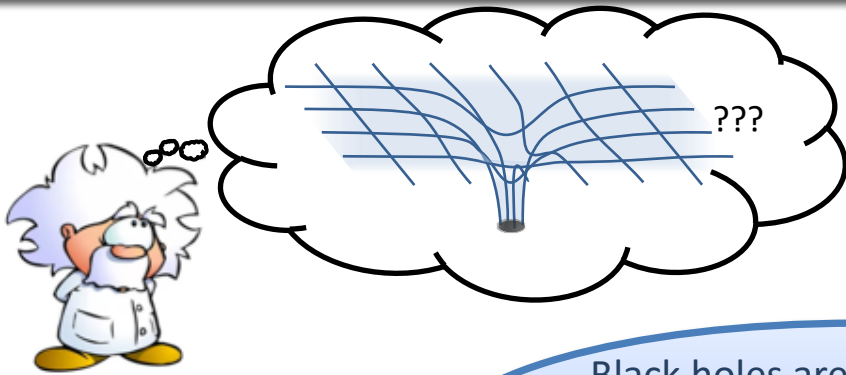


# Geometric Resolution of Schwarzschild Horizon

GGI Workshop,  
April 19 2023

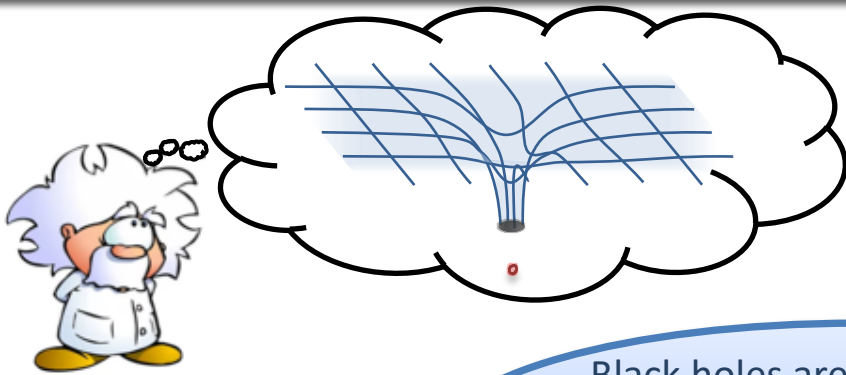
**Pierre Heidmann**

# Black Hole Puzzles



Black holes are at the center of  
**theoretical conflicts** between  
GR and QM

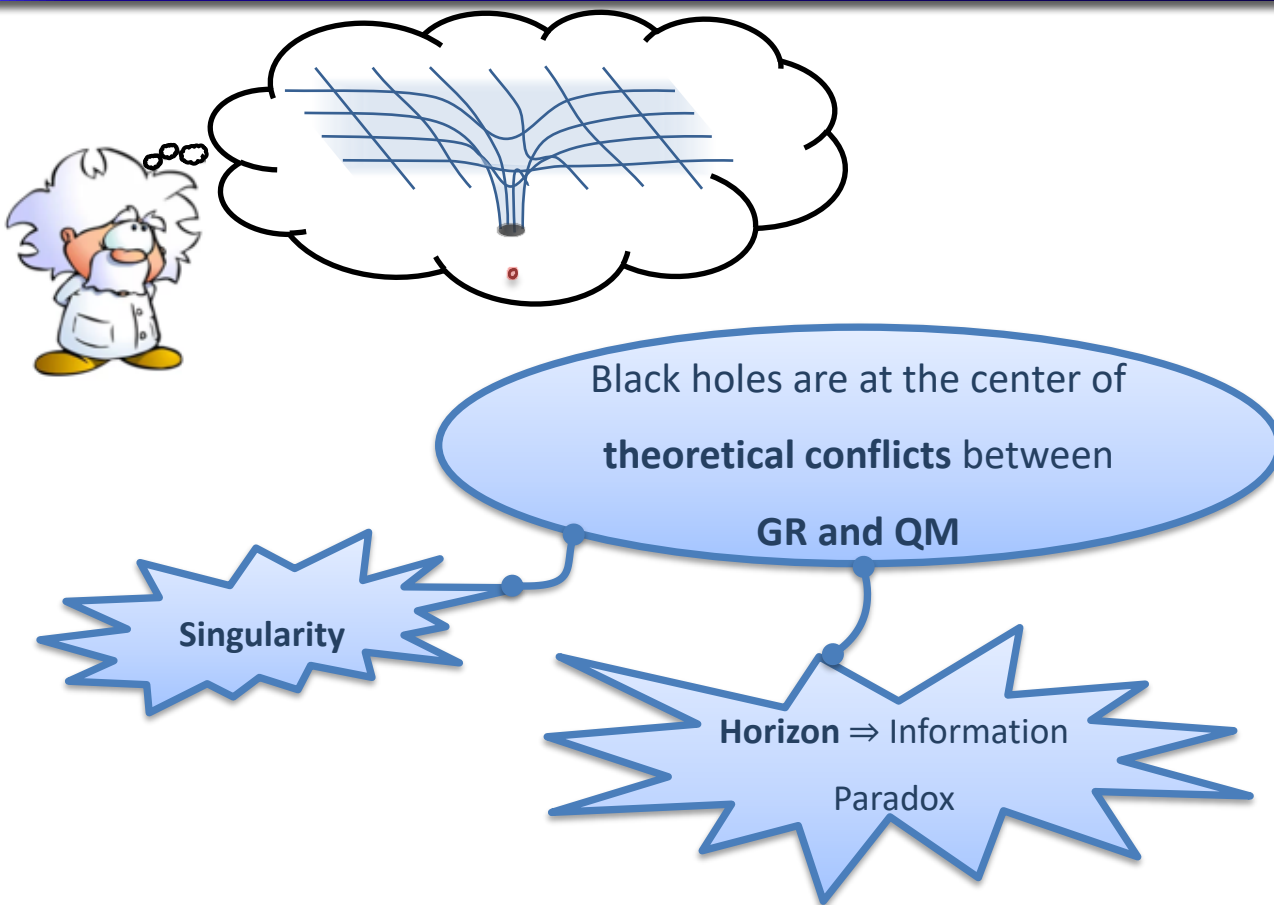
# Black Hole Puzzles



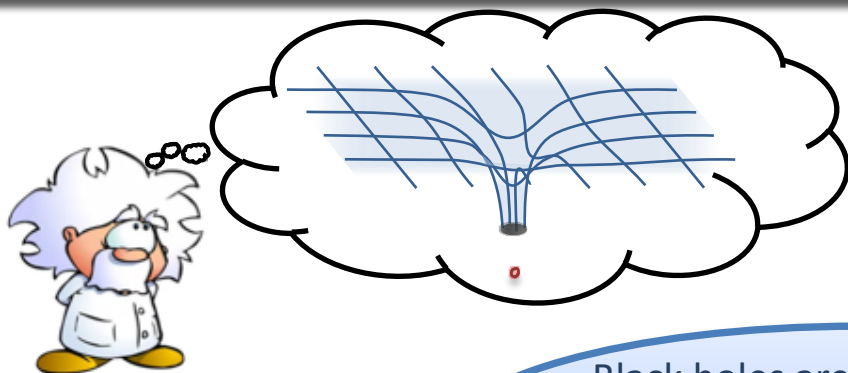
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Singularity

# Black Hole Puzzles



# Black Hole Puzzles



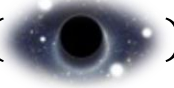
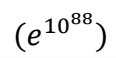
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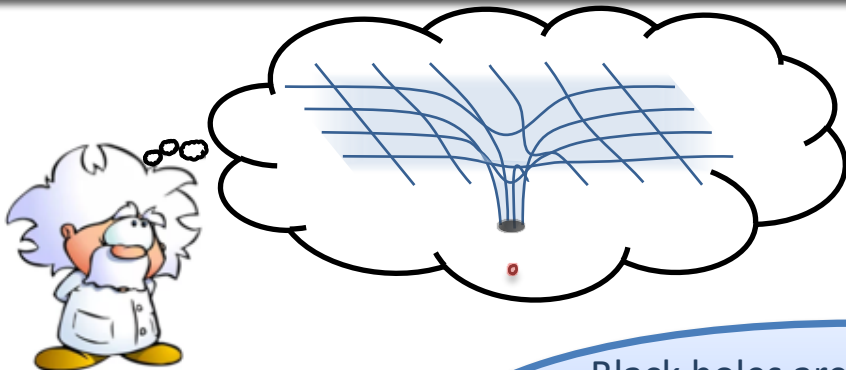
Horizon  $\Rightarrow$  Information  
Paradox

Microscopic Description of BH  
entropy

# states (  )  $\gg$  # states (  ) !

( $e^{10^{90}}$ ) ( $e^{10^{88}}$ )

# Black Hole Puzzles



Black Hole:  
Quantum Gravitational Phase of Matter?  
**New Quant. Grav. States!**

