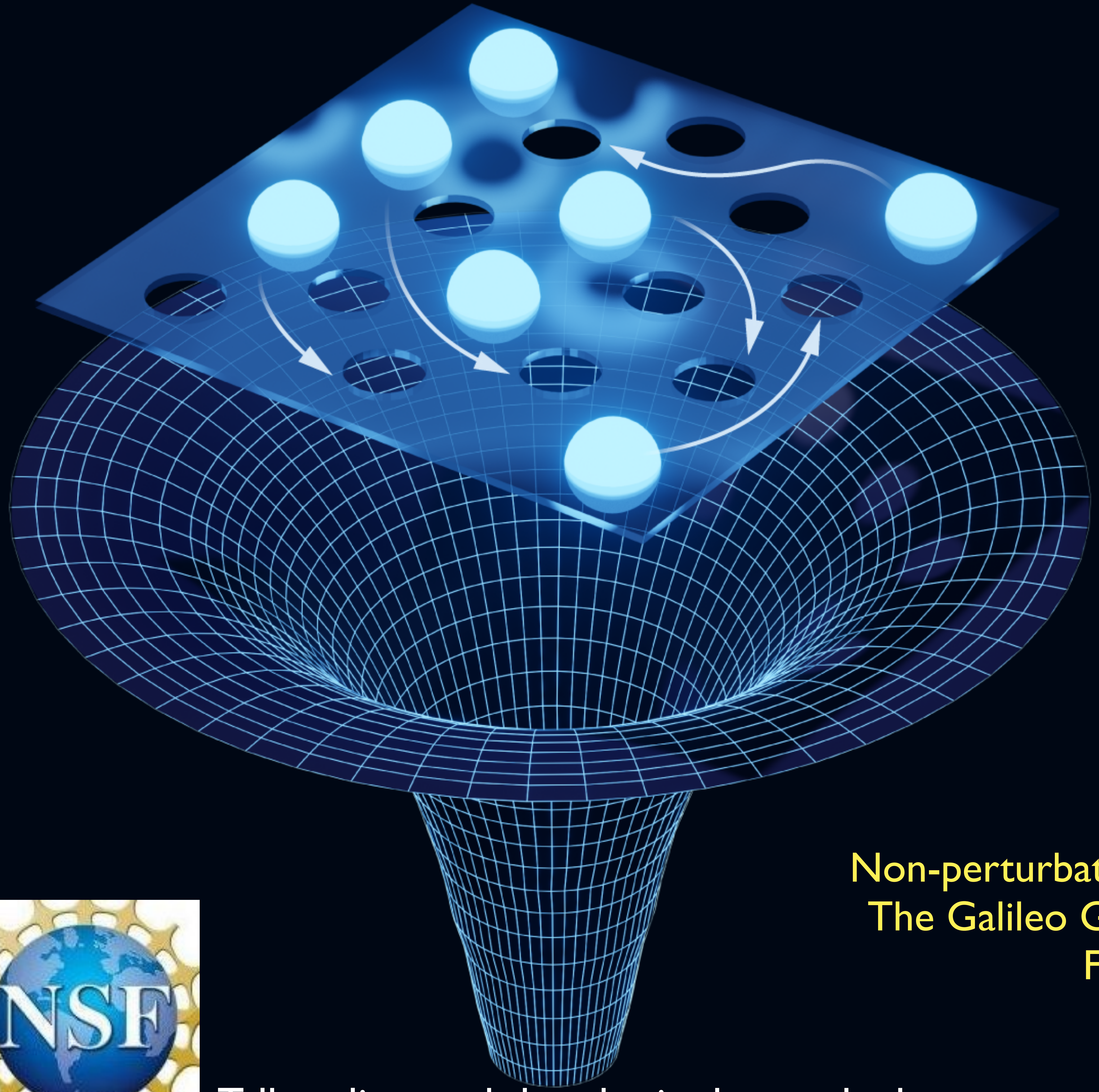


CFTs in 0+1 dimensions: implications for charged black holes and strange metals

Bootstrapping Nature:
Non-perturbative Approaches to Critical Phenomena
The Galileo Galilei Institute For Theoretical Physics
Firenze, October 20, 2022

Subir Sachdev

Talk online: sachdev.physics.harvard.edu

1. SYK model: a solvable and generic 0+1 dimensional CFT
2. Charged black holes
3. Strange metals
4. From Yukawa-SYK models to a universal theory of strange metals

D. Chowdhury, A. Georges, O. Parcollet, and S. S.,
Reviews of Modern Physics **94**, 035004 (2022)

SYK model: A solvable and generic 0+1 dimensional CFT

Operators in a generic CFT_1 are similar to heavy ('black hole') operators in CFT_D with $D \geq 2$

The Sachdev-Ye-Kitaev (SYK) model

(See also: the "2-Body Random Ensemble" in nuclear physics; did not obtain the large *N* limit;
T.A. Brody, J. Flores, J.B. French, P.A. Mello, A. Pandey, and S.S.M. Wong, Rev. Mod. Phys. **53**, 385 (1981))

$$\mathcal{H} = \frac{1}{(2N)^{3/2}} \sum_{\alpha,\beta,\gamma,\delta=1}^{N} U_{\alpha\beta;\gamma\delta}\, c^\dagger_\alpha c^\dagger_\beta c_\gamma c_\delta - \mu \sum_\alpha c^\dagger_\alpha c_\alpha$$

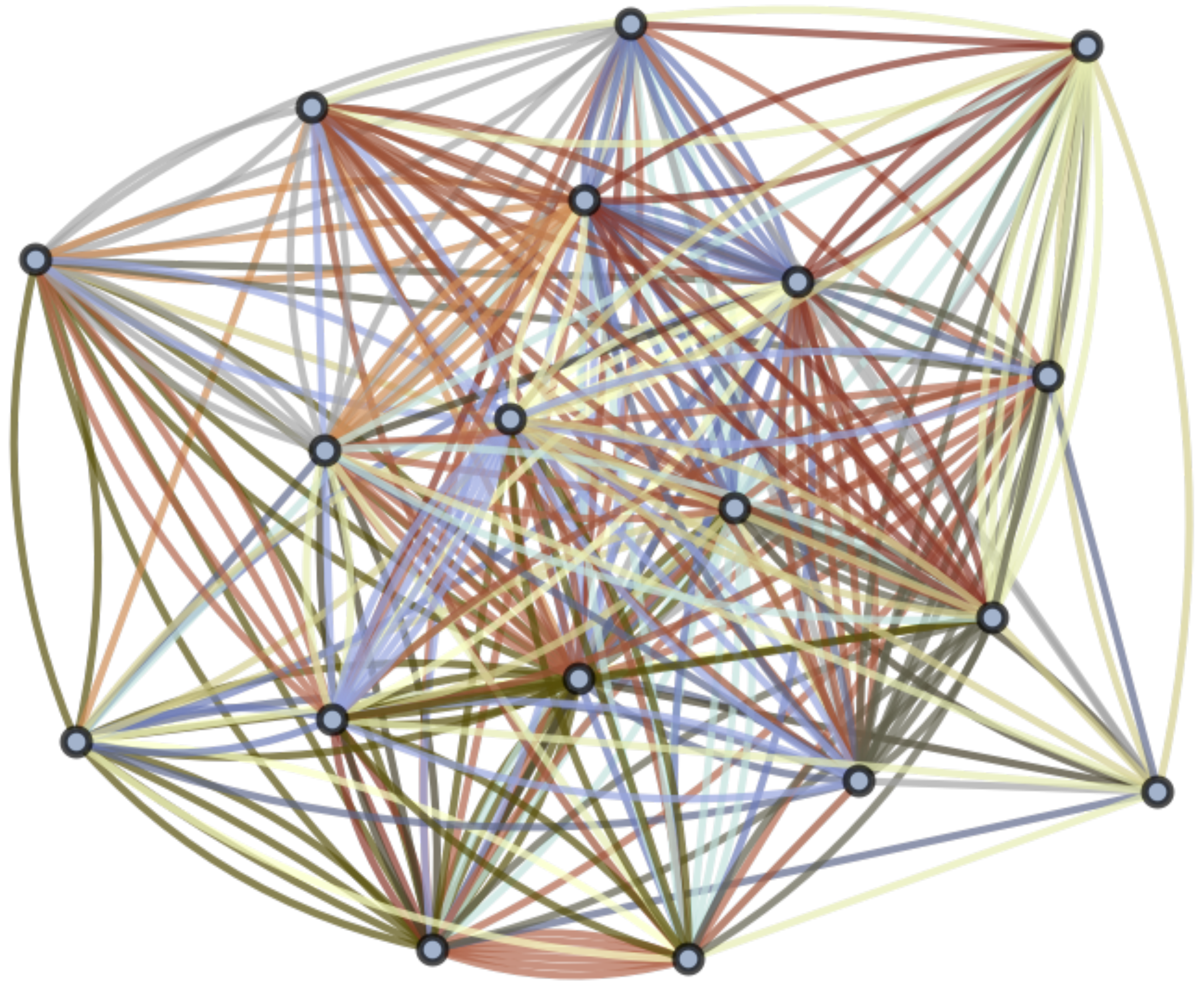
$$c_\alpha c_\beta + c_\beta c_\alpha = 0 \quad , \quad c_\alpha c^\dagger_\beta + c^\dagger_\beta c_\alpha = \delta_{\alpha\beta}$$

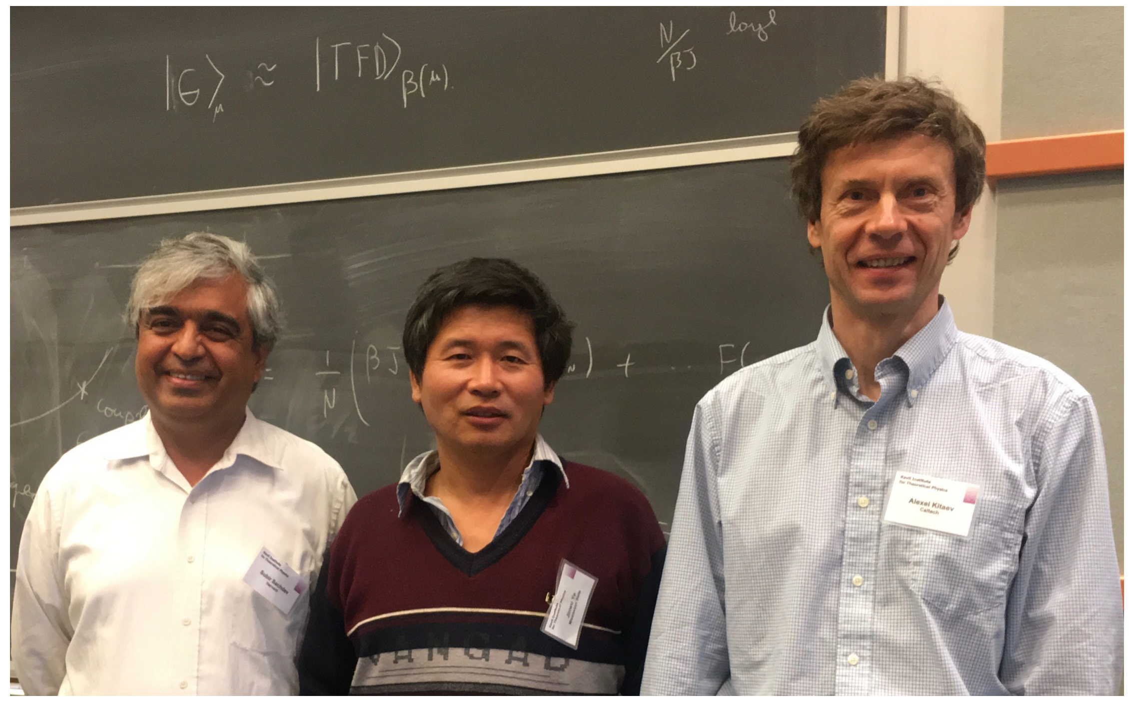
$$\mathcal{Q} = \frac{1}{N} \sum_\alpha c^\dagger_\alpha c_\alpha \,; \quad [\mathcal{H}, \mathcal{Q}] = 0 \,; \quad 0 \leq \mathcal{Q} \leq 1$$

$U_{\alpha\beta;\gamma\delta}$ are independent random variables with $\overline{U_{\alpha\beta;\gamma\delta}} = 0$ and $\overline{|U_{\alpha\beta;\gamma\delta}|^2} = U^2$
$N \to \infty$ yields critical strange metal.

S. Sachdev and J. Ye, PRL **70**, 3339 (1993)

A. Kitaev, unpublished; S. Sachdev, PRX **5**, 041025 (2015)



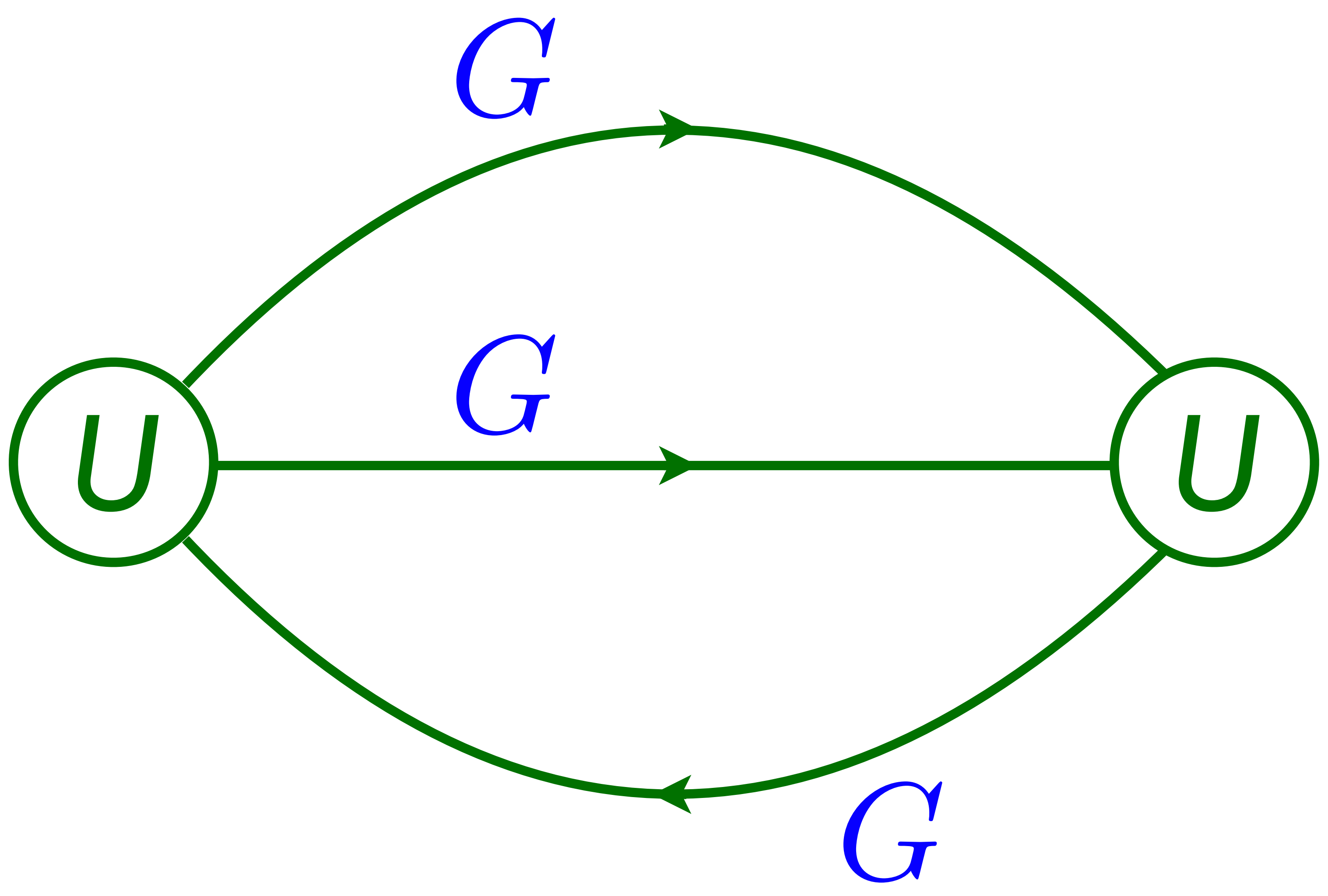


The Sachdev-Ye-Kitaev (SYK) model

Feynman graph expansion in $U_{\alpha\beta;\gamma\delta}$, and graph-by-graph average, yields exact equations in the large N limit:

$$G(i\omega) = \frac{1}{i\omega + \mu - \Sigma(i\omega)} \quad , \quad \Sigma(\tau) = -U^2 G^2(\tau) G(-\tau)$$
$$G(\tau = 0^-) = \mathcal{Q}.$$

($i\omega + \mu \to 0$ Bootstrap equations!)

$\Sigma =$ 

S. Sachdev and J. Ye, PRL **70**, 3339 (1993)

The complex SYK model

$$G_*(\tau) = -C \frac{e^{-2\pi\mathcal{E}T\tau}}{\sqrt{1+e^{-4\pi\mathcal{E}}}} \left(\frac{T}{\sin(\pi T\tau)}\right)^{1/2}.$$

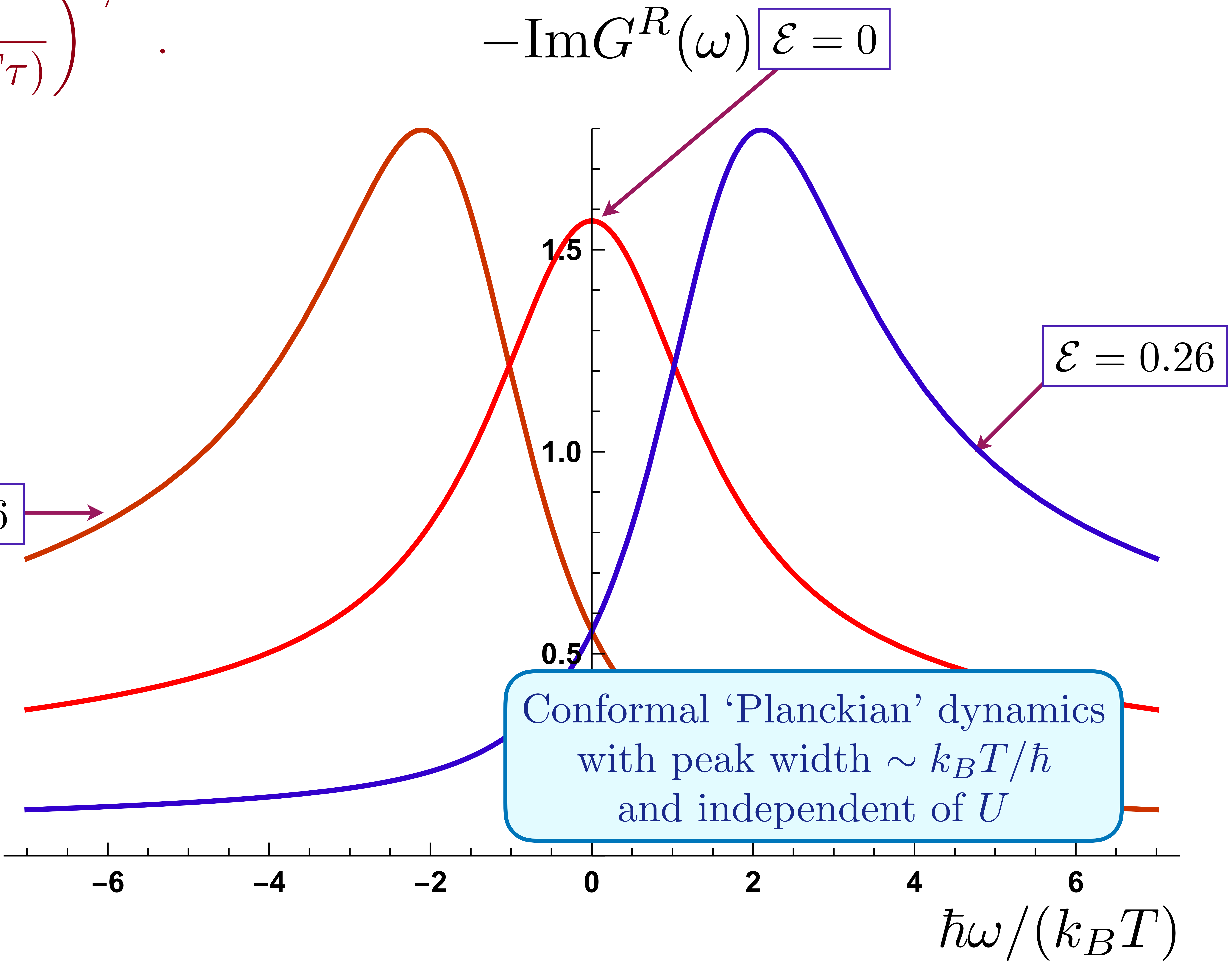
$$G_*^R(\omega) =$$

$$\frac{-iCe^{-i\theta}}{(2\pi T)^{1/2}} \frac{\Gamma\left(\frac{1}{4} - \frac{i\omega}{2\pi T} + i\mathcal{E}\right)}{\Gamma\left(\frac{3}{4} - \frac{i\omega}{2\pi T} + i\mathcal{E}\right)}.$$

$$e^{2\pi\mathcal{E}} = \frac{\sin(\pi/4+\theta)}{\sin(\pi/4-\theta)}$$

$$C = \left(\frac{\pi}{U^2\cos(2\theta)}\right)^{1/4}$$

$\mathcal{E}$ is a known function of $\mathcal{Q}$ (Luttinger relation)



S. Sachdev and J. Ye, PRL **70**, 3339 (1993)
A. Georges and O. Parcollet PRB **59**, 5341 (1999)
S. Sachdev, PRX **5**, 041025 (2015)

The Sachdev-Ye-Kitaev (SYK) model

$$\begin{aligned}
\mathcal{Z}(\mathcal{Q},T) &= \mathrm{Tr}_{\mathcal{Q}} \exp\left(-\frac{\mathcal{H}}{T}\right) = \exp(-F/T)\,; \;\; \text{Entropy } S = -\frac{\partial F}{\partial T}. \\
&\equiv \int_{E_0^-}^{\infty} D(E) e^{-E/T}\,; \quad D(E) = \sum_i \delta(E - E_i)\,; \quad \mathcal{H}\,|\Psi_i\rangle = E_i\,|\Psi_i\rangle
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A CFT in d spatial dimensions has an entropy density $S \sim T^d$. The SYK model is a 0+1 dimensional CFT, and we obtain a T-independent entropy:

$$\lim_{T\to 0} \lim_{N\to\infty} \frac{S}{N} = s_0$$

A. Georges, O. Parcollet, and S. Sachdev, PRB **63**, 134406 (2001)

The constant s_0 is a known universal number. At $\mathcal{Q} = 1/2$, we have

$$s_0 = \frac{\text{Catalan}}{\pi} + \frac{\ln 2}{4} = 0.464847699170805107492692486833\ldots\,.$$

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$$\lim_{T\to 0}\lim_{N\to\infty} \frac{S}{N} = s_0 \quad \boxed{\Rightarrow \quad D(E) \overset{?}{=} e^{N s_0}\delta(E - E_0)}$$

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Conformal Perturbation theory

$$S = S_{\rm CFT} + \sum_h \lambda_h \int_0^\beta d\tau O_h(\tau)$$

where $G_{\rm CFT} = G_* \sim {\rm sgn}(\tau)/\sqrt{|\tau|}$ and $\langle O_h(\tau) O_h(0)\rangle \sim 1/|\tau|^{2h}$

$$G(\tau) \sim \frac{{\rm sgn}(\tau)}{\sqrt{|\tau|}} \left(1 + \sum_h \frac{g_h}{|\tau|^{h-1}} + \dots \right) \quad , \quad S(T) = N \left[s_0 + \sum_h s_h T^{h-1} + \dots \right]$$

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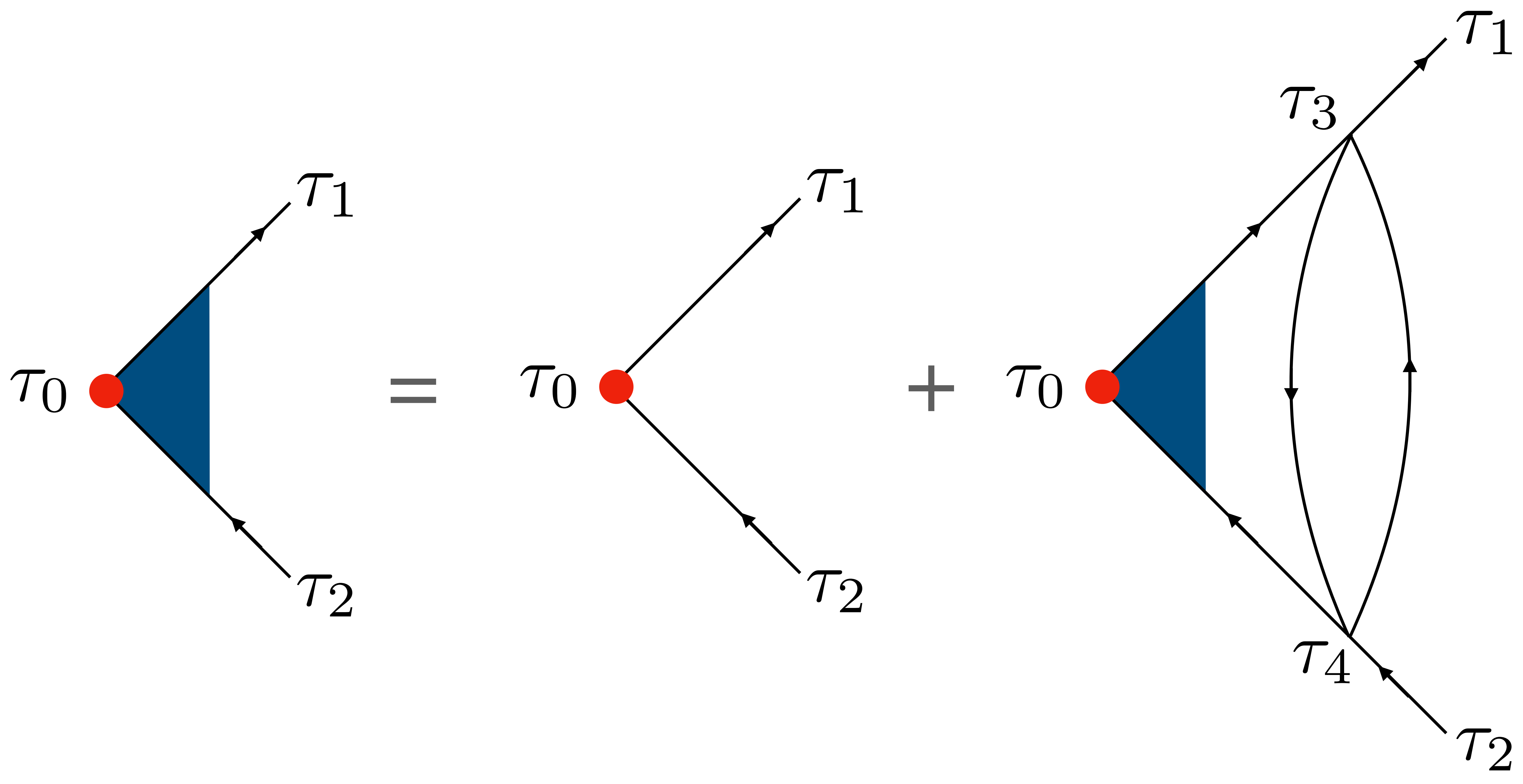

