Cracking the critical flavor number of QED_3 with conformal bootstrap

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Based on arXiv:1812.09281 (*ZL*), 2005.01721 (*ZL and Poland*), 2006.05119 (*ZL*), 2107.09020 (*ZL*), 2112.02106 (*Albayrak, Erramilli, ZL, Poland and Xin*)

Background: IR phases of QED_3

Three dimensional Quantum Electrodynamics (QED_3) coupled to N_f flavors of massless two-component Dirac fermions ψ^i:

$$L = \frac{1}{4e^2} F_{\mu\nu} F^{\mu\nu} + \sum_{i=1}^{N_f} \bar{\psi}_i \sigma^\mu (\partial_\mu + iA_\mu) \psi^i,$$

Interesting facts about this theory:

- The flavor number N_f 2 2Z to avoid parity anomaly (*Witten 2016*)
- QED_3 is asymptotically free (gauge coupling e^2 has mass unit)
- Its IR phase is solvable with large N_f or in 4 ϵ dimension
- Large N_f: IR fixed point at $e^2 = 6\pi^2/N_f$ (*Appelquist et al 1988*)
- Small N_f: Spontaneous fermion mass generalization and chiral symmetry breaking (*Appelquist et al 1985*)

A long-standing problem: what is the critical N_f which separates the conformal and chiral symmetry breaking phases?

Critical value N_f of $U(1)/R$ QED$_3$ (*Gukov 2016 +...*)

N_f (2-component Dirac fermion)	Method	Year and Reference
6.5	Schwinger-Dyson equations	1984-88 Pisarski,Appelquist et al
8.6	Schwinger-Dyson equations	1996-97 Maris,Aitchison, et al
3	thermal free energy	1999-2004 Appelquist et al
4	Hybrid Monte Carlo	2002-04 Hands et al
4.3	divergence of the chiral susceptibility	2002 Franz et al
8	covariant solutions for propagators	2004 Fischer et al
12	perturbative RG in the large-N_f limit	2004 Kaveh et al
$10 \ldots 13$	comparison to the Thirring model	2007-12 Christofi,Janssen, et al
3	lattice simulations	2008 Strouthos et al
$8 \quad N_f^{\chi\mathrm{SB}} \quad N_f^{\mathrm{conf}} \quad 20$	functional RG	2014 Braun et al
8	F-theorem	2015 Giombi et al
4	one-loop ϵ-expansion	2015 DiPietro et al
5.7	$1/N_f$ expansion	2016 Gusynin et al
5.8	ϵ-expansion	2016 Herbut et al
< 2	lattice simulations	2017 Qin et al
< 2	lattice simulations	2015-2020 Karthik et al

Why is it difficult to estimate N_f?

- A typical strong coupling problem!
  Perturbative approaches find at the IR fixed point of QED_3, the gauge coupling is $e^2 = 6\pi^2/N_f$, so the theory is strongly coupled for small N_f 10! Hard to estimate the contributions from higher order terms. A non-perturbative approach is needed!

- Why is it challenging for lattice simulation?
  - Computation demanding for fermionic systems
  - Near the critical flavor number N_f, hard to distinguish the continuous phase transition from the weakly first order phase transition
  - Not strict estimation on the errors

An efficient non-perturbative approach with strict control on the errors?
Conformal bootstrap can shed new light for this problem!

Outline

1. Conformality of $N_f = 2$ QED_3 and $O(4)/SO(5)$ DQCPs
2. A surprise in $O(N)$ vector bootstrap: new family of kinks
3. A novel algebraic structure in 4pt crossing equations
4. Towards the bootstrap island for the $N_f = 4$ QED_3
5. Outlook: towards a comprehensive understanding of conformal QED_3

1. Conformality of $N_f = 2$ QED$_3$ and $O(4)/SO(5)$ DQCPs

UV symmetry of compact QED$_3$ coupled to N_f 2-component fermions:

$$SU(N_f)_f \quad U(1)_t$$

where $U(1)_t$ is generated by the conserved current $j_t = \epsilon \quad F$.
$N_f = 2$ QED$_3$: a remarkably simple example for symmetry enhancement and dualities without SUSY? (*Chong Wang's lecture in the workshop*)

- Symmetry enhancement:

$$SU(2)_f \quad U(1)_t \; ! \; SU(2)_f \quad SU(2)_t \quad O(4)$$

- Bosonization:

$$\text{fermionic } N_f = 2 \text{ QED}_3 \quad ! \quad \text{bosonic easy-plane } N_f = 2 \text{ QED}_3$$

$SO(5)$ symmetric DQCP: $N_f = 2$ QED$_3$ coupled with a critical boson.

$N_f = 2$ QED3 and DQCPs: first or second order phase transitions?

