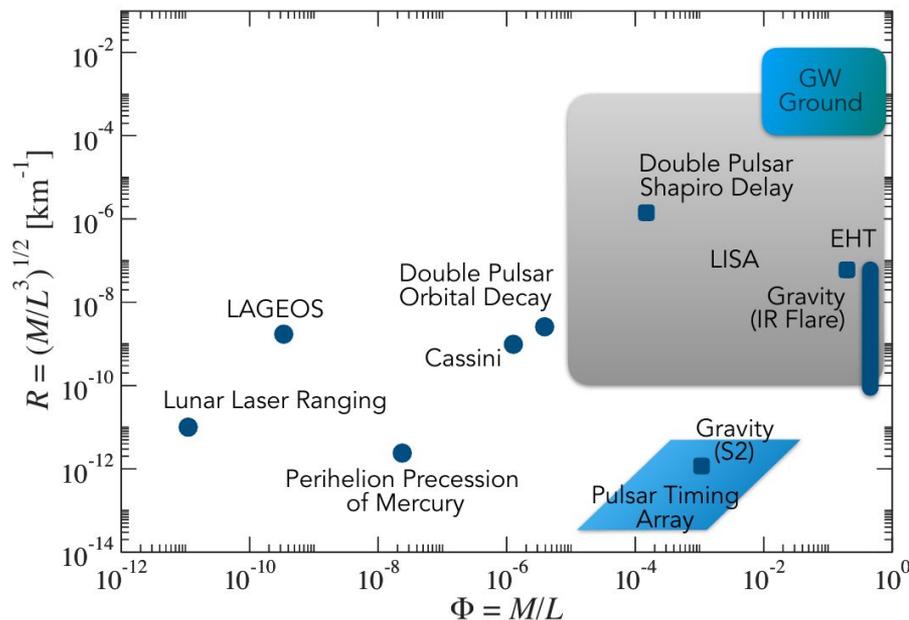


Black hole spectroscopy with the next generation of gravitational wave detectors

Costantino Pacilio (University of Milano Bicocca)

GGI, April 20 2023

Probing gravity with gravitational waves



Ground based GW detectors probe gravity in systems with both:

Strong **potential**

Strong **curvature**

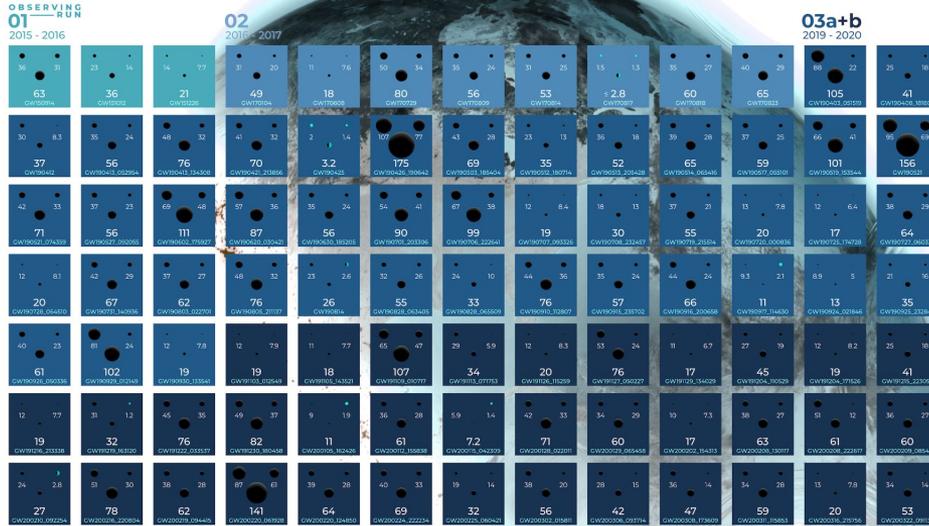
The Riemann curvature at the event horizon of a Schwarzschild metric scales as:

$$\sqrt{R^{abcd} R_{abcd}} \sim M^{-2}$$

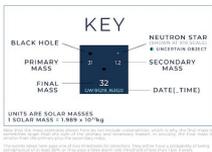
Baker+ (2014), [arxiv 1412.3455](https://arxiv.org/abs/1412.3455)

EXTREME GRAVITY AND FUNDAMENTAL PHYSICS, [arxiv 1903.09221](https://arxiv.org/abs/1903.09221)

Compact binary sources



Binary black holes dominate the catalog of compact binary sources of ground based detections



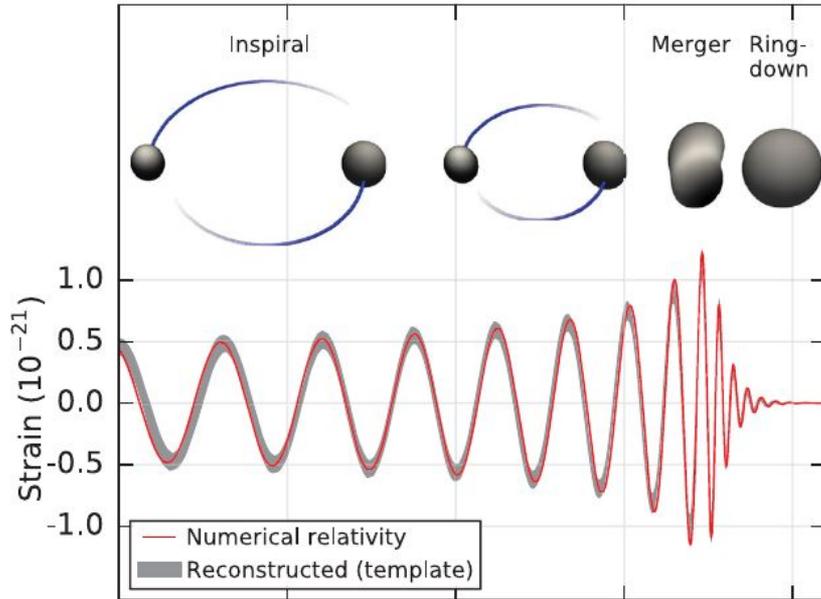
GRAVITATIONAL WAVE
MERGER
DETECTIONS
SINCE 2015



