Soft Stuff, Heavy Ions and Event Generators II

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Frontiers in Nuclear and Hadronic Physics
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Outline of Lectures

- Lecture I: Basics of Monte Carlo methods, the event generator strategy, matrix elements, LO/NLO, ...
- Lecture II: Parton showers, initial/final state, (matching/merging), hadronization, decays. ...
- Lecture III: Minimum bias, multi-parton interactions, pile-up, summary of general purpose event generators, ...
- Lecture IV: Protons vs. heavy ions, Glauber calculations, initial/final-state interactions, ...

Buckley et al. (MCnet collaboration), *Phys. Rep.* **504** (2011) 145.

Outline of Lecture II

- Final-State Showers
  - Angular Ordering
  - Evolution Variables
- Initial-State Showers
  - Parton densities
  - Backwards Evolution
- The Veto Algorithm
- Hadronization
  - Local Parton–Hadron Duality
  - Cluster Hadronization
  - String Hadronization
- Particle Decays
  - Standard Hadronic Decays

Tutorials/Slides

Tutorials:

```
% git clone https://gitlab.com/hepcedar/mcnet-schools/zakopane-2022.git
% git clone https://gitlab.com/Pythia8/tutorials.git
```

Slides

http://home.thep.lu.se/ leif/talks/GGI23-1.pdf

The purpose of parton showers is to generate real *exclusive* events on parton level down to a very low (almost non-perutbative) jet resolution scale μ.

Starting from an initial hard scattering eg. $\mathrm{e^+e^-} \to \mathrm{q\bar{q}}$ or $\mathrm{q\bar{q}} \to Z^0$, we basically need

$$
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\sigma_{+0} &= \sigma_0(1 + C_{01}\alpha_s + C_{02}\alpha_s^2 + C_{03}\alpha_s^3 + \ldots) \\
\sigma_{+1} &= \sigma_0(C_{11}\alpha_s + C_{12}\alpha_s^2 + C_{13}\alpha_s^3 + \ldots) \\
\sigma_{+2} &= \sigma_0(C_{22}\alpha_s^2 + C_{23}\alpha_s^3 + C_{24}\alpha_s^4 + \ldots) \\
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Tree-level generators only gives us inclusive events.

NLO generators only gives us one extra parton.

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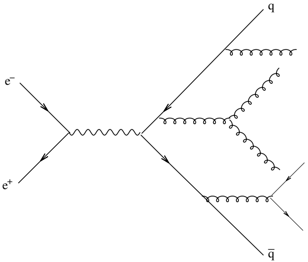
Final-State Showers

The tree-level matrix element for an *n*-parton state can be approximated by a product of splitting functions corresponding to a sequence of one-parton emissions from the zeroth order state.



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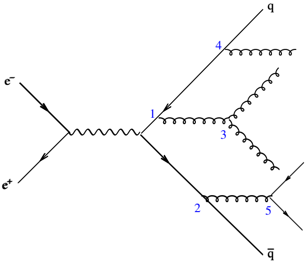

We can then order the emissions acording to some resolution scale, ρ, so that $\rho_1 \gg \rho_2 \gg \rho_3 \gg \ldots$

We have the standard DGLAP splitting kernels

$$\begin{aligned} P_{\mathrm{q}\to\mathrm{qg}}(\rho,z)d\rho dz &= \frac{\alpha_{\mathrm{s}}}{2\pi}\,dz\frac{d\rho}{\rho}\,C_F\,\frac{1+z^2}{1-z} \\ P_{\mathrm{g}\to\mathrm{gg}}(\rho,z)d\rho dz &= \frac{\alpha_{\mathrm{s}}}{2\pi}\,dz\frac{d\rho}{\rho}\,N_C\,\frac{(1-z(1-z))^2}{z(1-z)} \\ P_{\mathrm{g}\to\mathrm{q\bar{q}}}(\rho,z)d\rho dz &= \frac{\alpha_{\mathrm{s}}}{2\pi}\,dz\frac{d\rho}{\rho}\,T_R\,(z^2+(1-z)^2) \end{aligned}$$

where ρ is the squared invariant mass or transverse momentum, and z is the energy (or light-cone) fraction taken by one of the daugthers. (We ignore the ϕ-dependence here).