Black holes are at the center of theoretical conflicts between

GR and QM

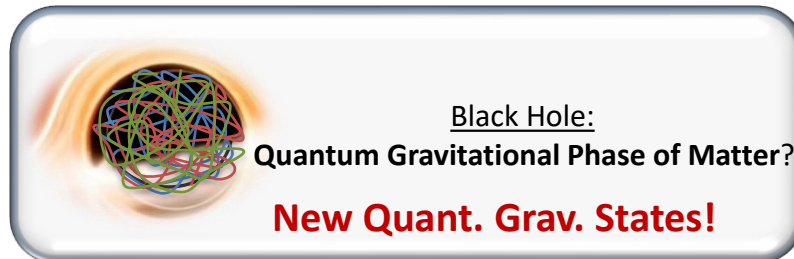
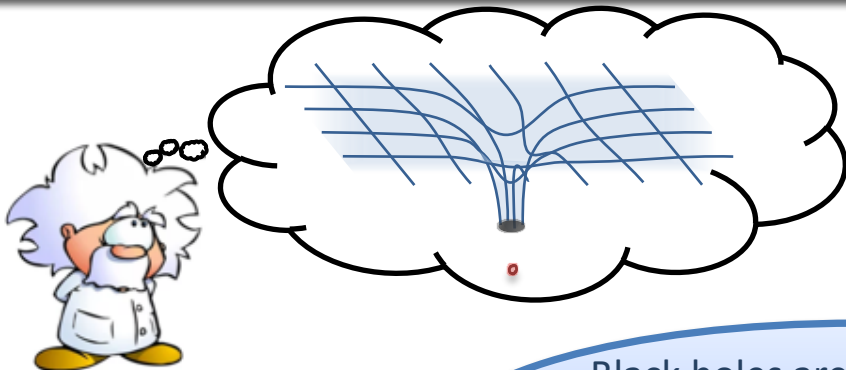
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$$\# \text{ states } \left( \begin{array}{c} \text{BH} \\ (e^{10^{90}}) \end{array} \right) \gg \# \text{ states } \left( \begin{array}{c} \text{ } \\ (e^{10^{88}}) \end{array} \right) !$$

# Black Hole Puzzles



Black holes are at the center of theoretical conflicts between

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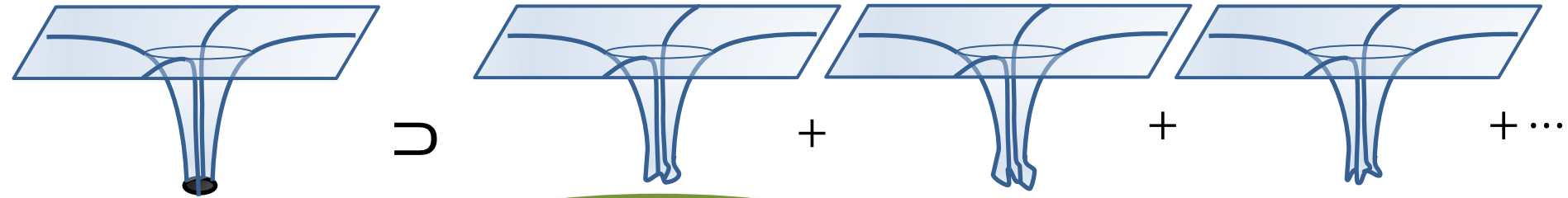
Singularity

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Microscopic Description of BH  
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What are the new degrees of freedom from Quantum Gravity that make black holes?

# Supersymmetric Paradigm



Part of the  $e^S$  microstates of  
SUSY BH can be described by  
**Gravitational Solitons**

Coherent states of  
quantum gravity

Topological deformations

Resolve the singularity &  
horizon into **smooth**  
**microstructure**

Extra compact dimensions

EM Flux

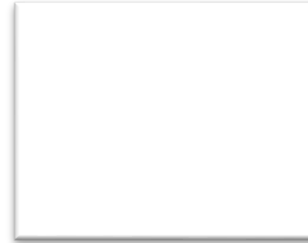
Do **coherent QG states** extend beyond supersymmetry? Can  
they resolve **horizons of non-extremal BH**?



- 1. Vacuum solitons in gravity**
- 2. Charged solitons in gravity**
- 3. Solitons that resolve Schwarzschild horizon**
- 4. Some gravitational properties**

# On Solitons in Gravity

- Soliton:
  - “perpetual” state of a non-linear theory
  - ~ fundamental particle but regular



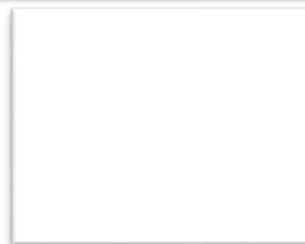
- Existence in 4d General Relativity?

Serini 1918, Einstein, Pauli '41, Breitenlohner, Maison, Gibbons '87

- $S = \int d^4x \sqrt{-g} R$
- Smooth & horizonless
- Pure deformation of spacetimes
- Finite energy & stable

# On Solitons in Gravity

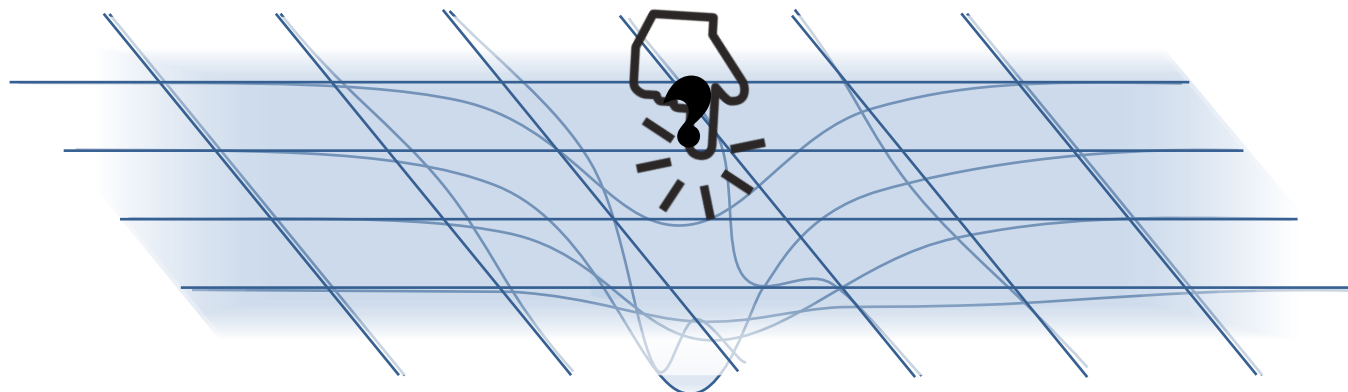
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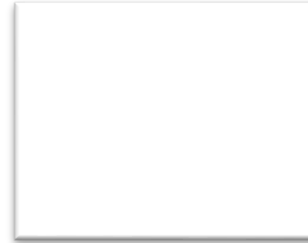
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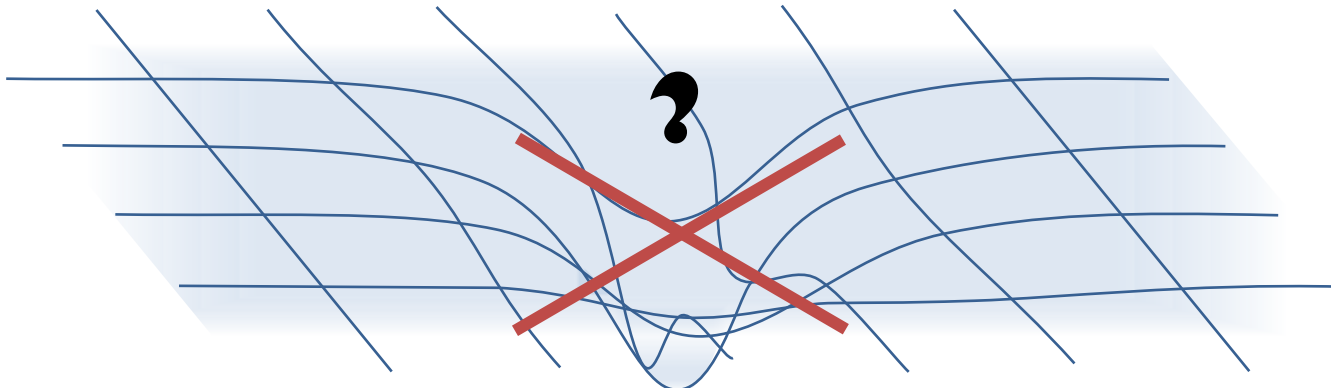


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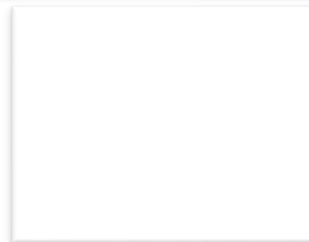
- $S = \int d^4x \sqrt{-g} R$
- Smooth & horizonless
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**No!**



# On Solitons in General Relativity

- Soliton:
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  - ~ fundamental particle but regular



- Existence in 4d General Relativity? Serini 1918, Einstein, Pauli '41, Breitenlohner, Maison, Gibbons '87

