The naturalness of vanishing black hole response

Riccardo Penco

GGI, Apr 18, 2023

Carnegie Mellon University
Mellon College of Science | **Physics**

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linear, (mainly) static, (mostly) conservative

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In collaboration with

Lam Hui
Columbia University

Austin Joyce
University of Chicago

Luca Santoni
APC, Paris

Adam Solomon
McMaster University

Based on 2010.00593, 2105.01069, 2203.08832

Example of static linear response

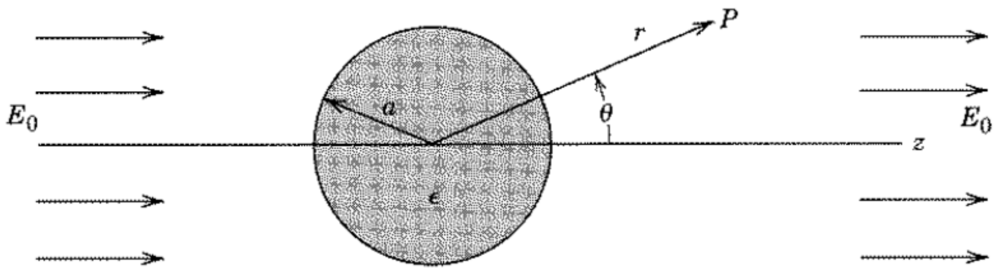


Figure 4.6

[Jackson]

Solution to $\nabla^2 \Phi = 0$ **:** $\Phi(r,\theta) = -E_0\, r \cos\theta + \dfrac{p\cos\theta}{r^2}$ (outside the sphere)

Growing = probe

Decaying = response

Susceptibility: $\chi = \dfrac{p}{E_0} = \left(\dfrac{\epsilon_r - 1}{\epsilon_r + 2}\right) a^3$

Static response of 4D GR Black Holes

Linear perturbations of 4D GR black holes are described by the **Teukolsky equation** for any s, ℓ, m. **Focus on** $\omega = 0$.

- boundary condition at the horizon: $\phi\Big|_{r=r_+}$ = finite
- at infinity: $\phi \sim r^\ell + \ldots + \dfrac{1}{r^{\ell+1}} + \ldots$
- **susceptibility** = $\dfrac{\text{coefficient of } 1/r^{\ell+1}}{\text{coefficient of } r^\ell} = \chi' + i\chi''$

$$\chi' = 0 \text{ for any } s, \ell, m$$

Static response of 4D GR Black Holes

A few comments:

- for $\vec{J}_{BH} = 0$, we also have $\chi'' = 0$
- for $s = 2$, $\chi' =$ **Love numbers**
- $\chi' \neq 0$ away from 4D or GR
- Some confusion in the literature on Kerr
    1. $\chi'' \neq 0$ *at* $\omega = 0$
    2. *growing solution also contains* $1/r^{\ell+1}$ *subleading terms*

Black holes are the only known objects with $\chi' = 0$ for any s, ℓ, m.

Observations: clear target

e.g. for stars, $\chi' = \mathcal{O}(1) \times$ *radius*$^{2\ell+1}$

Theory: naturalness problem

Point-particle EFT

Interactions between objects and **long wavelength** fields described by, e.g. for spin 1:

$$S = \int d\tau \left[-m + qU^{\mu}A_{\mu} + \left(c_1\eta^{\mu\nu} + c_2U^{\mu}U^{\nu}\right) F_{\mu}{}^{\lambda}F_{\nu\lambda} + \ldots\right]$$

Higher derivative terms are suppressed by:

size $\times \vec{\partial}$ (size / v) $\times \partial_t$

Consider object neutral ($q = 0$) and at rest ($U^{\mu} = \delta^{\mu}_{0}$).

Linear response in the p.p. EFT

Setting also $\overrightarrow{E} = \overrightarrow{E}_0, \overrightarrow{B} = 0$, our action reduces to

$$S = \int dt\,(-V_0), \quad \text{with} \quad V_0 = \text{constant}$$

In order to have $\chi' = 0$ for any s, ℓ, m,
an infinite number of Wilson coefficients
must vanish.

