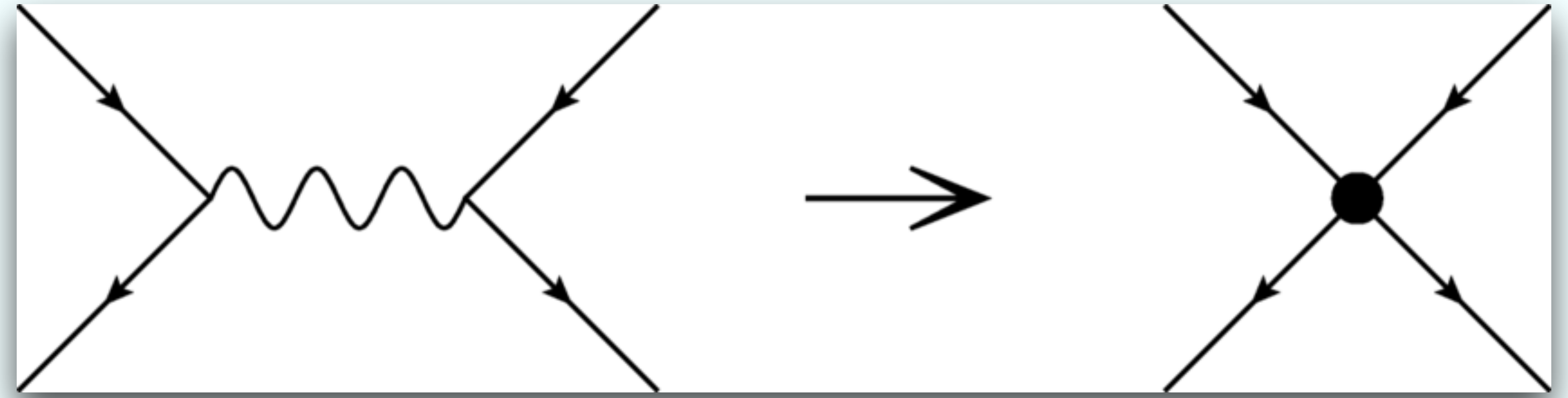
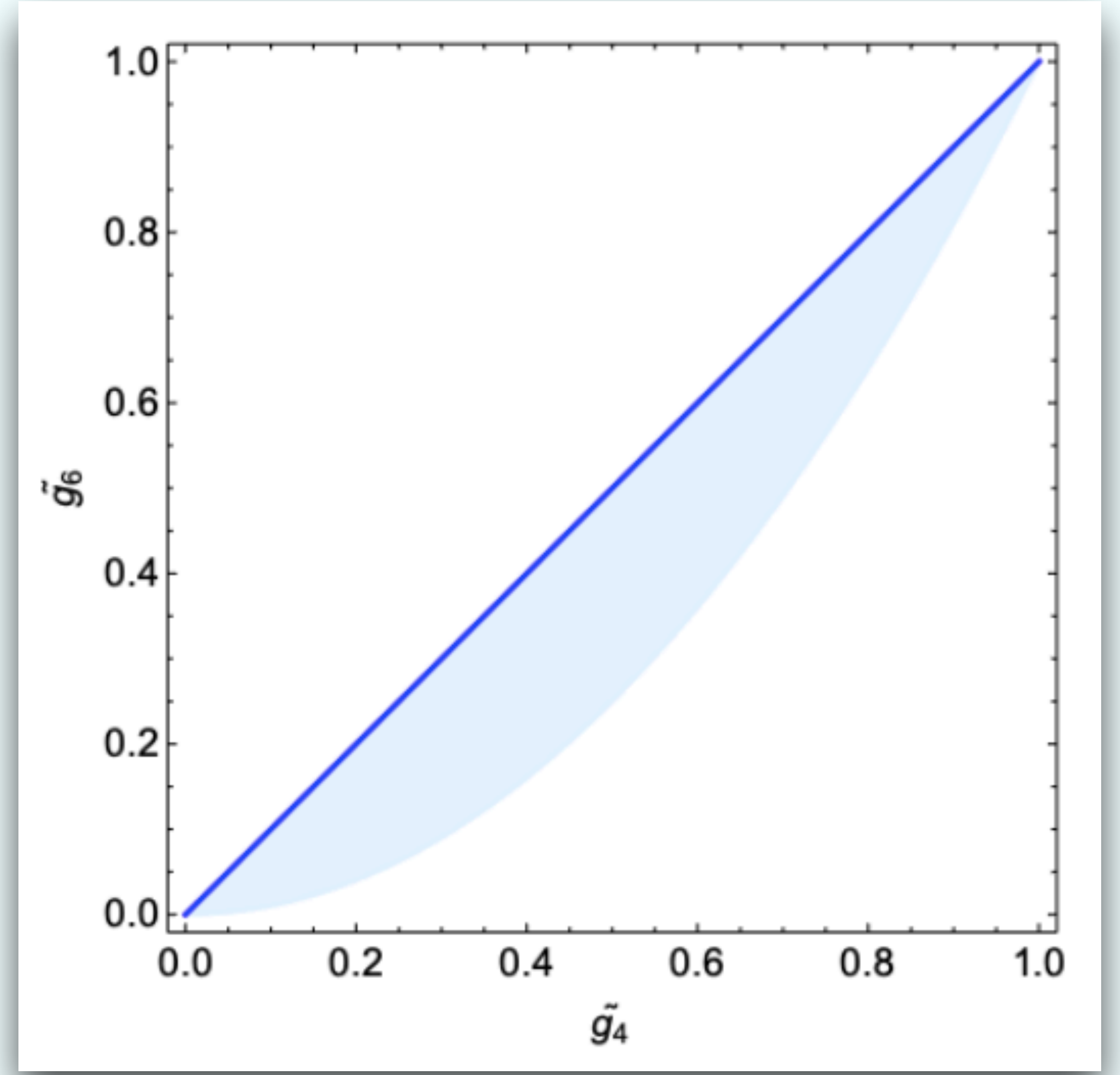
UV Constraints on IR EFTs





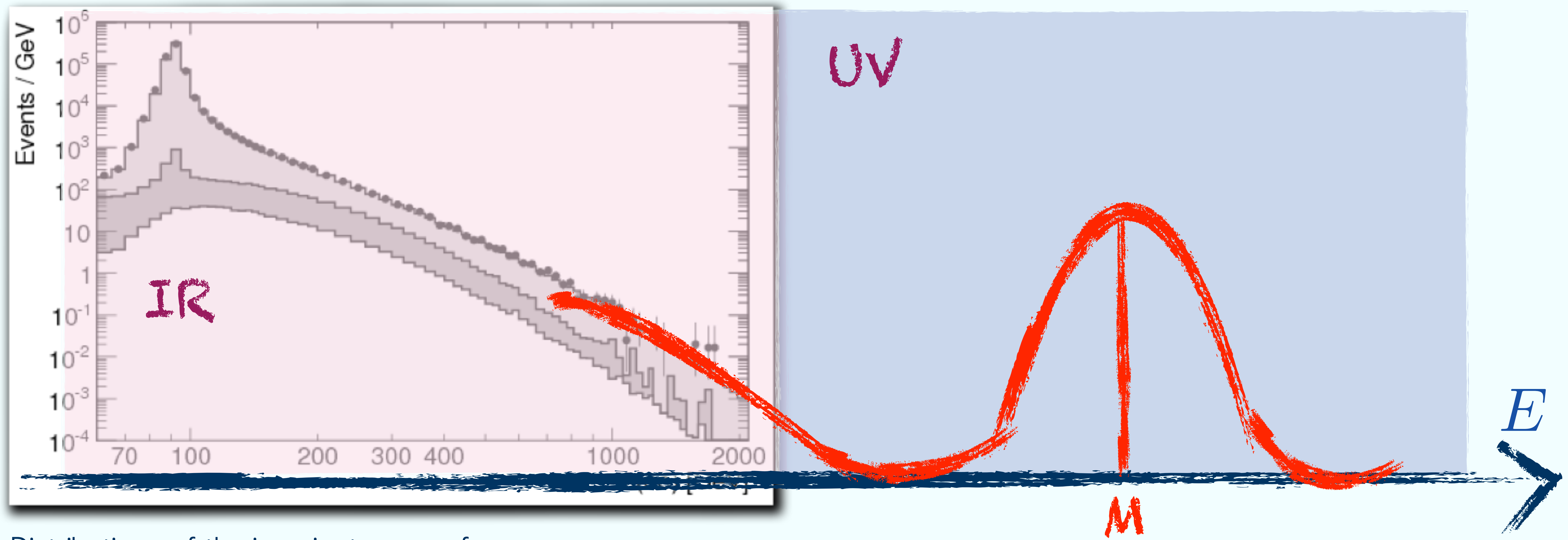
Positivity Bounds for a Scalar Field

Davide Maria
Lombardo

Boost Fellow
GGI/22

New Physics at HE and EFTs

- More-Microscopic Explanations in Particle Physics



Distributions of the invariant mass of electrons' pairs - CMS coll.