Credit: OzGRav/Carl Knox

Binary black holes and tests of GR

[GW150914 discovery paper](#)



Tests of GR with LVK event catalogs:

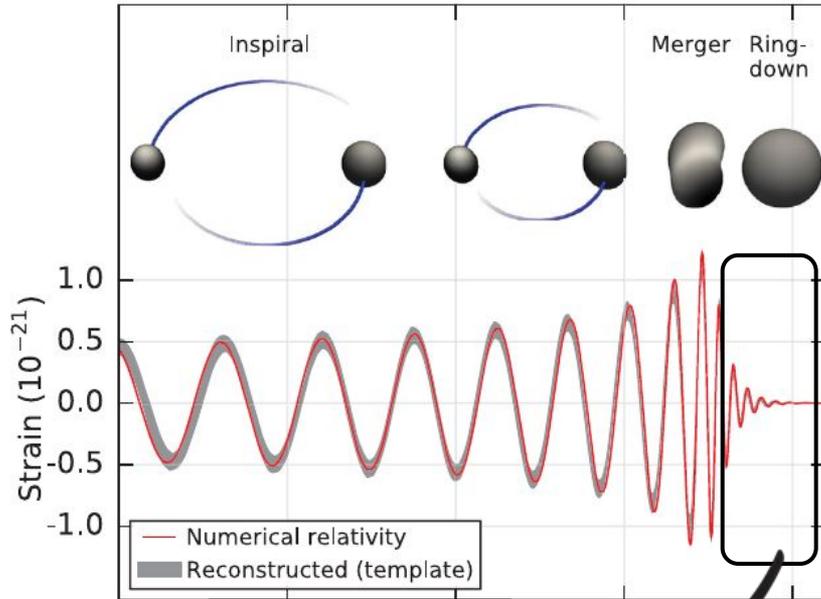
- LKV GWTC-2 (2020), [arxiv 2010.14529](#)
- LVK GWTC-3 (2021), [arxiv 2112.06861](#)

Ghosh (2022), [arxiv 2204.00662](#)

Test	Section	Quantity
RT	4.1	p -value
IMR	4.1	Fractional deviation in remnant mass and spin
PAR	4.2	PN deformation parameter
SIM	4.2	Deformation in spin-induced multipole parameter
MDR	4.3	Magnitude of dispersion, $ A_\alpha $
POL	4.4	Bayes Factors between different polarization hypotheses
RD	4.5	Fractional deviations in frequency (PYRING)
		Fractional deviations in frequency and damping time (PSEOB)
ECH	4.5	Signal-to-noise Bayes Factor

Binary black holes and tests of GR

[GW150914 discovery paper](#)



Black Hole Spectroscopy

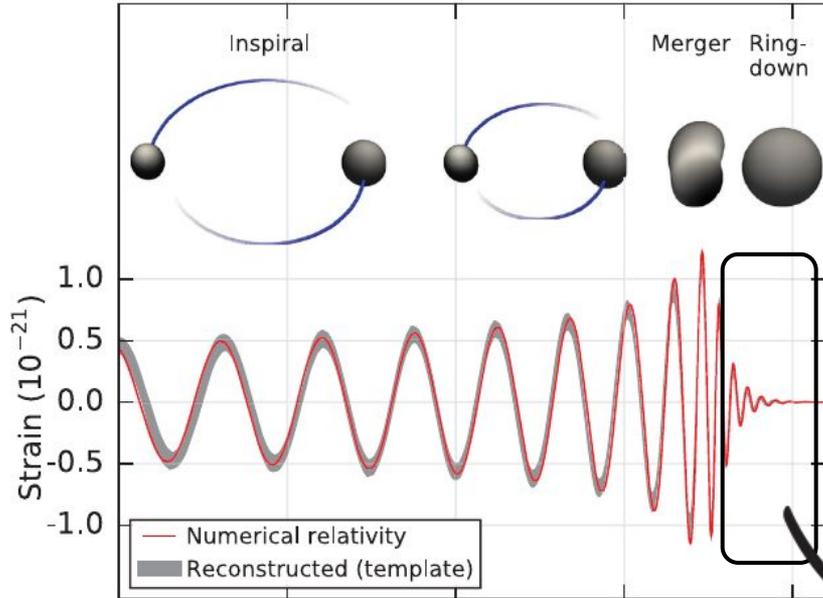
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Quasi-normal modes of black holes

