Conformal Field Theory and Chiral Melting in 2D

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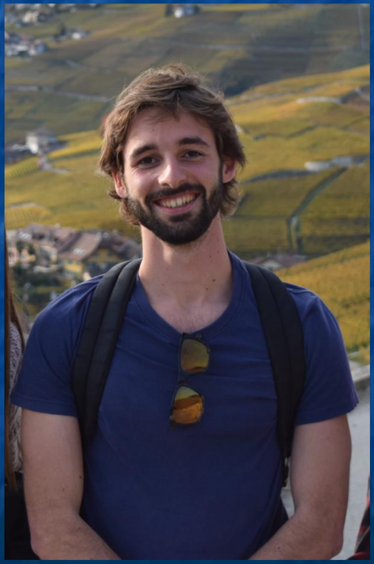

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Delft

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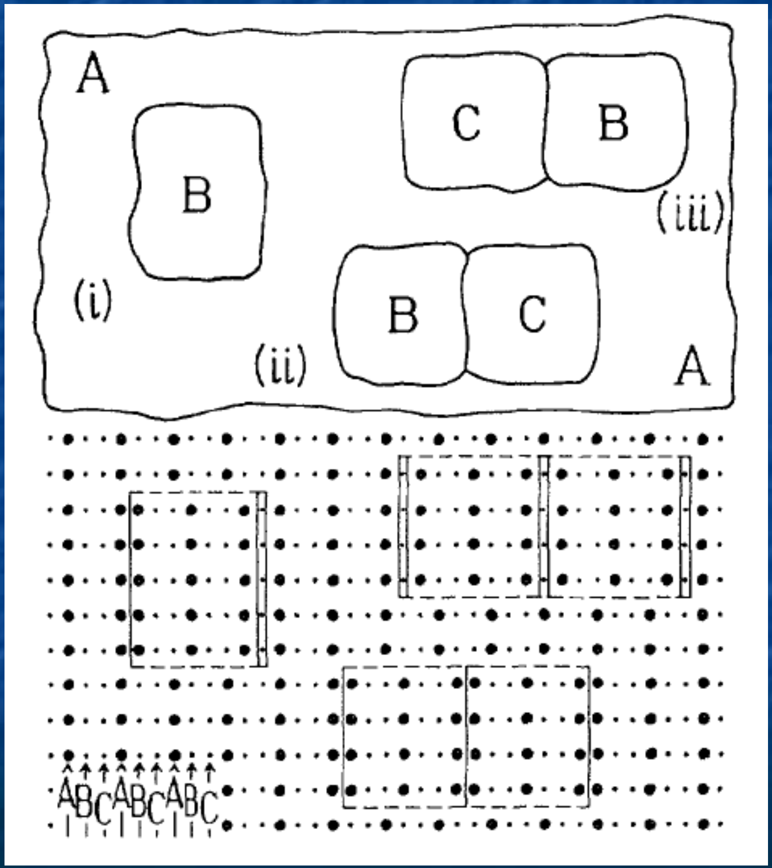
Scope

- A classical unsolved problem of the eighties
  → C-IC transition in 2D period-3 systems
- Quantum 1D version: Rydberg chains (2017)
  → DMRG investigation of constrained models
  → Chiral transition
- Chiral 3-state Potts model in 2D
  → CTMRG phase diagram
- Chiral transition and c-theorem
- Conclusions

Generalities on phase transitions

- Scale invariance at critical point (1965)
  → correlation length diverges with exponent ν
- Conformal invariance (1970)
  → exponent ν the same in all directions
- Conformal invariance in 2D (1984)
  → minimal models with well defined exponents
- Classical physics in 2D <=> quantum physics in 1D
  → quantum transitions in 1D belong to 2D universality classes

C-IC transition in adsorbed layers



3 types of domains
→ 3-state Potts?

Not so simple!

A B C ≠ A C B

Chiral perturbation

Huse-Fisher, 1982

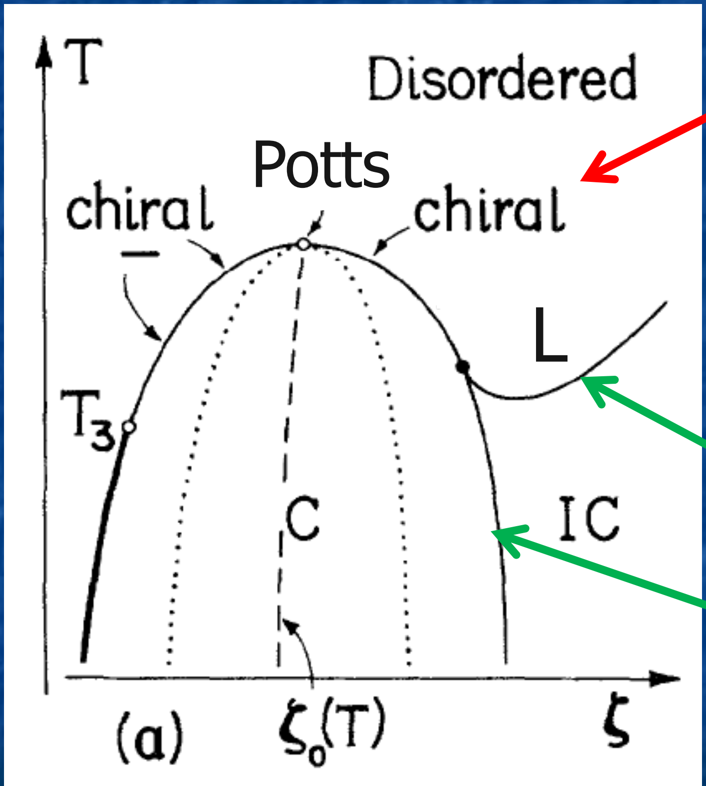
Chiral 3-state Potts in 2D

$$\mathscr{H} = -J_x \sum_{\langle ij \rangle}^{x} \cos\left[\frac{2\pi}{3}(n_i - n_j + \Delta)\right] - J_y \sum_{\langle ij \rangle}^{y} \cos\left[\frac{2\pi}{3}(n_i - n_j)\right]$$

n_i=0,1 or 2

- Huse-Fisher: possibility of a non-conformal chiral transition between a Potts point and a Lifshitz point
- Anisotropic scaling
  - → $\nu_x \neq \nu_y$ (correlation length exponents)
  - → dynamical exponent z >1
- Intermediate (floating) critical phase beyond Lifshitz point

Huse-Fisher phase diagram



Chiral: Anisotropic scaling

→ $\nu_x \neq \nu_y$

→ dynamical exponent z >1

Kosterlitz-Thouless

Pokrovsky-Talapov

Huse and Fisher, 1982

Landau-Ginsburg theory

Order parameter

$$\psi_j = \exp(2\pi i n_j / p)$$

Chiral term

$$\mathscr{H}[\psi(\vec{\mathrm{r}})] = \int d^d r \left[\tfrac{1}{2} \left| \vec{\nabla} \psi(\vec{\mathrm{r}}) \right|^2 + u_2 \left| \psi(\vec{\mathrm{r}}) \right|^2 + \Delta \left(\psi^*(\vec{\mathrm{r}}) \frac{\partial \psi(\vec{\mathrm{r}})}{\partial x} - \psi(\vec{\mathrm{r}}) \frac{\partial \psi^*(\vec{\mathrm{r}})}{\partial x} \right) \right.$$
$$\left. - h_p [\psi^p(\vec{\mathrm{r}}) + \psi^{*p}(\vec{\mathrm{r}})] + \sum_{k=2}^{\infty} u_{2k} \left| \psi(\vec{\mathrm{r}}) \right|^{2k} \right],$$