$$\mathbf{M \gg m_{SM}}$$

New Physics at HE and EFTs

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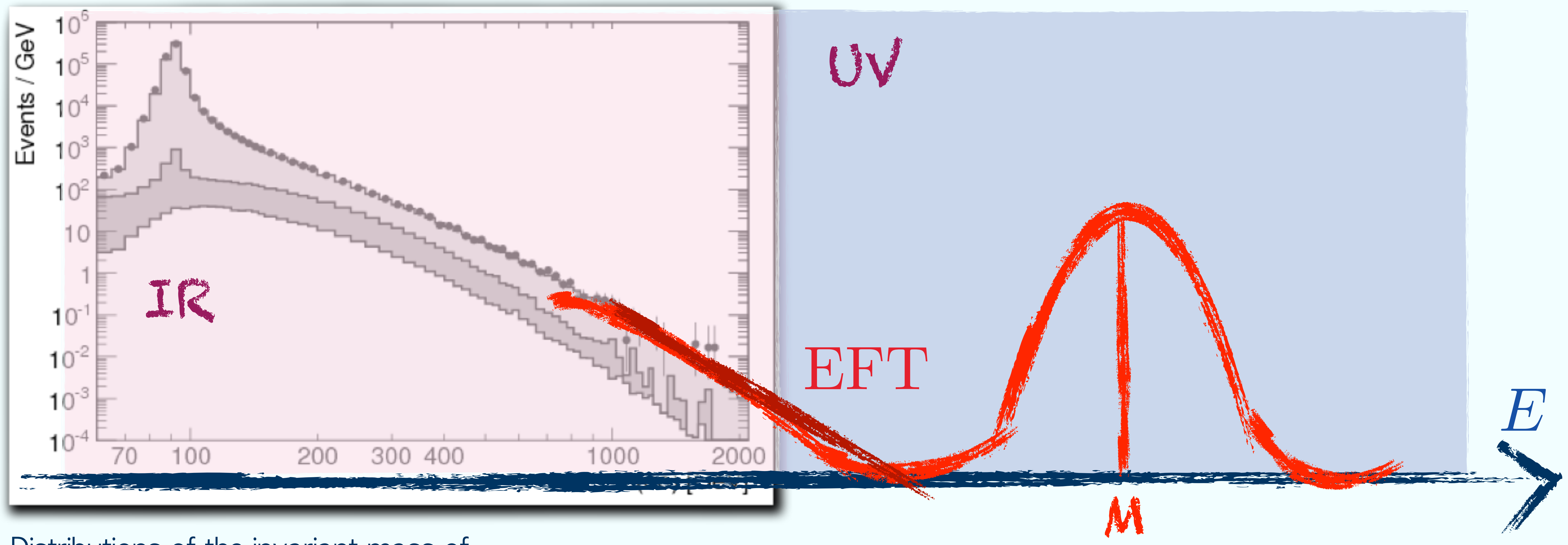


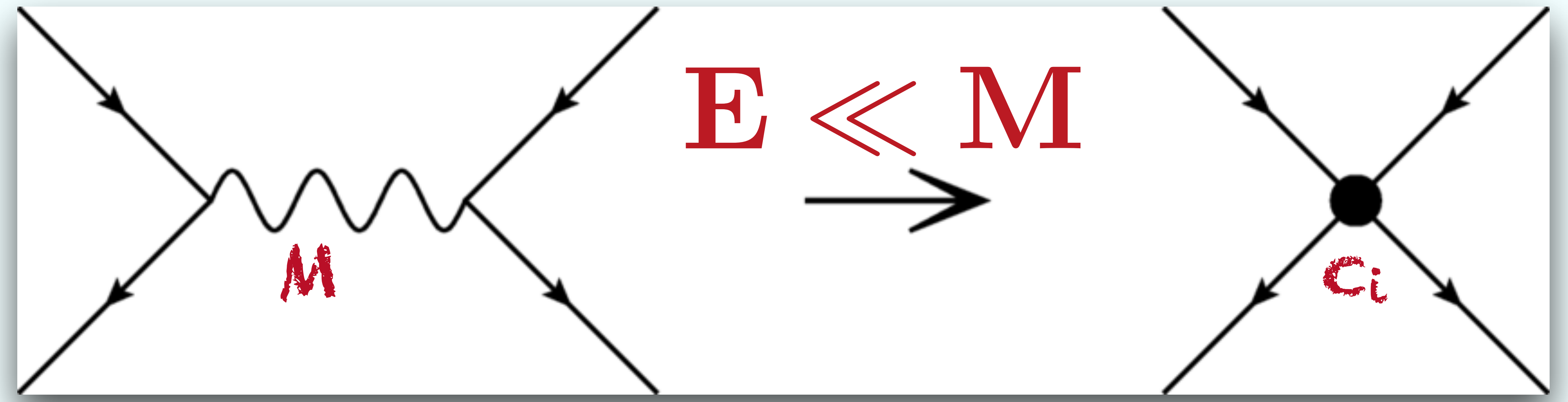
Distributions of the invariant mass of electrons' pairs - CMS coll.

$$\mathbf{M \gg m_{SM}}$$

- Low-Energy Expansion to Parametrise NP-Effects

- EFT-Coefficients Encode UV-Physics

$$\mathcal{L}_{\mathrm{EFT}} = \mathcal{L}_{d\leq 4} + \sum_{d>4,i} c_i^d \frac{\mathcal{O}_i^d}{M^{d-4}}$$