This is usually a consequence of symmetries...

Do black holes have more symmetries than regular objects?

YES!

1. Charalambous, Dubovsky, Ivanov [2103.01234, 2209.02091]
2. Hui, Joyce, RP, Santoni, Solomon [2105,01069, 2203,08832]

Setup

For simplicity, focus on **free scalar on Schwarzschild**:

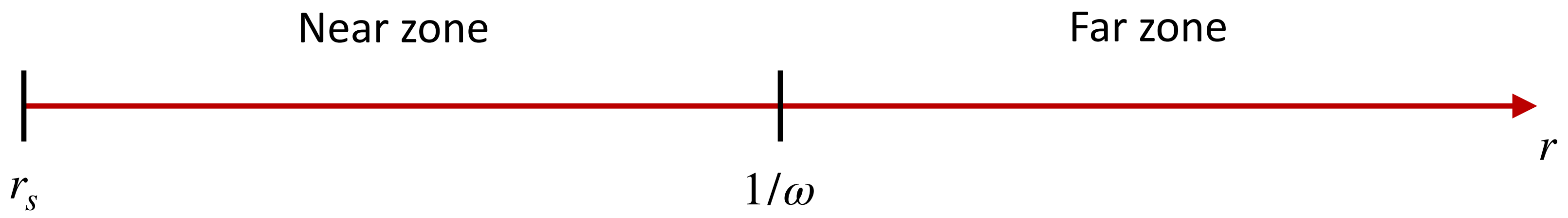
$$S = \frac{1}{2}\int dtdrd\Omega\left[\frac{r^4}{\Delta}(\partial_t\phi)^2 - \Delta(\partial_r\phi)^2 + \phi\,\nabla^2_{S_2}\phi\right], \qquad \Delta = r(r - r_s)$$

Ultimately interested in static fields.

For now, consider small, finite frequencies to discuss all symmetries in single framework.

Near-zone Approximation

Consider scalar modes with $r_s \ll 1/\omega$:



Near-zone approximation: $r \ll 1/\omega$

$$\frac{r^4}{\Delta}(\partial_t \phi)^2 \quad \to \quad \frac{r_s^4}{\Delta}(\partial_t \phi)^2$$

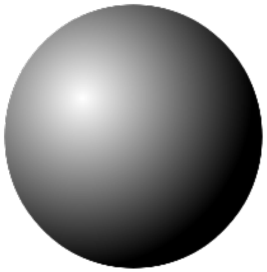
Effective Geometry

- ϕ is now minimally coupled to

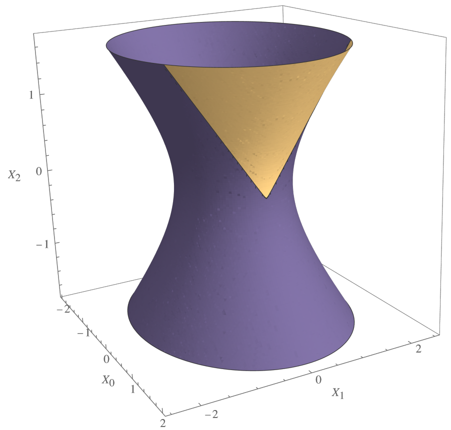
$$ds^2 \simeq \frac{\Delta}{r_s^2} dt^2 + \frac{r_s^2}{\Delta} dr^2 + r_s^2 d\Omega^2$$

$$(\Delta \equiv r^2 - r_s r)$$

S^2



AdS_2



- $AdS_2 \times S^2$: 6 Killing vectors
- Conformally flat: 9 conformal Killing vectors
- Ricci scalar = 0: ϕ is conformally coupled

Hidden Symmetries

Near-zone action is invariant under $SO(4,2)$:

$$\delta\phi = \xi^{\mu}\partial_{\mu}\phi + \frac{1}{4}\phi\nabla_{\mu}\xi^{\mu}$$

with ξ^{μ} = 3 S^2 isometries, 3 AdS_2 isometries, 9 CKVs

$SO(3,1)$ Symmetries

Hui, Joyce, RP, Santoni, Solomon [2105,01069, 2203,08832]