CFT point of view

- 3-state Potts universality class when there is no chiral pertubation

- Operators with scaling dimensions (7/5,2/5) and (2/5,7/5), hence with overall scaling dimension 9/5 < 2

  → relevant perturbations

Difference between the 3 cases

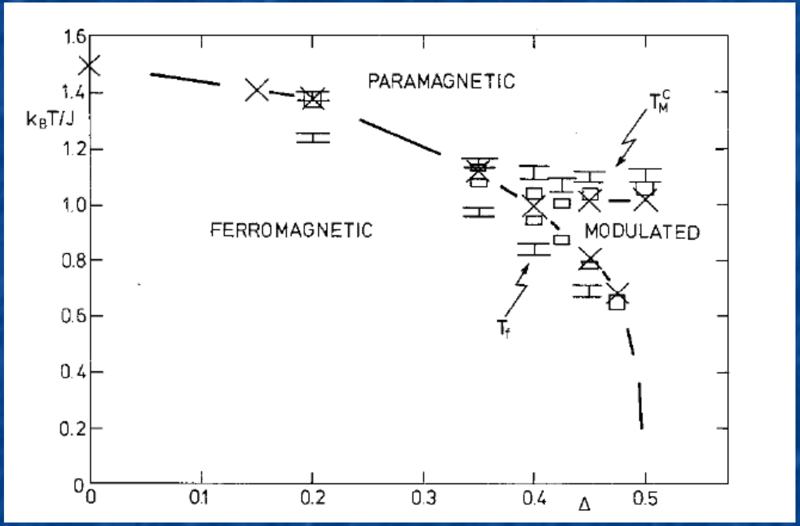
- Δq: distance to $2\pi/3$; ξ : correlation length
- $\Delta q\ \xi \ \rightarrow 0$ for Potts

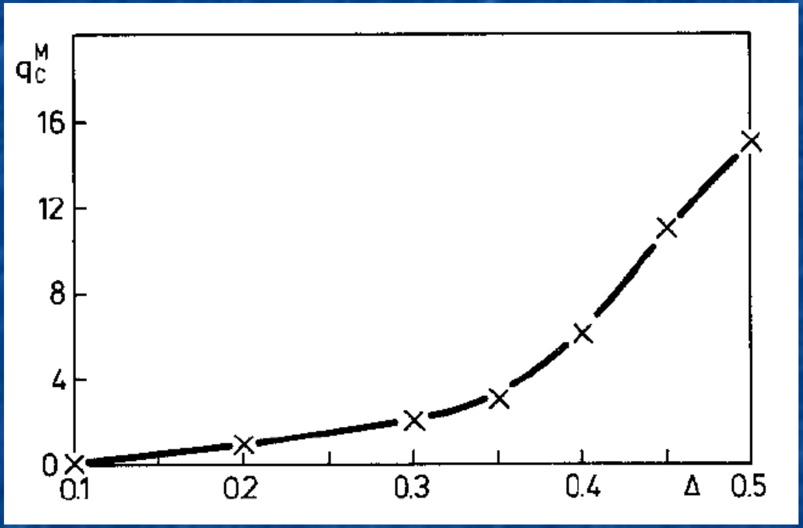
  $\rightarrow$ cst > 0 for chiral

  $\rightarrow + \infty$ for KT transition
- Monte Carlo simulations in the eighties

  $\rightarrow$ systems too small to extract Δq with sufficient precision Selke and Yeomans, 1982

MC simulations of asymmetric Potts model



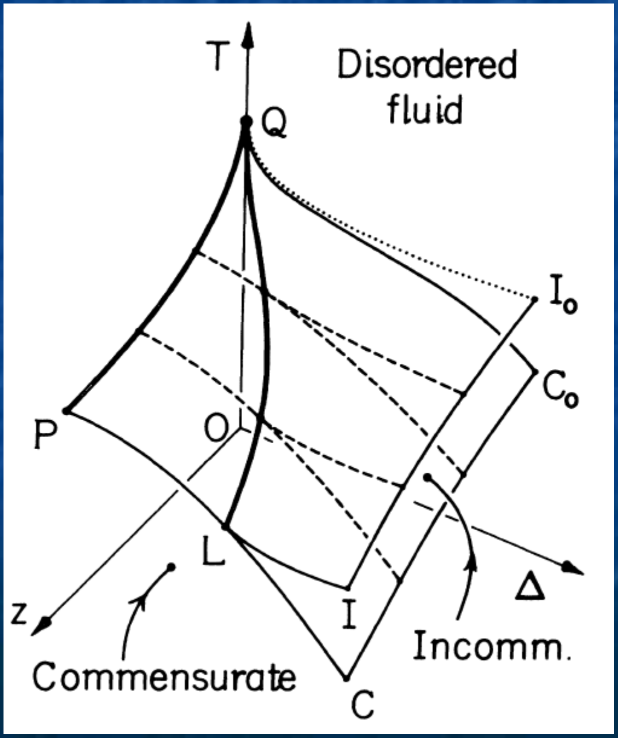


Selke and Yeomans, 1982

Field theory argument

Intermediate phase immediately away from Potts
(mapping on quantum sine-Gordon model)

Haldane, Bak, Bohr, 1983; Schulz, 1983



Not necessarily true
if dislocations are allowed

Huse-Fisher, 1984

Cardy’s self dual model

$$S = -\sum_{x,y} \left(K_x \cos(\Delta_x \theta_{x,y} - \delta_x) + K_y \cos(\Delta_y \theta_{x,y} - \delta_y) \right)$$

$$\delta_y = \pm i \delta_x$$

Coupling to (7/5,2/5) operator only

$$\nu_1 = 1 \qquad \nu_2 = 2/3$$

Cardy, Nuclear Phys B 1993

Si (113) 3 x 1

D. L. Abernathy, S. Song, K. I. Blum, R. J. Birgeneau, and S. G. J. Mochrie, PRB 1994

- Evidence of anisotropic scaling

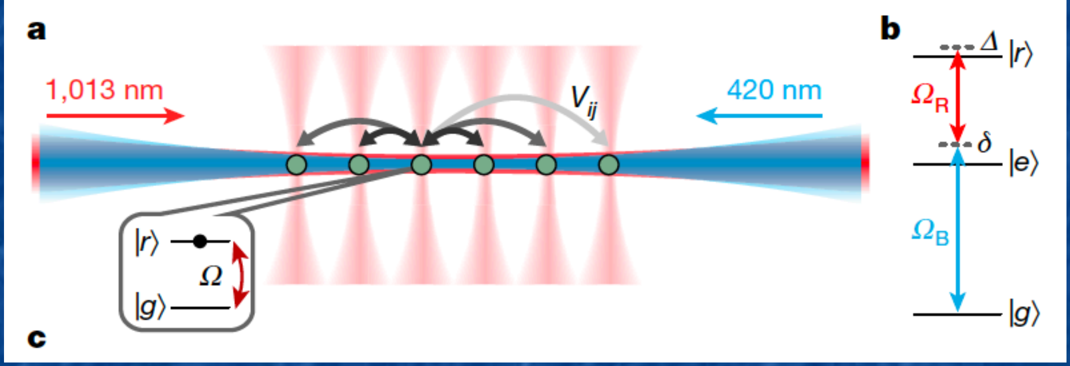
$$\nu_x \simeq 0.65,\ \nu_y \simeq 1.06$$

$$\beta \simeq 0.66 \quad \text{(exponent of } \Delta q\text{)}$$

$$z \simeq 1.6$$

NB: $\beta \simeq \nu_x \Longrightarrow \Delta q\ \xi \rightarrow \text{Cst} \Longrightarrow$ Chiral transition