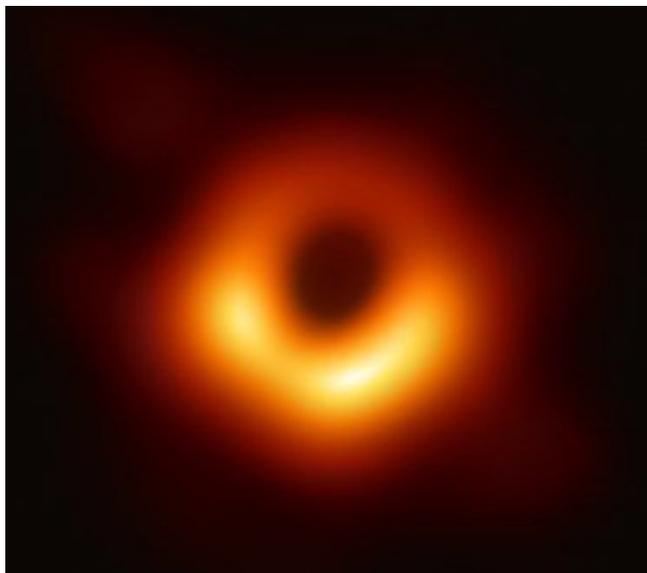


The ringdown is generated by a distorted BH relaxing towards the equilibrium

The signal can be expressed as a superposition of damped sinusoids each with a characteristic frequency and damping time (quasi-normal modes)

$$\omega_{lmn} = 2\pi f_{lmn} + i/\tau_{lmn}$$

No hair theorem



EHT Collaboration (2019),
[arxiv 1906.11238](https://arxiv.org/abs/1906.11238)

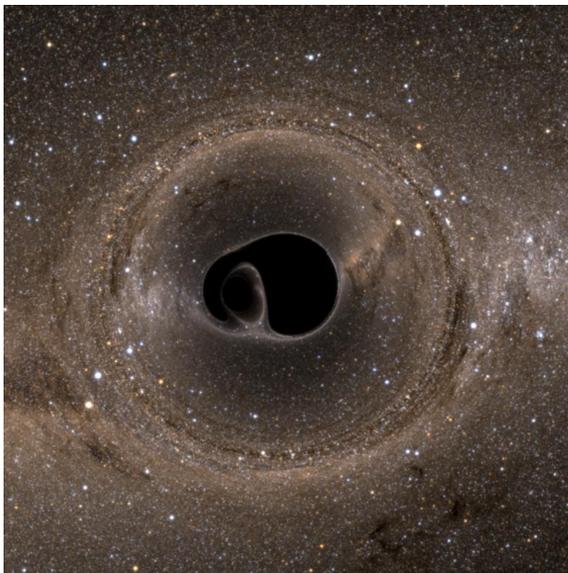
Uniqueness Theorem

Stationary black holes in vacuum GR are uniquely described by the Kerr spacetime metric

$$ds^2 = - \left(1 - \frac{2Mr}{\Sigma} \right) dv^2 + 2dvdr + \Sigma d\theta^2 + \frac{(r^2 + a^2)^2 - \Delta a^2 \sin^2 \theta}{\Sigma} \sin^2 \theta d\bar{\phi}^2 - 2a \sin^2 \theta dr d\bar{\phi} - \frac{4Mra}{\Sigma} \sin^2 \theta dv d\bar{\phi}.$$

Roy P. Kerr
Phys. Rev. Lett. 11, 237 (1963)

Black hole perturbations



Simulation by The SXS (Simulating eXtreme Spacetimes) Project

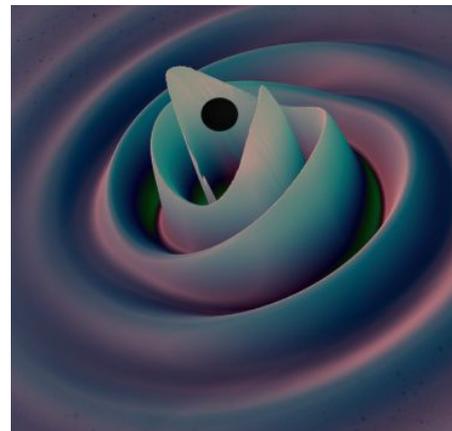
Black hole perturbations

Linear perturbations about black holes are described by Teukolsky's equation

Key prediction

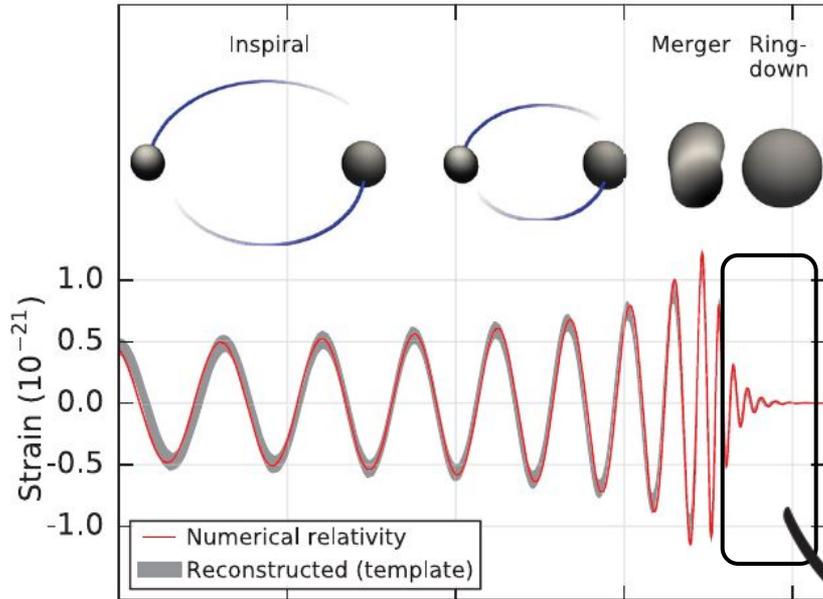
A perturbed black hole emits gravitational waves with a characteristic frequency spectrum

Teukolsky 2014, [arxiv 1410.2130](#)
Berti+ (2009), [arxiv 0905.2975](#)
Pani (2013), [arxiv 1305.6759](#)



