Women in Theoretical Physics
Premio Nazionale "Milla Baldo Ceolin" 2021

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About me

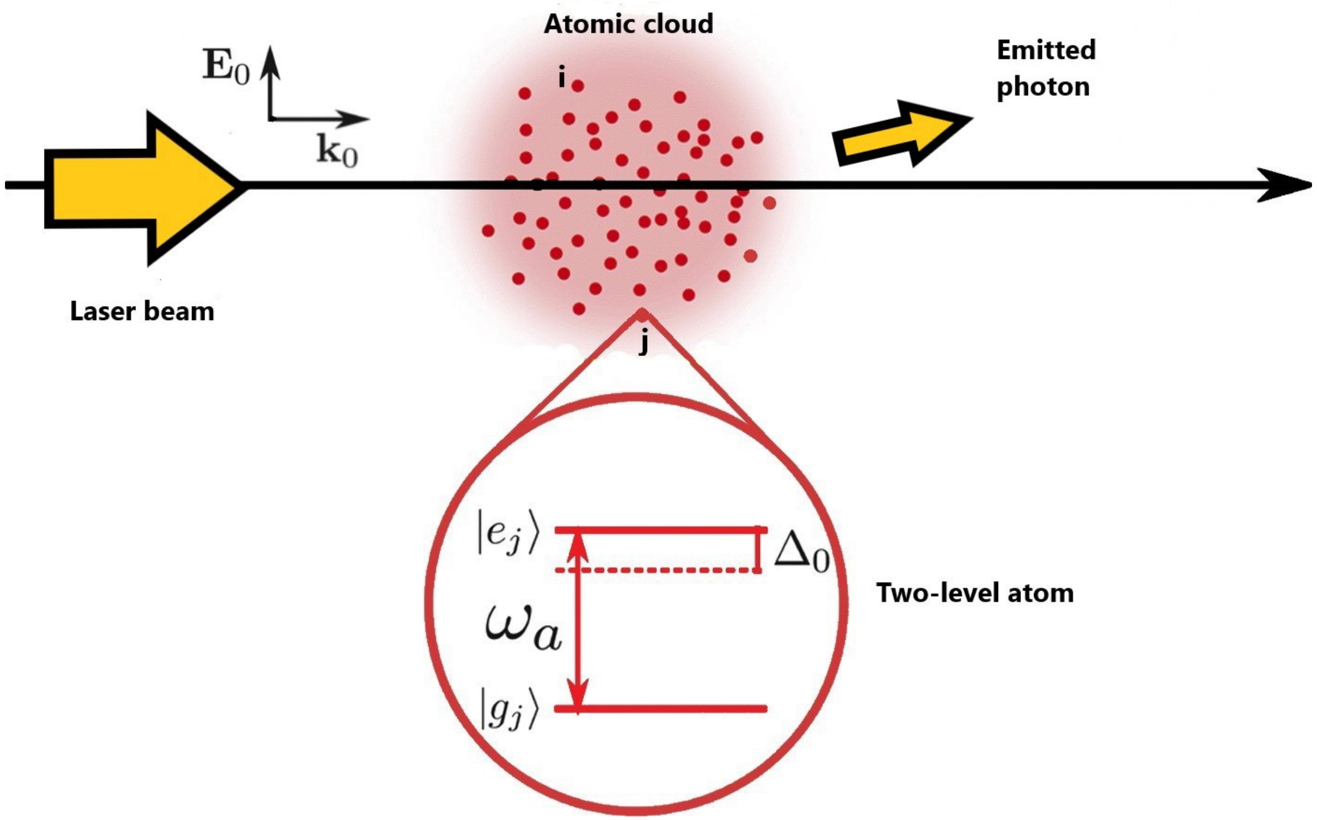
- June 2021: Master's Degree in Theoretical Physics at the University of Bari
  Thesis: "Cooperative effects in single photon emission" under the supervision of Prof. Saverio Pascazio

- July 2021: Master's Degree in Piano at the Conservatory of Music of Matera

- October 2021- at present: PhD student at the University of Bari, working on cooperative effects in atom-photon interactions in macroscopic quantum systems