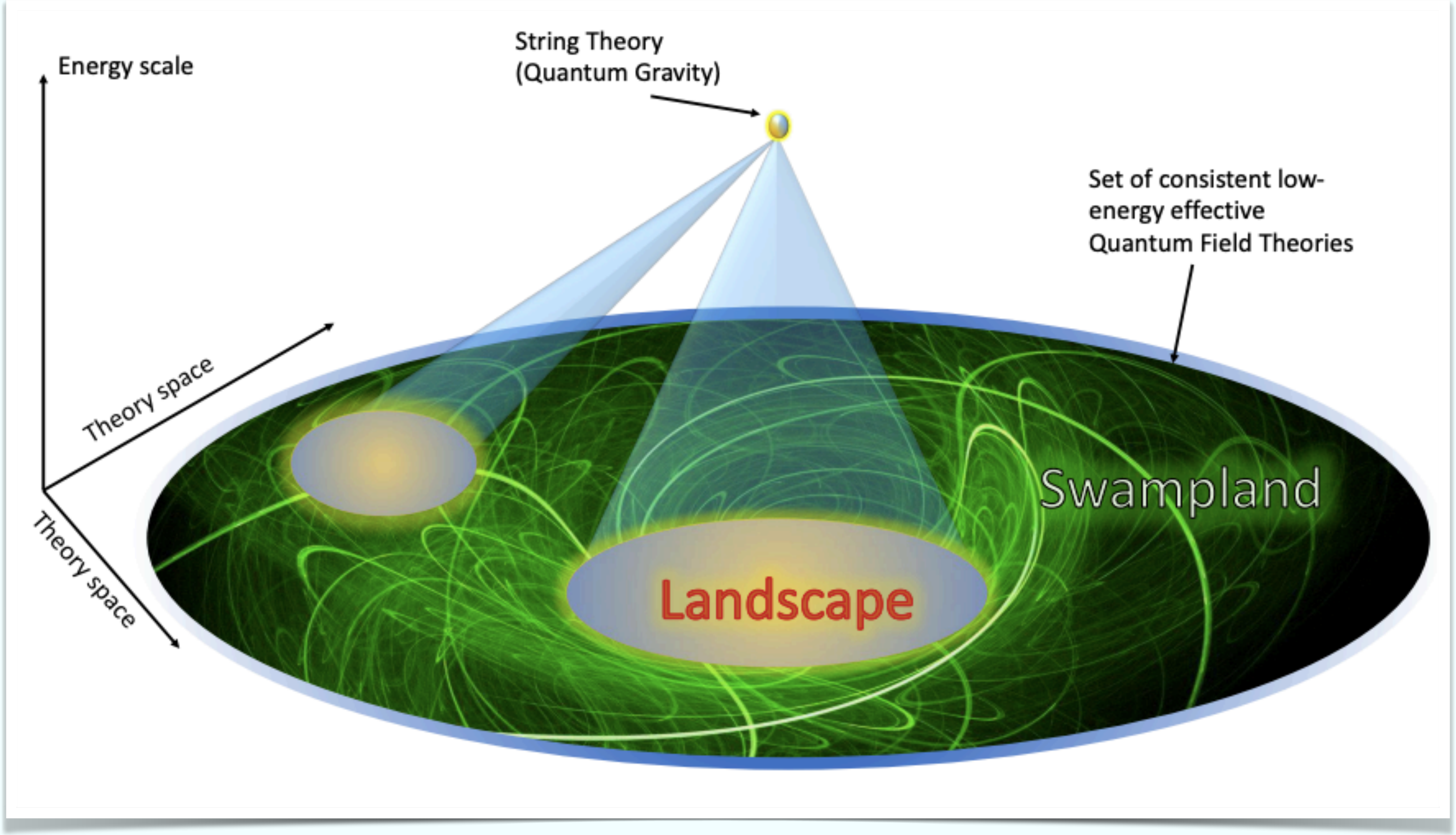


Identifying Viable Low-Energy EFTs

- Causality and Unitarity in the UV

- Constraints on the EFT Coefficients from these UV-Assumptions

- Ruling out some IR-descriptions



The Swampland Program — Palti - 2019

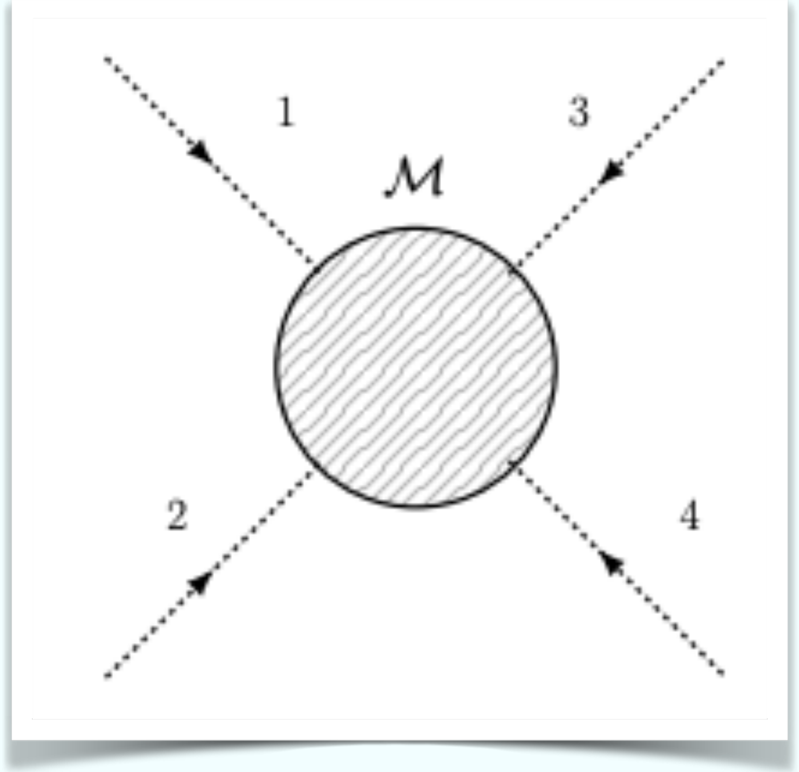
- Dispersion Relations on the Scattering Amplitudes

- Investigating Strong-Coupling Corrections

Defining a Sensible Low-Energy Scattering

Adams, Arkani-Hamed et al. - 2006
de Rham, Tolley et al. - 2017
Arkani-Hamed, Huang et al. - 2020
Bellazzini, Rattazzi, Riva et al. - 2020

- Assumptions on the Scattering Amplitude
- 2→2 Scattering Amplitude for Scalars: $\mathcal{M}(s,t)$



$s=(p_1+p_2)^2$

$t=(p_1-p_3)^2$

$u=(p_1-p_4)^2$

$s+t+u=4m^2$

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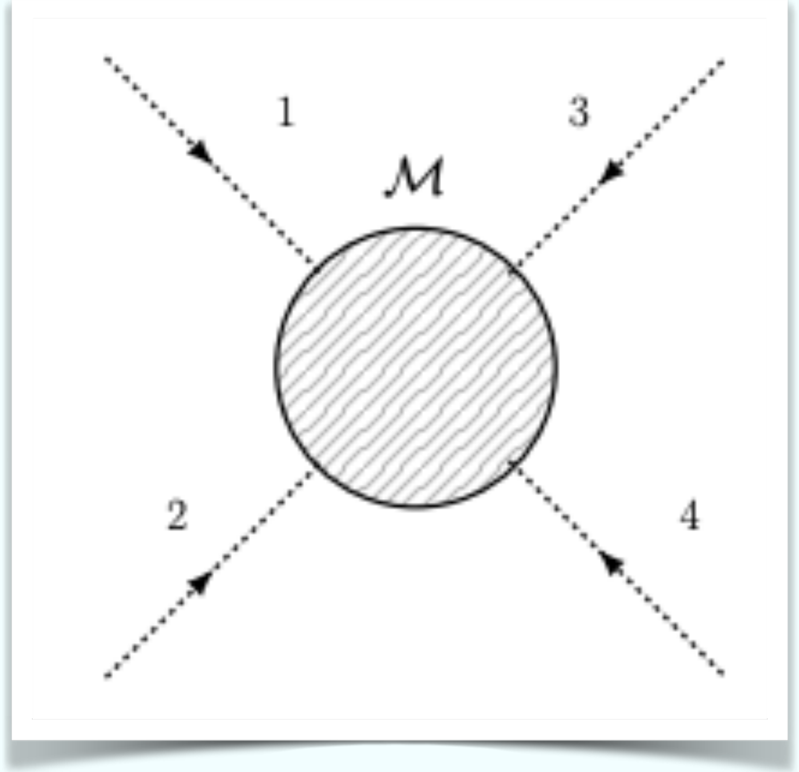

