QFT in AdS

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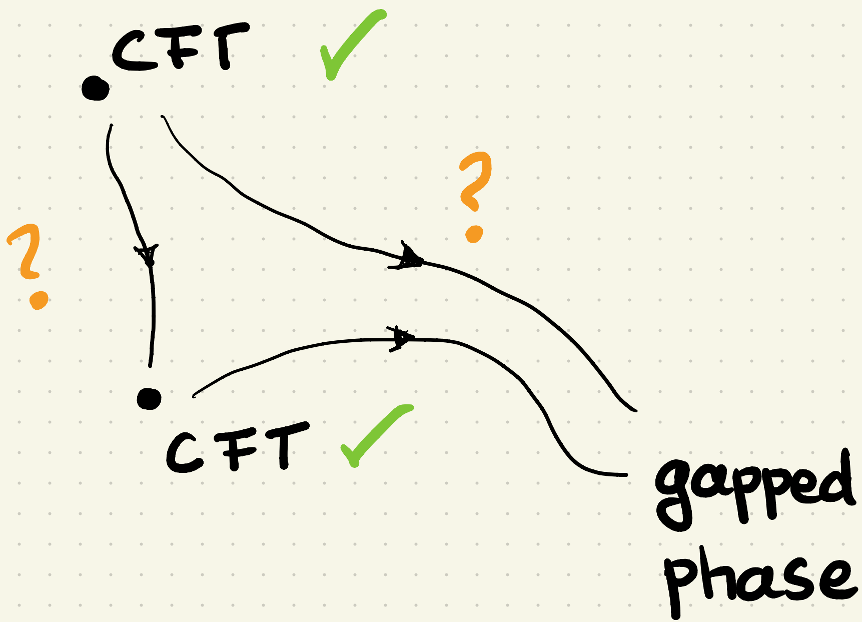
Ecole Polytechnique

based on work with:

A. Antunes, E. Lauria, M. Milam (to appear)

A. Antunes, M. Costa, J. Penedones, A. Salgarkar (appeared)

Motivation



Q: can we bootstrap an RG flow?

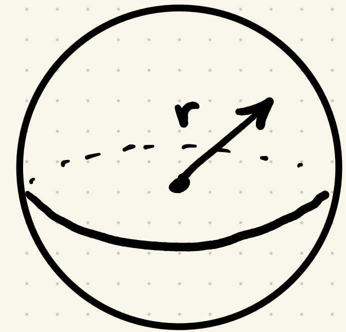
Q': are there any conformal structures along an RG flow?

AdS_{d+1}

$Ric \sim \frac{1}{R^2}$

$$ds^2 = dr^2 + \sinh^2(r/R)\, R^2\, d\Omega_d^2$$

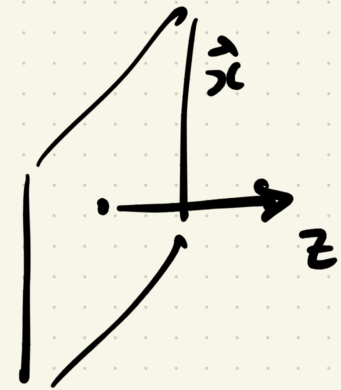
$r \geq 0$, $\vec{n}$ s.t. $|\vec{n}|^2 = 1$



$\updownarrow$ diffeo

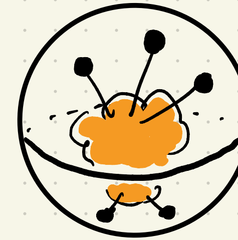
$$ds^2 = \frac{R^2}{z^2}\left(d\vec{x}^2 + dz^2\right)$$

$z > 0$, $\vec{x} \in \mathbb{R}^d$



isometries: $so(d+1,1)$ conf. tr. on $\vec{x}$!

AdS_{d+1}



consider, for a QFT in AdS_{d+1}, the correlator:

$$\lim_{r_i \to \infty} \left(\prod_i e^{+m_i r_i} \right) \langle \mathcal{O}_1(r_1, \vec{n}_1) \ldots \mathcal{O}(r_k, \vec{n}_k) \rangle$$

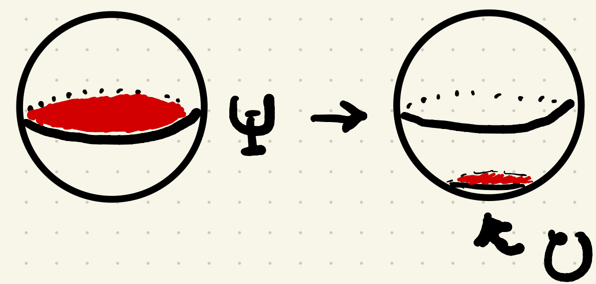
$$=: \langle \hat{\mathcal{O}}_1(\vec{n}_1) \ldots \hat{\mathcal{O}}_k(\vec{n}_k) \rangle$$

"boundary correlation function"

- conf. invt.
- state-op. correspondence

$\Rightarrow$ conf. block dec.

