The Correspondence Between Rotating Black Holes and Fundamental Strings

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Based on ongoing work with R. Emparan, A. Puhm, and M.Tomašević

New horizons for horizonless physics: from gauge to gravity and back again

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Motivation

- Black holes have large entropy

$$S_{BH} = \frac{A_H}{4\, G_D\, \hbar} \gg 1.$$

- Is there a statistical interpretation?
- String theory offers some insight into the microscopic picture:
  - Explicit constructions of microstates.
  - Counting arguments at weak string coupling.

Black holes in String Theory

- String coupling g controls the strength of gravitational interactions

$$G_D \sim l_{\mathrm{P}}^{D-2} \sim g^2 \, l_s^{D-2}$$

Black hole entropy
S_{BH}

Δg

Microscopic Entropy
S_{Micro}

$g \gg 1$ $\qquad$ $g \to 0$

If Δg adiabatic
$S_{\mathrm{BH}} \sim S_{\mathrm{Micro}}$

- But in general the properties are vastly different, for example in D=4

$$S_{\mathrm{BH}} \propto M^2 \,, \qquad S_{\mathrm{micro}} \propto M \,.$$

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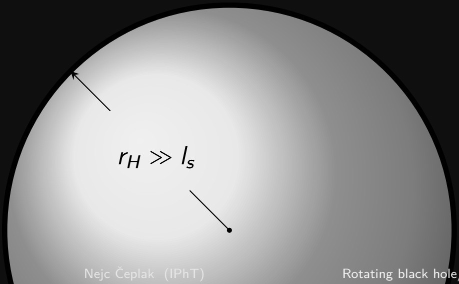
Starring

Schwarzschild Black Hole in D-dimensions with mass M

- $S_{\rm BH} \sim \left(\frac{M}{M_P}\right)^{\frac{D-2}{D-3}} \sim g^{\frac{2}{D-3}} \left(\frac{M}{M_s}\right)^{\frac{D-2}{D-3}}$,
- $r_H \sim \left(\frac{M}{M_P}\right)^{\frac{1}{D-3}} l_P \sim \left[g_c^2 \frac{M}{M_s}\right]^{\frac{1}{D-3}} l_s$
- $T_{\rm Haw} \sim \frac{1}{r_H}$



Highly excited fundamental string in D-dimensions with mass $M \sim \sqrt{N}\, M_s$

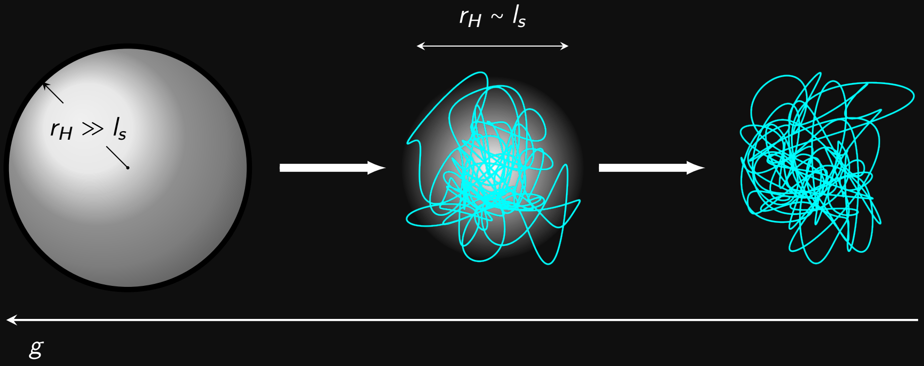
- $S_{\rm Micro} \sim \frac{M}{M_s} \sim \sqrt{N}$,
- $\langle r \rangle \sim \sqrt{\frac{M}{M_s}}\, l_s$,

$$g^2 \sim \left(\frac{l_P}{l_s}\right)^{D-2} \sim \left(\frac{M_s}{M_P}\right)^{D-2}$$

The correspondence

[Susskind, Horowitz+Polchinski, Damour+Veneziano. . .]

