Charged spinning operators in CFTs:
from superfluids to Regge theory

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based on 1711.02108 with ***A. De La Fuente****,* ***A. Monin****,* ***D. Pirtskhalava*** *and* ***R. Rattazzi***
& on a work in progress with ***Z. Komargodski***

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Introduction

Universality of CFT data at large quantum numbers

- Large spin J

Alday Maldacena 2007, Komargodski Zhiboedov 2012,
Fitzpatrick Kaplan Poland Simmons-Duffin 2012, Caron-Huot 2017,…

- Large internal charge Q

Hellerman Orlando Reffert Watanabe 2015, Monin Pirtskhalava Rattazzi Seibold 2016,…

- Large scaling dimension Δ

Lashkari Dymarsky Liu 2016, Cardy Maloney Maxfield 2017, Delacrétaz 2020,…

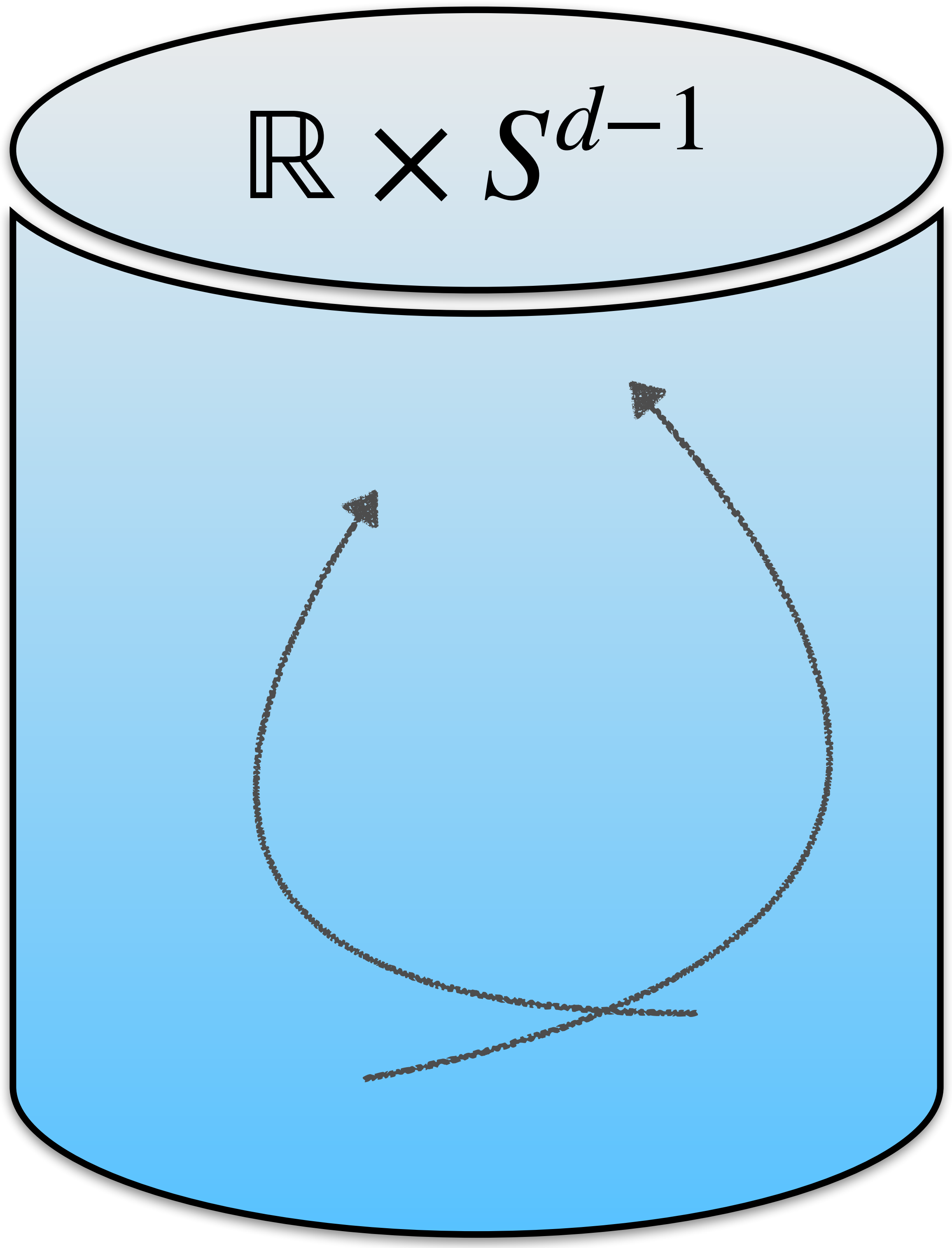
Why semiclassics at large quantum numbers?

$$\mathcal{O}_{\mathbb{R}^d}(x) \xleftrightarrow{\text{State-operator map}} |\mathcal{O}\rangle_{\mathbb{R}\times S^{d-1}}$$

Ex.: Alday-Maldacena "EFT" for large J double-trace op.s:

Alday Maldacena 2007

$$\mathcal{O}_{2,J} \sim \phi\partial^J\phi$$



- $|\phi\partial^J\phi\rangle \simeq 2$ free moving particles + small "Yukawa" exchanges

$$\Delta_{\phi\partial^J\phi} = E_{free} + \# e^{-\tau_{\min}\Delta\chi}$$

$$= J + 2\Delta_\phi + \frac{\#}{J^{\tau_{min}}}$$

- Result rigorously established via analytic bootstrap

Komargodski Zhiboedov 2012, Fitzpatrick Kaplan Poland Simmons-Duffin 2012…

Operators in the 3d O(2) CFT

We will focus on the critical point of $U(1)$ Landau-Ginzburg theory

$$\int d^3x \left(|\partial\phi|^2 + \frac{\lambda}{4}|\phi|^4 \right) \xrightarrow{IR} O(2)\ \text{CFT}$$

- Lowest dimension operator at given charge Q and spin J: $\mathcal{O}_{Q,J}(x)$

Question: what's the scaling dimension $\Delta_{Q,J}$ as a function of Q and J?

- For $Q \sim J \sim O(1)$ many data from the numerical bootstrap

Chester Landry Liu Poland Simmons-Duffin Su Vichi 2019

- We expect semiclassics allows to compute $\Delta_{Q,J}$ when $Q \gg 1$ and/or $J \gg 1$

Large charge operators in the 3d O(2) CFT

- For $Q =$ fixed and $J \to \infty$ Alday-Maldacena implies

$$\mathcal{O}_{Q,J} \sim \phi\partial^{J/Q}\phi \dots \partial^{J/Q}\phi \quad \Longrightarrow \quad \Delta_{Q,J} = J + Q\Delta_\phi + \dots$$

- For $Q \gg 1$ and $J = O(1)$: superfluid phase (see next slides)

Hellerman Orlando Reffert Watanabe 2015

$$\mathcal{O}_{Q,0} \sim \phi^Q \quad \Longrightarrow \quad \Delta_{Q,J} = \alpha Q^{3/2} + \dots$$

This talk: use EFT+semiclassics to compute $\Delta_{Q,J}$ for $Q \gg 1$ and $J \gg 1$ identifying $|\mathcal{O}_{Q,J}\rangle$ as a proper superfluid+vortices configuration