- $N_f = 2$ QED_3:
  - Spontaneous chiral symmetry breaking (*Hands et al. 2002; Strouthos and Kogut, 2008*)
  - Continuous phase transition and the critical indices support an enhanced $O(4)$ symmetry (*Qin et al, 2017; Karthik, Narayanan, 2015-2020)*
- $SO(5)$ symmetric DQCP:

  Unusual behavior in large-scale $SO(5)$ DQCP lattice simulation: drift of critical indices with scales. (*Nahum et al 2015*)

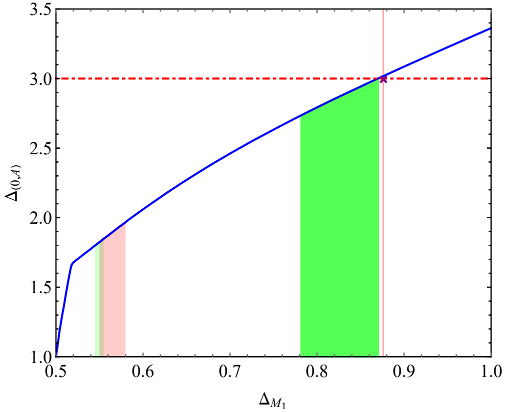
  Finite size effect? Weakly first order phase transition? Another relevant singlet scalar needs to be fine-tuned? etc...

Conformal bootstrap: nonperturbative approach with strict control on errors. (*Andreas Stergiou's lecture in the workshop)*

Bootstrap bounds on $N_f = 2$ QED$_3$ and $O(4)$ DQCP

Singlet bound coincidence: $O(4)$ vector $= SU(2)_f$ $U(1)_t$ monopole bootstrap
Irrelevance of lowest singlet $\Delta_S > 3$ leads to: $\Delta_{M_1} > 0.876$ (*Poland, ZL*)
Why irrelevant Δ_S:

- Extra fine tuning is needed with a relevant singlet scalar
- A relevant scalar could generate RG flow dissolving IR fixed point.

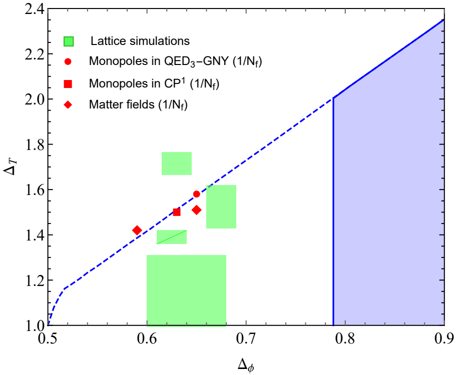


Pink (light green):
$\Delta_V = 0.550(5)(0.565(15))$,
Qin et. al, 2017;

Green:
$\Delta_{M_1} = 0.826(44)$,
Karthik and Narayana, 2019

Bootstrap bounds on $SO(5)$ DQCP

Blue dashed line: upper bound on the lowest scalar in the $SO(5)$ T sector;
Blue shadowed: allowed region with an assumption $\Delta_S > 3.0$.



Refs	Sandvik, 2007	Melko, 2008	Pujari, 2013	Nahum,2015	Dyer,2015	Dupuis,2021	Boyack,2018
Δ_ϕ	0.630^{15}	0.675^{15}	0.64^{4}	0.625^{15}	0.63	0.65	0.59/0.65
Δ_T	1.716^{50}	1.52^{9}	1.11^{20}	1.39^{3}	1.50	1.58	1.42/1.51

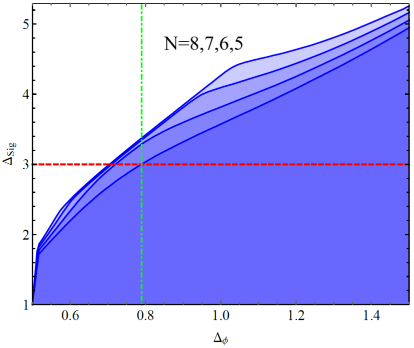
Bootstrap results on $O(4)/SO(5)$ DQCPs:

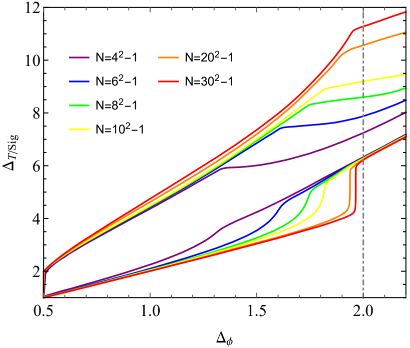
- The $SO(5)/O(4)$ DQCP CFT data obtained from lattice simulations is inconsistent with the bootstrap bounds associated with an irrelevant assumption, indicating the phase transitions observed in lattice simulations are not truly continuous. See also (*Nakayama, Poland, Simmons-Duffin, Ilisiu and Pufu, et al.*)
- A possible explanation is that the phase transitions are weakly first order, which are hard to be distinguished from second order phase transitions on finite size lattice system.
- More precise lattice simulations are useful to verify above conclusions: CFT data of presumed $N_f = 2$ QED_3 with higher precision? Fine tune a possible relevant singlet scalar in $SO(5)$ DQCP?