$\Rightarrow$ bootstrap!

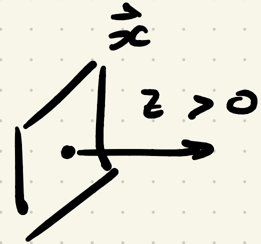


Expectations

1. BCFTs
2. RG flows

Expectations

BCFTs



$$ds^2 = \left(\frac{R}{z}\right)^2 \left(dz^2 + d\vec{x}^{\,2}\right) \underset{\text{Weyl}}{\longleftrightarrow} ds^2 = dz^2 + d\vec{x}^{\,2}$$

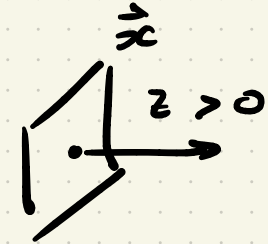
$$\langle \mathcal{O}(\vec{x}, z) \rangle_{\text{flat}} = \frac{a_{\mathcal{O}}}{z^{\Delta_{\mathcal{O}}}}$$

$$\langle \mathcal{O}_1(\vec{x}_1, z_1)\, \mathcal{O}_2(\vec{x}_2, z_2) \rangle_{\text{flat}} = f(\xi) / z_1^{\Delta_1} z_2^{\Delta_2}$$

$$\xi = \frac{(\vec{x}_1 - \vec{x}_2)^2 + (z_1 - z_2)^2}{4 z_1 z_2}$$

Expectations

BCFTs



$$ds^2 = \left(\frac{R}{z}\right)^2 \left(dz^2 + d\vec{x}^2\right) \xleftrightarrow{\text{Weyl}} ds^2 = dz^2 + d\vec{x}^2$$

$$\langle \mathcal{O}(\vec{x},z)\rangle_{AdS} = \frac{a_{\mathcal{O}}}{R^{\Delta_{\mathcal{O}}}} \longleftarrow \langle \mathcal{O}(\vec{x},z)\rangle_{flat} = \frac{a_{\mathcal{O}}}{z^{\Delta_{\mathcal{O}}}}$$

just constant

$$\langle \mathcal{O}_1(\vec{x}_1,z_1)\,\mathcal{O}(\vec{x}_2,z_2)\rangle_{AdS} = \longleftarrow \langle \mathcal{O}_1(\vec{x}_1,z_1)\,\mathcal{O}_2(\vec{x}_2,z_2)\rangle_{flat} =$$

$$f(\xi)/R^{\Delta_1+\Delta_2} \qquad\qquad f(\xi)/z_1^{\Delta_1} z_2^{\Delta_2}$$

$$2\xi + 1 = \cosh\left(d_{1,2}/R\right) \qquad\qquad \xi = \frac{(\vec{x}_1-\vec{x}_2)^2 + (z_1-z_2)^2}{4 z_1 z_2}$$

just geodesic distance

$\rightarrow$ at fixed R, no new conf. W.I.!

Boundary OPE

In BCFT:

$$\langle O(\vec{x}, z) \cdots \rangle_{\text{flat}} = \sum_{\hat{O}} z^{\Delta_{\hat{O}} - \Delta_O} b_O{}^{\hat{O}} \langle \hat{O}(\vec{x}) \cdots \rangle_{\text{flat}}$$

so in AdS:

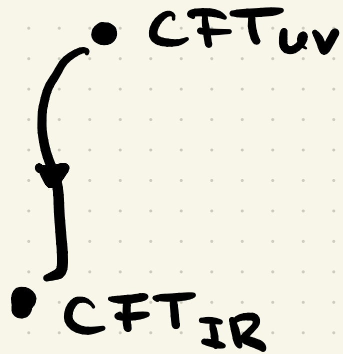
$$\langle O(\vec{x}, z) \cdots \rangle_{\text{AdS}} = \sum_{O} z^{\Delta_{\hat{O}}} R^{\Delta_O} b_O{}^{\hat{O}} \langle \hat{O}(\vec{x}) \cdots \rangle_{\text{AdS}}$$