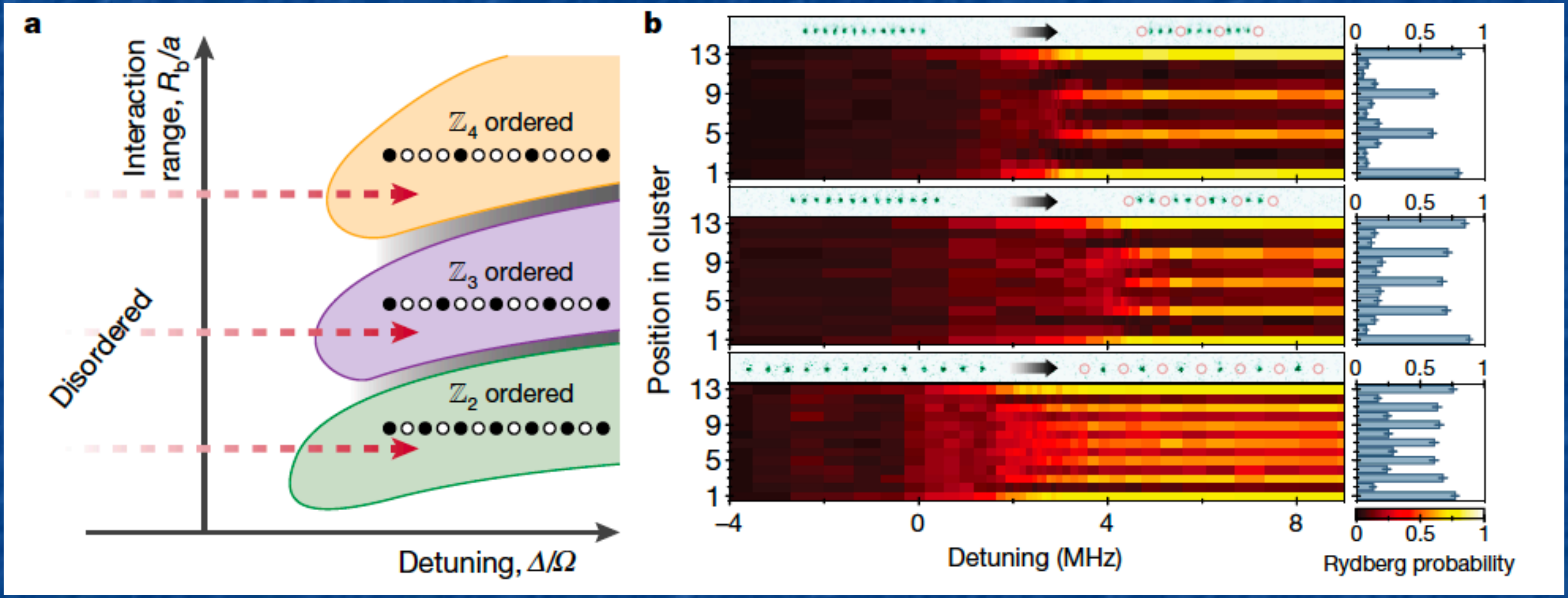
Rydberg atoms



$$\frac{\mathcal{H}}{\hbar} = \sum_i \frac{\Omega_i}{2}\sigma_x^i - \sum_i \Delta_i n_i + \sum_{i<j} V_{ij} n_i n_j$$

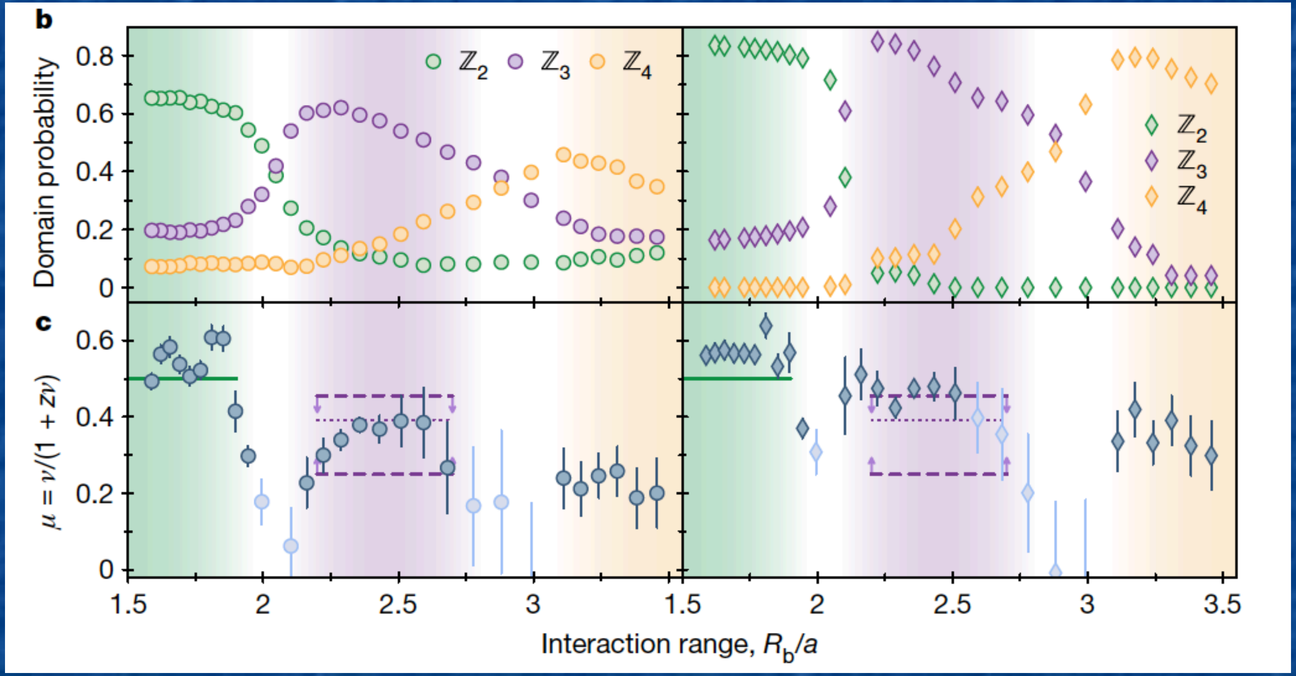
H. Bernien … M. Lukin, Nature 2017

Commensurate phases



H. Bernien … M. Lukin, Nature 2017

Kibble-Zurek exponent $\mu=\nu/(1+z\nu)$



Correlation length

$$\xi \ \alpha \ (1/s)^{\mu}$$

Sweeping rate

A. Keesling … M. Lukin, Nature 2019

Rydberg chains

$$H_{\mathrm{Ryd}} = \sum_i \left[-\frac{\Omega}{2}(d_i^\dagger + d_i) - \Delta n_i + \sum_{R=1}^{+\infty} V_R n_i n_{i+R} \right]$$

$$V_R = V\left(\frac{1}{R}\right)^6$$

- Blockade model for period p phase:

$$\begin{cases} V_R = \infty \text{ for } R < p-1 \\ V_{p-1} \neq 0 \\ V_R = 0 \text{ for } R \geq p \end{cases}$$