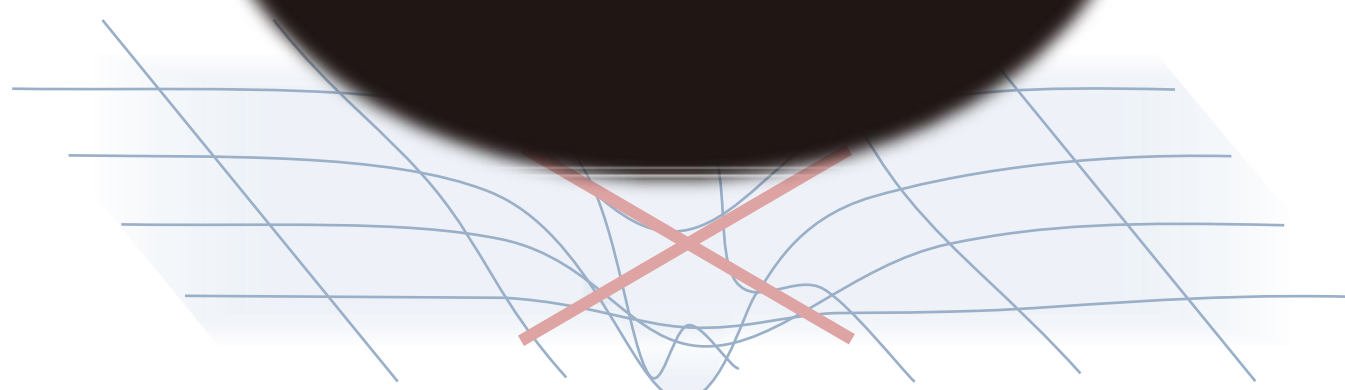
- $S = \int d^4x \sqrt{-g} \left( R - \frac{1}{4} F^2 \right)$

- Smooth & ~~no~~ holeless
- Pure deformation of
- Finite energy & s

**No!**

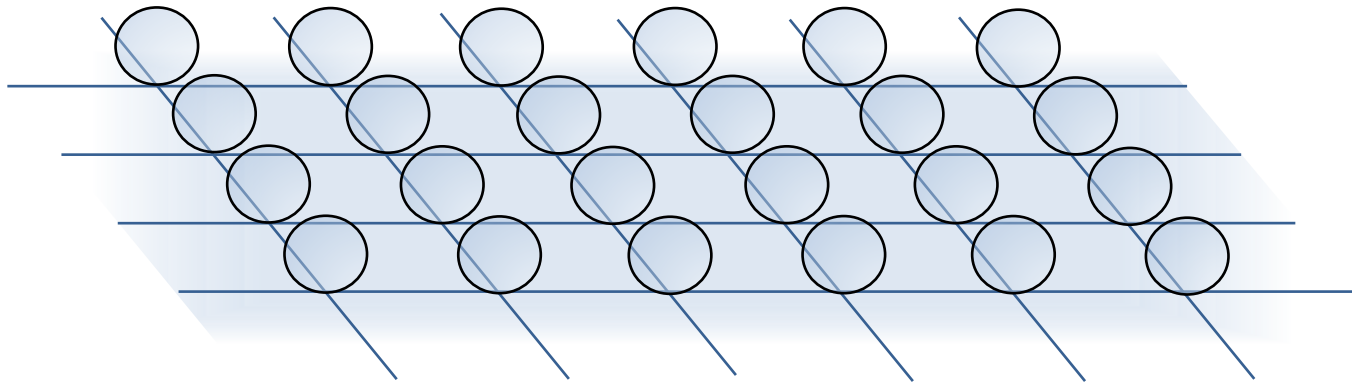


No-go Theorem *ns in 4d GR*

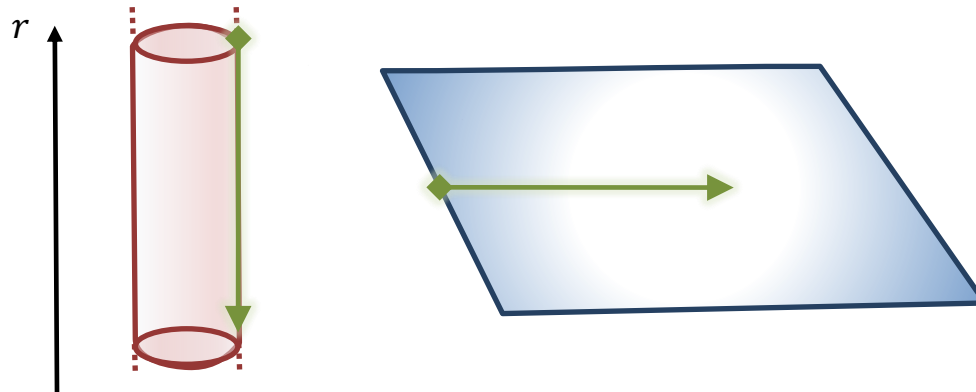


# Solitons with extra compact dimensions

- Gravity with extra compact dimensions:
  - 4d spacetime + a compact dimension in vacuum
  - Deformation of the circle  $\Rightarrow$  Deformation of the 4d spacetime
  - **Topological Soliton:** state of gravity produced by the degeneracy of the circle

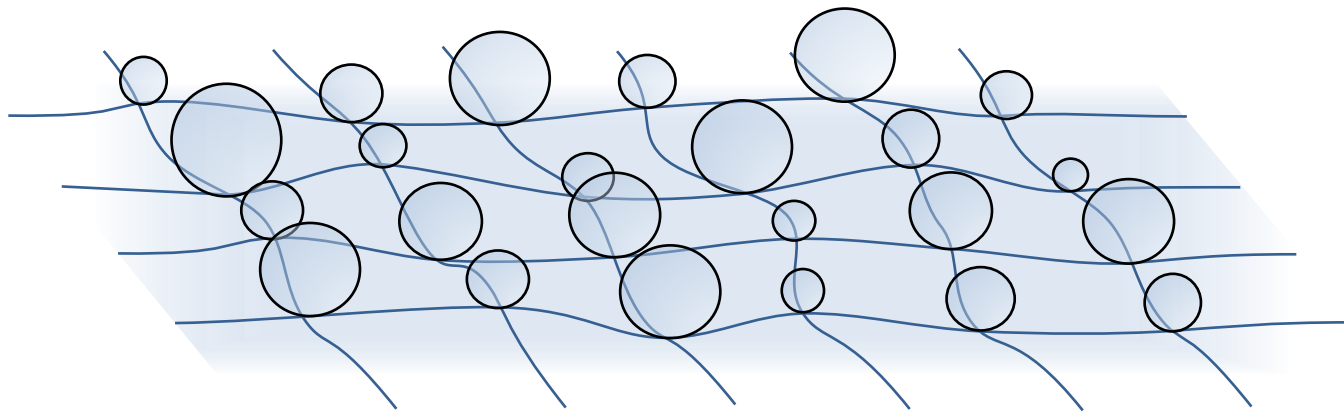


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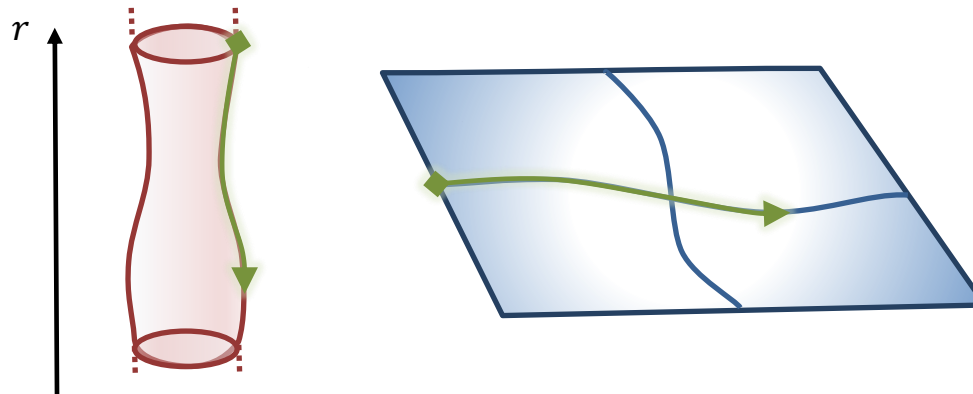


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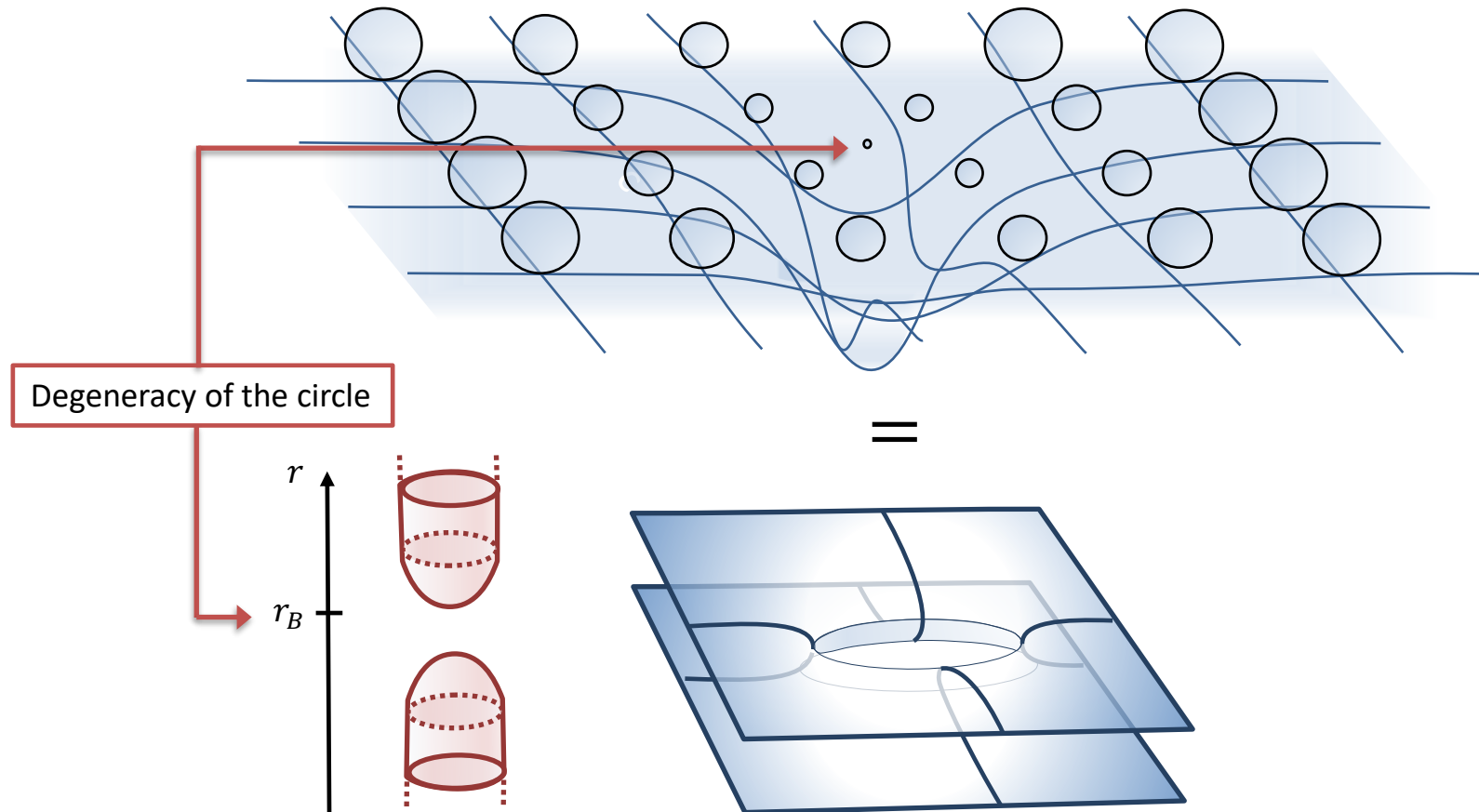