Gross, Rosenhaus (2017)
Klebanov, Tarnopolsky (2017)

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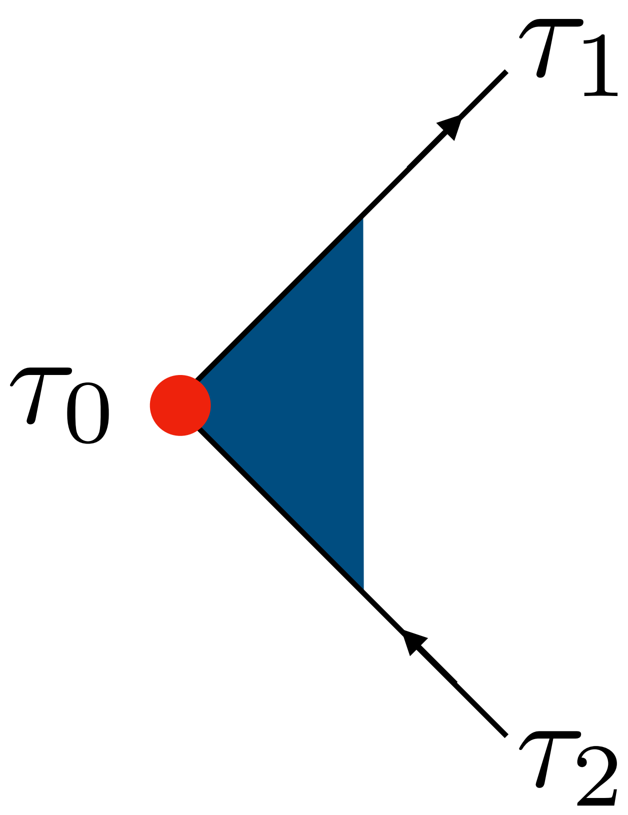
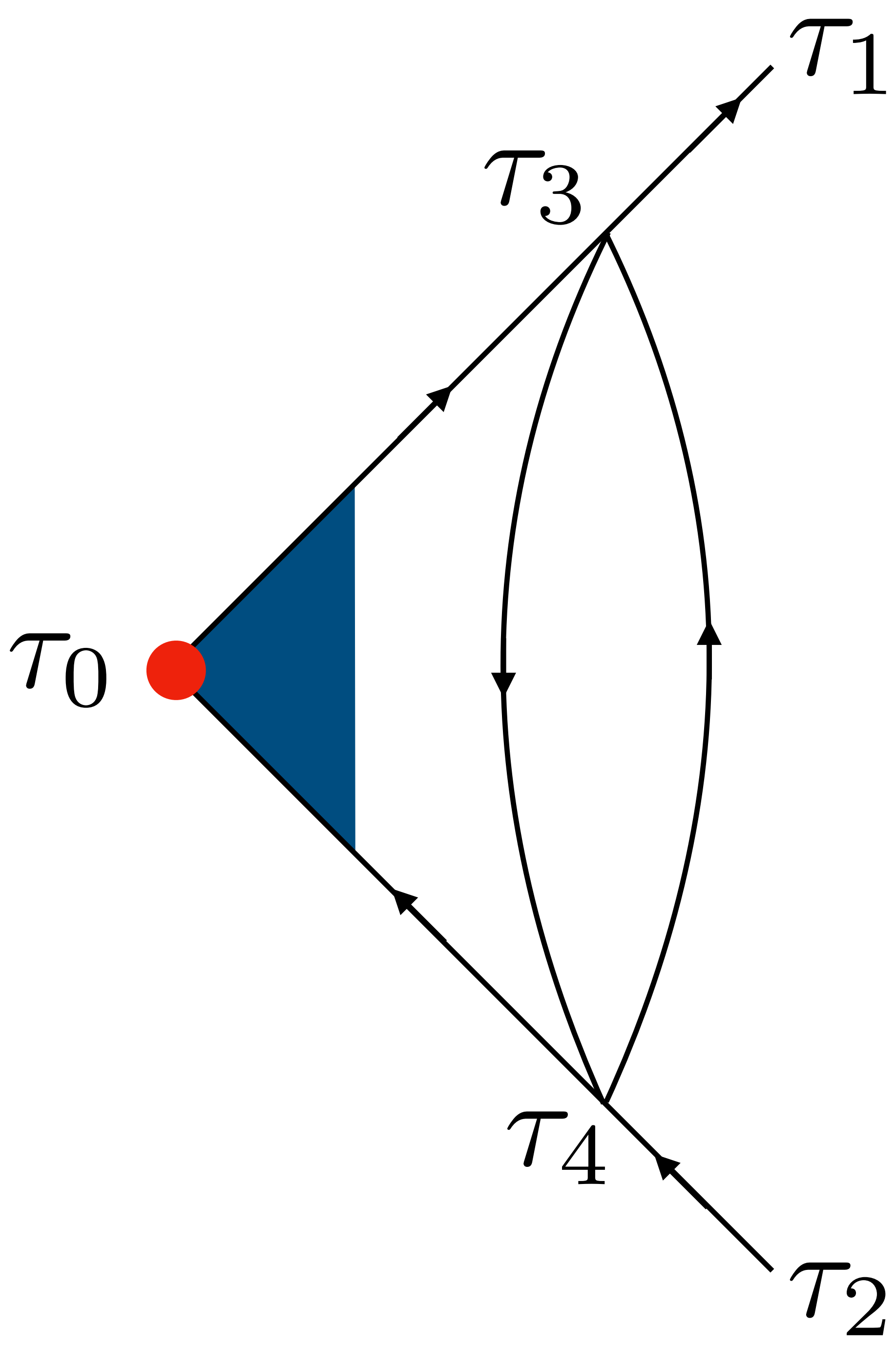
E 

$=$ 

(More bootstrap equations!)

Solution of eigenvalue equation with $E = 1$ yields a tower of O_h.

Gross, Rosenhaus (2017)
Klebanov, Tarnopolsky (2017)

Conformal Perturbation theory

We define the three point function

$$v_h(\tau_1, \tau_2, \tau_0) = \langle c(\tau_1) c^\dagger(\tau_2) O_h(\tau_0) \rangle \, .$$

In the long time scaling limit, we can drop the bare first time on the right hand side, and obtain the eigenvalue equation

$$E v(\tau_1, \tau_2, \tau_0) = \int d\tau_3 d\tau_4 K(\tau_1, \tau_2; \tau_3, \tau_4) v_h(\tau_3, \tau_4, \tau_0) \, ,$$

where the kernel K is

$$K(\tau_1, \tau_2; \tau_3, \tau_4) = -3U^2 G_*(\tau_{13}) G_*(\tau_{24}) G_*(\tau_{34})^2 \, ,$$

with $\tau_{ij} \equiv \tau_i - \tau_j$, and we are interested in the eigenvalue $E = 1$.

Conformal Perturbation theory

It is sufficient to solve the eigenvalue equation as $\tau_0 \to \infty$. Then, we can use the operator product expansion to write

$$c(\tau_1)c^\dagger(\tau_2) \sim \mathrm{sgn}(\tau_{12}) \left[\frac{1}{|\tau_{12}|^{1/2}} + \sum_h \frac{c_h}{|\tau_{12}|^{1/2-h}} O_h(\tau_1) + \ldots \right]$$

Inserting this into the defintion of v, we conclude that $v \sim \mathrm{sgn}(\tau_{12})/|\tau_{12}|^{1/2-h}$ as $\tau_0 \to \infty$. Then the eigenvalue equation simplifies to

$$E = -\frac{3\tan(\pi h/2 - \pi/4)}{2h-1} = 1 \,.$$

There are an infinite number of solutions, and the lowest values are $h = 2, 3.77354\ldots, 5.567946\ldots, 7.63197\ldots, \ldots$. Consequently, the low T behavior of the entropy is

$$S(T) = N \left[s_0 + \gamma\, T + \gamma_2\, T^{2.77354\ldots} + \ldots \right] \,.$$

The Sachdev-Ye-Kitaev (SYK) model

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$\gamma = \#/U$ is non-universal.

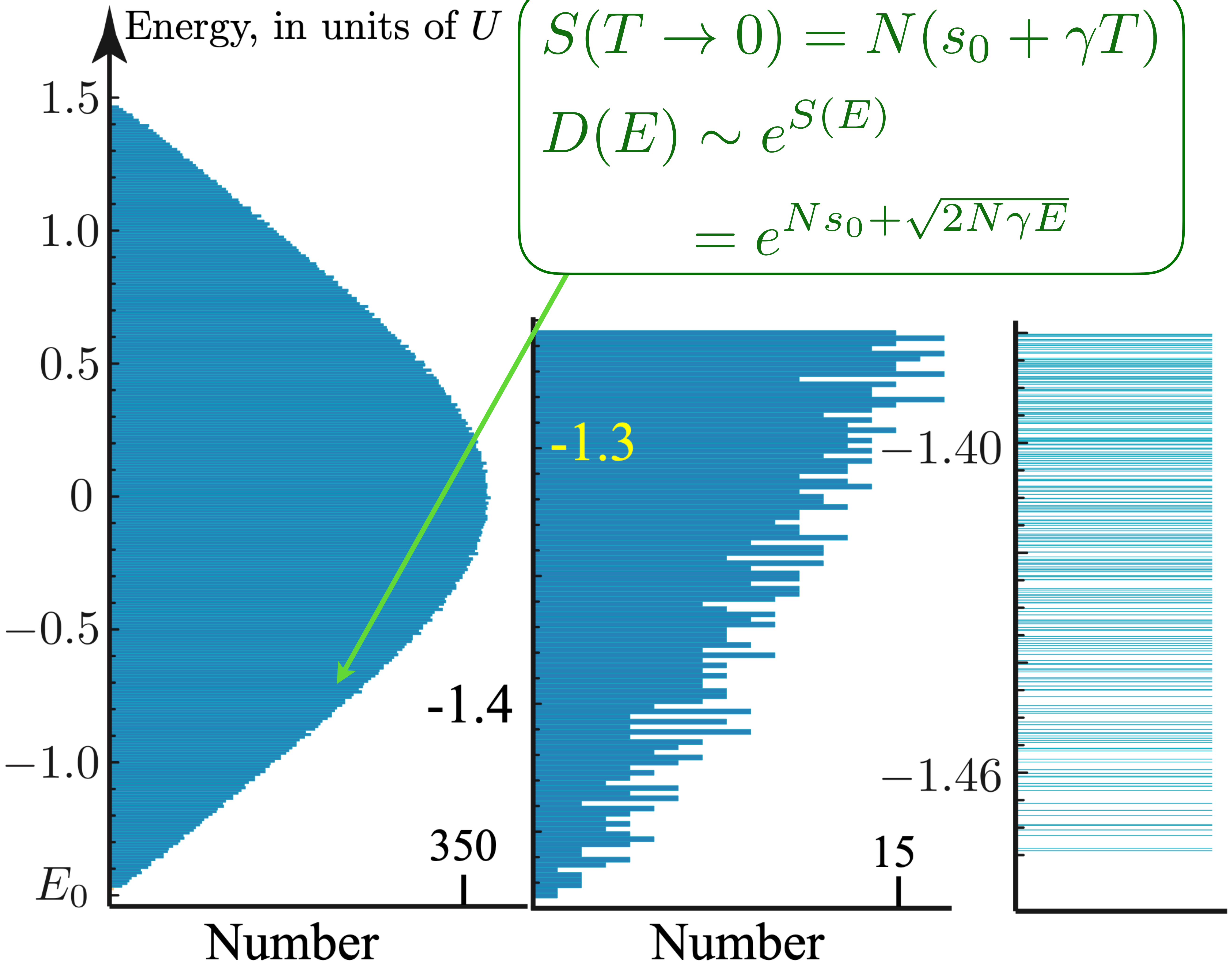
Many-body density of states

$$D(E) = \sum_i \delta(E - E_i); \quad E_0 + E_i \Rightarrow \text{Many body eigenvalue}$$

$$S(T \to 0) = N(s_0 + \gamma T)$$

$$D(E) \sim e^{S(E)}$$

$$= e^{N s_0 + \sqrt{2N\gamma E}}$$



Complex SYK model

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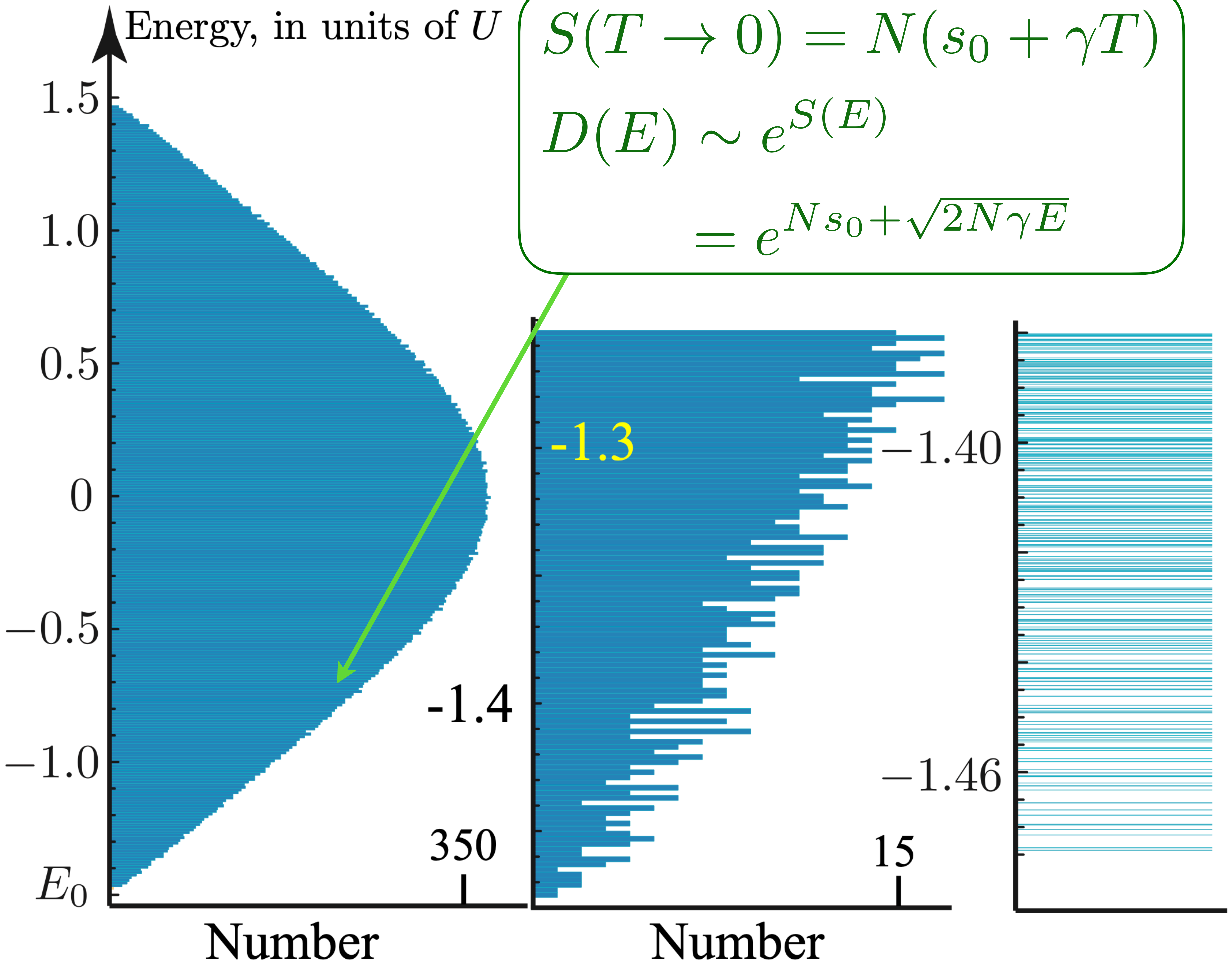

Energy level spacing $\sim e^{-N s_0}$!

Complex SYK model

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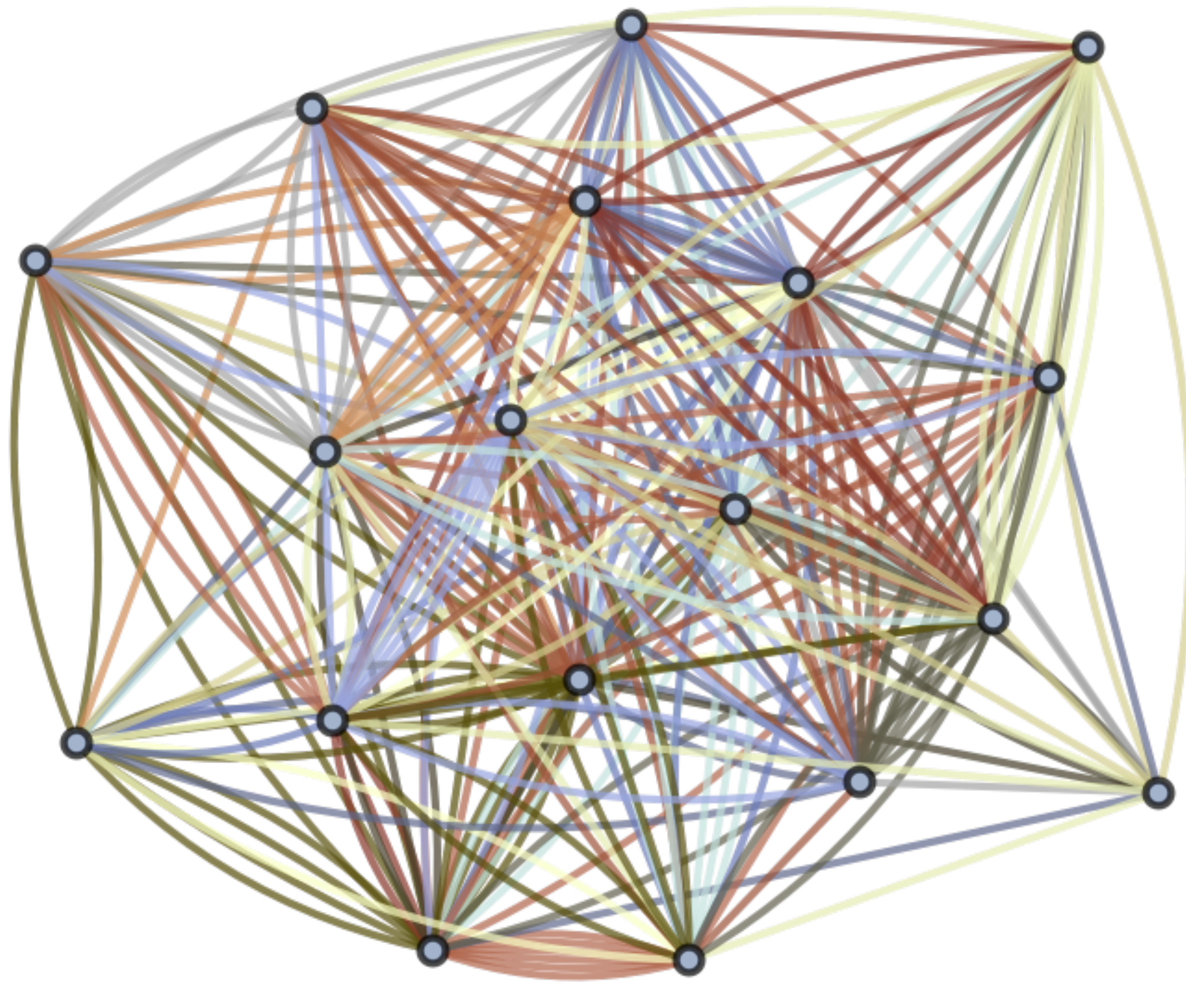
We will have a particular interest in the $h = 2$ operator. This operator is <u>irrelevant</u> for $U/N \ll T \ll U$, but <u>dangerously irrelevant</u> for $Ue^{-Ns_0} \ll T \ll U/N$.

The Sachdev-Ye-Kitaev (SYK) model



The disorder-averaged partition is exactly this G-Σ theory:

$$\mathcal{Z} = \int \mathcal{D}G(\tau_1, \tau_2)\mathcal{D}\Sigma(\tau_1, \tau_2) \exp(-NI)$$

$$I = \ln\det\left[\delta(\tau_1 - \tau_2)(\partial_{\tau_1} + \mu) - \Sigma(\tau_1, \tau_2)\right]$$
$$+ \int d\tau_1 d\tau_2 \left[\Sigma(\tau_1, \tau_2)G(\tau_2, \tau_1) + (U^2/2)G^2(\tau_2, \tau_1)G^2(\tau_1, \tau_2)\right]$$

Saddle-point equations for $G(\tau_1 - \tau_2)$ and $\Sigma(\tau_1 - \tau_2)$:

$$G(i\omega) = \frac{1}{i\omega + \mu - \Sigma(i\omega)} \quad , \quad \Sigma(\tau) = -J^2 G^2(\tau) G(-\tau)$$
$$G(\tau = 0^-) = \mathcal{Q}.$$

G-Σ path integral

At frequencies $\ll U$, the time derivative in the determinant is less important, and without it the path integral is invariant under the reparametrization and gauge transformations

$$
\begin{aligned}
\tau &= f(\sigma) \\
G(\tau_1, \tau_2) &= [f'(\sigma_1) f'(\sigma_2)]^{-1/4} \, e^{-i\phi(\sigma_1) + i\phi(\sigma_2)} \, G(\sigma_1, \sigma_2) \\
\Sigma(\tau_1, \tau_2) &= [f'(\sigma_1) f'(\sigma_2)]^{-3/4} \, e^{-i\phi(\sigma_1) + i\phi(\sigma_2)} \, \Sigma(\sigma_1, \sigma_2)
\end{aligned}
$$

where $f(\sigma)$ and $\phi(\sigma)$ are arbitrary functions.

A. Georges and O. Parcollet
PRB **59**, 5341 (1999)
A. Kitaev, 2015
S. Sachdev, PRX **5,** 041025 (2015)

G-Σ path integral

Reparametrization and phase zero modes

We find the conformal saddle point, G_*, Σ_*, and only focus on the "Nambu-Goldstone" modes associated with breaking reparameterization and U(1) gauge symmetries by writing

$$G(\tau_1, \tau_2) = [f'(\tau_1) f'(\tau_2)]^{1/4} G_*(f(\tau_1) - f(\tau_2)) e^{i\phi(\tau_1) - i\phi(\tau_2)}$$

(and similarly for Σ). Then the path integral is approximated by

$$\mathcal{Z} = \int \mathcal{D}f(\tau) \mathcal{D}\phi(\tau) e^{-E_0/T + N s_0 - N I_{\text{eff}}[f, \phi]},$$

where $E_0 \propto N$ is the ground state energy.

A. Kitaev (2015); J. Maldacena and D. Stanford, arXiv:1604.07818;
R. Davison, Wenbo Fu, A. Georges, Yingfei Gu, K. Jensen, S. Sachdev, arXiv.1612.00849;
S. Sachdev, PRX **5,** 041025 (2015)

$$
\begin{aligned}
\mathcal{Z} &= \operatorname{Tr} \exp\left(-\frac{\mathcal{H}}{k_B T}\right) \\
&\approx \exp\left(N\frac{s_0}{k_B}\right) \int \frac{\mathcal{D}f(\tau)\mathcal{D}\phi(\tau)}{||\mathrm{SL(2,R)}||} \exp\left(-\frac{1}{\hbar} I_{\text{eff}}\left[f(\tau), \phi(\tau)\right]\right)
\end{aligned}
$$

$$
I_{\text{eff}}[f,\phi] = \frac{NK}{2}\int_0^{1/T} d\tau (\partial_\tau \phi + i(2\pi\mathcal{E}T)\partial_\tau f)^2 - \frac{N\gamma}{4\pi^2}\int_0^{1/T} d\tau \left\{\tan(\pi T f(\tau)), \tau\right\},
$$

where $f(\tau)$ is a monotonic map from $[0, 1/T]$ to $[0, 1/T]$. The conformal group is d spatial dimensions is SO$(d+2, 1)$, and PSL(2,R)$\cong$SO(2,1), and the Schwarzian

$$
\{g,\tau\} \equiv \frac{g'''}{g'} - \frac{3}{2}\left(\frac{g''}{g'}\right)^2 ,
$$

vanishes for $g(\tau) = (a\tau + b)/(c\tau + d)$ a SL(2,R) transformation $(ad - bc = 1)$. The couplings K, γ, and $\mathcal{E}$ can be related to thermodynamic derivatives.

