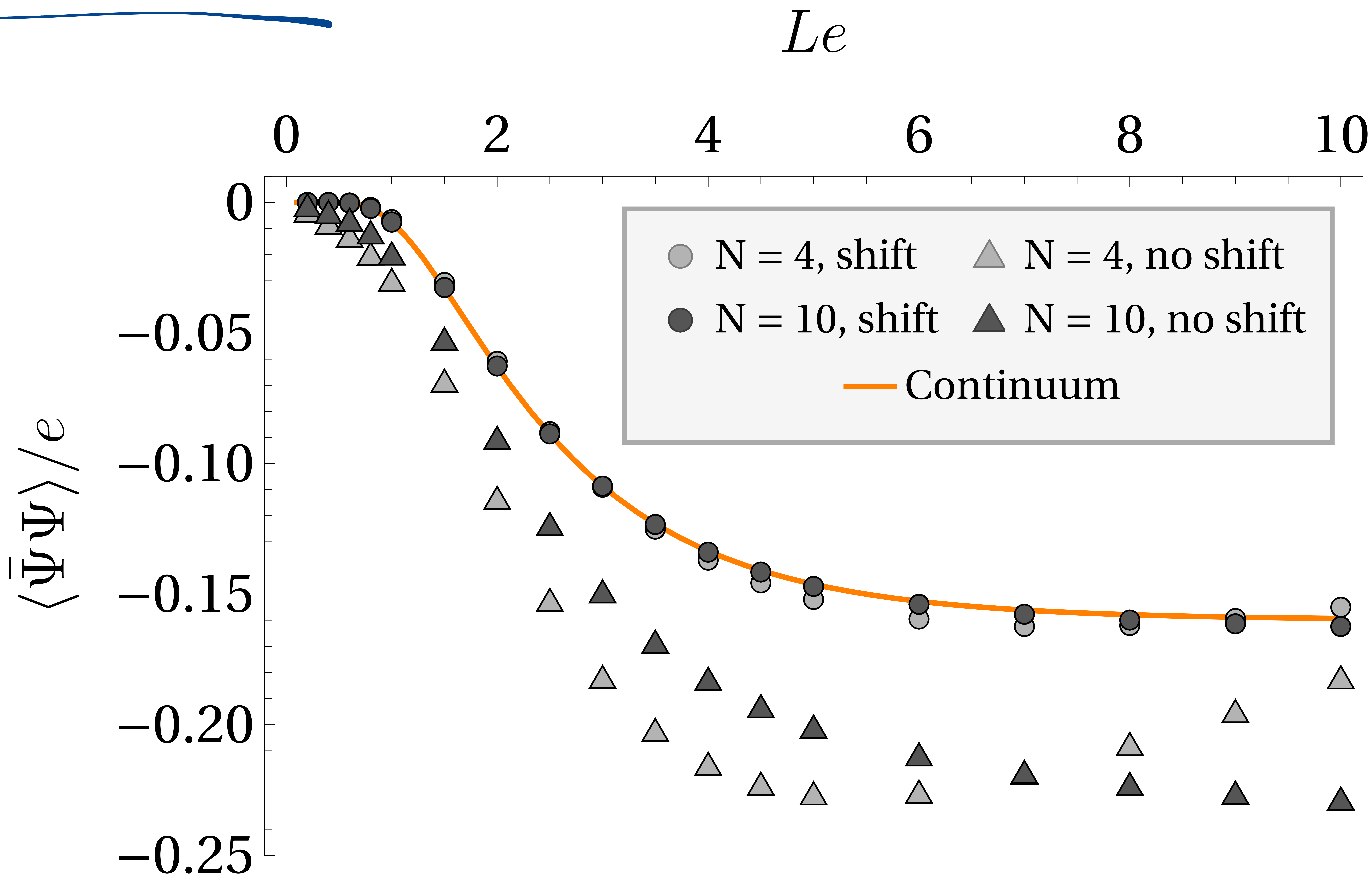
For us $\mathcal{E}_{MAX} = 5.5$, $N = 2, \dots, 18$.

