An Exotic Interaction-driven Metal-Insulator Transition and Quantum Bad Metal

Cenke Xu
许岑珂

University of California, Santa Barbara

An Exotic Interaction-driven Metal-Insulator Transition and Quantum Bad Metal

Collaborators:

Nayan Myerson-Jain, Yichen Xu, (Department of Physics, UCSB)

Xiao-Chuan Wu (UCSB → U. of Chicago)

Mengxing Ye (KITP, UCSB → U. of Utah)

Zhu-Xi Luo (KITP, UCSB → Harvard)

Chao-Ming Jian (Cornell University)

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Content:

1. Introduction: 2d resistivity as universal quantity; bad metal and previous theory of interaction-driven MIT;

2. Experiments on MIT in Transition Metal dichalcogenide moiré heterostructure; facts and puzzles;

3. Candidate theory for the exotic interaction-driven MIT;

4. Construction for a "quantum bad metal".

Reference:

Y. Xu et.al. arXiv:2106.14910, Phys. Rev. X 12, 021067 (2022);
N. Myerson-Jain, et.al. arXiv:2209.04472

The electric conductivity/resistivity in pure 2d systems is a dimensionless quantity, hence it could take a "universal value", meaning independent of microscopic details.

Example 1: quantum Hall effect: universality guaranteed by topology

$$J_i = \sigma_H \epsilon_{ij} E_j, \quad \sigma_H = \nu \frac{e^2}{h}$$

Example 2: critical conductivity (resistivity) at 2+1d conformal field theories, or quantum critical points: universality protected by scaling invariance (Fisher, et.al. 1990)

$$\sigma(x) \sim \frac{e^2}{h}, \quad x = \omega/T$$

$$\langle T_{\mu\nu}(x_1) T_{\lambda\rho}(x_2) \rangle = C_T \frac{I_{\mu\nu,\lambda\rho}(x_{12})}{(x_{12}^2)^d}$$

$$\langle J_\mu^a(x_1) J_\nu^b(x_2) \rangle = C_J \frac{I_{\mu\nu}(x_{12})}{(x_{12}^2)^{d-1}} \delta^{ab} .$$

Conductivity is one of the universal quantities associated with a CFT, see, e.g. Giombi, et.al. 2016 (the AC limit)

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Liu, et.al. 1991

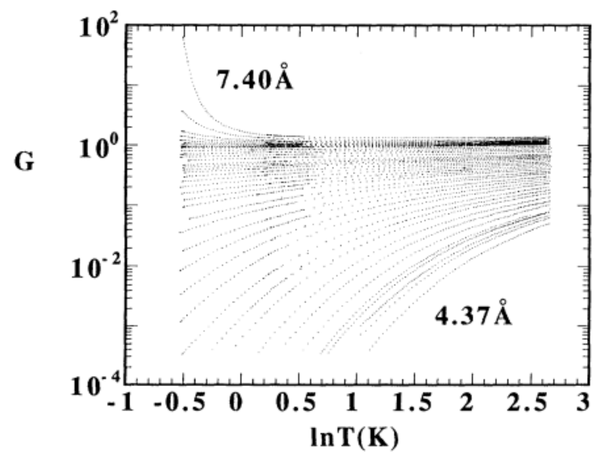

FIG. 1. Logarithm of the conductance G, in units of $4e^2/h$, vs $\ln(T)$ for a number of different Bi films. The thickness of the first (thinnest) and last (thickest) films are indicated.

The electric conductivity/resistivity in pure 2d systems is a dimensionless quantity, hence it could take a "universal value", meaning independent of microscopic details.

Example 3: threshold for "bad metal" (e.g. Emery, Kivelson, 1995)

$$\sigma \sim \frac{ne^2\tau}{m} \sim \frac{ne^2 l}{mv_F}$$

For the semiclassical theory of metal to "work", the mean free path l needs be greater than $\sim 1/k_F$. This means that there is a lower bound for conductivity, or upper bound for resistivity (Mott-Ioffe-Regel limit) captured by the rudimentary method: $\rho \sim h/e^2$

When the resistivity is larger than the MIR limit (or $k_F\, l > 1$), the (noninteracting) electrons are supposed to be insulating.

The electric conductivity/resistivity in pure 2d systems is a dimensionless quantity, hence it could take a "universal value", meaning independent of microscopic details.

Metal-insulator transition for noninteracting electrons:

2d free electron system at length scale far larger than the mean free path of disorder, is described by a 2d NLSM whose target space (in the replica space after disorder average) is decided by the symmetry of the system. The coupling constant g of the NLSM corresponds to the resistivity.

In the (rather singular) replica limit $n \rightarrow 0$, there may be a fixed point of the RG flow of the NLSM, which generally should happen when g is of order 1, which means the critical resistivity is of order $\rho \sim h/e^2$

Previous theory for interaction-driven continuous MIT at filling $v = 1$:
(S. S. Lee, P. A. Lee, 2007, Senthil, 2008, Witczak-Krempa, et.al. 2012)

Starting point: a parton construction $c_\alpha = b f_\alpha$

The bosonic parton b carries the charge, fermionic parton f_α carries spin-1/2. The fermionic sector of the theory does not change dramatically across the MIT; at the most basic level the MIT is just a phase transition of the bosonic sector only.