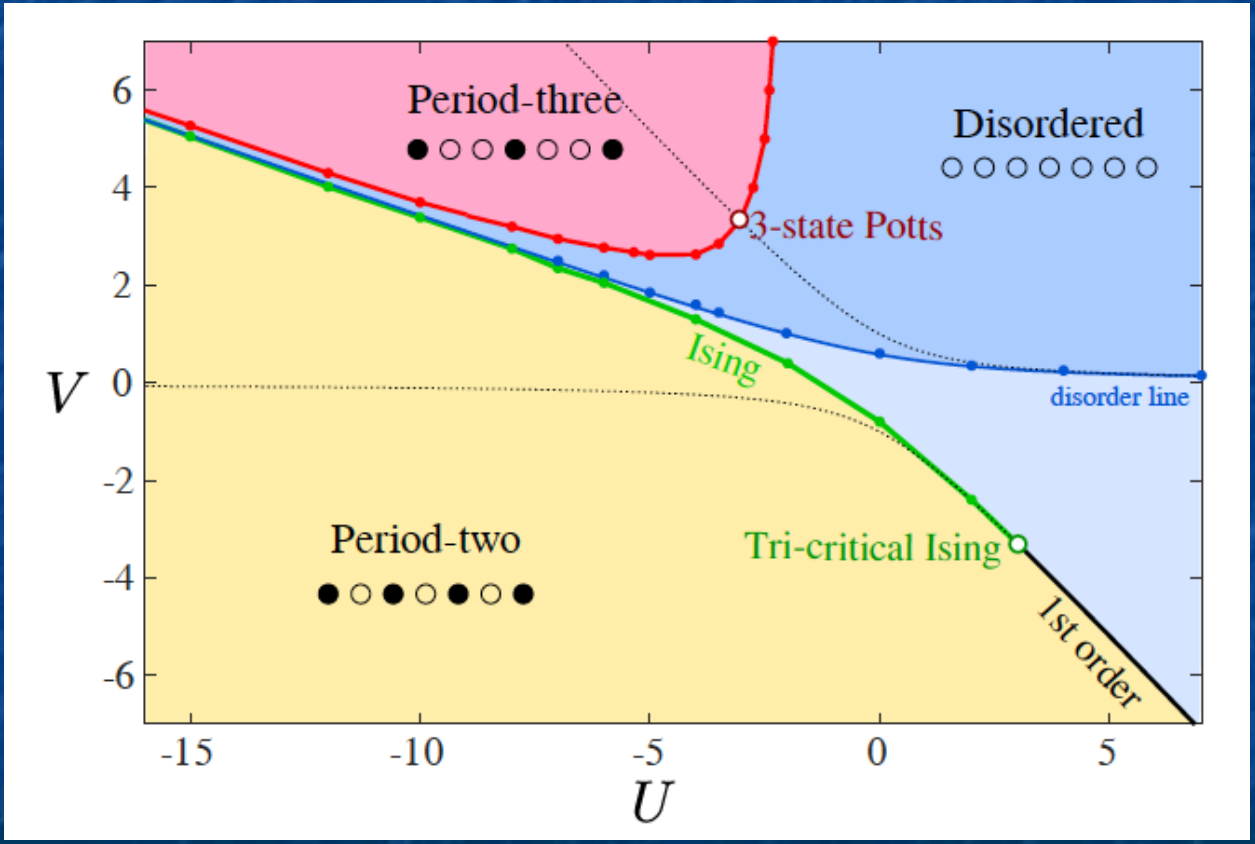
Hard boson model for period-3 phase

$$H_{\mathrm{HB}} = \sum_j \left[-w(d_j^\dagger + d_j) + U n_j + V n_{j-1} n_{j+1} \right]$$

- Two constraints $n_j(n_j - 1) = 0$ $n_j n_{j+1} = 0$
- Hilbert space: grows as Fibonacci number

Fendley, Sengupta, Sachdev 2004

Hard-bosons: phase diagram



Fendley et al, PRB 2004; Chepiga and FM, PRL 2019

Phase transitions

- Transition out of period-2 phase:
  → Ising
  → Tricritical Ising point
  → First order
- Disorder line: does not cross the boundaries of the ordered phases
- Transition out of period-3 phase: Commensurate-Incommensurate transition

Transition out of period 3

- Fendley-Sengupta-Sachdev (2004)

  → Intermediate phase for $U \to -\infty$

  → Probable intermediate phase up to Potts

- Samajdar, Choi, Pichler, Lukin, Sachdev (2018)

  → Evidence of non-integer dynamical exponent between Potts and $V \to +\infty$

  → Chiral transition between Potts and $V \to +\infty$

DMRG

- Algorithm that takes the constraint into account when building the MPS

  → Huge reduction of Hilbert space

$$\Omega_0(N_r) \approx \varphi^{N_r} \approx 1.6^{N_r} \text{ instead of } \Omega(N_r) \approx 8^{N_r}$$

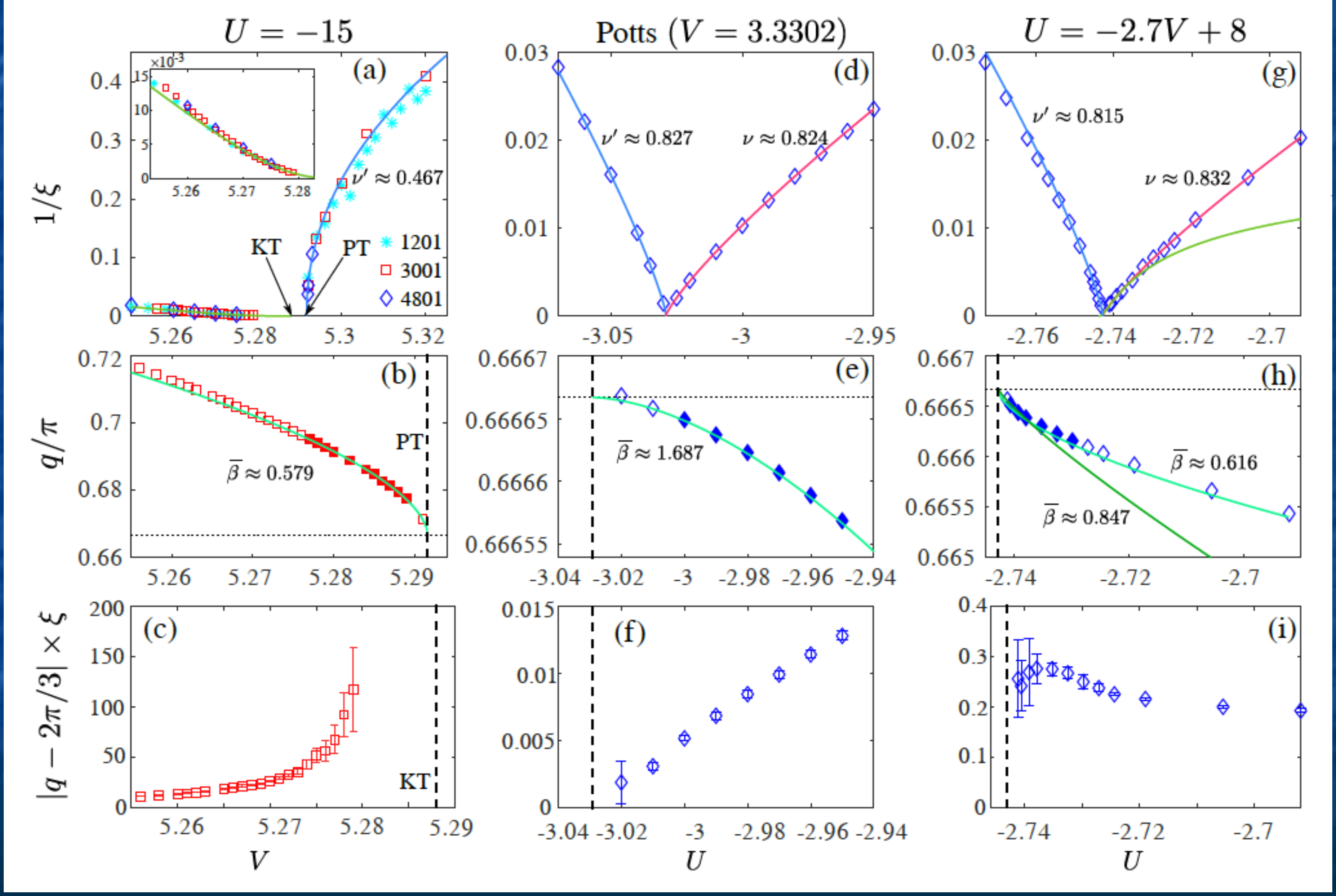
  → Simulations up to 9'000 sites

- Δq: extremely precise results (third digit)
- ξ: access to values of several hundreds