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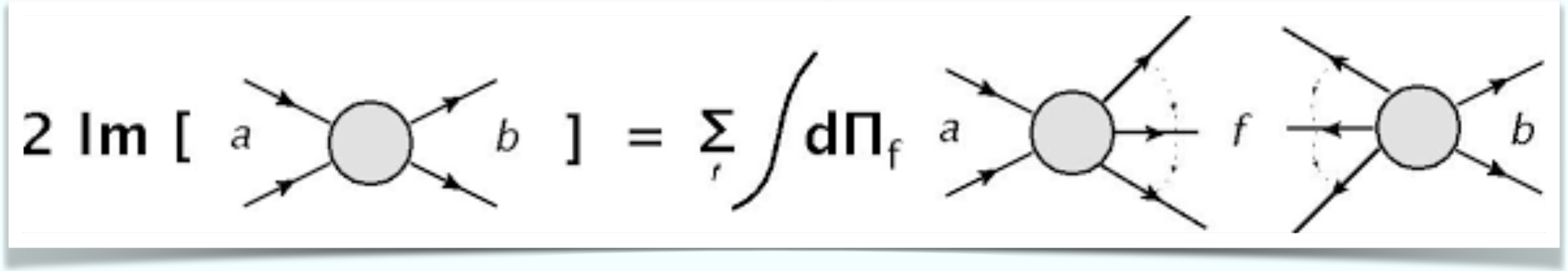
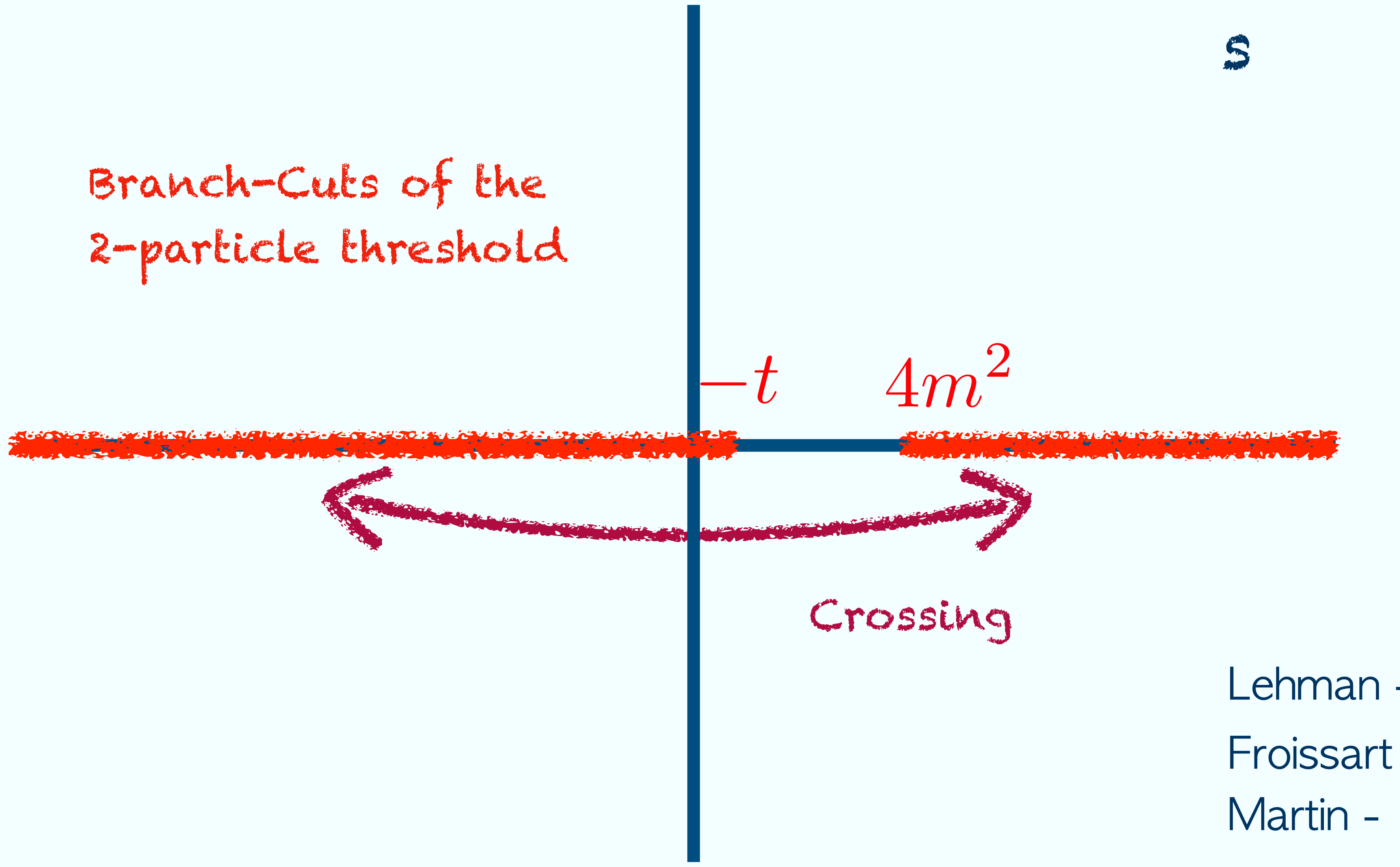
- Assumptions in the Complex-s Plane
  - Optical Theorem: $\mathrm{Im}(\mathcal{M}(s,t=0))>0$



  - Analyticity and Singularities Structure in s for $0\le t\le 4m^2$
  - Crossing: $\mathcal{M}(s,t)=\mathcal{M}^*(4m^2-s^*-t,t)$
  - Asymptotic Boundedness for $0\le t\le 4m^2$: $\mathcal{M}(s,t)/s^2 \xrightarrow{|s|\to\infty} 0$