Overview

1. The conformal superfluid EFT for large charge operators

2. Charged spinning operators and rotating superfluids

3. Comments on large multitraces from the large spin expansion

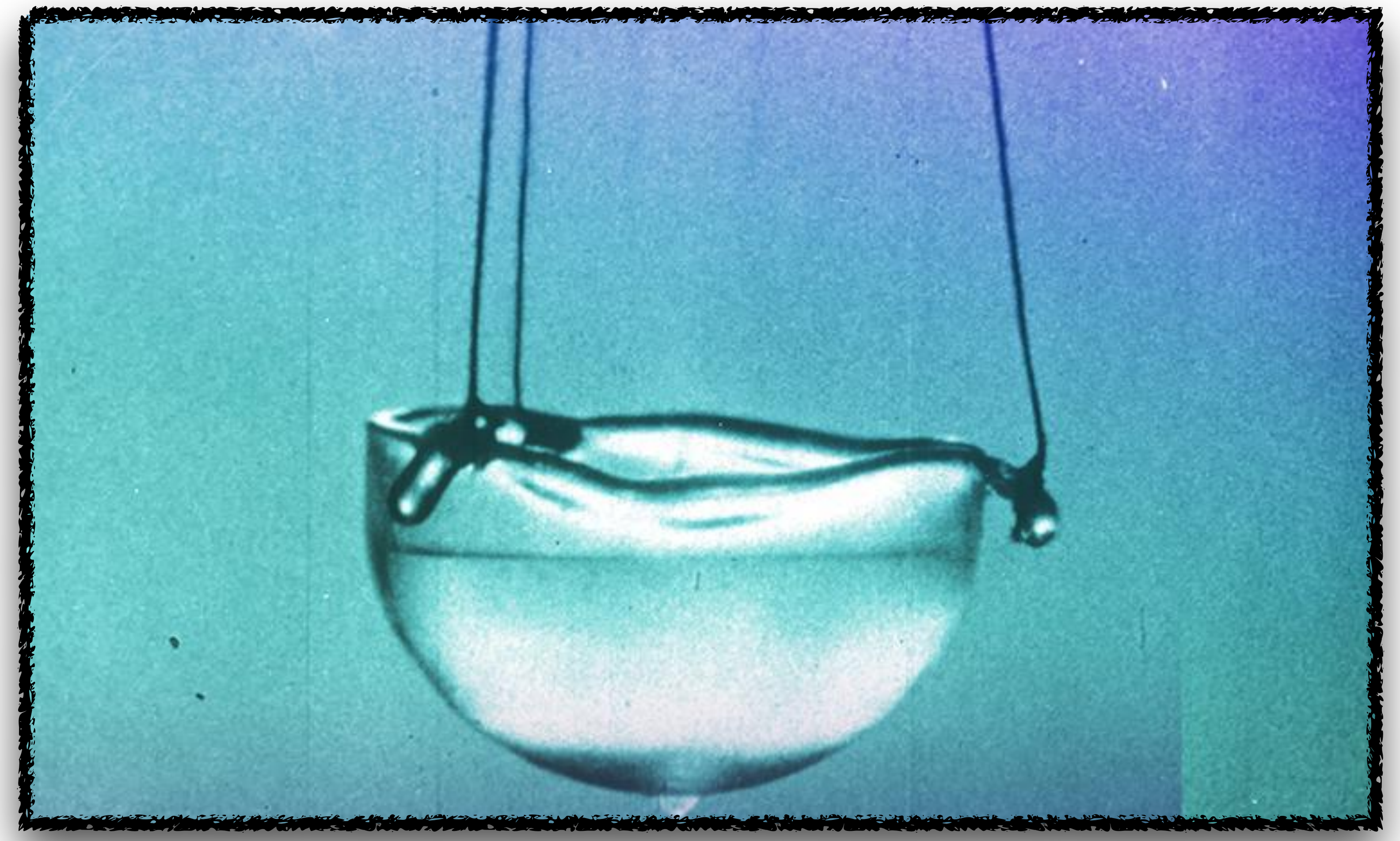
4. Summary and outlook

The conformal superfluid EFT for large charge operators

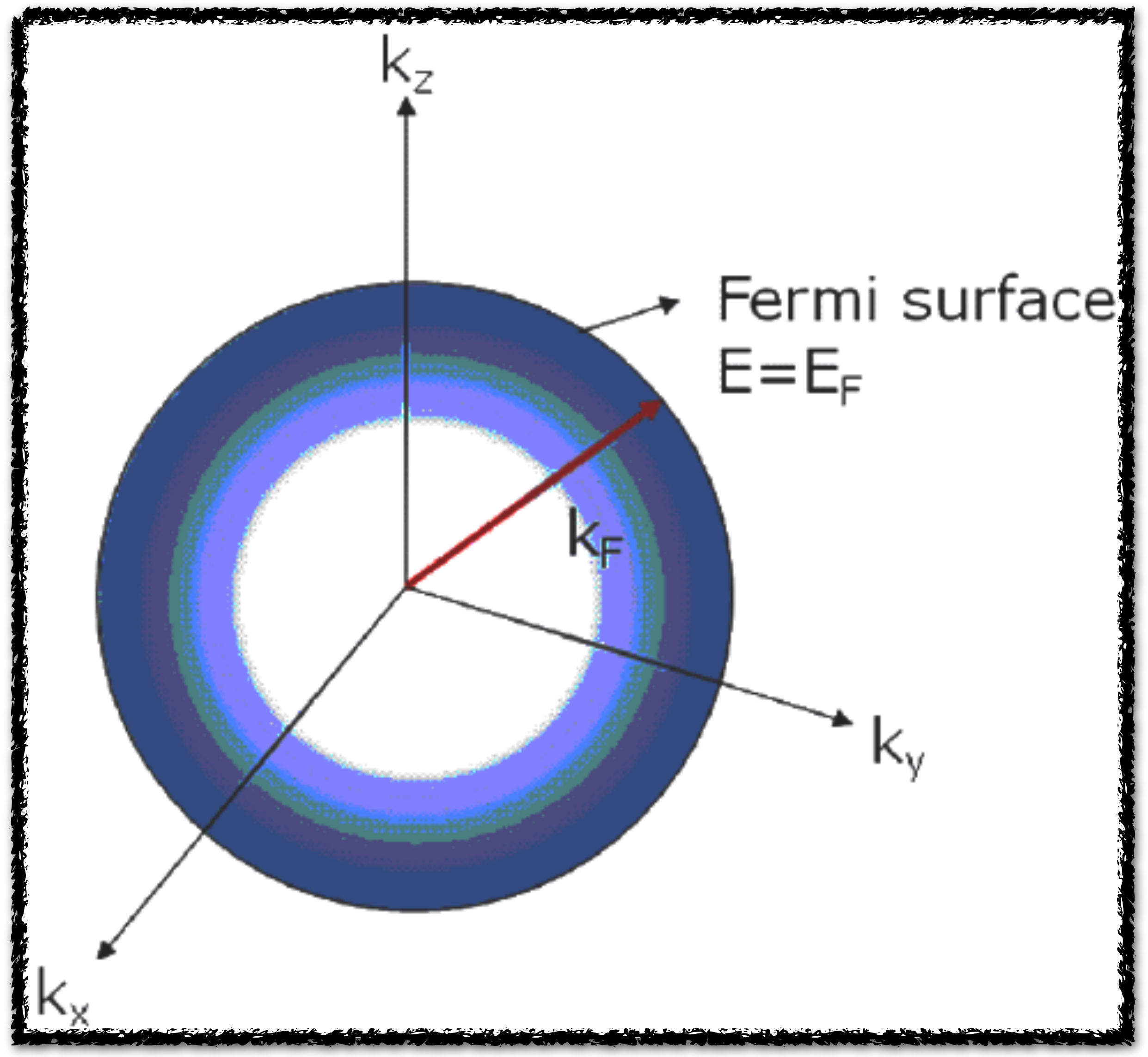
Large charge state

Consider $J = 0$: $\mathcal{O}_{Q,J=0} \sim \phi^Q \leftrightarrow |Q\rangle$

- charge density $j_0 \sim Q/R^2 \propto \mu^2$: *condensed matter* state. Which?