Simulation by Georgia Tech, MAYA Collaboration

Quasi normal modes and the no-hair theorem



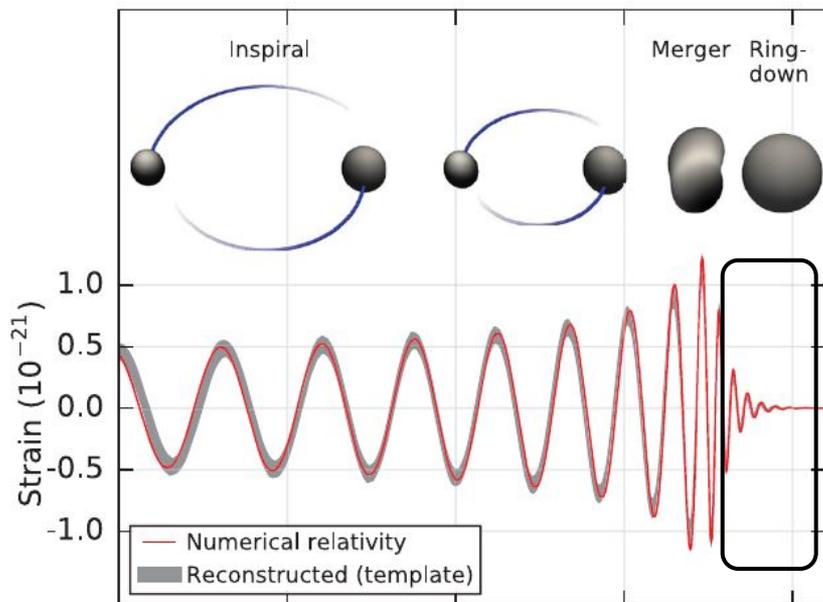
As a consequence of the no-hair theorem, the quasi normal modes depend only on the mass and spin of the BH remnant

Quasi normal modes are countably infinite, but only two of them are independent

$$f_{lmn} \equiv f_{lmn}(M, \chi)$$

$$\tau_{lmn} \equiv \tau_{lmn}(M, \chi)$$

Testing no-hair with black hole spectroscopy



BH spectroscopy:
measuring the frequency and damping times of a BH from its GW signal

GR test:
check that there is no “third” independent frequency or damping time

Berti, Cardoso, Will (2006), [arxiv gr-qc/0512160](#)

Berti+ (2016), [arxiv 1605.09286](#)

Baibhav+ (2023), [arxiv 2302.03050](#)

Gossan+ (2011), [arxiv 1111.5819](#)

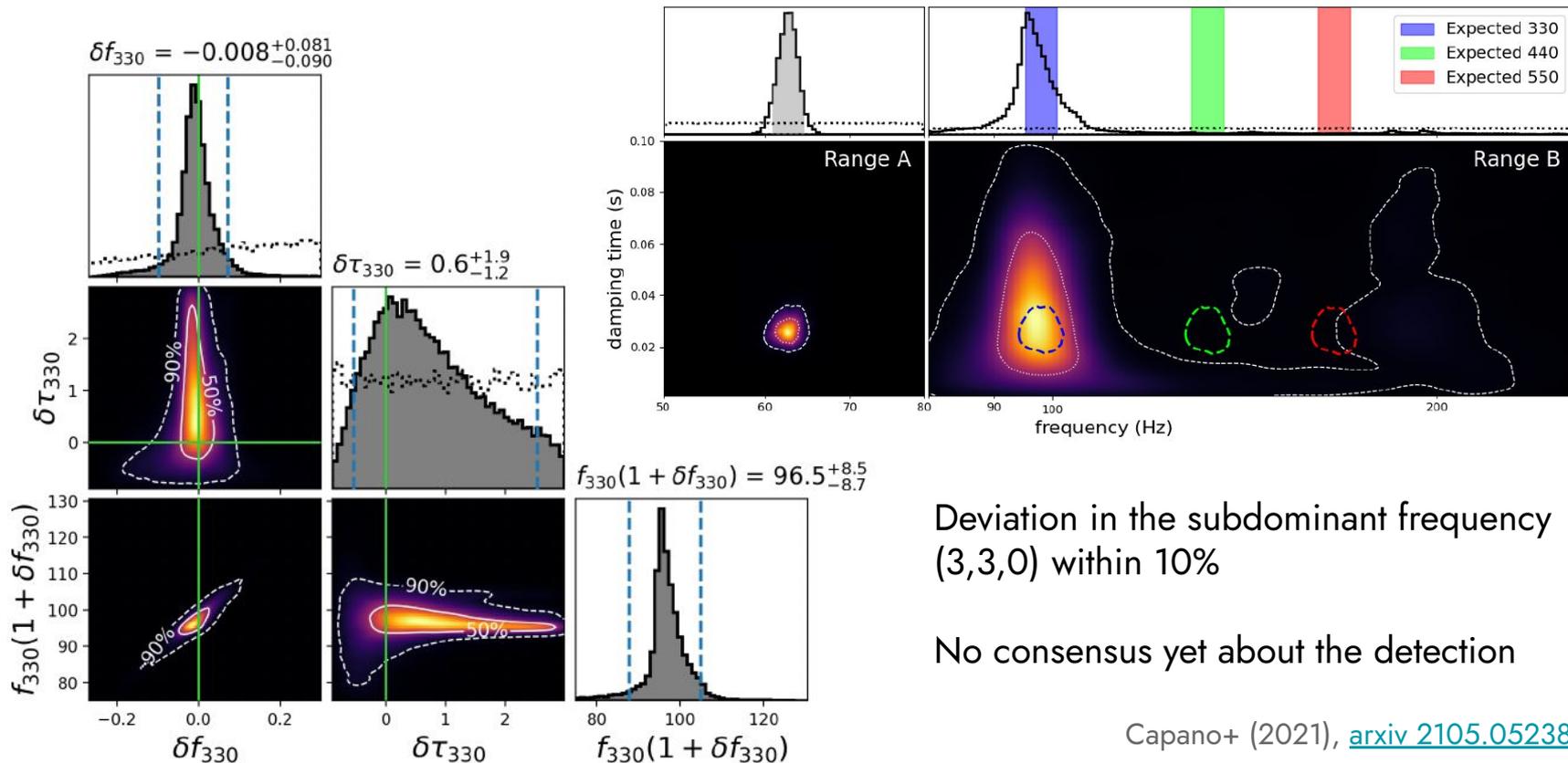
Meidam+ (2014), [arxiv 1406.3201](#)

