

# New horizons: testing the black hole paradigm



*“Caribbean Sea,  
Jamaica, 1980.”  
Photograph by  
Hiroshi Sugimoto*

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**[Niels Bohr Institute & Técnico Lisboa]**

# Singularities & Cosmic Censorship

Theorem (Penrose 1965; 1969):

For “reasonable” matter, trapped surface formation results in “singularity,” where at least one of the following holds:

- a. Negative local energy occurs.
- b. Einstein's equations are violated.
- c. The space-time manifold is incomplete.
- d. The concept of space-time loses its meaning at very high curvatures – possibly because of quantum phenomena.

Conjecture (Penrose 1969):

No singularity is visible from future null infinity (weak CCC)

General Relativity is deterministic (strong CCC)

# Uniqueness: the perfect laboratory

Theorem (*Carter 1971; Robinson 1975; Chrusciel & Costa 2012*):

A stationary, asymptotically flat, vacuum BH solution must be Kerr

$$ds^2 = \frac{\Delta - a^2 \sin^2 \theta}{\Sigma} dt^2 + \frac{2a(r^2 + a^2 - \Delta) \sin^2 \theta}{\Sigma} dt d\phi$$
$$- \frac{(r^2 + a^2)^2 - \Delta a^2 \sin^2 \theta}{\Sigma} \sin^2 \theta d\phi^2 - \frac{\Sigma}{\Delta} dr^2 - \Sigma d\theta^2$$

$$\Sigma = r^2 + a^2 \cos^2 \theta, \quad \Delta = r^2 + a^2 - 2Mr$$

Describes a rotating BH with mass  $M$  and angular momentum  $J=aM$ , iff  $a < M$

$$M_{2\ell} = (-1)^\ell M a^{2\ell}$$
$$S_{2\ell+1} = (-1)^\ell M a^{2\ell+1}$$

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“In my entire scientific life, extending over forty-five years, the most shattering experience has been the realization that an exact solution of Einstein’s equations of general relativity provides the *absolutely exact representation* of untold numbers of black holes that populate the universe.”

*S. Chandrasekhar*, The Nora and Edward Ryerson lecture, Chicago April 22 1975

# Black holes are black

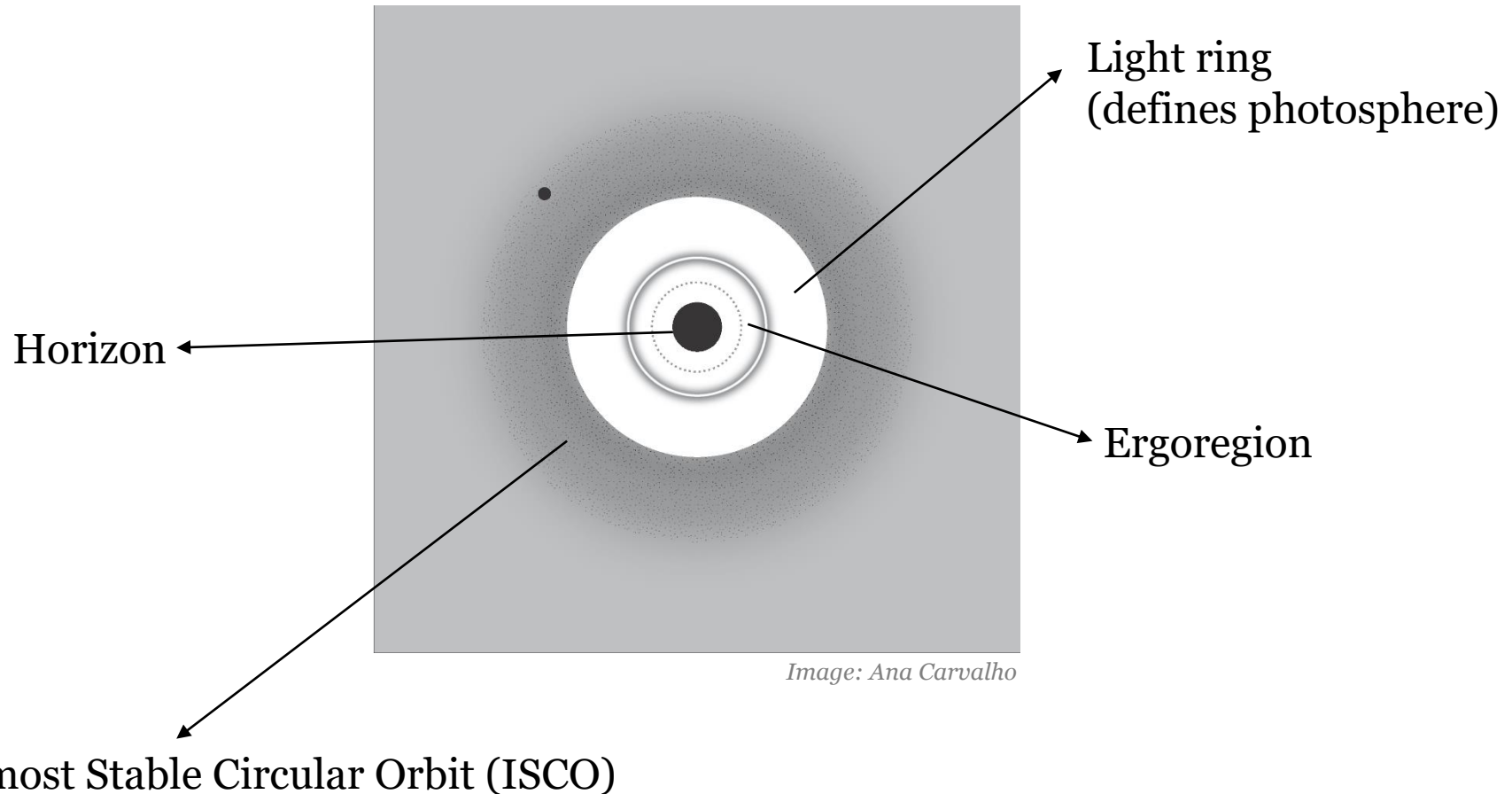


Image: Ana Carvalho

# Black holes are black

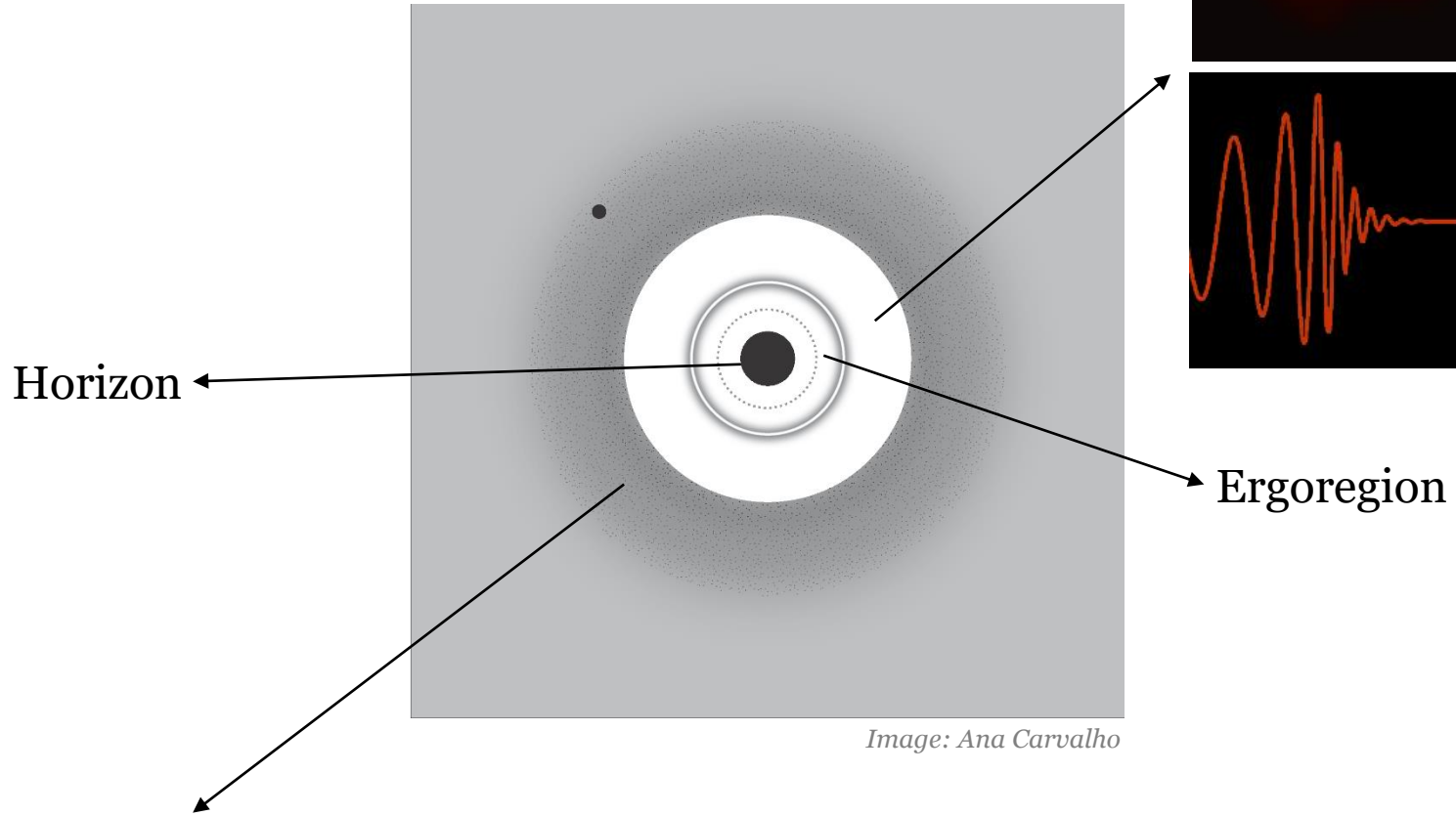
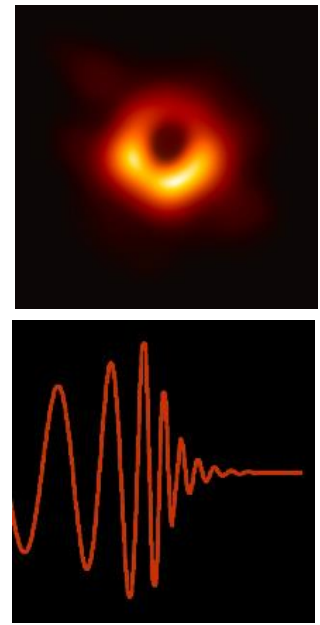


Image: Ana Carvalho

# Fundamental questions

Is it a Kerr black hole? Can we constrain alternatives?

*Cardoso+ CQG33:174001 (2016); Barack+ CQG36:143001 (2019); Cotesta PRL129:111102 (2022)*

What if horizons are transient properties?