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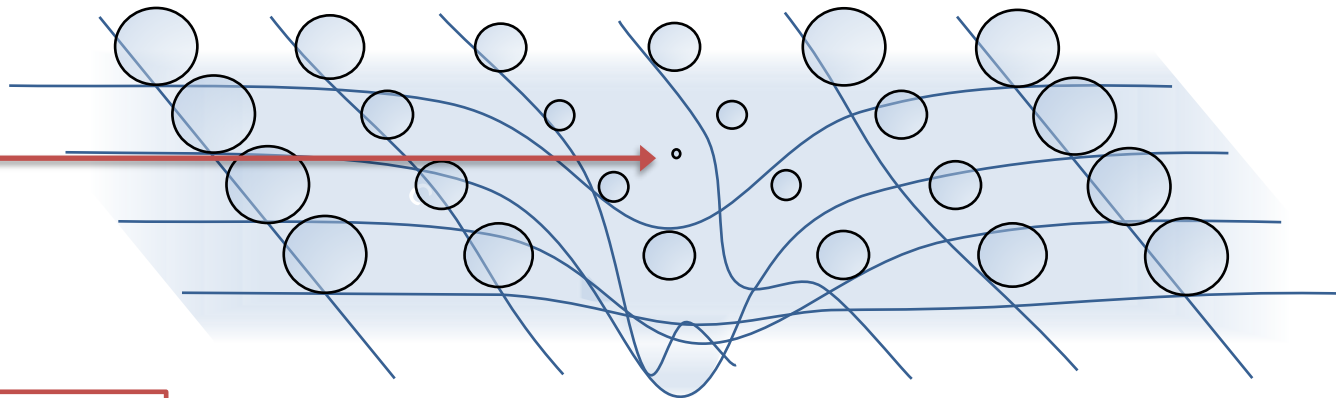
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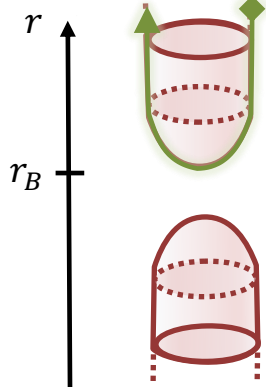


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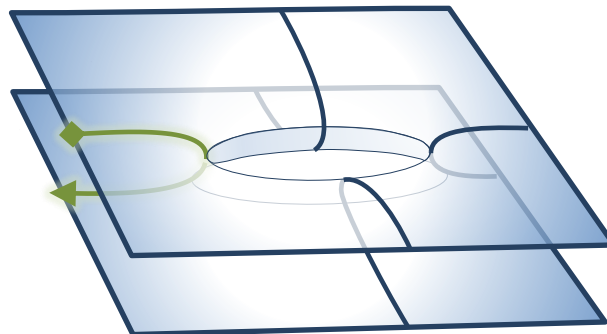
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Degeneracy of the circle

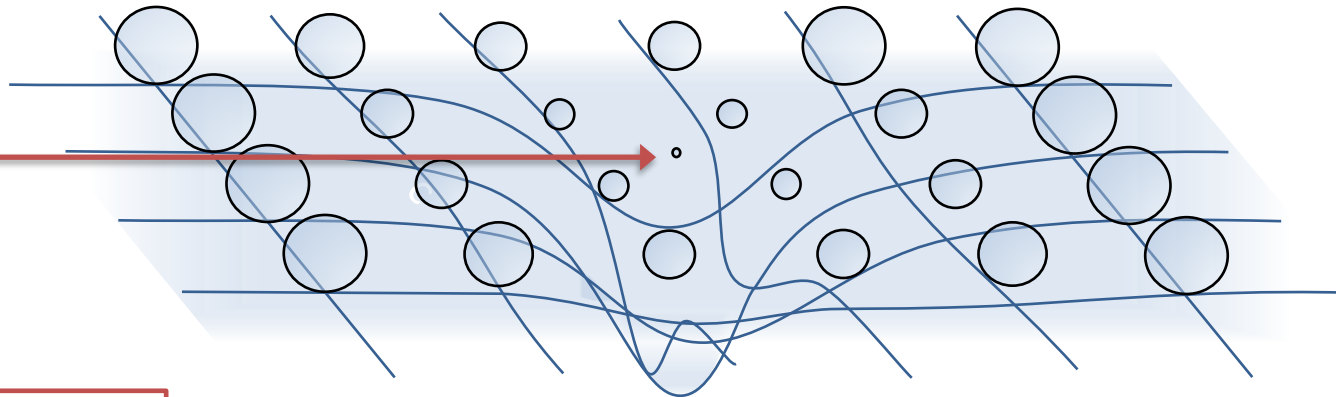


=  
Spacetime "bubble of nothing" Witten '81



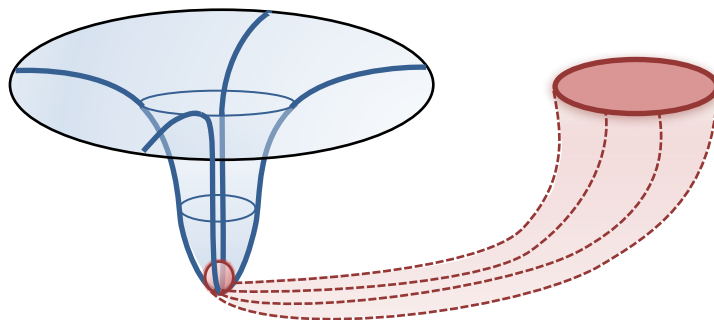
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Degeneracy of the circle

=



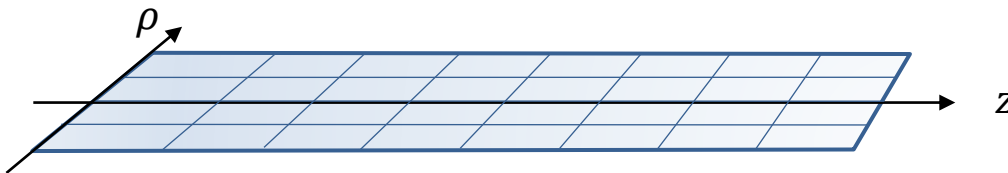
This is the  
**gravitational  
phase of matter**  
we are looking for!