Is this a bitter end for the fantastic adventure of $N_f = 2$ QED$_3$?

2. A surprise in $O(N)$ vector bootstrap: new family of kinks

Kinks (type I) deformed from free boson : 3D $O(N)$ vector models! *(Kos, Poland, Simmons-Duffin 2013)*
Kinks (type II) deformed from free fermion bilinear: fermionic gauge theories? *(ZL 2018)*





Are the new kinks related to the DQCPs with larger symmetries?

Interesting properties of the type II kinks:

1. In the large N limit, they approach free fermion theory, it is natural to expect their finite N analogies relate to **fermionic gauge theories**.
2. **Strongly coupled theories**: The family of kinks distributed in a wide range with large anomalous dimensions! In contrast the kinks of $O(N)$ vector models are near free boson Δ 0.5.
3. **Conformal window: $N > 6$:** The $SO(5)/O(4)$ DQCP is just below the window of this family of kinks, consistent with the expectation that they are of weakly first order phase transition.
4. **Merger and annihilation of FPs:** Near $N = 6$, the kinks approach the marginality condition $\Delta_S = 3$ and disappear after crossing the marginality condition. Mechanism for the loss of conformality?
5. **Bound coincidence:** $O(N_f^2 \quad 1)$ vector bootstrap bounds coincide with the $SU(N_f)$ adjoint $O_{\rm ad}$ bootstrap bounds. **For different N_fs the kinks have $\Delta_{O_{\rm ad}}$ close to the large N_f results of QED_3.**
6. More interesting properties. (*ZL and Poland; He, Rong and Su, 2020*)

3. A novel algebraic structure in 4pt crossing equations

How can different crossing equations generate the same bounds?
Hidden $O(N)$ symmetric structure in the non-$O(N)$ crossing equations!

Let us consider $SU(N_f)$ fundamental bootstrap. There are 6 $SU(N_f)$ irreps in the crossing equations, which can be written into a 6   6 matrix

$$M_{SU(N_f)} \quad \begin{pmatrix} 0 & 0 & F^+_{\text{Adj}} & F_{\text{Adj}} & F^+_T & F_A \\ 0 & 0 & H^+_{\text{Adj}} & H_{\text{Adj}} & H^+_T & H_T \\ F^+_S & F_S & \left(1 \quad \frac{1}{N_f}\right) F^+_{\text{Adj}} & \left(1 \quad \frac{1}{N_f}\right) F_{\text{Adj}} & 0 & 0 \\ H^+_S & H_S & \left(1 + \frac{1}{N_f}\right) H^+_{\text{Adj}} & \left(1 + \frac{1}{N_f}\right) H_{\text{Adj}} & 0 & 0 \\ F^+_S & F_S & \frac{1}{N_f} F^+_{\text{Adj}} & \frac{1}{N_f} F_{\text{Adj}} & F^+_T & F_A \\ H^+_S & H_S & \frac{1}{N_f} H^+_{\text{Adj}} & \frac{1}{N_f} H_{\text{Adj}} & H^+_T & H_T \end{pmatrix} :$$

In contrast, the $SO(N)$ vector crossing equations are

$$M_{SO(N)} \quad \begin{pmatrix} 0 & F^+_T & F_A \\ F^+_S & \left(1 \quad \frac{2}{N}\right) F^+_T & F_A \\ H^+_S & \left(1 + \frac{2}{N}\right) H^+_T & H_A \end{pmatrix} :$$

A novel algebraic structure in the 4pt crossing equations

$M_{SU(N_f)}$ and $M_{SO(2N_f)}$ are related through a linear map! *(ZL, Poland 2020)*

$$T_{SU(N_f)} \quad \begin{pmatrix} 1 & 0 & \frac{1}{1 \quad 2N_f} & 0 & \frac{1}{2N_f \quad 1} & 0 \\ 0 & 0 & \frac{1}{2N_f \quad 1}+1 & 0 & \frac{1}{1 \quad 2N_f}+1 & 0 \\ 0 & \frac{2}{2N_f \quad 1} & 0 & \frac{1}{1 \quad 2N_f}+1 & 0 & \frac{1}{2N_f \quad 1}+1 \end{pmatrix} :$$

$$T_{SU(N_f)} \quad M_{SU(N_f)} \quad =$$

$$\begin{pmatrix} 0 & F_S & F^+_{\mathrm{Adj}} & F_{\mathrm{Adj}} & F^+_T & F_A \\ F^+_S & F_S & \left(1 \quad \frac{1}{N_f}\right) F^+_{\mathrm{Adj}} & F_{\mathrm{Adj}} & \left(1 \quad \frac{1}{N_f}\right) F^+_T & F_A \\ H^+_S & H_S & \left(1+\frac{1}{N_f}\right) H^+_{\mathrm{Adj}} & H_{\mathrm{Adj}} & \left(1+\frac{1}{N_f}\right) H^+_T & H_A \end{pmatrix}$$

$$\mathrm{diag} f \mathbf{1}; \mathbf{y_1}; \mathbf{x_1}; \mathbf{y_2}; \mathbf{x_2}; \mathbf{y_3} g$$

$$(y_1;\ x_1;\ y_2;\ x_2;\ y_3) = \left(\frac{1}{2N_f \quad 1}; \frac{N_f \quad 1}{2N_f \quad 1}; \frac{N_f^2 \quad 1}{N_f(2N_f \quad 1)}; \frac{N_f}{2N_f \quad 1}; \frac{N_f \quad 1}{2N_f \quad 1} \right) :$$