A. Kitaev (2015); J. Maldacena and D. Stanford, arXiv:1604.07818;
R. Davison, Wenbo Fu, A. Georges, Yingfei Gu, K. Jensen, S. Sachdev, arXiv.1612.00849;
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Conformal perturbations of saddle-point theory:

$$\boxed{S = N(s_0 + \gamma T) \quad \Rightarrow \quad D(E) \sim \exp\left(N s_0 + \sqrt{2N\gamma E}\right)}$$

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Exact path integral over Schwarzian quantum gravity theory:

$$\boxed{S = N(s_0 + \gamma T) - \frac{3}{2}\ln\left(\frac{U}{T}\right) - \frac{\ln N}{2} \quad \Rightarrow \quad D(E) \sim N^{-1} \exp\left(N s_0\right) \sinh(\sqrt{2N\gamma E})}$$

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A. Georges, O. Parcollet, and S. Sachdev, PRB **63**, 134406 (2001)

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J. S. Cotler et al., JHEP 05 (2017) 118

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$$\equiv \int_{E_0^-}^{\infty} D(E) e^{-E/T}\,; \quad D(E) = \sum_i \delta(E - E_i)\,; \quad \mathcal{H}\,|\Psi_i\rangle = E_i\,|\Psi_i\rangle$$

Conformal perturbations of saddle-point theory:

$$\boxed{S = N(s_0 + \gamma T) \quad \Rightarrow \quad D(E) \sim \exp\left(N s_0 + \sqrt{2N\gamma E}\right)}$$

$\gamma = \#/U$ is non-universal.

Yingfei Gu, A. Kitaev, S. Sachdev, and G. Tarnopolsky, JHEP 02 (2020) 157

A. Georges, O. Parcollet, and S. Sachdev, PRB **63**, 134406 (2001)

J. S. Cotler et al., JHEP 05 (2017) 118

Exact path integral over Schwarzian quantum gravity theory:

$$\boxed{S = N(s_0 + \gamma T) - \frac{3}{2}\ln\left(\frac{U}{T}\right) - \frac{\ln N}{2} \quad \Rightarrow \quad D(E) \sim N^{-1} \exp\left(N s_0\right) \sinh(\sqrt{2N\gamma E})}$$

Many-body density of states

$$D(E) = \sum_i \delta(E - E_i); \quad E_0 + E_i \Rightarrow \text{Many body eigenvalue}$$

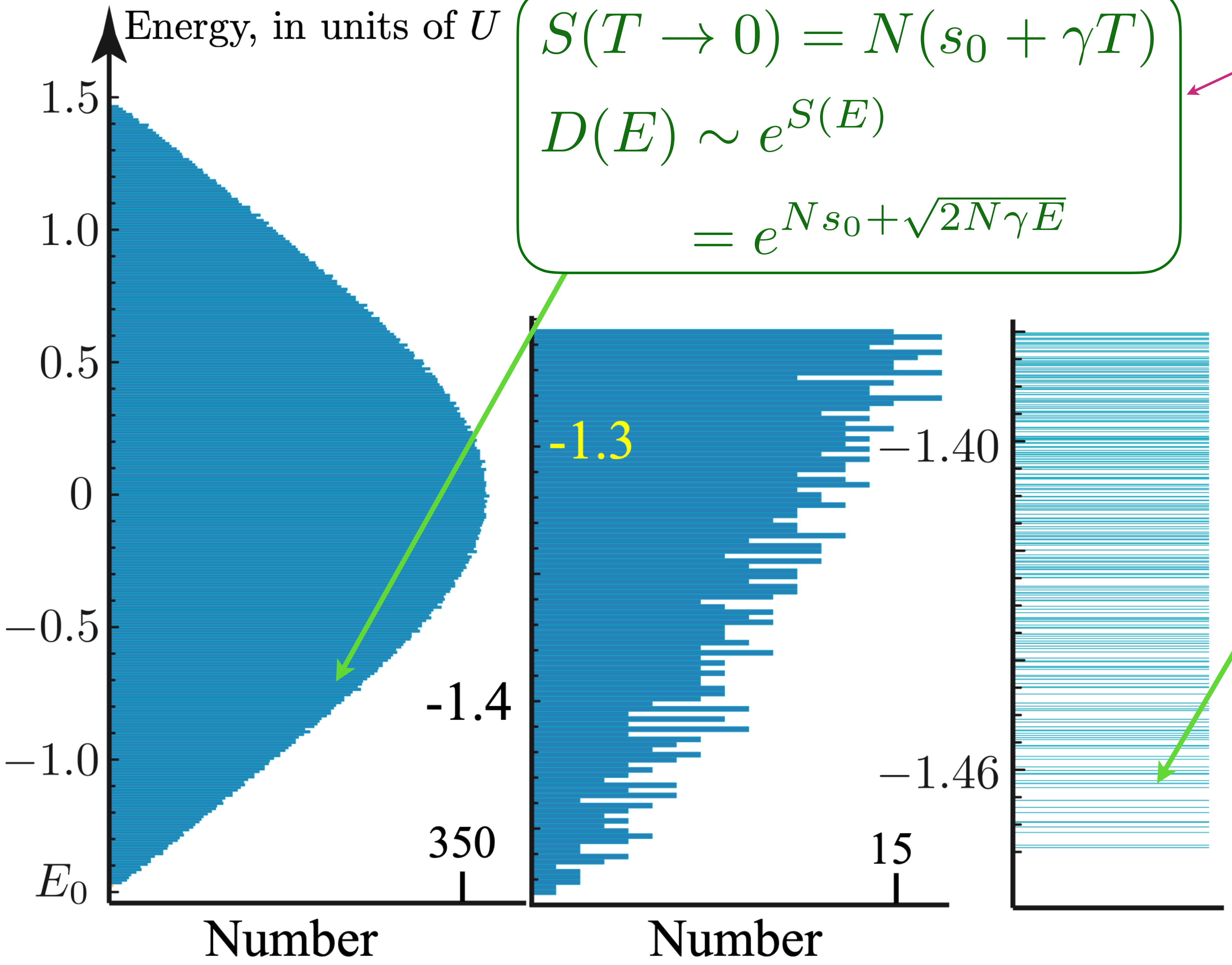
$$D(E) \sim N^{-1}\, e^{N s_0} \sinh(\sqrt{2N\gamma E})$$

$$S(T \to 0) = N(s_0 + \gamma T)$$

$$D(E) \sim e^{S(E)}$$

$$= e^{N s_0 + \sqrt{2N\gamma E}}$$

$$D(E) \sim e^{N s_0} \sqrt{2\gamma E/N}$$



Complex SYK model

Many-body density of states

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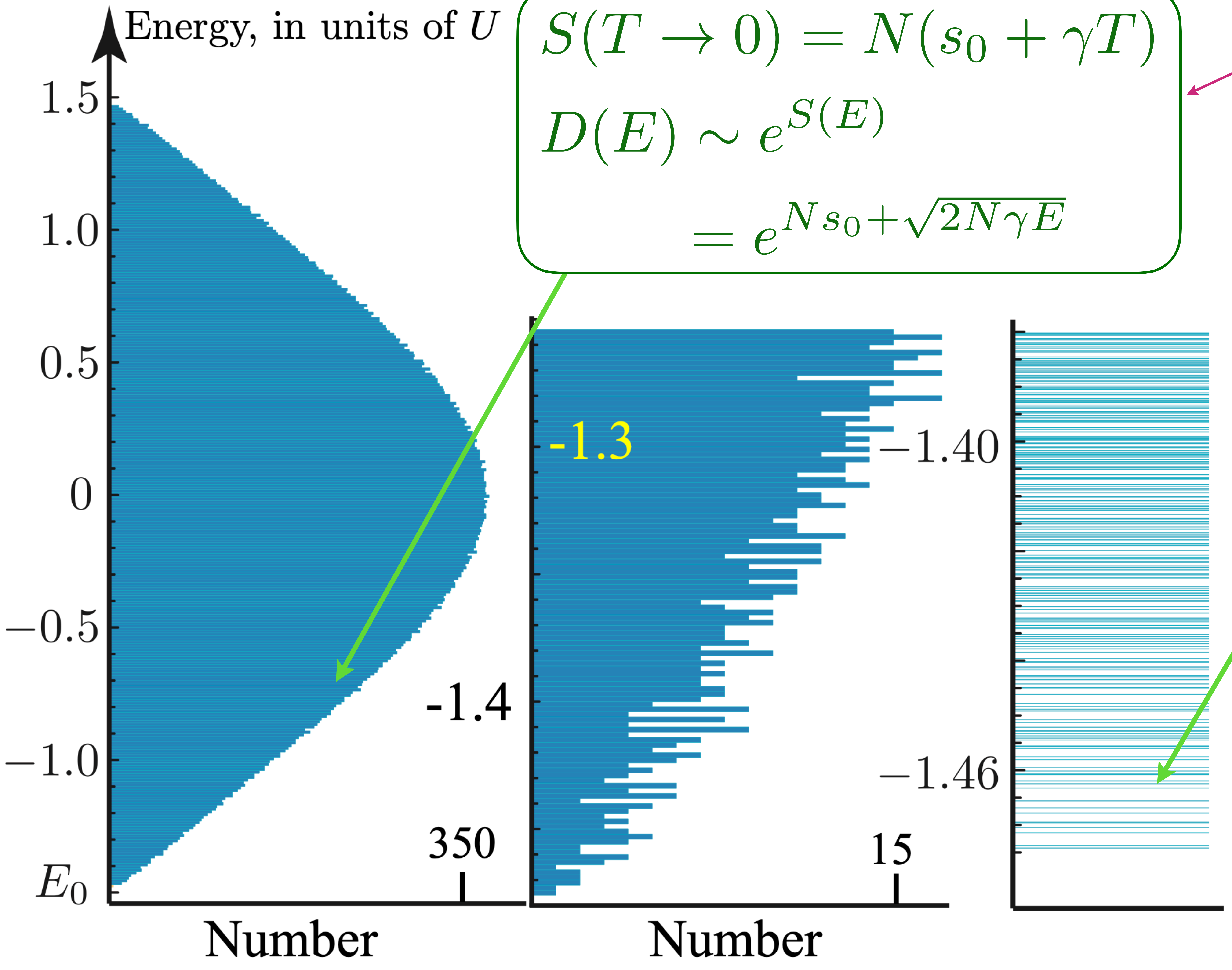


$$D(E) \sim e^{N s_0} \sqrt{2 \gamma E / N}$$

No exponentially large degeneracy, but exponentially small level spacing!
No quasiparticle decomposition: wavefunctions change chaotically from one state to the next.

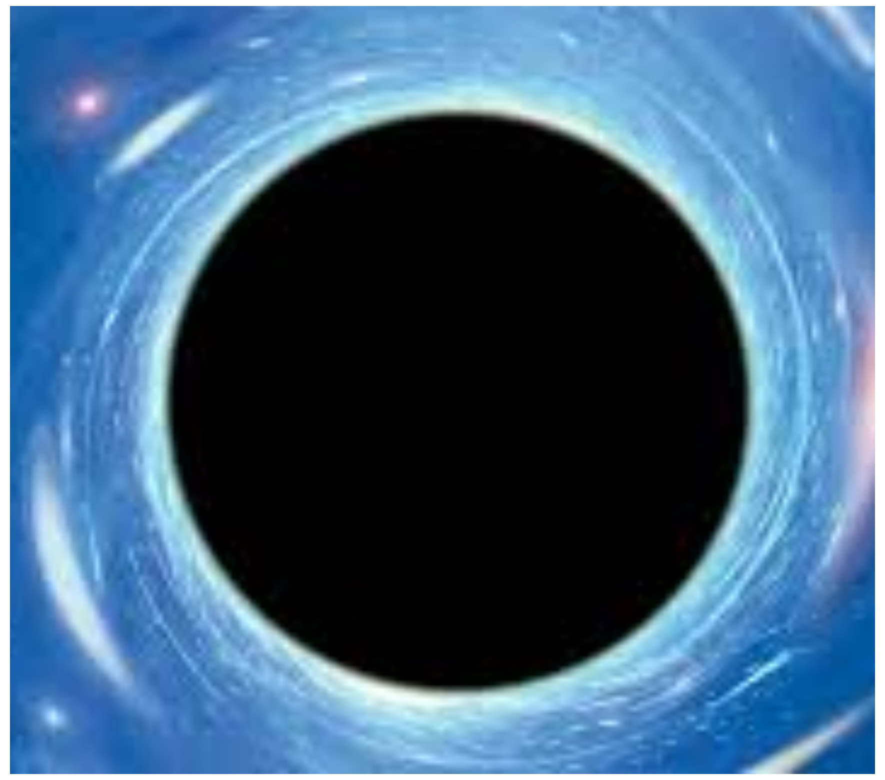
Complex SYK model

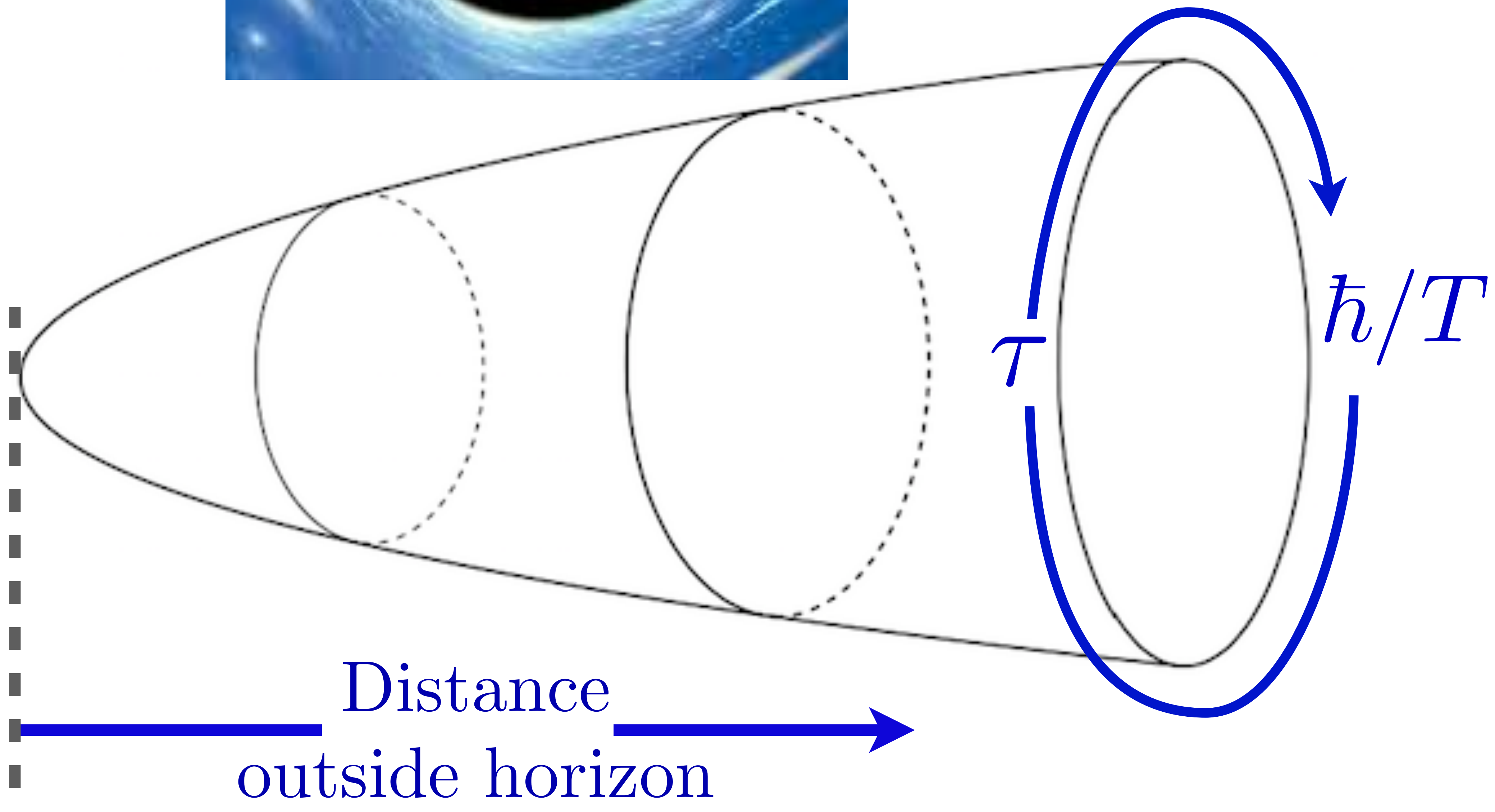
Charged black holes

S.S., arXiv:2205.02285

Thermodynamics of quantum black holes with charge $\mathcal{Q}$:

$$\mathcal{Z}(\mathcal{Q},T) = \int \mathcal{D}g_{\mu\nu}\mathcal{D}A_\mu \exp\left(-\frac{1}{\hbar}\, I^{(3+1)}_{\text{Einstein gravity+Maxwell EM}}[g_{\mu\nu},A_\mu]\right)$$





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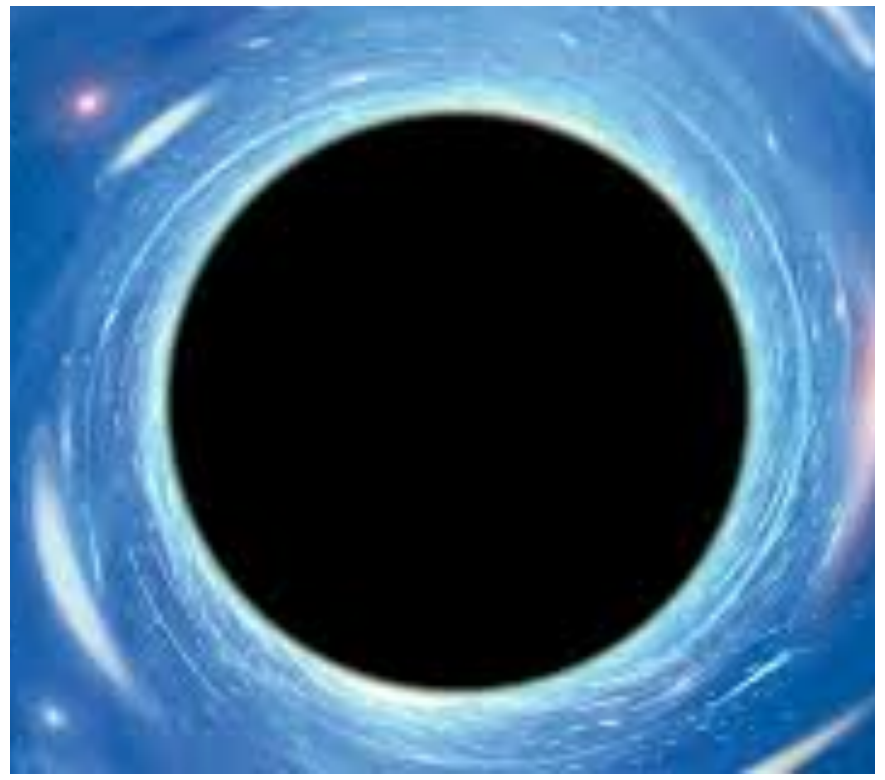
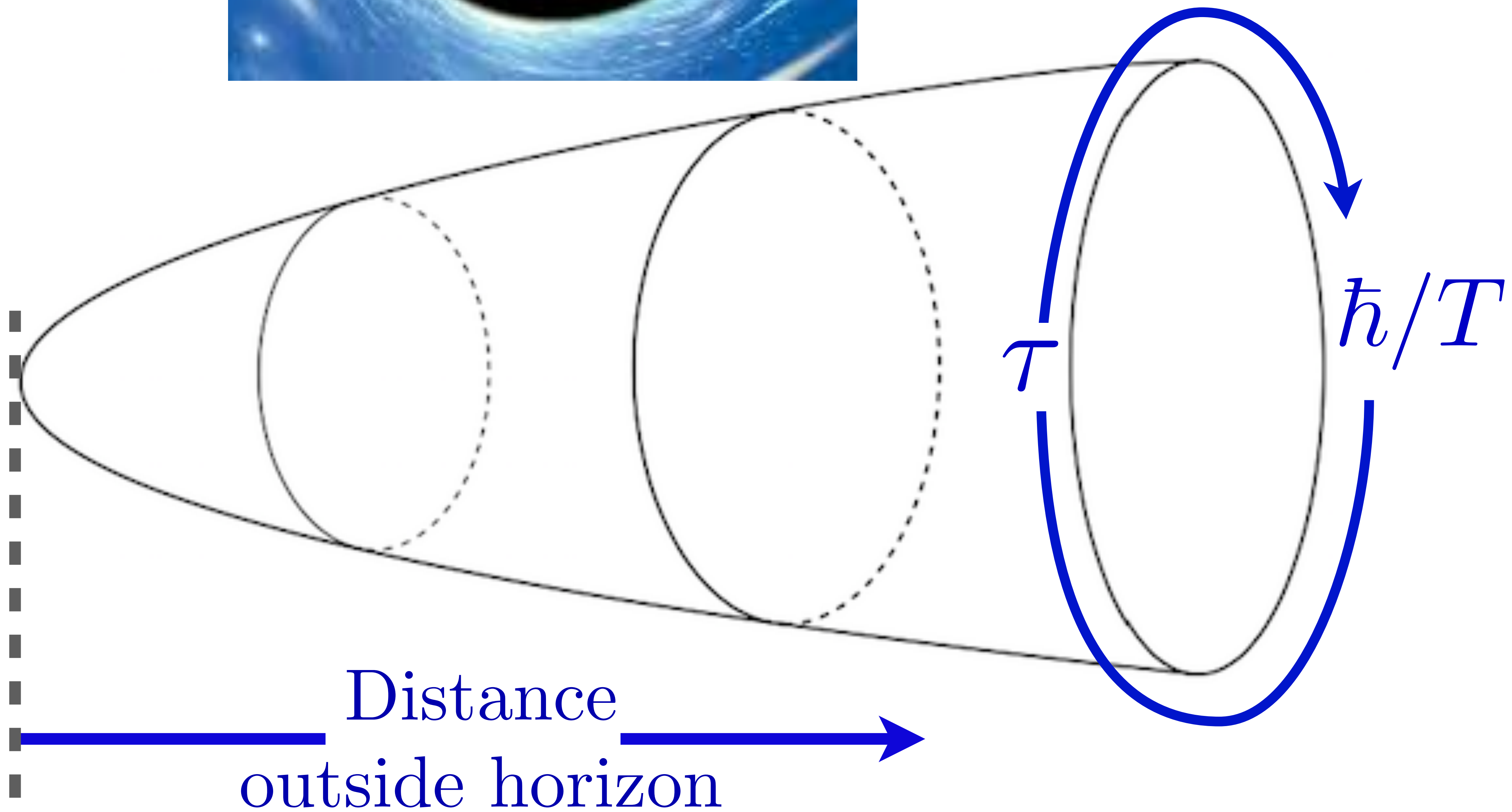
$$= \exp\Big(S_{BH}\Big) \times \left(\quad ....????... \quad \right)$$

Gibbons, Hawking (1977)
Chambin, Emparan, Johnson, Myers (1999)

$$S_{BH}(T\to 0, \mathcal{Q}) = \frac{A(T)c^3}{4G\hbar} = \frac{A_0 c^3}{4G\hbar}\left(1 + \frac{2(\pi A_0)^{1/2}T}{\hbar c}\right)$$



$A_0 = 2G\mathcal{Q}^2/c^4$ is the area of the charged black hole horizon at $T=0$.



Obtained from the saddle-point of the gravity path integral in the imaginary time spacetime outside the black hole.