$\Delta_{\hat{O}} > 0$, so falloff as $z \to 0$

BCFT "UV" $\longrightarrow$ IR in AdS

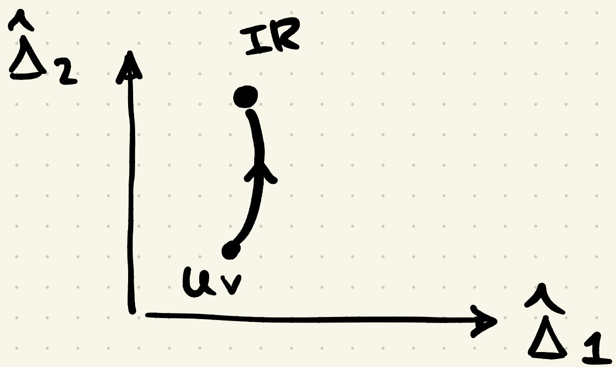
RG flows (naive)

In AdS,



is normally covariant,

so bdy. correlators should be conformal:

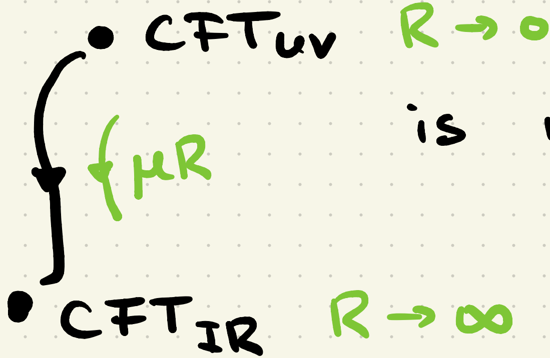


RG flows (naive)

$$ds^2 = dr^2 + \sinh^2(r/R)\, R^2 d\Omega^2$$

$$Ric \sim 1/R^2$$

In AdS,



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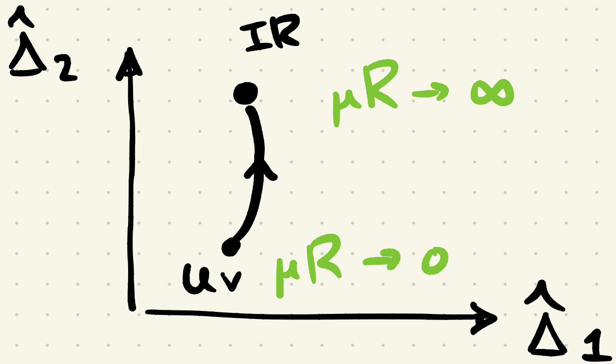


RG flows (less naive)

$$\begin{tikzcd} \bullet\ CFT_{UV} \arrow[d] \\ \bullet\ CFT_{IR} \end{tikzcd}$$

flat space

$$\begin{tikzcd} CFT_{UV} + B.C.\ 1 \arrow[r] \arrow[d] & CFT_{UV} + B.C.\ 2 \arrow[d] \\ CFT_{IR} + B.C.\ 1' \arrow[r] & CFT_{IR} + B.C.\ 2' \end{tikzcd}$$

AdS

$$\begin{tikzcd} CFT_{UV} + B.C.\ 1 \arrow[r] \arrow[d] & CFT_{UV} + B.C.\ 2 \arrow[d] \\ CFT_{IR} + B.C.\ 1' \arrow[r] & CFT_{IR} + B.C.\ 2' \end{tikzcd}$$

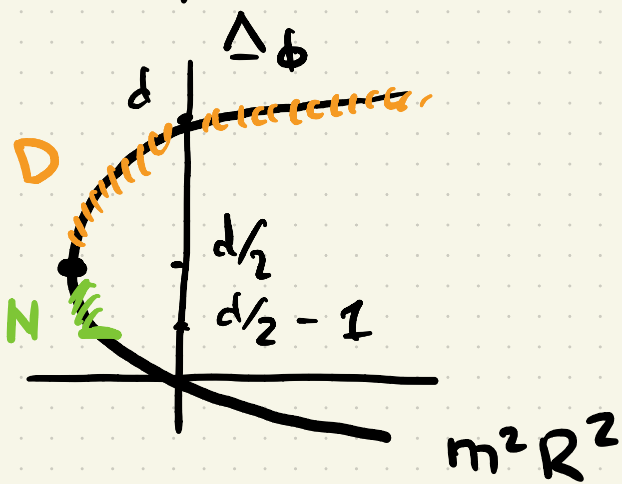
claim : bdy flows are not covariant $\Rightarrow$ bdy follows bulk.