Even if the background is Kerr, no-hair is violated if the perturbation dynamics differ from Teukolsky's

Barausse & Sotriou (2004), [arxiv 0803.3433](#)

Tattersall+ (2017), [arxiv 1711.01992](#)

GW190521: a case study for LVK

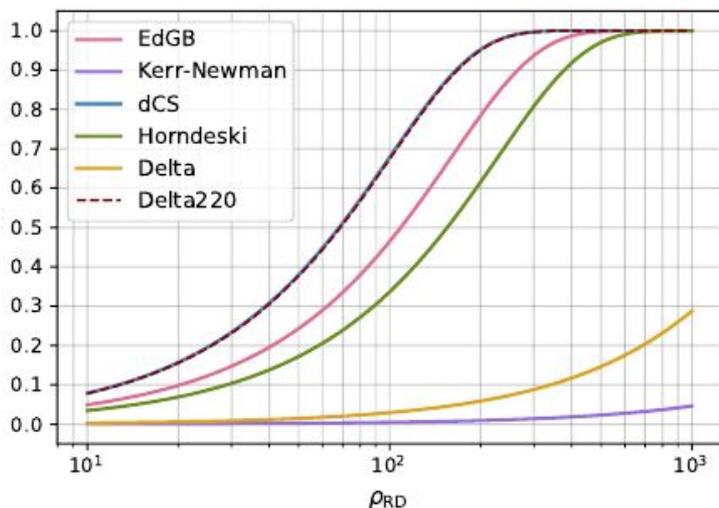


Deviation in the subdominant frequency (3,3,0) within 10%

No consensus yet about the detection

SNR requirements for detecting modified spectra

Computing modified spectra at the non-perturbative level in the spin is challenging



CP & Bhagwat (2023), [arxiv 2301.02267](https://arxiv.org/abs/2301.02267)

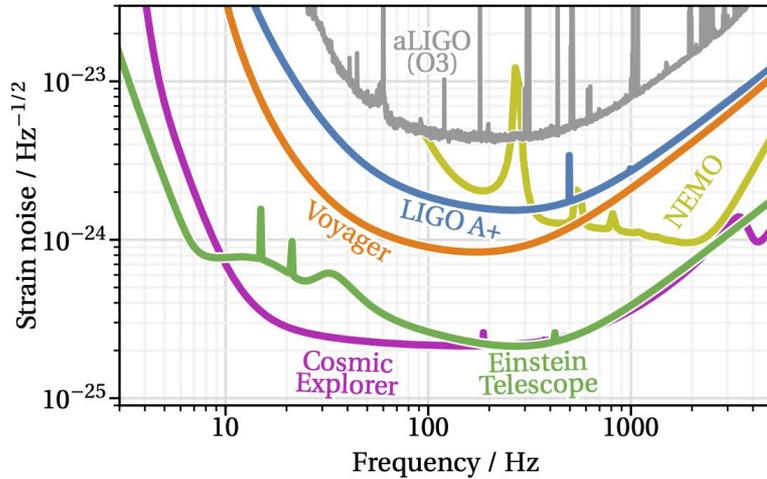
Kerr-Newman: Dias+ (2015) [arxiv 1501.04625](https://arxiv.org/abs/1501.04625)
Horndeski: Tattersall+ (2018) [arxiv 1804.08950](https://arxiv.org/abs/1804.08950)
dCS: Wagle+ (2022), [arxiv 2103.09913](https://arxiv.org/abs/2103.09913)
EdGB: Pierini & Gualtieri (2022), [arxiv 2207.11267](https://arxiv.org/abs/2207.11267)
Recent results:
Cano+ (2023), [arxiv 2304.02663](https://arxiv.org/abs/2304.02663)
Ghosh+ (2023) [arxiv 2303.00088](https://arxiv.org/abs/2303.00088)

There are very degenerate spectra (Delta-like) or less degenerate spectra (Delta₂₂₀-like)

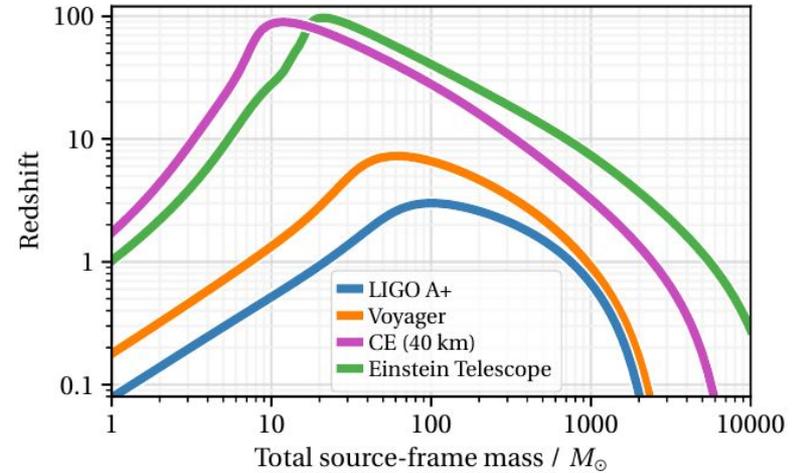
SNR larger than ~200 is required to confidently tell non-GR apart from GR

From a great power ...