# Solution Generating Technique

- Challenge: Einstein equations
- Weyl formalism in 4d GR: [Weyl 1917](#)

$$ds_4^2 = -W_0 dt^2 + \frac{1}{W_0} [e^{2\nu} (d\rho^2 + dz^2) + \rho^2 d\phi^2]$$

- Generalized Weyl formalism [Empanan, Reall '02](#)



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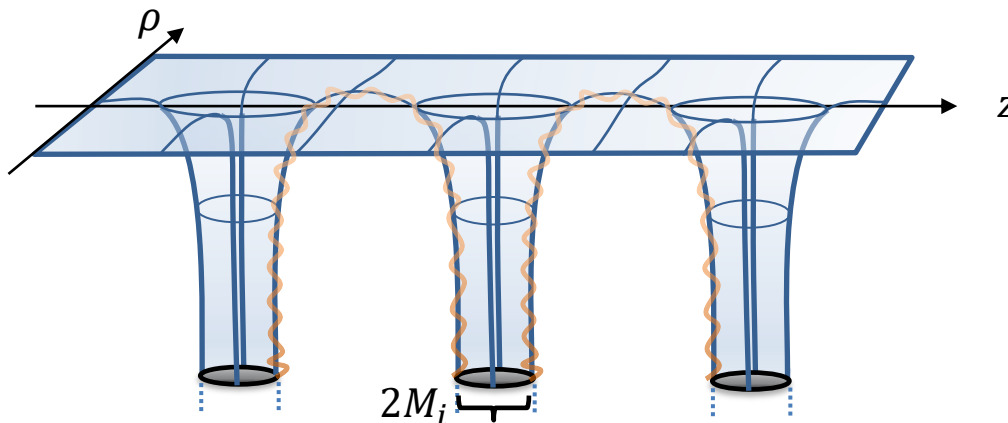
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[Bah, PH '20](#)

Linear Einstein Equations!

$$W_0 = \prod_i \left( 1 - \frac{2M_i}{r_i(\rho, z)} \right)$$



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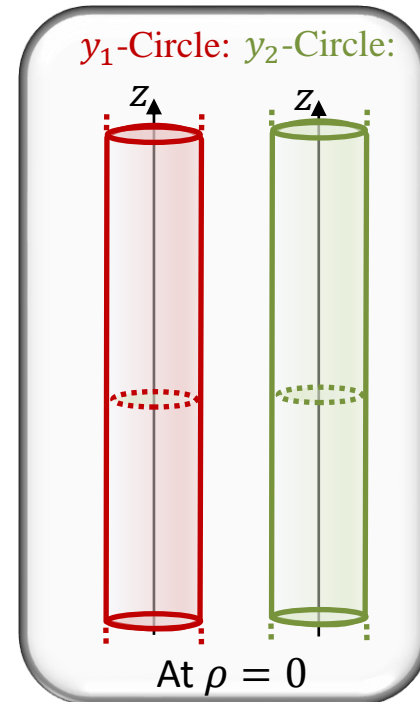
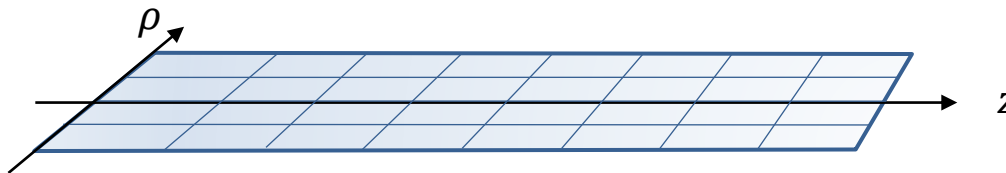
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Linear Einstein Equations!

$$W_\Lambda = \prod_i \left( 1 - \frac{2M_i}{r_i(\rho, z)} \right)^{D_i^{(\Lambda)}}$$

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$$ds_D^2 = -W_0 dt^2 + \frac{1}{\prod W_\Lambda} [e^{2\nu}(d\rho^2 + dz^2) + \rho^2 d\phi^2] + \sum_I W_I dy_I^2$$



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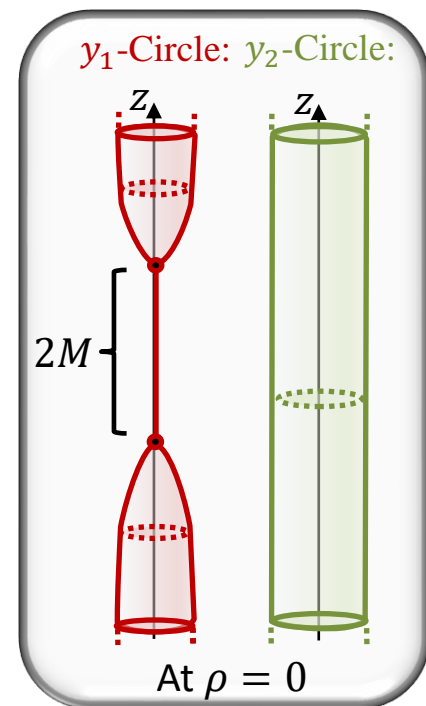
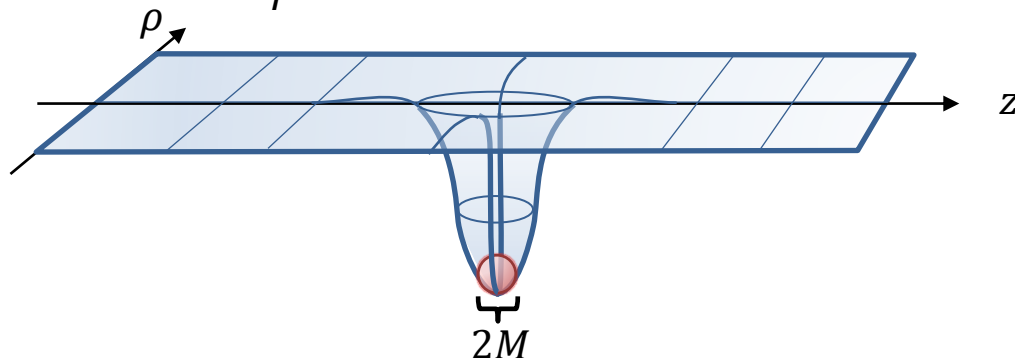
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Contains the bubble of nothing

Regularity:  $2M = R_{y_1}$

$$ds_D^2 = -dt^2 + \frac{dr^2}{1 - \frac{2M}{r}} + r^2 (d\theta^2 + \sin^2 \theta d\phi^2) + \left( 1 - \frac{2M}{r} \right) dy_1^2 + \dots$$



# Solution Generating Technique

- Challenge: Einstein equations

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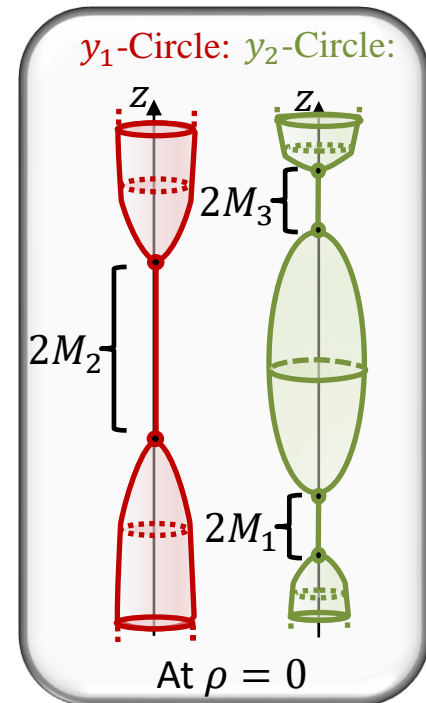
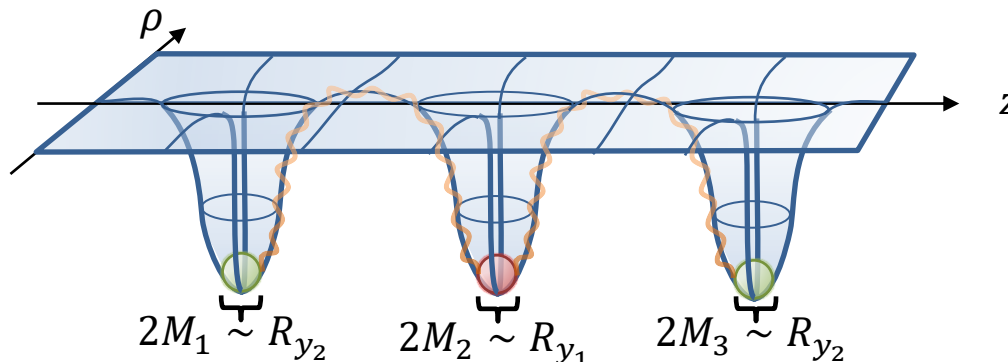
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Contains the bubble of nothing

Bound states of bubbles



# Solution Generating Technique

- Challenge: Einstein equations

- Weyl formalism in 4d GR: Weyl 1917

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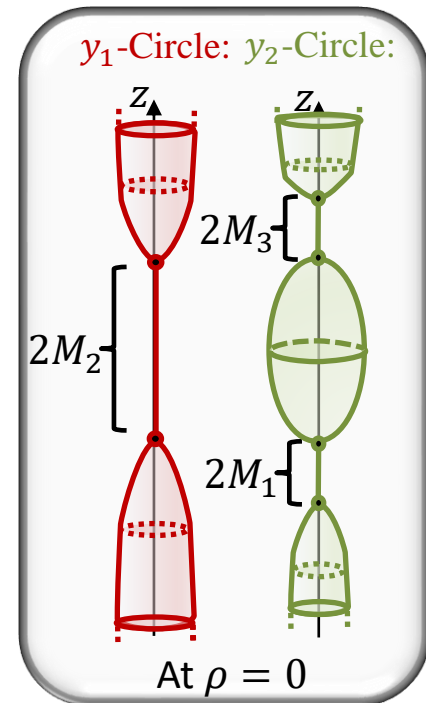
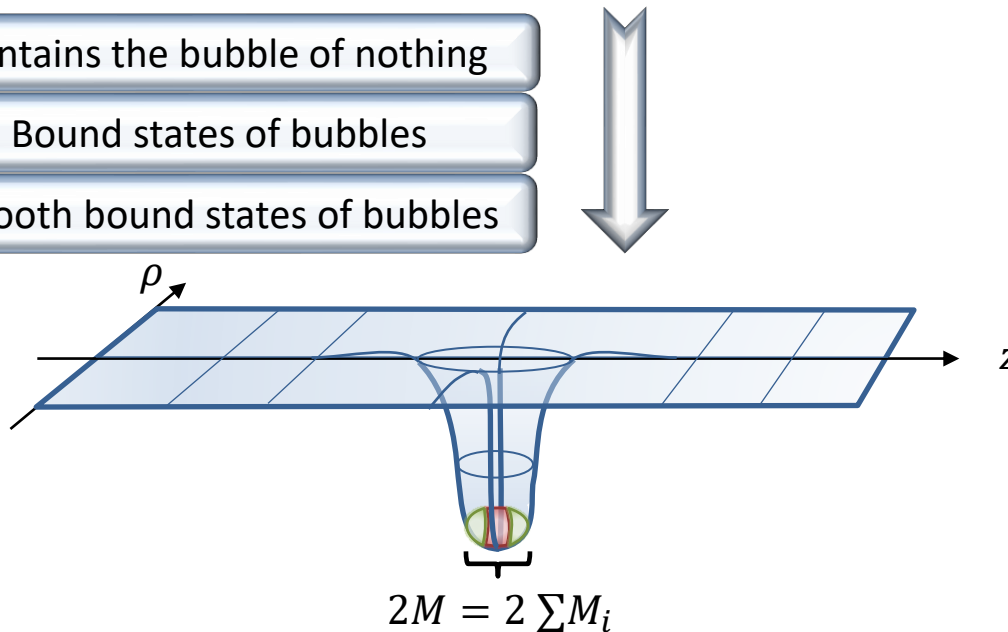
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- Generalized Weyl formalism Empanan, Reall '02  
Bah, PH '20