Where is the +-description: $1/(1-z)_+$?

We now to make the events exclusive. This is done by saying that the first emission at some ρ_1 is given by the splitting kernel multiplied by the probability that there has been no emission above that scale.

In a given interval $d\rho$ we have the no-emission probability

$$1 - d\rho \sum_{bc} \int dz\, P_{a\to bc}(z,\rho)$$

Integrating from ρ_1 up to some maximum scale, ρ_0 we get

$$\Delta(\rho_0,\rho_1) = \exp\left(-\sum_{bc}\int_{\rho_1}^{\rho_0} d\rho \int dz\, P_{a\to bc}(z,\rho)\right)$$

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(a.k.a. Sudakov form factor)

In the same way we get the probability to have the *n*th emission at some scale ρ_n

$$P(\rho_n) = \sum_{abc} \int dz\, P_{a\to bc}(\rho_n, z) \times \exp\left(-\sum_{abc} \int_{\rho_n}^{\rho_{n-1}} d\rho' \int dz'\, P_{a\to bc}(z', \rho') \right)$$

Integrating we get schematically

$$\sigma_{+0} = \sigma_0 \Delta_{S0} = \sigma_0(1 + C_{01}^{PS}\alpha_s + C_{02}^{PS}\alpha_s^2 + \ldots)$$
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We still need a cutoff, ρ_{cut}, and the coefficients C_{nn}^{PS} diverges as $\log^{2n} \rho_{\text{max}}/\rho_{\text{cut}}$

but the no-emission probabilities corresponds to the an approximate resummation of all virtual terms and makes things finite, and we can use $\rho_{\text{cut}} \sim 1$ GeV.

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The divergencies comes from the soft and collinear poles in the splitting kernels, eg.

$$\int_{\rho_c}^{\rho_0} d\rho \int dz\, P_{q\to qg}(\rho, z) \sim \int_{\rho_c}^{\rho_0} \frac{\alpha_s d\rho}{\rho} \ln(\rho_0/\rho) \sim \alpha_s \ln^2(\rho_0/\rho_c)$$

Parton showers systematically resums all orders of $\alpha_s^n \ln^{2n}(\rho_0/\rho_c)$ which is the main part of the higher order corrections.
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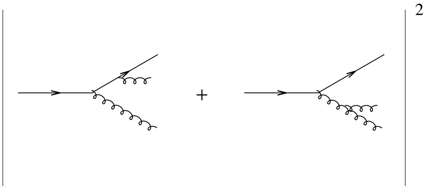
However if there is no strong ordering, $\rho_1 \gg \rho_2 \gg \rho_3 \gg \ldots$, the PS approximation breaks down

Parton showers cannot model several hard jets very well. Especially the correlations between hard jets are poorly described.

Angular Ordering

The splitting probabilities means that coherence effects are not taken into account



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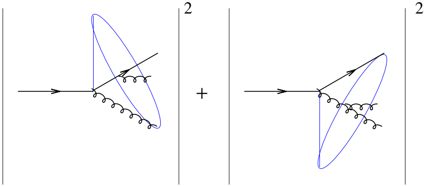

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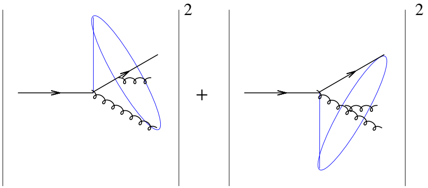

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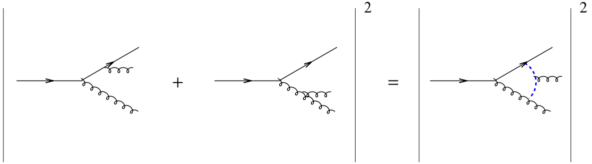
Some angular correlations can also be taken into account by adjusting the azimuthal angles after a shower is generated.

Coherence effects can be included directly, by considering gluon radiation from colour dipoles between colour-connected partons.