*Cardoso, Costa, Natário, Zhong (2023)*

Are we really observing black holes at all?

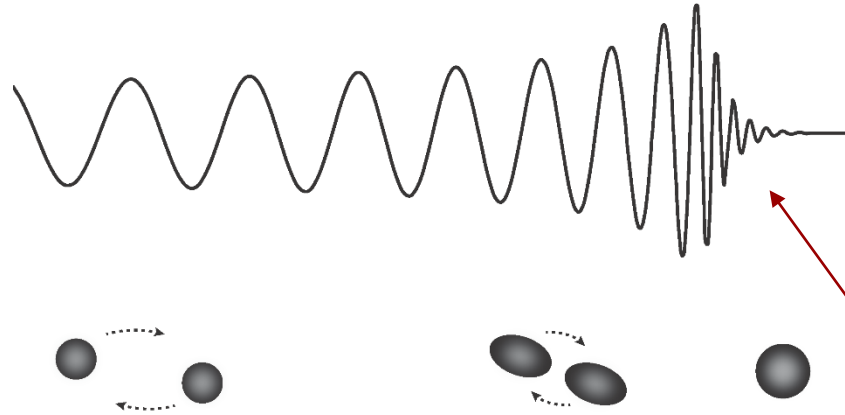
*Cardoso+ PRL116: 171101 (2016); Nature Astronomy 1: 586 (2017); Liv. Rev. Rel. 22 (2019)*

Can we do galaxy tomography, or constrain dark matter?

*Barausse+ PRD89:104059 (2014); Cardoso+ AA644:147 (2020); Cardoso+ PRDL105:104023 (2022)*

Answer requires understanding of theoretical framework, PDEs, precise modelling,  
challenging simulations & challenging data analysis techniques

# I. Spectroscopy: testing the Kerr nature



$$f = \omega_R/2\pi = 1.207 \left( \frac{10 M_\odot}{M} \right) \text{ kHz}$$

$$\tau = 1/|\omega_I| = 0.5537 \left( \frac{M}{10 M_\odot} \right) \text{ ms.}$$

*Ringdown*

$$\frac{\partial^2 \Psi}{\partial r_*^2} - \frac{\partial^2 \Psi}{\partial t^2} - V(r_*)\Psi = S \quad \sim e^{-i\omega t}$$

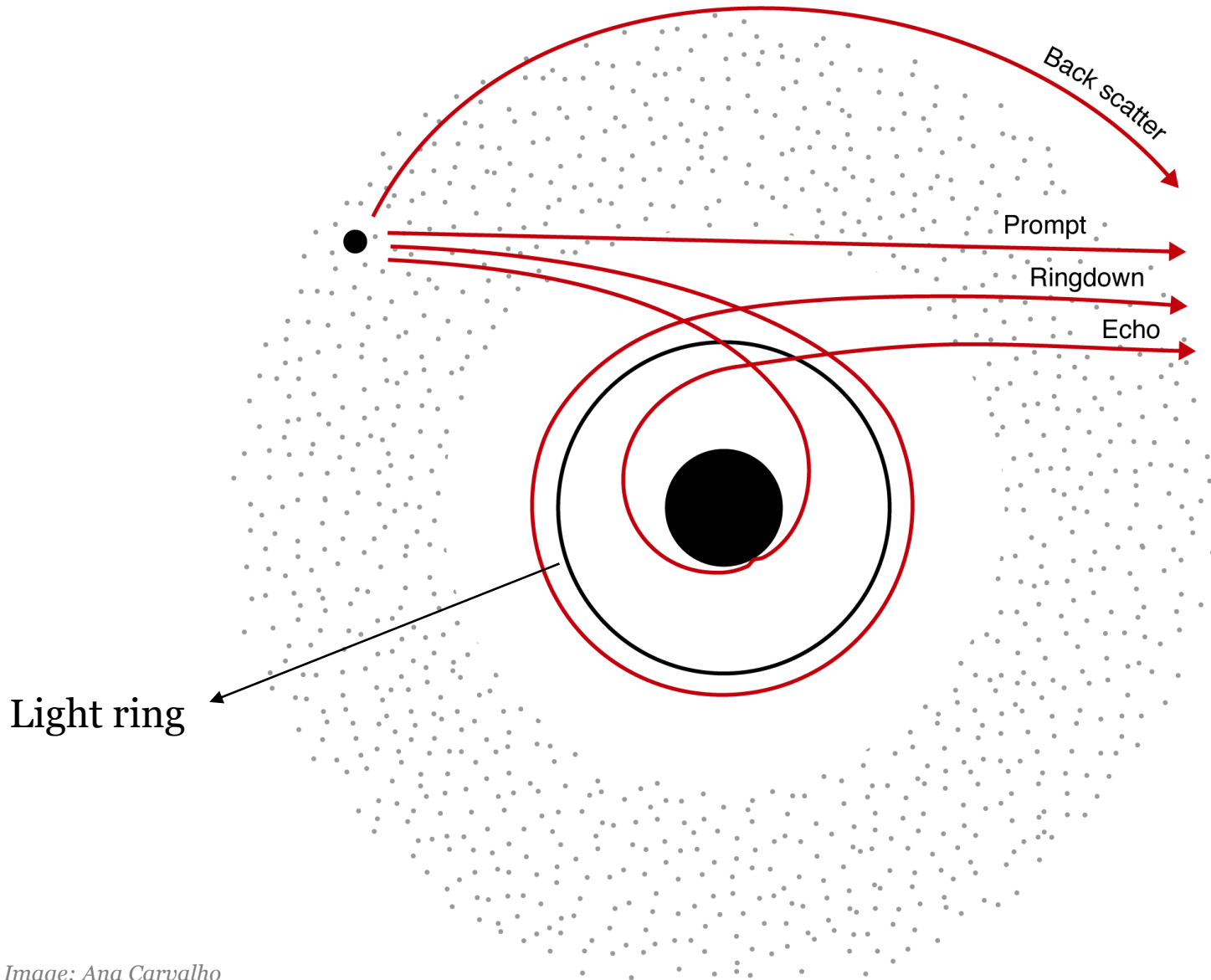
$$(\omega^2 - \mathcal{L}) \psi = s \rightarrow \psi = (\omega^2 - \mathcal{L})^{-1} s$$

*poles = QNMs*

$$h = \sum_{nlm} A_{nlm} e^{-t/\tau_{nlm}} \sin(2\pi f_{nlm} t) Y_{lm}(\theta, \phi)$$

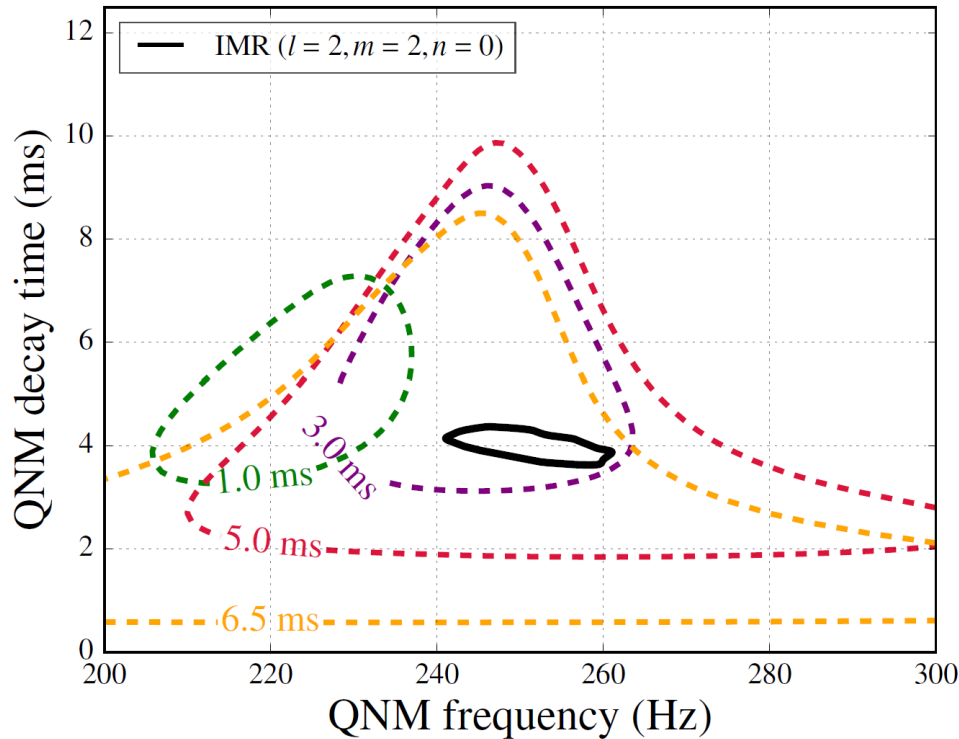
*Leaver PRD34:384 (1986); Berti + PRD73: 064030 (2006); PRL 117:101102 (2016);  
Cardoso & Gualtieri CQG33: 174001 (2016); Chen + PRL (in press, 2023); Baibhav + arXiv:2302.03050*

# When is a linear ringdown description valid?





# One and two-mode estimates: the start of spectroscopy

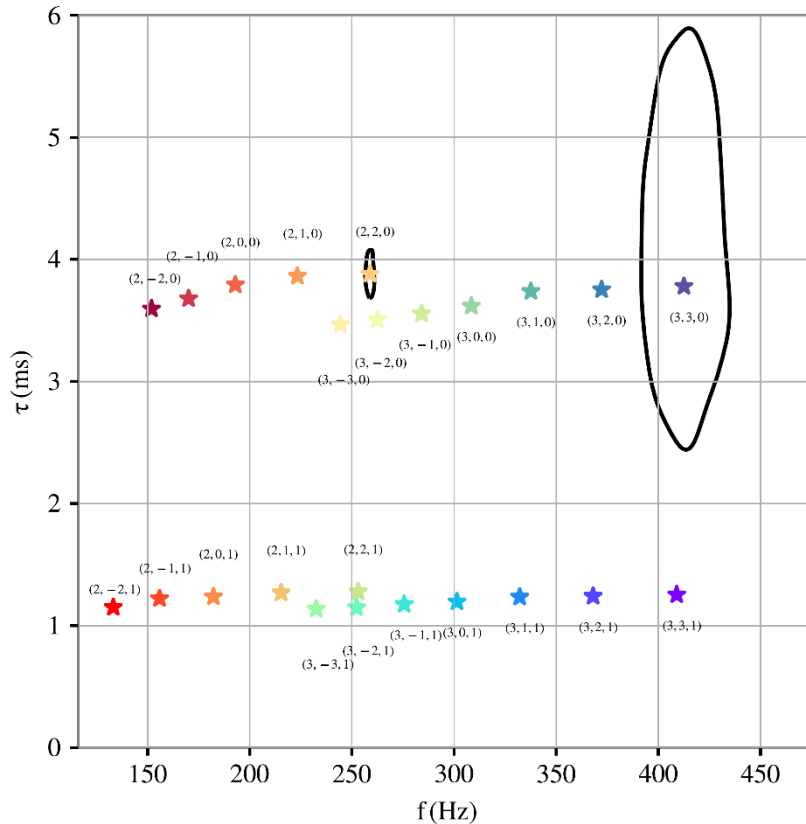


90% posterior distributions.

