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AN IMPROVED EFFECTIVE-ONE-BODY MODEL FOR COALESCING BLACK HOLE BINARIES

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Based on A. Albertini, A. Nagar, P. Rettegno, S. Albanesi, and R. Gamba,
"Waveforms and fluxes: Towards a self-consistent effective one body waveform model for nonprecessing, coalescing black-hole binaries for third generation detectors"
*Phys. Rev. D **105**, 084025 (2022)*

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OUTLINE

- Gravitational waves from compact binary coalescences
- The effective-one-body (EOB) approach to the two-body problem
- TEOBResumS: EOB model for binary black hole coalescences
- Testing the angular momentum flux with respect to numerical relativity (NR) simulations
- Improving TEOBResumS: radiation reaction & spin-orbit sector
- Increased dynamical consistency and faithfulness to NR

COMPACT BINARY COALESCENCES

- Coalescence of two compact objects (BH/NS): emission of gravitational radiation
- Faithful waveform templates needed to detect the signal through matched filtering and infer the sources parameters
- Einstein's equations can be solved numerically
  ➡ but high computational costs
  ➡ impossible to cover the whole parameter space
- Analytical approaches: allow a fast and accurate waveform generation

THE EFFECTIVE-ONE-BODY FORMALISM

mapping the two-body dynamics in general relativity in the motion of a particle with the reduced mass of the system moving in an effective metric

Mass ratio $q = \frac{m_1}{m_2}$, $m_1 > m_2$ Symmetric mass ratio $\nu \equiv \frac{m_1 m_2}{(m_1 + m_2)^2}$

Continuous deformation in ν of a Schwarzschild metric:

$G = c = 1$
$u = 1/r$

$$ds^2_{\mathrm{eff}} = g^{\mathrm{eff}}_{\mu\nu} dx^\mu_{\mathrm{eff}} dx^\nu_{\mathrm{eff}} = -A(r)dt^2 + B(r)dR^2 + R^2(d\theta^2 + \sin^2\theta d\phi^2)$$

$$A^{\mathrm{PN}}_{\mathrm{orb}}(u) = 1 - 2u + 2\nu u^3 + \nu a_4 u^4 + \nu\left[a^c_5(\nu) + a^{\log}_5 \ln u\right] u^5 + \nu\left[a^c_6(\nu) + a^{\log}_6 \ln u\right] u^6$$

THEORETICAL FRAMEWORK

- Hamiltonian: $\hat{H}_{\rm EOB} \equiv \dfrac{H_{\rm EOB}}{\mu} = \dfrac{1}{\nu}\sqrt{1 + 2\nu\left(\hat{H}_{\rm eff} - 1\right)}$

$$\hat{H}_{\rm eff} = \boxed{\sqrt{p_{r_*}^2 + A\left(1 + \frac{p_\varphi^2}{r_c^2} + 2\nu(4-3\nu)\frac{p_{r_*}^4}{r_c^2}\right)}} + \boxed{p_\varphi\left(G_S\hat{S} + G_{S_*}\hat{S}_*\right)}$$

orbital + spin-orbit

- Hamiltonian equations of motion complemented by the **radiation reaction**:

$$\left.\begin{aligned}
\frac{d\varphi}{dt} &= \frac{\partial \hat{H}_{\rm EOB}}{\partial p_\varphi} = \Omega \\
\frac{dr}{dt} &= \left(\frac{A}{B}\right)^{1/2} \frac{\partial \hat{H}_{\rm EOB}}{\partial p_{r_*}} \\
\frac{dp_\varphi}{dt} &= \hat{\mathcal{F}}_\varphi \\
\frac{dp_{r_*}}{dt} &= -\left(\frac{A}{B}\right)^{1/2} \frac{\partial \hat{H}_{\rm EOB}}{\partial r}
\end{aligned}\right\}$$

the phase space variables enter the evaluation of the waveform:

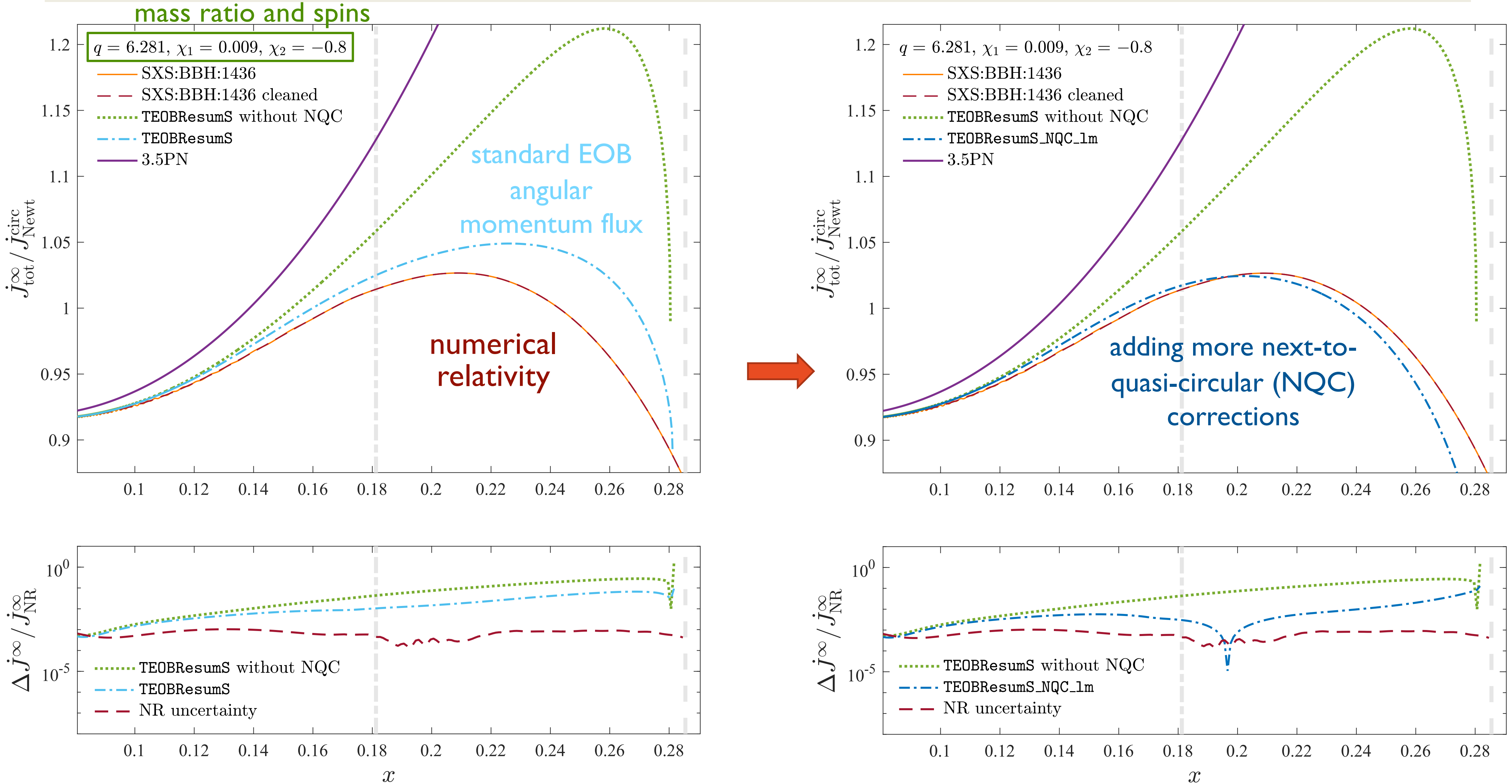
$$h_+ - i h_\times = \frac{1}{\mathcal{D}_L} \sum_{\ell=2}^{\infty} \sum_{m=-\ell}^{\ell} h_{\ell m}\, {}_{-2}Y_{\ell m}$$