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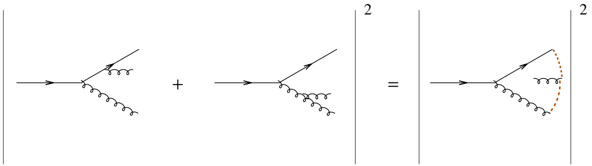


Rather than iterating 1 → 2 parton splitting we iterate 2 → 3 splittings. Each emission from a dipole will create two new dipoles, each of which may continue radiating.

This was first implemented in the ARIADNE generator. Recently similar schemes have been implemented in PYTHIA, HERWIG, SHERPA and VINCIA.

Evolution Variables

How do we choose the evolution variable, ρ?

The most natural choice is to choose a variable which isolates both the soft and collinear poles in the splitting kernel. This is the case for $\rho = p_\perp^2$ as used in eg. ARIADNE.

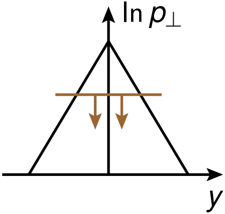
In old versions of PYTHIA and SHERPA the evolution variable is the virtuality Q^2 which in principle is fine except that $\alpha_s(p_\perp^2)$ may diverge for any given Q^2. Also angular ordering needs to be imposed in separately.

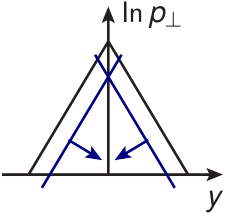
In HERWIG the ordering is in angle, which ensures angular ordering, but does not isolate the soft pole, and an additional cutoff is needed.

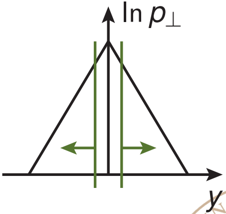
Transverse momentum
$\rho = p_\perp^2$

Virtuality
$\rho = Q^2 \sim \frac{p_\perp^2}{z(1-z)}$

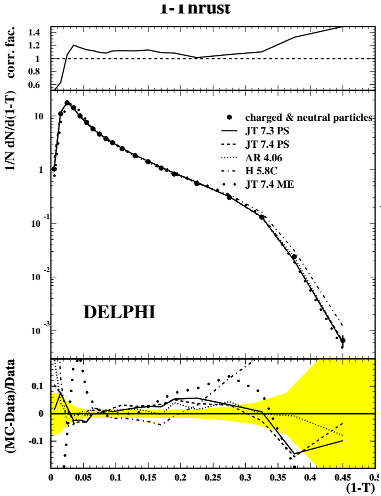
Angle
$\rho \sim E^2\theta^2 \sim \frac{p_\perp^2}{z^2(1-z)^2}$

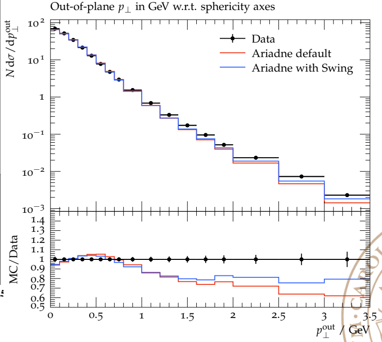






Final-state parton showers did really well at LEP



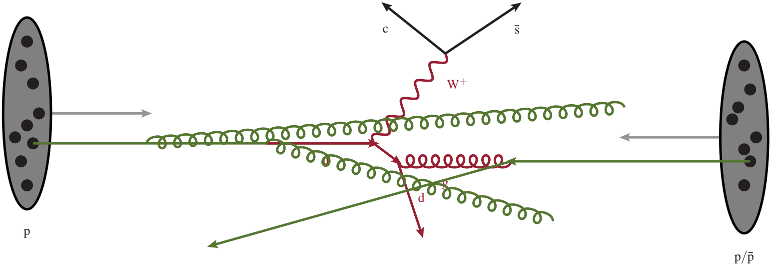


Initial-State Showers

For incoming hadrons, we need to consider the evolution of the parton densities. Using collinear factorization and DGLAP evolution we have (with $t = \log k_\perp^2/\Lambda^2$)

$$\frac{df_b(x,t)}{dt} = \sum_a \int \frac{dx'}{x'} f_a(x',t) \frac{\alpha_s}{2\pi} P_{a\to b}\left(\frac{x}{x'}\right)$$