- Keep the entropy S fixed and change the string coupling g.
- The black hole and free string descriptions change when the curvature at the horizon of the black hole becomes of the string scale.
- For Schwarzschild Black holes the correspondence point is when



[Susskind]

- Fix the entropy of a large black hole

$$S_{\rm BH} \sim \left(\frac{M}{M_P}\right)^{\frac{D-2}{D-3}} \sim g^{\frac{2}{D-3}} \left(\frac{M}{M_s}\right)^{\frac{D-2}{D-3}}$$

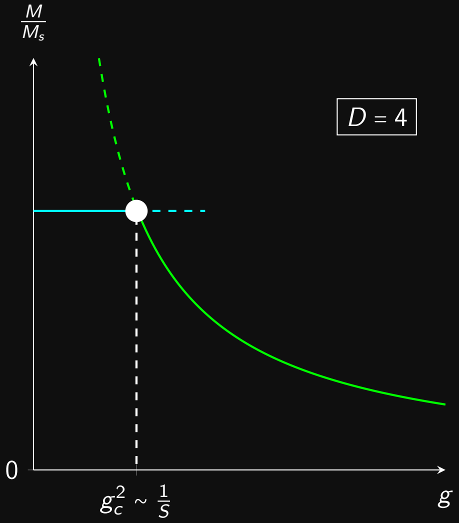
- At which value of string coupling is $r_H \sim l_s$

$$r_H \sim \left[g_c^2 \frac{M}{M_s}\right]^{\frac{1}{D-3}} l_s ,$$

$$\boxed{g_c^2 \sim \frac{M_s}{M} \sim \frac{1}{S_{\rm BH}} \ll 1}$$

- At $g = g_c$

$$\boxed{S_{\rm BH} \sim \frac{M}{M_s} \sim S_{\rm Micro}}$$



Other properties

- Hawking temperature increases to the Hagedorn scale

$$T_{\text{Haw}} \sim \frac{1}{r_H}\Big|_{g=g_c} \sim \frac{1}{l_s} \sim T_{\text{Hag}}$$

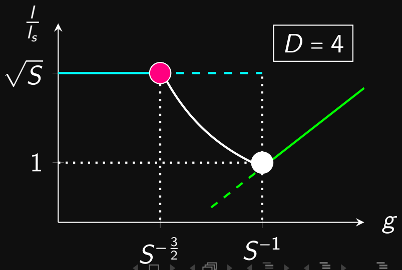
- Sizes do not match – string is much larger

$$r_H \sim l_s \quad \nleftrightarrow \quad \langle r \rangle \sim \sqrt{\frac{M}{M_s}}\, l_s .$$

- One needs to include the effects of self-interaction.

[Horowitz+Polchniski, Damour+Veneziano, Chen+Maldacena+Witten, Brustein et al., . . .]

- Modelled using a winding condensate near the Hagedorn temperature.
- Strong dependence on the dimension D.
- Upshot: Self-interactions interpolate between black hole and free string sizes

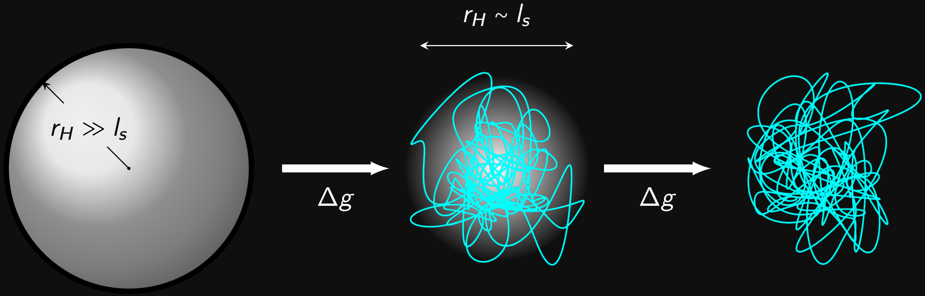


Intermezzo – Dynamical evaporation

- Hawking evaporation: Fix g at a large value and decrease M and S.
- When $r_H \sim l_s$

$$M \sim \frac{1}{g^2} M_s\,, \qquad S_{\rm BH} \sim \frac{1}{g^{\frac{2}{D-3}}}\,, \qquad T_{\rm Haw} \sim \frac{1}{l_s} \sim T_{\rm Hag}\,,$$

you can think of the black hole becoming a hot soup of weakly interacting strings ⇒ Possible endpoint of BH evaporation



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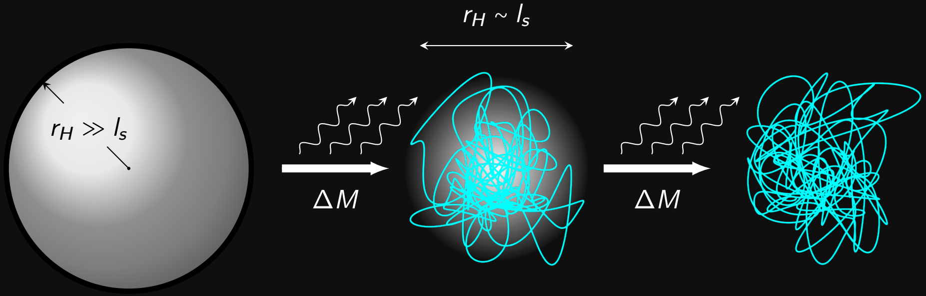

Correspondence Priniciple [Susskind, Horowitz+Polchinski, Damour+Veneziano. . .]