A novel algebraic structure in the 4pt crossing equations

$SO(2N_f)$! $SU(N_f)$ branching rules in $T_{SU(N_f)}$ $M_{SU(N_f)}$! $M_{SO(2N_f)}$

$SO(2N)$		$SU(N)$
S	!	S,
T	!	Adj T ($\mathbf{x_i}$),
A	!	S Adj A ($\mathbf{y_i}$).

Positivity: The coefficients y_i, x_i are all positive!

Conclusion: The $SU(N_f)$ crossing equations $M_{SU(N_f)}$ have an $SO(2N_f)$ symmetric positive structure!

Similar linear transformations between G-symmetric crossing equations and the $SO(N)$ vector's can be found for general G, case by case.*(ZL 2020, Manenti, Reehorst et al. 2020)*

The new algebraic structure and $N_f = 2$ QED$_3$ self-duality

$O(4)$ positive structure in the $SU(2) \quad U(1)$ crossing equations of $\mathcal{M}_1$:

$O(4)$	$SU(2)_f$	$SU(2)_t$		$SU(2)_f$	$SO(2)_t$	
	S	$(\mathbf{0}, \mathbf{0})$	!	$(\mathbf{0}, A)$,		
	T	$(\mathbf{1}, \mathbf{1})$	!	$(\mathbf{1}, S)$	$(\mathbf{1}, T)$,	
A	$(\mathbf{1}, \mathbf{0})$	$(\mathbf{0}, \mathbf{1})$	!	$(\mathbf{1}, A)$	$(\mathbf{0}, S)$	$(\mathbf{0}, T)$.

Fun fact: matches $N_f = 2$ QED$_3$ self-duality: $SU(2)_f \quad U(1)_t$! $O(4)$.

(*Wang, Nahum, Xu, Son, Hsin, Seiberg, et al.*)

- $O(4)$ V scalar: $\mathcal{M}_1$ in $(\frac{\mathbf{1}}{\mathbf{2}}, V)$ of $SU(2)_f \quad SO(2)_t$! V of $O(4)$.
- $O(4)$ T scalar: $(\mathbf{1}, S)$ fermion mass + $(\mathbf{1}, T)$ monopole.
- $O(4)$ current $J_{A=(\mathbf{1};\mathbf{0})+(\mathbf{0};\mathbf{1})}$: $J_{(\mathbf{1};\mathbf{0})} = j_{SU(2)_f}$, $J_{(\mathbf{0};\mathbf{1})} = (j_t\ ,\ \mathcal{M}_2)$,
  where the monopole $\mathcal{M}_2$ has quantum numbers:
  $SU(2)_f \quad SO(2)_t : (\mathbf{0}, T)$; $\Delta_{\mathcal{M}_2} = 2 \quad \ell = 1$.
  Can we find such a monopole in the perturbative results?

How is the non-$O(N)$ symmetric conformal bootstrap affected by the $O(N)$ positive structure in the crossing equations?

The crossing equations do NOT "know" the non-$O(N)$ symmetry!
How to bootstrap these non-$O(N)$ symmetric CFTs with conformal bootstrap?

- A straightforward method: Introduce gap assumptions in the different representations which break the $O(N)$ symmetry explicitly.
  —Will be discussed with more details for $N_f = 4$ QED_3 bootstrap!
- Mixing with non-$O(N)$ symmetric conserved currents, which could introduce more constraints due to the Wald identities.
- Other possibilities?

Bootstrap Ising/$O(N)$ vector models: get something out of nothing!
Bootstrap non-$O(N)$ CFTs: get sth out of sth—inputs are necessary!

4. Towards the bootstrap island for the $N_f = 4$ QED$_3$

- How to break the enhanced $SO(N)$ symmetry in crossing equations and implement the proper symmetry of QED$_3$?
- How to distinguish QED$_3$ from QCD$_3$?