Boundary RG flows - example

free scalar in AdS_{d+1}

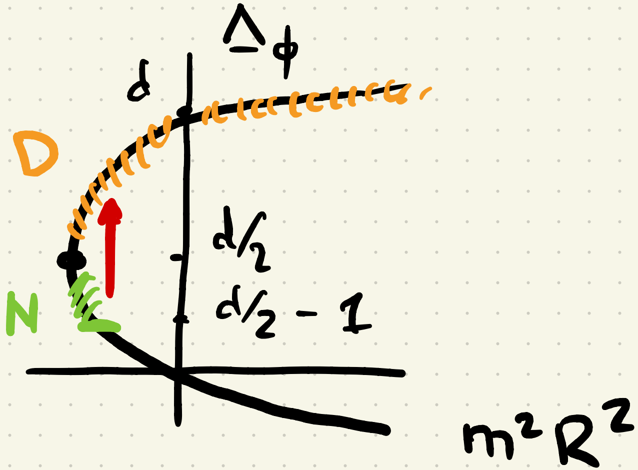
$$S = \frac{1}{2}\int d^d\vec{x}\, dz \sqrt{g}\left(\partial_\mu \Phi \partial^\mu \Phi + m^2 \Phi^2\right)$$

bdy. correlators : G.F.F. ϕ



$$\left(\langle \phi_1 \dots \phi_n \rangle_{\text{connected}}^{n>2} = 0 \right)$$

$$\Delta_\phi(\Delta_\phi - d) = m^2R^2$$



$\uparrow$: RG flow by $\int d^d\vec{x}\,\phi^2$ (at bdy!)

$$\left\langle \phi(\vec{x})\,\phi(0)\,\exp\left(-g\int d^d\vec{x}\,\phi^2\right)\right\rangle$$

$$\left(\propto \int d^d\vec{k}\, e^{i\vec{k}\cdot\vec{x}}\,\frac{\#\,k^{2\Delta-d}}{1+\#\,g\,k^{2\Delta-d}}\right)$$

$$\rightarrow \begin{cases} 1/x^{2\Delta_\phi^-} & \text{as } x\to 0 \\ 1/x^{2\Delta_\phi^+} & \text{as } x\to\infty \end{cases}$$

is not conformal!

(i.e. not covariant)

Bulk flows – boundary terms

BCFT + relevant deformation in AdS

$$\left\langle \ldots \exp\left(-\lambda \int d^d\vec{x} \int_0 dz \sqrt{g}\, O(\vec{x}, z)\right)\right\rangle$$

$$\supset \left\langle \ldots \lambda \int d^d\vec{x} \int_0 dz \left(\frac{z}{R}\right)^{-(d+1)} z^{\hat{\Delta}_{\hat{O}}} R^{-\Delta_O}\, b_O{}^{\hat{O}}\, \hat{O}(\vec{x})\right\rangle$$

$\Rightarrow$ IR divergence if $\hat{\Delta}_{\hat{O}} \leq d$. Covariant counterterm:

$$ds^2\Big|_{z=\epsilon} = \frac{d\vec{x}^2}{(z/R)^2} \rightsquigarrow \sqrt{h} = \left(\frac{z}{R}\right)^{-d}$$

$$S_{ct} = -\frac{\lambda}{\hat{\Delta}_{\hat{O}} - d} \int d^d\vec{x} \sqrt{h}\, O(\vec{x}, z)\Big|_{z=\epsilon}$$

bulk operator.

Recap

Callan-Wilczek 1990: .. constant negative curvature provides a very convenient IR regulator..

RG flows in AdS can have IR divergences, but $\exists$ covariant RG flows in AdS, where bdy follows the bulk.

(q: what happens for $\hat{\Delta}_{\hat{O}} = d$?)

Q1 : perturbative checks?

Q2 : can we bootstrap a flow?

Perturbative checks

- WF in AdS
- Minimal models in AdS

Subtle : $\log(d_{1-2}/R)$, $\log(\mu R)$

so RG improvement is of limited use.