We can interpret this as during a small increase dt there is a probability for parton a with momentum fraction x' to become resolved into parton b at $x = zx'$ and another parton c at $x' - x = (1-z)x'$.


c
s̄
W+
d
p
p/p̄

In a backward evolution scenario we start out with the hard sub-process at some scale $t_{\max}$

$$\sigma_0 \propto \hat{\sigma}_{ab\to X} f_a(x_a, t_{\max}) f_b(x_b, t_{\max})$$

and we get the relative probability for the parton a to be *unresolved* into parton c during a decrease in scale dt

$$d\mathcal{P}_a = \frac{df_a(x_a,t)}{f_a(x_a,t)} = |dt| \sum_c \int \frac{dx'}{x'} \frac{f_c(x',t)}{f_a(x_a,t)} \frac{\alpha_s}{2\pi} P_{c\to a}\left(\frac{x_a}{x'}\right)$$

Summing up the cumulative effect of many small changes dt, the probability for no radiation exponentiates and we get a no-emission probabilities

$$\Delta_{S_{+a}}(x_a, t_{\max}, t) = \exp\left\{ -\int_t^{t_{\max}} dt' \sum_c \int \frac{dx'}{x'} \frac{f_c(x',t')}{f_a(x_a,t')} \frac{\alpha_s(t')}{2\pi} P_{c\to a}\left(\frac{x_a}{x'}\right)\right\}$$

This now gives us the probability for the *first* backwards initial-state splitting

$$d\mathcal{P}_{ca} = \frac{\alpha_s}{2\pi} P_{ac}(z) \frac{f_c(x_a/z, t)}{f_a(x_a, t)} dt \frac{dz}{z} \times \Delta_{S_{+a}}(x_a, t_{\max}, t)$$

In a hadronic collision we first generate the hard scattering, then evolve the incoming partons backward to lower scales.

This is like undoing the evolution of the PDFs

Then alow for a final-state shower from all partons from the hard scattering and from the initial-state shower.

Questions!

How do we generate a parton shower emission?

$$\mathcal{P}(t) = P(t) \exp\left(-\int_t^{t_{\max}} dt' P(t')\right)$$

$\mathcal{P}(t)$ is a probability distribution, so we can do the standard transformation method

$$1 - r = \int_r^1 dt\, p_R(t) = \int_t^{t_{\max}} dt \mathcal{P}(t) = 1 - \exp\left(-\int_t^{t_{\max}} dt' P(t')\right)$$

So if P has a simple primitive function F we get

$$t = F^{-1}(F(t_{\max}) - \ln r)$$

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Assume g is a simple function with a simple primitive function G such that $g(t) \geq P(t), \forall t$. Then we can use the following algorithm

- start with $t_0 = t_{\max}$;
- select $t_i = G^{-1}(G(t_{i-1}) - \ln R)$,
- compare a (new) R with the ratio $P(t_i)/g(t_i)$; if $P(t_i)/g(t_i) \leq R$, then return to point 2 for a new try, $i \to i + 1$;
- otherwise t_i is retained as final answer.

If $t_i < t_{\rm cut}$, there is no emission and the shower is done.

Consider the various ways in which one can select a specific scale t. The probability that the first try works, $t = t_1$, i.e. that no intermediate t values need be rejected, is given by

$$p_0(t) = e^{-\int_t^{t_{\max}} g(t')\,dt'}\, g(t)\,\frac{P(t)}{g(t)} = P(t) e^{-\int_t^{t_{\max}} g(t')\,dt'}$$

The probability that we have thrown away one intermediate value t_1

$$p_1(t) = \int_t^{t_{\max}} dt_1\, e^{-\int_{t_1}^{t_{\max}} g(t')\,dt'} g(t_1) \left[1 - \frac{P(t_1)}{g(t_1)}\right] \times \times e^{-\int_t^{t_1} g(t')\,dt'} g(t)\,\frac{P(t)}{g(t)}$$