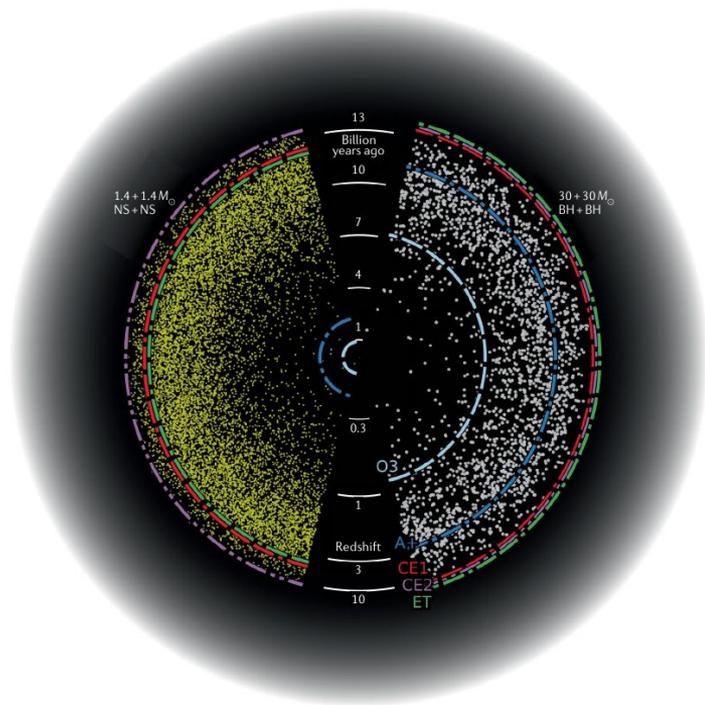
3G detectors will improve sensitivity by a factor of ~ 10



They will be sensitive to sources at cosmological distances

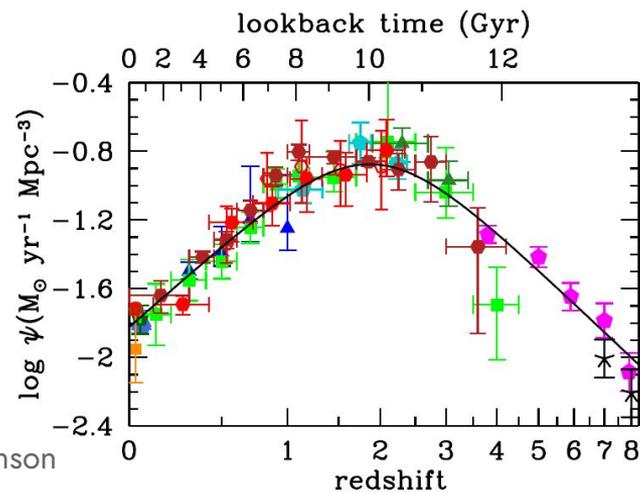


... come responsible expectations



Most of the events are placed at redshift larger than 1 and therefore, they will present low SNR

This can be traced back to the star formation rate



Madau & Dickinson
(2014), [arxiv](#)
[1403.0007](#)

The landscape of black hole spectroscopy

Bhagwat, CP+ (2023) , [arxiv 2304.02283](https://arxiv.org/abs/2304.02283)

With: S. Bhagwat, P. Pani, M. Mapelli

We consider a population of stellar mass binary black holes in agreement with the LVK population constraints



Generated with the pop synth code MOBSE and described in Mapelli et al. 2021, [arxiv 2109.06222](https://arxiv.org/abs/2109.06222)



Compute the expected SNR for each event in the catalog

Compute the constraints in the deviation of the subdominant modes from the GR predictions



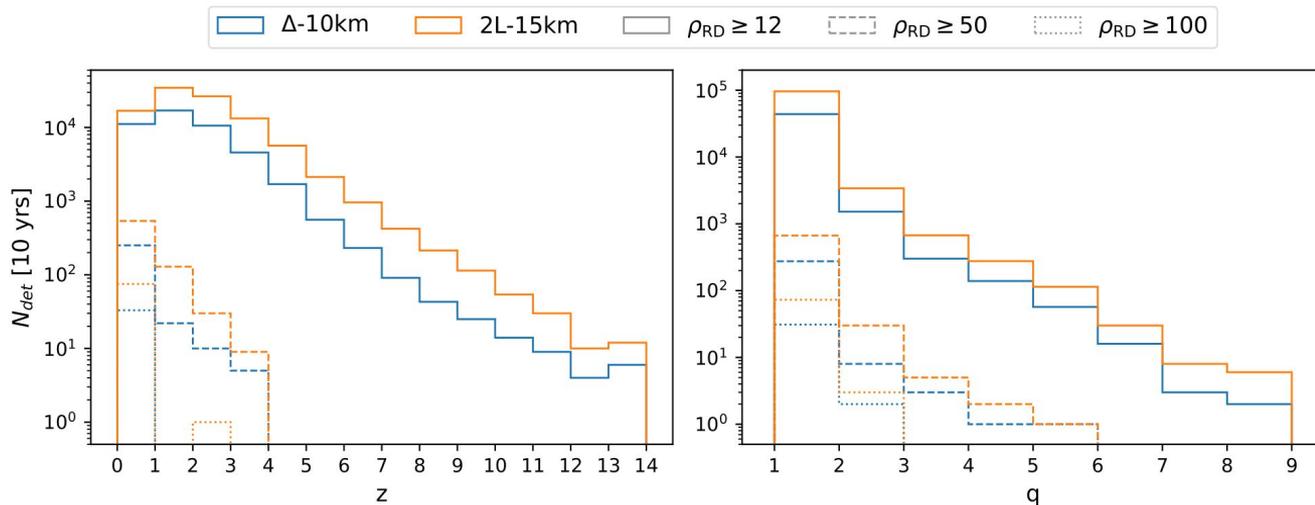
$\sim O(10^5)$ events / yr

Total duration: 10 years

Einstein Telescope: SNR prospects

NOTE: We use the updated sensitivities and detector designs from the recent ET paper [arxiv 2303.15923](https://arxiv.org/abs/2303.15923)