In this theory, the metal phase becomes the "superfluid" phase of b; the Mott insulator phase of electrons becomes the MI phase of b at $v = 1$.

$$\mathcal{L} = |(\partial_\mu - \mathrm{i}a_\mu)b|^2 + r|b|^2 + u|b|^4 + \mathcal{L}_F(f_\alpha, a_\mu)$$

Dynamical gauge field,
overdamped by fermions

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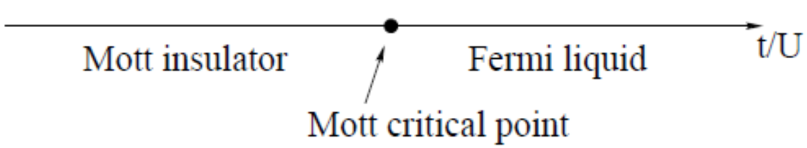
Starting point: a parton construction $c_\alpha = b f_\alpha$

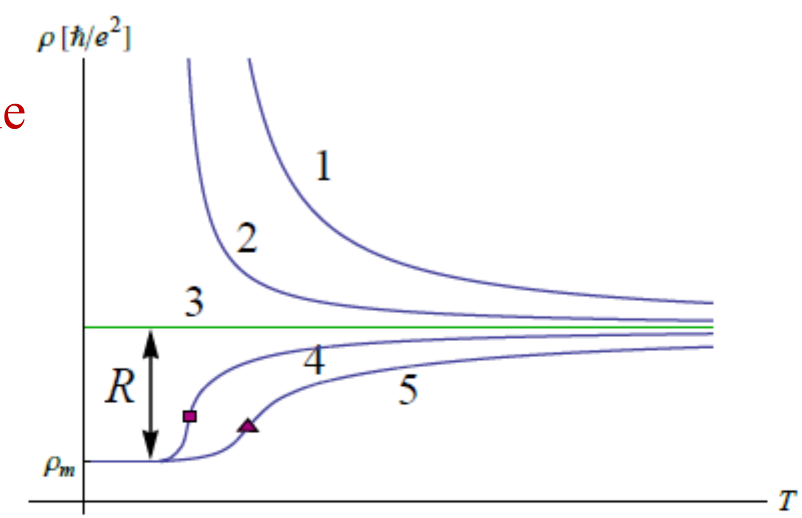
In this theory, the metal phase becomes the "superfluid" phase of *b*, the Mott insulator phase of electrons becomes the MI phase of *b* at $\nu = 1$.

Ioffe-Larkin rule: $\rho = \rho_b + \rho_f$

The critical resistivity should be at the order of h/e^2 (with a factor of 7~8 with large-*N* calculation)

Bose MI of *b* "Superfluid" of *b*





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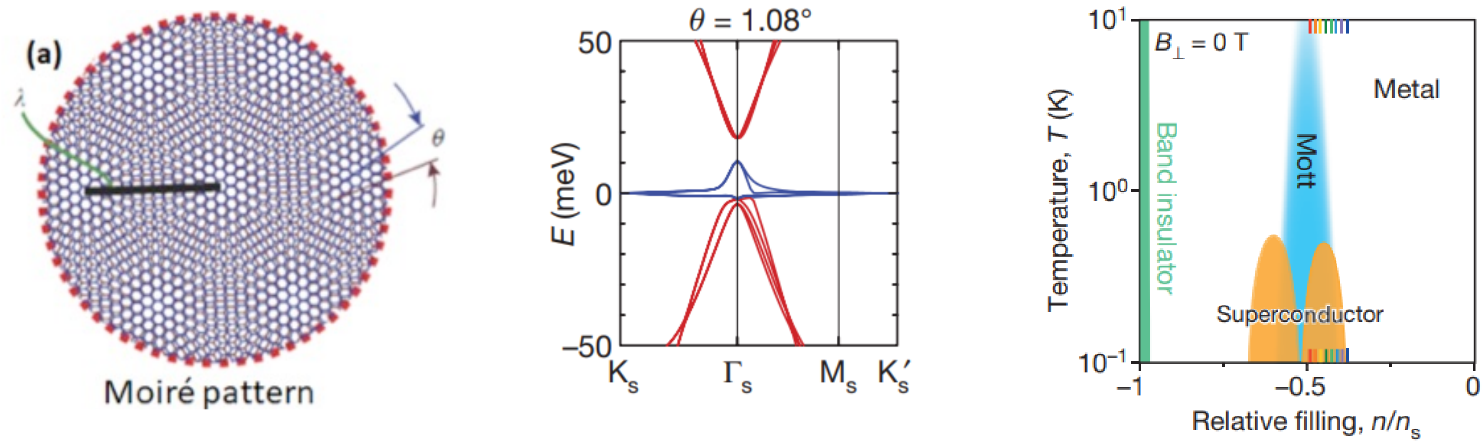
Moiré: a perfect playground for 2d strongly interacting electrons

Started with twisted bilayer graphene, (Cao, et.al. 2018)



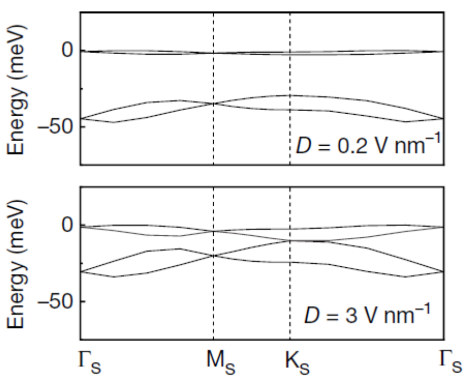
Many more systems in the moiré family, many more striking phenomena: correlated insulator, high Tc superconductor, anomalous quantum Hall effect, strange metal, spin-triplet SC, nematicity…