  → meaningful evaluation of Δq ξ

Three cases

Far left from Potts | Potts | Just above Potts



Three cases

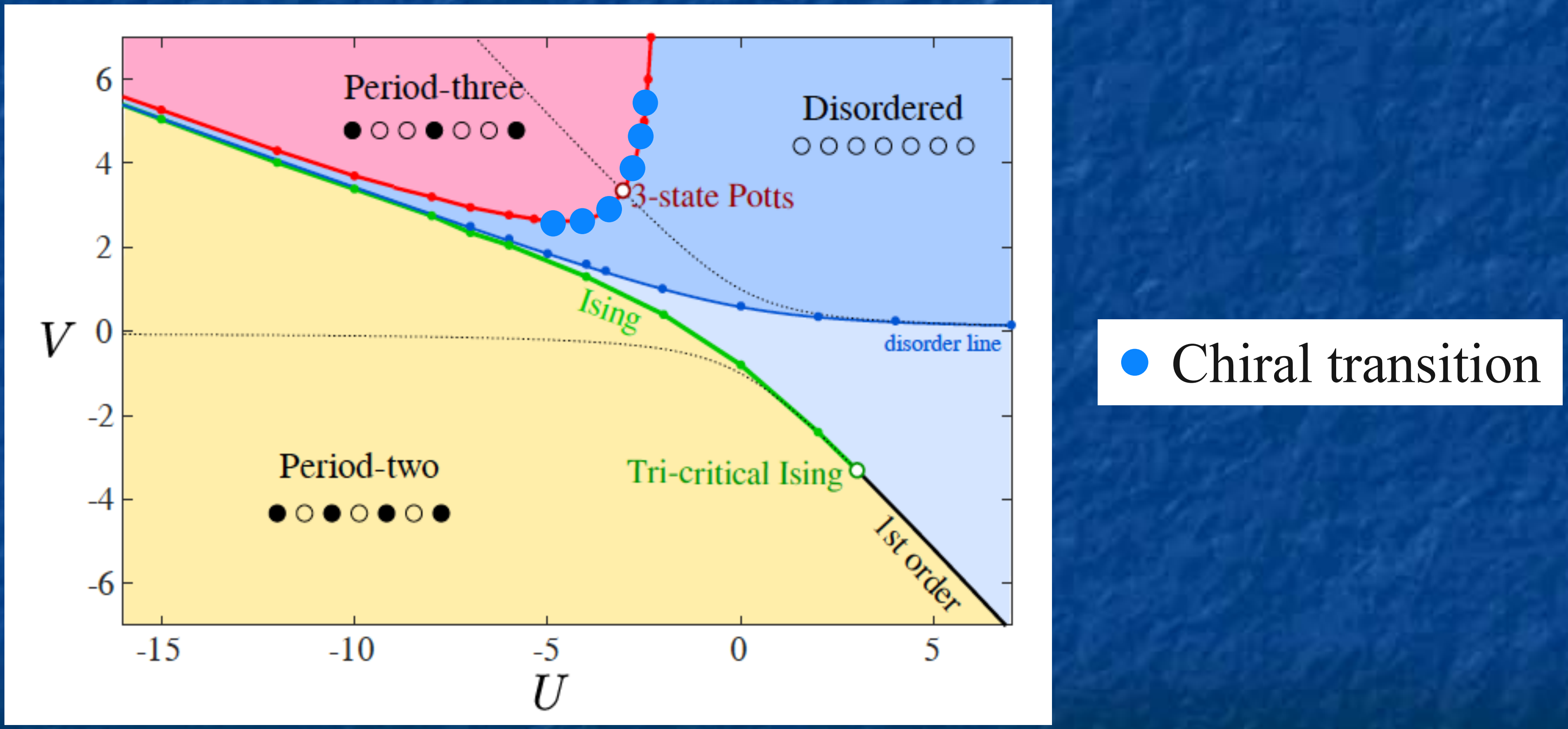
- Potts: Δq goes to zero with exponent 5/3, ξ diverges with exponent 5/6

  ➔ $\Delta q\ \xi \to 0$
- Far from Potts: Two transitions : KT and PT, and intermediate critical phase in both directions
- Vicinity of Potts: consistent with single transition