- S^2 isometries + 3 CKVs:

$$J_{01} = -\frac{2\Delta}{r_s}\cos\theta\,\partial_r - \frac{\partial_r\Delta}{r_s}\sin\theta\,\partial_\theta$$

$$J_{02} = -\cos\phi\left[\frac{2\Delta}{r_s}\sin\theta\,\partial_r - \frac{\partial_r\Delta}{r_s}\left(\frac{\tan\phi}{\sin\theta}\partial_\phi - \cos\theta\partial_\theta\right)\right]$$

$$J_{03} = -\sin\phi\left[\frac{2\Delta}{r_s}\sin\theta\,\partial_r - \frac{\partial_r\Delta}{r_s}\left(\frac{\cot\phi}{\sin\theta}\partial_\phi + \cos\theta\partial_\theta\right)\right]$$

- This $SO(3,1)$ is an **exact symmetry of the static sector**
- **NOTE:** doesn't rely on near-zone approximation

Vanishing Response in the IR

Regular $r_s \to 0$ **limit:** special conformal transformations (SCTs)

$$\delta\phi \to c_i \left(x^i - \vec{x}^2 \partial^i + 2x^i \, \vec{x} \cdot \vec{\partial} \right) \phi \qquad \text{(weight 1/2)}$$

Must be symmetries of the static long-distance EFT of BHs:

$$S = -\frac{1}{2} \int d^4x \, (\nabla_i \phi)^2 - m \int dt \left[1 + \frac{\lambda_2}{2} (\nabla_i \phi)^2 + \frac{\lambda_4}{2} (\nabla_i \nabla_j \phi)^2 + \ldots \right]$$

These worldline couplings would break SCTs $\to \lambda_2 = \lambda_4 = \ \ldots \ = 0$.

For a star, SCTs are explicitly broken at scale $r_\star$ $\quad \to \lambda_2 \sim r_\star^4 , \ \ldots$

$SO(3,1)$ Symmetries in the UV

Static equations:

$$H_\ell \, \phi_\ell = 0\,, \qquad \text{with} \qquad H_\ell = -\,\Delta \left[\partial_r(\Delta \partial_r) - \ell(\ell+1)\right]$$

The J_{0i}'s mix modes with different ℓ:

$$\delta\phi_\ell = c_{\ell-1} D^+_{\ell-1}\phi_{\ell-1} - c_\ell D^-_{\ell+1}\phi_{\ell+1}$$

Ladder algebra:

$$H_{\ell+1} D^+_\ell = D^+_\ell H_\ell \qquad \Longrightarrow \qquad H_{\ell+1}(D^+_\ell \phi_\ell) = 0$$

$$H_{\ell-1} D^-_\ell = D^-_\ell H_\ell \qquad \Longrightarrow \qquad H_{\ell-1}(D^-_\ell \phi_\ell) = 0$$