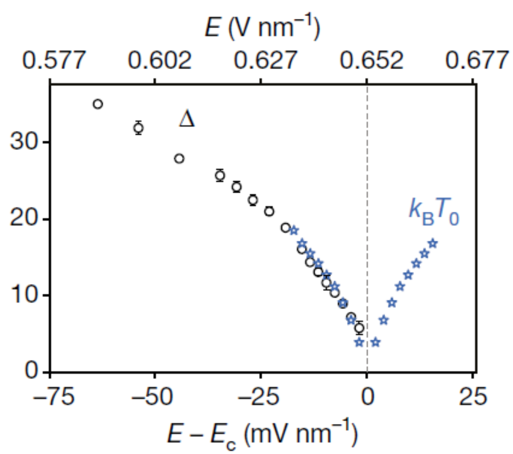
Experiment on interaction driven MIT in Transition Metal dichalcogenide (TMD) moiré heterostructure $MoTe_2/WSe_2$

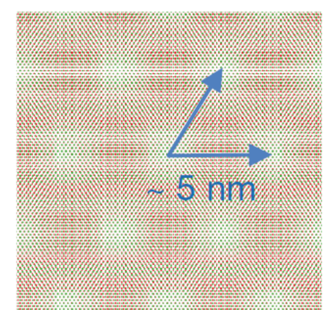
T. Li, et.al. Nature 597, 350, 2021

Main messages from the experimental report:

1. The physics at the mini moiré band is captured by an (extended) Hubbard model on a triangular lattice with “spin-1/2” fermions (F. Wu, et.al. 2018)







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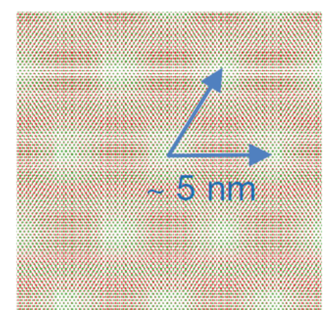
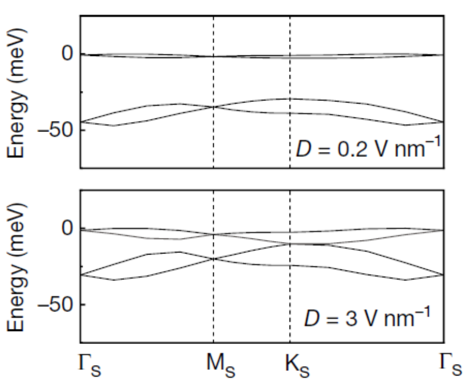
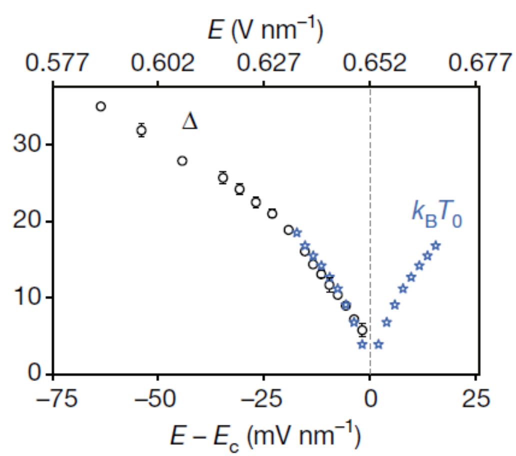
Main messages from the experimental report:

2. By tuning the displacement field (bandwidth), there is an interaction-driven continuous MIT; disorder plays a "perturbative role".







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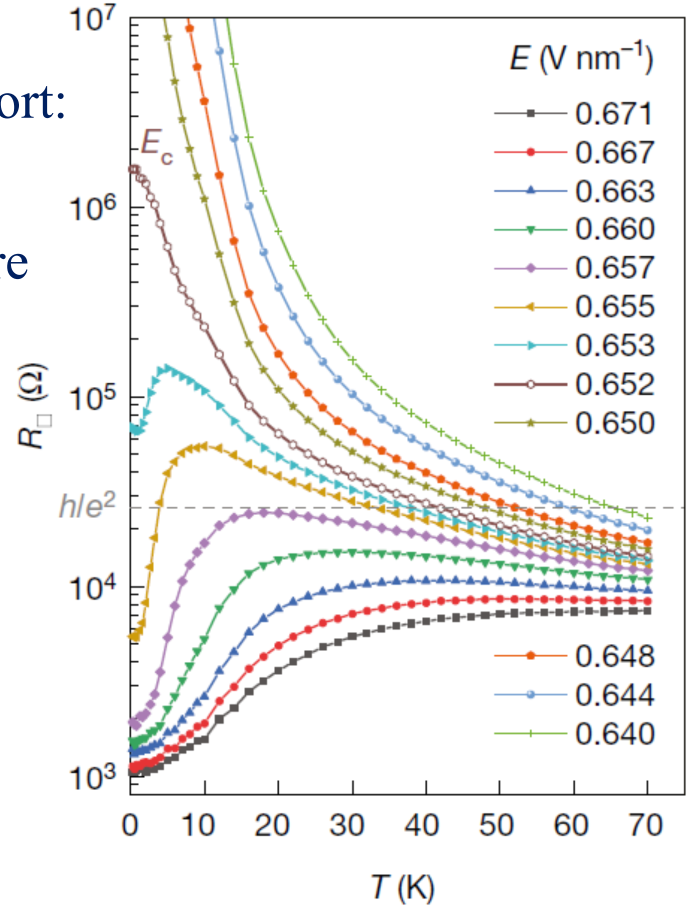
Main messages from the experimental report:

3. Spin susceptibility changes smoothly across the MIT, compatible with the picture that the insulator has a Fermi surface of spinon f_α.

4. But: The critical resistivity is exceptionally large compared with prediction of the previous theory.

A new theory is needed.