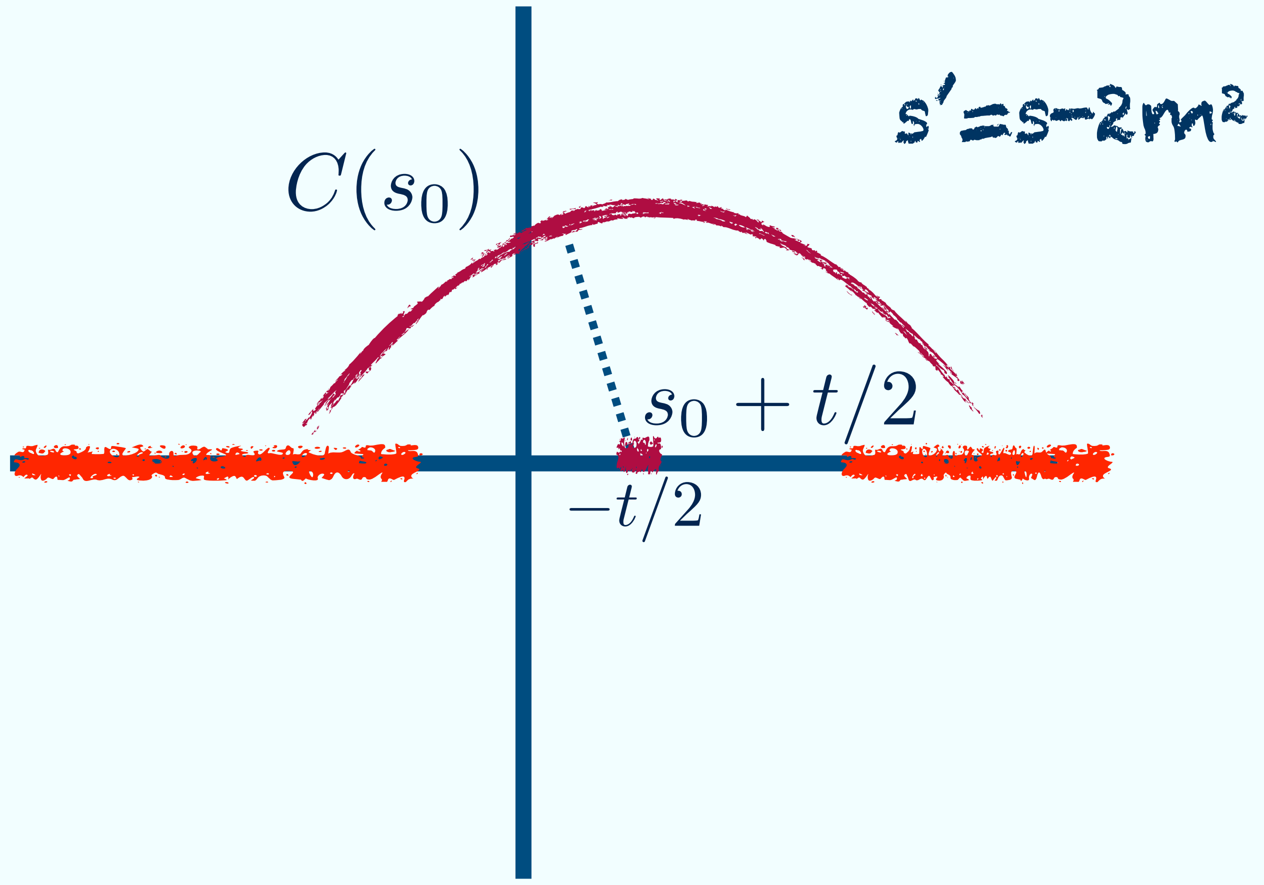


Lehman - 1958
Froissart - 1961
Martin - 1963

Arcs to Parametrise Low-Energy Scattering

$$a_k(s_0,t) \equiv (i\pi)^{-1} \oint_{C_{(s_0)}} \frac{\mathcal{M}(s',t)}{(s'+t/2)^{k+1}} ds'$$

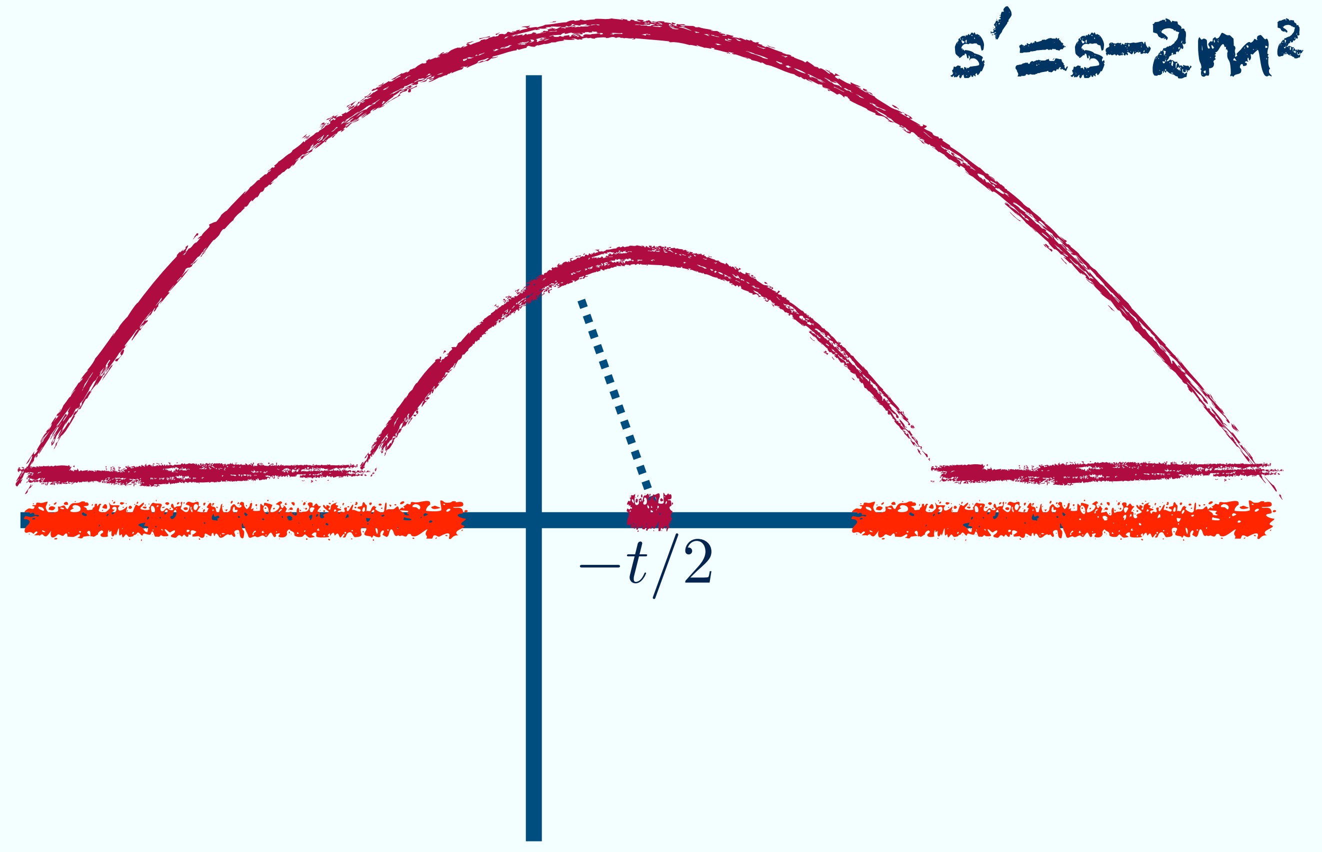
- Complete Parametrisation
- Non-Perturbative



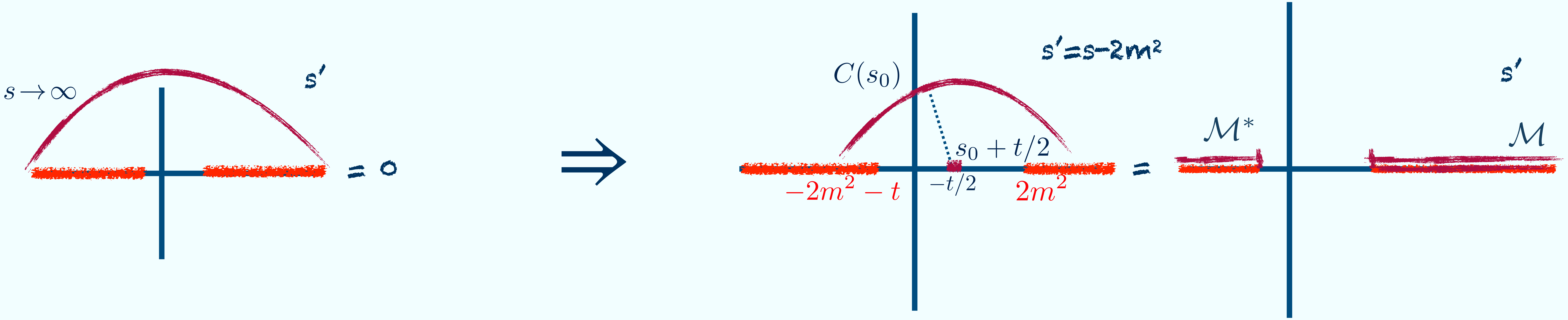
Arcs to Parametrise Low-Energy Scattering

$$a_k(s_0,t) \equiv (i\pi)^{-1} \oint_{C_{(s_0)}} \frac{\mathcal{M}(s',t)}{(s'+t/2)^{k+1}} ds' = \frac{2}{\pi} \int_{s_0}^{\infty} \frac{\mathrm{Im}[\mathcal{M}(s',t)]}{(s'+t/2)^{k+1}} ds'$$

At Weak Coupling the arcs are the EFT-coefficients



- Complete Parametrisation
- Non-Perturbative
- Dispersive Integrals to Relate IR and UV



Causality and Unitarity Mandate Bounds on the Arcs

Caron-Huot, Van Duong — 2020

- Our Method to Read the Bounds: Expansion Around $t=0$

- Partial Waves Decomposition

$$\mathrm{Im}[\mathcal{M}(s,t)] = \sum_J (2J+1)\rho_J(s)\mathcal{P}_J\left(1+\frac{2t}{s}\right)$$