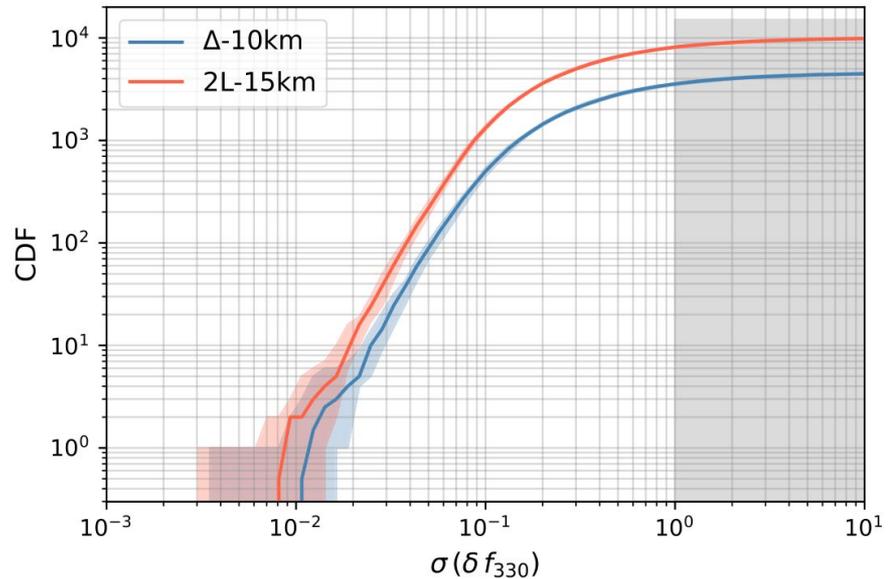
Configuration	$\rho_{\text{RD}} \geq 12 \text{ yr}^{-1}$	$\rho_{\text{RD}} \geq 50 \text{ yr}^{-1}$	$\rho_{\text{RD}} \geq 100 \text{ yr}^{-1}$	$\max(\rho_{\text{RD}})$
Δ -10km	4594 ± 61	28 ± 7	3 ± 1	1134
2L-15km	10071 ± 88	70 ± 9	7 ± 3	1262



ET: measuring deviations from GR

$$(l, m, n) \neq (2, 2, 0) \quad f_{lmn} = f_{lmn}^{\text{GR}} (1 + \delta f_{lmn}) \quad \tau_{lmn} = \tau_{lmn}^{\text{GR}} (1 + \delta \tau_{lmn})$$

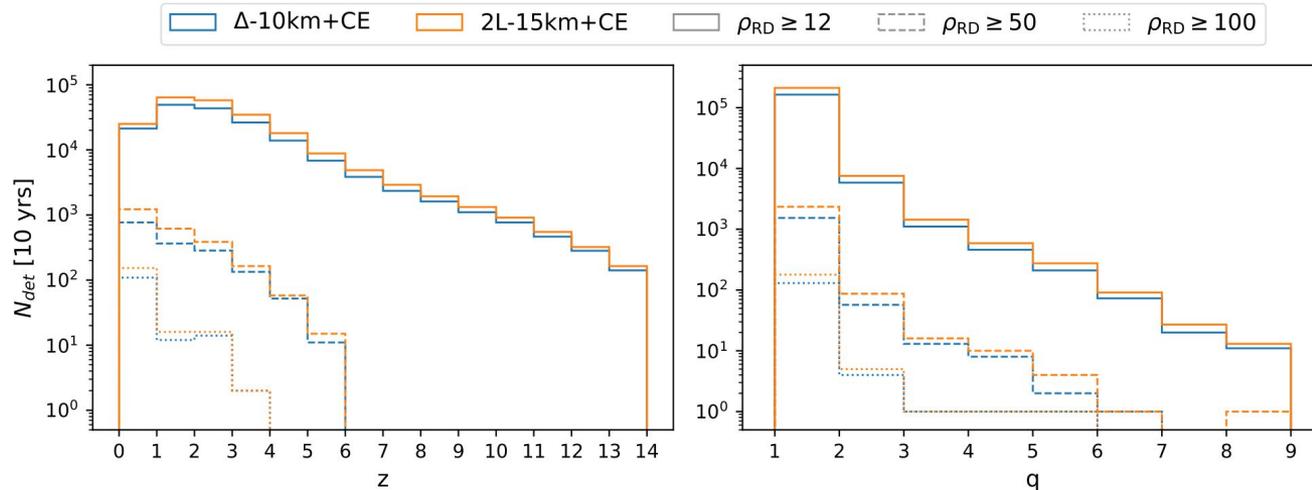
- Damping times are poorly measured overall the catalog
- 500-to-1000 events/yr with $\mathcal{O}(10)$ % measurability
- 1-to-few events/yr with $\mathcal{O}(1)$ % measurability (but larger variance within the catalog)