- A proposal on how to relate black hole and free string regimes.
- Allows for a parametric match between black hole and string states

$$S_{\mathrm{BH}} \sim S_{\mathrm{Micro}} \,.$$

Successes

- Works for a black holes with a wide variety of charges.
- Provides a microscopic understanding of the black hole entropy (at $g = 0$).
- Can be seen as a model for the endpoint of black hole evaporation.

Limitations

- In general does not capture numerical $\mathcal{O}(1)$ factors.
  - In supersymmetric configurations these can be reproduced.

[Strominger+Vafa, Sen, . . .]

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Adding Angular momentum

- Fixing the angular momentum should not modify the black hole/fundamental string correspondence too much.

  ⇒ Neutral rotating black objects should be related to rotating stringy objects.

- Kerr bound and Regge bound are qualitatively similar

$$J_{\mathrm{Kerr}} = \frac{M^2}{M_P^2}\,, \qquad J_{\mathrm{Regge}} = \frac{M^2}{M_s^2}\,,$$

but ultimately expressed in different units.

Difficulties

- At weak string coupling $g \ll 1$

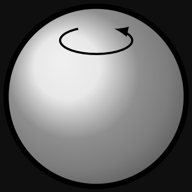
$$J_{\rm Kerr} \sim g^2 \frac{M^2}{M_s^2} \ll \frac{M^2}{M_s^2} = J_{\rm Regge}\,.$$

- There exist stringy objects which have violate the Kerr bound.
- The two bounds are saturated by completely different objects:

$J = J_{\rm Kerr}$:

Extremal Kerr solution

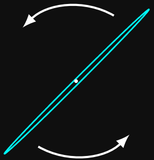
- Large entropy $S_{\rm BH} \propto M^2$
- Still spherical



$J = J_{\rm Regge}$:

Rigid Rods

- Non-degenerate
- Highly non-spherical



Even more difficulties

Black Hole side

- Kerr Bound only for $D = 4$.
- Increasing the number of dimensions add to complexity:
  - Kerr bound get replaced by stability bounds.
  - More allowed angular momenta $\lfloor \frac{D-1}{2} \rfloor$
    $\Rightarrow$ More complicated black objects (black rings, ...)

[Myers+Perry, Emparan+Reall, ...]

- Do all such objects have a corresponding string counterpart?

String side

- Regge bound is independent of number of dimensions.
- In $D > 4$ more planes of rotation allow for stringy objects stabilised by angular momentum (plasmid strings)

Stringy objects ($g = 0$)

- Highly excited strings behave like random walks with $M \sim \sqrt{N}\, M_s$ steps

[Mitchel+Turok, ...]

$$\langle (\Delta r)^2 \rangle \sim \frac{M}{M_s}\, l_s^2 , \qquad S \sim \frac{M}{M_s} \sim \sqrt{N} ,$$

- Strings with no rotation are isotropic in all directions: string balls.

- Slow rotating strings ($J < \sqrt{N}$): Corrections quadratic in J but heavily suppressed

$$\langle (\Delta r)^2 \rangle \sim \left[\sqrt{N} \pm \mathcal{O}\left(\frac{J^2}{N} \right) \right] l_s^2 , \qquad S \sim \sqrt{N} - \frac{J^2}{N} ,$$

Slowly rotating strings essentially stay string balls.