  Joint work with Fabio D. Cunden, Paolo Facchi, Saverio Pascazio, Francesco V. Pepe (Bari University) and Robin Kaiser (Nice University) in the framework of the QuantERA Project PACE-IN

Cooperative scattering by a cold atomic cloud



N identical two-level atoms with fixed positions $\mathbf{r_j}$, j=1,…,N, sampled from a 3D Gaussian distribution with zero mean and variance σ^2

Huge number of atoms interacting with the quantized electromagnetic field: collective effects

Subradiance and superradiance: suppression and enhancement of the decay rate of the cloud with respect to the case of an isolated atom

Cooperativity parameter $b_0 = \frac{N}{(k_a\sigma)^2}$

Wavefunction in the linear regime of weak excitation

$$|\Psi(t)\rangle = \alpha(t)\,|g\rangle \otimes |\text{vac}\rangle + \sum_{j=1}^{N} \beta_j(t)\,|j\rangle \otimes |\text{vac}\rangle + \sum_{\boldsymbol{k}} \gamma_{\boldsymbol{k}}(t)\,|g\rangle \otimes |\boldsymbol{k}\rangle$$

Total excitation probability

$$P(t) = \sum_{j=1}^{N} |\beta_j(t)|^2 = \boldsymbol{\beta}^\dagger(t)\boldsymbol{\beta}(t) \quad \Rightarrow \quad \dot{P}(t) = -\Gamma\boldsymbol{\beta}^\dagger(t) S \boldsymbol{\beta}(t)$$

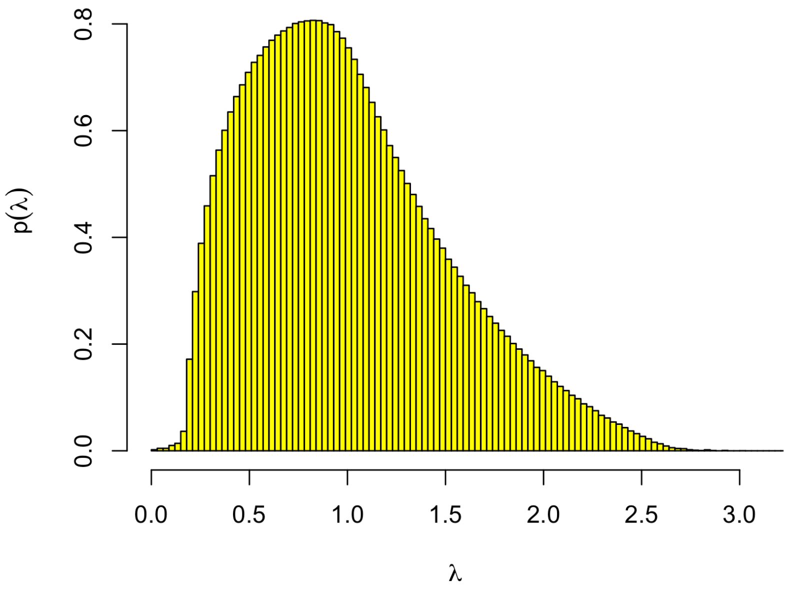
Γ decay rate of an isolated atom

$$S_{jk} = \mathrm{sinc}\left(\sqrt{\frac{N}{b_0}}\, x_{jk}\right) \quad \forall\ j,k = 1,\dots,N \qquad x_{jk} = |\boldsymbol{x}_j - \boldsymbol{x}_k| \qquad \boldsymbol{x}_j \sim \mathcal{N}(\mathbf{0},\mathbf{1})$$

Spectrum of S:

- Subradiance $0 \le \lambda < 1$
- Superradiance $1 < \lambda \le N$
- Normal decay rate $\lambda = 1$

Eigenvalue distribution for $b_0 = 1$



S is a Euclidean random matrix (ERM):
N^2 entries but only O(N) random degrees of freedom

Universality of random matrix ensembles $\Rightarrow$ useful to simulate complex systems with a huge number of interacting components

Complication: few results available for ERMs

Numerical study of the matrix S with large size N using the ReCas Data Center in Bari

Study of the spectral properties of S

Study of the nearest neighbour spacing distribution (NNSD):
pdf of spacings between adjacent eigenvalues