Perturbative checks - minimal models

- flat space BCFT

$$\Delta_{\phi_{(1,3)}} = 2 - \frac{\#}{m}$$

$$\hat{\Delta}_{\hat{\varphi}_{(1,3)}} = 1 - \frac{\#/2}{m}$$

Fredenhagen, Gaberdiel, Schmidt-Colinet '09

$$\begin{tikzcd}
m^{th} \quad \cdots \arrow[r] & \bigoplus_{\ell}^{\min(a_1,a_2)} (a_1+a_2+1-2\ell,1) \arrow[d] & (a_1,a_2) \arrow[l] \arrow[r] \arrow[d] & \bigoplus_{\ell} (a_1+a_2-2\ell,1) \arrow[d] & (a_1,a_2-1) \cdots \arrow[l] \\
(m-1)^{th} \quad \cdots & \bigoplus_{\ell} (1,a_1+a_2+1-2\ell) \arrow[r] & (a_2-1,a_1) & \bigoplus_{\ell} (1,a_1+a_2-2\ell) \arrow[l] \arrow[r] & \cdots
\end{tikzcd}$$

$\phi_{(1,3)}$

not perturbative

- AdS $(a_1,a_2) \downarrow (a_2,a_1)$

Lauria, Milam, BvR '22

- bdy spectrum : $\psi^{m}_{r,s} \to \psi^{m-1}_{s,r}$ ✓
- 1-pt. functions ?

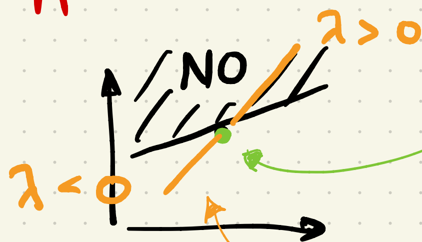
Numerical results

① EFT - type bounds

② RG flow constraints

EFT - type bounds

idea:



QFT in AdS

+

irrelevant bulk interaction

$$\lambda \int d^{d+1}x \sqrt{g}\, \mathcal{O}$$

implies $\lambda < 0$ necessarily!

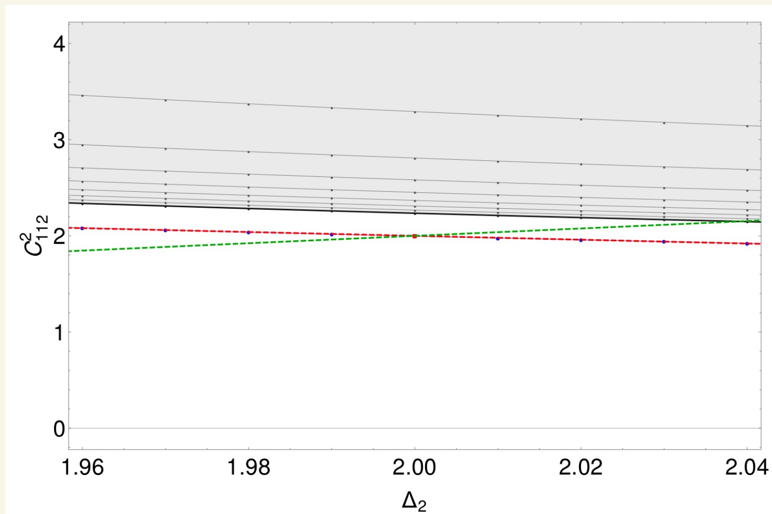
EFT - type bounds

① $S = \int d^{d+1}x \sqrt{g} \left(\frac{1}{2} |\partial\Phi|^2 - \tilde{\lambda} |\partial\Phi|^4 \right)$

pert thy : $(\Delta_\phi, \Delta_{\phi^2}, c^2_{112}) = \left(1, 2 - \frac{\tilde{\lambda}}{6\pi}, 2 - \tilde{\lambda}\frac{23}{36\pi}\right) + \mathcal{O}(\tilde{\lambda}^2)$

numerics:

$\left(\begin{matrix} \langle \phi\phi\phi\phi \rangle \\ \text{in 1d} \end{matrix} \right)$



$\Rightarrow$ $\boxed{\tilde{\lambda} \geq 0}$

Antunes, Costa, Penedones, Salgarkar, BvR '21

EFT-type bounds

Antunes, Lauria, Milam, BvR (to appear)

② $S = S_{\substack{\text{min.}\\ \text{model}}} + \lambda \int T\bar{T}\, d^2x$

pert. thy: $(\hat{\Delta}_{\hat{\phi}}, \hat{\Delta}_D, \hat{\Delta}_{D^2}) = (\hat{\Delta}_{\hat{\phi}} + \frac{\lambda\pi}{2}\hat{\Delta}_{\hat{\phi}}(\hat{\Delta}_{\hat{\phi}} - 1), 2 + \lambda\pi, 4 + 6\lambda\pi) + O(\lambda^2)$

numerics:

$\langle\phi\phi\phi\phi\rangle$

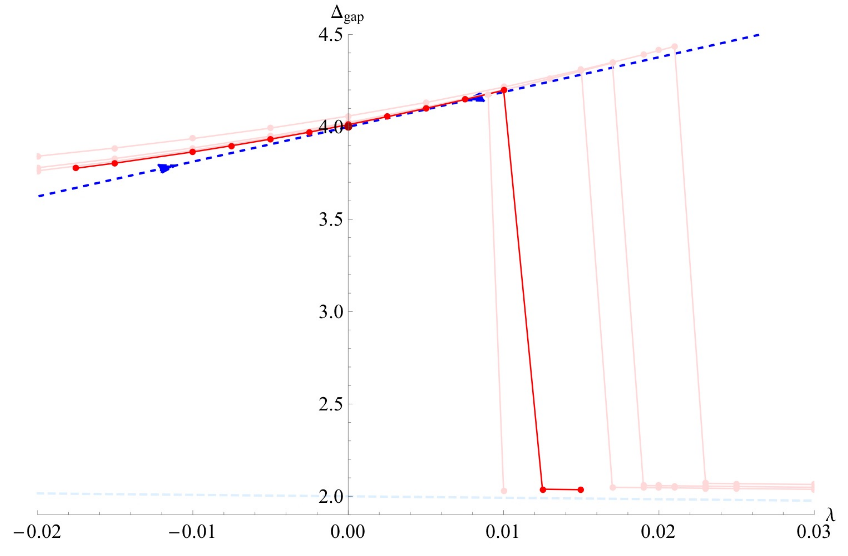
$\phi\times\phi = 1 + D + \text{gap}$

$\hat{\Delta}_\phi = \frac{1}{2} - \frac{\lambda\pi}{8}$

$\hat{\Delta}_D = 2 + \lambda\pi$

max Δ_{gap}

($\phi = \varphi_{(1,3)}$ in Ising)



$\Rightarrow$ $\boxed{\lambda \leq 0 \text{ for 2d Ising}}$

EFT-type bounds

numerics:

$\langle \phi\phi\phi\phi \rangle$

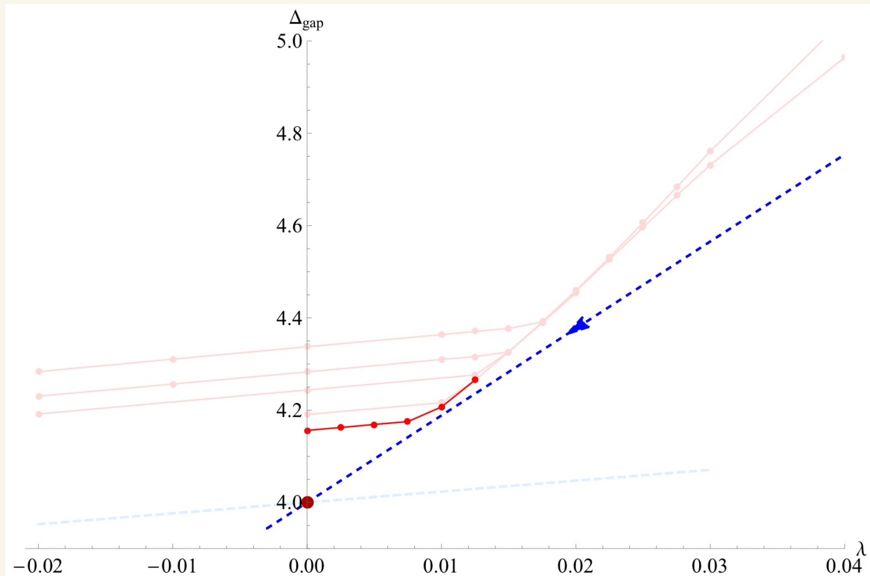
$\phi \times \phi = 1 + D + \text{gap}$

$\hat{\Delta}_\phi = \frac{3}{2} + \frac{3\lambda\pi}{8}$

$\hat{\Delta}_D = 2 + \lambda\pi$

max Δ_{gap}

($\phi = \psi_{3,1}$ in tri-crit. Ising)



no bound for tri-crit. Ising (from this corr.)