Superfluid: broken $U(1)$



FS: unbroken $U(1)$

The conformal superfluid

Breaking pattern: $H \times U(1) \longrightarrow \bar{H} = H + \mu Q$

- 1 Goldstone mode $\chi(x) = \mu t + \pi(x)$
- "radial" modes: expectedly gapped at $\mu \sim \dfrac{\sqrt{Q}}{R}$

(But moduli in SCFTs: Hellerman Maeda Orlando Reffert Watanabe 2017-2021)

Write $\mathscr{L}\left(\chi\right)$ systematically in a derivative expansion $\partial/\mu \propto R\omega/\sqrt{Q}$

Hellerman Orlando Reffert Watanabe 2015, Monin Pirtskhalava Rattazzi Seibold 2016

Action for $\chi(x) = \mu t + \pi(x)$:

$$\left.\mathcal{L} = c(\partial\chi)^3 \right\} = Q^{\frac{3}{2}}$$

$$\left.\begin{aligned} &+c_1(\partial\chi)\left\{\mathcal{R} + 2\frac{[\partial_\mu(\partial\chi)]^2}{(\partial\chi)^2}\right\} \\ &+c_2(\partial\chi)\mathcal{R}_{\mu\nu}\frac{\partial^\mu\chi\partial^\nu\chi}{(\partial\chi)^2} + \dots \end{aligned}\right\} = Q^{\frac{1}{2}}$$

$$\left. + \dots \right\} = Q^{-\frac{1}{2}}$$

Simple prediction for the ground state energy on $\mathbb{R} \times S^2$:

$$\boxed{\Delta_0(Q) = \underbrace{\alpha Q^{3/2} + \beta Q^{1/2}}_{\text{classical}} \underbrace{-0.0937\dots}_{\text{1-loop}} + O\left(Q^{-1/2}\right)}$$

Hellerman Orlando Reffert Watanabe 2015, Monin 2016

Sound speed of the phonon fluctuations fixed by conformal invariance:

$$S[\pi] = 3c\mu \int d^3x \sqrt{g} \left[\dot{\pi}^2 + \frac{1}{2}(\partial_i \pi)^2 \right]$$

$$\omega_J^2 = \frac{J(J+1)}{2R^2}$$

Non-trivial informations about the spectrum

$$\Delta(Q, \{n_J\}) = \Delta_0(Q) + \sum n_J R \omega_J + \dots$$

$$J = 1 \qquad \text{descendants} \qquad \begin{cases} R\omega_1 = 1, \\ a_{1,m} \propto K_m, \quad a^\dagger_{1,m} \propto P_m; \end{cases}$$

$$J > 1 \qquad \text{new primaries.}$$

Hellerman Orlando Reffert Watanabe 2015

Charged spinning operators and rotating superfluids

Adding spin to the superfluid

Consider $\mathcal{O}_{Q,J}$ for $J > 0$. Look at phonon states with $J \gg 1$:

1 phonon with spin=J

$$\delta E_Q\, R = \frac{J}{\sqrt{2}}\sqrt{1+\frac{1}{J}}$$

$$J \ll \sqrt{Q}\,.$$

$$<$$

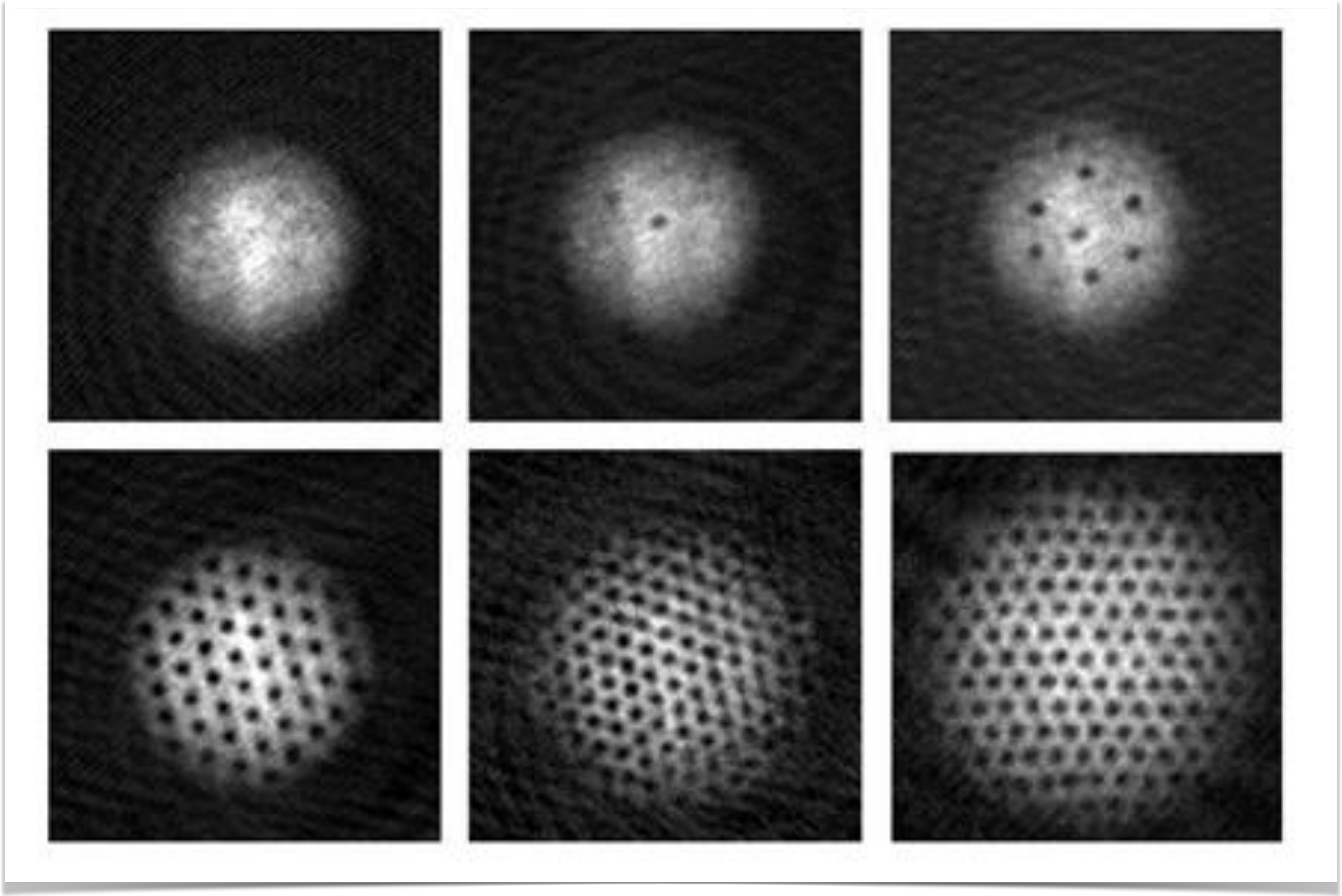
n phonons with spin=J/n

$$\delta E_Q\, R = \frac{J}{\sqrt{2}}\sqrt{1+\frac{n}{J}}$$

$$\begin{cases} J/n \ll \sqrt{Q}, \qquad n \ll Q, \\ \implies \qquad J \ll Q^{3/2}. \end{cases}$$

$$\overset{?}{\implies} \qquad \Delta(Q,J) = \alpha_1 Q^{3/2} + \frac{J}{\sqrt{2}}$$

Experiments show that spinning superfluids develop vortices..