Black solid is 90% posterior of QNM as derived from the posterior mass and spin of remnant

*LSC PRL116:221101 (2016); arXiv:2010.14529;  
For future detectors, Berti+ PRL117:10102 (2016)*

# One and two-mode estimates: the start of spectroscopy



90% posterior distributions.

Black solid is 90% posterior of QNM from a future event with SNR=40 in ringdown.

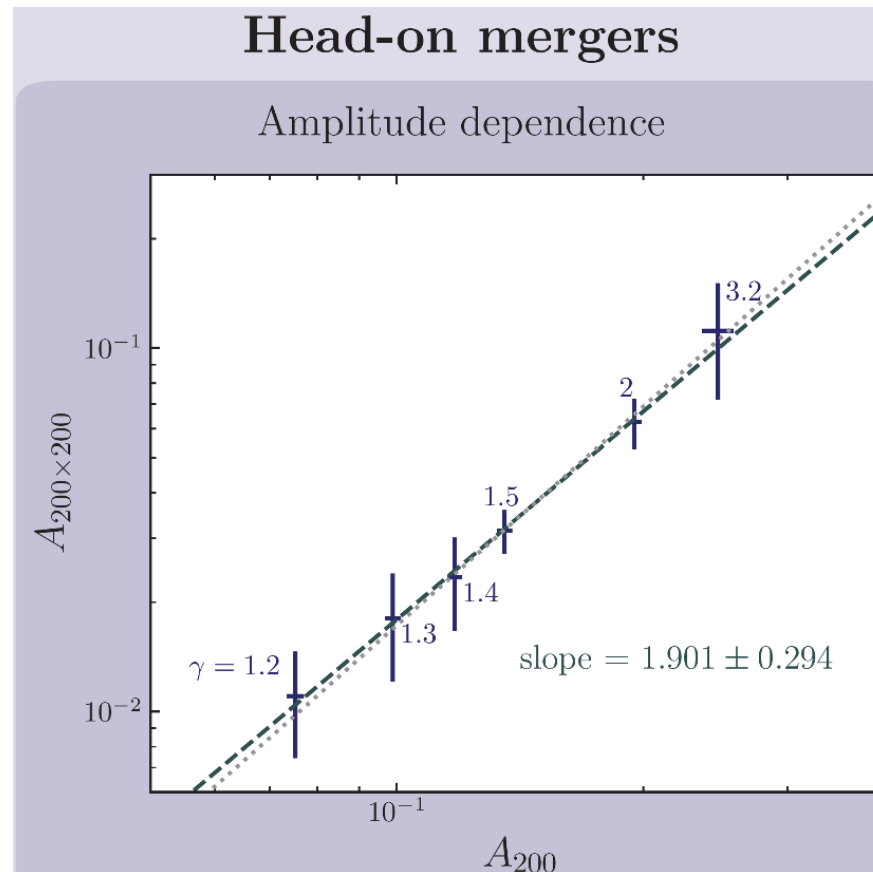
LISA will see SNRs of thousands...

*Courtesy of Gregorio Carullo*

*See also Berti+ PRL117:10102 (2016); Bhagwat + arXiv:2304.02283*

# Addenda i. Nonlinearities in ringdown?

$$\frac{\partial^2 \Psi^{(2)}}{\partial r_*^2} - \frac{\partial^2 \Psi^{(2)}}{\partial t^2} - V \Psi^{(2)} \sim \left( \Psi^{(1)} \right)^2$$

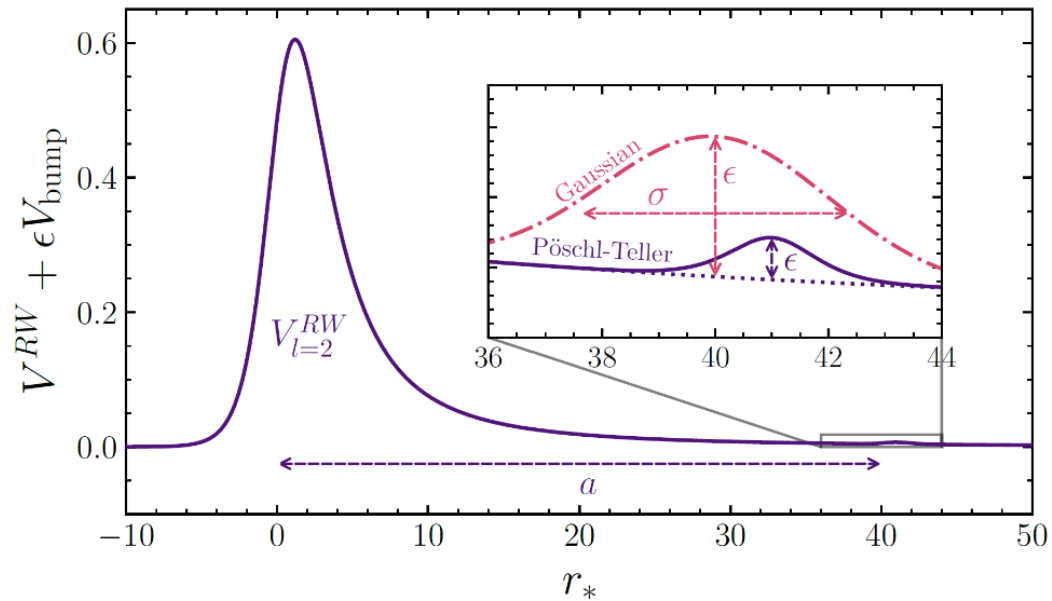


*Cheung + PRL130:8 (2023);*

*also Ma + arXiv 2207.10870; Mitman+ arXiv 2208.07380; see Kehagias + arXiv:2301.09345 for Kerr/CFT*

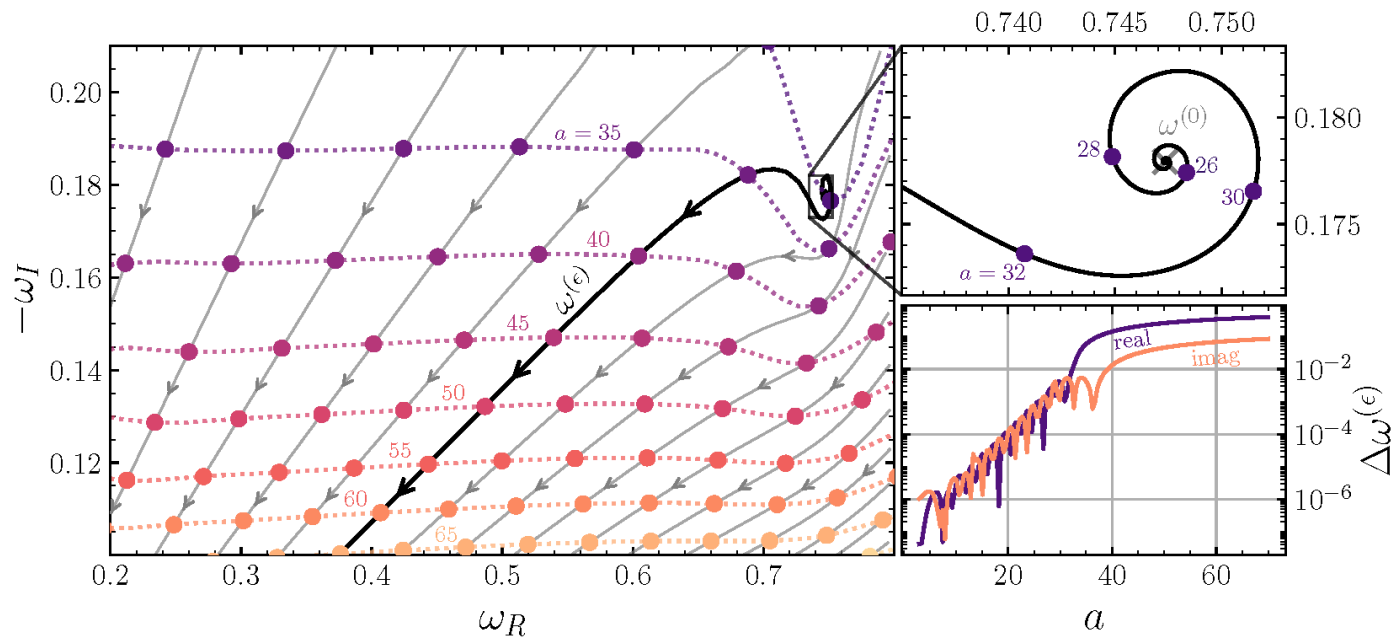
# Addenda ii. Spectral stability

Spectrum is unstable: Cheung+ PRL128:111103 (2022); PRD106:084011 (2022)