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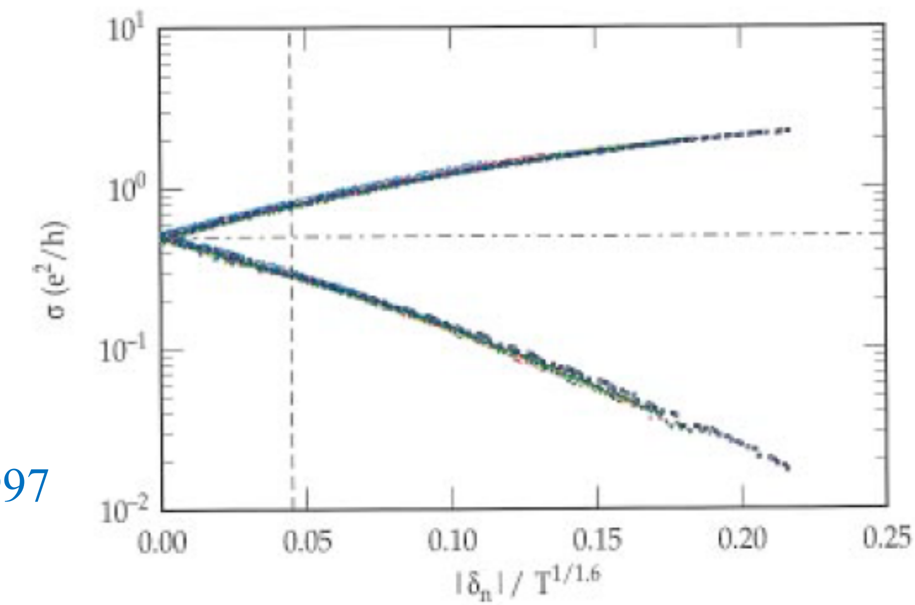
Stark contrast with MIT in another system: 2dEG, where the critical resistivity is indeed order of h/e^2.

Popovic, et.al. 1997



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Assumption of our theory: what was observed is indeed a **continuous**, **interaction-driven** MIT, and **disorder plays "perturbative role"**.
(If disorder/inhomogeneity is important, Kim, et.al. arXiv:2204.10865)

A simple alternative parton construction:

Introduce a charged parton for each spin/valley flavor:
This formalism breaks spin SU(2) symmetry, allowed in the current system due to "spin"-orbit coupling

$$c_{\boldsymbol{r},\alpha} = b_{\boldsymbol{r},\alpha} f_{\boldsymbol{r},\alpha}.$$

For filling factor $\nu = 1$ (one electric charge per moiré unit cell), each boson flavor is at half-filling;

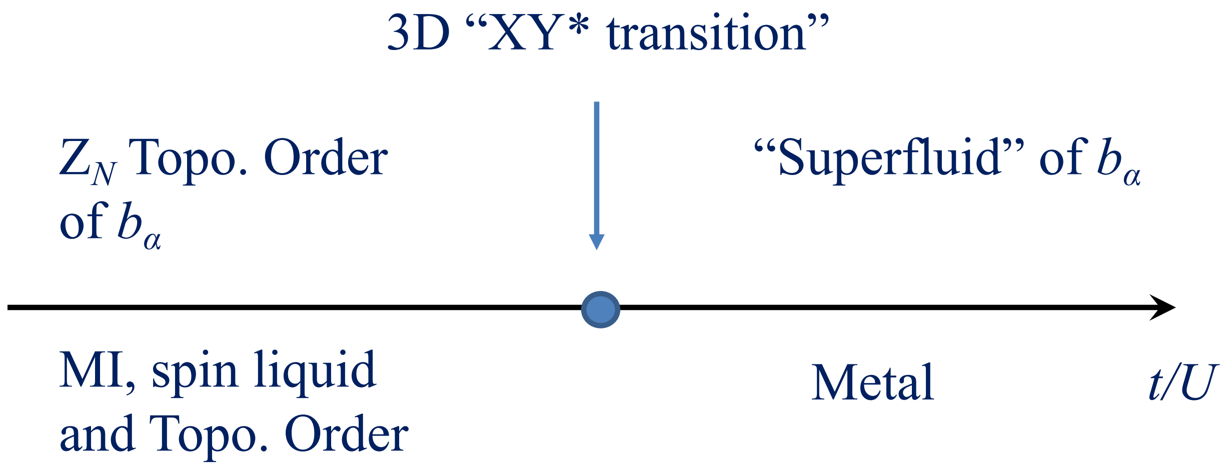
⇓

then according to the Lieb-Schultz-Mattis theorem, the boson MI cannot be "featureless", i.e. it must either spontaneously break translation symmetry (certain density wave), or have topological order.

Consequence of a "nontrivial" Mott insulator of b_α ?

Charge fractionalization: each elementary charge carrier at the MIT (e-anyon of the Z_N topological order) carries charge $e^* = e/N$, hence the critical resistivity should be at the order of

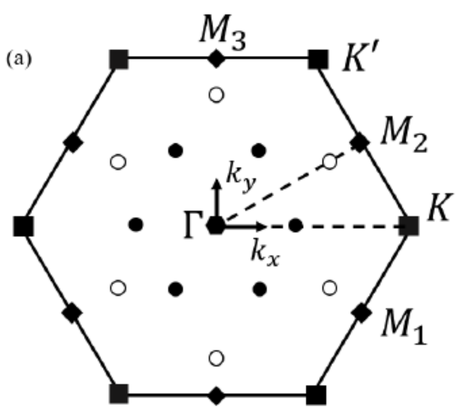
$$\rho \sim \frac{h}{e_*^2} = N^2 \frac{h}{e^2}$$



Another (more natural) possibility:

The nontrivial MI has some density wave that spontaneously breaks the translation symmetry. Then the MIT corresponds to a SF - density wave transition of b_α, which resembles the deconfined QCP between Neel and VBS orders of quantum magnets.

The SF – density wave transition is most naturally described using the boson-vortex duality (Burkov, Balents, 2004): The MI with density wave corresponds to vortex condensation; the superfluid of b_α, i.e. the metal phase, is the "insulator" of vortex.