Thermodynamics of quantum black holes with charge $\mathcal{Q}$:

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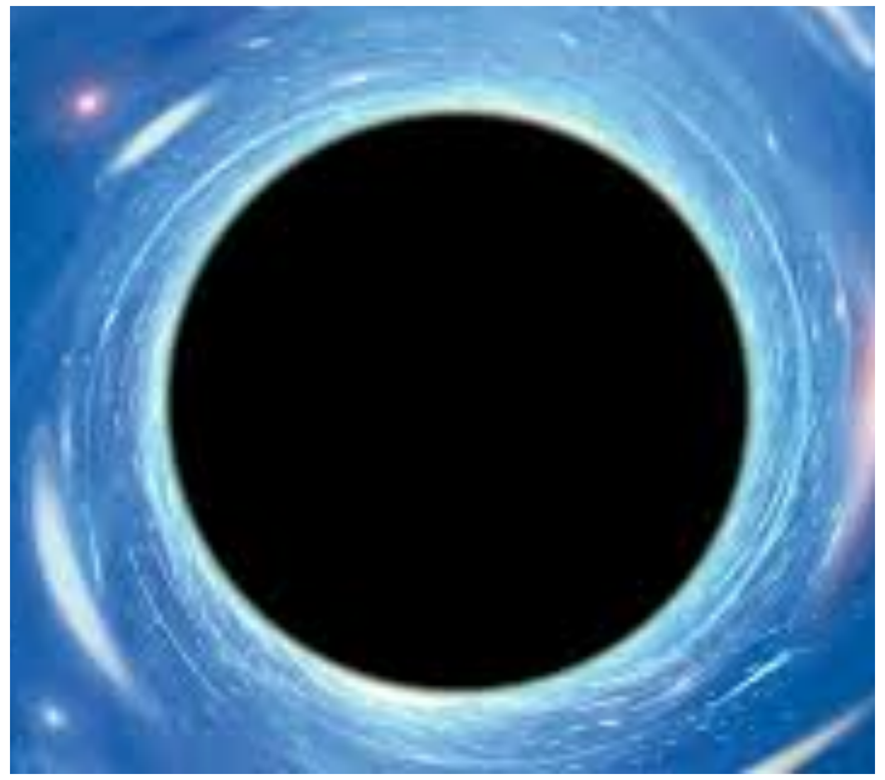

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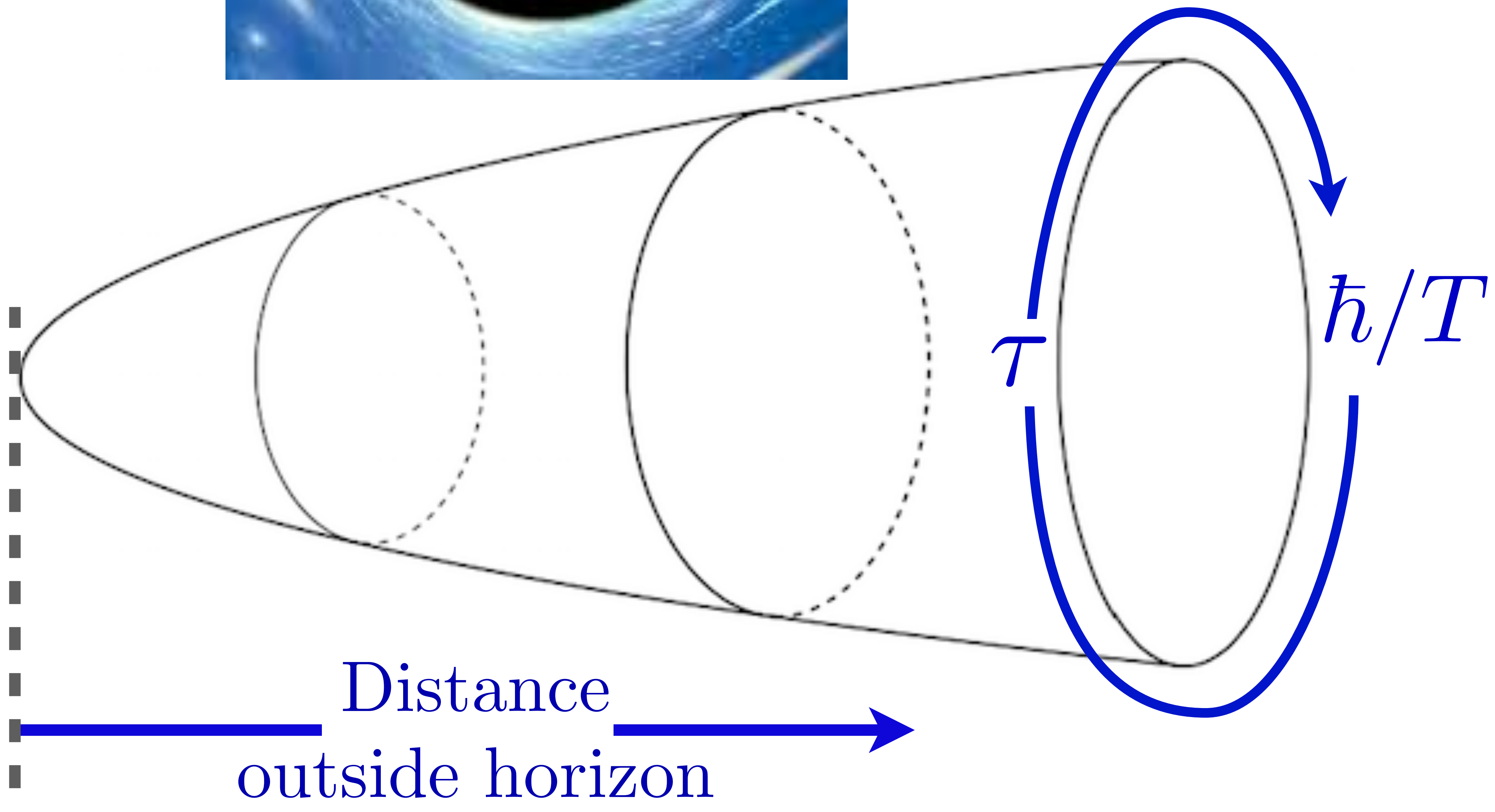
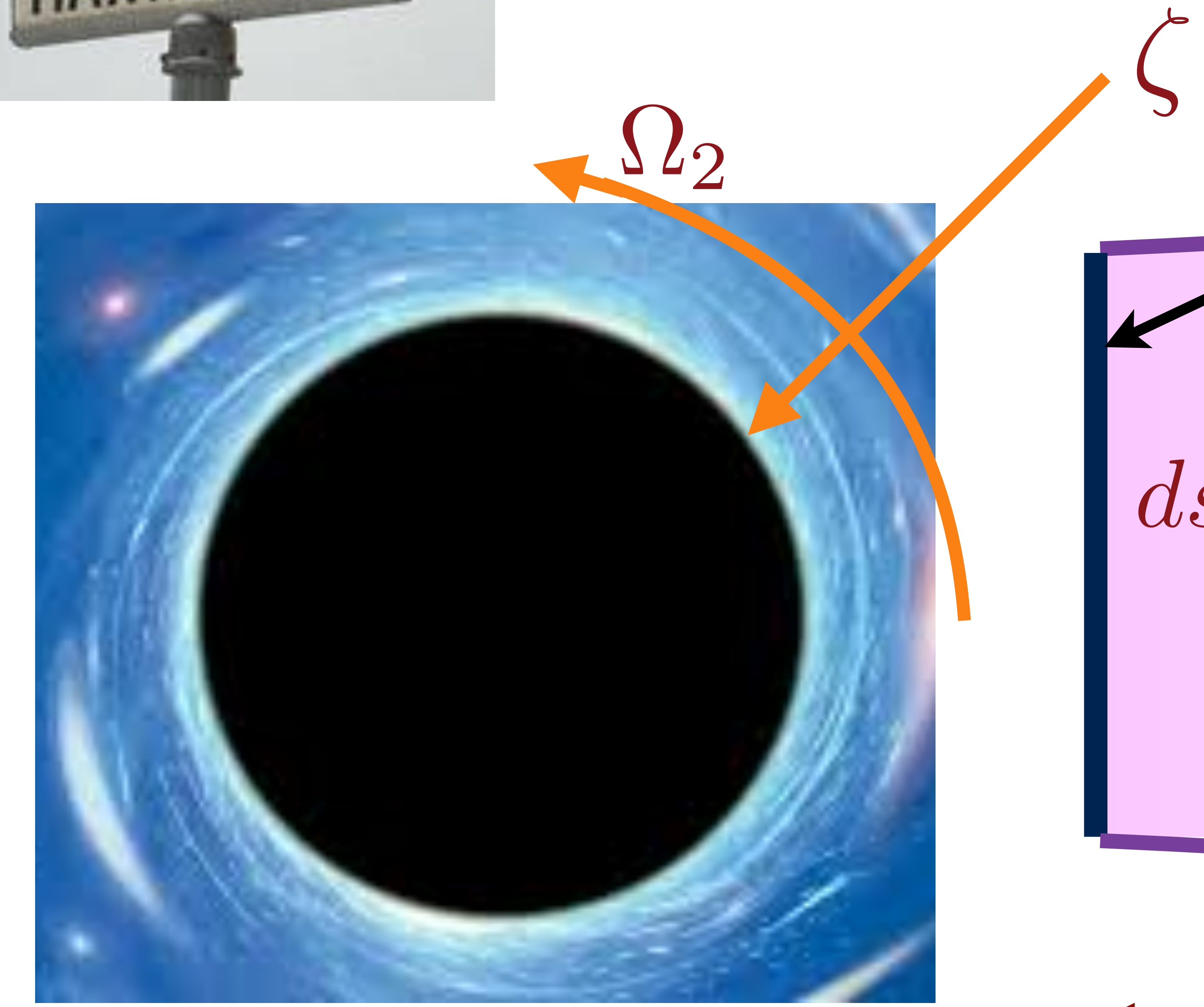
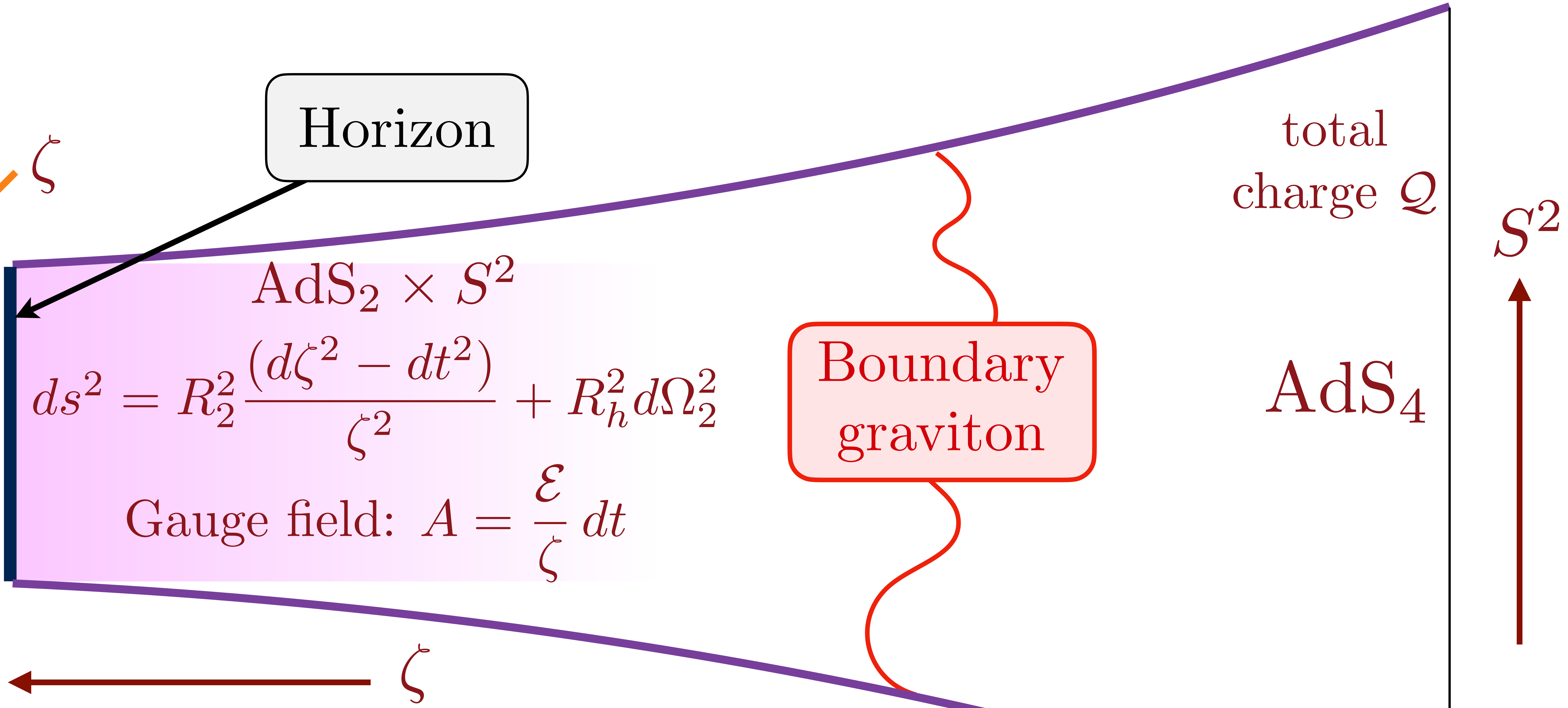


Note the similarity to the large *N* entropy of the SYK model ! (along with other similarities)

Sachdev PRL 2010

Reissner-Nordstrom black hole of Einstein-Maxwell theory





Dimensional reduction from 3+1 dimensions to 1+1 dimensions (AdS_2) at low energies!

The isometry group of AdS_2 is the 0+1 dimensional conformal group SL(2,R).

Faulkner, Liu, McGreevy, Vegh 2009; Sachdev 2010

Thermodynamics of quantum black holes with charge $\mathcal{Q}$:

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Saddle-point:

$$S_{BH}(T\to 0,\mathcal{Q}) = \frac{A(T)c^3}{4G\hbar} = \frac{A_0c^3}{4G\hbar}\left(1+\frac{2(\pi A_0)^{1/2}T}{\hbar c}\right)$$

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Maldacena, Stanford, Yang (2016)

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$$\approx \exp\left(\frac{A_0 c^3}{4\hbar G}\right)\int \mathcal{D}g_{\mu\nu}\mathcal{D}A_\mu \exp\left(-\frac{1}{\hbar}\, I^{(1+1)}_{\text{JT gravity of AdS}_2\text{+boundary graviton}}[g_{\mu\nu},A_\mu]\right)$$

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$$= \int \mathcal{D}f(\tau)\mathcal{D}\phi(\tau)\exp\left(-\frac{1}{\hbar} I_{\text{SYK}}[\text{time reparameterizations } f(\tau), \text{ phase rotations } \phi(\tau)]\right)$$

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Sachdev (2010); Kitaev (2015); Sachdev (2015); Bagrets, Altland, Kamenev (2016); Maldacena, Stanford, Yang (2016); Moitra, Trivedi, Vishal (2018) ; Gaikwad, Joshi, Mandal, Wadia (2018); Sachdev (2019); Iliesiu, Turaci (2020)

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$$S(T\to 0,\mathcal{Q}) = \frac{A(T)c^3}{4G\hbar} - \frac{3}{4}\ln\left(\frac{\hbar c^5}{GT^2}\right)$$

The $\ln T$ term is the SYK/boundary-graviton correction to Bekenstein-Hawking.

There is also a

$$-\frac{559}{180}\ln\left(\frac{A_0 c^3}{\hbar G}\right)$$

term from other massless modes; Sen (2011) Iliesiu, Murthy, Turiaci (2022)

Sachdev (2010); Kitaev (2015); Sachdev (2015); Bagrets, Altland, Kamenev (2016); Maldacena, Stanford, Yang (2016); Moitra, Trivedi, Vishal (2018) ; Gaikwad, Joshi, Mandal, Wadia (2018); Sachdev (2019); Iliesiu, Turaci (2020)

Black hole questions and answers

Can we find a quantum simulation of the inside of a black hole whose $D(E)$ matches the Bekenstein-Hawking entropy computed outside the black hole?

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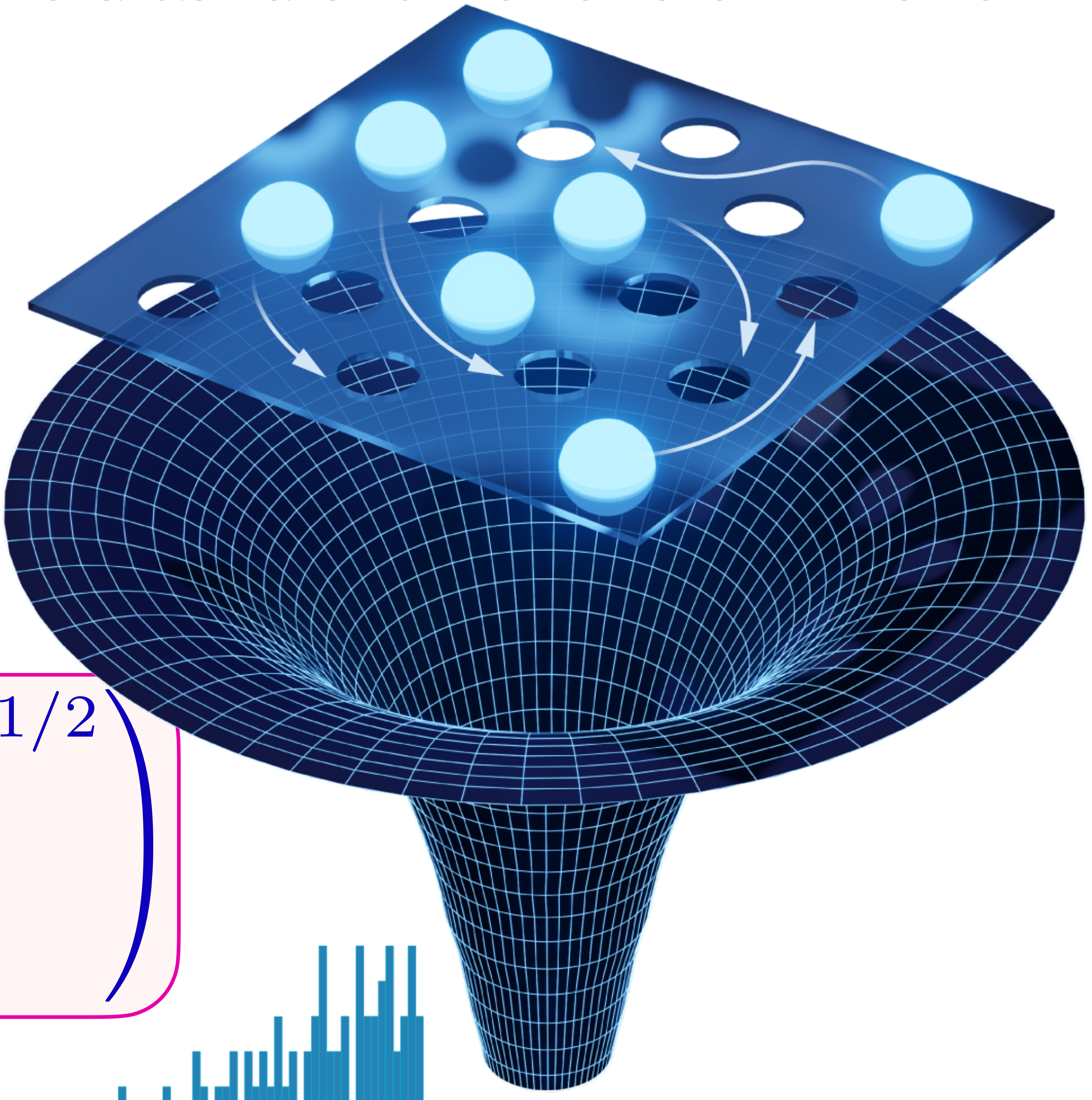
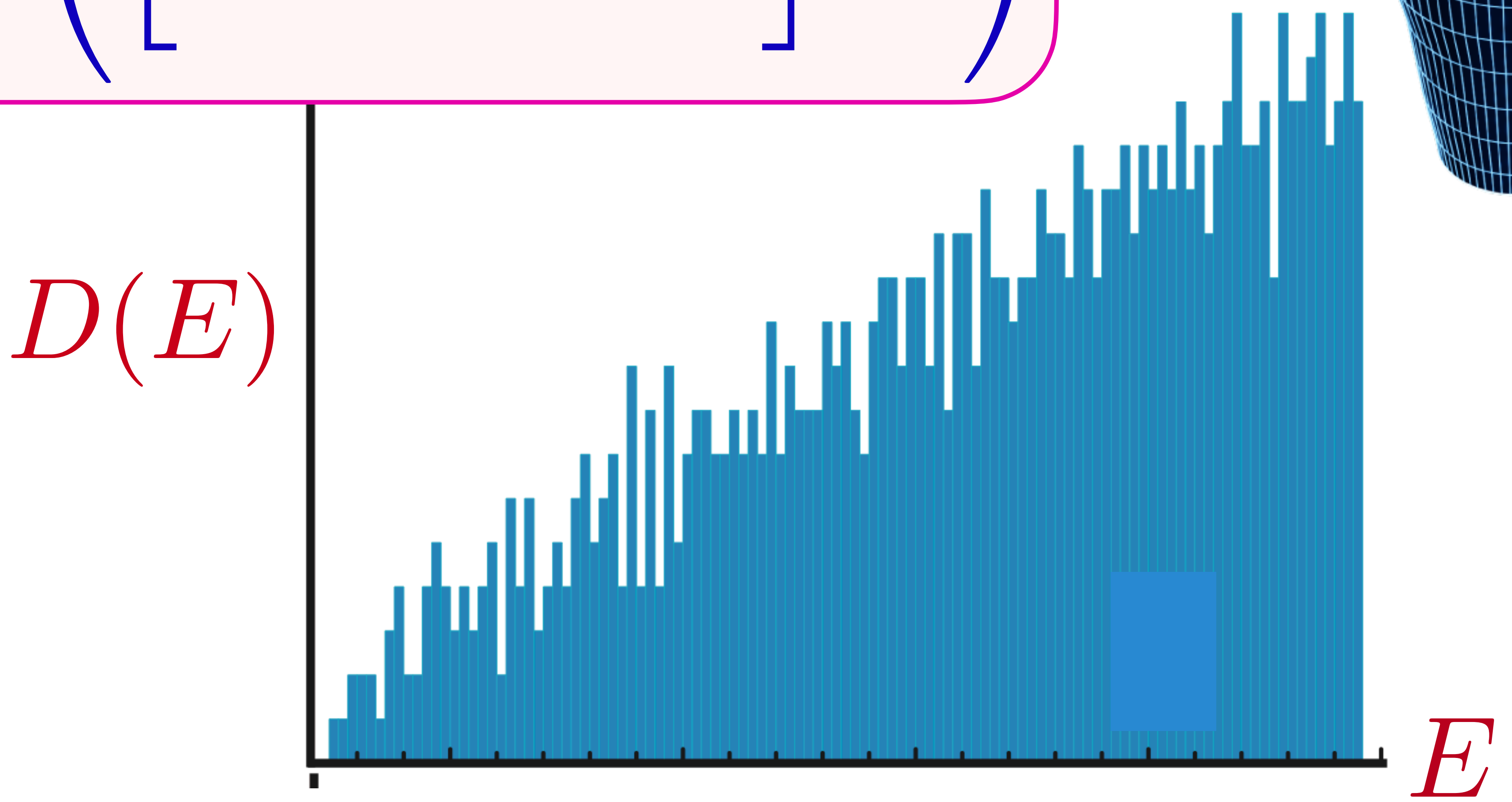
Yes, for charged black holes:



- For generic charged black holes in 3+1 dimensions, the SYK model yields, in terms of $A_0 = 2GQ^2/c^4$ the horizon area at $T=0$:

$$\boxed{D(E) \sim \left(\frac{A_0 c^3}{\hbar G}\right)^{-559/180} \exp\left(\frac{A_0 c^3}{4\hbar G}\right) \sinh\left(\left[\frac{\sqrt{\pi} A_0^{3/2} c^2}{\hbar^2 G} E\right]^{1/2}\right)}$$

There is no degeneracy, but an exponentially small level spacing down to the ground state.



Black hole questions and answers

Can we find a quantum simulation of the inside of a black hole whose $D(E)$ matches the Bekenstein-Hawking entropy computed outside the black hole?

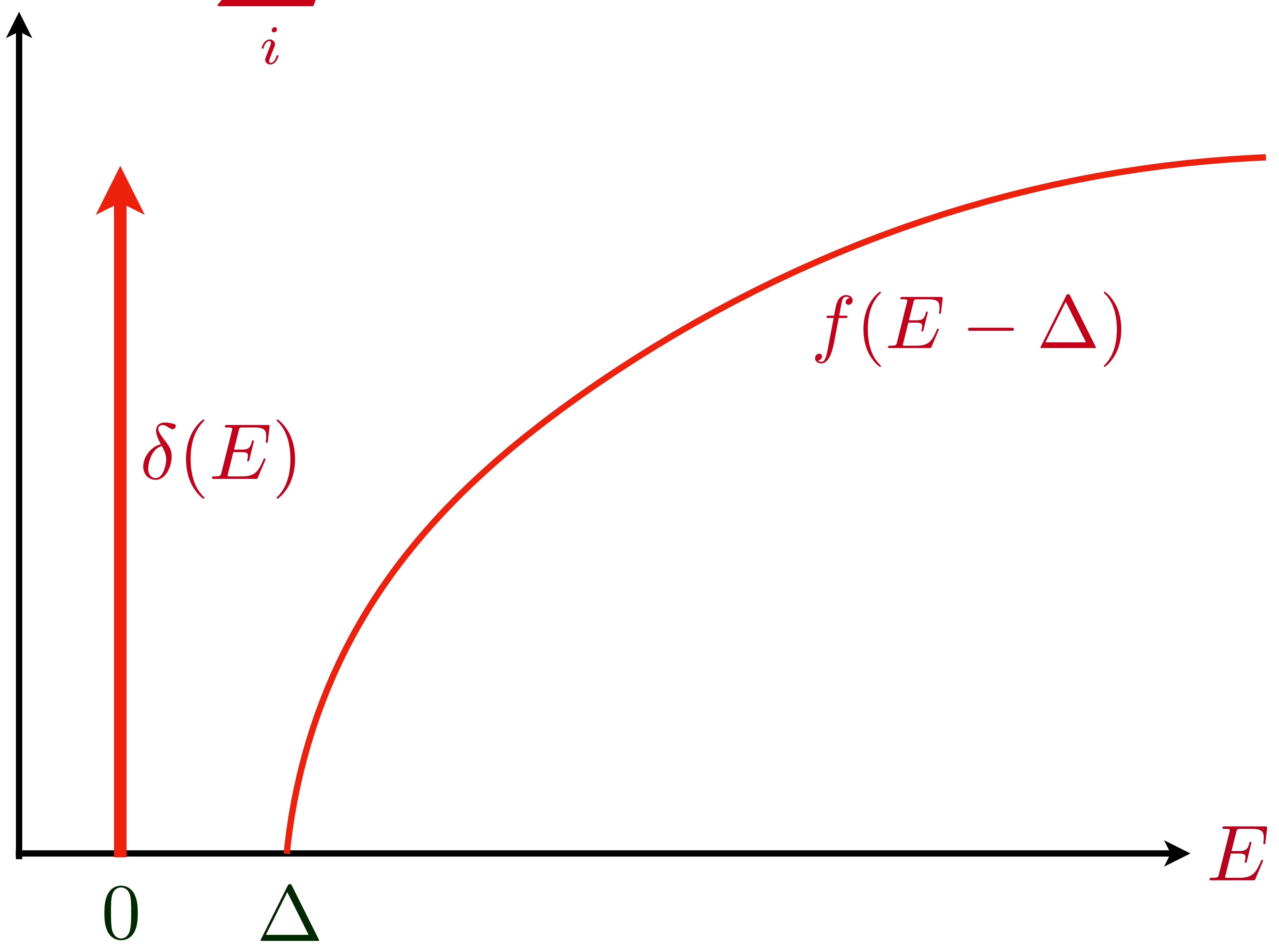
Yes, for charged black holes:

- With sufficient low energy supersymmetry, string theory yields:

$$D(E) = \exp\left(\frac{A_0 c^3}{4\hbar G}\right)\delta(E) + \theta(E-\Delta)f(E-\Delta) + \ldots$$

There are exponentially many degenerate BPS ground states, and an energy gap Δ above the ground state.