A Bose-Einstein condensate in a magnetic trap develop an increasing number of vortices as the angular velocity is increased.

Vortex EFT and particle-vortex duality

It is convenient to introduce a dual gauge field:

$$\mathcal{L} = c(\partial\chi)^3 \qquad \Longleftrightarrow \qquad \mathcal{L} = -\kappa F^{3/2} \qquad \begin{cases} F = \sqrt{F_{\mu\nu}F^{\mu\nu}} \\ F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \\ \kappa = \frac{1}{2^{5/4}(3\pi)^{3/2}\sqrt{c}} \end{cases}$$

The U(1) current provides the explicit relation:

$$j^\mu = 3c(\partial\chi)\partial_\mu\chi = \frac{1}{4\pi}\frac{\epsilon^{\mu\nu\lambda}}{\sqrt{g}}F_{\nu\lambda}$$

$$\langle j_0 \rangle = \frac{Q}{4\pi R^2} \qquad \Longleftrightarrow \qquad \langle F_{\theta\phi} \rangle = B\sin\theta = \frac{Q}{2R^2}\sin\theta\,.$$

Cutoff: $\Lambda \sim \sqrt{Q}/R \sim \sqrt{B}$. $B =$ monopole field.

Vortices=charged particles

$$S = -\kappa \int d^3x \sqrt{g} F^{3/2} - \sum_p q_p \int A_\mu dX_p^\mu - \sum_p \gamma_p \cancel{\int d\tau \sqrt{F} \sqrt{\dot{X}_p^2}} + \dots$$

Horn Nicolis Penco 2015

- Effective vortex mass: $m_p = \gamma_p \sqrt{B} \sim \sqrt{Q}$
- Physically: Landau levels (LLs) separated by $\omega_L = B/m_p \sim$ cutoff
- Integrate out all LLs but the first $\longrightarrow$ EFT for the lowest LL
- In practice treat mass term as higher derivative term (leading single derivative kinetic term from the monopole connection)

Classical analysis: electrostatics on the sphere

To leading order, a simple electrostatic problem:

$$E^i = (\dot{X}_p)_j F^{ji}, \qquad \frac{1}{e^2}\nabla_i E^i = \rho, \qquad (e^2 \sim \sqrt{Q})$$

$$\Longrightarrow \qquad \vec{E} \sim \sqrt{Q}/d, \qquad \dot{\vec{X}} \sim \frac{1}{d\sqrt{Q}}.$$

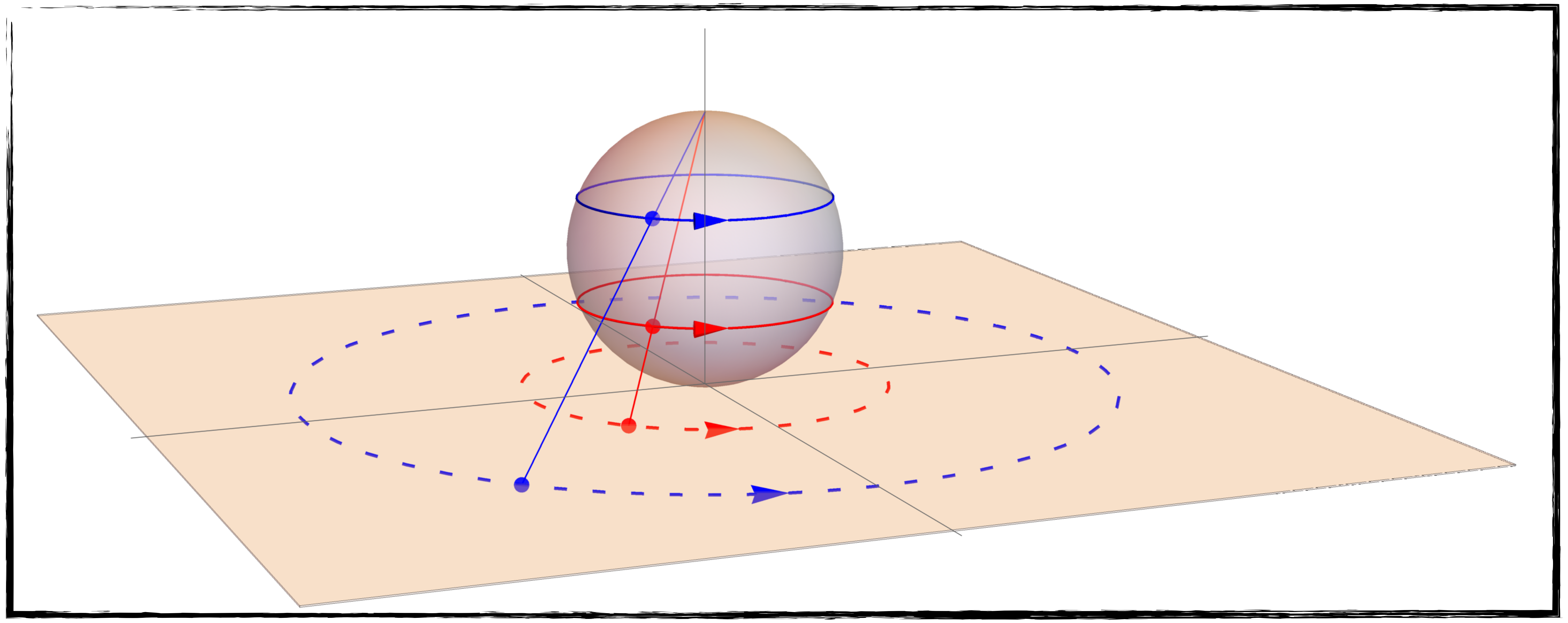
Gauss law implies $\sum_p q_p = 0$.

$$\Delta = \alpha Q^{3/2} - \frac{\sqrt{Q}}{12\alpha}\sum_{p\neq r} q_p q_r \log Q(\vec{R}_p - \vec{R}_q)^2 + \ldots$$

$$\vec{J} = -\frac{Q}{2}\sum_p q_p \vec{R}_p = \sum_p \vec{J}_p$$

Results: vortex-antivortex pair

$$q = \pm 1 \qquad \Longrightarrow \qquad \vec{J} = \frac{Q}{2}\left(\vec{R}_- - \vec{R}_+\right)$$



$$\boxed{\Delta = \alpha Q^{3/2} + \frac{\sqrt{Q}}{6\alpha}\log\frac{J^2}{Q}\,, \qquad \sqrt{Q} \ll J \leq Q.}$$