ET + Cosmic Explorer: SNR prospects

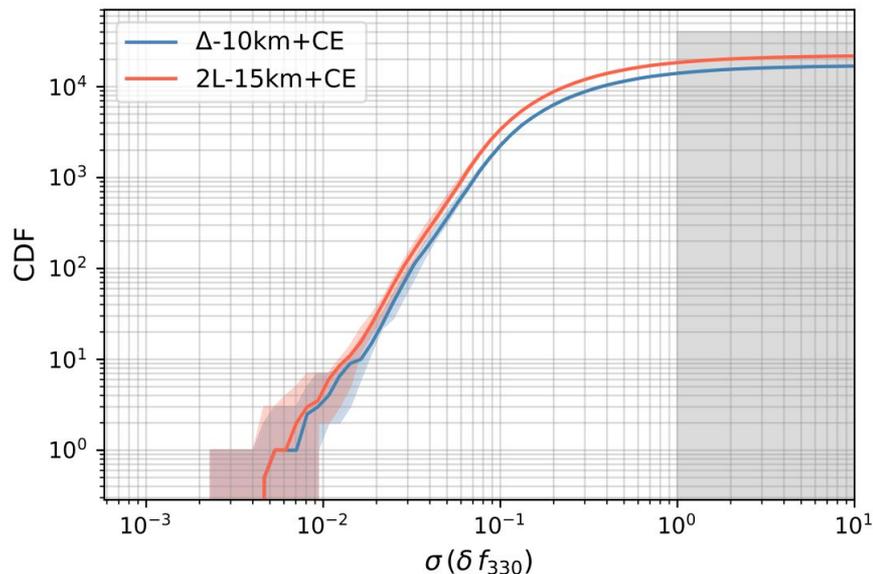
NOTE: We use the baseline 40km CE

Configuration	$\rho_{\text{RD}} \geq 12 \text{ yr}^{-1}$	$\rho_{\text{RD}} \geq 50 \text{ yr}^{-1}$	$\rho_{\text{RD}} \geq 100 \text{ yr}^{-1}$	$\max(\rho_{\text{RD}})$
Δ -10km+CE	17174 ± 115	161 ± 14	13 ± 5	1508
2L-15km+CE	22144 ± 122	246 ± 16	18 ± 7	1607



ET + CE: measuring deviations from GR

- 2000-to-4000 events/yr with $O(10)$ % measurability
- ~ 10 events/yr with $O(1)$ % measurability (but larger variance within the catalog)
- Back of the envelope **stacking**: constraining GR at the **sub-percent** level

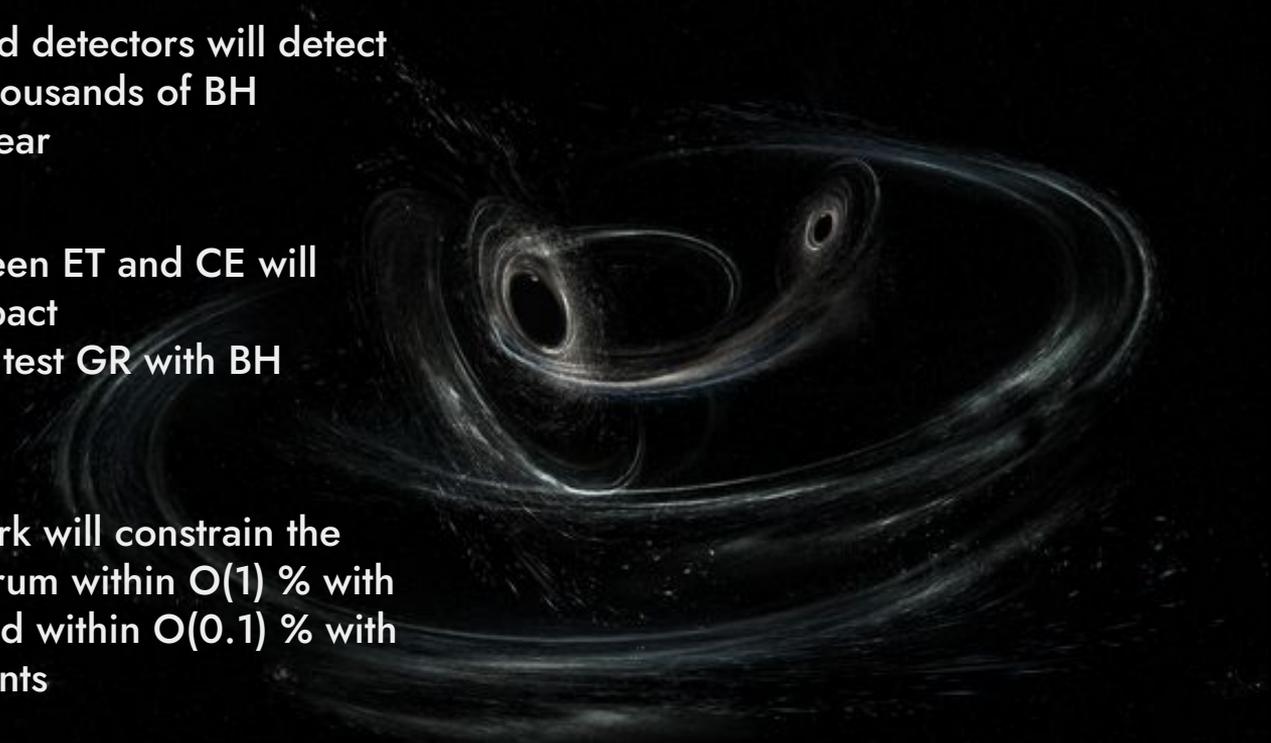


Conclusions

3G ground based detectors will detect hundreds—to—thousands of BH ringdowns per year

A synergy between ET and CE will have a large impact on our ability to test GR with BH spectroscopy

A full 3G network will constrain the ringdown spectrum within $O(1)$ % with single events and within $O(0.1)$ % with a catalog of events



Thank You !