$$p_1(t) = p_0(t) \int_t^{t_{\max}} dt_1 \left[g(t_1) - P(t_1)\right]$$

Similarly we get

$$\begin{aligned}
p_2(t) &= p_0(t) \int_t^{t_{\max}} dt_1 \left[g(t_1) - P(t_1)\right] \int_t^{t_1} dt_2 \left[g(t_2) - P(t_2)\right] \\
&= p_0(t) \frac{1}{2} \left(\int_t^{t_{\max}} \left[g(t') - P(t')\right] dt' \right)^2
\end{aligned}$$

$$\begin{aligned}
p_{tot}(t) &= \sum_{n=0}^{\infty} p_n(t) = p_0(t) \sum_{n=0}^{\infty} \frac{1}{n!} \left(\int_t^{t_{\max}} \left[g(t') - P(t')\right] dt' \right)^n \\
&= P(t) e^{-\int_t^{t_{\max}} g(t')\, dt'} e^{\int_t^{t_{\max}} \left[g(t') - P(t')\right] dt'} \\
&= P(t) e^{-\int_t^{t_{\max}} P(t')\, dt'}
\end{aligned}$$

Also if several things may happen, $P_1(t)$, $P_2(t)$, $P_3(t)$, ... the probability of i happening first is

$$P_i(t) \times \prod_j e^{-\int_t^{t_{\max}} P_j(t')\,dt'}$$

Simply generate a scale for each i according to

$$P_i(t) \times e^{-\int_t^{t_{\max}} P_i(t')\,dt'}$$

and pick the process with the largest scale.

Hadronization

Now that we are able to generate partons, both hard, soft, collinear and from multiple scatterings, we need to convert them to hadrons.

This is a non-perturbative process, and all we can do is to construct models, and try to include as much as possible of what we know about non-perturbative QCD.

Local Parton–Hadron Duality

An analytic approach ignoring non-perturbative difficulties.

Run shower down to scales $\sim \Lambda_{\mathrm{QCD}} \sim m_\pi$.

Each parton corresponds to one (or 1.something) hadron.

Can describe eg. momentum spectra surprisingly well.

Can be used to calculate power corrections to NLO predictions for event shapes,

- $\langle 1 - T \rangle = c_1 \alpha_{\mathrm{s}}(E_{\mathrm{cm}}) + c_2 \alpha_{\mathrm{s}}^2(E_{\mathrm{cm}}) + c_p / E_{\mathrm{cm}}$

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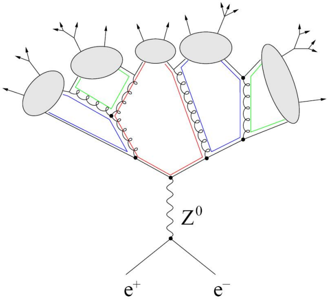
Cannot generate real events with this though.

Cluster Hadronization

Close to local parton–hadron duality in spirit. Based on the idea of Preconfinement:

The pattern of perturbative gluon radiation is such that gluons are emitted mainly between colour-connected partons. If we emit enough gluons the colour-dipoles will be small.



After the shower, force $g \to q\bar{q}$ splittings giving low-mass, colour-singlet clusters

Decay clusters isotropically into two hadrons according to phase space weight $\sim (2s_1 + 1)(s_2 + 1)(2p/m)$

Cluster hadronization is very simple and clean.
Maybe too simple...

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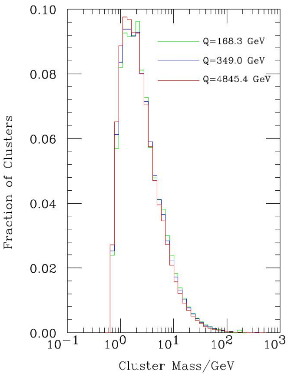
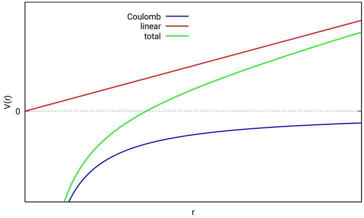


- Cluster masses can be large (finite probability for no gluon emission): Introduce *string-like* decays of heavy clusters into lighter ones (with special treatment of proton remnant).
- In clusters including a heavy quark (or a di-quark) the heavy meson (or baryon) should go in this direction: introduce anisotropic cluster decays.
- …

String Hadronization

What do we know about non-perturbative QCD?