$$D(E) = \sum_i \delta(E - E_i)$$



M. Heydeman, L.V. Iliesiu, G. J. Turiaci, and W. Zhao, 2020
L.V. Iliesiu, S. Murthy, G. J. Turiaci, 2022

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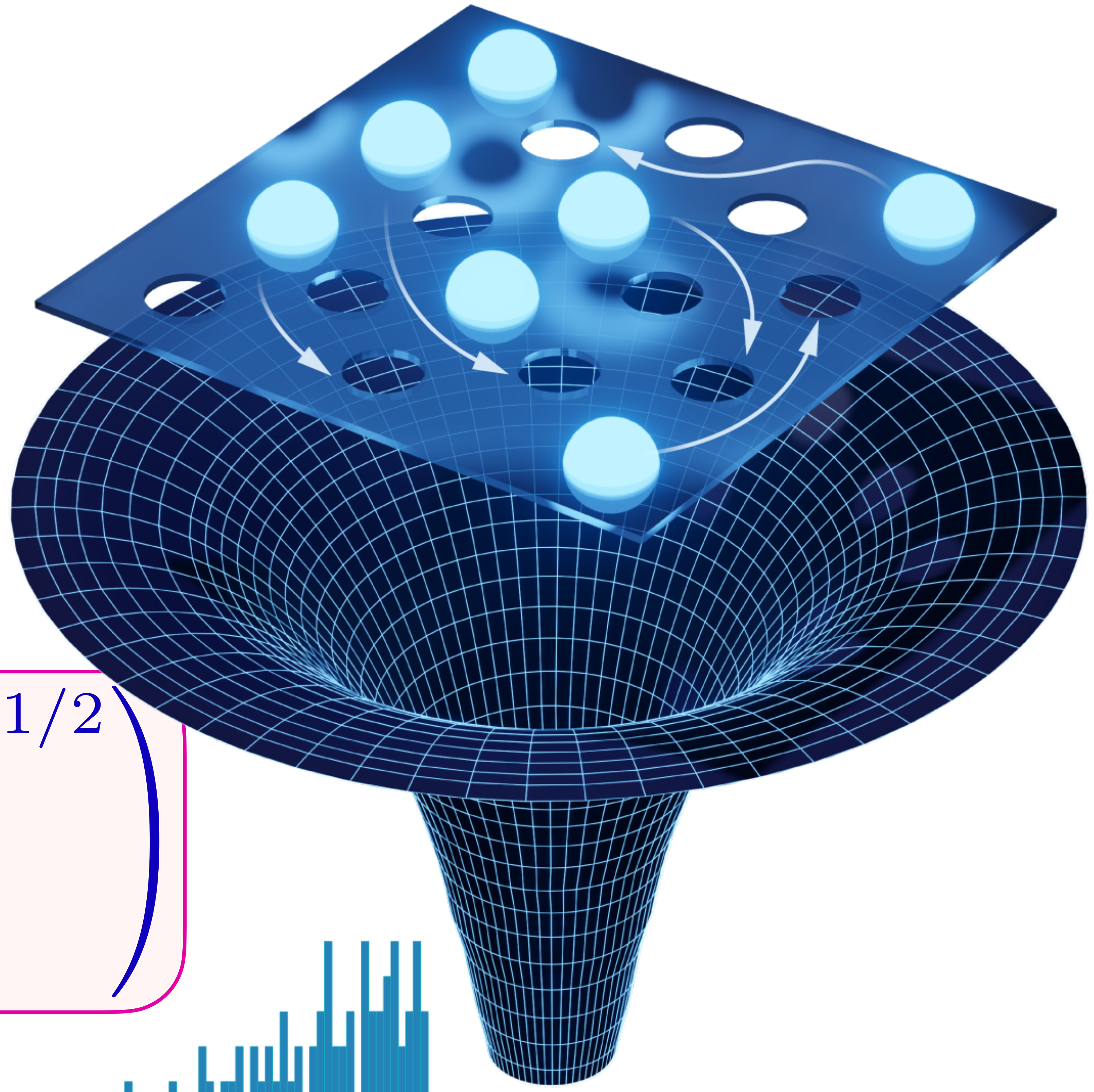

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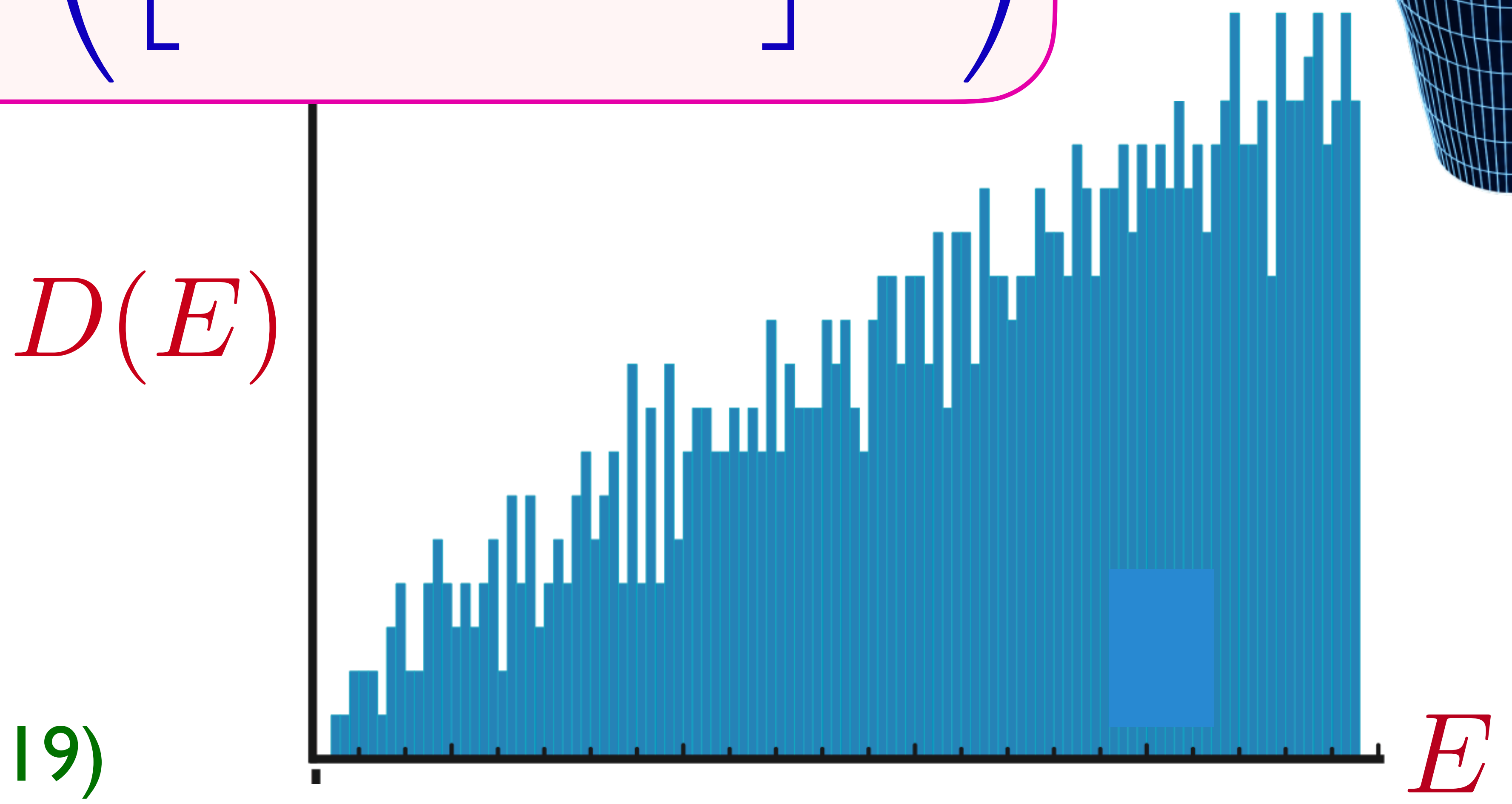
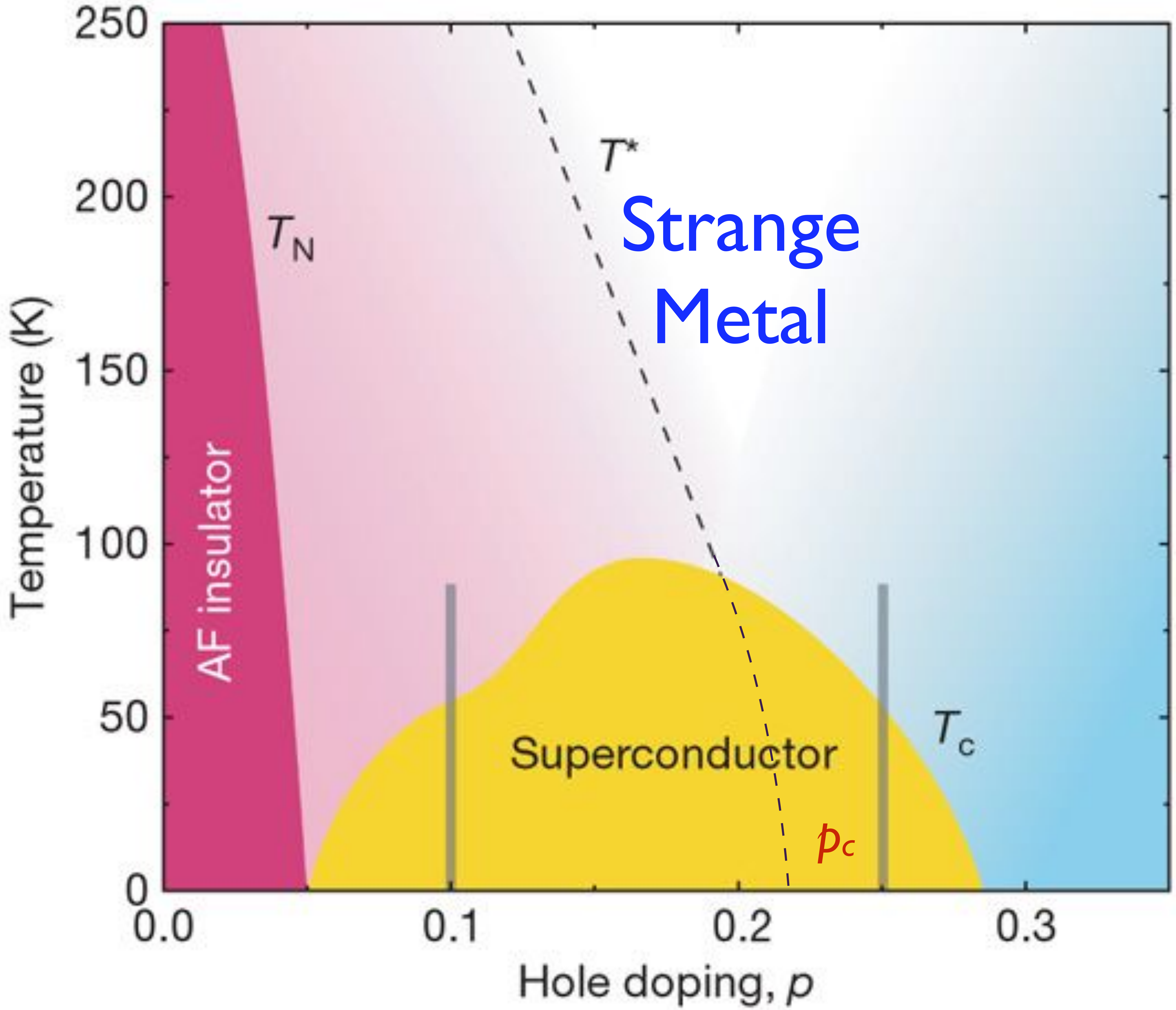
- ‘Wormhole’ contributions to this quantum simulation have led to an understanding of the Page curve of entanglement entropy of evaporating black holes.

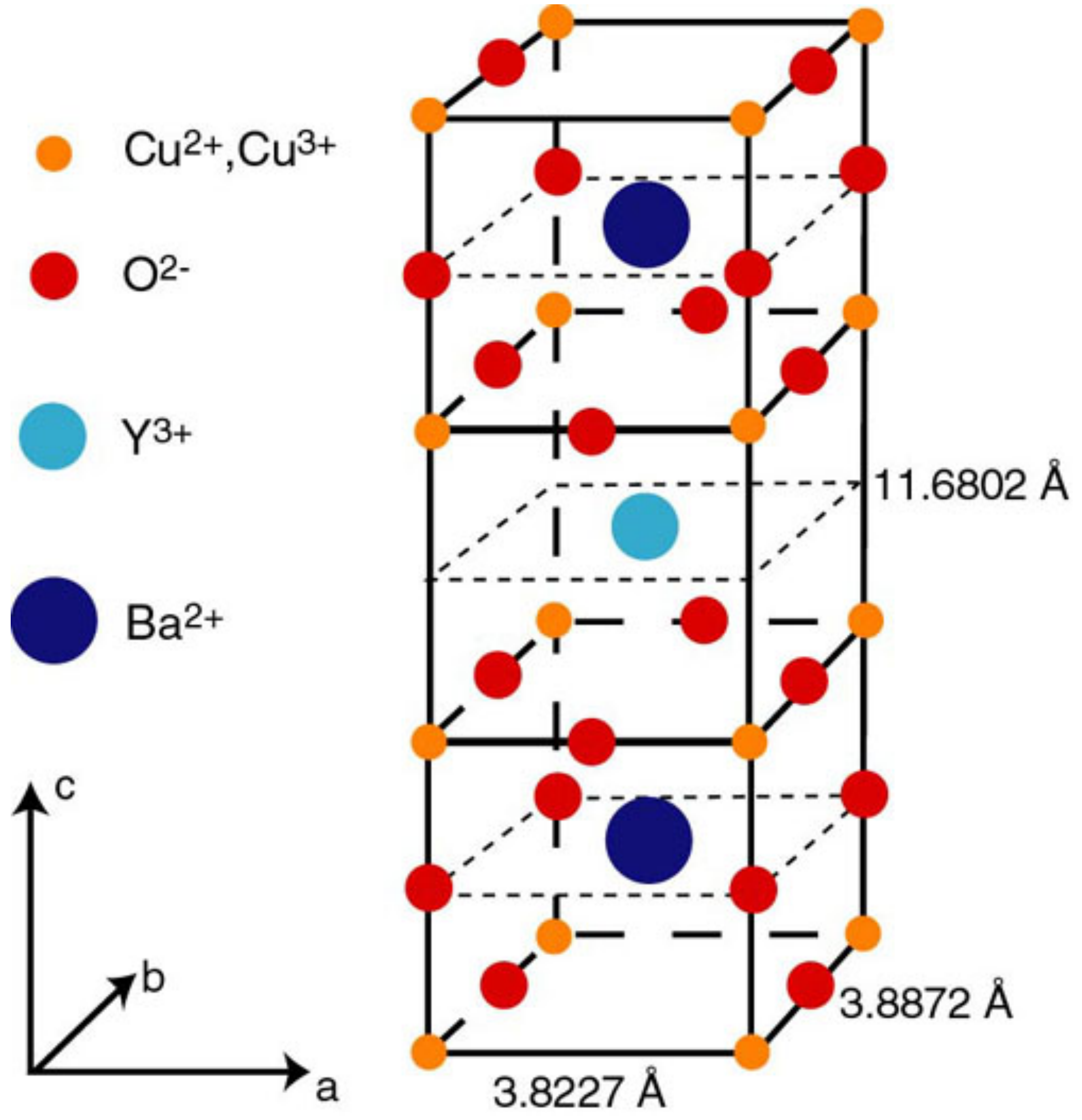


Saad, Shenker, Stanford (2019)

Strange metals



250
200
150
100
50
0
0.0
0.1
0.2
0.3
Temperature (K)
Hole doping, p
AF insulator
T_N
T^*
Strange Metal
Superconductor
T_c
p_c



Cu^{2+},Cu^{3+}
O^{2-}
Y^{3+}
Ba^{2+}
c
b
a
11.6802 Å
3.8872 Å
3.8227 Å

Properties of a strange metal:

1. Resistivity $\rho(T) = \rho_0 + AT + \ldots$ as $T \to 0$
   and $\rho(T) < h/e^2$ (in $d = 2$).
   Metals with $\rho(T) > h/e^2$ are bad metals.

2. Specific heat $\sim T \ln(1/T)$ as $T \to 0$.

S.A. Hartnoll and A.P. MacKenzie, arXiv:2107.07802

3. Optical conductivity

$$\sigma(\omega) = \frac{K}{\dfrac{1}{\tau_{\rm trans}(\omega)} - i\omega \dfrac{m^*_{\rm trans}(\omega)}{m}} \quad ; \quad \frac{1}{\tau_{\rm trans}(\omega)} \sim |\omega| \Phi_\sigma \left(\frac{\hbar\omega}{k_B T} \right)$$

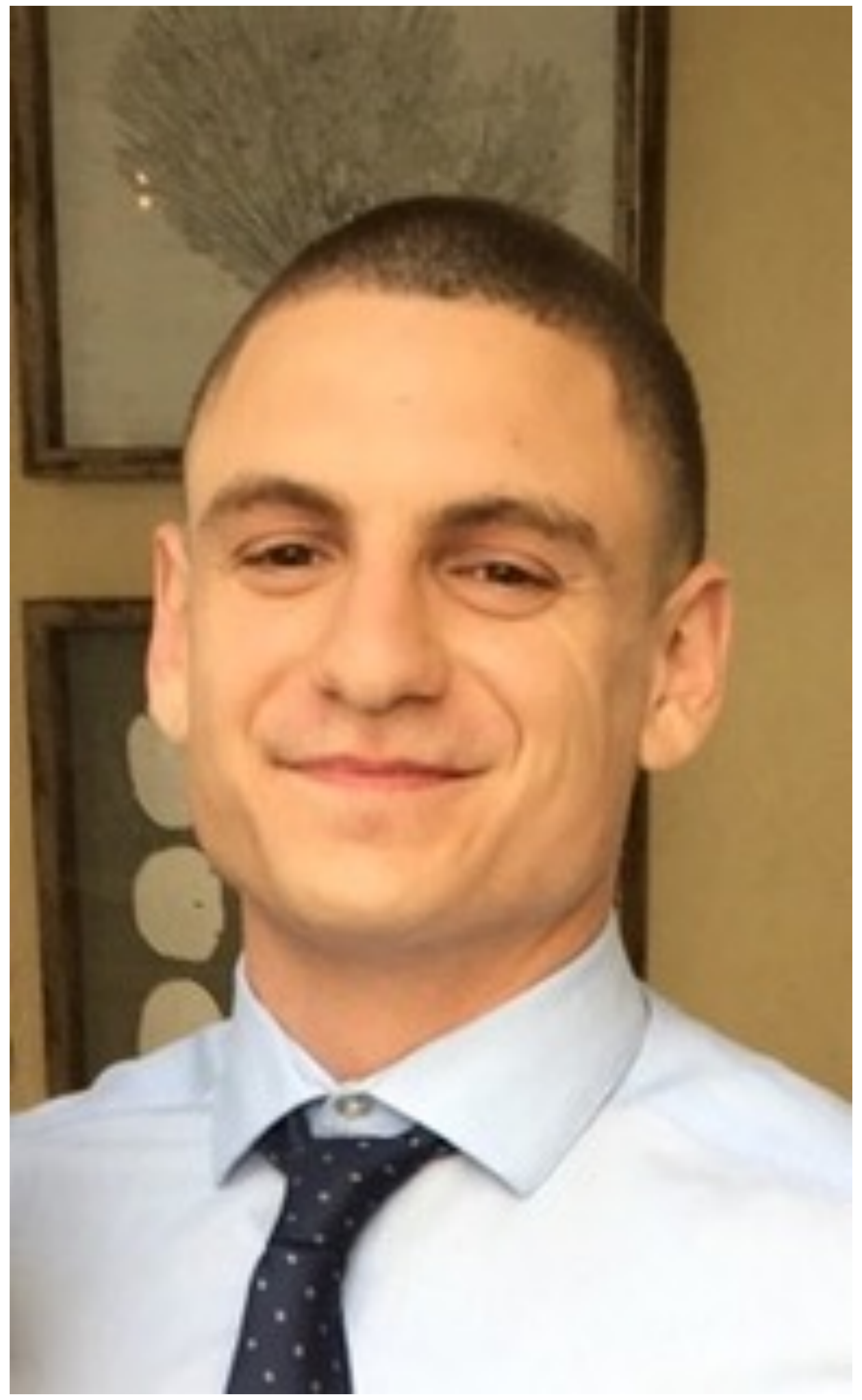
B. Michon……A. Georges, arXiv:2205.04030

4. Photoemission: nearly "marginal Fermi liquid" electron spectral density:

$$\mathrm{Im}\Sigma(\omega) \sim |\omega|^{2\alpha} \Phi_\Sigma \left(\frac{\hbar\omega}{k_B T} \right) \quad \text{with } \alpha \approx 1/2$$

T.J. Reber….D. Dessau, Nature Communications **10**, 5737 (2019)

From Yukawa-SYK models to a universal theory of strange metals



Aavishkar Patel
Flatiron Institute, NYC

Haoyu Guo
Harvard

Ilya Esterlis
Harvard → Wisconsin

arXiv: 2103.08615, 2203.04990, 2207.08841

E. E. Aldape, T. Cookmeyer, Aavishkar A. Patel, and Ehud Altman, arXiv:2012.00763

Yukawa-SYK models

$$\mathcal{H} = -\mu \sum_i \psi_i^\dagger \psi_i + \sum_\ell \frac{1}{2}\left(\pi_\ell^2 + \omega_0^2 \phi_\ell^2\right) + \frac{1}{N} \sum_{ij\ell} g_{ij\ell} \psi_i^\dagger \psi_j \phi_\ell \,,$$

$g_{ij\ell}$ independent random numbers with zero mean. Large N limit leads to Migdal-Eliashberg equations $\Sigma_\psi \sim g^2 G_\psi G_\phi$, $\Sigma_\phi \sim g^2 G_\psi G_\psi$.

W. Fu, D. Gaiotto, J. Maldacena, and S. Sachdev, PRD **95**, 026009 (2017)
J. Murugan, D. Stanford, and E. Witten, JHEP 08, 146 (2017)
A. A. Patel and S. Sachdev, PRB **98**, 125134 (2018)
E. Marcus and S. Vandoren, JHEP 01, 166 (2018)
Yuxuan Wang, PRL **124**, 017002 (2020)
I. Esterlis and J. Schmalian, PRB **100**, 115132 (2019)
Yuxuan Wang and A. V. Chubukov, PRR **2**, 033084 (2020)
E. E. Aldape, T. Cookmeyer, A. A. Patel, and E. Altman, arXiv:2012.00763
Jaewon Kim, E. Altman, and Xiangyu Cao, PRB **103**, 081113 (2021)
W. Wang, A. Davis, G. Pan, Yuxuan Wang, and Zi Yang Meng, PRB **103**, 195108 (2021)
I. Esterlis, H. Guo, A. A. Patel, and S. Sachdev, PRB **103**, 235129 (2021).

Yukawa-SYK models

$$\mathcal{H} = -\mu \sum_i \psi_i^\dagger \psi_+ \sum_\ell \frac{1}{2}\left(\pi_\ell^2 + \omega_0^2 \phi_\ell^2\right) + \frac{1}{N} \sum_{ij\ell} g_{ij\ell} \psi_i^\dagger \psi_j \phi_\ell$$

with $g_{ij\ell}$ independent random numbers with zero mean.