$m=0$ spectrum



$\frac{3}{\sqrt{\pi}}$
$\frac{2}{\sqrt{\pi}}$
$\frac{1}{\sqrt{\pi}}$
Mass/e
1 particle
w/ $p=\frac{2\pi}{L}$
→2-particle
→1-particle
Lattice
Continuum
0
2
4
6
8
10
Le

$m=0, \quad \langle\bar{\psi}\psi\rangle$



Phase transition at $\theta = \pi$

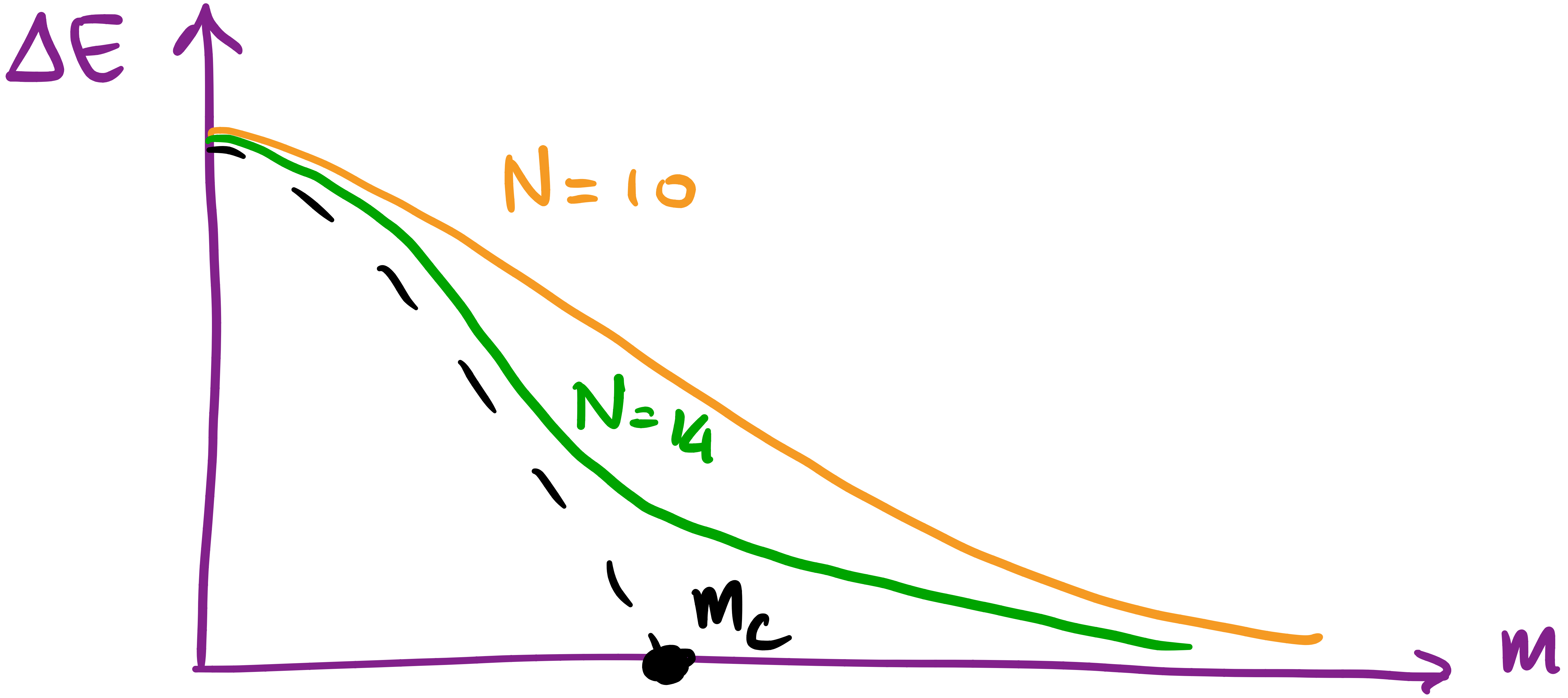
$$\Sigma_{eff} \equiv \Sigma + \frac{\theta}{2\pi}$$