- At small distances we have a Coulomb-like asymptotically free theory
- At larger distances we have a linear confining potential

For large distances, the field lines are compressed to vortex lines like the magnetic field in a superconductor

1+1-dimensional object $\sim$ a massless relativistic string

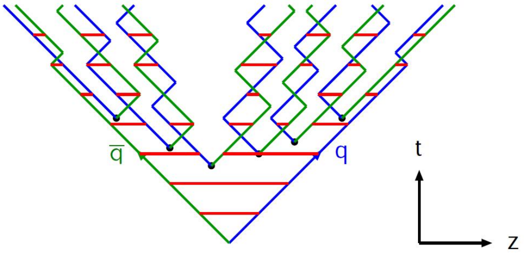
As a $\mathrm{q}\bar{\mathrm{q}}$-pair moves apart, they are slowed down and more and more energy is stored in the string.

If the energy is small, the $\mathrm{q}\bar{\mathrm{q}}$-pair will eventually stop and move together again. We get a “YoYo”-state which we interpret as a meson.

If high enough energy, the string will break as the energy in the string is large enough to create a new $\mathrm{q}\bar{\mathrm{q}}$-pair.

The energy in the string is given by the string tension

$$\kappa = \left|\frac{dE}{dz}\right| = \left|\frac{dE}{dt}\right| = \left|\frac{dp_z}{dz}\right| = \left|\frac{dp_z}{dt}\right| \sim 1\text{GeV/fm}$$



The quarks obtain a mass and a transverse momentum in the breakup through a tunneling mechanism

$$\mathcal{P} \propto e^{-\frac{\pi m_{q\perp}^2}{\kappa}} = e^{-\frac{\pi m_q^2}{\kappa}} e^{-\frac{\pi p_\perp^2}{\kappa}}$$

Gives a natural supression of heavy quarks
$\mathrm{d\bar{d}} : \mathrm{u\bar{u}} : \mathrm{s\bar{s}} : \mathrm{c\bar{c}} \sim 1 : 1 : 0.3 : 10^{-11}$

The break-ups starts in the middle and spreads outward, but they are causually disconnected. So we should be able to start anywhere.

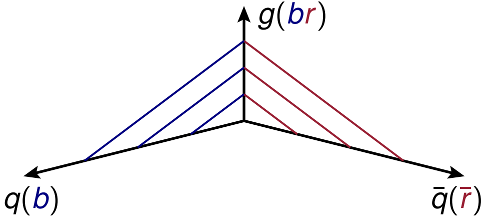
In particular we could start from either end and go inwards.

Requiring left-right symmetry we obtain a unique *fragmentation function* for a hadron taking a fraction z of the energy of a string end in a breakup

$$p(z) \propto \frac{(1-z)^a}{z} e^{-bm_\perp^2/z}$$

The Lund symmetric fragmentation function.

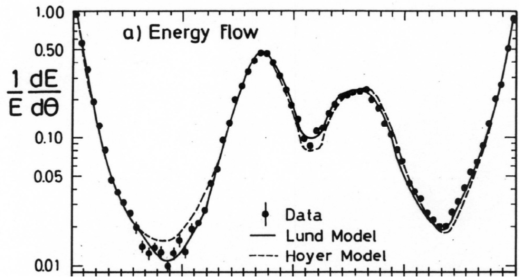
Gluons complicates the picture somewhat. They can be interpreted as a “kinks” on the string carrying energy and momentum



The gluon carries twice the charge ($N_C/C_F \to 2$ for $N_C \to \infty$)

A bit tricky to go around the gluon corners, but we get a consistent picture of the energy–momentum structure of an event with no extra parameters.

The Lund string model predicted the string effect measured by Jade.



In a three-jet event there are more energy between the $\mathrm{g}-\mathrm{q}$ and $\mathrm{g}-\bar{\mathrm{q}}$ jets than between $\mathrm{q}-\bar{\mathrm{q}}$.

For the flavour structure the picture becomes somewhat messy.

Baryons can be produced by having $\mathrm{qq}-\bar{\mathrm{q}}\bar{\mathrm{q}}$-breakups (diquarks behaves like an anti-colour), but more complicated mechanisms (“popcorn”) needed to describe baryon correlations.