The disorder-averaged partition function is a G-Σ-D-Π theory:

$$\mathcal{Z} = \int \mathcal{D}G\, \mathcal{D}\Sigma\, \mathcal{D}D\, \mathcal{D}\Pi \exp(-N S_{\text{all}})$$

$$\begin{aligned} S_{\text{all}} = &- \ln\det(\partial_\tau - \mu + \Sigma) + \frac{1}{2} \ln\det(-\partial_\tau^2 + \omega_0^2 - \Pi) \\ &+ \int d\tau \int d\tau' \Big[-\Sigma(\tau';\tau) G(\tau,\tau') + \frac{1}{2}\Pi(\tau'\tau) D(\tau,\tau') \\ &\qquad + \frac{g^2}{2} G(\tau,\tau') G(\tau',\tau) D(\tau,\tau') \Big] \,. \end{aligned}$$

Yukawa-SYK models

$$\mathcal{H} = -\mu \sum_i \psi_i^\dagger \psi_+ \sum_\ell \frac{1}{2}\left(\pi_\ell^2 + \omega_0^2 \phi_\ell^2\right) + \frac{1}{N} \sum_{ij\ell} g_{ij\ell} \psi_i^\dagger \psi_j \phi_\ell$$

with $g_{ij\ell}$ independent random numbers with zero mean. The large N saddle point equations are

$$\boxed{\begin{aligned} G(i\omega_n) &= \frac{1}{i\omega_n + \mu - \Sigma(i\omega_n)} \quad , \quad D(i\omega_n) = \frac{1}{\omega_n^2 + \omega_0^2 - \Pi(i\omega_n)} \\ \Sigma(\tau) &= g^2 G(\tau) D(\tau) \quad , \quad \Pi(\tau) = -g^2 G(\tau) G(-\tau) \end{aligned}}$$

Make the low frequency ansatz

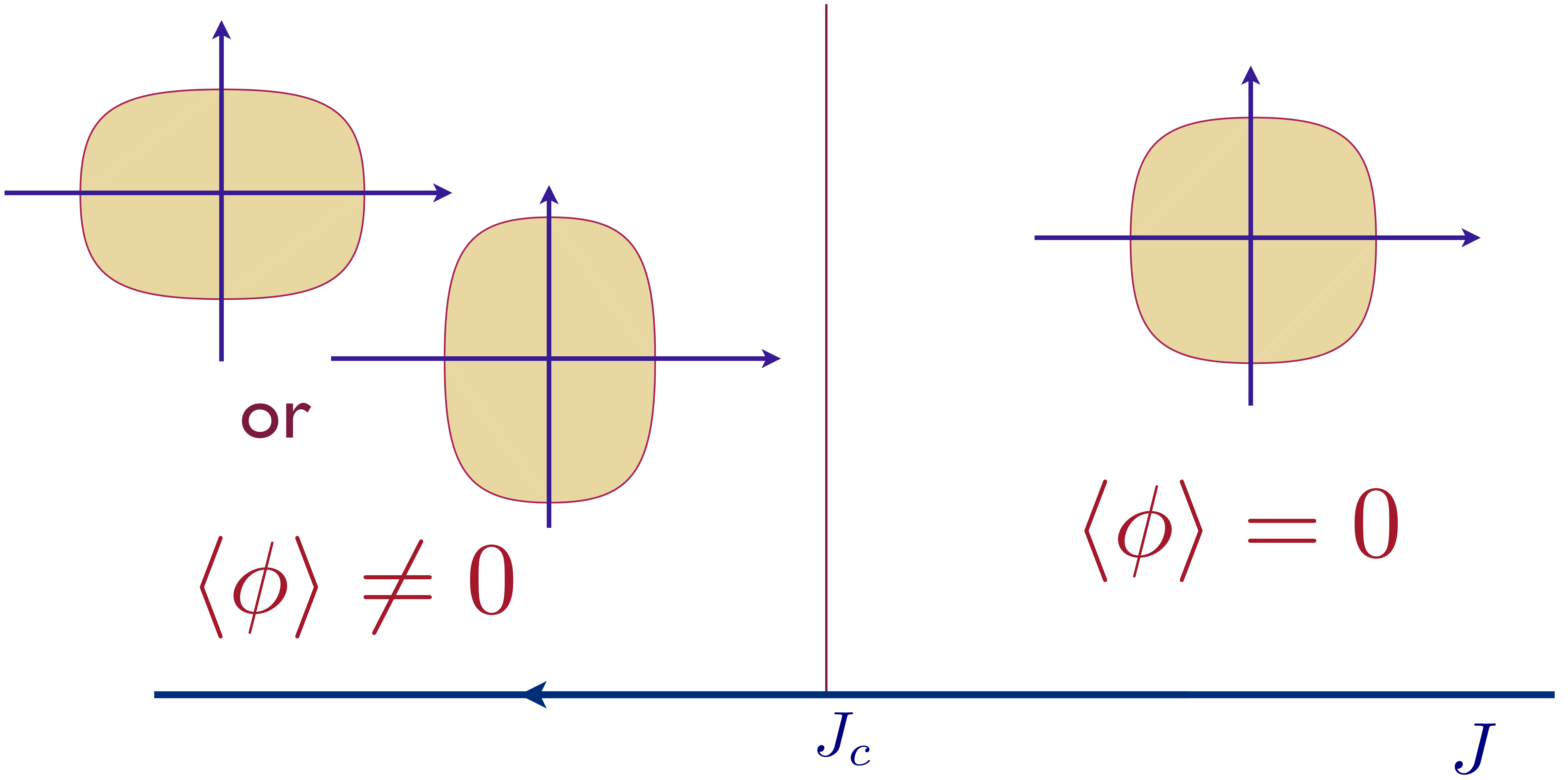
$$G(i\omega) \sim -i\,\mathrm{sgn}(\omega)|\omega|^{-(1-2\Delta)} \quad , \quad D(i\omega) \sim |\omega|^{1-4\Delta} \quad , \quad \frac{1}{4} < \Delta < \frac{1}{2}$$

A consistent solution exists for

$$\frac{4\Delta - 1}{2(2\Delta - 1)[\sec(2\pi\Delta) - 1]} = 1 \quad , \quad \Delta = 0.42037\ldots$$

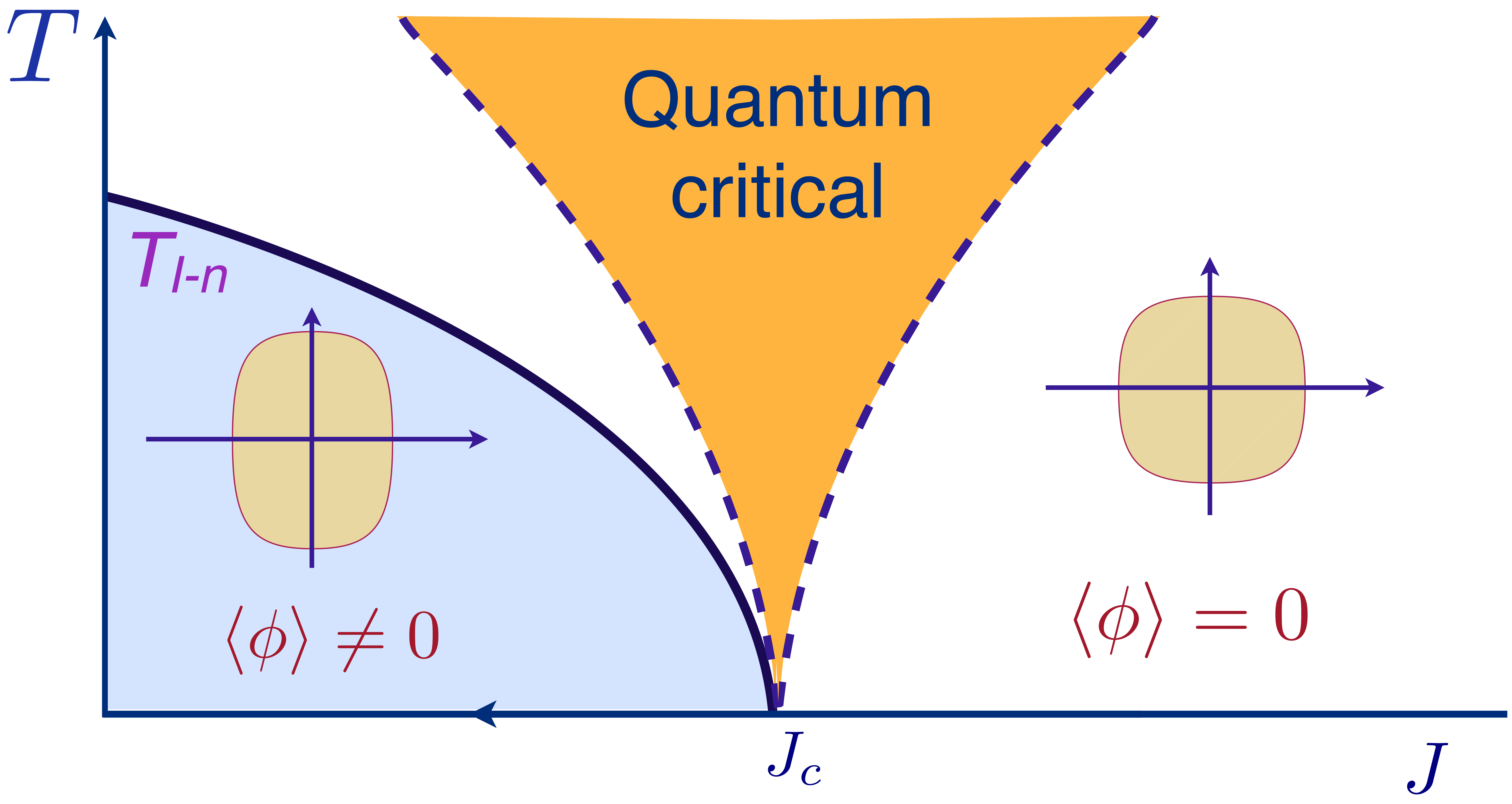
I. Esterlis and J. Schmalian, PRB **100**, 115132 (2019)
See also Yuxuan Wang, PRL **124**, 017002 (2020)

Quantum criticality of Ising-nematic ordering in a metal



Pomeranchuk instability as a function of coupling J

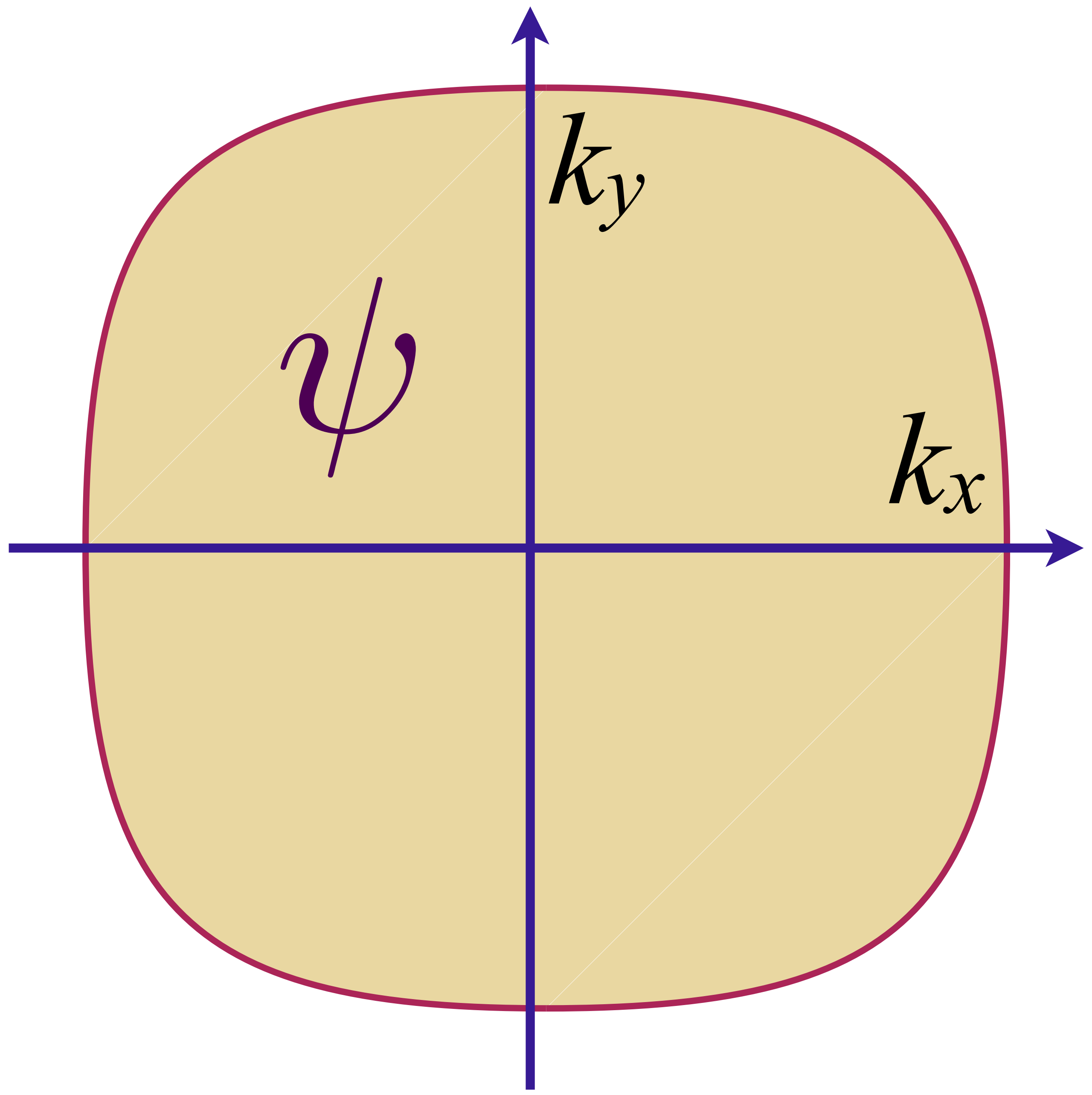
Quantum criticality of Ising-nematic ordering in a metal



Phase diagram as a function of T and J

Fermi surface

$$\mathcal{L}_\psi = \psi_{\boldsymbol{k}}^\dagger \left(\frac{\partial}{\partial \tau} + \varepsilon(\boldsymbol{k}) \right) \psi_{\boldsymbol{k}}$$

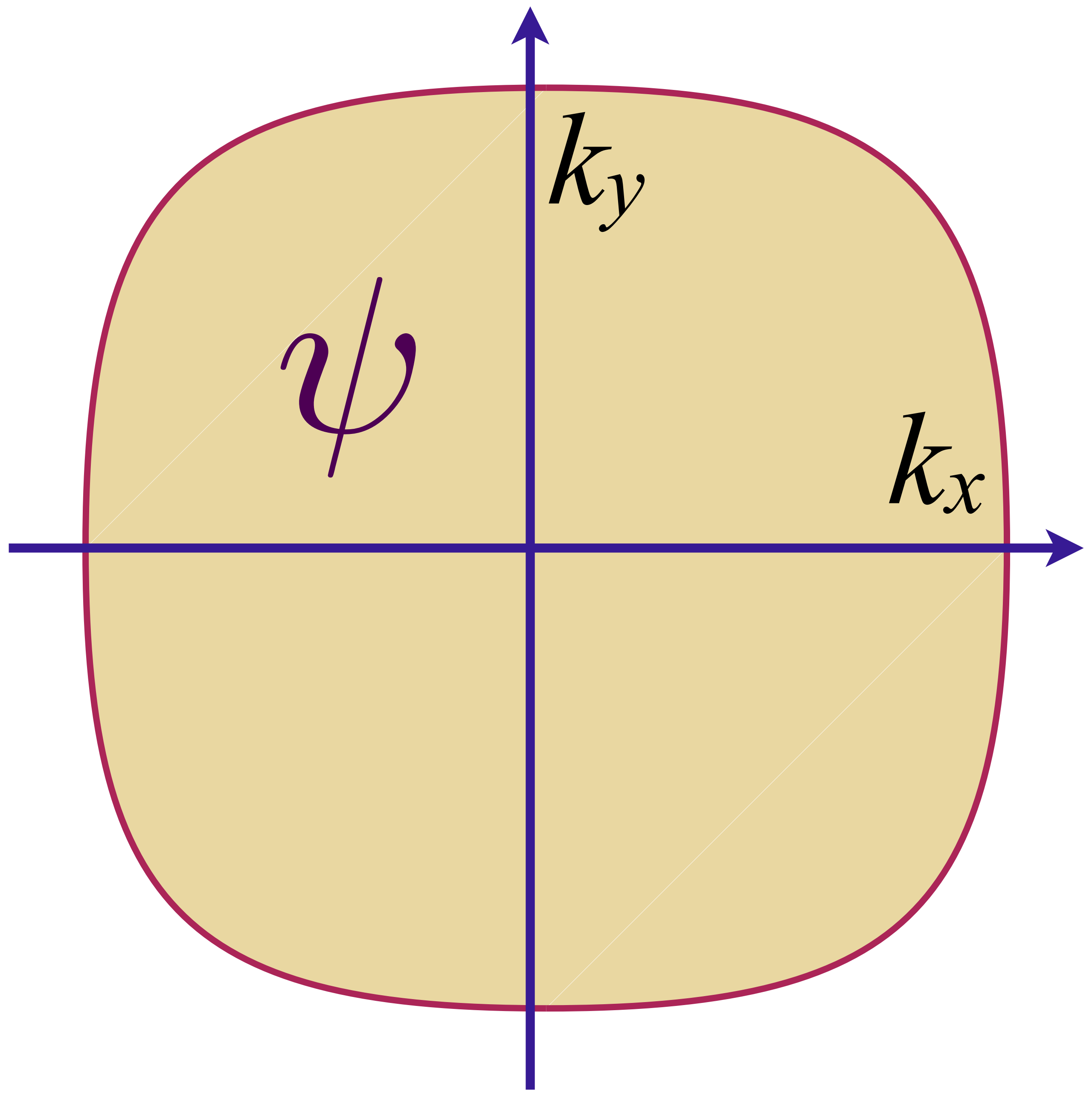


$$-J\, \psi^\dagger(\boldsymbol{r})\psi^\dagger(\boldsymbol{r})\psi(\boldsymbol{r})\psi(\boldsymbol{r})$$

Fermi surface coupled to a critical boson

$$\mathcal{L}_\psi = \psi_{\boldsymbol{k}}^\dagger \left(\frac{\partial}{\partial \tau} + \varepsilon(\boldsymbol{k}) \right) \psi_{\boldsymbol{k}}$$

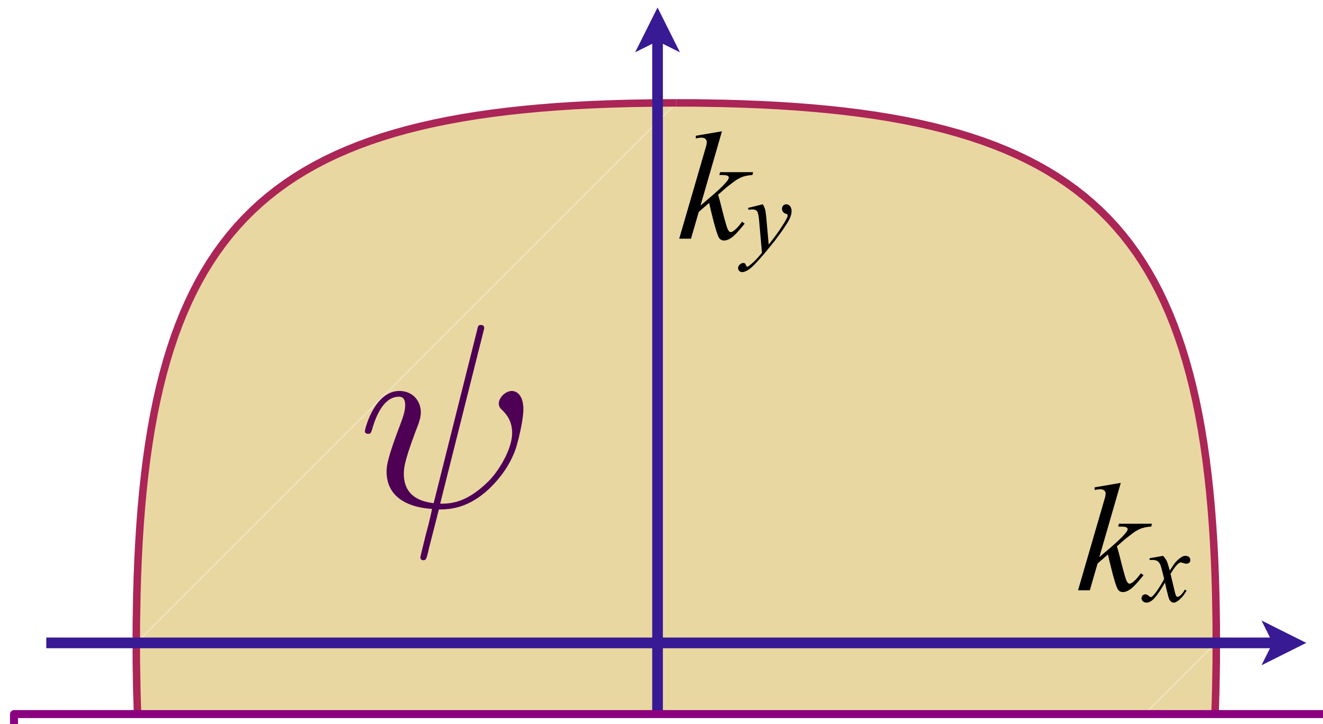
a critical boson ϕ
e.g. Ising-nematic order



$$\frac{[\phi(\boldsymbol{r})]^2}{J} + \psi^\dagger(\boldsymbol{r})\psi(\boldsymbol{r})\,\phi(\boldsymbol{r})$$

Fermi surface coupled to a critical boson

$$\mathcal{L}_\psi = \psi^\dagger_{\boldsymbol{k}} \left(\frac{\partial}{\partial \tau} + \varepsilon(\boldsymbol{k}) \right) \psi_{\boldsymbol{k}}$$



a critical boson ϕ
e.g. Ising-nematic order

$$\frac{[\phi(\boldsymbol{r})]^2}{J} + \psi^\dagger(\boldsymbol{r})\psi(\boldsymbol{r})\,\phi(\boldsymbol{r})$$

Solve in a large N limit with Yukawa coupling

$$\frac{g_{ij\ell}}{N} \int d^2r d\tau\, \psi^\dagger_i(r,\tau)\psi_j(r,\tau)\phi_l(r,\tau) \quad , \quad \overline{g_{ijl}} = 0 \quad , \quad \overline{|g_{ijl}|^2} = g^2$$

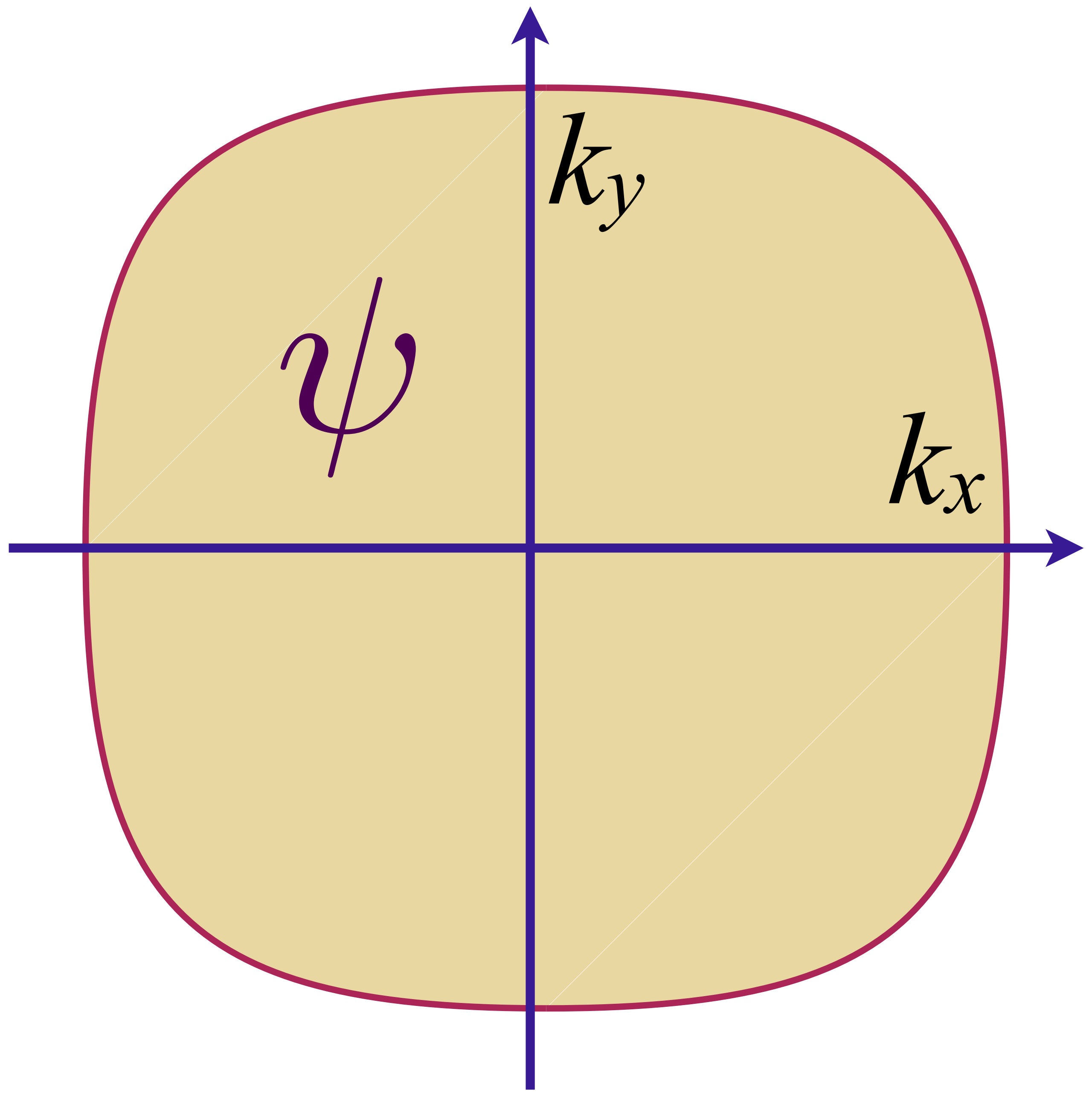
to obtain Eliashberg solution for electron (G) and boson (D) Green's functions at small ω:

$$\Sigma(\hat{\boldsymbol{k}}, i\omega) \sim -i\text{sgn}(\omega)|\omega|^{2/3}, \quad G(\boldsymbol{k}, i\omega) = \frac{1}{i\omega - \varepsilon(\boldsymbol{k}) - \Sigma(\hat{\boldsymbol{k}}, i\omega)}, \quad D(\boldsymbol{q}, i\Omega) = \frac{1}{\Omega^2 + q^2 + \gamma|\Omega|/q}$$

P.A. Lee (1989)

Fermi surface coupled to a critical boson

$$\mathcal{L}_\psi = \psi^\dagger_{\boldsymbol{k}} \left(\frac{\partial}{\partial \tau} + \varepsilon(\boldsymbol{k}) \right) \psi_{\boldsymbol{k}}$$



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$$\frac{[\phi(\boldsymbol{r})]^2}{J} + \psi^\dagger(\boldsymbol{r})\psi(\boldsymbol{r})\,\phi(\boldsymbol{r})$$

S. A. Hartnoll, P. K. Kovtun, M. Muller, and S.S. PRB **76**, 144502 (2007)
D. L. Maslov, V. I. Yudson, and A. V. Chubukov PRL **106**, 106403 (2011)
S. A. Hartnoll, R. Mahajan, M. Punk, and S.S. PRB **89**, 155130 (2014)
A. Eberlein, I. Mandal, and S.S. PRB **94**, 045133 (2016)

Transport—a perfect metal!