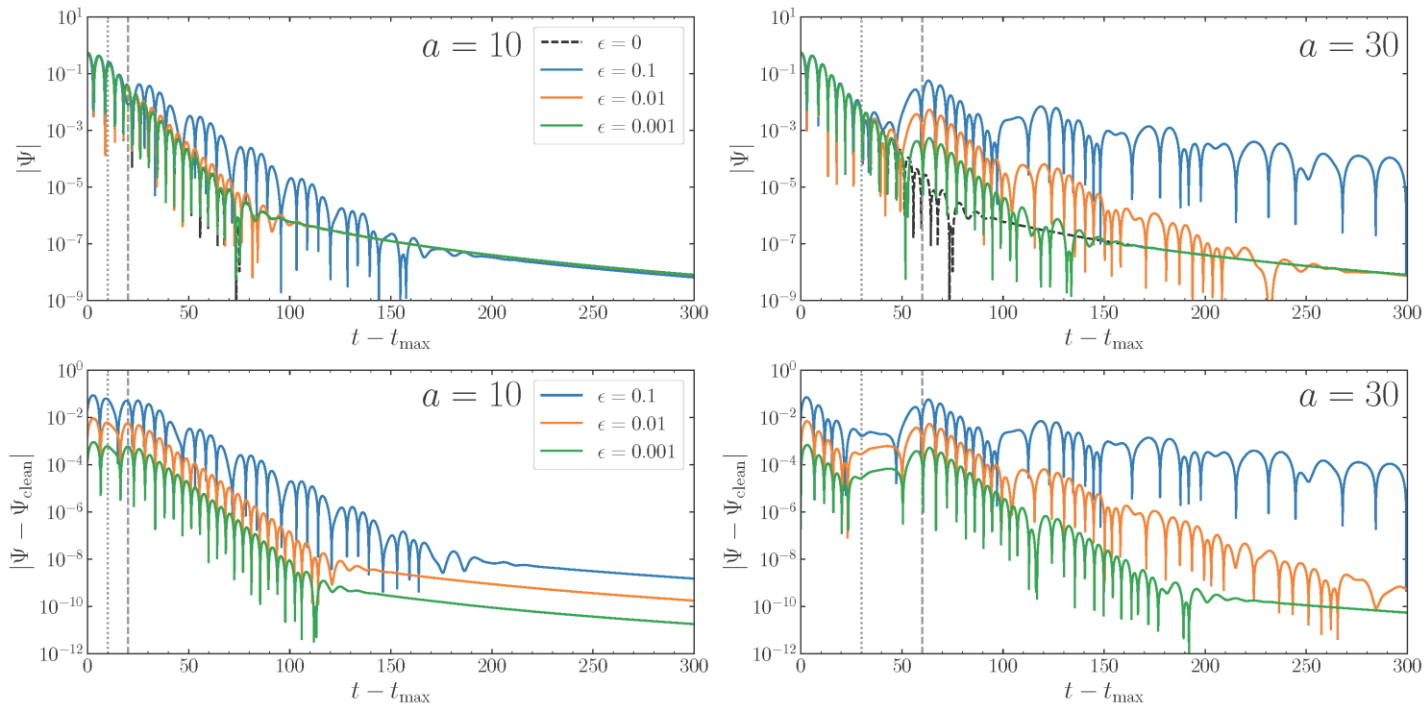
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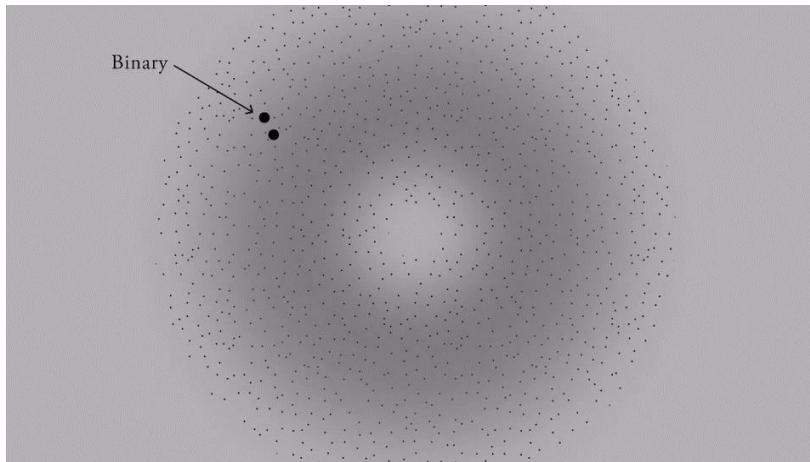
Spectrum is unstable: Cheung+ PRL128:111103 (2022); PRD106:084011 (2022)



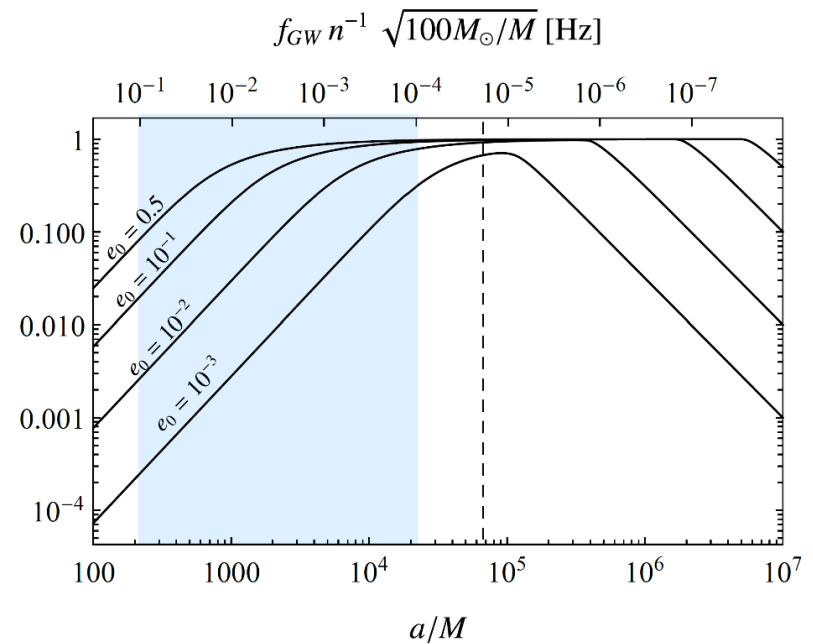
# Addenda iii. Environmental effects

Inspiral occurs in DM-rich environment, within galaxies, and may modify the way inspiral proceeds, given dense-enough media.

*Eda + PRL110:221101 (2013); Macedo + ApJ774:48 (2013); Barausse+PRD89:104059 (2014); Cardoso + AA644: A147 (2020); Annulli + PRD102: 063022 (2020); Cardoso+ PRL129:241103 (2022)*



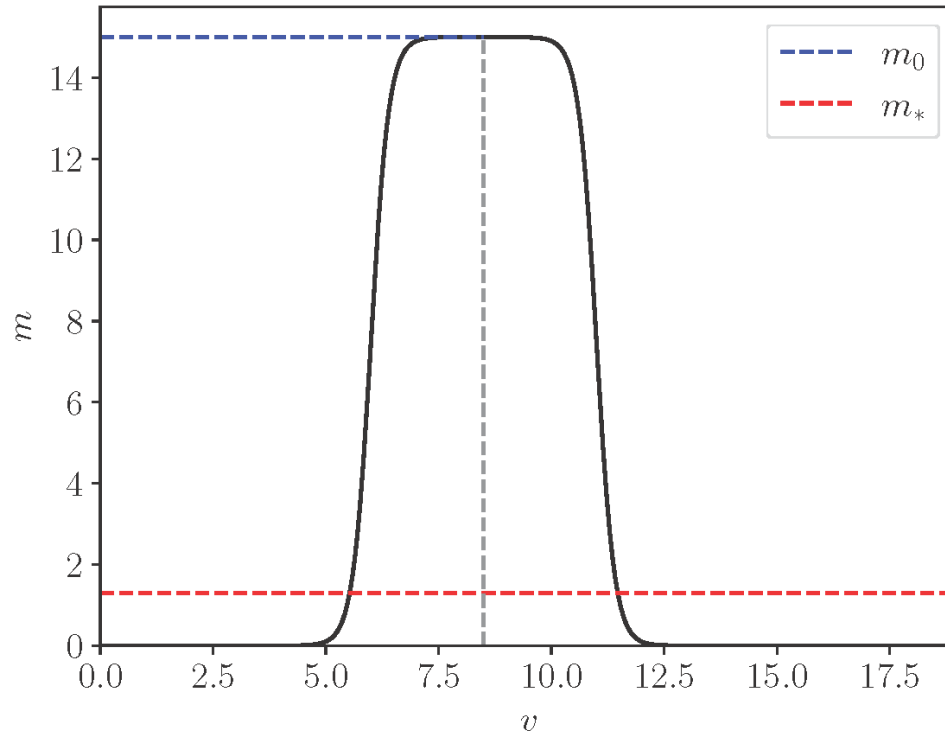
*Animation by Ana Carvalho*



*Cardoso + PRD103: 023015 (2021); PRDL105:104023 (2022)*

## II. When horizons die young

Cardoso, Costa, Natário, Zhong, to appear; Hayward PRL96:031103 (2006)



$$ds^2 = -F(v, r)dv^2 + 2 dv dr + r^2 d\Omega^2$$

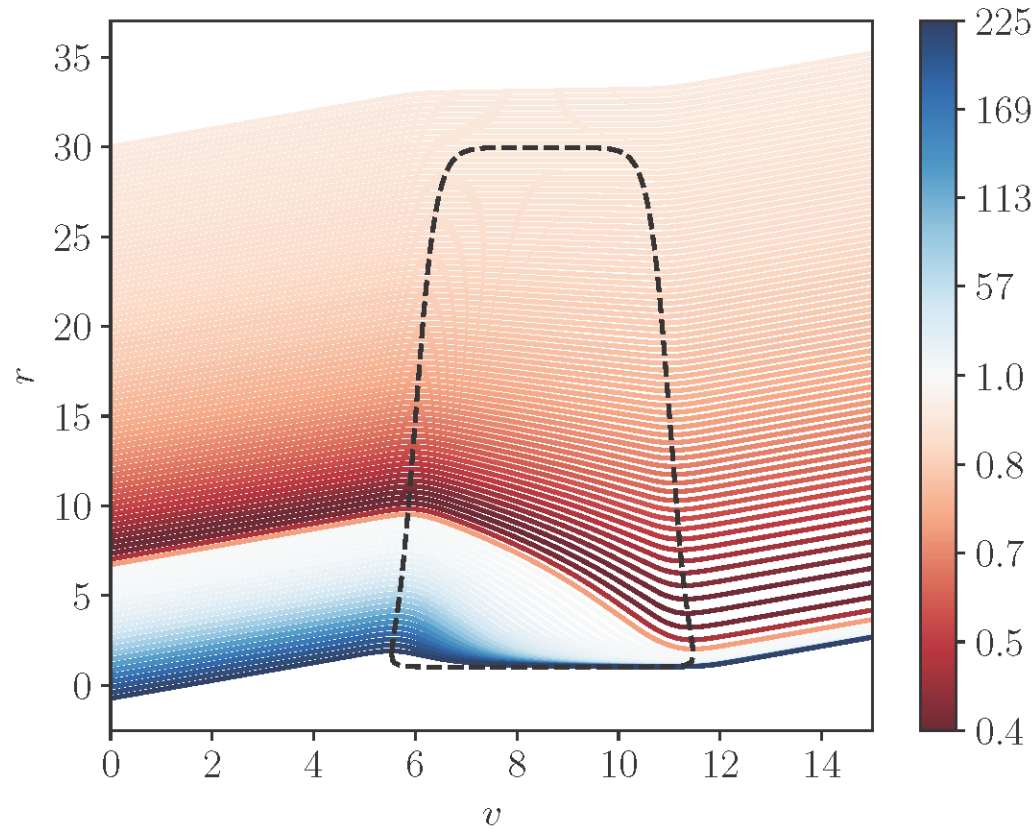
$$F = 1 - \frac{2 m(v)r^2}{r^3 + 2l^2 m(v)}$$