multipoles

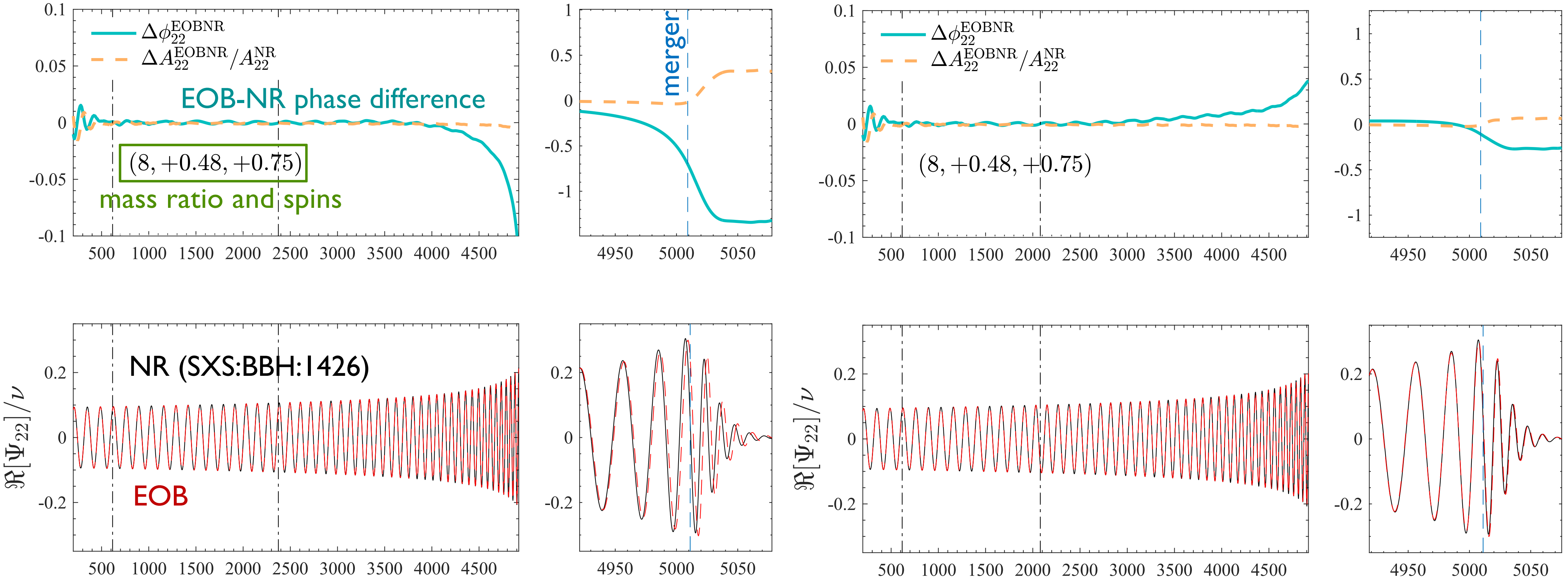
TEOBRESUMS: CHECKING THE DYNAMICS

- TEOBResumS is an EOB waveform model: we work with the quasi-circular version for spin-aligned black hole binaries
- Some parameters are tuned to numerical relativity (NR) (orbital sector, spin-orbit, merger & ringdown)
- Checking the dynamics: comparing the angular momentum flux (radiation reaction) to NR results

IMPROVING THE ANGULAR MOMENTUM FLUX (RADIATION REACTION)