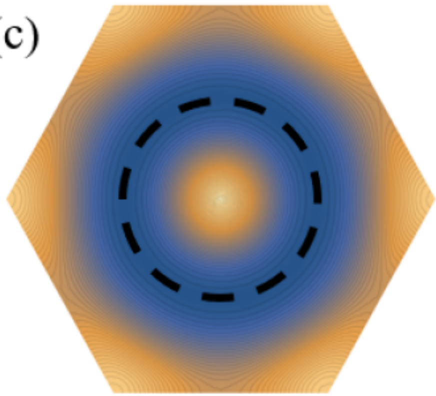


The vortex band structure must have multiple minima in the Brillion zone due to fractional filling of the boson b_α.

When there are N minima in the vortex band structure, the charge carrier carries charge $e^* = e/N$, analogous to the case with Z_N topological order.

Depending on the microscopic model, N can take different values, or even "infinity", which corresponds to a "ring" degenerate minima in the Brillouin zone.

(c)



Unlike the case with Z_N topological order, the resistivity will acquire a factor of N rather than N^2 due to charge fractionalization.

Lagrangian for the vortex condensation:

$$\mathcal{L}^{(1)} = \sum_{j=0}^{N-1} (|(\partial_\mu - \mathrm{i}A_\mu)\psi_j|^2 + r|\psi_j|^2) + \cdots \qquad \frac{1}{2\pi} dA = \bar{J}_b$$

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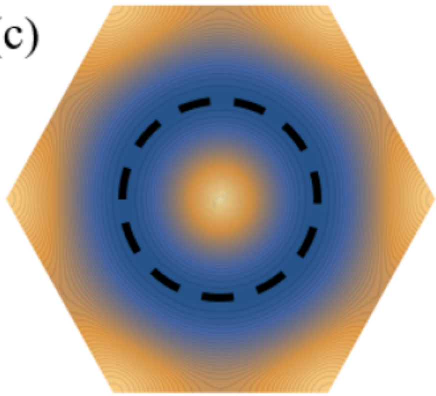

Unlike the case with Z_N topological order, the resistivity will acquire a factor of N rather than N^2 due to charge fractionalization.

Simple evaluation of the bosonic parton contribution to the AC resistivity shows that, the J_b correlation (dA correlation) is suppressed by $1/N$, hence in the limit $\omega/T \rightarrow$ infty:

$$\rho_b(\infty) \sim \frac{4N}{\pi}\frac{h}{e^2}$$

When there are N minima in the vortex band structure, the charge carrier carries charge $e^* = e/N$, analogous to the case with Z_N topological order.

The bosonic parton contribution to the DC resistivity at the MIT reads

$$\rho_b(0) = \Delta\rho = \left(R^{(0)} + R^{(1)}(N-1)\right)\frac{h}{e^2},$$

$R^{(0)} \sim 3.62$, $R^{(1)} \sim 1.68$

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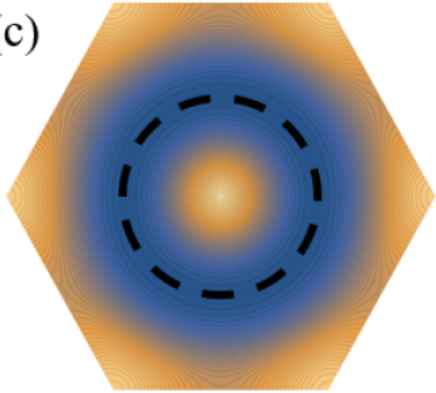

The bosonic parton contribution to the DC resistivity can be nonzero even without considering disorder, due to an emergent PH symmetry; The fermionic parton contribution to the resistivity mainly comes from scattering with disorder, and is supposedly weaker.

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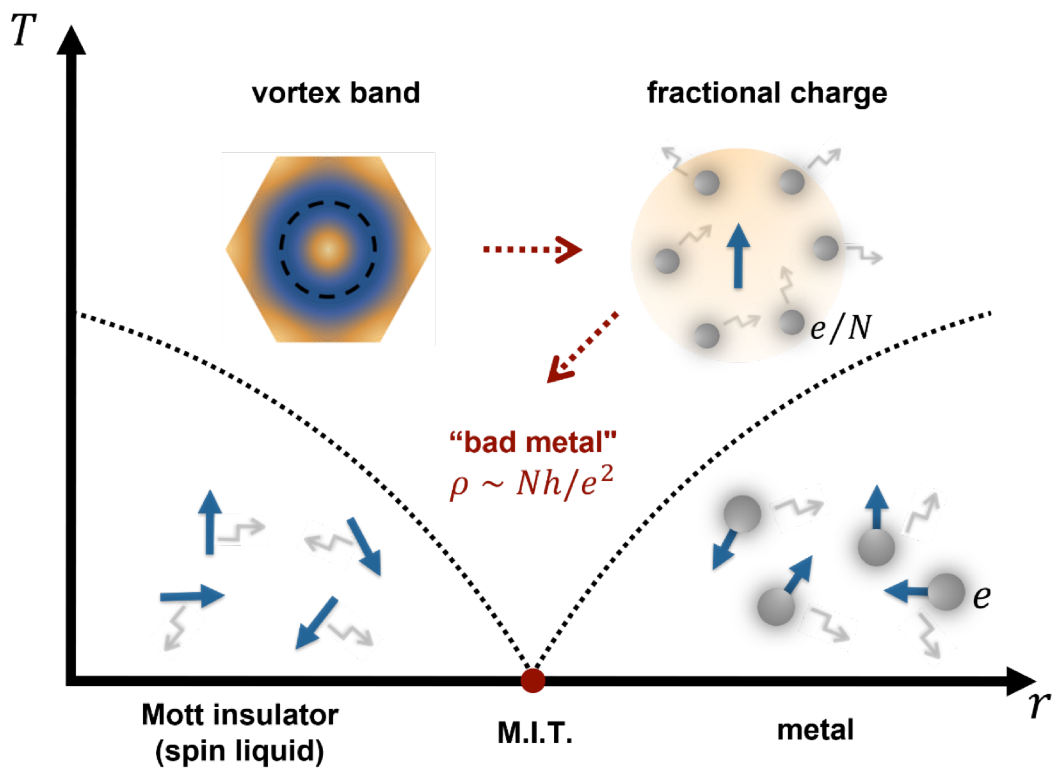