- Increasing rotation lowers the entropy [Russo+Susskind]

$$S \sim \sqrt{N - J}\,,$$

and pancakes the string

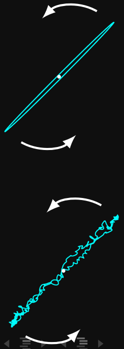
$$\left\langle (\Delta r)^2 \right\rangle_{\perp} \sim \sqrt{N - J}\,, \qquad \left\langle (\Delta r)^2 \right\rangle_{\parallel} \sim \sqrt{N}$$

Pancakes

Plasmids

[Blanco-Pillado+Emparan+Iglesias]

Bars



Black objects in $D = 4$

- Kerr black hole
- Radius of the outer horizon

$$r_+ = \frac{M}{M_P}\left[1 + \sqrt{1 - \frac{M_P^4}{M^4}\, J^2}\,\right] l_P \,,$$

- Entropy

$$S_{\mathrm{BH}} = \frac{M^2}{M_P^2}\left[1 + \sqrt{1 - \frac{M_P^4}{M^4}\, J^2}\,\right],$$

- Temperature

$$T_{\mathrm{Haw}} \sim \frac{\sqrt{1 - \frac{M_P^4}{M^4}\, J^2}}{r_+}$$

- Kerr Bound

$$J_{\mathrm{Kerr}} \leq \frac{M^2}{M_P^2}\,.$$

$D \geq 5$

- I will focus on black objects with only one plane of rotation.
- Simplest solutions are Myers-Perry black holes [Myers+Perry]

$$ds^2 = -dt^2 + \frac{\mu}{r^{D-5}\,\Sigma}\left(dt - a\sin^2\theta\, d\phi\right)^2 + \frac{\Sigma}{\Delta}\, dr^2 + \Sigma\, d\theta^2 + \left(r^2 + a^2\right)\sin^2\theta\, d\phi^2$$
$$+ r^2\cos^2\theta\, d\Omega_{D-4}^2\,,$$

where

$$\Sigma = r^2 + a^2\cos^2\theta\,, \qquad \Delta = r^2 + a^2 - \frac{\mu}{r^{D-5}}\,,$$

and

$$\mu = \frac{16\,\pi\, G}{(D-2)\Omega_{D-2}}\, M\,, \qquad a = \frac{D-2}{2}\,\frac{J}{M}\,.$$

Myers-Perry Black holes

- The event horizon is determined by

$$r_0^2 + a^2 - \frac{\mu}{r_0^{D-5}} = 0\,,$$

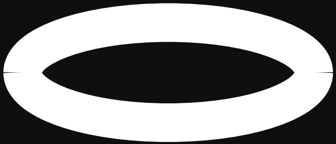
- The entropy is proportional to

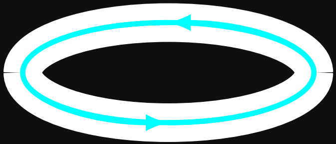
$$S_{\mathrm{MP}} \sim r_0^{D-4}\,(r_0^2 + a^2)\,.$$

Black rings ($D \geq 5$) [Emparan+Reall]

- Gravitational attraction is balanced out by angular momentum.
- Can also have arbitrary large angular momentum.

- We will consider two cases:
  - Neutral black rings
  - Dipole black rings (additional fundamental string dipole charge) [Emparan]





Stability

[Emparan+Harmark+Niarchos+Obers]

- Which of these objects are stable?
- Mass and angular momentum length scales

$$L_M \equiv \left(\frac{M}{M_P}\right)^{\frac{1}{D-3}} l_P\,, \qquad L_J \equiv \frac{J}{M}\,,$$

- Stability bounds replace the Kerr bound in $D \geq 5$

$$L_J \lesssim L_M\,, \qquad \Longleftrightarrow \qquad \boxed{J \lesssim S}$$

- Spheroidal objects (Kerr, gently spinning MP) are stable
- Ultraspinnging objects ($J > S$) are unstable: They can fragment.
- Exception: Dipole rings can be stable even if highly spinning.

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General principle

- Adiabatic change of string coupling g, keep S and J fixed.
- Follow strings states as g increases.
- Follow black objects as g decreases.
- Identify which string objects get mapped to which object at strong coupling.
- We have to consider the relevant timescale for stability: At finite g both black holes and strings radiate.
- Radiation sets the scale of stability.

Correspondence in $D = 4$

- Only the Kerr black hole at $g \gg 1$.
- Rotating sting balls, pancakes and rods on the string side.