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- Contains the bubble of nothing
- Bound states of bubbles
- Smooth bound states of bubbles





# Vacuum Topological Solitons

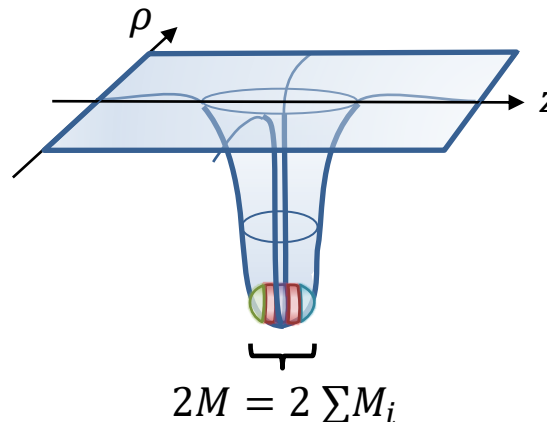
- Class of **Vacuum Topological Solitons** induced by the degeneracy of compact directions Bah, PH '20

$$ds_D^2 = -dt^2 + H \left[ \frac{dr^2}{1 - \frac{2M}{r}} + r^2 d\theta^2 \right] + r^2 \sin^2 \theta d\phi^2 + \sum_I \prod_{i=1}^N \left( 1 - \frac{2M_i}{r_i} \right)^{D_i^{(\Lambda)}} dy_i^2 \quad D_i^{(\Lambda)} = 0 \text{ or } 1.$$

accounts for **interaction between internal bubbles** of different kinds

Sizes of internal bubbles constrained by **bubble equations** in terms of radii of extra dimensions  $R_{y_I}$

- Reduction to 4d: **Singular BH with scalars** of mass  $\frac{1}{2} M = \frac{1}{2} \sum M_i$



# Vacuum Topological Solitons

- Class of **Vacuum Topological Solitons** induced by the degeneracy of compact directions Bah, PH '20 '21

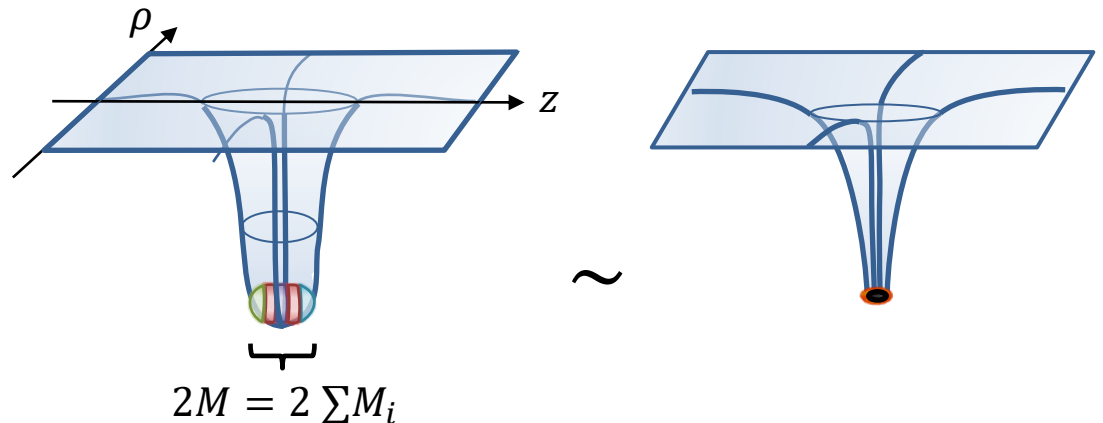
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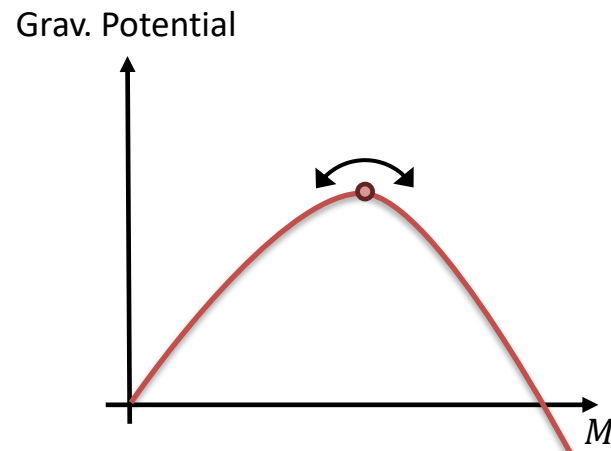
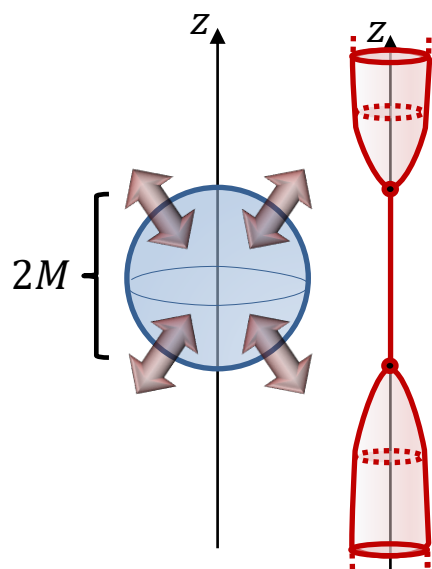
- Reduction to 4d: **Singular BH with scalars** of mass  $\frac{1}{2}M = \frac{1}{2}\sum M_i$

$$ds_4^2 = \sqrt{1 - \frac{2M}{r}} \left[ -dt^2 + H \left[ \frac{dr^2}{1 - \frac{2M}{r}} + r^2 d\theta^2 \right] + r^2 \sin^2 \theta d\phi^2 \right] + \text{bunch of scalar fields}$$



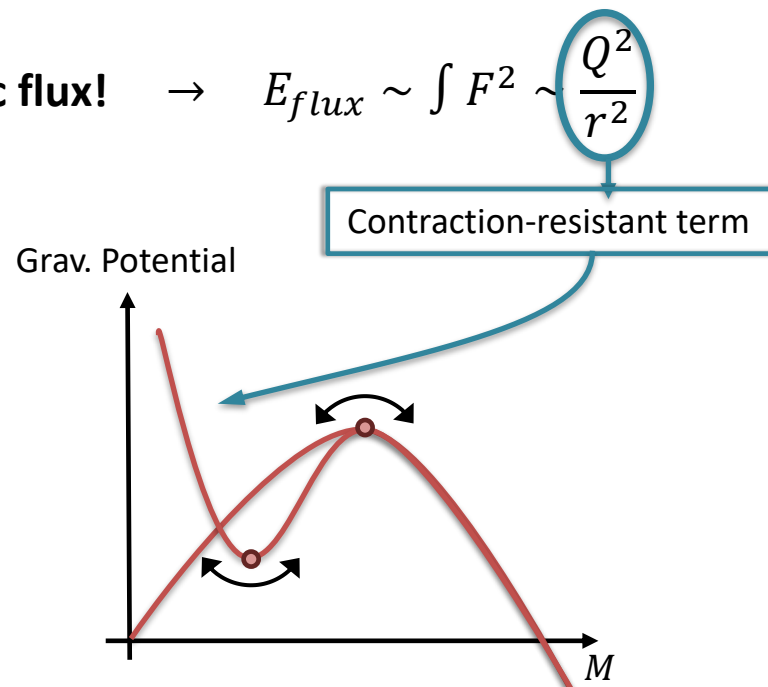
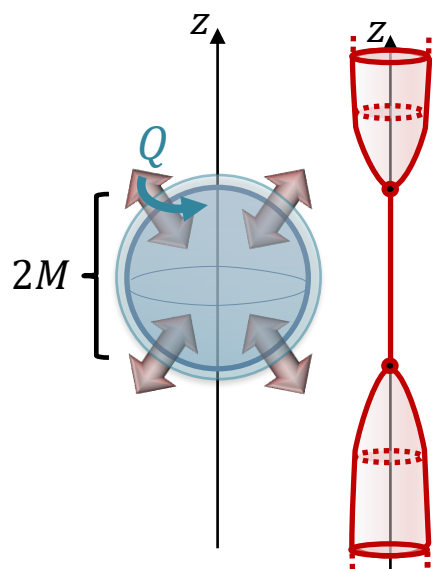
# Instability of Vacuum Solitons

- But... Stability issue! Witten '81, Gross, Perry, Yaffe '82
  - Bubble of nothing mediates the decay of Kaluza-Klein vacuum spacetimes: it **grows indefinitely or collapses**
  - Another ingredient is required...