- Average on the UV-States

$$\langle(\cdots)\rangle \equiv \frac{1}{\pi}\sum_J (2J+1)\int_{s_0}^{\infty}\frac{ds}{s}\rho_J(s)(\cdots)$$

Positive For Unitarity

$$a_k(s_0,t) = \sum_l 2\left\langle\frac{2\mathcal{P}_J^{(l)}(1)}{s^{(k+l)}}\right\rangle t^l \equiv \sum_l \langle A_{k,l}(s,J)\rangle t^l$$

$$\sum_l (a_{k,l} - \langle A_{k,l}\rangle)t^l = 0$$

at t=0 the standard Positivity Bounds follow

- Arcs are UV-Averaged Coefficients

- UV-Assumptions constrain IR-Scattering

Pions Scattering at Large-N

- Interesting for Phenomenology
- Bootstrap Approach to QCD
- Large-N QCD:
  - At LO Pions Interact at Tree-Level
  - No Exotic Mesons: Arcs with Odd-Subtractions
  - Boundedness is Relaxed to: $\mathcal{M}(s,t)/s \xrightarrow{|s|\to\infty} 0$

$g \sim$ LECs $\sim c_i/M^{d_i}$

Low-Energy Coefficients

$g_{1,0} = f_\pi^{-2}$

$$\mathcal{M}(s,t) = g_{1,0}\, s + g_{2,1}\, st + g_{3,1}\, st^2 + g_{3,1}\, s^2 t + \ldots = \sum_{n,l} g_{n,l}\, s^{n-l} t^l$$

$\sim L_i$

Weak Coupling Analysis

- Tree-Level: Arcs=EFT-Coefficients $a_{n,l} = g_{n,l}$

$$\mathcal{M}(s,t) = g_{1,0}s + g_{2,1}st + g_{3,1}st^2 + g_{3,1}s^2t + \ldots = \sum_{n,l} g_{n,l}\, s^{n-l}t^l$$

$g \sim$ LECs
Low-Energy Coefficients

NC

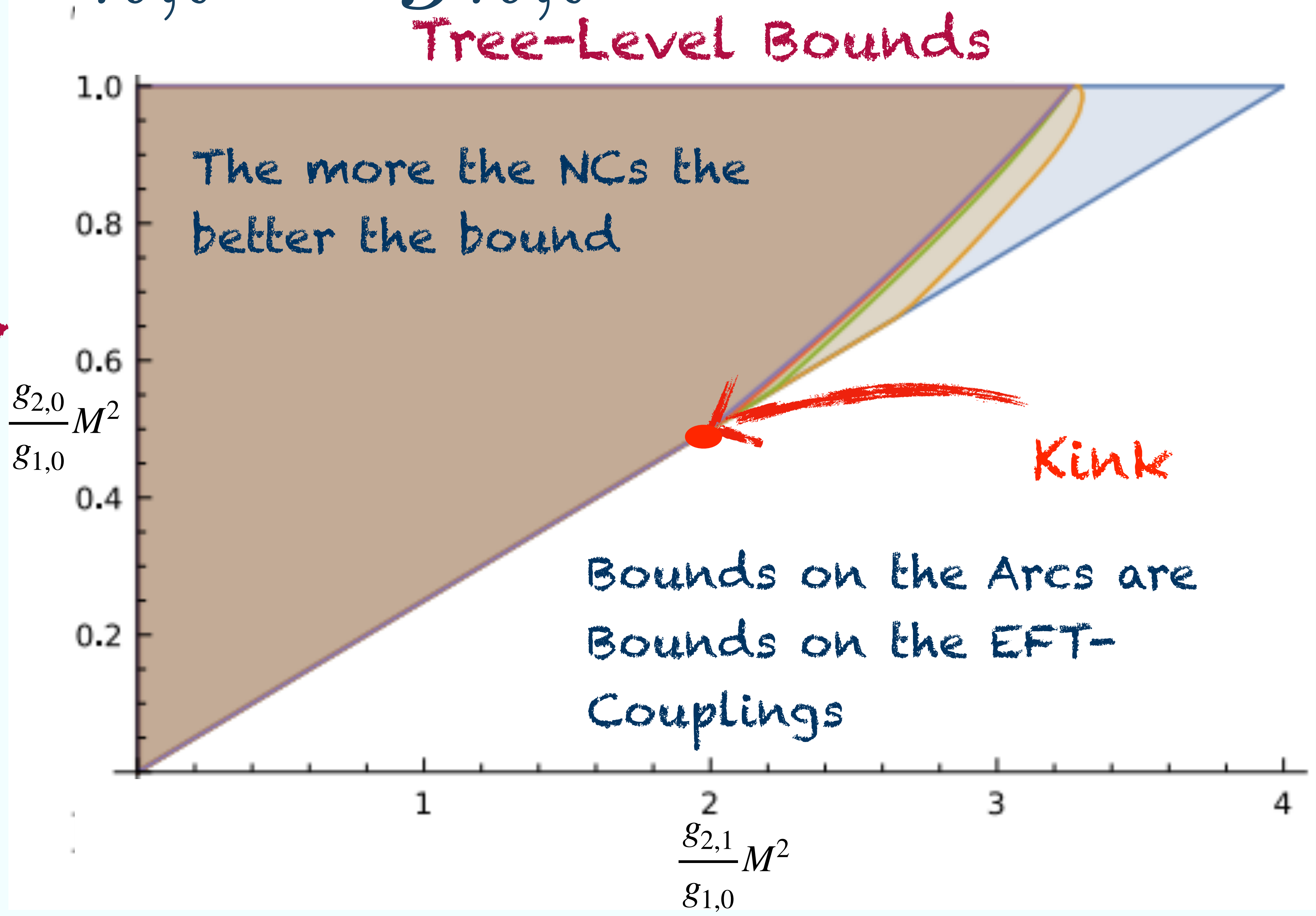
- Crossing implies Null Constraints (NC) on the Arcs E.G. : $a_{n,l} = a_{n-l,l}$

- Numerical Procedure to Bound the Arcs Caron-Huot, Van Duong - 2020

- Special Kink at LO Pomarol, Riva et al. - 2022

Bounds on the Lowest LECs=Arcs

$a_{n,l} = g_{n,l}$ Rastelli, et al. - 2022



Including Loop Corrections

- Including the First Loop Corrections: $O(f_\pi^{-4})$

$$\delta\mathcal{M}(s,t) = L\left(-s^2\log\left(\frac{-s}{M^2}\right) - (s+2t)\left[t\log\left(\frac{-t}{M^2}\right) + (s+t)\log\left(\frac{s+t}{M^2}\right)\right]\right)$$

$L = (4\pi f_\pi^2)^{-2}$, Intensity of Loop-Effects

Bellazzini, Riembau, Riva - 2021

Including Loop Corrections

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$L = (4\pi f_\pi^2)^{-2}$, Intensity of Loop-Effects