Other predictions:

1. The transport gap v.s. Tunneling gap

In the MI phase, if there are deconfined fractional charges, then the activation gap for transport corresponds to the gap of the fractional charge carrier; while the electron tunneling gap (for example from STM) is given by N fractional charge carriers; hence these two gaps should differ (roughly) by a factor of N.

2. Quasi-particle weight scaling near the MIT:

The quasi-particle weight will vanish approaching the MIT, with scaling

$$\sqrt{Z} \sim \langle \varphi_\alpha^N \rangle \sim |r|^{\beta_N},$$

β_N should also increase with N, i.e. much larger than previous theory.

3. Shot noise (analogous to probing fractional charge in FQH)

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Summary of the MIT:

1. Interaction driven continuous MIT observed in TMD moiré heterostructure; critical resistivity far exceeds the previous prediction;

2. A new theory for interaction driven MIT where the charge fractionalizes due to the nontrivial nature of the Mott insulator, which leads to a large critical resistivity; Other predictions such as quasi-particle weight scaling, etc.

Related: Metal-Wigner crystal transition at fractional electron filling (arXiv:2106.14910, Y. Xu et.al.; arXiv:2111.09894 Musser et.al.); Large critical resistivity at the 3D XY* transition of bosons (Wang, et.al. Nat. Comm. 12, 5347, 2021); Another theory for the observed MIT in TMD, which relies on strong inhomogeneity (Kim, et.al. arXiv:2204.10865).

Supports: NSF, Simons Foundation, Moore Foundation.

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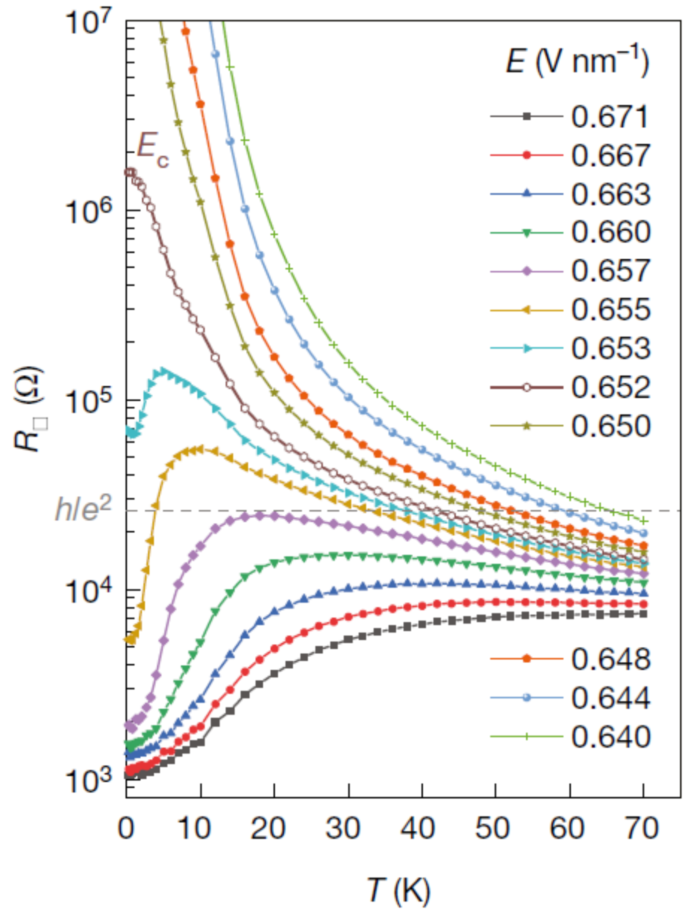
Goal: a stable bad metal phase with strong interaction

The experiment reported a MIT whose critical resistivity at zero and low T is far beyond the MIR limit.

We can ask if there exists a stable bad metal phase whose resistivity remains finite, but far exceeds the MIR limit, at zero and low temperature.

Different from the original bad metal physics in cuprates, where the resistivity increases linearly with temperature and cross the MIR limit at finite T.



An Exotic Interaction-driven Metal-Insulator Transition and Quantum Bad Metal

Goal: a stable bad metal phase with strong interaction

Strongly interacting "electron liquids" have no microscopic quasiparticles. It may be described by hydrodynamics, holographic dual, etc. (reviews, Hartnoll, Lucas, Sachdev, arXiv:1612.07324)

But some strongly interacting systems may have a weakly interacting dual description, there could be a trackable microscopic description in terms of quasiparticles in the dual picture.