Conservation of momentum and fermion-boson drag imply:

$$\mathrm{Re}\,[\sigma(\omega)] = D\delta(\omega) + \ldots$$

Fermi surface coupled to a critical boson

$$\mathcal{L}_\psi = \psi_{\boldsymbol{k}}^\dagger \left(\frac{\partial}{\partial \tau} + \varepsilon(\boldsymbol{k}) \right) \psi_{\boldsymbol{k}}$$

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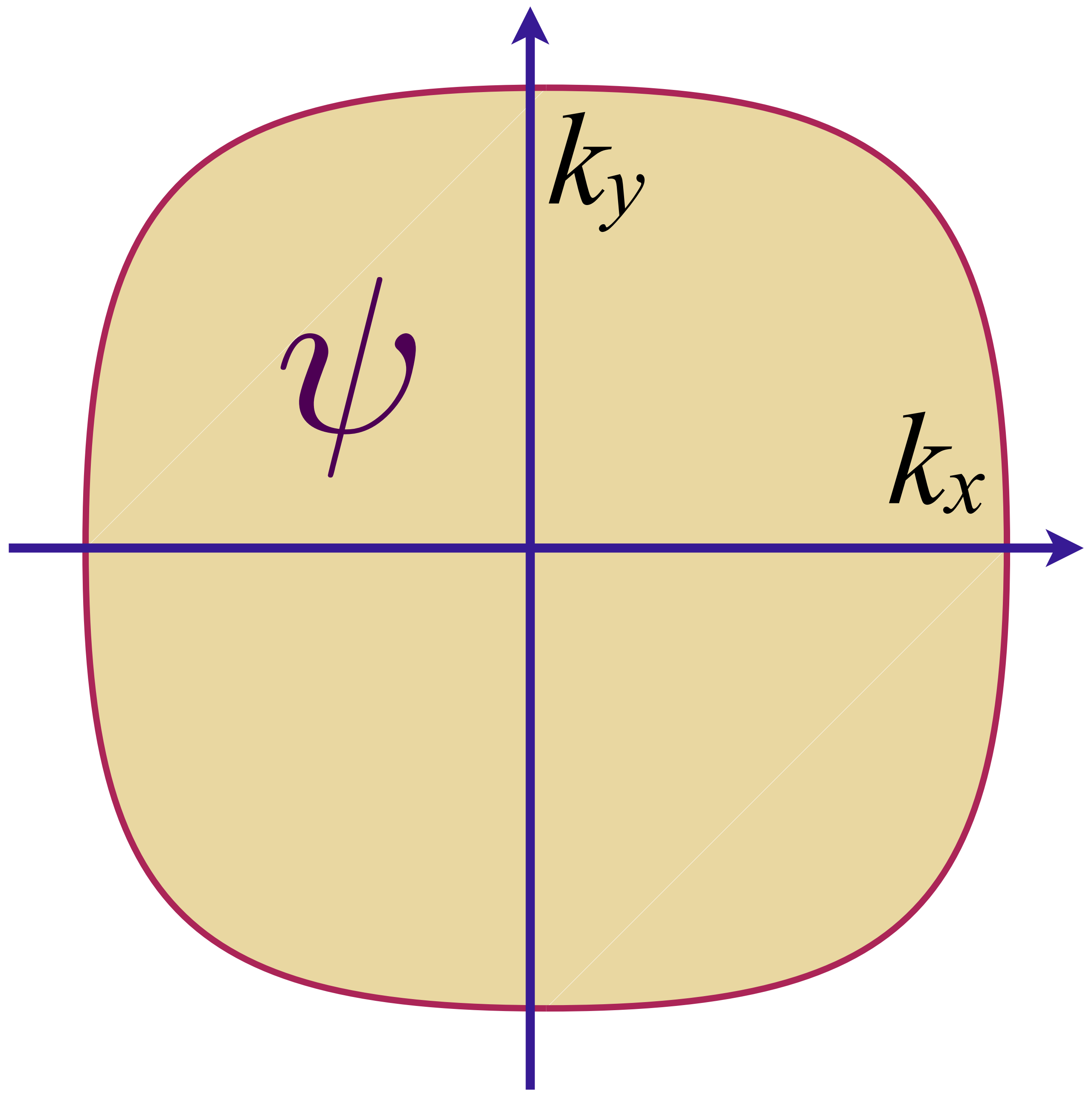

Transport—a perfect metal!
Conservation of momentum and fermion-boson drag imply:

$$\sigma(\omega) \sim \frac{1}{-i\omega} + |\omega|^0 + \cdots \quad (\omega^{-2/3} \text{ term has vanishing co-efficient})$$

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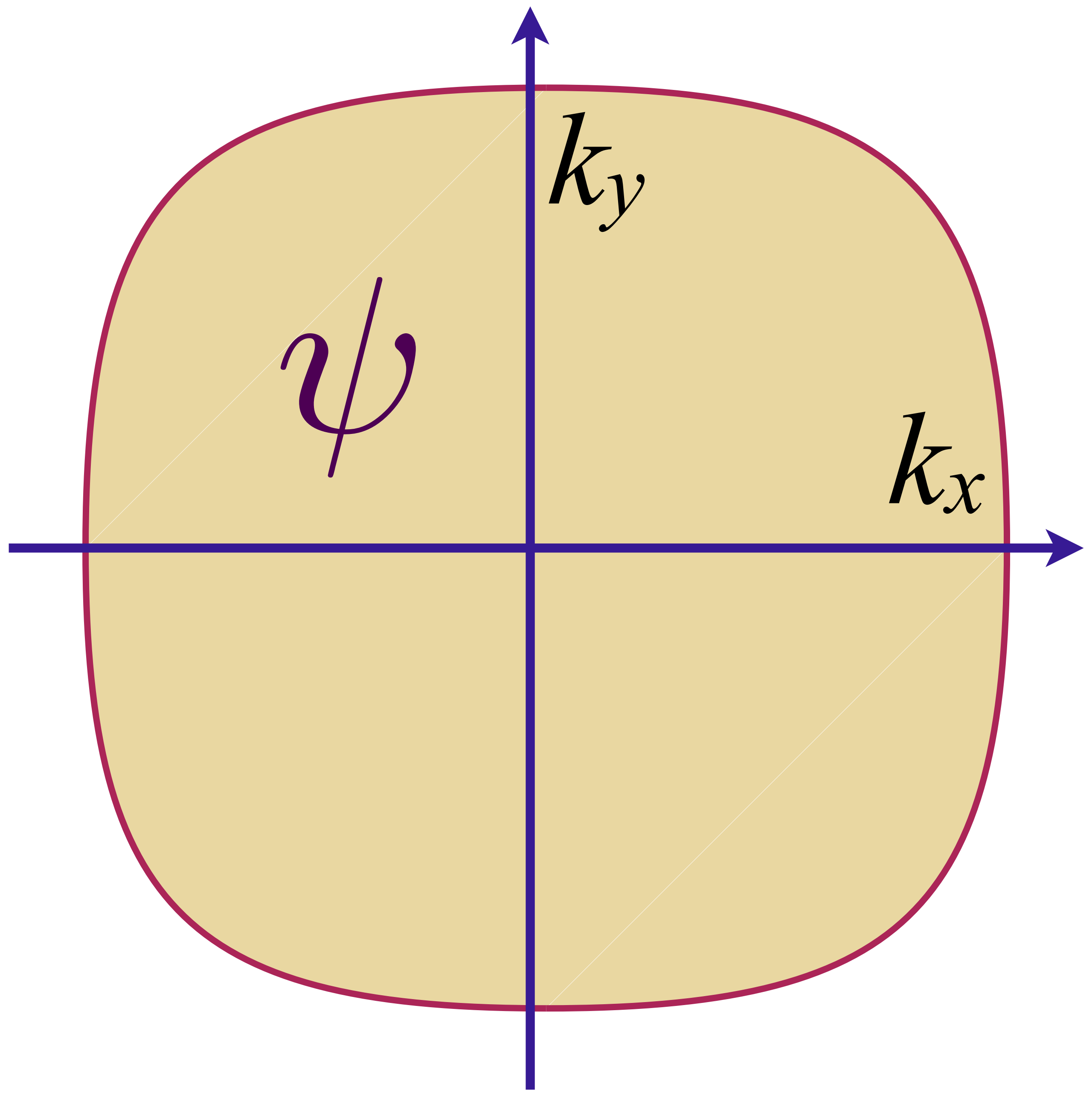
Haoyu Guo, Aavishkar Patel, Ilya Esterlis, S.S. PRB **106**, 115151 (2022)

Fermi surface coupled to a critical boson: No spatial disorder

A non-Fermi liquid but NOT a strange metal

Fermi surface coupled to a critical boson with disorder

$$\mathcal{L}_\psi = \psi_{\boldsymbol{k}}^\dagger \left(\frac{\partial}{\partial \tau} + \varepsilon(\boldsymbol{k}) \right) \psi_{\boldsymbol{k}}$$



a critical boson ϕ
e.g. Ising-nematic order

$$\frac{[\phi(\boldsymbol{r})]^2}{J} + \psi^\dagger(\boldsymbol{r})\psi(\boldsymbol{r})\,\phi(\boldsymbol{r})$$

$$+v(\boldsymbol{r})\psi^\dagger(\boldsymbol{r})\psi(\boldsymbol{r})$$

Spatially random potential $v(\boldsymbol{r})$ with $\overline{v(\boldsymbol{r})} = 0$, $\overline{v(\boldsymbol{r})v(\boldsymbol{r}')} = v^2\delta(\boldsymbol{r} - \boldsymbol{r}')$

Fermi surface coupled to a critical boson with disorder

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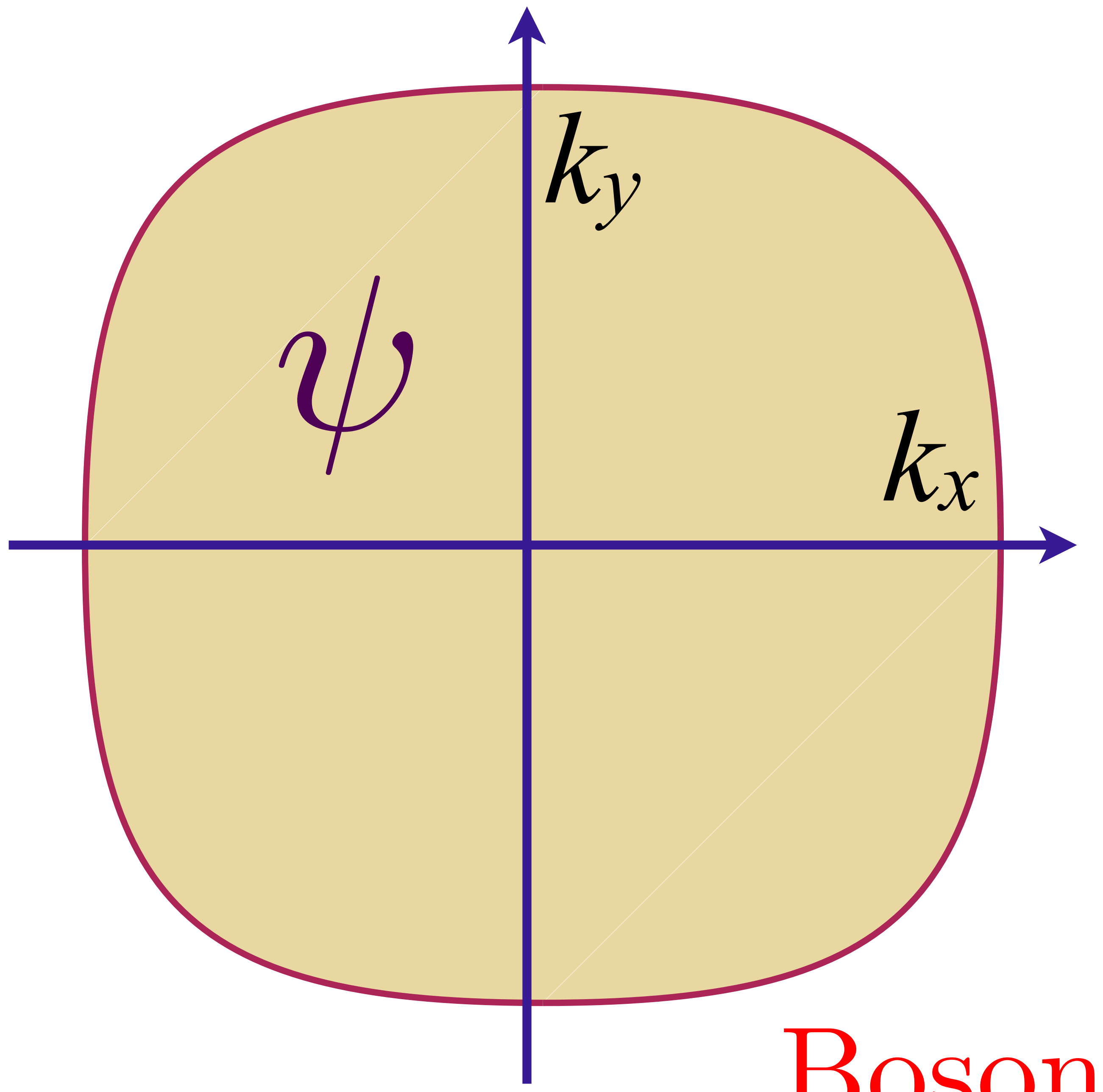


$$\frac{[\phi(\boldsymbol{r})]^2}{J} + \psi^\dagger(\boldsymbol{r})\psi(\boldsymbol{r})\,\phi(\boldsymbol{r})$$
$$+v(\boldsymbol{r})\psi^\dagger(\boldsymbol{r})\psi(\boldsymbol{r})$$

Boson Green's function: $D(q, i\Omega) \sim 1/(q^2 + \gamma|\Omega|)$

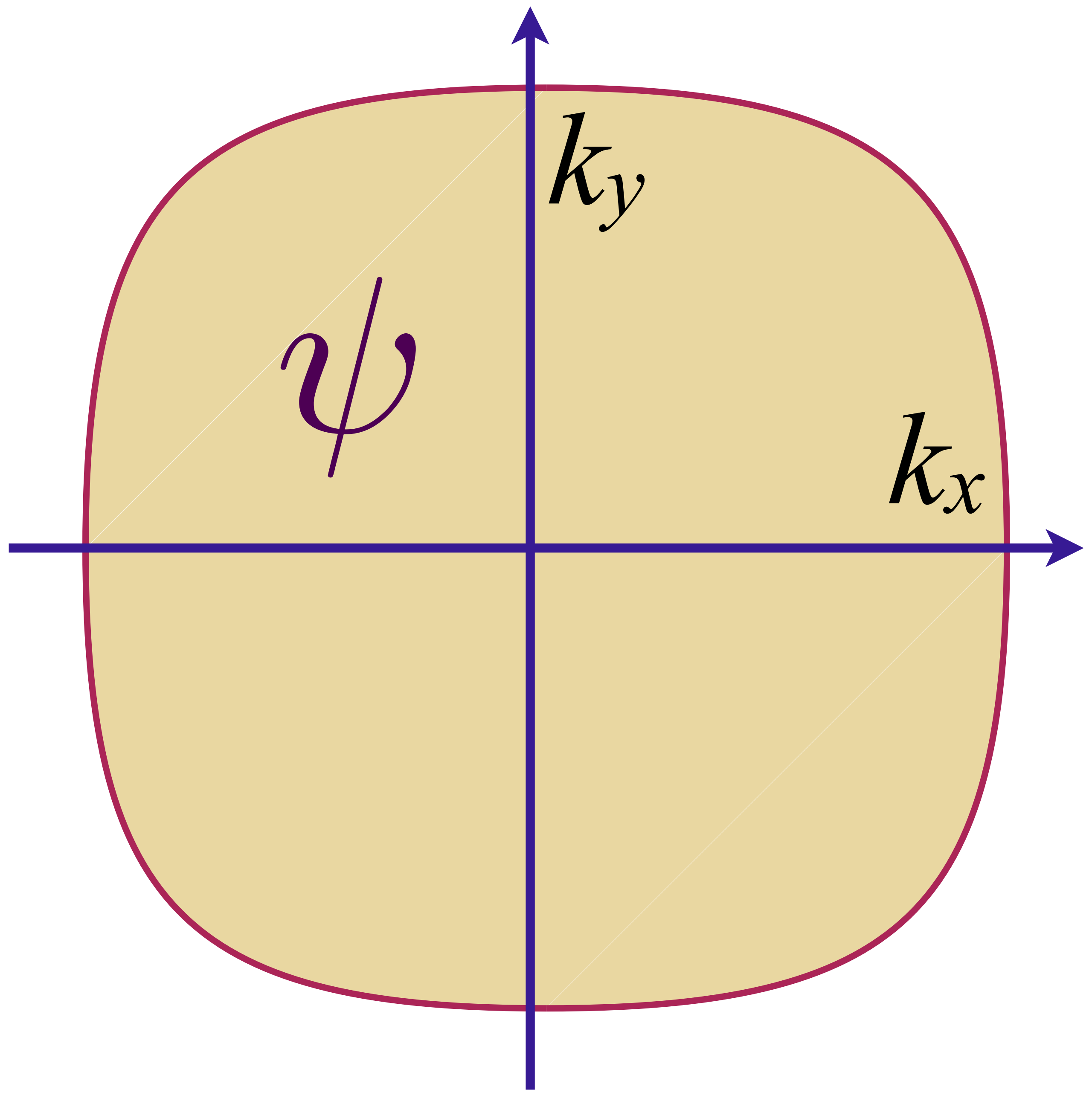
Fermion self energy: $\Sigma(i\omega) \sim -iv^2 \text{sgn}(\omega) - i\dfrac{g^2}{v^2}\omega \ln(1/|\omega|)$; $\quad \dfrac{1}{\tau(\omega)} \sim \dfrac{g^2}{v^2}|\omega|$

Marginal Fermi liquid self energy and $T\ln(1/T)$ specific heat

Halperin, Lee, Read (1993)

Fermi surface coupled to a critical boson with disorder

$$\mathcal{L}_\psi = \psi_{\boldsymbol{k}}^\dagger \left(\frac{\partial}{\partial \tau} + \varepsilon(\boldsymbol{k}) \right) \psi_{\boldsymbol{k}}$$



a critical boson ϕ
e.g. Ising-nematic order

$$\frac{[\phi(\boldsymbol{r})]^2}{J} + \psi^\dagger(\boldsymbol{r})\psi(\boldsymbol{r})\,\phi(\boldsymbol{r})$$

$$+v(\boldsymbol{r})\psi^\dagger(\boldsymbol{r})\psi(\boldsymbol{r})$$

But resistivity and optical conductivity are like a Fermi liquid, with residual resistivity $\sim v^2$.

Haoyu Guo, Aavishkar Patel, Ilya Esterlis, S.S. PRB **106**, 115151 (2022)
T. C. Wu, Y. Liao, and M. S. Foster, PRB **106**, 155108 (2022)

Fermi surface coupled to a critical boson:

No spatial disorder

A non-Fermi liquid but NOT a strange metal

Fermi surface coupled to a critical boson:

Potential disorder v

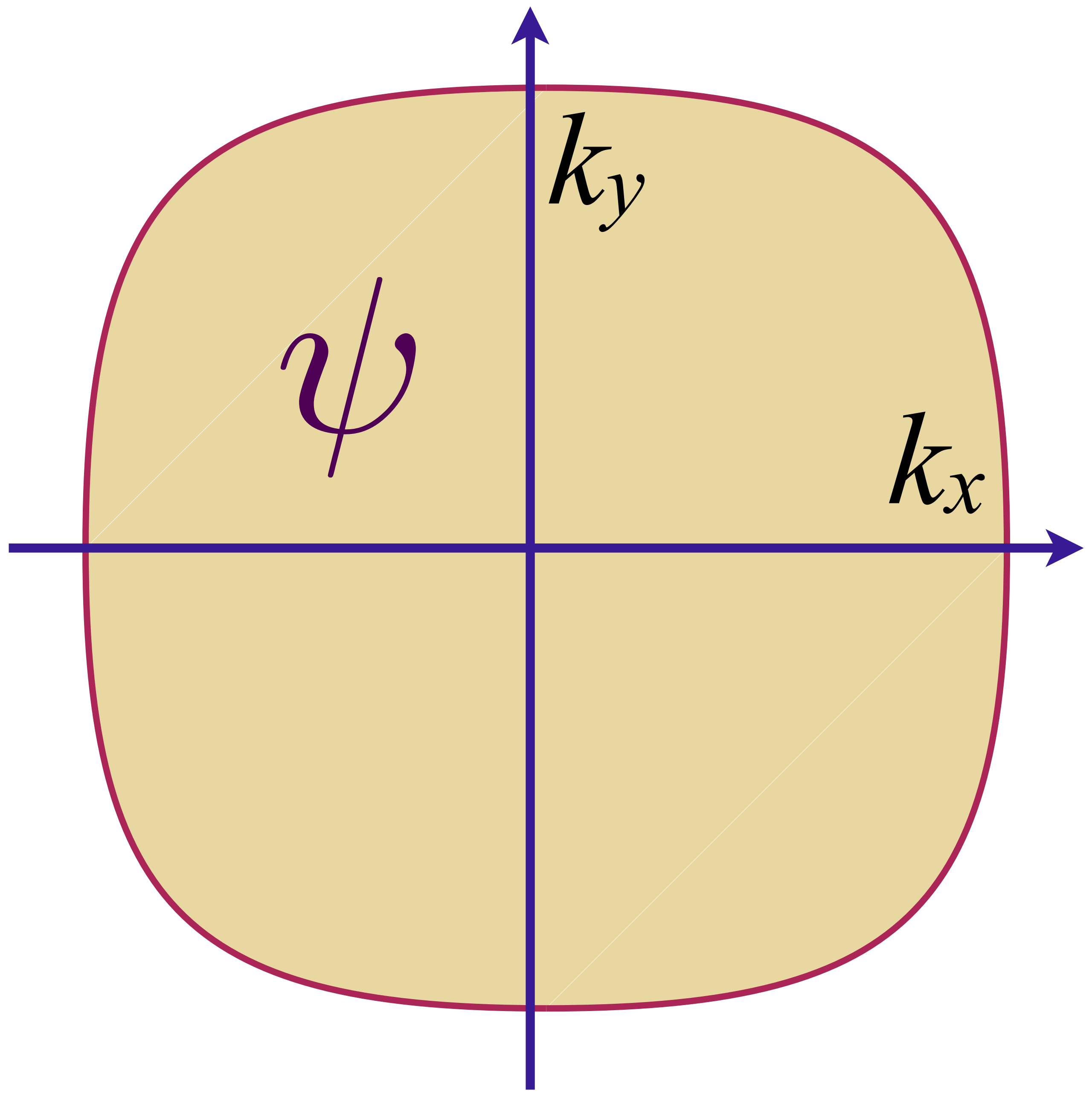
A marginal Fermi liquid but NOT a strange metal

Fermi surface coupled to a critical boson with disorder

$$\mathcal{L}_\psi = \psi^\dagger_{\boldsymbol{k}} \left(\frac{\partial}{\partial \tau} + \varepsilon(\boldsymbol{k}) \right) \psi_{\boldsymbol{k}}$$

a critical boson ϕ

e.g. Ising-nematic order



$$\frac{[\phi(\boldsymbol{r})]^2}{J + J'(\boldsymbol{r})} + \psi^\dagger(\boldsymbol{r})\psi(\boldsymbol{r})\,\phi(\boldsymbol{r})$$

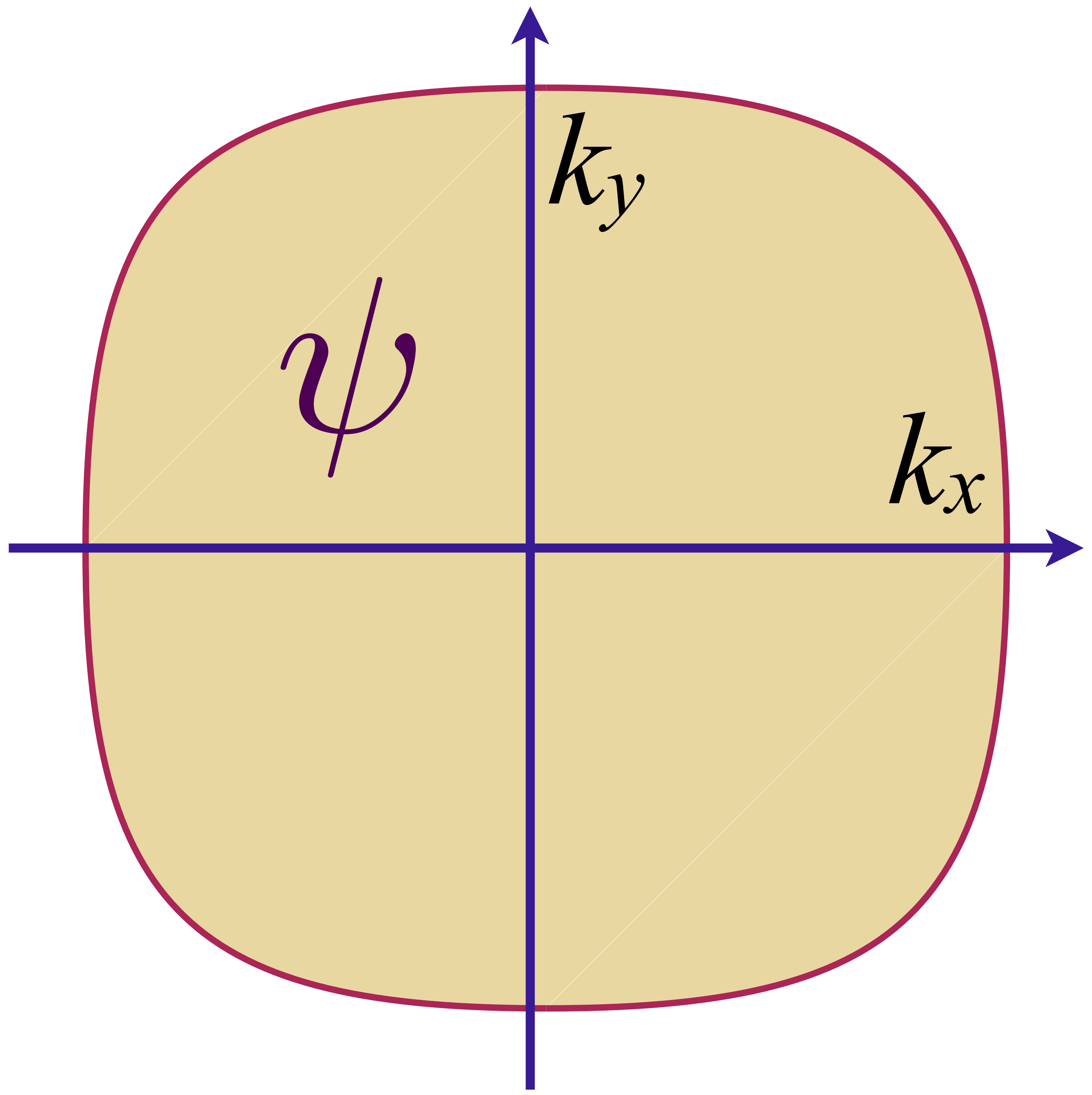
$$+ v(\boldsymbol{r})\psi^\dagger(\boldsymbol{r})\psi(\boldsymbol{r})$$