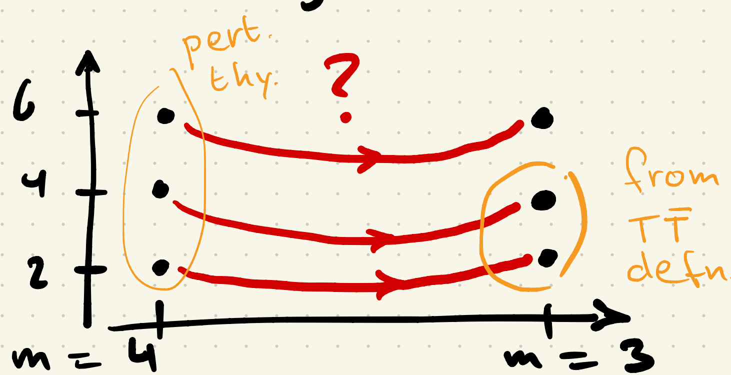
RG flow constraints

minimal model flow

$$m=4 \text{ tri-crit. Ising} \longrightarrow m=3 \text{ Ising}$$

$$(2,1)\ \Big|\ \mathbb{1},\ \varphi_{3,1}\ \Big|\ \Delta_{3,1}=\tfrac{3}{2} \qquad (1,2)\ \Big|\ \mathbb{1},\ \varphi_{1,3}\ \Big|\ \Delta_{1,3}=\tfrac{1}{2}$$

singlets



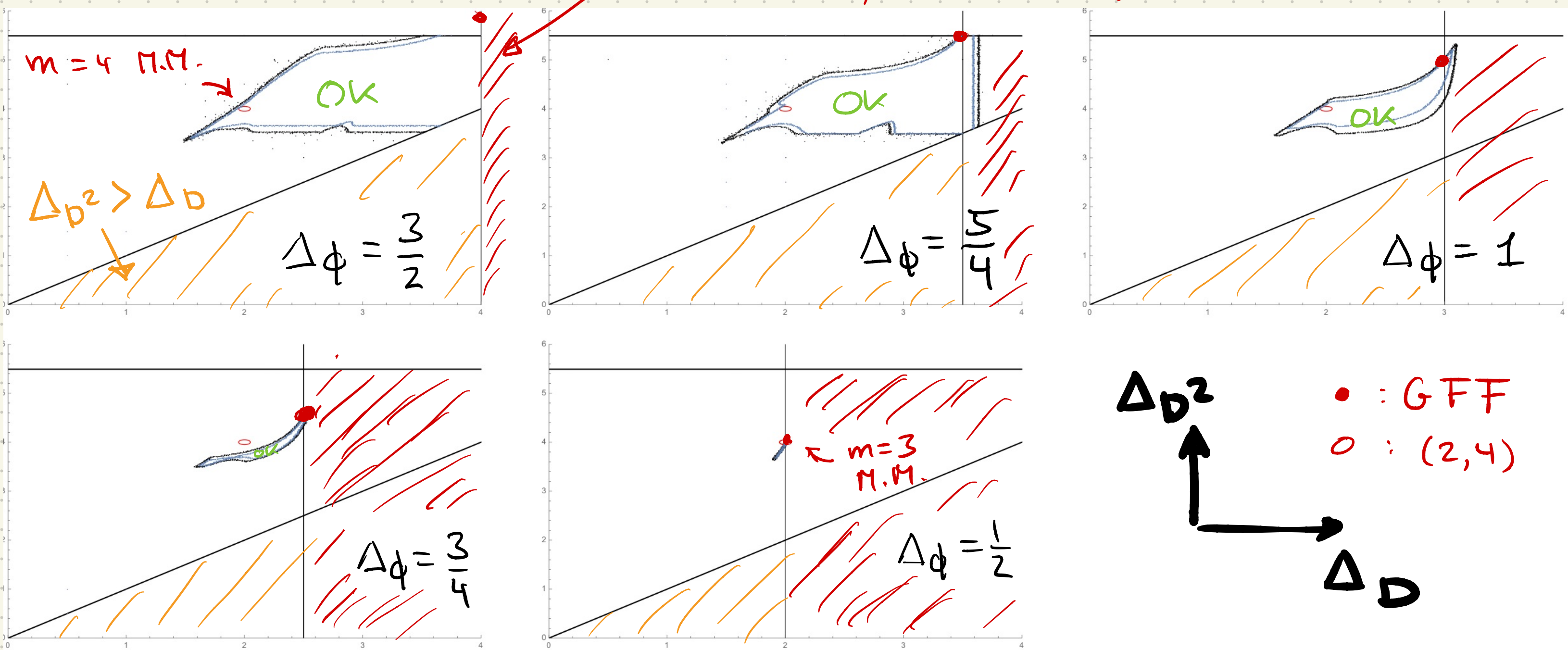
RG flow constraints - numerics

$\langle \phi\phi\phi\phi \rangle, \quad \langle \phi\phi D D \rangle, \quad \langle D D D D \rangle$