Outer horizon  $2m_0$ , inner at  $l$



# Bouncing geometries: energy amplification

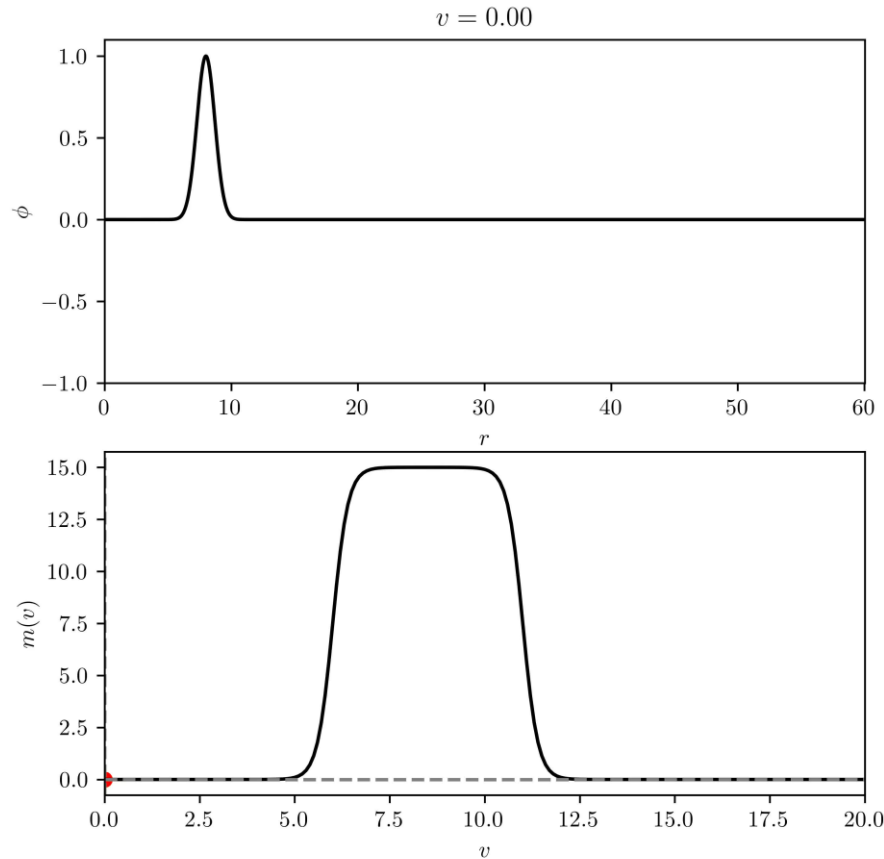
Cardoso, Costa, Natário, Zhong, to appear



$$A = \frac{E_{\infty}}{E_0} = \exp(\text{horizon lifetime}/l)$$

# Bouncing geometries: energy amplification

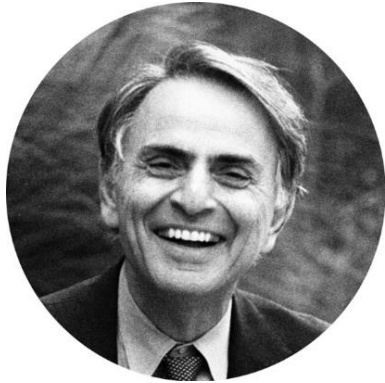
Cardoso, Costa, Natário, Zhong, to appear



$$A = \frac{E_\infty}{E_0} = \exp(10^{61}) \quad \text{For Hubble-old geometries}$$

# III. Testing the black hole paradigm

1. BH exterior is pathology-free, interior is not.



*“Extraordinary claims require extraordinary evidence.”*  
Carl Sagan

2. Quantum effects not fully understood. Non-locality to solve information paradox?
3. Tacitly assumed quantum effects at Planck scales. Planck scale could be significantly lower (*Arkani-Hamed+ 1998; Giddings & Thomas 2002*). Even if not, many orders of magnitude standing, surprises can hide (*Bekenstein & Mukhanov 1995*).
4. *Dark matter exists, and interacts gravitationally. Are there compact DM clumps?*
5. Physics is experimental science. We can test exterior. Aim to quantify evidence for horizons. Similar to quantifying equivalence principle.

# Some challenges

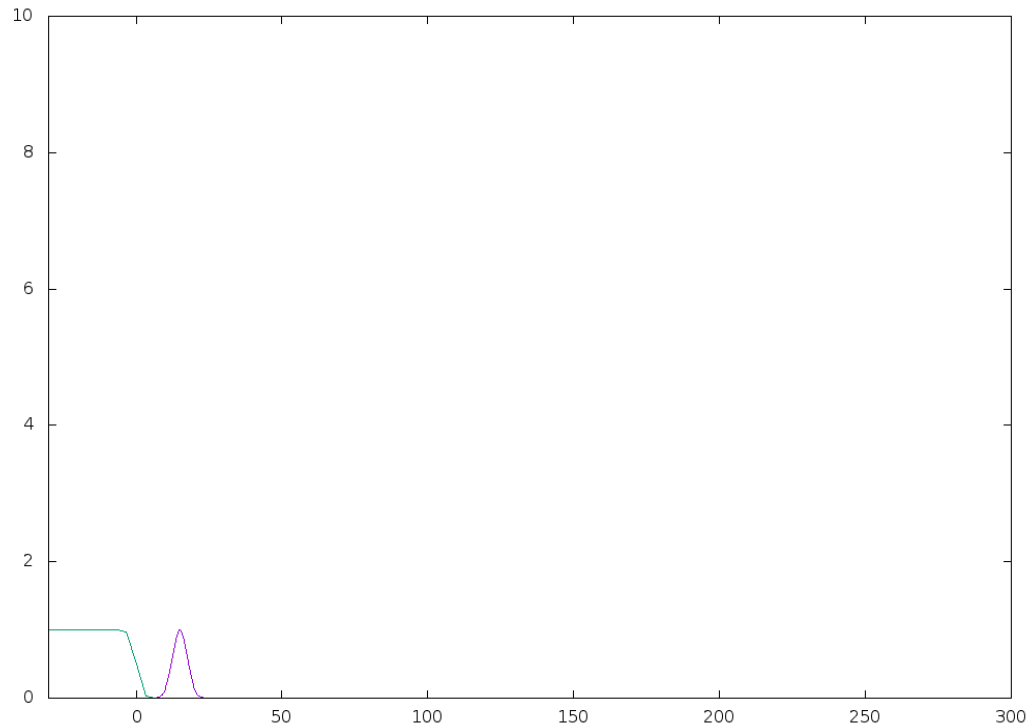
- i. Are there alternatives?
- ii. Do they form dynamically under reasonable conditions?
- iii. Are they stable?
- iv. How do they look like? Is GW or EM signal similar to BHs?
- v. Observationally, how close do we get to horizons?

Answer requires understanding of theoretical framework, PDE analysis, precise modelling, observations, challenging simulations & data analysis techniques

# Stability of objects with ergoregions

*AS flat, horizonless spacetimes with ergoregions are linearly unstable*

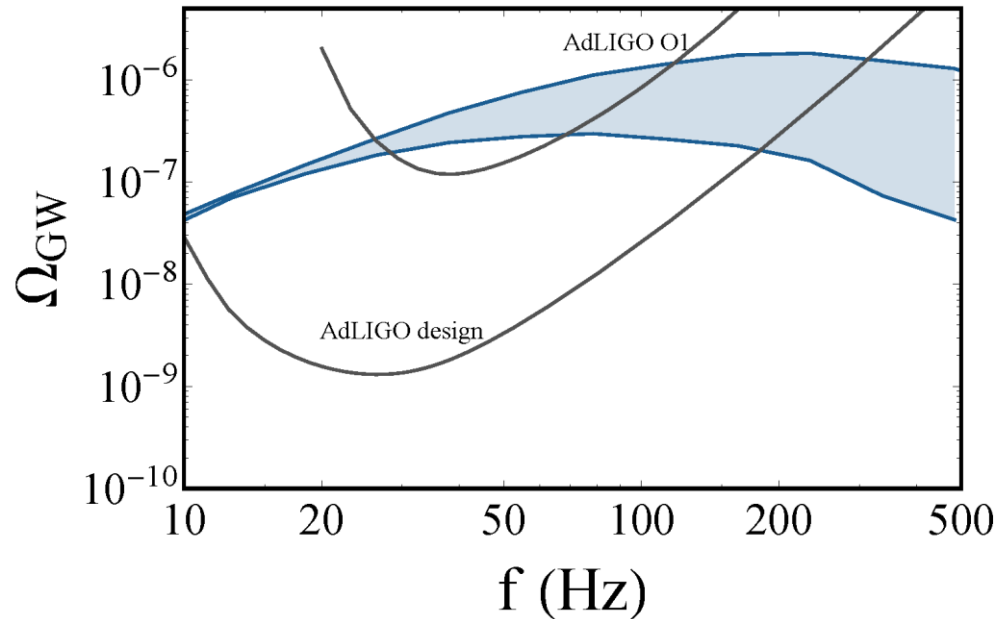
Friedmann *Comm. Math. Phys.* 63:243 (1978); Moschidis *Comm. Math. Phys.* 358: 437 (2016)



Vicente + *PRD*97:084032 (2018); Brito+ *Lect. Notes Phys* 906 (2015)

# Stochastic background of GWs

$$r_{\text{surface}} = r_+ (1 + 10^{-40})$$



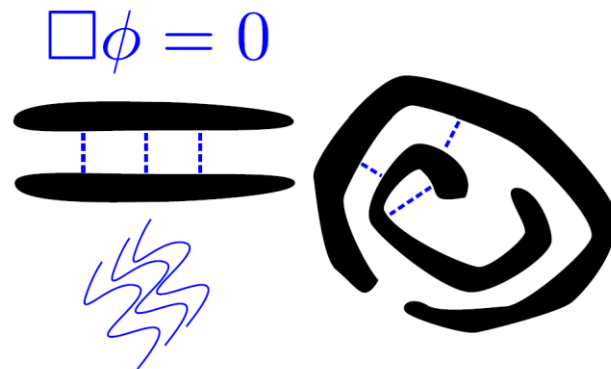
Blue bands bracket population models, from optimistic to pessimistic  
**Must we get away from Kerr in modeling? Do we fully understand when it shuts off?**

# Stability of objects with photospheres

Static objects: *No uniform decay estimate with faster than logarithmic decay can hold for axial perturbations of ultracompact objects.*