We also need special suppression of strange mesons, baryons. Parameters for different spin states, . . .

There are *lots* of parameters i PYTHIA.

Strings vs. Clusters

Model	string (PYTHIA)	cluster (HERWIG)
energy–momentum picture	powerful, predictive few parameters	simple, (unpredictive) more parameters
flavour composition	messy, unpredictive many parameters	simple, reasonably predictive few parameters

There will always be parameters. . .

Most hadronization parameters have been severely constrained by LEP data. Does this mean we can use the models directly at LHC?

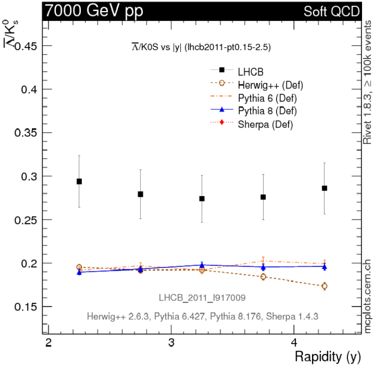
Jet universality

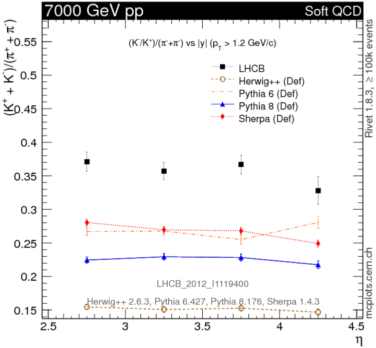
There may be problems with flavour and meson/baryon issues.

Also at LEP there were mainly quark jets, gluon jets are softer and not very well measured.

At LHC there will be very hard gluon jets.

We need to check that jet universality works.


7000 GeV pp
Soft QCD
$\bar{\Lambda}$/K0S vs |y| (lhcb2011-pt0.15-2.5)
LHCB
Herwig++ (Def)
Pythia 6 (Def)
Pythia 8 (Def)
Sherpa (Def)
$\bar{\Lambda}/K^0_s$
Rapidity (y)
LHCB_2011_I917009
Herwig++ 2.6.3, Pythia 6.427, Pythia 8.176, Sherpa 1.4.3
Rivet 1.8.3, ≥ 100k events
mcplots.cern.ch


7000 GeV pp
Soft QCD
$(K^-/K^+)/(\pi^-+\pi^-)$ vs |y| (p_T > 1.2 GeV/c)
LHCB
Herwig++ (Def)
Pythia 6 (Def)
Pythia 8 (Def)
Sherpa (Def)
$(K^+ + K^-)/(\pi^+ + \pi^-)$
η
LHCB_2012_I1119400
Herwig++ 2.6.3, Pythia 6.427, Pythia 8.176, Sherpa 1.4.3
Rivet 1.8.3, ≥ 100k events
mcplots.cern.ch

The PDG decay tables

The Particle Data Group has machine-readable tables of decay modes.

But they are not complete and cannot be used directly in an event generator.

- Branching ratios need to add up to unity.
- Some decays are listed as $B^{\star 0} \to \mu^+ \nu_\mu X$.
- ...

Most decays need to be coded by hand

Particle Decays

Not the most sexy part of the event generators, but still essential.

$$B^{\star 0} \to \gamma \; B^0$$
$$\hookrightarrow \overline{B}^0 \to e^- \bar{\nu}_e \; D^{\star +}$$
$$\hookrightarrow \pi^+ \; D^0$$
$$\hookrightarrow K^- \; \rho^+$$
$$\hookrightarrow \pi^+ \; \pi^0$$
$$\hookrightarrow e^+ e^- \gamma$$

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$\hookrightarrow \pi^+ \;\; D^0$

$\hookrightarrow K^- \;\; \rho^+$

$\hookrightarrow \pi^+ \;\; \pi^0$

$\hookrightarrow e^+ e^- \gamma$

EM decays

Particle Decays

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but still essential.