# Instability of Vacuum Solitons

- But... Stability issue! Witten '81, Gross, Perry, Yaffe '82
  - Bubble of nothing mediates the decay of Kaluza-Klein vacuum spacetimes: it **grows indefinitely or collapses**
  - Another ingredient is required: **Electromagnetic flux!**  $\rightarrow E_{flux} \sim \int F^2 \sim \frac{Q^2}{r^2}$



Expectation: meta-stable **Gravitational Solitons** with compact dimensions and EM flux could exist!

Question: Can we actually construct them? What is their physics?

1. Vacuum solitons in gravity
2. Charged solitons in gravity
3. Solitons that resolve Schwarzschild horizon
4. Some gravitational properties

# Electrostatic Ernst Formalism in 4d

- Challenge: Einstein Maxwell equations

- Electrostatic Ernst formalism in 4d: Ernst '68

$$ds_4^2 = -\frac{1}{Z} dt^2 + Z[e^{2\nu}(d\rho^2 + dz^2) + \rho^2 d\phi^2]$$

$$F = dA \wedge dt$$

⇒

Non-linear Ernst equations  
but **integrable!**

- Solution-generating techniques:

- Inverse scattering: Generic solutions with an arbitrary number of sources (BH here) induced by a mass parameter  $M_i$  and a charge  $Q_i$  Ruiz, Manko, Martin '95

- Charged Weyl: Linear solutions for an arbitrary number of sources with  $M_i/Q_i = cst$

Bah, PH '20, '21

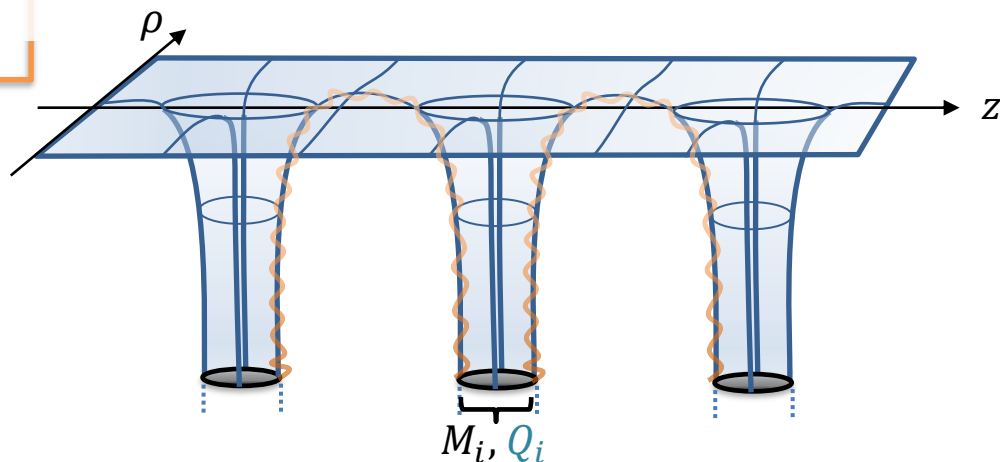
$$Z = \frac{e^{\rho W^{-1}} - e^{-bW}}{2 \sinh b},$$

$$A = \frac{\sqrt{1 + \sinh^2 b} Z^2}{Z}$$

$$W_\Lambda = \prod_i \left(1 - \frac{2M_i}{r_i}\right)$$

charge-to-mass ratio  $M_i/Q_i = \cosh b$ :

- $b \rightarrow \infty$  vacuum limit
- $b \rightarrow 0$  extremal limit



# Generalized Ernst Formalism

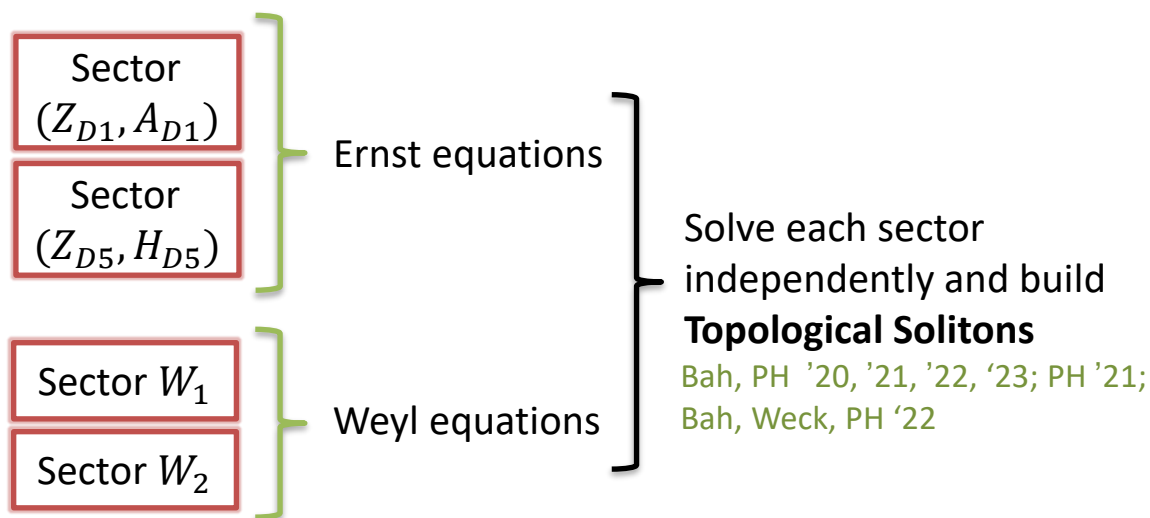
- Classification of frameworks that **decompose into set of Ernst Equations:**

PH '21

- M-theory on  $T^6 \times S^1$  with M2-M2-M2-KKm flux
- Type IIB on  $T^4 \times S^1 \times S^1$  with D1-D5-P-KKm flux
- ⋮
- D1-D5 system on  $S^1 \times S^1$  and rigid  $T^4$

$$ds_6^2 = \frac{1}{\sqrt{Z_{D1}Z_{D5}}} \left[ -W_1 dt^2 + \frac{dy_1^2}{W_1} \right] + \sqrt{Z_{D1}Z_{D5}} \left[ \frac{1}{W_2} [e^{2\nu} (d\rho^2 + dz^2) + \rho^2 d\phi^2] + W_2 dy_2^2 \right],$$

$$F_3 = d[H_{D5} d\phi \wedge dy_2 - A_{D1} dt \wedge dy_1],$$



# A Topological Star

- Source the previous framework by a **single charged bubble** Bah, PH PRL '20

- The metric:  $r_B > r_S$

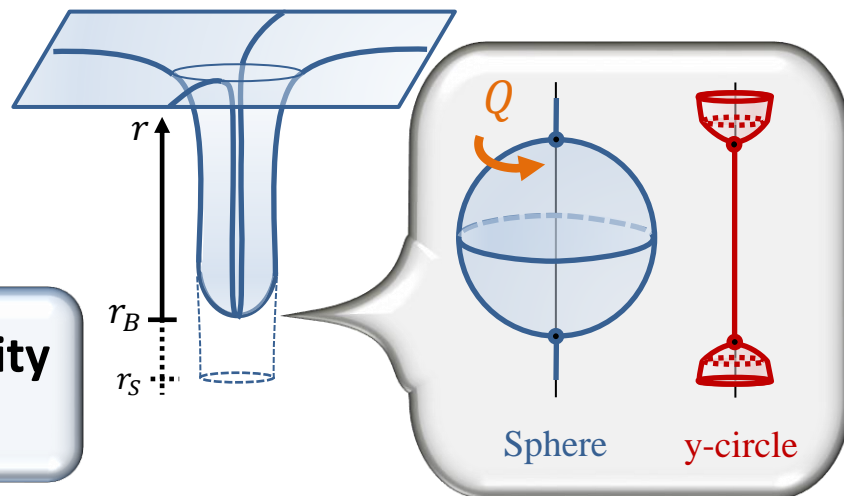
$$ds^2 = -\left(1 - \frac{r_S}{r}\right) dt^2 + \underbrace{\left(1 - \frac{r_B}{r}\right) dy^2}_{\text{Extra Dimension}} + \frac{dr^2}{\left(1 - \frac{r_S}{r}\right) \left(1 - \frac{r_B}{r}\right)} + \underbrace{r^2 [d\theta^2 + \sin^2\theta d\phi^2]}_{\text{Sphere}}$$

- Soliton is located at  $r = r_B$  and carries **D1-D5 charges**  $Q = \sqrt{3r_S r_B}$

- Regularity**  $\Rightarrow r_B, r_S \sim R_y$   
Extra dimension size

- $R_y \sim 10^3 \ell_P \Rightarrow \begin{cases} \text{Mass} \sim 10^{21} \text{ protons} \\ \sim 10^{-36} M_\odot \\ \text{Size, } r_B \sim 10^{-23} r_{\text{proton}} \end{cases}$

Topological star  $\sim$  **Topological Particle of Gravity**  
How to make **macroscopic solitons**?