- Because of the Kerr bound

$$J_{\mathrm{Kerr}} \leq \frac{M^2}{M_P^2},$$

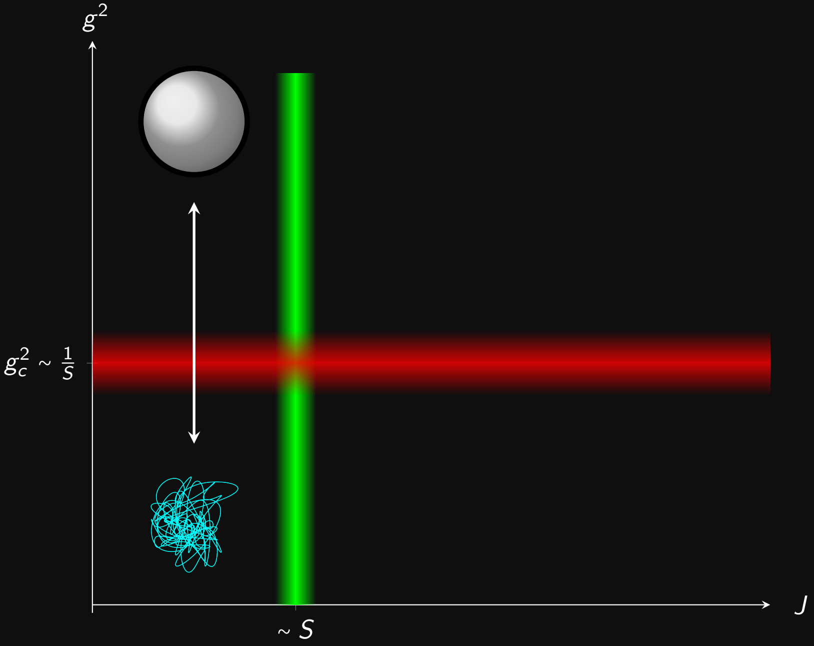
angular momentum effects give an $\mathcal{O}(1)$ correction

$$r_+ = \frac{M}{M_P}\left[1 + \sqrt{1 - \frac{M_P^4}{M^4} J^2}\right] l_P , \qquad S_{\mathrm{BH}} = \frac{M^2}{M_P^2}\left[1 + \sqrt{1 - \frac{M_P^4}{M^4} J^2}\right],$$

- To leading order we get the same results as in the non-rotating case:

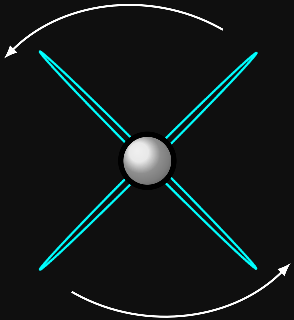
$$g_c^2 \sim \frac{1}{S}, \qquad \text{and} \qquad J \lesssim S$$

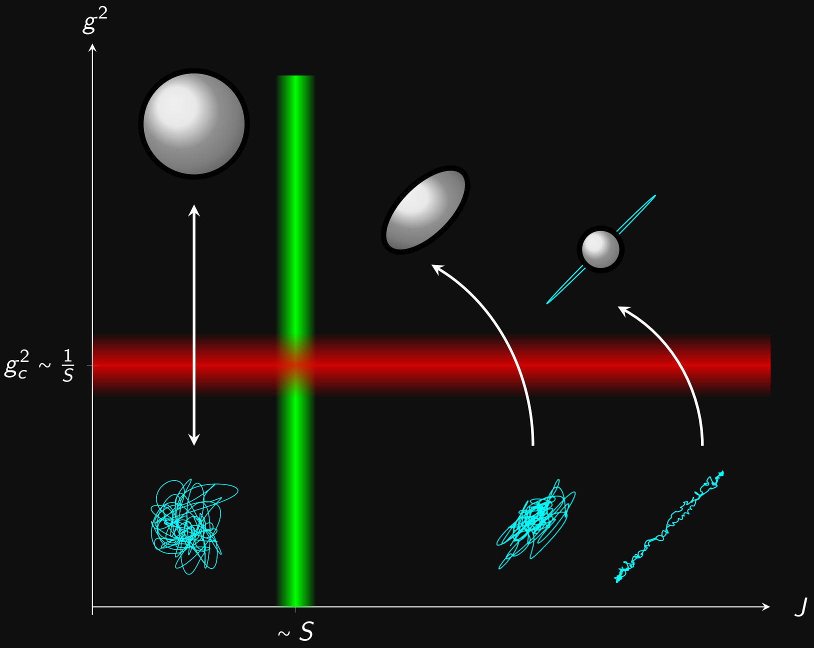
Plump rotating black holes are matched with slowly rotating black strings.


g^2
$g_c^2 \sim \frac{1}{S}$
J
$\sim S$

What about highly rotating strings?