Consider the $SU(N_f)$ adjoint 4pt correlator: $hO_{\rm ad}O_{\rm ad}O_{\rm ad}O_{\rm ad}i$. Its crossing equations can be mapped to the $SO(N_f^2 \quad 1)$ vector crossing equations, associated with the branching rules

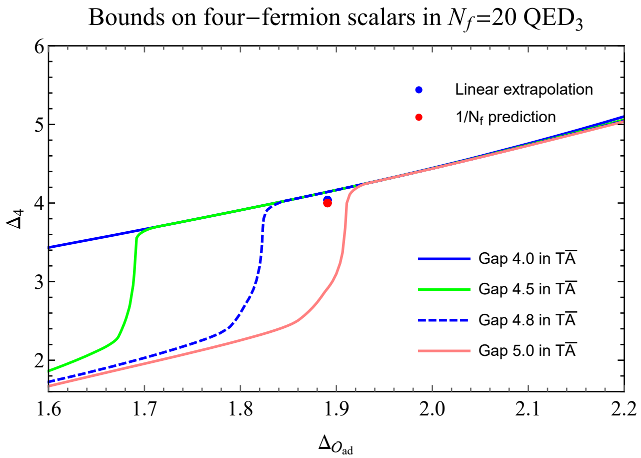
$$
\begin{array}{rcl}
SO(N_f^2-1) & & SU(N_f) \\
\mathrm{Sig} & \longleftrightarrow & \mathbf{1}\,, \\
T & \longleftrightarrow & \mathrm{Ad}^+ \oplus A\bar{A} \oplus T\bar{T}\,, \\
A & \longleftrightarrow & \mathrm{Ad} \quad \oplus T\bar{A}\,.
\end{array}
$$

To bootstrap QED$_3$, the $SO(N_f^2 \quad 1)$ symmetry needs to be broken. QED$_3$ physical spectrum *(Gracey, Xu, Chester & Pufu, et al.)*

$$
\begin{aligned}
(\Delta_{\mathrm{Ad}^+}, \Delta_{A\bar{A}}, \Delta_{T\bar{T}}) &\simeq (4-\frac{185}{3\pi^2 N_f}, 4-\frac{64}{\pi^2 N_f}, 4+\frac{64}{3\pi^2 N_f}) \\
(\Delta_{\mathrm{Ad}} \quad, \Delta_{T\bar{A}}) &\simeq (2,\ 5+O(\frac{1}{N_f}))
\end{aligned}
$$

Kinks in the $SU(N_f)$ adjoint fermion bilinear bootstrap

Bootstrap condition: $(\Delta_{\mathrm{Ad}^+}, \Delta_{A\bar{A}}, \Delta_{T\bar{T}}) > (\Delta_4 \quad \frac{185}{3\pi^2 N_f}, \Delta_4 \quad \frac{64}{\pi^2 N_f}, \Delta_4 + \frac{64}{3\pi^2 N_f})$



Prediction: $\Delta_{T\bar{A}} = 5 \quad \frac{jcj}{N_f}$! More precise matches for larger $N_f = 50, 100, 200$.

$SU(4)$-ad bootstrap: kinks but not enough for islands

$SU(N_f)$-ad fermion bilinear bootstrap:
can generate sharp kinks but not enough to produce closed island!

- By introducing the information on physical QED_3 spectrum which breaks the $O(N_f^2 \quad 1)$ symmetry explicitly, bootstrap bounds are nearly saturated by QED_3 with sharp kinks.
- The four-fermion scalar operators break the $SO(N_f^2 \quad 1)$ symmetry weakly—only at the subleading order corrections.

Question: Are there 4pt correlators in QED_3 in which the physical spectrum strongly breaks the hidden $O(N)$ symmetry?

This could help us to *get strong bootstrap results using physically reliable assumptions.*

Monopole crossing equations and parity symmetry

Parity: $U(1)$ flux q changes its sign under parity P: $P M_q P \ ! \ M \ \ q$.
Monopoles M_q *have no definite parity charges.*

Monopole OPE: $SU(4)_f \quad SO(2)_t$ irreps in $M_1 \quad M_1$ (*Chester and Pufu*)

$$SU(4): \quad (110) \quad (110) = (000) \quad (211) \quad (220)\,,$$
$$SO(2): \quad V \quad V = S \quad A \quad T\,,$$

Crossing equations of $h M_1 M_1 M_1 M_1 i$ can be mapped to the $SO(12)$ vector crossing equations $M_{SO(12)}$ associated with the branching rules

$SO(12)$		$SU(4) \quad SO(2)$			
S	!	$S_{(000)}\,,$			
T	!	$S_{(220)}$	$A_{(211)}$	$T_{(000)}$	$T_{(220)}\,,$
	$\Delta:$	$4+\ldots$	$1.4+\ldots$	$4.4+\ldots$	$2.5+\ldots$
A	!	$S_{(211)}$	$A_{(000)}$	$A_{(220)}$	$T_{(211)}\,.$