Quantization: fuzzy sphere

Quantization from $SU(2)$ algebra:

$$\vec{J} = -\frac{Q}{2}\sum_p q_p \vec{R}_p = \sum_p \vec{J}_p \qquad \Longrightarrow \qquad [J_p^i, J_p^j] = i\epsilon_{ijk} J_p^k$$

- R_p^i are non-commuting coordinates
- Quantum corrected energy for a vortex-antivortex pair

$$\Delta = \alpha Q^{3/2} + \frac{\sqrt{Q}}{6\alpha}\log\frac{J(J+1)}{Q}$$

Results: vortex distribution

- $J \geq Q \qquad \Longrightarrow \qquad n_{\rm V} > 1$
- $J \gg Q \qquad \Longrightarrow \qquad n_{\rm V} \gg 1$: approximate by smooth distribution ρ

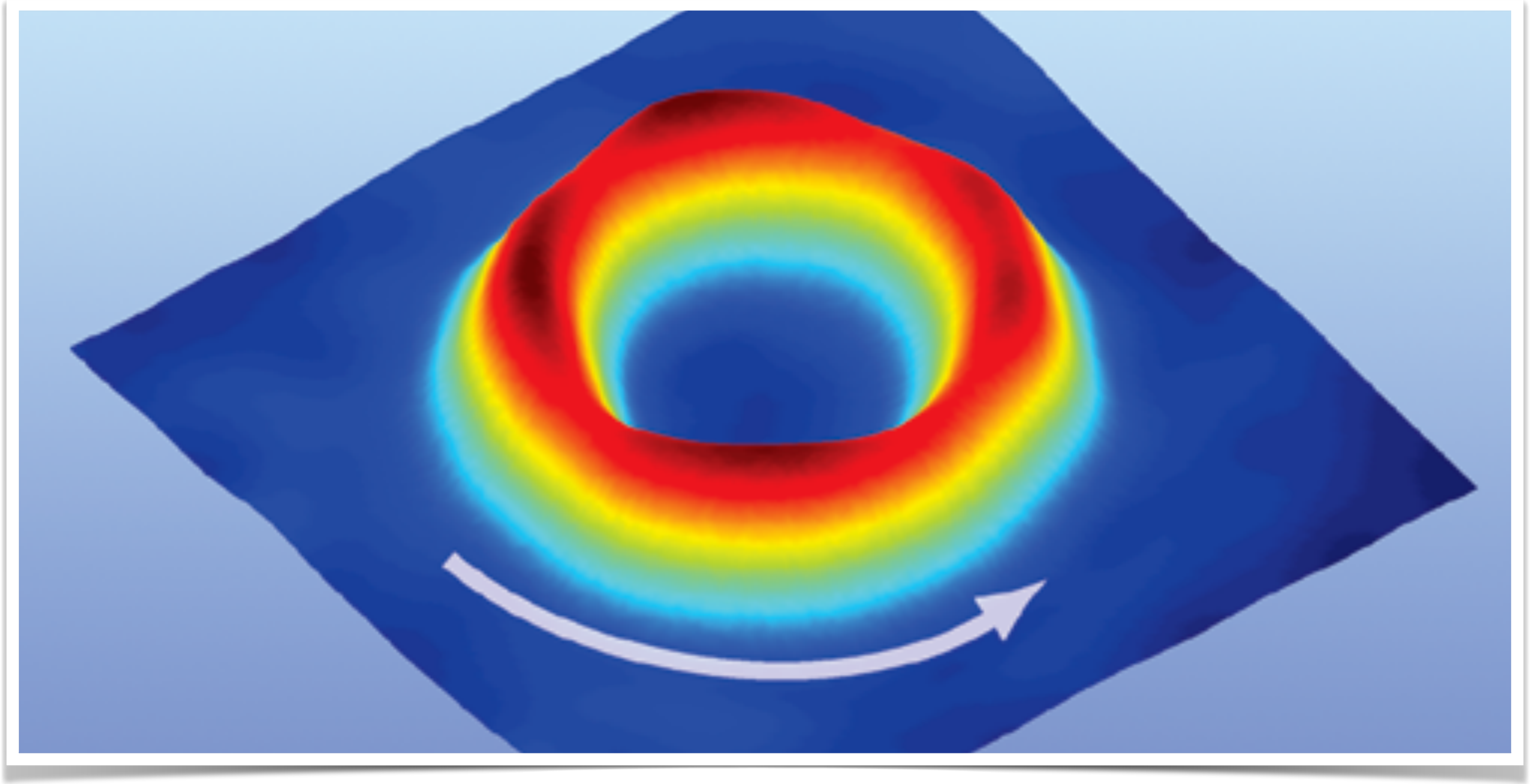
Minimize Δ at fixed J :

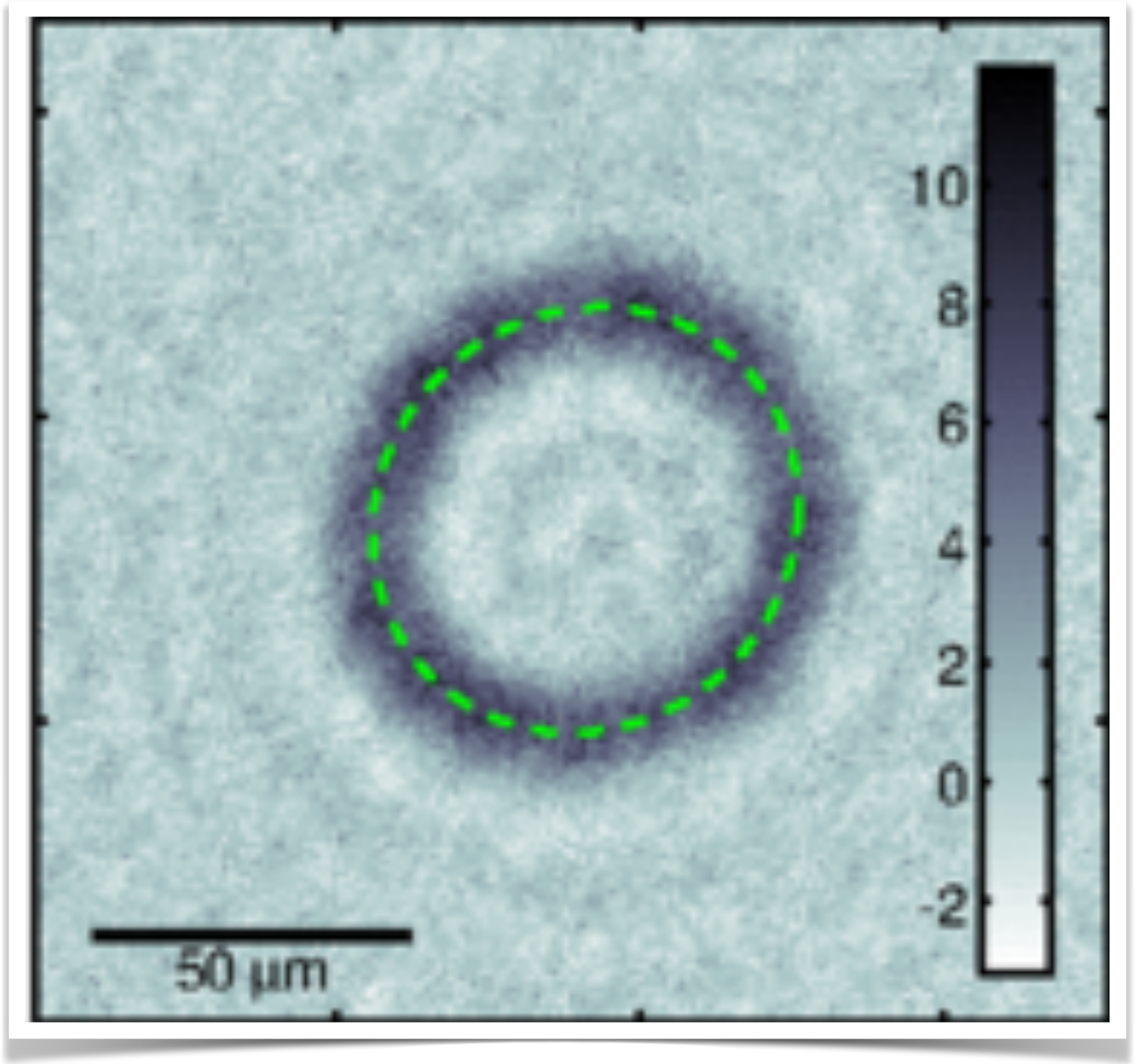
$$\rho = \frac{3}{2\pi R^2}\frac{J}{Q}\cos\theta$$