TUNING TO NUMERICAL RELATIVITY



$$\Delta\phi_{22}^{\mathrm{EOB-NR}}\Big|_{\mathrm{merger}} = -0.70$$

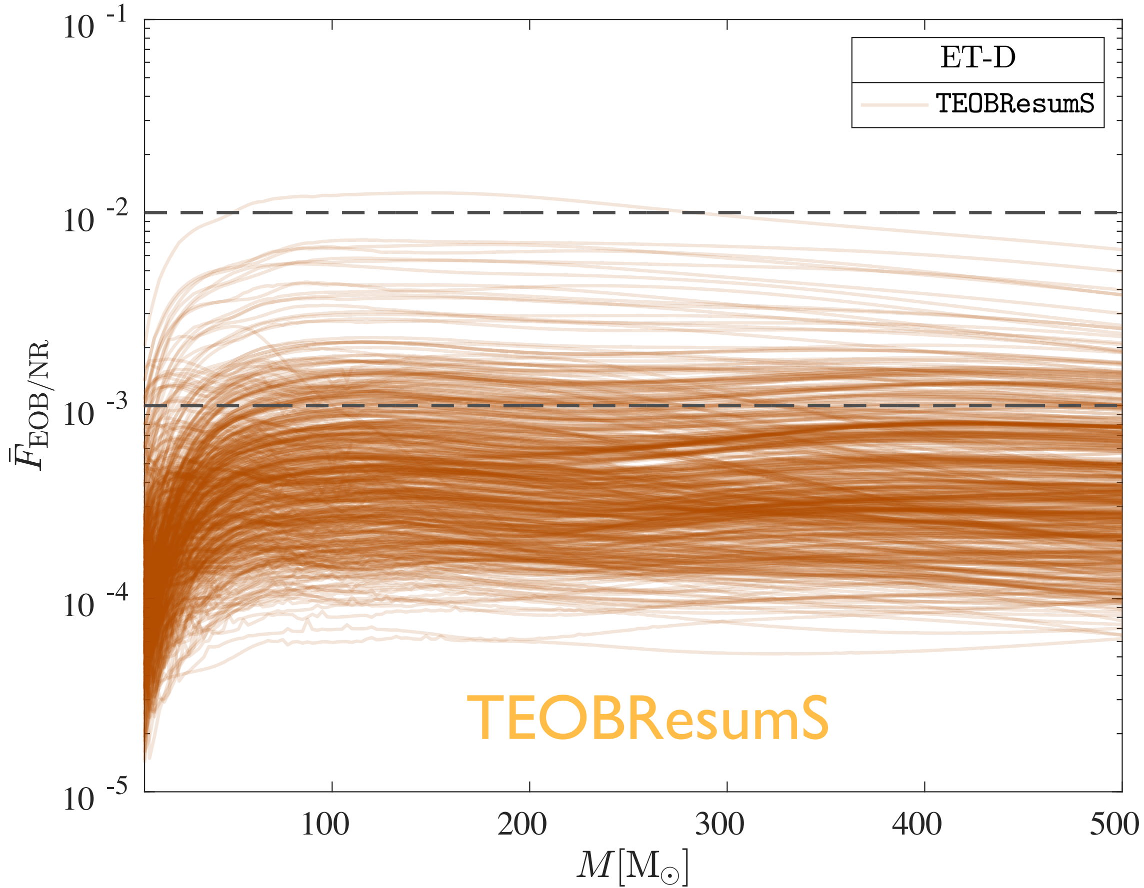
re-calibrating the effective spin-orbit parameter (c_3)