Some perturbative spectrum of $N_f = 4$ QED$_3$

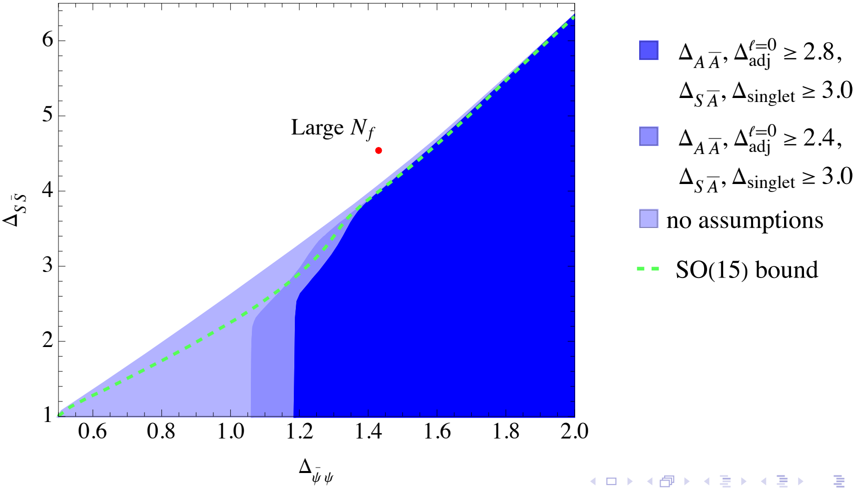
$SO(2)$	$SU(4)$ rep	Δ_1-leading	Δ_1-sub	Δ_2-leading	Refs
A	(211) (Adj)	2	0.540	4	*Gracey*
S	(000) (Sig)	4	0.349	5	*Chester & Pufu*
S	(211) (Adj)	4	1.563	5	*Chester& Pufu*
S	(220) ($A\bar{A}$)	4	1.621	6	*Xu, Chester& Pufu*
S	(422) ($S\bar{S}$)	4	+0.540	6	*Xu*
V	(110) (Anti)	1.060	+0.038	3.888	*Pufu, Dupuis et al.*
T	(000) (Sig)	4.424		6.156	*Pufu, Dupuis et al.*
T	(220) ($A\bar{A}$)	2.693	0.194	6.156	*Pufu, Dupuis et al.*

1. **Four-fermion scalars:** large subleading corrections at $N_f = 4$! (Away from physical value?)
2. **Monopole sectors:** weak subleading order corrections. (Close to the physical value?)

Nonperturbative checks?

Nonperturbative check of perturbative spectrum of QED$_3$

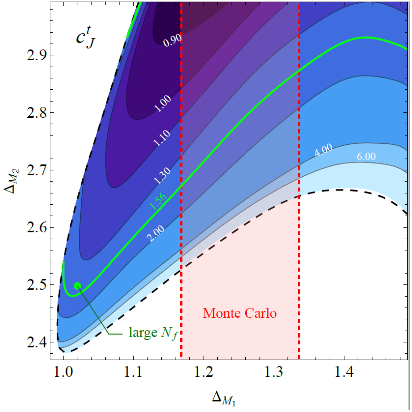
Another conclusion on QED$_3$ from bootstrap: *perturbative result of 4-fermion scalar ($S\bar{S}$) at the subleading order can be excluded for $N_f = 4$.* (Kinks appear with gaps related to QED$_3$ spectrum!)



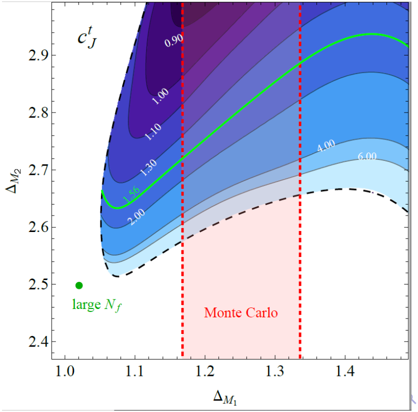
Peninsula structure in the monopole bootstrap

- Gap in $(220)A\bar{A}$ sector is important! Large N_f: 2.4 —but not reliable!
- $\Delta_{M_2^0}$ 6.16. Subleading order corrections are mild in monopole sectors.
- Lattice data $\Delta_{M_1} = 1.252(84)$ is from *Karthik and R. Narayanan, 2019*.

$\Delta^{\ell=0}_{S(000)}$ 3:0 $\Delta^{\ell=0}_{S(220)}$ 2:8; $\Delta_{M_2^0}$ 5:0

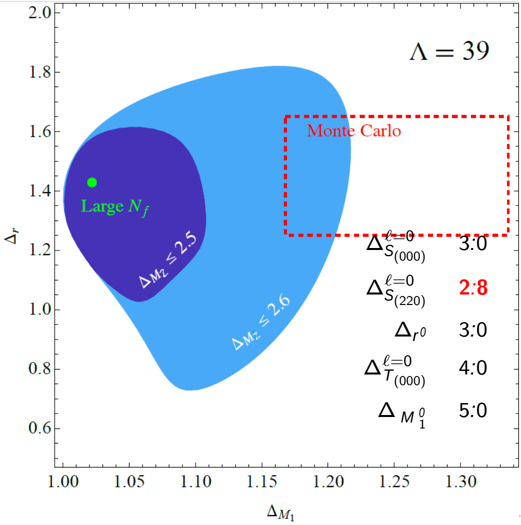


$\Delta^{\ell=0}_{S(000)}$ 3:0; $\Delta^{\ell=0}_{S(220)}$ 3:0; $\Delta_{M_2^0}$ 5:0