$$\boxed{\Delta = \alpha Q^{3/2} + \frac{1}{2\alpha}\frac{J^2}{Q^{3/2}}, \qquad Q \ll J \ll Q^{3/2}.}$$

Constant angular velocity ($\sim$ rigid body). ***What happens beyond this regime?***

Let's look again at experiments…





When the angular velocity exceeds the speed of sound in BECs the vortex lattice becomes unstable towards the formation of a coherent "giant" vortex annulus.

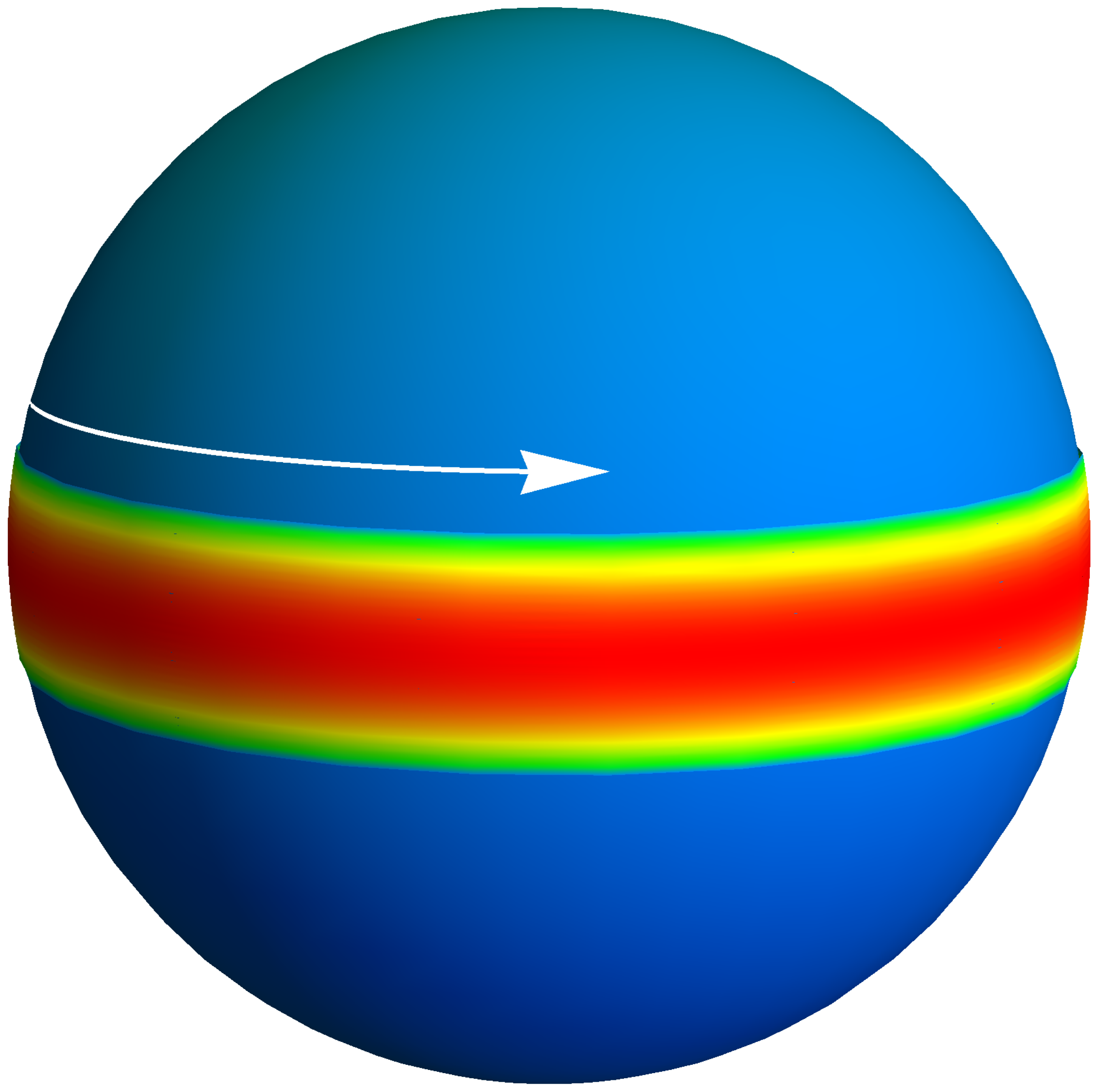
Theory: *Fischer Baym 2003, Fetter Jackson Stringari 2005*
Experiment: *Guo Dubessy de Herve Kumar Badr Perrin Longchambon Perrin 2019*
Non-technical review: *Sophia Chen - Physics 2020*

A giant vortex in the O(2) model

For $J/Q \in \mathbb{Z}$ a natural candidate for the "giant vortex" profile is

$$\chi = \mu t - \ell\phi \quad \Longrightarrow \quad J = \ell Q$$

$$\langle j_0 \rangle = \begin{cases} 3c\mu^2\sqrt{1 - \frac{\ell^2/\mu^2}{R^2 \sin^2\theta}} & \sin^2\theta \geq \frac{\ell^2}{R^2\mu^2}, \\ 0 & \sin^2\theta < \frac{\ell^2}{R^2\mu^2}. \end{cases}$$