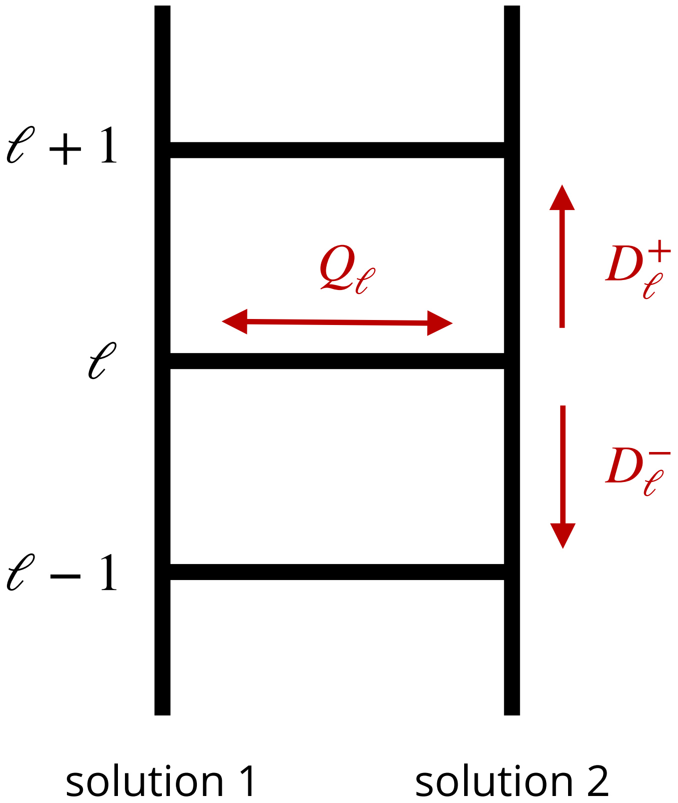
$SO(3,1)$ Symmetries in the UV

We can also define recursively the following operators:

$$Q_0 = \Delta \partial_r, \qquad\qquad Q_\ell = D^+_{\ell-1} Q_{\ell-1} D^-_\ell$$

which generate symmetries at fixed ℓ: $\quad [H_\ell, Q_\ell] = 0$.

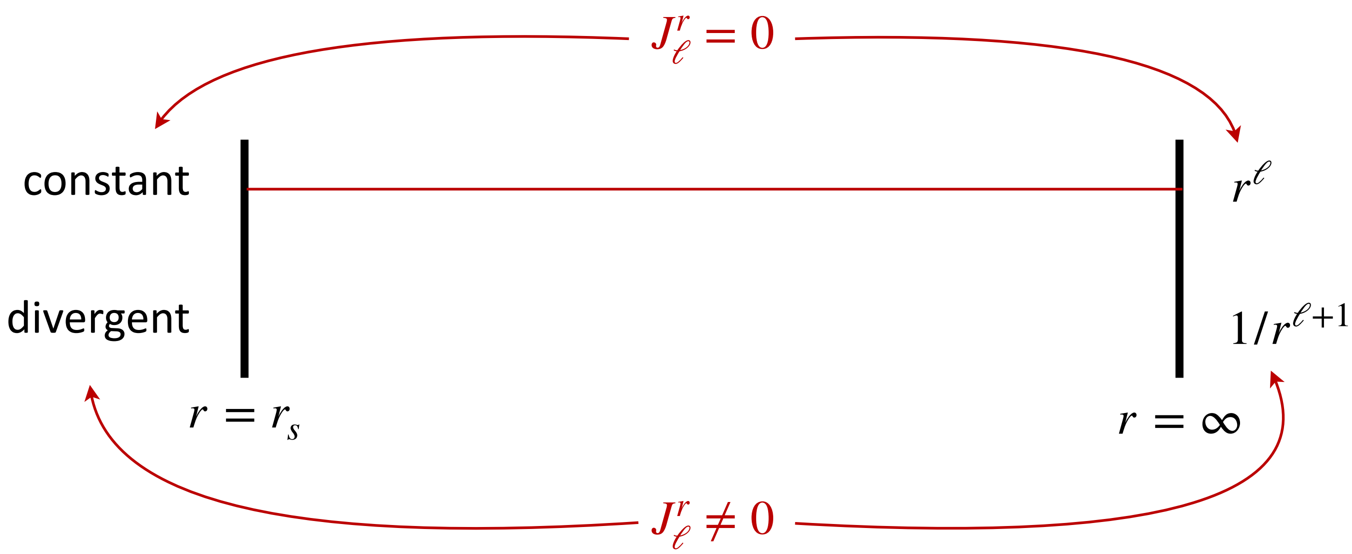
Ladder Symmetries



Vanishing Response in the UV

Current conservation for Q_ℓ symmetries: $\partial_r J^r_\ell = 0$

$$J^r_\ell = \left[\Delta \partial_r \left(D_1^- \dots D_\ell^- \phi_\ell\right)\right]^2$$



$SO(3,1)$ Symmetries

A few comments:

- exact same story for scalar on Kerr, but $J_\ell^r = (\chi'')^2$
- For $s = 1,2$, symmetries of the Teukolsky equation

see also: **Kehagias, Perrone & Riotto [2211.02384]**

- Also a ladder in spin: $\psi_{\ell,s+1} = E_s^+ \psi_{\ell,s}, \qquad \psi_{\ell,s-1} = E_s^- \psi_{\ell,s}$
- Similar ladder structure in AdS_{2n+1}: **transparency**

Compton & Morrison [2003.08023]

Conclusions

Hidden symmetries of black holes:

- conformal Killing vectors of the near-zone metric
  ensure that black holes have vanishing response.