Brody distribution $p_{\mathrm{br}}(q,s) = \alpha(q+1)\, s^q \,\exp\!\left(-\alpha s^{q+1}\right)$ $\qquad \alpha = \left[\Gamma\left(\frac{q+2}{q+1}\right)\right]^{q+1}$

Interpolates between the two universality classes of NNSD:

q=0 is the Poisson distribution:
- Spacing for iid random variables
- Integrable systems
- Localized eigenstates

q=1 is the Wigner-Dyson distribution:
- Real random matrices
- Chaotic systems
- Delocalized eigenstates

Result

For the central part of the spectrum of S, the behaviour is chaotic close to Wigner-Dyson and with delocalized states

Localization properties of the eigenstates of S

Participation ratio

$$\mathrm{PR}(\Psi) = \frac{1}{\sum_{i=1}^{N} |\Psi_i|^4}$$

$\Psi_i,\ i = 1, \ldots, N$ components of the eigenstate Ψ

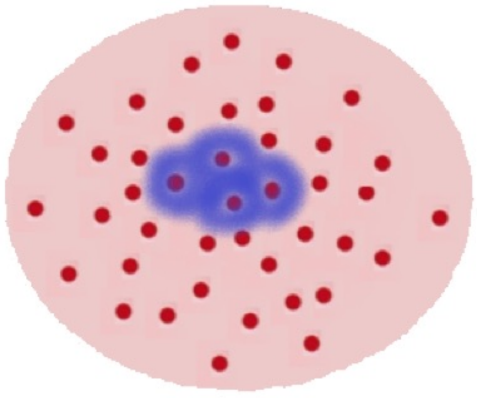
Moments of the eigenstates

$$M_q = \left\langle \sum_{i=1}^{N} |\Psi_i|^{2q} \right\rangle \xrightarrow[N \to +\infty]{} C_q N^{-\tau(q)}$$

Fractal dimension

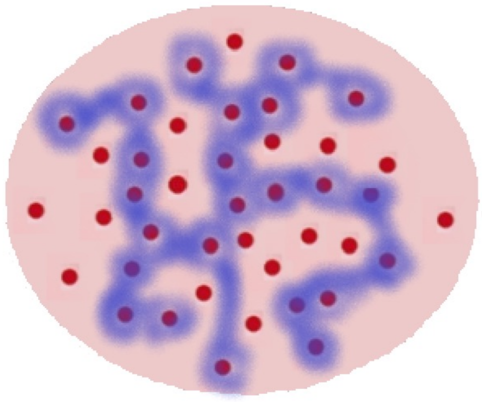
$$D_q = \frac{\tau(q)}{q-1}$$

$0 < D_q < 1$: systems with non trivial fractal dimension



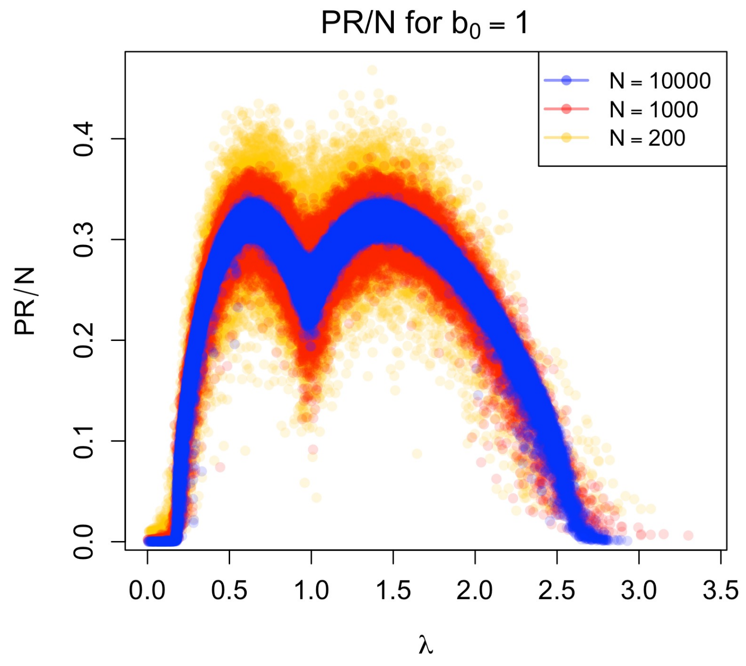
Localized state

PR=1 D_q=0



Delocalized state

PR=N D_q=1



Results: appropriate scaling PR ~ N for random delocalized states!