$m \to -\infty$, one vacuum

$m \to \infty$, two vacuums , $\langle \Sigma_{eff} \rangle = \pm \frac{1}{2}$

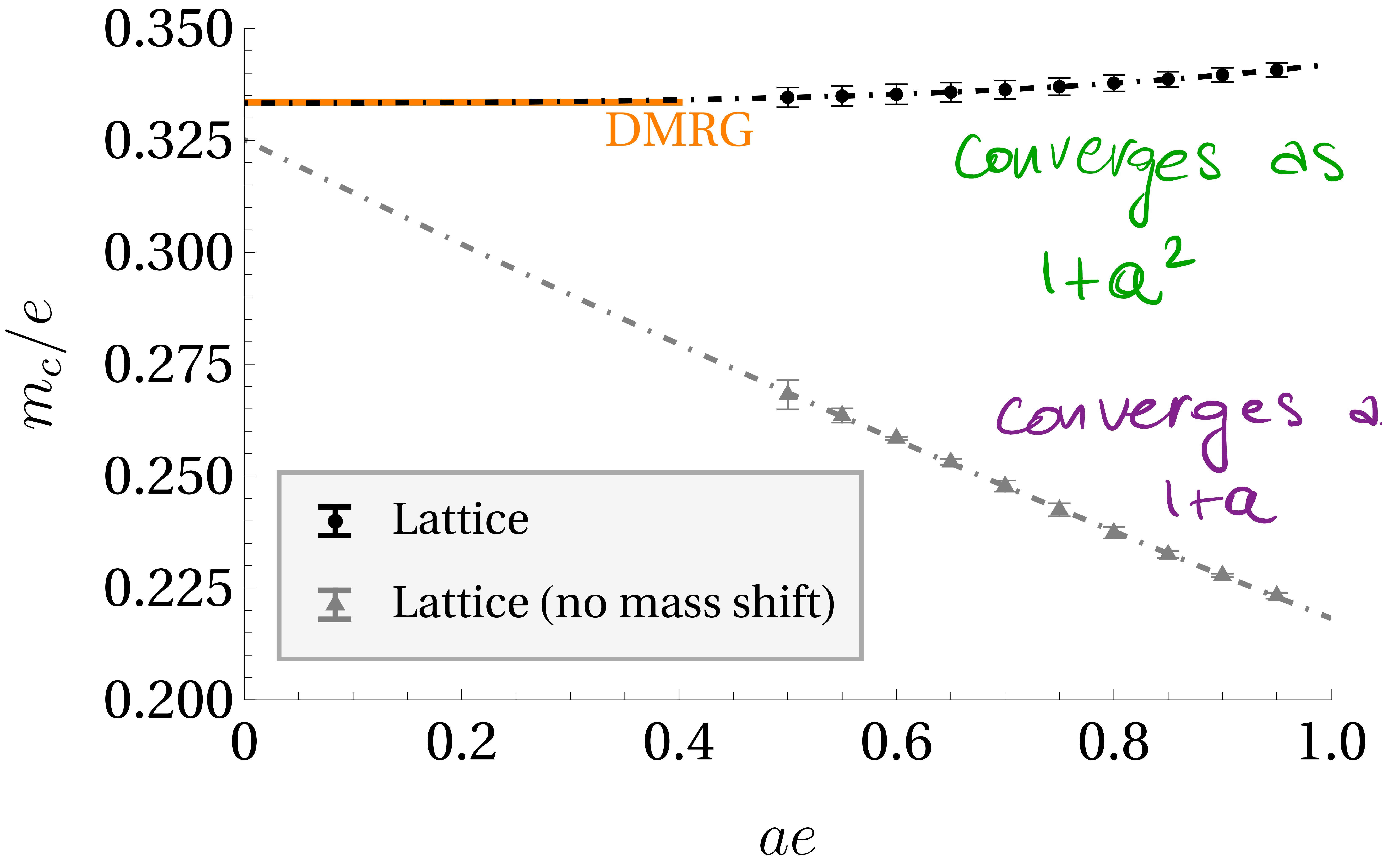
$\mathbb{Z}_2$ breaking $\to$ Ising

Qualitatively



Critical mass

Fit



Converges as $1+a^2$

Converges as $1+a$

$\rightarrow \frac{m_c}{e} = 0.333(5)$

$\rightarrow \frac{m_c}{e} = 0.325(20)$

Best way to get these results : MPS/DMRG

Previous best result

$$m_{c}/e = 0.3335(2)$$

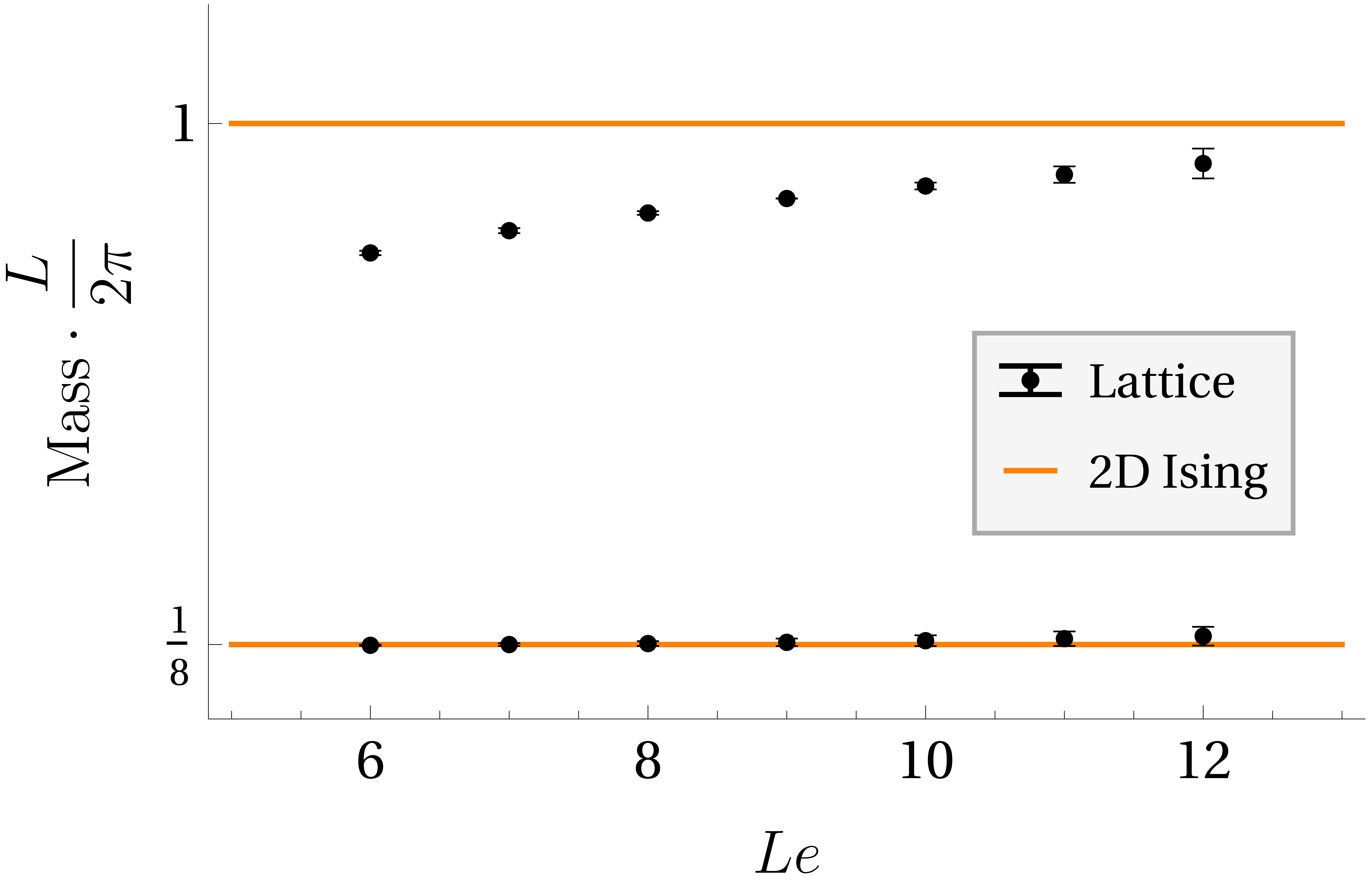
T. Byrnes, P. Sriganesh, R. J. Bursill, and C. J. Hamer, "Density matrix renormalization group approach to the massive Schwinger model," *Nucl. Phys. B Proc. Suppl.* **109** (2002) 202–206, hep-lat/0201007.

Our MPS result

$$m_{c}/e = 0.33334(3)$$

¿is it 1/3?

Mass spectrum at m_c : Ising universality class.



(Exact when $L \to \infty$)

Other models:

Charge $-q$ Schwinger model

$$Q_n \to q Q_n$$

$$\chi_n^\dagger U_n \chi_{n+1} \to \chi_n^\dagger (U_n)^q \chi_{n+1}$$

The numerics are the same!

$$e \to eq$$

$$\theta \to \theta/q$$

Mass shift

$$m_{lat} = m - q^2 \frac{e^2 a}{8}$$

But, different symmetry interpretation

$$V H_\theta V^{-1} = H_{\theta + q\pi}$$

q even $\rightarrow$ actual $\mathbb{Z}_2$ symmetry

Continuum

$$\mathbb{Z}_q^{(1)} \times \mathbb{Z}_q$$

$\longrightarrow$

Lattice

$$\mathbb{Z}_q^{(1)} \times \mathbb{Z}_2$$

One form symmetry $\mathbb{Z}_q^{(1)}$

Topological operator $W = \exp\left(\frac{2\pi i}{q} L_n\right)$

$WHW^{-1} = H \longrightarrow q$ different universes

→ If q is even $VWV^{-1} = -W$

Shift by 1 site takes us from

$m^{th} \rightarrow \left(m + \frac{q}{2}\right)^{th}$ universe : they are degenerate.