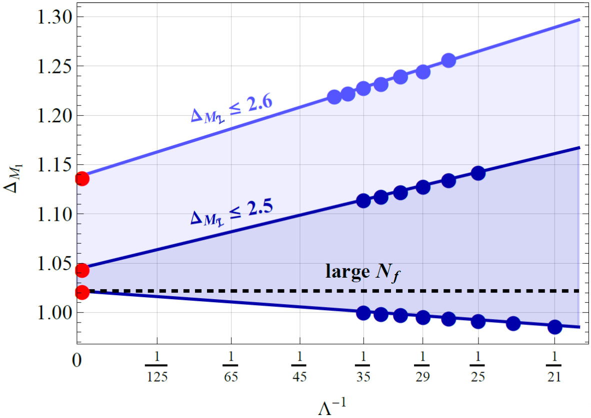
Peninsula to islands of the $N_f = 4$ QED$_3$

Bootstrap islands for $(\Delta_{M_1}, \Delta_{O_{\mathrm{ad}}=r})$ from peninsula structure with "interval positivity" conditions Δ_{M_2} 6 $2.5, 2.6$. (*Albayrak, Erramilli, ZL, Poland, Xin 2021*)



Will the island disappear with high precision? Like in 5D

Island size can be reduced significantly with higher numerical precision.



Perturbative predictions: $(\Delta_{M_1}, \Delta_{M_2})$ ' $(1.022, 2.499)$.
After imposing the condition Δ_{M_2} 6 2.5, the bootstrap island shrinks to Δ_{M_1} 2 [1.02, 1.04] remarkably close the perturbative results!

QED$_3$ or QCD$_3$: central charges

How do we know the island realted to QED$_3$ instead of QCD$_3$-like theories?

- Similar low-lying spectrum in QED$_3$ and QCD$_3$: $\psi_i \bar{\psi}^j$ *vs* $\psi_i^c \bar{\psi}_c^j$.
- Monopoles appear both in QED$_3$ and QCD$_3$: $U(1) \quad SU(N_c)$.

Central charges $c_{J;J^t;T}$**:** useful to distinguish QED$_3$ and QCD$_3$:

$$\text{QED}_3: \ \psi_i, A \ ; \quad \text{QCD}_3: \ \psi_i^a, A_{ab}, \quad \text{a,b are color indices.}$$

Central charges in QED$_3$ (*Giombi et al. 2016*):

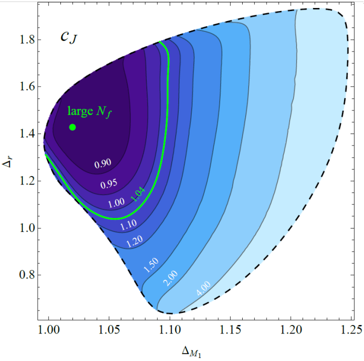
$$c_J = c_{J0}\left(1 + \frac{0.1429}{N_f} + O(1/N_f^2)\right),$$

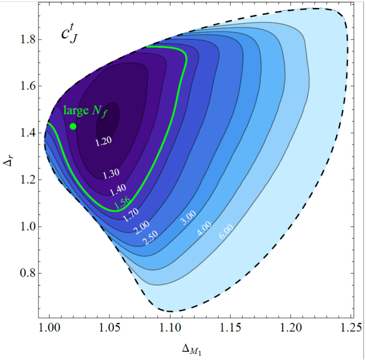
$$c_T = c_{T0}\left(1 + \frac{0.7193}{N_f} + O(1/N_f^2)\right).$$

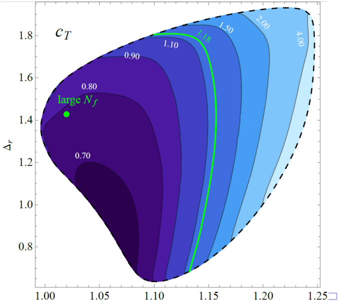
Central charges in QCD$_3$ (*Giombi et al. 2016*):

$$c_J = N_c c_{J0}\left(1 + \frac{0.1429}{N_f}\frac{N_c^2 \quad 1}{N_c} + O(1/N_f^2)\right),$$

$$c_T = N_c c_{T0}\left(1 + \frac{0.7193}{N_f}\frac{N_c^2 \quad 1}{N_c} + O(1/N_f^2)\right).$$


c_J
large N_f
0.90
0.95
1.00
1.04
1.10
1.20
1.50
2.00
4.00
1.8
1.6
1.4
1.2
1.0
0.8
Δ_r
1.00
1.05
1.10
1.15
1.20
1.25
Δ_{M_1}


c_J^t
large N_f
1.20
1.30
1.40
1.56
1.70
2.00
2.50
3.00
4.00
6.00
1.8
1.6
1.4
1.2
1.0
0.8
Δ_r
1.00
1.05
1.10
1.15
1.20
1.25
Δ_{M_1}


c_T
large N_f
0.70
0.80
0.90
1.10
1.18
1.50
2.00
4.00
1.8
1.6
1.4
1.2
1.0
0.8
Δ_r
1.00
1.05
1.10
1.15
1.20
1.25