Numerical study of higher moments and fractal dimension corroborates the delocalization of states

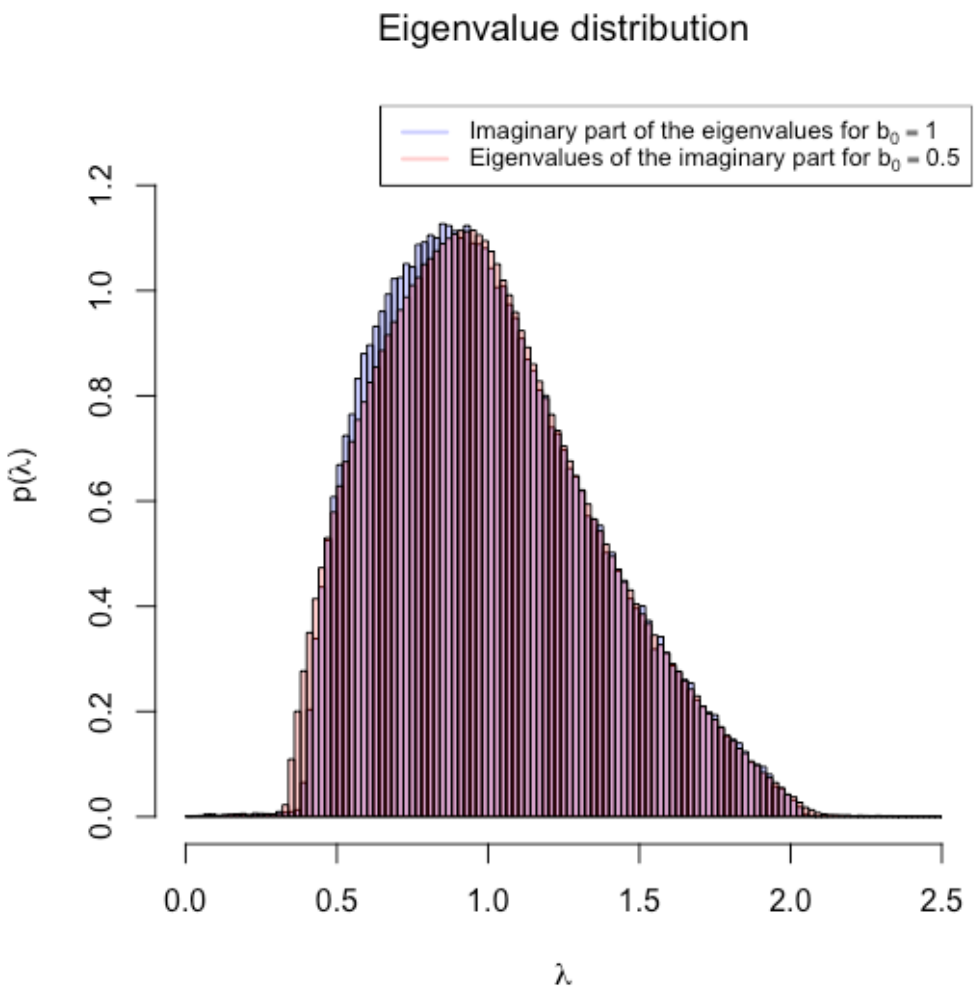
Current work and perspectives

- Study of a non-Hermitian Euclidean random matrix ruling the dynamics of the system and its connections with the spectral properties of S

$$\dot{\boldsymbol{\beta}}(t) = -iM\boldsymbol{\beta}(t)$$

$$M'_{jk} = -\frac{2M_{jk}}{\Gamma} = \begin{cases} \dfrac{e^{i\sqrt{\frac{N}{b_0}}x_{jk}}}{\sqrt{\frac{N}{b_0}}x_{jk}} & \text{if } j \neq k \\ i & \text{if } j = k \end{cases}$$

$$S = \mathrm{Im} M'$$



- Deeper investigation of subradiance

- A combination of analytical results of random matrix theory and numerical simulations can be used to study a variety of systems with collective effects in atom-photon interactions

Thank you!