*Keir CQG33: 135009 (2016); Cardoso + PRD90:044069 (2014)*

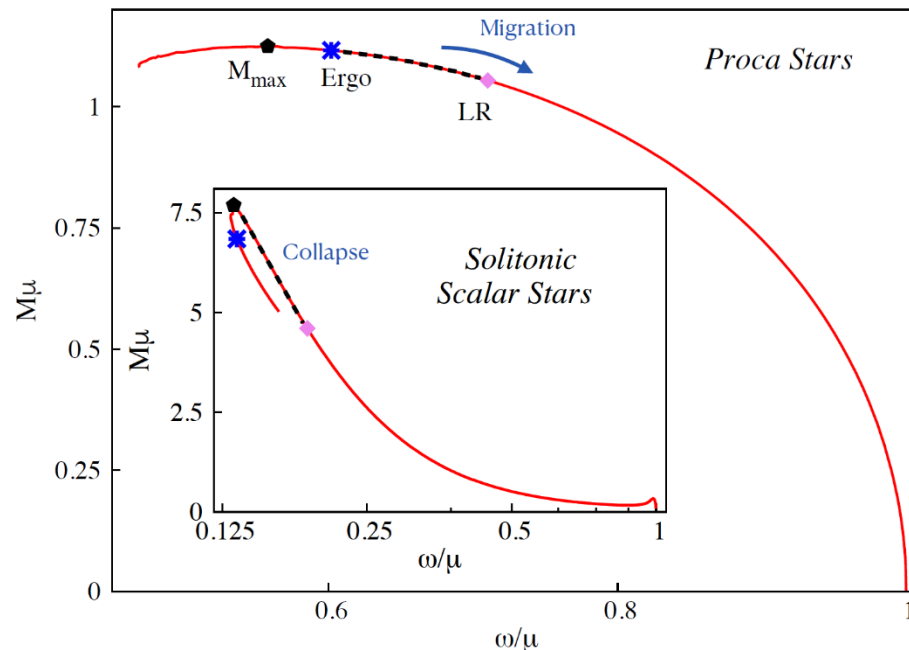
$$\mathcal{E}_{\text{local}}^{(N)}(t) \lesssim \frac{1}{(\log(2+t))^2} \mathcal{E}_{(2)}^{(N)}(0)$$



*Burq, Acta Mathematica 180: 1 (1998)*

*...such objects have long-lived fluctuations which may fragment the star and make it less compact on long time scales. Alternatively, weak turbulence might lead to collapse of the star into a BH.*

*Cardoso + PRD90:044069 (2014)*



*...we confirm the LR's triggered instability, identifying two possible fates: migration to non-ultracompact configurations or collapse to BHs.*

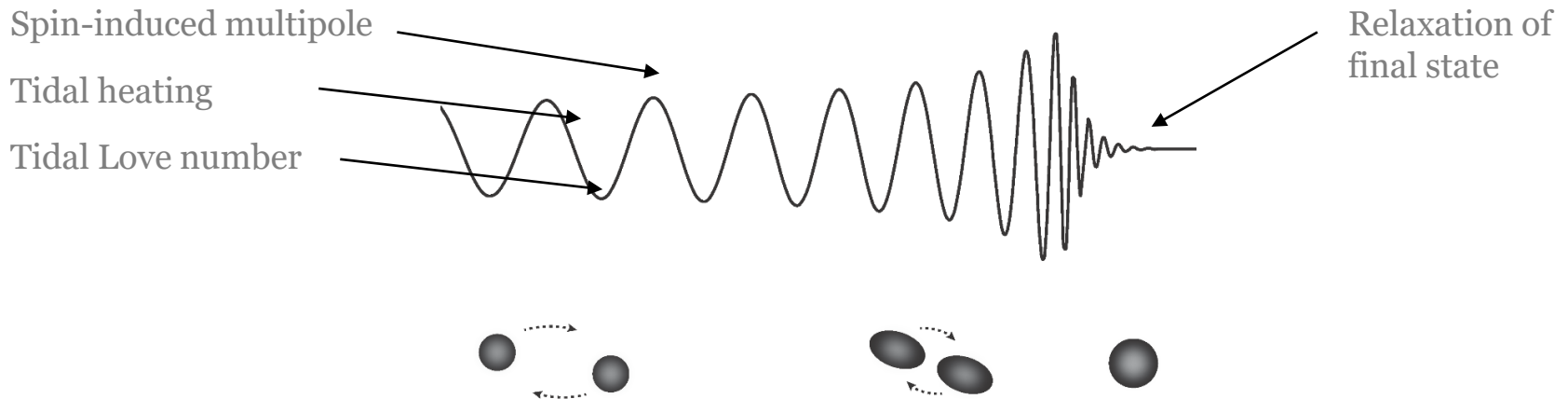
*Cunha + arXiv:2207.13713*

*But beware: unknown mechanism! numerical issues?  
Is it nonlinear? Is it specific to boson stars?*



# Gravitational waves

going down 6 to 20 orders of magnitude



But BHs in GR are simple objects:

Multipolar structure entirely dependent on mass and spin

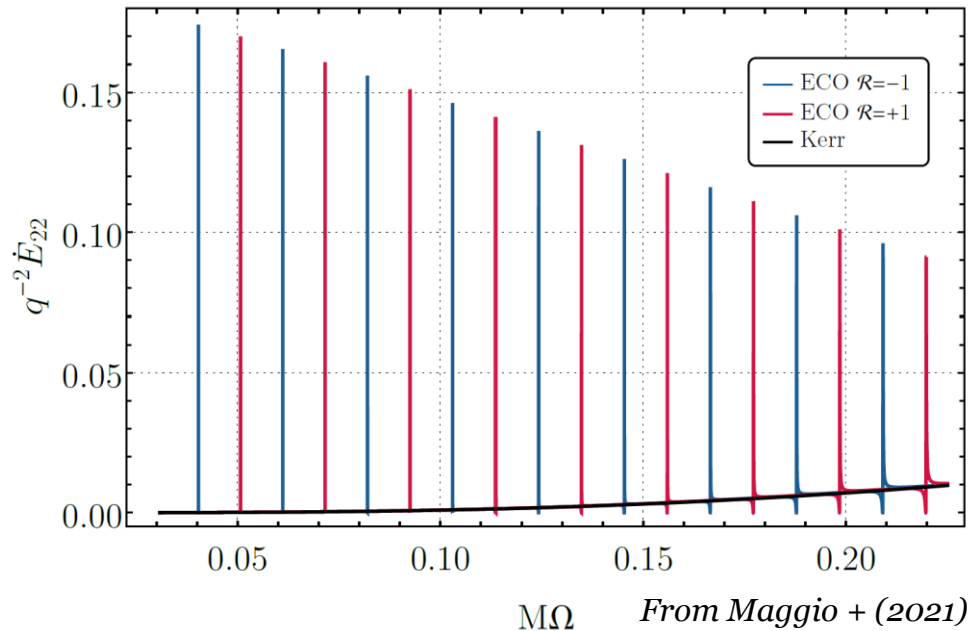
Tidal Love numbers vanish (black holes don't "polarize")

Relaxation depends only on mass and spin...

*Cardoso + PRD95:084014 (2017); Sennett + PRD96:024002 (2017)*  
*Maselli+ PRL120:081101 (2018); Cardoso & Pani, Nature Astronomy 1: 586 (2017);*  
*Living Reviews in Relativity 22: 1 (2019)*

# Early inspiral: resonances

$$r_{\text{surface}} = r_+ (1 + 10^{-10})$$



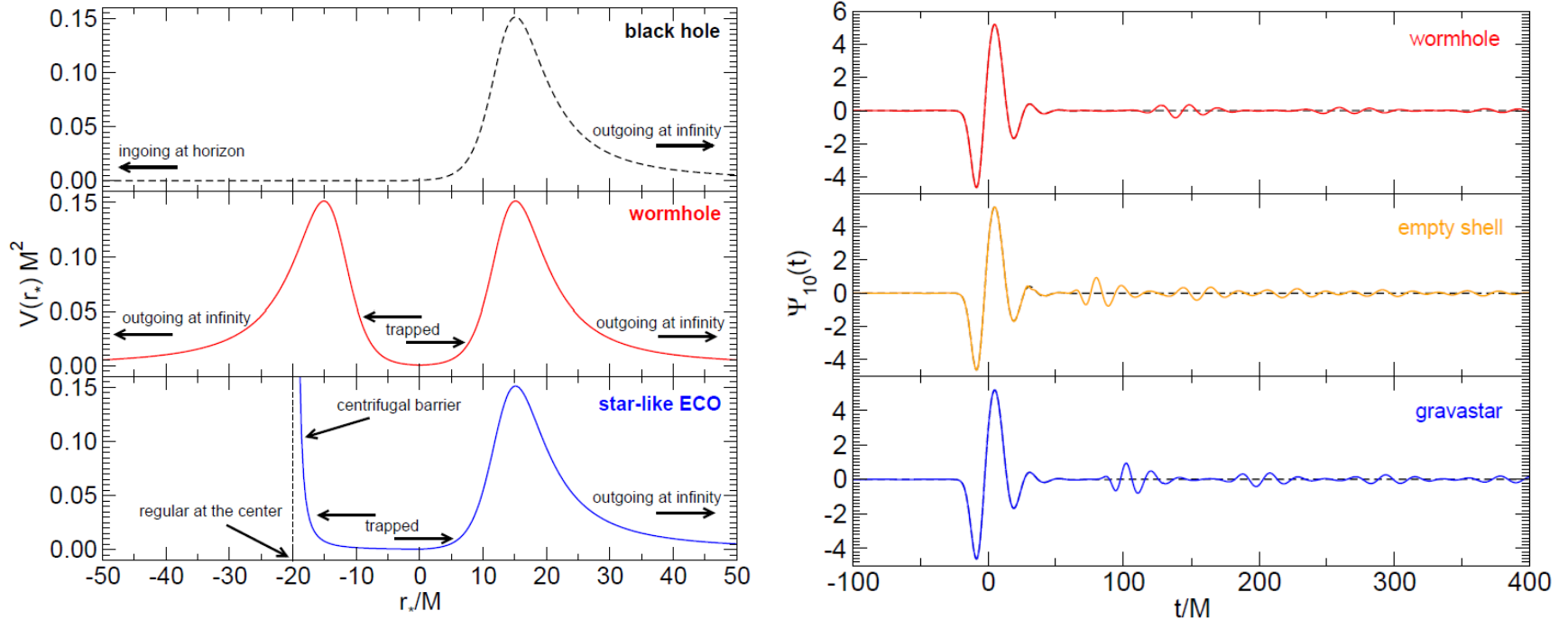
*Cardoso, del Rio & Kimura PRD101:069902 (2020)*