Bootstrap results of the $N_f = 4$ QED$_3$: a summary

- **Fermion bilinear bootstrap**: the enhanced $SO(N)$ symmetry in the crossing equations is mildly broken by QED$_3$ spectrum. With this information the fermion bilinear bootstrap can generate sharp kinks nearly saturated by conformal QED$_3$.
- **Monopole bootstrap**: the enhanced $SO(N)$ symmetry in the crossing equations is strongly broken by QED$_3$ spectrum. With gaps inspired by the perturbative CFT data, the parameters can be isolated into a closed region which shows strong connection with QED$_3$!
  The results suggest: *the $N_f = 4$ QED$_3$ monopole CFT data provides a unitary solution to the crossing equations, and crucially, this solution can be captured by bootstrap!*

Our $N_f = 4$ QED$_3$ island is different from the celebrated islands of 3D Ising and $O(N)$ vector models, as it does not provide a numerical solution to the theory. Nevertheless, our bootstrap results have been shown closely contacted with the strongly coupled QED$_3$ dynamics, therefore provide a substantial pivot for future studies.

Outlook: towards a comprehensive understanding of conformal QED_3

Questions towards a comprehensive understanding of conformal QED_3:

1. What is the critical flavor number N_f of QED_3?
2. Near N_f by which mechanism the conformality is lost?
3. How to precisely estimate critical indices of strongly coupled QED_3?

Problem 1: Bootstrap has provided a new estimate for N_f *2* $(2, 4)$:

- *Bootstrap + lattice:* $N_f = 2$ QED_3 critical indices from lattices simulations are inconsistent with bootstrap bounds associated with suitable assumptions. The phase transitions are not truly continuous.
- *Indirect evidence:* The proposed $SO(5)/O(4)$ DQCPs are just below a new family of bootstrap kinks, indicating the $SO(5)/O(4)$ DQCPs are slightly below the conformal window.
- *Bootstrap + large N_f:* $N_f = 4$ QED_3 CFT data can be isolated into a closed island using bootstrap, which supports the theory provides a unitary solution to crossing equations.

N_f (2-component Dirac fermion)	Method	Year and Reference
$\frac{64}{\pi^2}$ 6.5	Schwinger-Dyson equations	1984-88 Pisarski,Appelquist et al
8.6	Schwinger-Dyson equations	1996-97 Maris,Aitchison, et al
3	thermal free energy	1999-2004 Appelquist et al
4	hybrid Monte Carlo	2002-04 Hands et al
4.3	divergence of the chiral susceptibility	2002 Franz et al
8	covariant solutions for propagators	2004 Fischer et al
12	perturbative RG in the large-N_f limit	2004 Kaveh et al
10. . . 13	comparison to the Thirring model	2007-12 Christofi,Janssen, et al
3	lattice simulations	2008 Strouthos et al
8 $N_f^{\chi SB}$ N_f 20	functional RG	2014 Braun et al
8	F-theorem	2015 Giombi et al
4	one-loop ϵ-expansion	2015 DiPietro et al
5.7	$1/N_f$ expansion	2016 Gusynin et al
5.8	ϵ-expansion	2016 Herbut et al
< 2	lattice simulations	2017 Qin et al
< 2	lattice simulations	2015-2020 Karthik et al
$2 < N_f < 4$	**conformal bootstrap +**	**2018-2021 Li & Yale Group**

Problem 2: Near N_f by which mechanism the conformality is lost? The Type II kinks disappear slightly above $N = 5$ when approaching the marginality condition $\Delta_S = 3$. A signal of the "merger and annihilation mechanism" (*Kubota and Terao, Kaveh and Herbut, Gies and Jaeckel, Kaplan, Lee, Son, Gorbenko, Rychkov, Zan et al.*). More precise CFT data is needed for a solid answer!

Problem 3: How to precisely estimate critical indices of strongly coupled QED_3? This question relates to reducing the size of our $N_f = 4$ QED_3 bootstrap island. Next step: mixing with conserved currents.

- The conserved currents are generators of global symmetries. Their Wald identities could help to fix the non-$O(N)$ global symmetries.
- Our results have shown that the bootstrap bounds on the current central charges are rather restrictive. By mixing with conserved currents, these constraints could be exploited further.

With a much smaller island, we could verify our gap assumptions further and provide more characteristic properties of QED_3.

Outlook: towards a comprehensive understanding of conformal QED_3

Properties of different approaches to study conformal QED_3:

	Conformal bootstrap	Lattice simulation	Large N_f
Non-perturbative	p	p	
Theory-specified		p	p
Strict errors	p		
Many spectrum	p		p
Computation eff	p		

Conformal bootstrap + lattice + large N_f =

a systematical study of QED_3 with strict control on the errors!

Thank You!