  ➔ Δq goes to zero with exponent smaller than 1

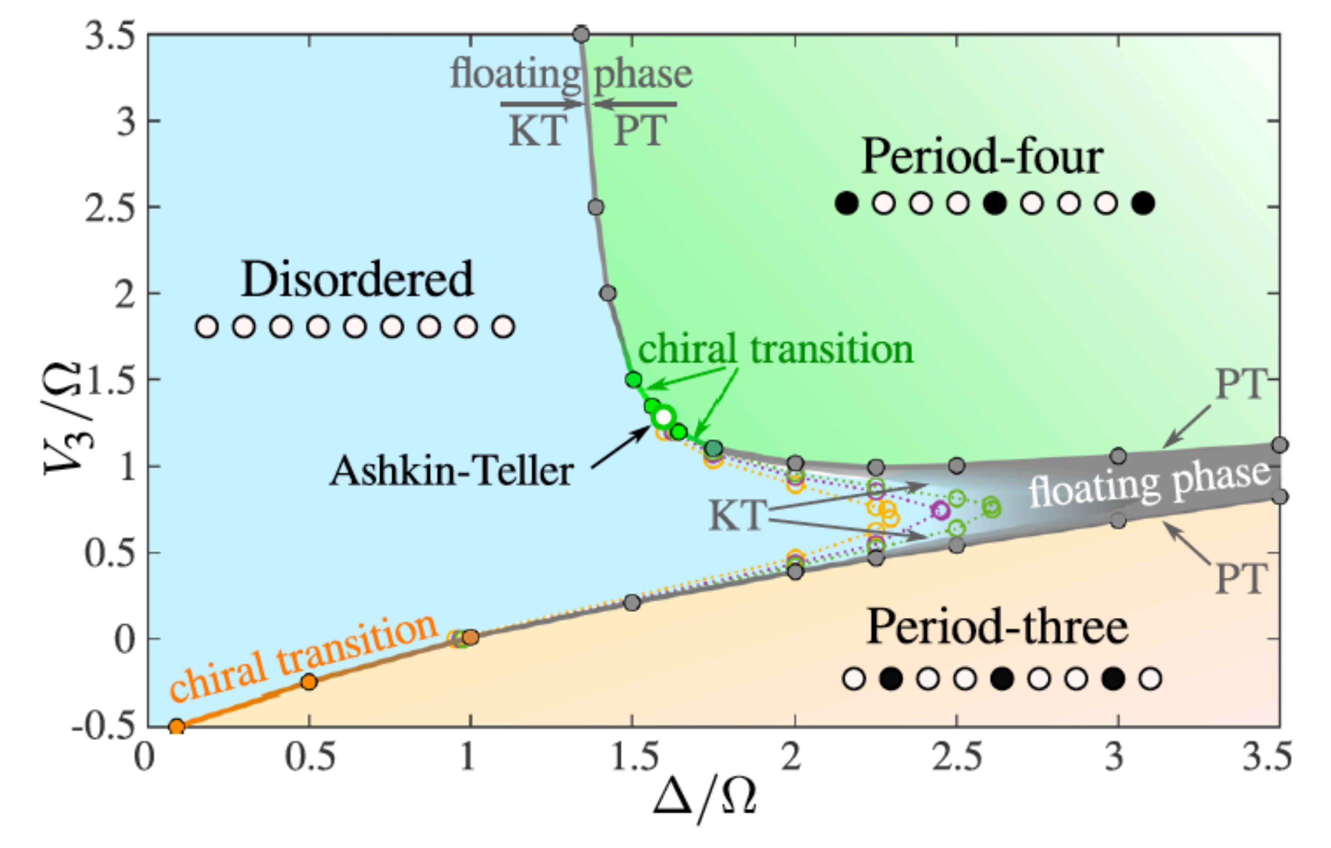
  ➔ $\Delta q\ \xi \to$ constant

Hard-bosons: phase diagram



Chepiga and FM, PRL 2019

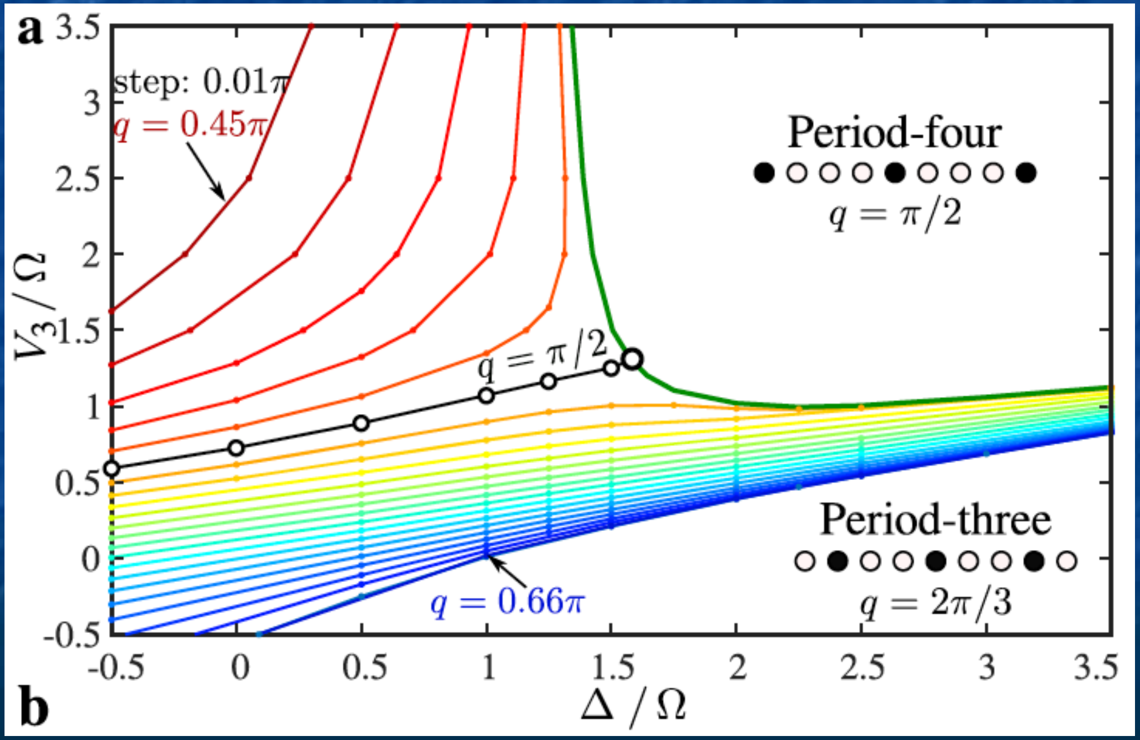
Period-4 phase



N. Chepiga and FM, Nat. Commun. 2021

Ashkin-Teller point

Along commensurate line, the transition must be conformal → Ashkin-Teller universality class

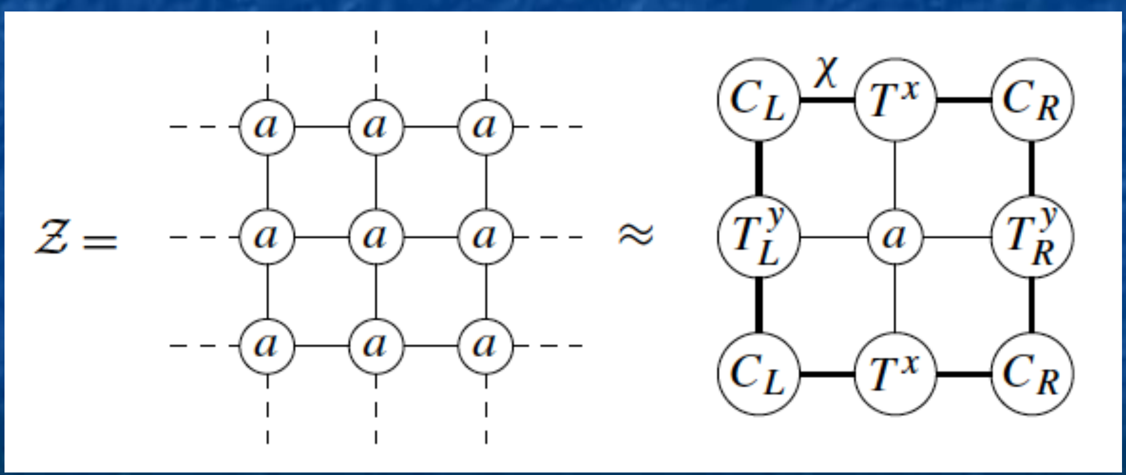


What about the classical 2D case?

CTMRG for 2D classical systems

Corner Transfer Matrix Renormalization Group

Nishino, 1995; Nishino and Okunishi, 1996

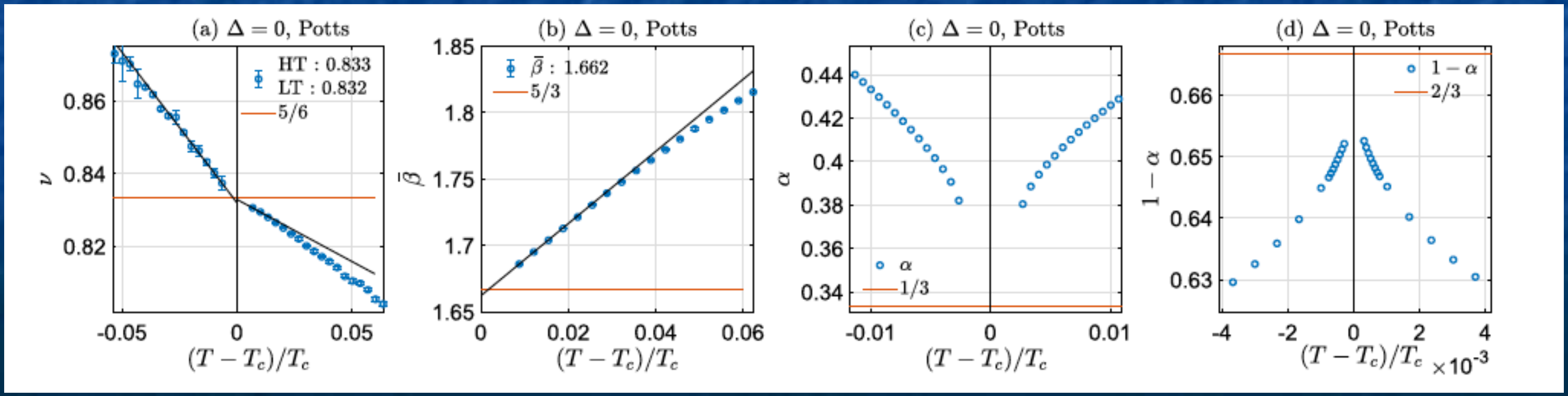


Benchmark: 3-state Potts model

Effective exponents (instead of fit of finite window)