Fermi surface coupled to a critical boson with disorder

$$\mathcal{L}_\psi = \psi^\dagger_{\boldsymbol{k}} \left(\frac{\partial}{\partial \tau} + \varepsilon(\boldsymbol{k}) \right) \psi_{\boldsymbol{k}}$$

a critical boson ϕ

e.g. Ising-nematic order



$$[\phi(\boldsymbol{r})]^2 + [g + g'(\boldsymbol{r})]\, \psi^\dagger(\boldsymbol{r})\psi(\boldsymbol{r})\, \phi(\boldsymbol{r})$$

$$+ v(\boldsymbol{r})\psi^\dagger(\boldsymbol{r})\psi(\boldsymbol{r})$$

ϕ^2 "mass" disorder $J'(\boldsymbol{r})$ is strongly relevant;
rescale ϕ to move disorder to the Yukawa coupling;

Spatially random Yukawa coupling $g'(\boldsymbol{r})$ with $\overline{g'(\boldsymbol{r})} = 0$, $\overline{g'(\boldsymbol{r})g'(\boldsymbol{r}')} = g'^2 \delta(\boldsymbol{r} - \boldsymbol{r}')$

Spatially random potential $v(\boldsymbol{r})$ with $\overline{v(\boldsymbol{r})} = 0$, $\overline{v(\boldsymbol{r})v(\boldsymbol{r}')} = v^2 \delta(\boldsymbol{r} - \boldsymbol{r}')$

Fermi surface coupled to a critical boson with disorder

All results are obtained from the large N saddle-point and response functions of this G-Σ-D-Π theory:

$$\mathcal{Z} = \int \mathcal{D}G\,\mathcal{D}\Sigma\,\mathcal{D}D\,\mathcal{D}\Pi\,\exp(-NS_{\rm all})$$

$$\begin{aligned}
S_{\rm all} = &-\ln\det(\partial_\tau + \varepsilon(\mathbf{k}) - \mu + \Sigma) + \frac{1}{2}\ln\det(-\partial_\tau^2 + \mathbf{q}^2 + m_b^2 - \Pi) \\
&+ \int d\tau d^2r \int d\tau' d^2r' \Big[-\Sigma(\tau',\mathbf{r}';\tau,\mathbf{r})G(\tau,\mathbf{r};\tau',\mathbf{r}') + \frac{1}{2}\Pi(\tau',\mathbf{r}';\tau,\mathbf{r})D(\tau,\mathbf{r};\tau',\mathbf{r}') \\
&+ \frac{g^2}{2}G(\tau,\mathbf{r};\tau',\mathbf{r}')G(\tau',\mathbf{r}';\tau,\mathbf{r})D(\tau,\mathbf{r};\tau',\mathbf{r}') + \frac{v^2}{2}G(\tau,\mathbf{r};\tau',\mathbf{r}')G(\tau',\mathbf{r}';\tau,\mathbf{r})\delta(\mathbf{r}-\mathbf{r}') \\
&+ \frac{g'^2}{2}G(\tau,\mathbf{r};\tau',\mathbf{r}')G(\tau',\mathbf{r}';\tau,\mathbf{r})D(\tau,\mathbf{r};\tau',\mathbf{r}')\delta(\mathbf{r}-\mathbf{r}')\Big] \,.
\end{aligned}$$

Fermi surface coupled to a critical boson with disorder

All results are obtained from the large N saddle-point and response functions of this G-Σ-D-Π theory:

$$\mathcal{Z} = \int \mathcal{D}G\, \mathcal{D}\Sigma\, \mathcal{D}D\, \mathcal{D}\Pi \exp(-N S_{\text{all}})$$

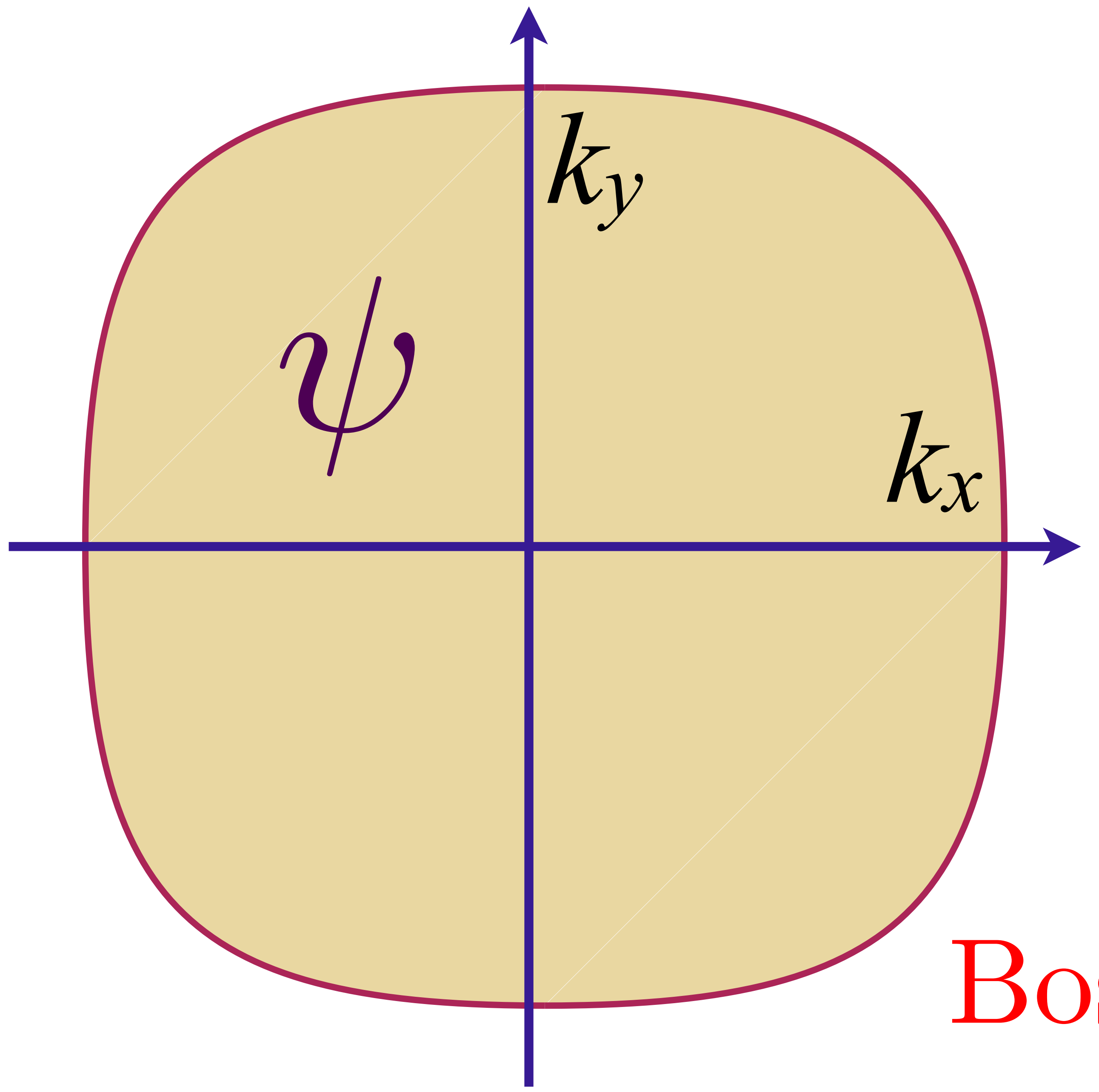
Saddle-point equations

$$\Sigma(\tau, \mathbf{r}) = g^2 D(\tau, \mathbf{r}) G(\tau, \mathbf{r}) + v^2 G(\tau, \mathbf{r}) \delta^2(\mathbf{r}) + g'^2 G(\tau, \mathbf{r}) D(\tau, \mathbf{r}) \delta^2(\mathbf{r}),$$
$$\Pi(\tau, \mathbf{r}) = -g^2 G(-\tau, -\mathbf{r}) G(\tau, \mathbf{r}) - g'^2 G(-\tau, \mathbf{r}) G(\tau, \mathbf{r}) \delta^2(\mathbf{r}),$$
$$G(i\omega, \mathbf{k}) = \frac{1}{i\omega - \varepsilon(\mathbf{k}) + \mu - \Sigma(i\omega, \mathbf{k})},$$
$$D(i\Omega, \mathbf{q}) = \frac{1}{\Omega^2 + \mathbf{q}^2 + m_b^2 - \Pi(i\Omega, \mathbf{q})}.$$

Fermi surface coupled to a critical boson with disorder

$$\mathcal{L}_\psi = \psi_{\boldsymbol{k}}^\dagger \left(\frac{\partial}{\partial \tau} + \varepsilon(\boldsymbol{k}) \right) \psi_{\boldsymbol{k}}$$

a critical boson ϕ
e.g. Ising-nematic order



$$[\phi(\boldsymbol{r})]^2 + [g + g'(\boldsymbol{r})]\, \psi^\dagger(\boldsymbol{r})\psi(\boldsymbol{r})\, \phi(\boldsymbol{r})$$

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Boson Green's function: $D(q, i\Omega) \sim 1/(q^2 + \gamma|\Omega|)$
Fermion self energy:

$$\Sigma(i\omega) \sim -iv^2 \text{sgn}(\omega) - i \left(\frac{g^2}{v^2} + g'^2 \right) \omega \ln(1/|\omega|); \quad \frac{1}{\tau(\omega)} \sim \left(\frac{g^2}{v^2} + g'^2 \right) |\omega|$$

Marginal Fermi liquid self energy and $T \ln(1/T)$ specific heat

Fermi surface coupled to a critical boson with disorder

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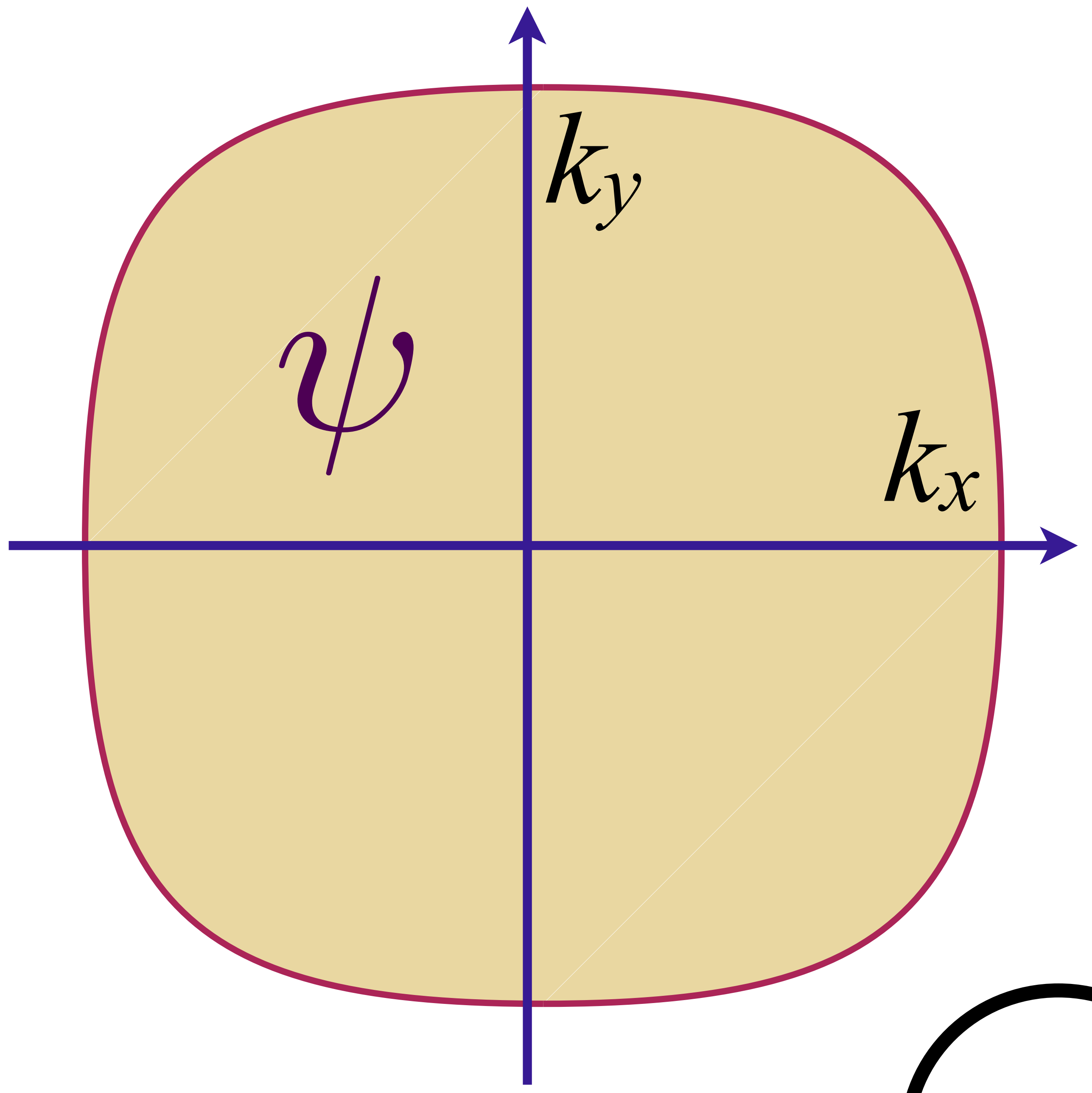
a critical boson ϕ
e.g. Ising-nematic order



$$[\phi(\boldsymbol{r})]^2 + [g + g'(\boldsymbol{r})]\, \psi^\dagger(\boldsymbol{r})\psi(\boldsymbol{r})\, \phi(\boldsymbol{r})$$

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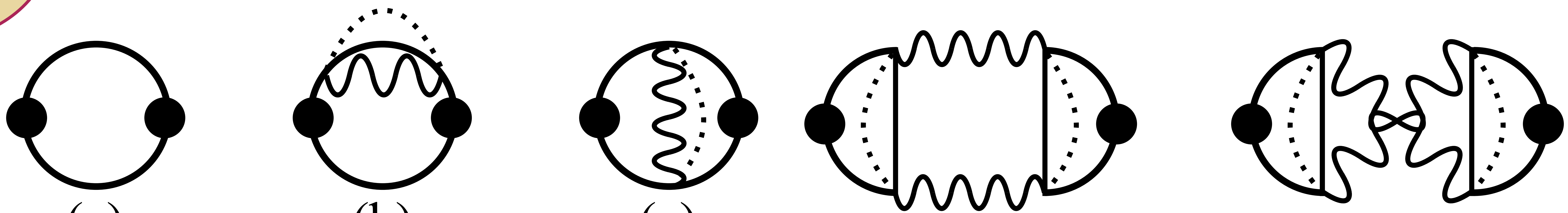
Conductivity:



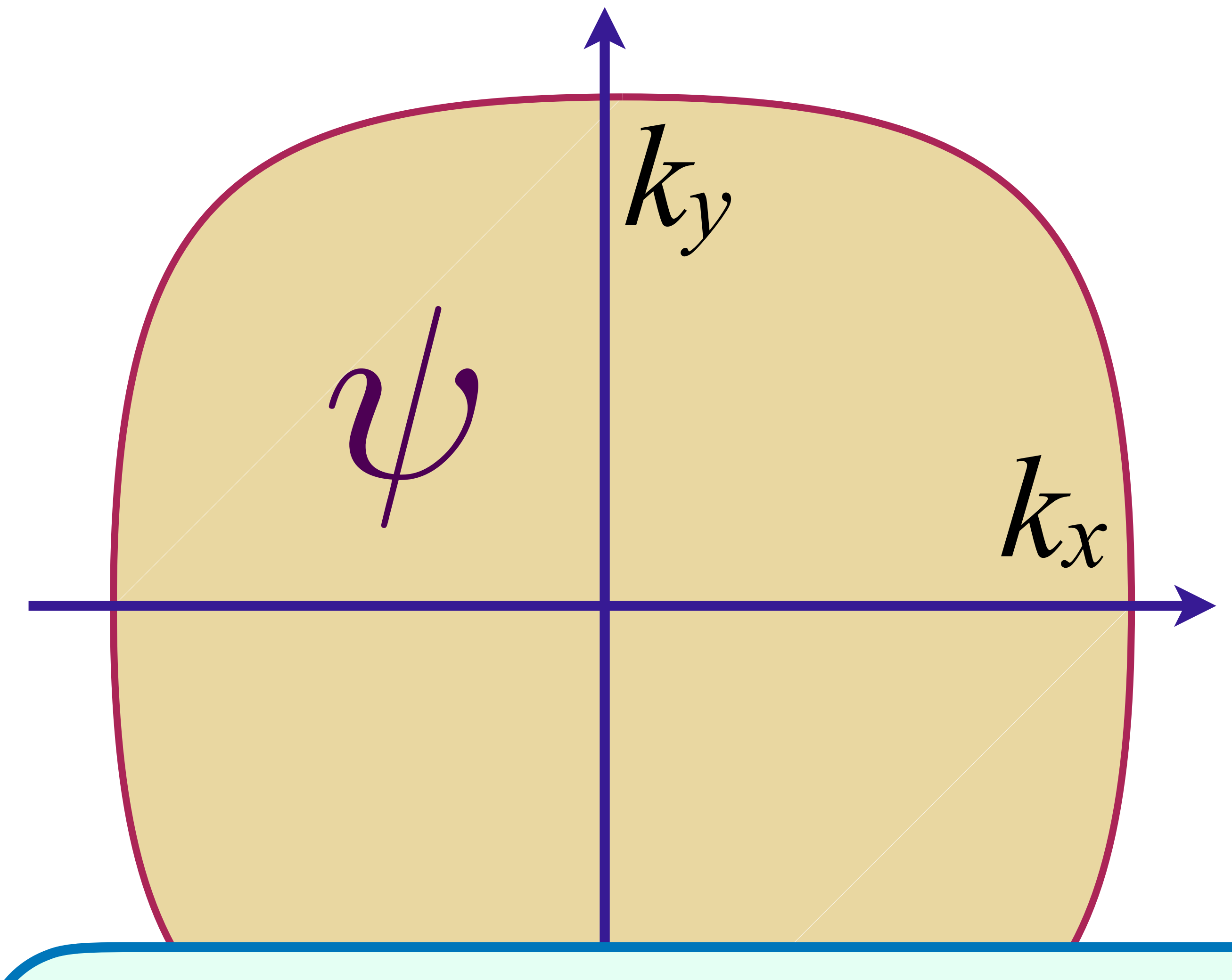
(a) σ_v (b) $\frac{\sigma_{\Sigma,g}}{2}, \frac{\sigma_{\Sigma,g'}}{2}$ (c) $\sigma_{V,g}$ (d) (e)

+ all ladders and bubbles…..

Fermi surface coupled to a critical boson with disorder

$$\mathcal{L}_\psi = \psi^\dagger_{\boldsymbol{k}} \left(\frac{\partial}{\partial \tau} + \varepsilon(\boldsymbol{k}) \right) \psi_{\boldsymbol{k}}$$

a critical boson ϕ
e.g. Ising-nematic order



$$[\phi(\boldsymbol{r})]^2 + [g + g'(\boldsymbol{r})]\, \psi^\dagger(\boldsymbol{r})\psi(\boldsymbol{r})\, \phi(\boldsymbol{r})$$

$$+ v(\boldsymbol{r})\psi^\dagger(\boldsymbol{r})\psi(\boldsymbol{r})$$

Conductivity: $\sigma(\omega) \sim [1/\tau_{\rm trans}(\omega) - i\omega\, m^*_{\rm trans}(\omega)/m]^{-1}$

$$\frac{1}{\tau_{\rm trans}(\omega)} \sim v^2 + g'^2|\omega| \quad ; \quad \frac{m^*_{\rm trans}(\omega)}{m} \sim \frac{2g'^2}{\pi} \ln(\Lambda/\omega)$$

Residual resistivity is determined by v^2; Linear-in-T resistivity determined by g'^2; Transport insensitive to g; Marginal Fermi liquid self energy and $T\ln(1/T)$ specific heat.

Fermi surface coupled to a critical boson:

<u>No spatial disorder</u>

A non-Fermi liquid but NOT a strange metal

Fermi surface coupled to a critical boson:

<u>Potential disorder v</u>

A marginal Fermi liquid but NOT a strange metal

Fermi surface coupled to a critical boson:

<u>Interaction disorder g'</u>

A marginal Fermi liquid AND a strange metal

Summary

- SYK: a solvable toy model without particle-like excitations, exhibiting thermalization and many-body chaos in a time of order $\hbar/(k_B T)$, independent of microscopic energy scales.

Summary

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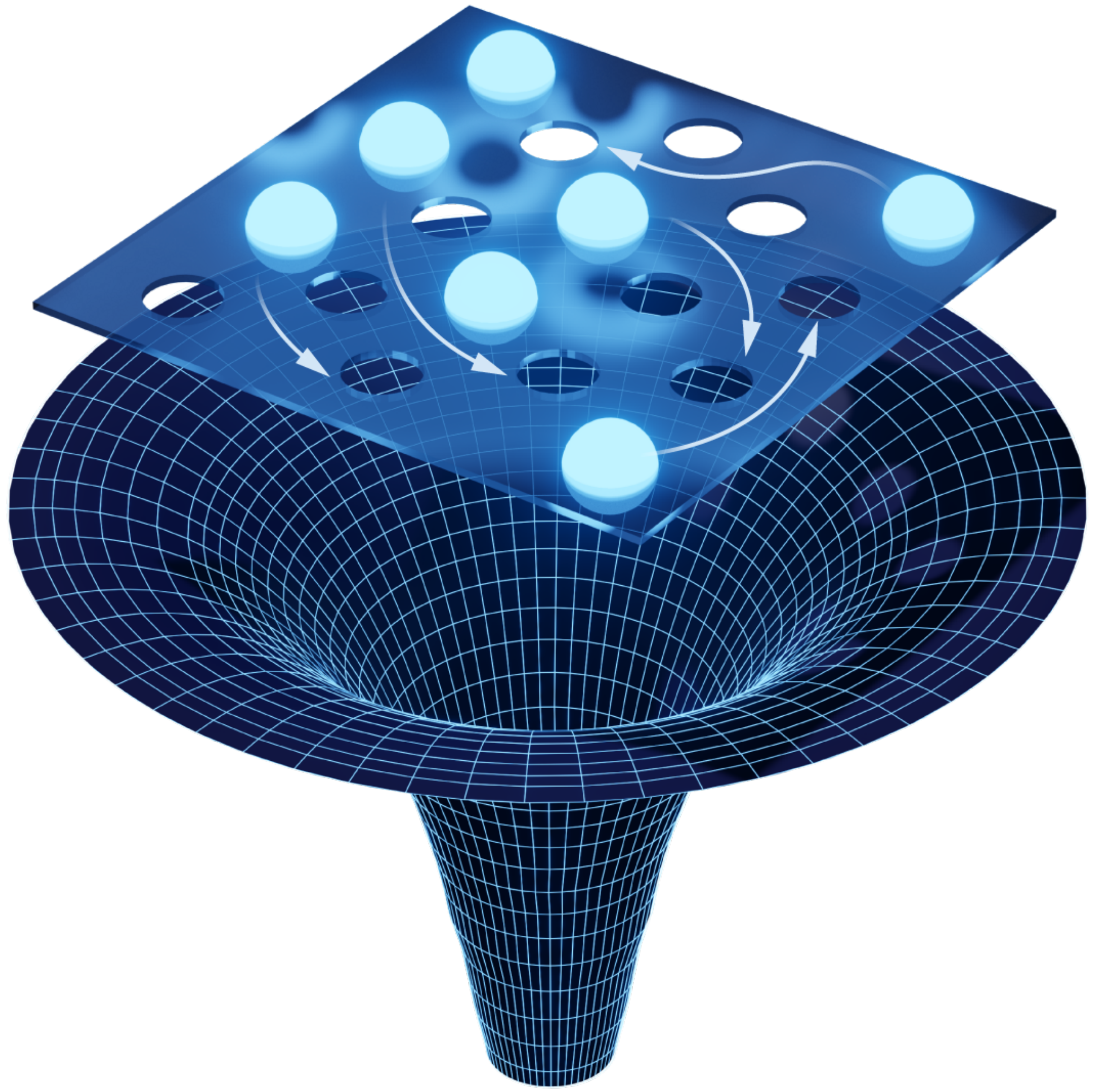
- Toy SYK model captures the correct universal low energy quantum theory of charged black holes, and provides a Hamiltonian realization of black hole microstates.

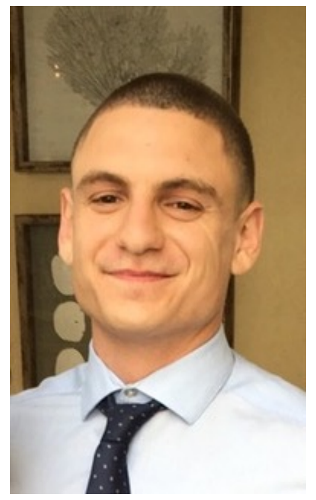


Summary

- SYK: a solvable toy model without particle-like excitations, exhibiting thermalization and many-body chaos in a time of order $\hbar/(k_B T)$, independent of microscopic energy scales.

- Toy SYK model captures the correct universal low energy quantum theory of charged black holes, and provides a Hamiltonian realization of black hole microstates.





- Linear-T resistivity, $T \ln(1/T)$ specific heat, $\sim 1/\omega$ optical conductivity, and marginal Fermi liquid electron spectrum *all* arise from a SYK-like model with spatially random interactions in a two-dimensional quantum-critical metal.

Aavishkar Patel, Haoyu Guo, Ilya Esterlis, S.S. arXiv: 2203.04990