- At NLO: Arcs≠EFT-Coefficients

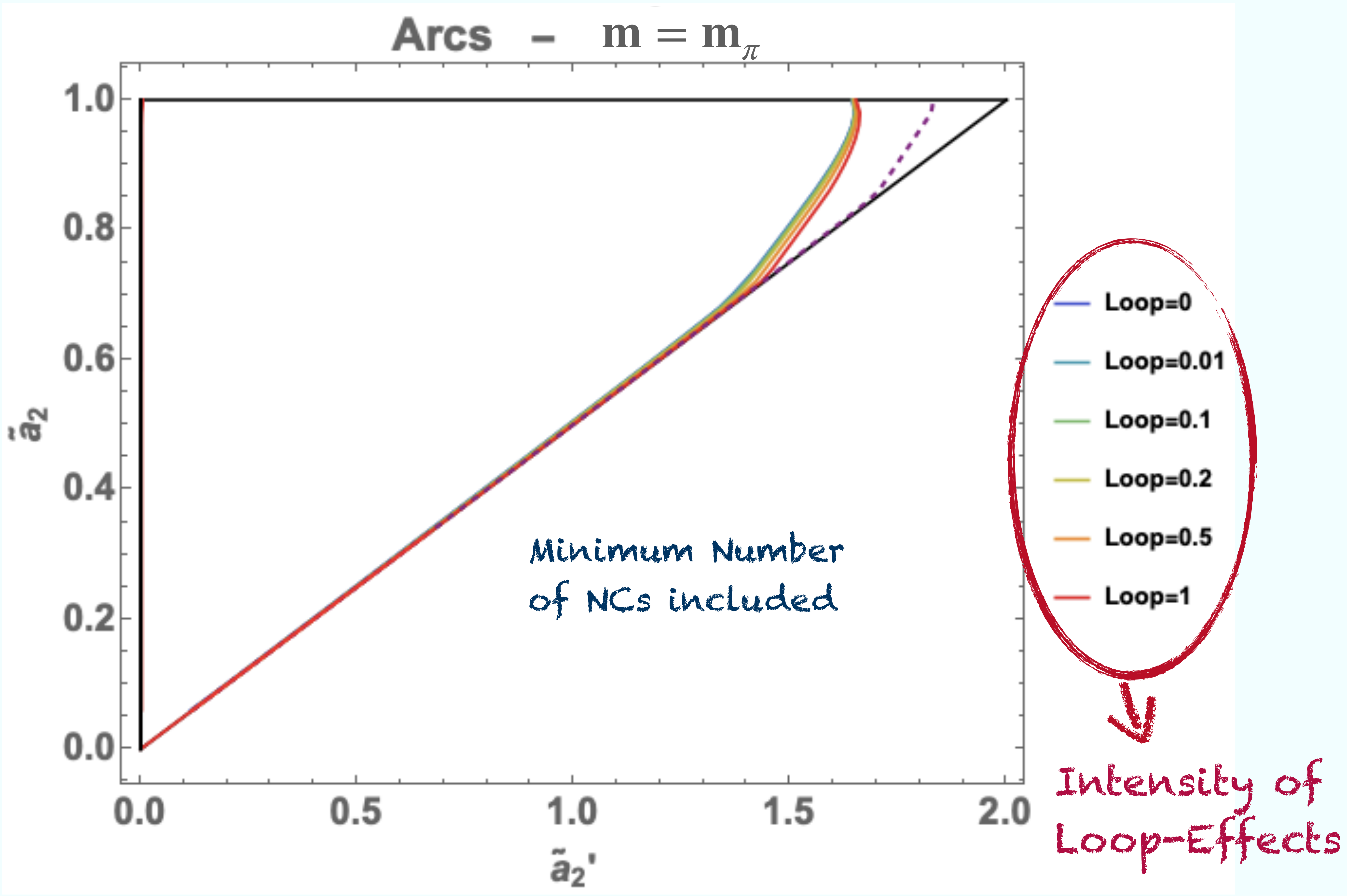
- IR-Singularities: Pion Mass to Cut-off

$$\delta a_1(s_0,t) = \frac{L}{24}\left(2t\log\left(-\frac{s_0}{4m^2+t}\right) - \frac{t^3}{4s_0^2} + \frac{t^2}{s_0} - 4s_0\right)$$

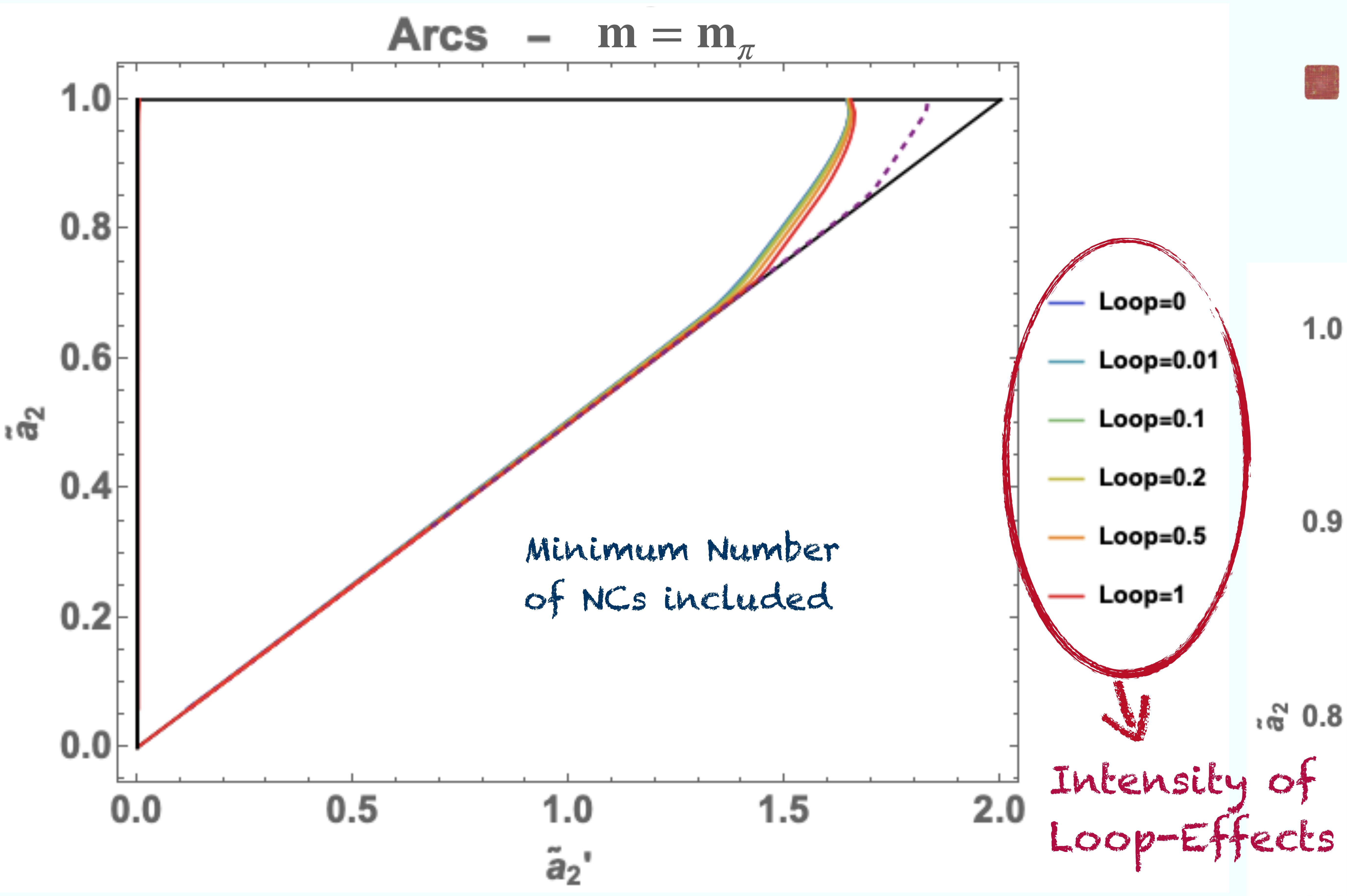
- Finite Corrections Relax the NCs
- Improvement of the Procedure including Loops
- Bounds on the EFT-Coefficients at NLO