*Maggio, van de Meent & Pani PRD104:104026 (2021)*

*Sago & Tanaka PRD104:064009 (2021); Sago & Tanaka arXiv:2202.04249*

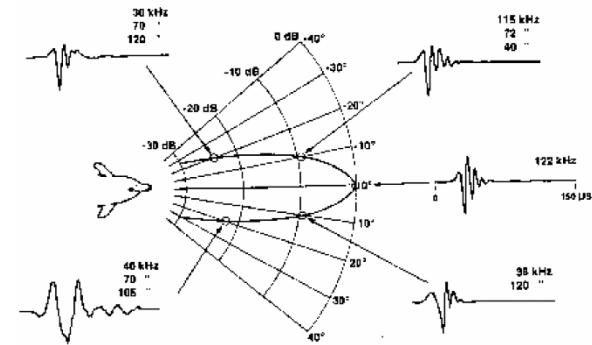
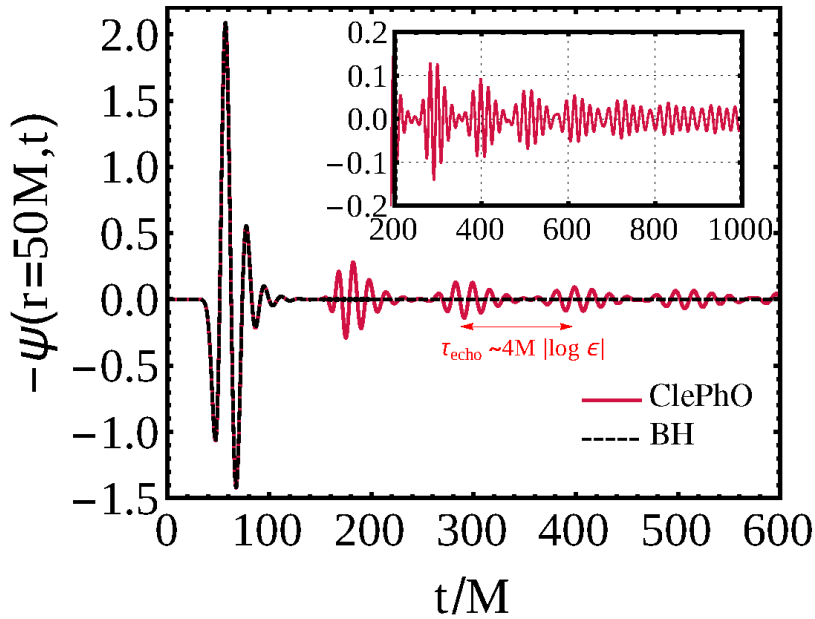
*Caution with frequency-domain calculations: see Cardoso & Duque PRD105:104023 (2022)*

# Echoes: point particles



Cardoso + PRL116:171101 (2016); Nature Astronomy 1 (2017); Living Reviews in Relativity 22:1 (2019)  
Abedi + PRD96:082004 (2016)  
Searches for echoes were conducted by LIGO/Virgo Collaboration arXiv:2010:14529; arXiv:2112.06861

# Echoes



Target echo in reverberation at the dolphin's threshold of detection (from Au [8]).

$$M\omega_R \sim |\log \epsilon|^{-1}$$

$$M\omega_I \sim |\log \epsilon|^{-(2l+3)}$$

Au+ (J. Acoust. Soc. Am. 1992)

# Conclusions: exciting times!

Gravitational wave astronomy *will* become a precision discipline, mapping compact objects throughout the entire visible universe.

Strong field gravity is a fascinating topic. From precise maps of Universe to tests of Cosmic Censorship or constraints on dark matter, possibilities are endless & exciting.

Black holes remain the most outstanding object in the universe. Black hole spectroscopy will allow to test GR and provide strong evidence for the presence of horizons... improved sensitivity pushes putative surface closer to horizon, like probing short-distance structure with accelerators.

They respond in simple way to external perturbations, and may serve as detectors for nontrivial environments.

**Thank you**



# II. Modeling black holes in galaxies

*Cardoso+ PRDLO61501 (2022); PRL129:241103 (2022); Figueiredo + arXiv:2303.08183*

## Black holes in galaxies: an Einstein Cluster prescription (Einstein 1939)

Assume averaged stress-tensor  $\langle T^{\mu\nu} \rangle = \frac{n}{m_p} \langle P^\mu P^\nu \rangle \Leftrightarrow T_\nu^\mu = \text{diag}(-\rho, 0, P_t, P_t)$

Impose spherical symmetry  $ds^2 = -f dt^2 + \frac{dr^2}{1 - 2m(r)/r} + r^2 d\Omega^2$

Assign mass function  $m(r) = M_{\text{BH}} + \frac{Mr^2}{(a_0 + r)^2} \left(1 - \frac{2M_{\text{BH}}}{r}\right)^2$   
*Hernquist ApJ356:359 (1990)*

Solve field equations  $f = \left(1 - \frac{2M_{\text{BH}}}{r}\right) e^\Upsilon$   
 $\Upsilon = -\pi \sqrt{\frac{M}{\xi}} + 2 \sqrt{\frac{M}{\xi}} \arctan \frac{r + a_0 - M}{\sqrt{M\xi}}$   
 $\xi = 2a_0 - M + 4M_{\text{BH}}$

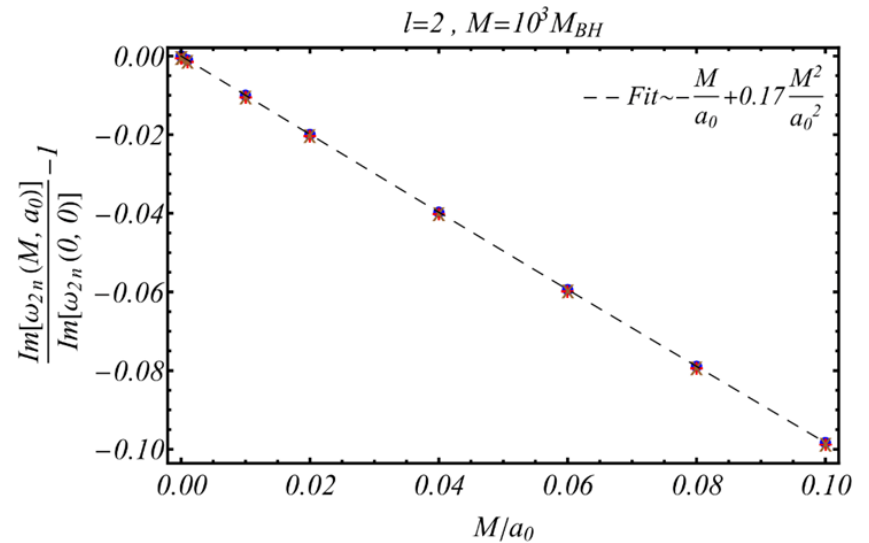
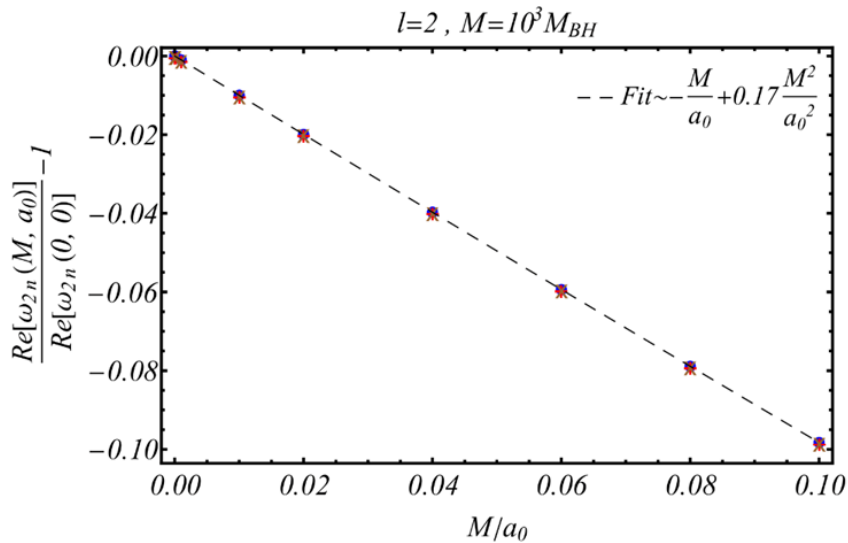
*Generalization to other profiles is straightforward, Figueiredo + arXiv:2303.08183*  
*see also Jusufi arXiv:2202.00010; Igata+arXiv:2202.00202; Konoplya+ arXiv:2202.02205*

Eikonal limit Cardoso + PRD 79, 064016 (2009)

$$\omega_{\text{QNM}} = [l - i(n + 1/2)] \Omega_{\text{LR}}$$

Light-ring corrections

$$M_{\text{BH}} \Omega_{\text{LR}} \sim \frac{1}{3\sqrt{3}} \left( 1 - \frac{M}{a_0} - \frac{M^2}{6a_0^2} \right) \sim M_{\text{BH}} \Omega_{\text{LR}}^{\text{Schw}} \left( 1 - \frac{M}{a_0} - 0.17 \frac{M^2}{a_0^2} \right)$$





# BH environment: galaxy tomography?

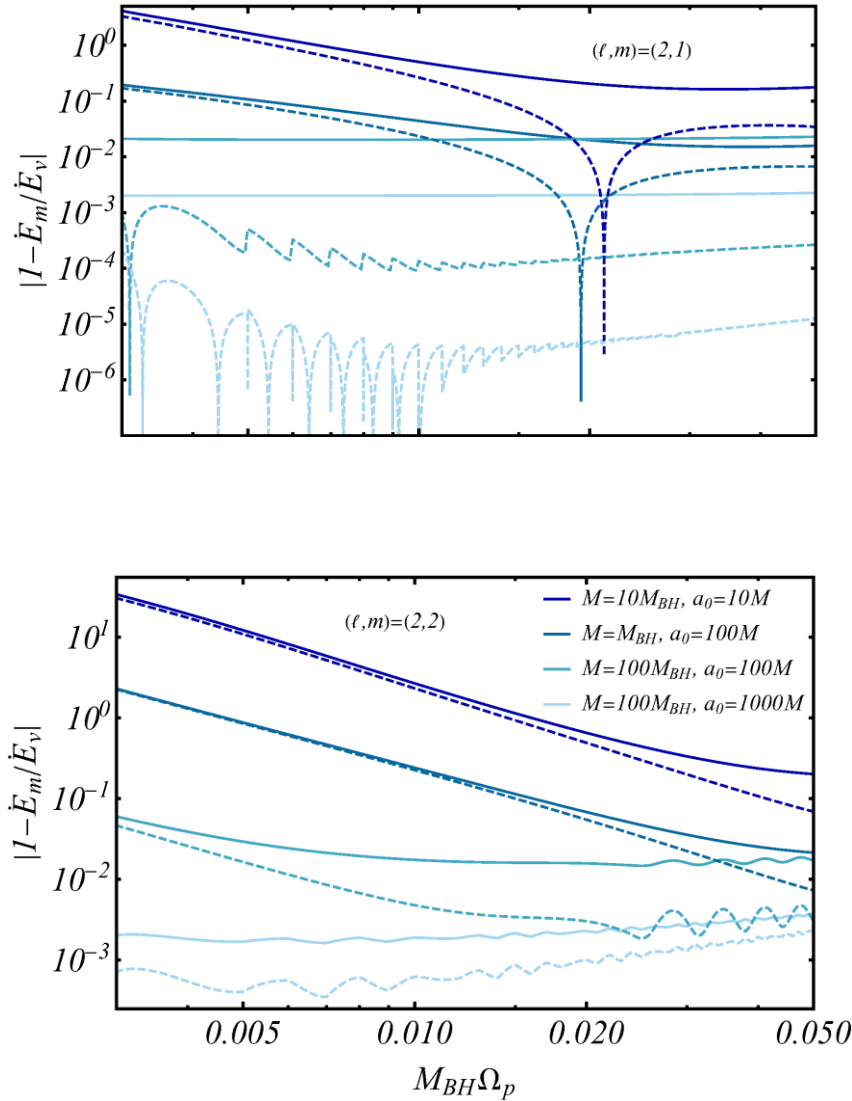
*Cardoso+ PRDLO61501 (2022); PRL129:241103 (2022); Figueiredo + arXiv:2303.08183*

$$b_{\text{crit}} = 3\sqrt{3}M_{\text{BH}} \left( 1 + \frac{M}{a_0} + \frac{M(5M - 18M_{\text{BH}})}{6a_0^2} \right)$$

Thus EHT physics affected to levels of  $10^{-8}$  only *for expected parameters*  
(tests on nature of compact objects can be done to very good precision)

# BH environment: GW dephasing

Cardoso+ *PRL*129:241103 (2022); Figueiredo + *arXiv*:2303.08183



# Surprises?

*Bekenstein & Mukhanov 1995*

*Kleban+2019; Cardoso+ JCAP08:006 (2019); Agullo+ PRL126:041302 2021*

i. Postulate area quantization

$$A = \alpha l_P^2 N = \alpha \frac{\hbar G}{c^2} N$$

$$\Delta A = \alpha \frac{\hbar G}{c^3} \Delta N = 32\pi \frac{G^2}{c^4} M \Delta M$$

ii. Compute absorbed energy of graviton

$$\Delta M = \alpha \frac{c\hbar}{32\pi G} \frac{\Delta N}{M}$$

$$\omega_n = \frac{\Delta M c^2}{\hbar} = \frac{n\alpha}{32\pi} \frac{c^3}{MG}$$

Classical!