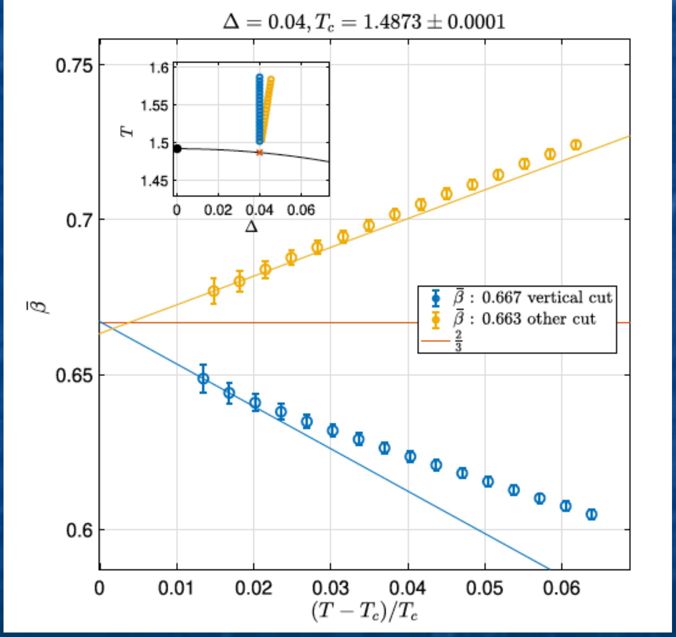
$$A \propto |t|^{-\theta} \qquad \theta(|t|) = -\frac{d\ln A}{d\ln|t|} \qquad \theta = \lim_{|t|\to 0} \theta(|t|)$$

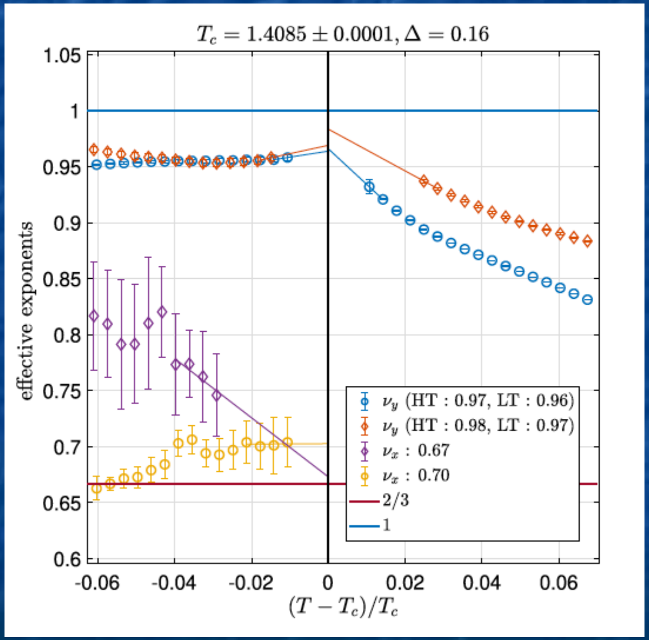
Possible thanks to high precision



Chiral 3-state Potts model

$$E = -\sum_{\vec{r}} \cos[2\pi/3(n_{\vec{r}+\vec{x}} - n_{\vec{r}} + \Delta)] - \sum_{\vec{r}} \cos[2\pi/3(n_{\vec{r}+\vec{y}} - n_{\vec{r}})]$$





S. Nyckees, J. Colbois, FM, Nuclear Phys B 2021

Chiral 3-state Potts model

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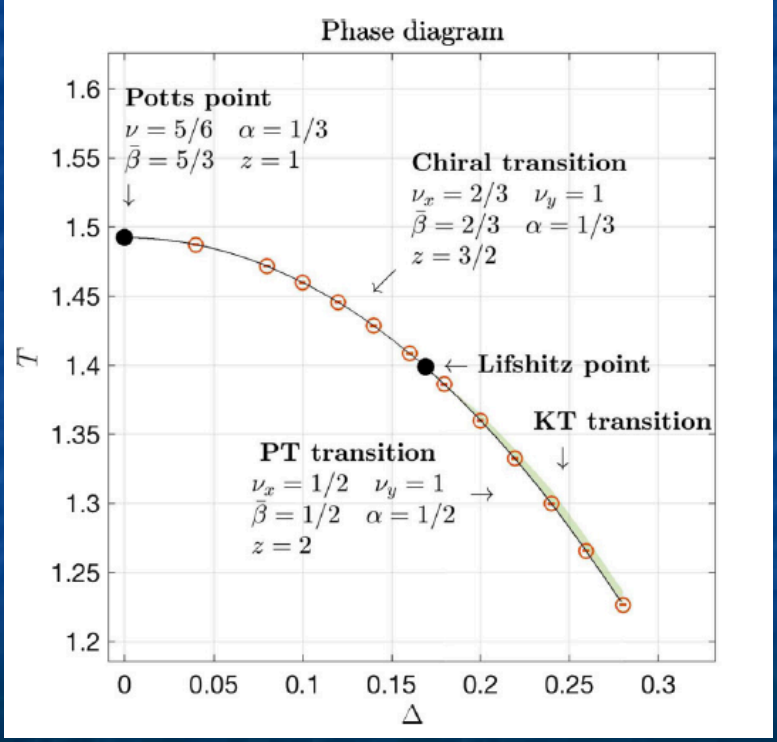
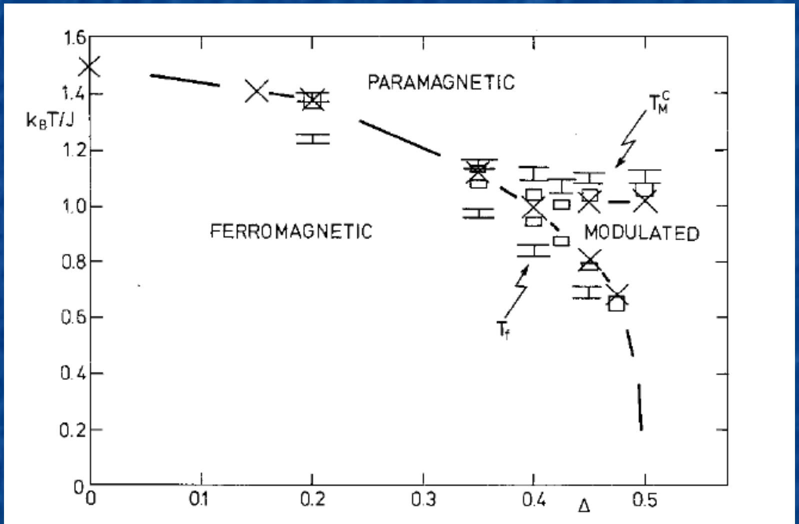