Examples of particle-vortex duality: superfluid-photon duality in 2+1d (Peskin, 1978; Dasgupta, Halperin, 1981; Fisher, Lee, 1989), N=1 QED to free Dirac fermion duality in 2+1d (D. T. Son 2015, Wang, Senthil, 2015, Metlitski, Vishwanath, 2015), Chern-Simons matter theory to free Dirac/Majorana fermion duality (Aharony, 2015, and many others)

Observations:

1. At least for the standard boson-vortex duality, when the "charge vortex" is in a good metal, the charged bosons will be in a bad metal, (Fisher, et.al. 1990) :

$$\sigma_e \sim 1/\sigma_v$$

2. When charges are strongly correlated, the vortices will be weakly interacting, which may justify a rudimentary description based on quasiparticles of vortices.

$$\sum_{i,j} V_{i,j} n_i n_j \sim \int d^2x \, \frac{1}{g} (\vec{\nabla} \times \vec{a})^2;$$

The momentum of the vortices can be transferred to the dual photons, and relax through (weak) disorder before "feeding back" to the vortices.

How do we make the "charge vortex" a fermion?

The standard SU(2) slave rotor for electrons (Wen, Lee, 1996; etc.):

$$\begin{aligned} c_\uparrow &= f_\uparrow z_1 - f_\downarrow^\dagger z_2^\dagger, \\ c_\downarrow &= f_\downarrow z_1 + f_\uparrow^\dagger z_2^\dagger. \end{aligned}$$

This formalism can maximally host a SU(2) charge symmetry; SU(2) spin symmetry, and a SU(2) gauge transformation (Affleck, et.al. 1988; Dagotto, et.al. 1988;). For convenience, we assume SU(2) charge and SU(2) gauge are broken to U(1).

$$\begin{aligned} \mathrm{U}(1)_e &: z_1 \to e^{\mathrm{i}\theta_e} z_1, \quad z_2 \to e^{-\mathrm{i}\theta_e} z_2, \quad f_\alpha \to f_\alpha; \\ \mathrm{U}(1)_g &: z_1 \to e^{\mathrm{i}\theta_g} z_1, \quad z_2 \to e^{\mathrm{i}\theta_g} z_2, \quad f_\alpha \to e^{-\mathrm{i}\theta_g} f_\alpha, \\ \mathrm{U}(1)_s &: z_\alpha \to z_\alpha, \quad f_\uparrow \to e^{\mathrm{i}\theta_s} f_\uparrow, \quad f_\downarrow \to e^{-\mathrm{i}\theta_s} f_\downarrow. \quad (3) \end{aligned}$$

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\end{aligned}
$$

This formalism allows us to construct many states of electrons using the states of bosonic chargon z_α and fermionic spinon f_α.

For example, if z_α are in a trivial insulator, and spinon f_α has an ordinary mean field band structure, the system becomes a Mott insulator, and it is also a spin liquid.

The last 10 years our understanding of states of matter has significantly broadened, including interacting symmetry protected topological states (Chen, et.al. 2011, and many others)

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Introduce another two composite bosonic fields: $\phi_e = z_1^\dagger z_2, \ \phi_g = z_1 z_2.$

ϕ_e, ϕ_g carry $U(1)_e$ and $U(1)_g$ charges (2, 0) and (0, 2) respectively.

Now we put ϕ_e, ϕ_g in a bosonic symmetry protected topological state. The physics of this bSPT state is that, the vortex of ϕ_e carries charge of ϕ_g , and vice versa (Levin, Senthil, 2012, Lu, Vishwanath, 2012).

$$\mathcal{L}_{\mathrm{bSPT}} = \frac{\mathrm{i}K^{IJ}}{2\pi}\tilde{a}_I \wedge d\tilde{a}_J + \frac{\mathrm{i}2}{2\pi}\tilde{a}_1 \wedge da + \frac{\mathrm{i}2}{2\pi}\tilde{a}_2 \wedge dA^e,$$

Lagrangian description of the bSPT:

$$\mathcal{L}_{\mathrm{bSPT}} = \frac{\mathrm{i}K^{IJ}}{2\pi}\tilde{a}_I \wedge d\tilde{a}_J + \frac{\mathrm{i}2}{2\pi}\tilde{a}_1 \wedge da + \frac{\mathrm{i}2}{2\pi}\tilde{a}_2 \wedge dA^e,$$

A bSPT is gapped, with nondegenerate ground state, safe to integrate out:

$$\mathcal{L} = \mathcal{L}_F(f_\alpha, a_\mu) + \frac{4\mathrm{i}}{2\pi} a \wedge dA^e + \frac{1}{g}(\vec{\nabla}\times\vec{a})^2 + \cdots$$

Consequences:
1, the gauge flux of gauge field *a* carries electric charge $4e$, *i.e.* the bSPT of the bosonic partons makes gauge field *a* the “dual” of charge.

2, the fermionic parton f_α, which couples minimally to *a*, now automatically becomes the “charge vortex”, as f_α also carries flux of the EM field, seen by any charge carrier.

Lagrangian description of the bSPT:

$$\mathcal{L}_{\mathrm{bSPT}} = \frac{\mathrm{i}K^{IJ}}{2\pi}\tilde{a}_I \wedge d\tilde{a}_J + \frac{\mathrm{i}2}{2\pi}\tilde{a}_1 \wedge da + \frac{\mathrm{i}2}{2\pi}\tilde{a}_2 \wedge dA^e,$$