- ladder symmetries
  shed further light on solutions.

Future directions:

- Understanding at the level of the action beyond scalars?
- Do ladder symmetries constrain dissipative response in EFT?
- Systematic EFT understanding of near-zone approximation?

$SO(2,1)$ Symmetries

Charalambous, Dubovsky, Ivanov [2103.01234, 2209.02091]

AdS_2 isometries:

$$T = 2r_s\,\partial_t \qquad\qquad L_\pm = e^{\pm t/2r_s}(2r_s\,\partial_r\sqrt{\Delta}\partial_t \mp \sqrt{\Delta}\partial_r)$$

Solutions finite at r_s belong to finite dim. irreps of $SO(2,1)$

$SO(2,1)$	$SO(3)$
T	J_3
$L_\pm$	$J_\pm$
static	$m=0$
ℓ	ℓ

$SO(2,1)$ Symmetries

Charalambous, Dubovsky, Ivanov [2103.01234, 2209.02091]

A few comments:

- originally derived as symmetries of Teukolsky eq., exist for Kerr and any integer s

- $L_{\pm}$ take static solutions into solutions $\sim e^{\pm t/r_s}$
  - *mix IR & UV outside near-zone regime: **formal trick***

- purely UV argument, no point-particle counterpart.

Conformal Killing Vectors

$$J_{01} = -\frac{2\Delta}{r_s}\cos\theta\,\partial_r - \frac{\partial_r\Delta}{r_s}\sin\theta\,\partial_\theta$$

$$J_{02} = -\cos\phi\left[\frac{2\Delta}{r_s}\sin\theta\,\partial_r - \frac{\partial_r\Delta}{r_s}\left(\frac{\tan\phi}{\sin\theta}\partial_\phi - \cos\theta\partial_\theta\right)\right]$$

$$J_{03} = -\sin\phi\left[\frac{2\Delta}{r_s}\sin\theta\,\partial_r - \frac{\partial_r\Delta}{r_s}\left(\frac{\cot\phi}{\sin\theta}\partial_\phi + \cos\theta\partial_\theta\right)\right]$$

Exact symmetries of the static sector

$$K_\pm = e^{\pm t/2r_s}\frac{\sqrt{\Delta}}{r_s}\cos\theta\left(\frac{r_s^3}{\Delta}\partial_t \mp \partial_r\Delta\partial_r \mp 2\tan\theta\partial_\theta\right)$$

$$M_\pm = e^{\pm t/2r_s}\cos\phi\left[\frac{r_s^2}{\sqrt{\Delta}}\sin\theta\partial_t \mp \frac{\sqrt{\Delta}\partial_r\Delta\sin\theta}{r_s}\partial_r \pm \frac{2\sqrt{\Delta}}{r_s}\cos\theta\partial_\theta \mp \frac{2\sqrt{\Delta}}{r_s}\frac{\tan\phi}{\sin\theta}\partial_\phi\right]$$

$$N_\pm = e^{\pm t/2r_s}\sin\phi\left[\frac{r_s^2}{\sqrt{\Delta}}\sin\theta\partial_t \mp \frac{\sqrt{\Delta}\partial_r\Delta\sin\theta}{r_s}\partial_r \pm \frac{2\sqrt{\Delta}}{r_s}\cos\theta\partial_\theta \pm \frac{2\sqrt{\Delta}}{r_s}\frac{\cot\phi}{\sin\theta}\partial_\phi\right]$$

Ladder Symmetries in de Sitter

Compton and Morrison [2003.08023]

Massive scalar in dS_{d+1}: $\left[-\partial_t^2 - \frac{\nu(\nu+1)}{\cosh^2 t} - k^2\right]\phi = 0, \qquad (\nu = \ell - 1 + d/2)$

For $d+1 =$ odd: **dS transparency**

Similar ladder structure.