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$$\hookrightarrow \overline{B}^0 \to e^- \bar{\nu}_e \; D^{\star +}$$
$$\hookrightarrow \pi^+ \; D^0$$
$$\hookrightarrow K^- \; \rho^+$$
$$\hookrightarrow \pi^+ \; \pi^0$$
$$\hookrightarrow e^+ e^- \gamma$$

Weak mixing

Particle Decays

Not the most sexy part of the event generators,
but still essential.

$$B^{\star 0} \to \gamma \; B^0$$
$$\hookrightarrow \overline{B}^0 \to e^- \bar{\nu}_e \; D^{\star +}$$
$$\hookrightarrow \pi^+ \; D^0$$
$$\hookrightarrow K^- \; \rho^+$$
$$\hookrightarrow \pi^+ \; \pi^0$$
$$\hookrightarrow e^+ e^- \gamma$$

Weak decay, displaced vertex, $|\mathcal{M}|^2 \propto (p_{\bar{B}} p_{\bar{\nu}})(p_e p_{D^\star})$

Particle Decays

Not the most sexy part of the event generators, but still essential.

$$B^{\star 0} \to \gamma \; B^0$$
$$\hookrightarrow \overline{B}^0 \to e^- \bar{\nu}_e \; D^{\star +}$$
$$\hookrightarrow \pi^+ \; D^0$$
$$\hookrightarrow K^- \; \rho^+$$
$$\hookrightarrow \pi^+ \; \pi^0$$
$$\hookrightarrow e^+ e^- \gamma$$

Strong decay

Particle Decays

Not the most sexy part of the event generators,
but still essential.

$$B^{\star 0} \to \gamma \; B^0$$
$$\hookrightarrow \overline{B}^0 \to e^- \bar{\nu}_e \; D^{\star +}$$
$$\hookrightarrow \pi^+ \; D^0$$
$$\hookrightarrow K^- \; \rho^+$$
$$\hookrightarrow \pi^+ \; \pi^0$$
$$\hookrightarrow e^+ e^- \gamma$$

Weak decay, displaced vertex, ρ mass smeared

Particle Decays

Not the most sexy part of the event generators, but still essential.

$$B^{\star 0} \to \gamma \;\; B^0$$
$$\hookrightarrow \overline{B}^0 \to e^- \bar{\nu}_e \;\; D^{\star +}$$
$$\hookrightarrow \pi^+ \;\; D^0$$
$$\hookrightarrow K^- \;\; \rho^+$$
$$\hookrightarrow \pi^+ \;\; \pi^0$$
$$\hookrightarrow e^+ e^- \gamma$$

ρ polarized, $|\mathcal{M}|^2 \propto \cos^2\theta$ in ρ rest frame

Particle Decays

Not the most sexy part of the event generators,
but still essential.

$$
\begin{aligned}
&B^{\star 0} \rightarrow \gamma \;\; B^0 \\
&\qquad \hookrightarrow \overline{B}^0 \rightarrow e^- \bar{\nu}_e \;\; D^{\star +} \\
&\qquad\qquad \hookrightarrow \pi^+ \;\; D^0 \\
&\qquad\qquad\qquad \hookrightarrow K^- \;\; \rho^+ \\
&\qquad\qquad\qquad\qquad \hookrightarrow \pi^+ \;\; \pi^0 \\
&\qquad\qquad\qquad\qquad\qquad \hookrightarrow e^+ e^- \gamma
\end{aligned}
$$

Dalitz decay, $m_{e^+e^-}$ peaked

Summary II

Parton showers corresponds to an approximate all order resummation of the perturbative series in α_S. The correctness of the resummation is quantified by the number of powers of logarithms correctly reproduced for each order in α_S.

Most parton shower programs are correct to (N)LL.

The interplay between matrix elements and partons showers, is the key to understand and improve the formal precision of event generators.

Questions!

Outline of Lectures

- Lecture I: Basics of Monte Carlo methods, the event generator strategy, matrix elements, LO/NLO, . . .
- Lecture II: Parton showers, initial/final state, (matching/merging), hadronization, decays. . . .
- Lecture III: Minimum bias, multi-parton interactions, pile-up, summary of general purpose event generators, . . .
- Lecture IV: Protons vs. heavy ions, Glauber calculations, initial/final-state interactions, . . .

Buckley et al. (MCnet collaboration), *Phys. Rep.* **504** (2011) 145.