$\phi \times \phi \sim \underline{1} + D + D^2 +$ operators with $\Delta > 5.5$

$D \times D \sim$ "

$\Delta_D < \Delta_{D,GFF} = 2\Delta_\phi + 1$



RG flow constraints - numerics

$\langle \phi\phi\phi\phi \rangle, \quad \langle \phi\phi D D \rangle, \quad \langle D D D D \rangle$

$\phi \times \phi \sim 1 + D + D^2 + \text{operators with } \Delta > 5.5$

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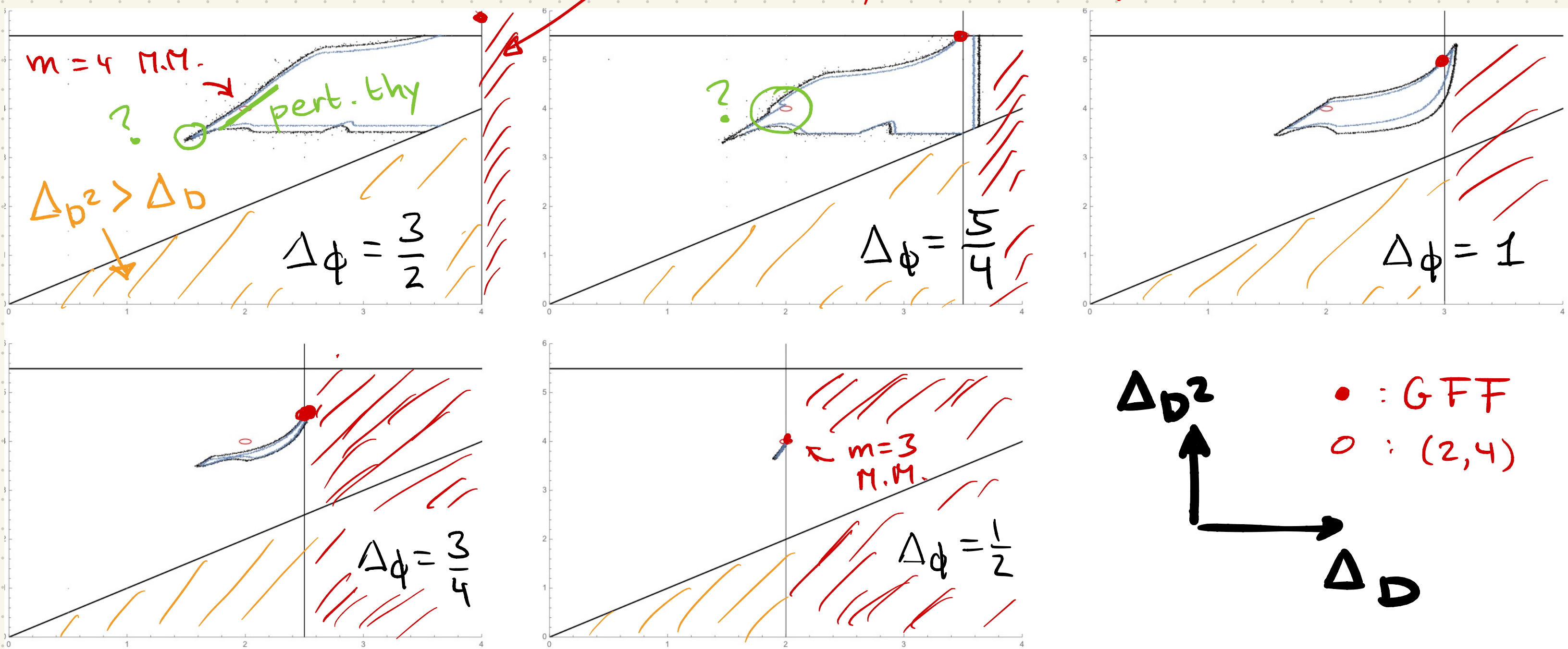
$\Delta_D < \Delta_{D,GFF} = 2\Delta_\phi$



Conclusions

QFT in AdS offers a promising way to constrain RG flows.

- Systematize EFT-type bounds?

  Caron-Huot, Mazac, Rastelli, Simmons-Duffin '21
- physics of extremal allowed points?
- connect with other analyses?

  Carmi, Di Pietro, Komatsu '18 (large N)

  Hogervorst, Meineri, Penedones, Salehi Vaziri '21 (HT)
- flat-space limit and S-matrix bootstrap

Thank you!