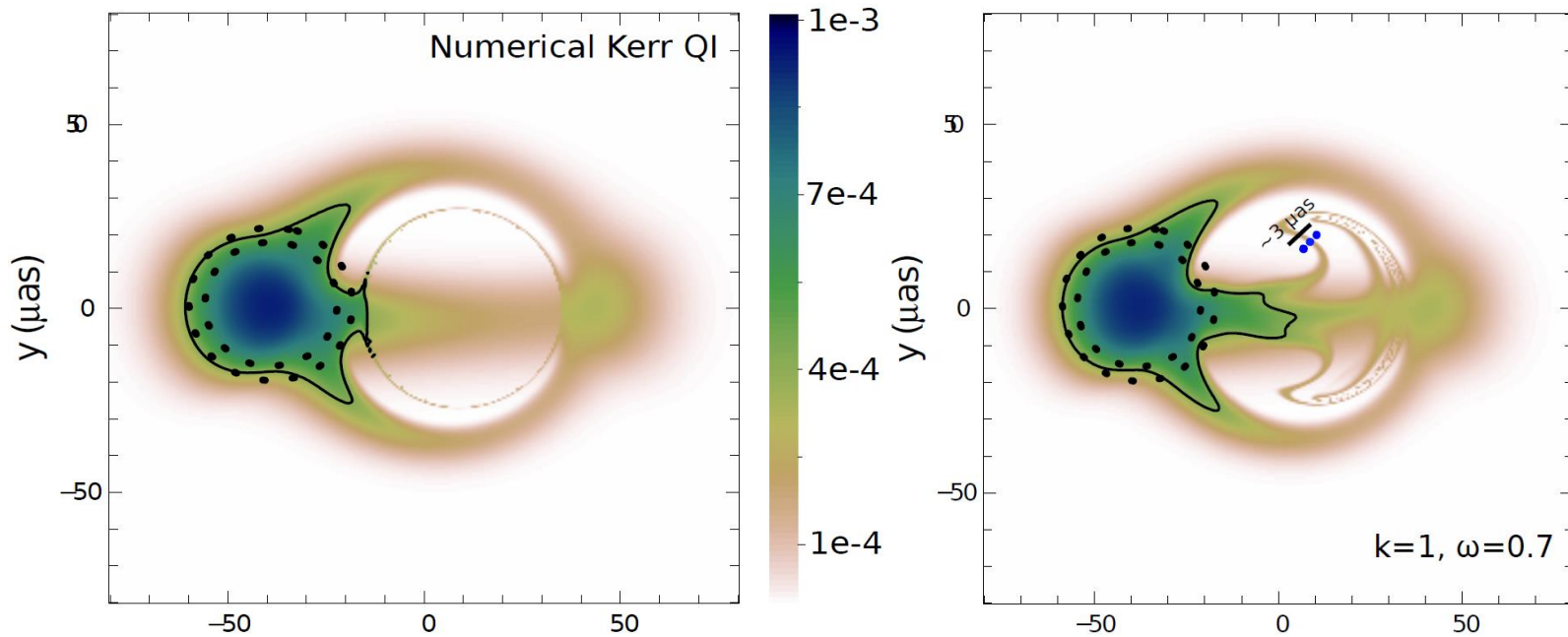


# The evidence for black holes

Cardoso and Pani, Living Reviews in Relativity 22:1 (2019)

	Constraints		Source
	$\epsilon(\lesssim)$	$\frac{\nu}{\nu_\infty}(\gtrsim)$	
1.	$\mathcal{O}(1)$	1.4	Sgr A* & M87
2.	$\mathcal{O}(0.01)$	10	GW140915
3.	$10^{-4.4}$	158	All with $M > 10^{7.5} M_\odot$
4.	$10^{-14}$	$10^7$	Sgr A*
5.	$10^{-40}$	$10^{20}$	All with $M < 100 M_\odot$
	Effect and caveats		
1.	<p>Uses detected structure in “shadow” of SgrA and M87. Spin effects are poorly understood; systematic uncertainties not quantified.</p>		
2.	<p>Uses same ringdown as BH and lack of echoes. ?</p>		
3.	<p>Lack of optical/UV transients from tidal disruption events. Assumes: all objects are horizonless, have a hard surface, spherical symmetry, and isotropy.</p>		
4.	<p>Uses absence of relative low luminosity from Sgr A*, compared to disk. Spin effects and interaction of radiation with matter poorly understood; assumes spherical symmetry.</p>		
5.	<p>Uses absence of GW stochastic background (from ergoregion instability). Assumes: hard surface (perfect reflection); exterior Kerr; all objects are horizonless.</p>		

# Shadows: soft changes



Vincent+ CQG 33:105015 (2016)  
*challenging to distinguish*

# Shadows: hard hanges

$$r = 2M (1 + \epsilon)$$

Extreme lensing makes thermodynamic equilibrium challenging

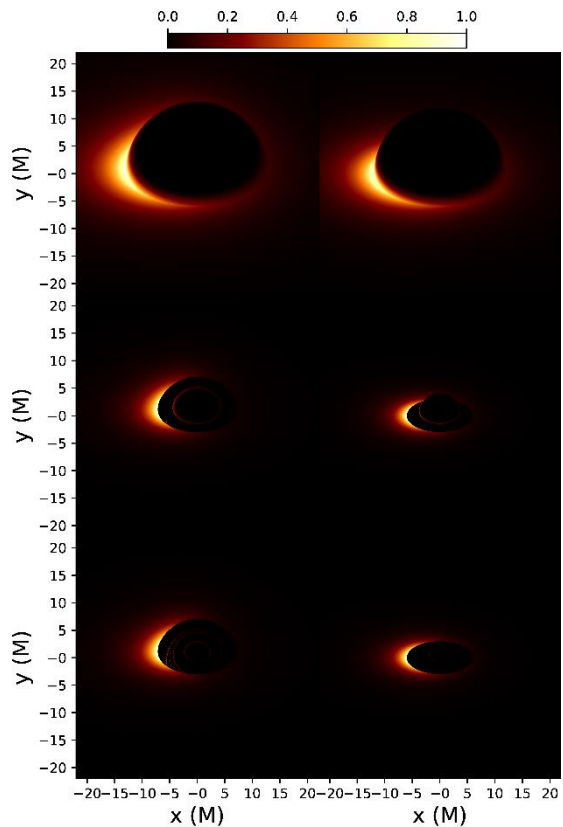
Cardoso&Pani, Living Reviews Relativity (2019)

Extreme blueshift makes compact objects particle factories, branching-ratios to other SM particle not negligible Carballo-Rubio+ (to appear)

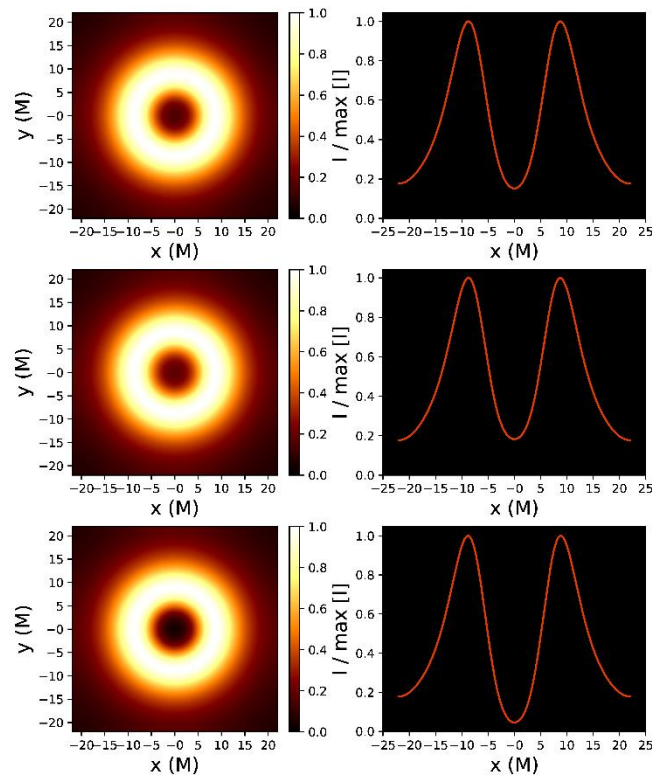
$$\begin{aligned} E_{\text{CM}} &= m_0 \sqrt{2} \sqrt{1 - g_{\mu\nu} u_{(1)}^\mu u_{(2)}^\nu} \\ &\approx \sqrt{2E} \frac{m_0}{\epsilon^{1/4}} \end{aligned}$$

Carballo-Rubio+ arXiv:2208.00704 (2022)

# Shadows



Images for spherical mirrors in Schwarzschild (left) and flat (right) spacetimes, with  $i = 60^\circ$  and  $\varepsilon = 5, 1, 0.1$ , top to bottom



Results of Gaussian filtering with EHT angular resolution of  $20 \mu\text{as}$  for  $\Gamma = 1, \eta = 0$  (top),  $\Gamma = 0.5, \eta = 10^{-1}$  (middle) and  $\Gamma = 0, \eta = 0$  (bottom).  
Need large dynamic range

*Carballo-Rubio+ arXiv:2208.00704 (2022)*  
*See also Vincent+ AA646:A37 (2021), but central object too cold*  
*challenging to distinguish, but ngEHT can tell  $\eta=0.001$ !*