Falkowski, Henning, DML, Riva,
Rodriguez-Sanchez

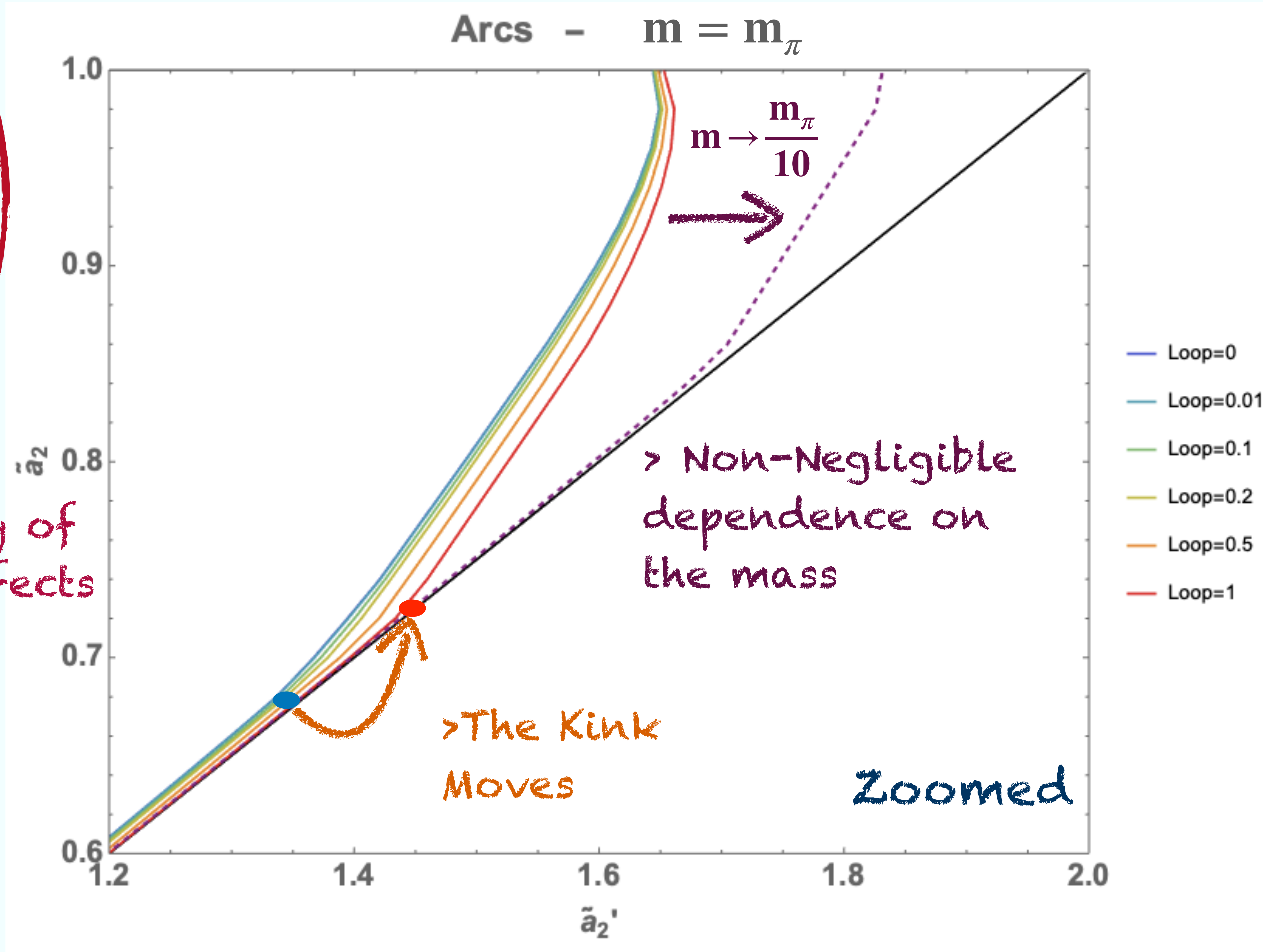
Arc Bounds at NLO



Arc Bounds at NLO



- Loops Weaken the Bounds

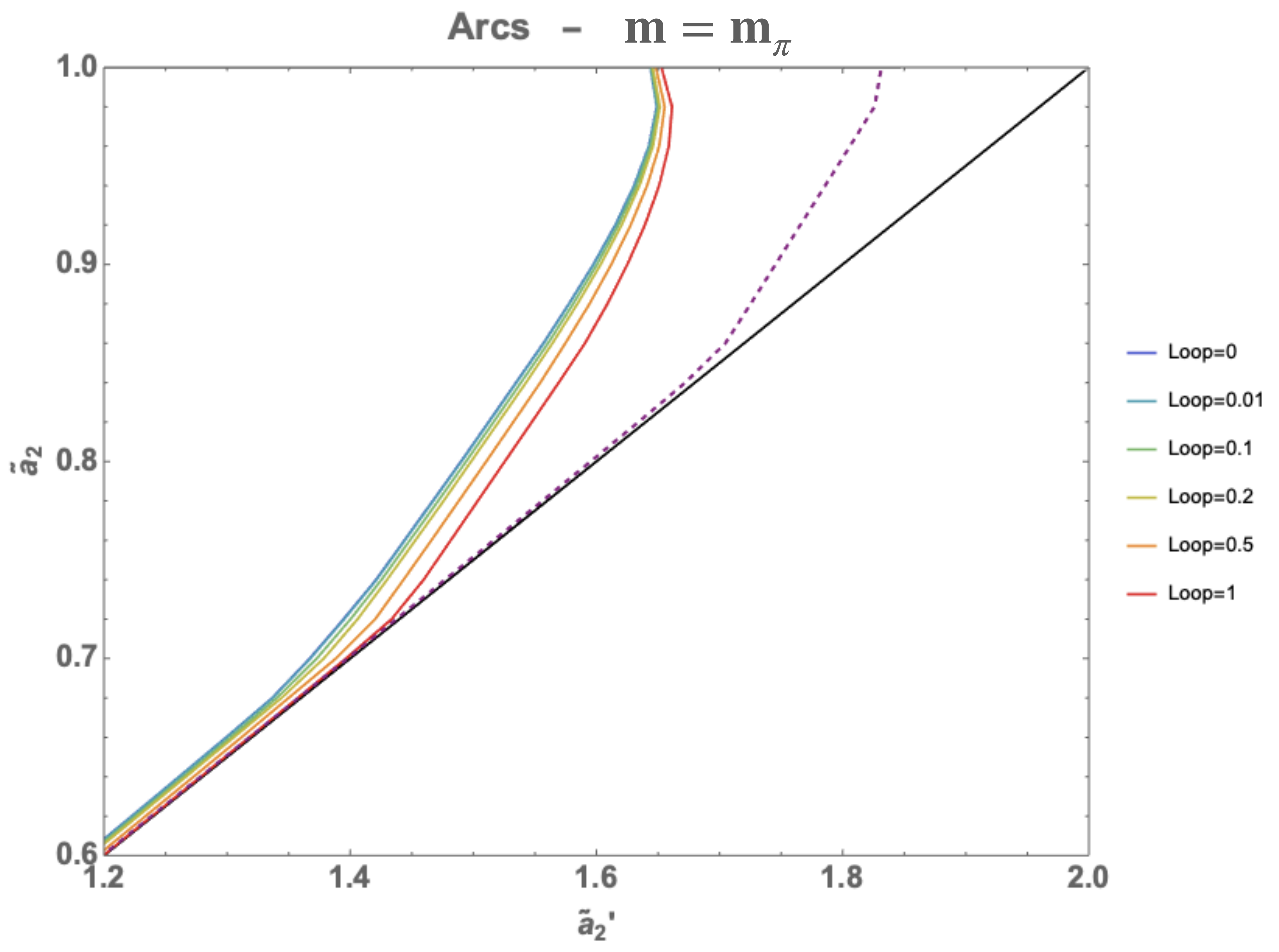


- The Kink is not Special at NLO
- IR-Singularities Spoil the Method in the massless case

Conclusions

- UV-Theory Bounds IR-EFTs
- Dispersion Relations for the Scattering Amplitudes
- Causality&Unitarity-Bounds on Pion Scattering
- Arcs' Approach to Study Loop Effects
  - Strong Coupling Corrections to the Bounds
- Current Improvements:
  - New method for $t \neq 0$
  - Arcs' computation for $m > 0$

$$a_k(s_0, t) \equiv (i\pi)^{-1} \oint_{C_{(s_0)}} \frac{\mathcal{M}(s', t)}{(s' + t/2)^{k+1}} ds'$$



Thank You!