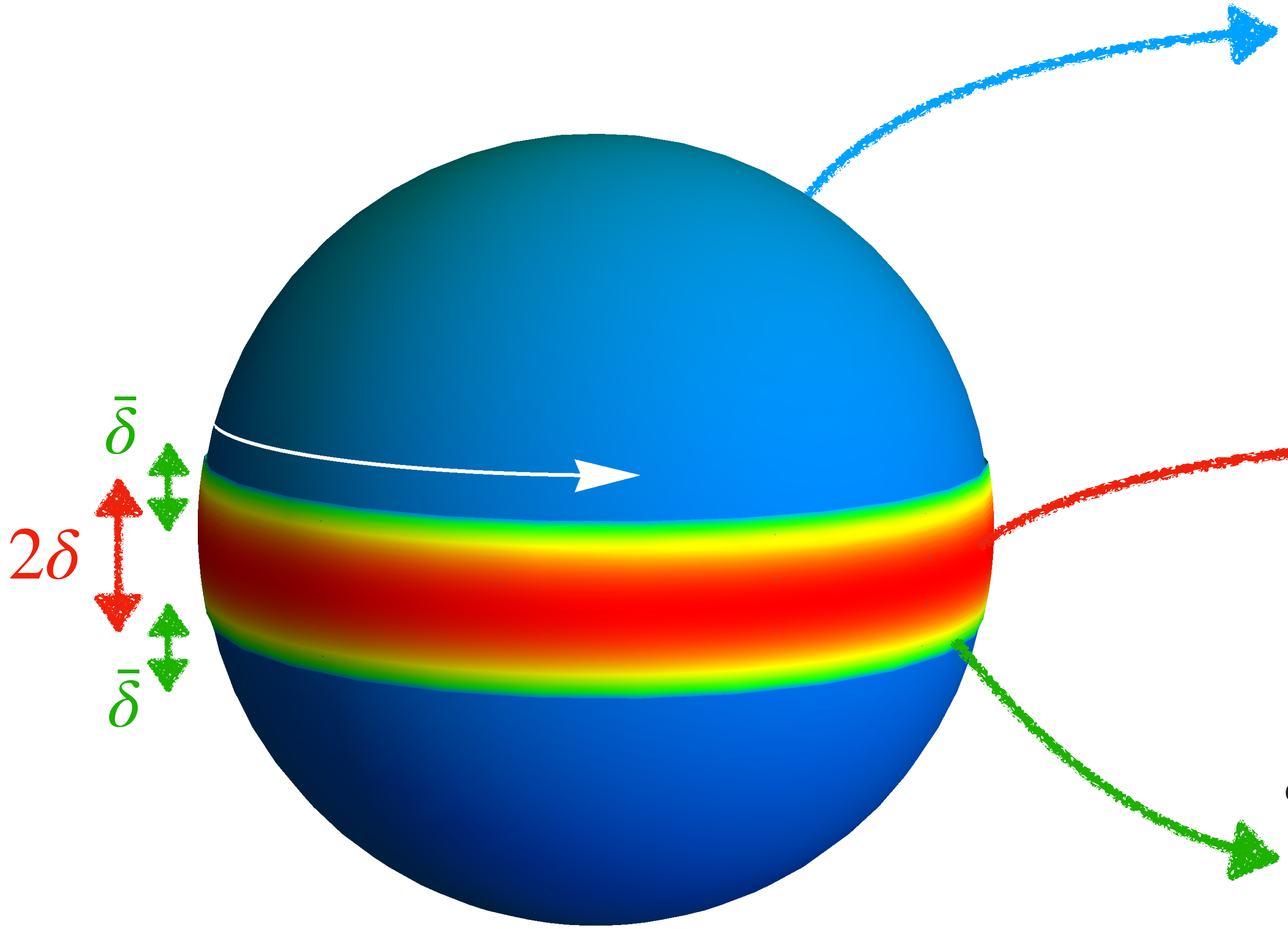


For $J \gg Q^{3/2}$:

$$R\mu = \frac{J}{Q}\left[1 + \frac{Q^3}{6\pi^2 c J^2} + \dots\right],$$

$$\frac{\ell}{R\mu} = 1 - \frac{Q^3}{6\pi^2 c J^2} + \dots$$

Three physically distinct regions in terms of $M^2 = -(\partial\chi)^2$



- Away from the equator the centrifugal potential
$$M^2 \sim V(\theta) \sim \frac{\ell^2}{\sin^2\theta} \text{ gaps all excitations .}$$

- The charge is localized around the equator on a strip of size $\delta \sim Q^{3/2}/J$. Cutoff $\Lambda = |M| \sim \sqrt{Q}$.
EFT for $\delta \gg \Lambda^{-1} \implies Q^2 \gg J$.

- $M^2 \sim 0$ in a small strip of size $\bar{\delta} \sim Q^{1/6}/J^{1/3}$:
effective boundary for $\bar{\delta}/\delta \sim (J/Q^2)^{2/3} \ll 1$

~ Hellerman Swanson 2020, GC Mezei Raviv-Moshe 2021

$\boxed{\text{EFT regime: } Q^{3/2} \ll J \ll Q^2}$

Results: the giant vortex energy

For $Q^{3/2} \ll J \ll Q^2$ we can compute the energy of the ground state:

$$\boxed{\Delta = J + \frac{9\alpha^2}{4\pi}\frac{Q^3}{J} + \dots}$$

- In the limit $J \gg Q^{3/2}$ the result approaches the expectation for $\phi\partial^{J/Q}\phi\ldots\partial^{J/Q}\phi$!
- The first correction depends on the same parameter α controlling the homogeneous superfluid energy $\Delta_{J=0} = \alpha Q^{3/2} + \ldots$
- Subleading corrections depend on novel Wilson coefficients associated with the effective boundary.

Results: fluctuations

In the limit $Q^3/J^2 \to 0$ the spectrum of fluctuations is given by

$$\omega = m + n\,, \quad J = \ell Q + m\,, \quad m \in \mathbb{Z}\,,\ n \in \mathbb{N}$$

- Expected spectrum of a (free) multi-trace!
- Corrections in Q^3/J^2 lift the apparent degeneracy, e.g. the gap of the next-to-lowest dimensional state with the same spin J of the giant vortex is:

$$\delta\Delta = \frac{18\alpha^2}{\pi}\frac{Q^3}{J^2}$$

Comments on large multi-trace operators from the large spin expansion

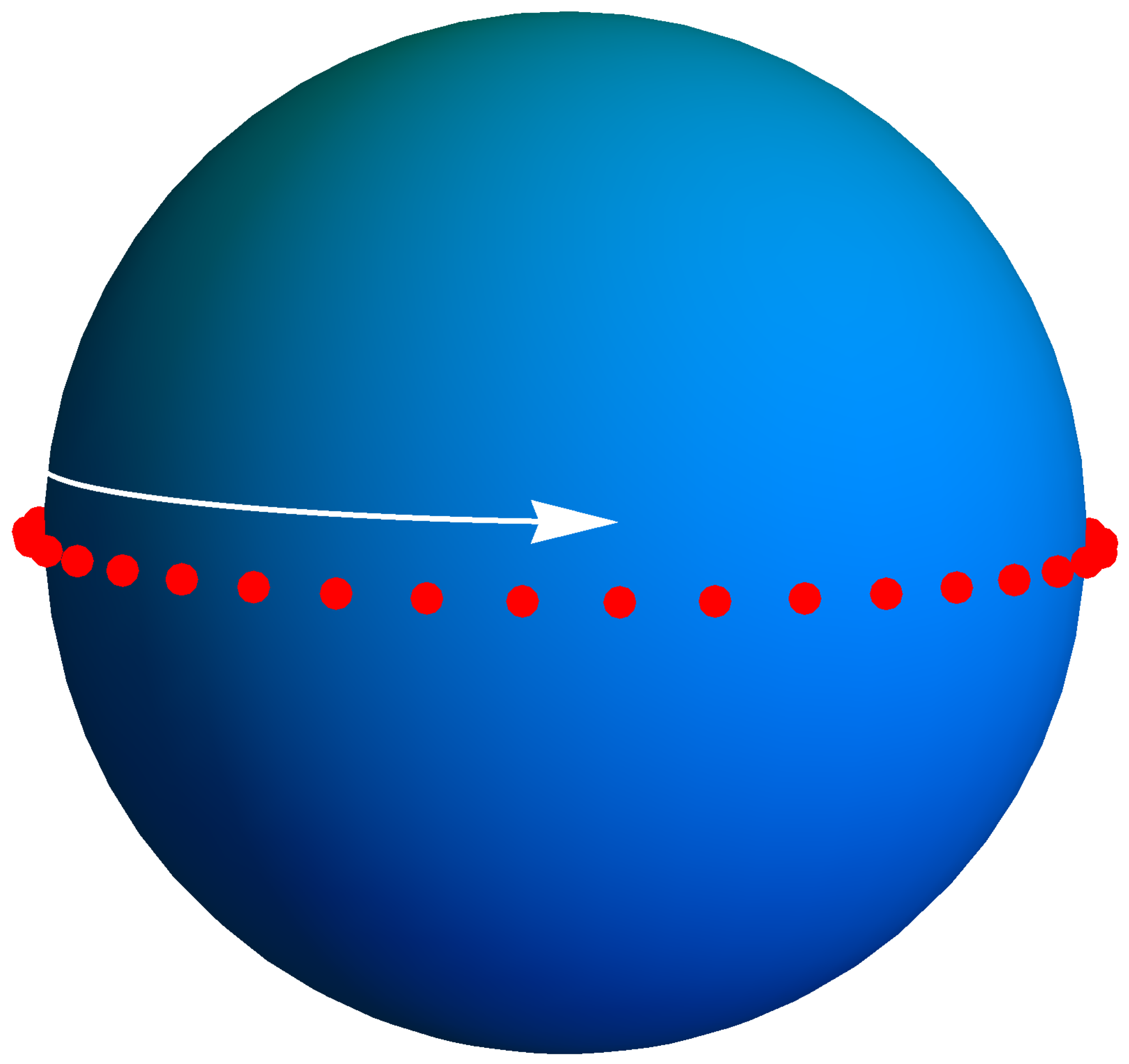
Large spin multi-trace operators

In the $J \to \infty$ the state breaks into Q *quasi*-free partons

$$\mathcal{O}_{Q,J} \sim \phi\partial^{J/Q}\phi\ldots\partial^{J/Q}\phi \qquad \Longrightarrow \qquad \Delta_{Q,J} = J + Q\Delta_\phi + \ldots$$

- Potential from *nearest neighbor* exchanges of $T_{\mu\nu}$ and j_μ

$$V \sim \sum_{\langle ij \rangle} e^{-\tau_{min}\Delta\chi_{ij}} \sim \frac{Q^2}{J}$$



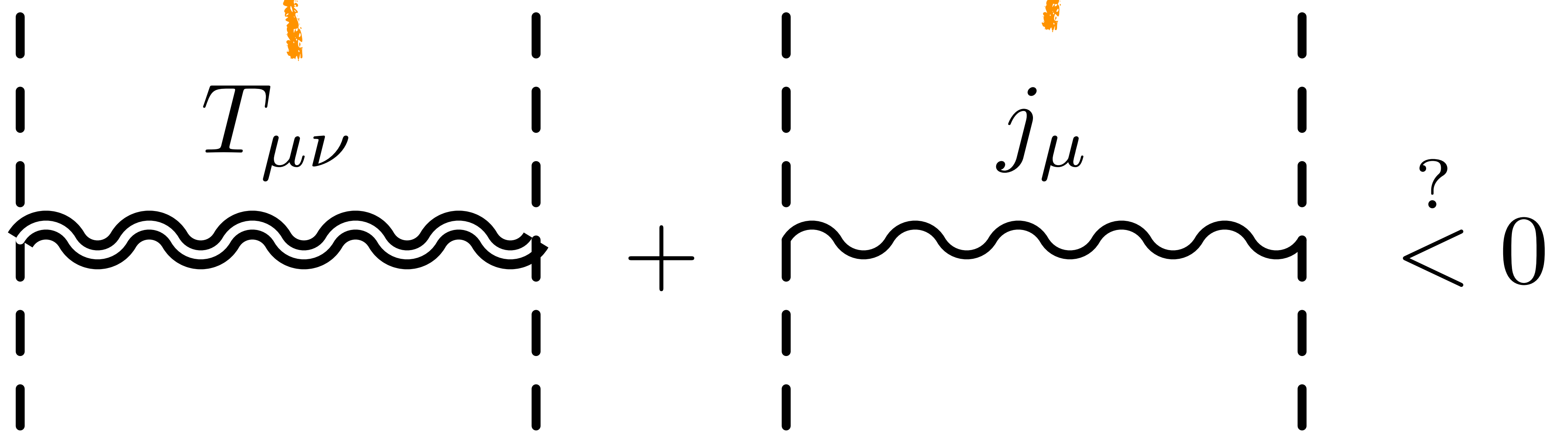
- Quasi-free dynamics for $V \ll 1 \quad \Longrightarrow \quad J \gg Q^2$: consistent with the upper limit of superfluid theory!

A *gravitational* collapse?

Is the interaction attractive or repulsive?

< 0 : attractive

> 0 : repulsive



Inequality quantified via the analytic bootstrap:

Komargodski Zhiboedov 2012

$$\frac{\Delta_\phi}{\sqrt{C_T}} \geq \frac{1}{\sqrt{6}\sqrt{C_J}}$$

✔ in the O(2) model

Chester Landry Liu Poland Simmons-Duffin Su Vichi 2019

Summary of results

Summary

The lowest dimensional operator at fixed $Q \gg 1$ and J in the O(2) model corresponds to

- $0 \leq J \ll \sqrt{Q}$: homogeneous superfluid +1 phonon $\qquad \Delta = \alpha Q^{3/2} + \frac{\sqrt{J(J+1)}}{\sqrt{2}}$
- $\sqrt{Q} \ll J \leq Q$: vortex-antivortex pair $\qquad \Delta = \alpha Q^{3/2} + \frac{\sqrt{Q}}{6\alpha} \log \frac{J^2}{Q}$
- $Q \ll J \ll Q^{3/2}$: regular vortex distribution $\qquad \Delta = \alpha Q^{3/2} + \frac{1}{2\alpha} \frac{J^2}{Q^{3/2}}$
- $Q^{3/2} \ll J \ll Q^2$: giant vortex state $\qquad \Delta = J + \frac{9\alpha^2 Q^3}{4\pi J}$
- $Q^2 \ll J$: Alday-Maldacena multi-trace $\qquad \Delta = J + Q\Delta_\phi$

Some open questions

- Giant-vortex for $J/Q \notin \mathbb{N}$?

- Parity-violating CFTs? 4d? SCFTs?

  GC Delacrétaz Mehta 2020, GC 2019, Hellerman Maeda Orlando Reffert Watanabe 2017-2021,…

- Supperadiant transition for $J \sim Q^{3/2}$ at weak coupling

- What's the order of the various transitions as J/Q changes?

- Gravitational collapse for $J \to Q^2$ from large spin EFT in AdS?

THANK YOU!