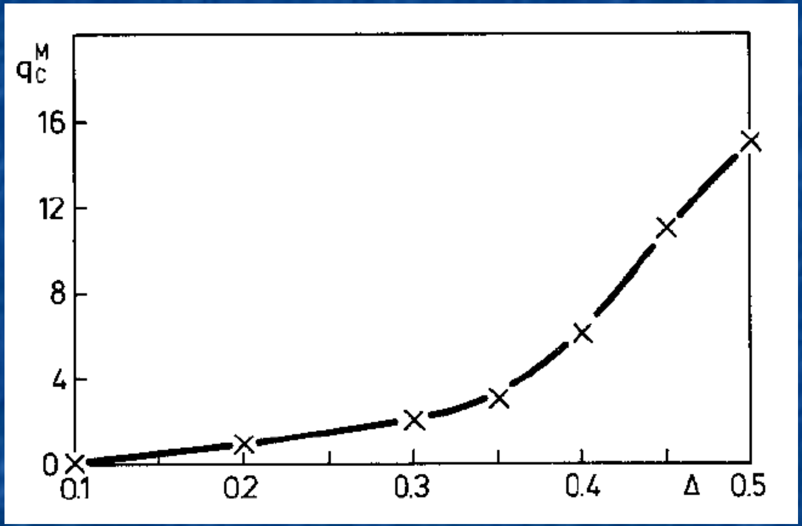
- Exponents similar to those of the self-dual version of the model with complex *Δ* (Cardy, 1993)
- Consistent with the results on Si(113) 3 x 1
- Reasonable agreement with recent field theory results (Whitsitt, Samajdar, Sachdev, 2018)

S. Nyckees, J. Colbois, FM, Nuclear Phys B 2021

MC simulations of chiral Potts model

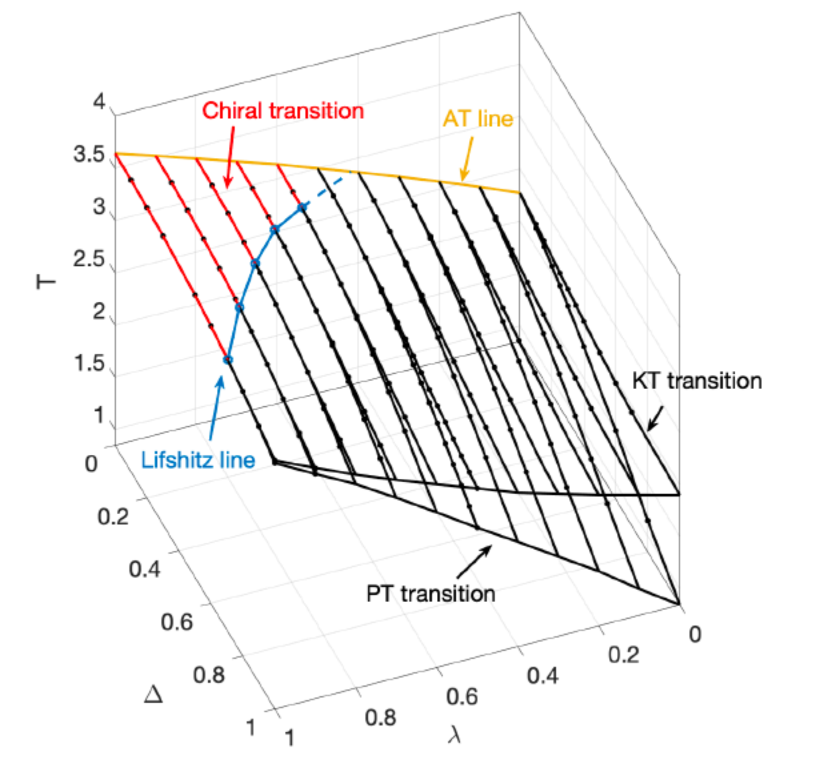
$$\mathscr{H} = -J_x \sum_{\langle ij \rangle}^{x} \cos\left[\frac{2\pi}{3}(n_i - n_j + \Delta)\right] - J_y \sum_{\langle ij \rangle}^{y} \cos\left[\frac{2\pi}{3}(n_i - n_j)\right]$$





Selke and Yeomans, 1982

Period-4 case: Ashkin-Teller



$$H_0 = -\sum_{\langle i,j \rangle} \sigma_i \sigma_j + \tau_i \tau_j + \lambda \sigma_i \sigma_j \tau_i \tau_j$$

Chiral perturbation

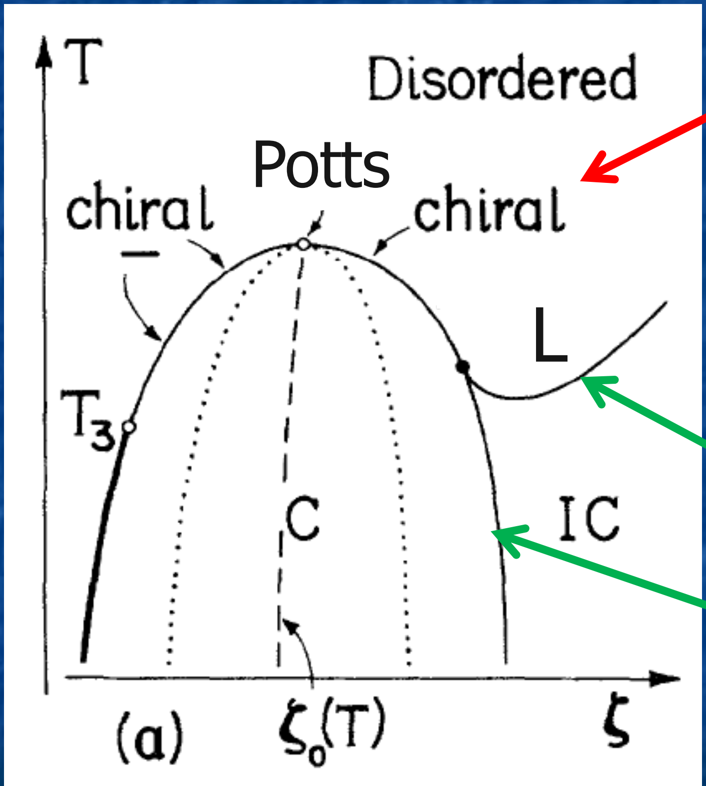
$$\Delta \sum_{x,y} (\tau_{x+1,y} \sigma_{x,y} - \sigma_{x+1,y} \tau_{x,y})$$

S. Nyckees and FM, Phys Rev Research 2022

CFT and chiral transition

- Chiral transition not conformal
- Yet, CFT might have something to say
  → Potts point: central charge $c=4/5$
  → Intermediate phase: $c=1$ (Luttinger liquid)
- Can the Potts point touch the Luttinger liquid phase?

Huse-Fisher phase diagram



Chiral: Anisotropic scaling
→ $\nu_x \neq \nu_y$
→ dynamical exponent z >1

Kosterlitz-Thouless

Pokrovsky-Talapov

Huse and Fisher, 1982

Chiral transition and c-theorem

- c-theorem: c decreases along an RG flow
- Potts point unstable due to the chiral perturbation, which is relevant
- Naive conclusion: a direct transition between Potts ($c=4/5$) and intermediate phase ($c=1$) excluded by c-theorem
- Why naive? One of the hypotheses of the c-theorem not satisfied (Lorentz invariance is not preserverd along the flow)

Conclusions

- Modern numerical approaches
  → strong evidence of chiral transition
- Classical 2D models: CTMRG
  → Chiral 3-state Potts model
  → Asymmetric Ashkin-Teller model
- Blockade models of Rydberg chains
  → Chiral transition for period-3 and period-4 phases from DMRG

Open questions

- CFT proof that there must be a chiral transition between the Potts point and the intermediate phase?
- Nature of Lifshitz point?
- Constant or variable exponents along the chiral transition?