$\hookrightarrow \mathbb{Z}_q^{(1)} \times \mathbb{Z}_2$

Multi-flavor Schwinger model

N_f flavors $\rightarrow$ $SU(N_f)$ global symmetry

Again $V H_\theta V^{-1} = H_{\theta + N_f \pi}$

$N_f = 2$ $\longrightarrow$ true symmetry

Mass shift improves the numerics greatly!

Non-abelian gauge theories

Gauss law is not staggered

→ no mass shift

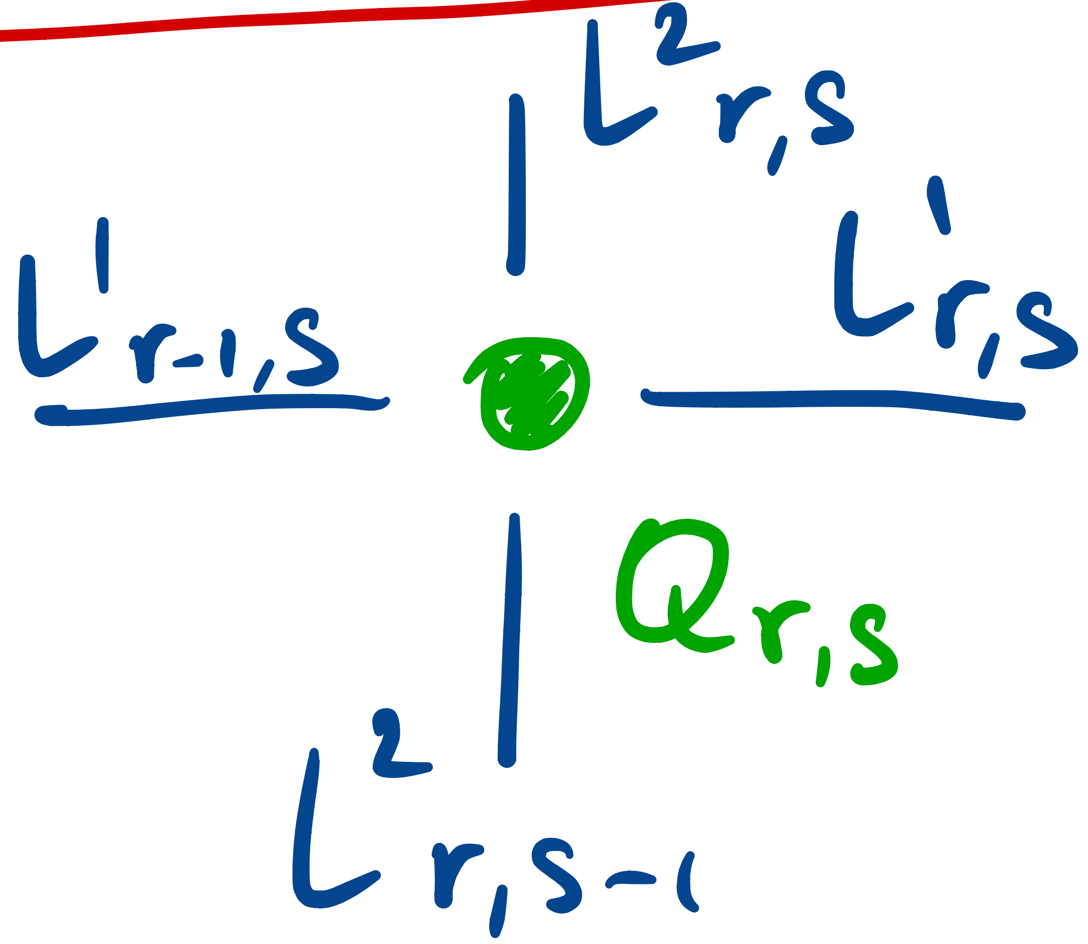
Can still use MPS methods

SU(2), fundamental fermions → [Bañuls et al., 2017]

SU(3), " " → [Silvi et al., 2017]

Abelian gauge theories in 2+1 d

Gauss law :



$$L^1_{r,s} - L^1_{r-1,s} + L^2_{r,s} - L^2_{r,s-1} = Q_{r,s}$$

$$Q_{r,s} = \chi^\dagger_{r,s}\,\chi_{r,s} - \frac{1-(-1)^{r+s}}{2}$$

Need mass shift!