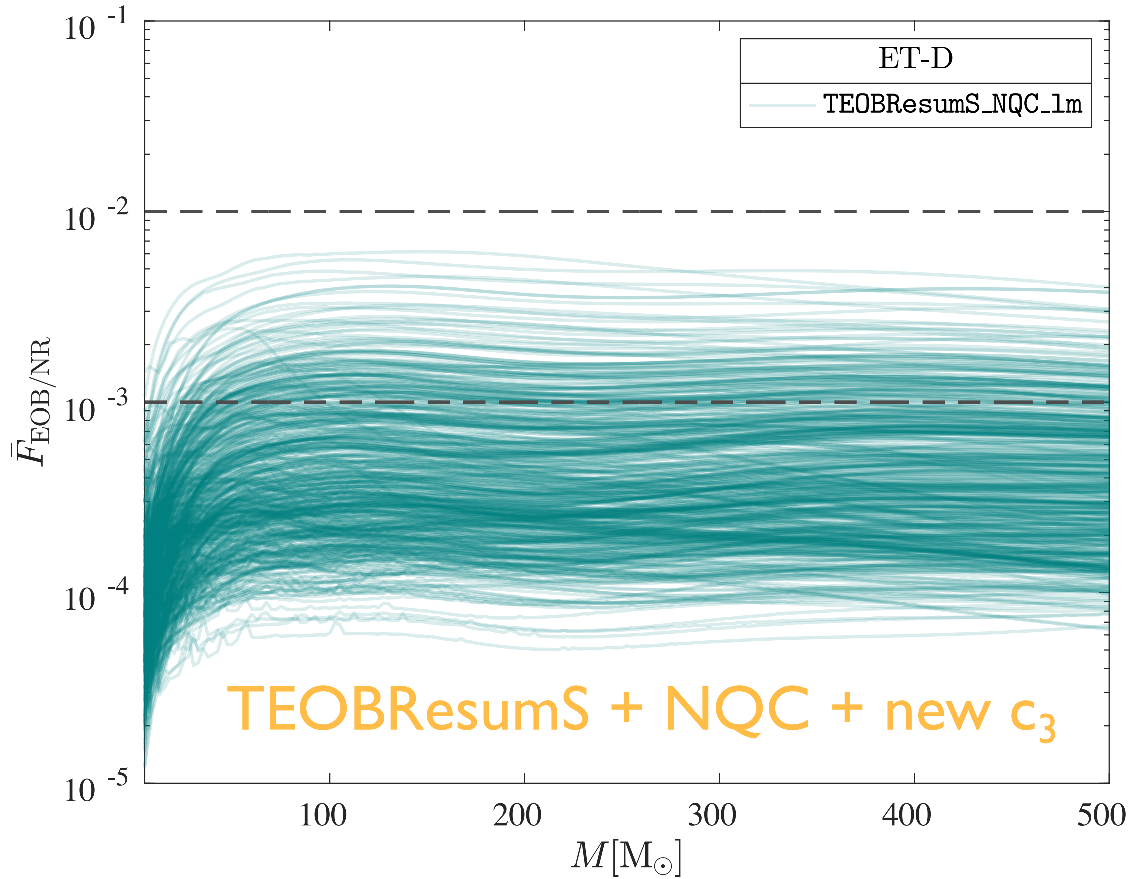
$$\Delta\phi_{22}^{\mathrm{EOB-NR}}\Big|_{\mathrm{merger}} = -0.11$$

UNFAITHFULNESS IMPROVEMENT

$$\bar{F}(M) \equiv 1 - F = 1 - \max_{t_0,\phi_0} \frac{\langle h_{22}^{\rm EOB}, h_{22}^{\rm NR}\rangle}{\|h_{22}^{\rm EOB}\|\|h_{22}^{\rm NR}\|} \qquad \|h\| \equiv \sqrt{\langle h, h\rangle} \qquad \langle h_1, h_2\rangle \equiv 4\Re \int_{f_{\rm min}^{\rm NR(M)}}^{\infty} \frac{\tilde{h}_1(f)\tilde{h}_2^*(f)}{\boxed{S_n(f)}} df$$

noise spectral density of Einstein Telescope





OVERVIEW

TEOBResumS: state-of-the-art EOB waveform model for spin-aligned coalescing black hole binaries on quasi-circular orbits

angular momentum flux comparison with NR simulations lead to two improvements:

addition of next-to-quasi-circular corrections to the radiation reaction	new calibration of the effective spin-orbit parameter $\mathbf{c_3}$
increased dynamical consistency and agreement with NR	lowered the unfaithfulness of problematic configurations

promising model for the next generation of gravitational wave detectors

Thanks for your attention!