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Consequences:

$$\mathcal{L}_{\mathrm{res}} = \sum_{\omega} \frac{1}{2}\frac{4}{\pi^2}\frac{\omega^2}{\Pi_f(\omega)}|\vec{A}^{e,t}(\omega)|^2.$$

$$\downarrow$$

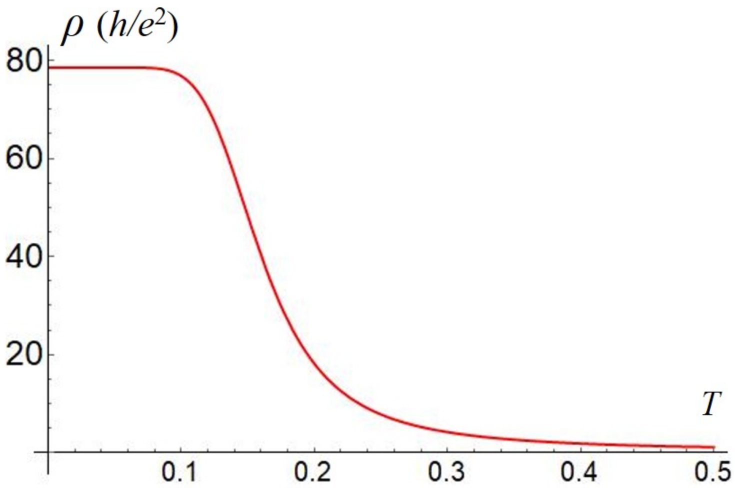
$$\rho_e(\omega) = \frac{\pi^2}{4}\sigma_f(\omega) = \frac{\pi^2}{4}\left(\frac{\sigma_0}{1 - \mathrm{i}\omega\tau_v}\right)$$

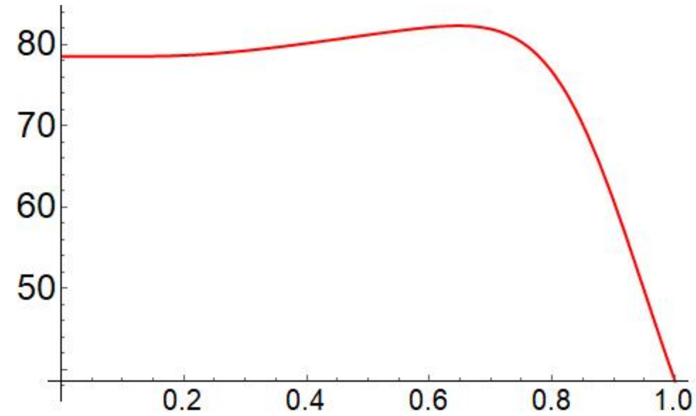
The σ_0 can be large (meaning the resistivity can far exceed the MIR limit), and it can be evaluated through the rudimentary Drude theory.

If we assume the bosons have a finite (but small) conductivity:

$$\sigma_e = \sigma_b + \frac{4}{\pi^2}\left(\frac{1}{\sigma_f + \sigma_b}\right)$$

At finite T, Assuming the conductivity of the boson follows an activated behavior $\sigma_b \sim \exp(-\Delta_b/T)$, and the conductivity of fermionic spinons f_α (now also the charge vortices) follows the standard Fermi liquid, then this composition rule will lead to (in unit of h/e^2):





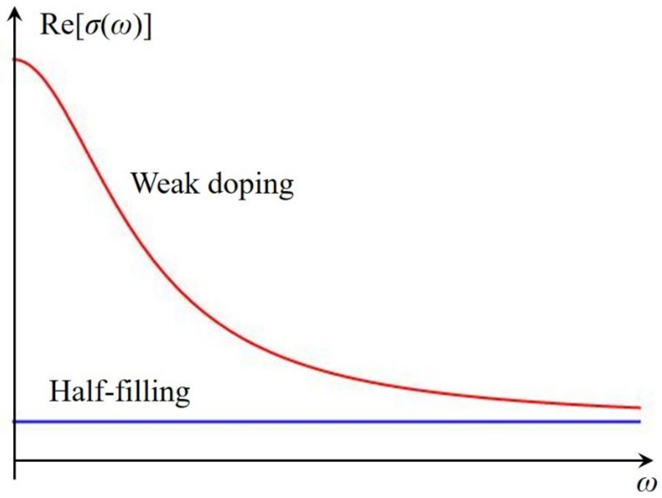
So far we have assumed one electron per site (half-filling of a lattice); in the dual picture this means that the gauge field a has zero average flux. Under weak doping, the fermionic spinons f_α (charge vortices) will see a nonzero background flux.

Vortex conductivity: $\sigma_f(\omega) = \left(\frac{\frac{1}{\tau_v} - i\omega}{\omega_c^2 + (\frac{1}{\tau_v} - i\omega)^2}\right)\frac{1}{\tau_v}\sigma_0$

$$\sigma_e(\omega) \sim \frac{4}{\pi^2\sigma_f(\omega)} = \frac{4\tau_v}{\pi^2\sigma_0}\left[\frac{\omega_c^2}{\frac{1}{\tau_v} - i\omega} + \left(\frac{1}{\tau_v} - i\omega\right)\right]$$

(Some subtlety: the Hall effect is cancelled due to discrete symmetries in the construction.)

A small Drude weight $\sim (\delta\, n_e)^2$ emerges under weak doping:



Many other effects:
Example: Wiedemann-Franz law violation, as vortices do not carry charges, but carry entropy (similar effect, Wang, Senthil, 2015)

Other vortex liquids:

1. The Dirac composite fermion at half-filled Landau level as vortex liquid, and similar bosonic state at filling 1, (HLR, 1993, Read, 1998, Son 2015, Wang, Senthil 2015-2017); (need T, P breaking, and longitudinal resistivity is actually small due to transverse transport)

2. (fermionized) vortex to understand the emergent anomalous metal observed in amorphous 2d thin films (between the SC and insulator) with anomalously large conductivity. Galitski, et.al. 2005;
"vortex glass" Wu, Phillips, 2006.

Summary of the quantum bad metal:

1. A construction of bad metal of charges can be made, by making the charge vortices in a (weakly interacting) good metal phase, with fermionic vortices that form a Fermi surface;

2. By making the bosonic partons a bSPT state, the fermionic spinons automatically becomes the charge vortices, which can naturally form a Fermi surface.

3. Features of the bad metal includes, an exceedingly large resistivity, an emergent Drude weight under doping, a strong Wiedemann-Franz law violation.