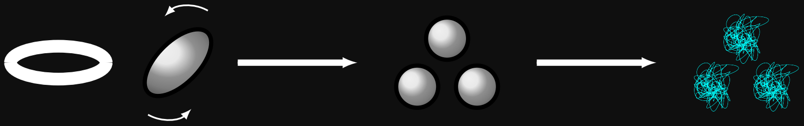
- Even in $D = 4$ we have string bars and pancakes.
- These objects should not have a stable counterpart at strong coupling.
- At $g > 0$ such objects radiate and lose angular momentum. [Iengo+Russo]
- Above the correspondence point, they either become non-stable objects or possibly even stringy hybrids [Deng+Gruzinov+Levin+Vilenkin]




g^2
$g_c^2 \sim \frac{1}{S}$
$\sim S$
J

$D \geq 5$

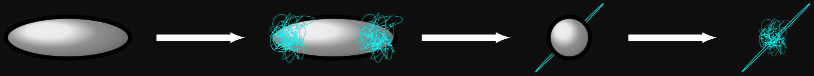
- The results from $D = 4$ naturally extend to higher dimensions.
- But we also have some new ingredients:
  - Ultraspinning Myers-Perry black holes ($J > S$).
  - Ring like configurations (string and black hole side).
- Most $J > S$ objects are unstable to fragmentation.



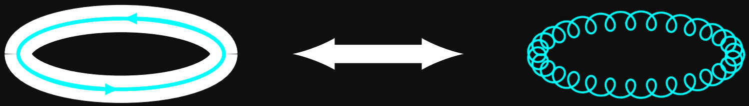
- Each individual fragment will transition into a (slowly rotating) string.

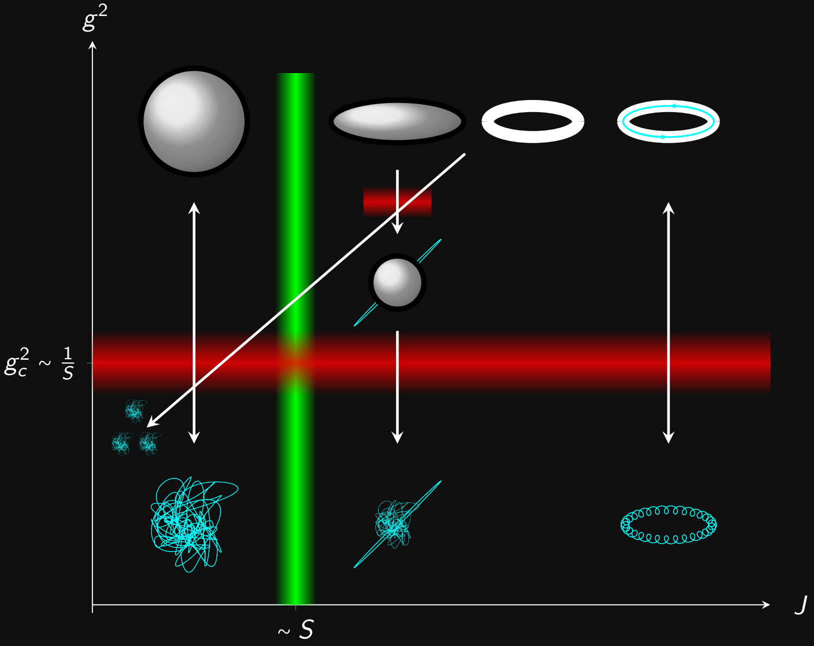
$D \geq 5$

- For long lived ellongated Myers-Perry black holes, end points can transition earlier than the poles



- Hybrids can potentially become stringy bars with suitably localised excitations.
- Dipole rings can be stable and transition into plasmids.





g^2
$g_c^2 \sim \frac{1}{S}$
$\sim S$
J

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Summary

- The black hole/string correspondence provides stringy insight into the black hole degrees of freedom.
- We characterised the correspondence between rotating black holes and fundamental strings with momentum in arbitrary ($D \geq 4$) dimensions.
- For slowly rotating black holes $J < S$, the correspondence is an extension of the non-rotating case.
- For higher angular momentum, there are several non-trivial transitions that depend on the configurations.
- We find that some transitions depend on the direction in which we change the coupling: Non-reversible changes.

Outlook

- Details of transitions?
- What happens near the extremal bound?
- Other spacetimes (AdS/dS)?
- Adding Ramond charges?
- For which configurations with angular momentum can one find bound states at $g > 0$? [Horowitz+Polchinski]