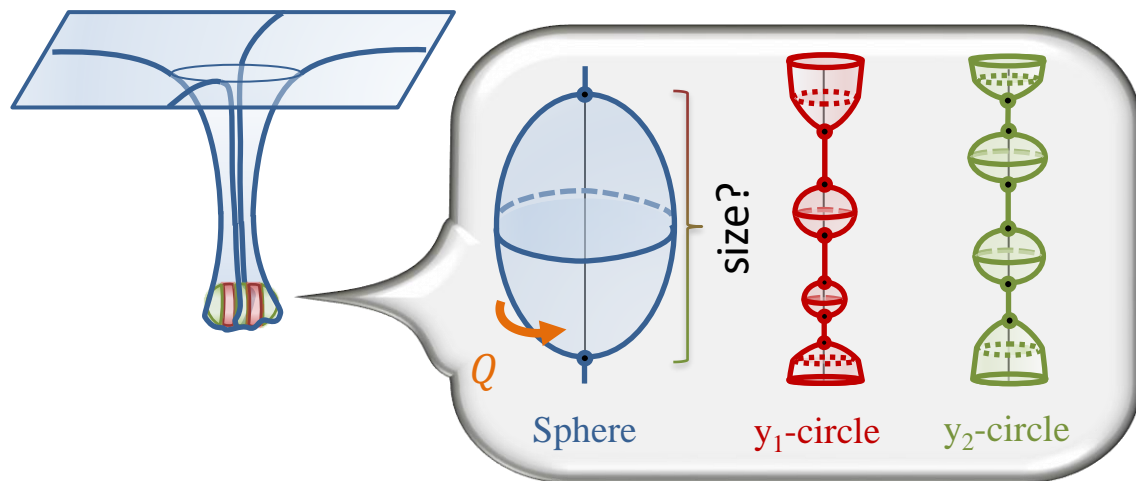




# Bubble Bag Ends

- **Bound states of topological stars using at least two extra dimensions**

Bah, PH '20, '21, '22; PH '21



- Use the Charged Weyl Formalism to solve EOM analytically with a linear ansatz
- Smooth bound states of  $N$  topological stars with  $Q = M / \cosh b$ 
  - Size  $\sim N^{1/4} R_y \rightarrow$  macroscopic!
  - Like non-extremal black holes + singular scalars in 4d but free from singularities and horizons in 6d!
  - Clear origin as M2-M2-M2 or D1-D5-KKm non-supersymmetric bubbling geometries

$$\begin{aligned}
 ds_6^2 &= \frac{1}{\mathcal{Z}_1} [-dt^2 + U_1 dy_1^2] + \frac{U_2 \mathcal{Z}_1}{\mathcal{Z}_0} (dy_2 + H_0 d\phi)^2 + \frac{\mathcal{Z}_0 \mathcal{Z}_1}{U_1 U_2} [e^{2\nu} (d\rho^2 + dz^2) + \rho^2 d\phi^2], \\
 F_3 &= dH_1 \wedge d\phi \wedge dy_2 + dT_1 \wedge dt \wedge dy_1,
 \end{aligned} \tag{2.14}$$

where the quantities,<sup>8</sup> which are independent of the gauge fields, are given by

$$\begin{aligned}
 U_1 &= \prod_{i=1}^{N+1} \left( 1 - \frac{2M_{2i-1}}{r_+^{(2i-1)} + r_-^{(2i-1)} + M_{2i-1}} \right), & U_2 &= \prod_{i=1}^N \left( 1 - \frac{2M_{2i}}{r_+^{(2i)} + r_-^{(2i)} + M_{2i}} \right), \\
 e^{2\nu} &= \frac{E_{-+}^{(1,n)}}{\sqrt{E_{++}^{(n,n)} E_{--}^{(1,1)}}} \prod_{i=1}^N \prod_{j=1}^{N+1} \sqrt{\frac{E_{--}^{(2i,2j-1)} E_{++}^{(2i,2j-1)}}{E_{-+}^{(2i,2j-1)} E_{+-}^{(2i,2j-1)}}},
 \end{aligned} \tag{2.15}$$

and the ones depending on the gauge fields are

$$\begin{aligned}
 \mathcal{Z}_0 &= \frac{e^{b_0} - e^{-b_0} U_1 U_2^2}{2 \sinh b_0}, & \mathcal{Z}_1 &= \frac{e^{b_1} - e^{-b_1} U_1}{2 \sinh b_1}, \\
 H_0 &= \frac{1}{2 \sinh b_0} \left[ r_-^{(1)} - r_+^{(2N+1)} + \sum_{i=1}^N (r_-^{(2i)} - r_+^{(2i)}) \right], \\
 H_1 &= \frac{1}{2 \sinh b_1} \sum_{i=1}^{N+1} (r_-^{(2i-1)} - r_+^{(2i-1)}), & T_1 &= -\sinh b_1 \frac{e^{b_1} + e^{-b_1} U_1}{e^{b_1} - e^{-b_1} U_1},
 \end{aligned} \tag{2.16}$$

We define the distances to the endpoints  $r_{\pm}^{(i)}$  and the generating functions  $E_{\pm\pm}^{(i,j)}$  such as

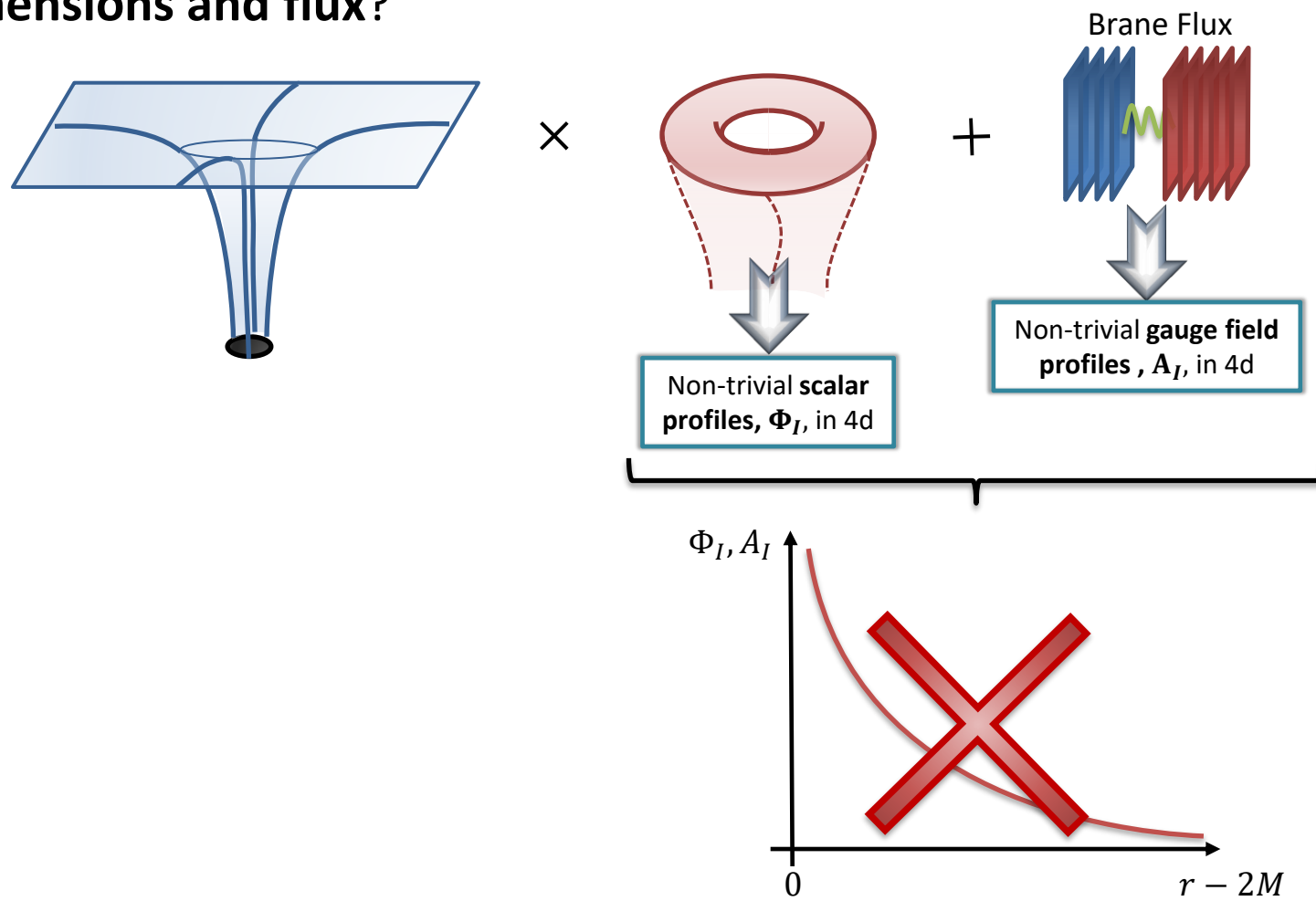
$$r_{\pm}^{(i)} \equiv \sqrt{\rho^2 + (z - z_i^{\pm})^2}, \quad E_{\pm\pm}^{(i,j)} \equiv r_{\pm}^{(i)} r_{\pm}^{(j)} + (z - z_i^{\pm})(z - z_j^{\pm}) + \rho^2, \tag{2.10}$$

But.... The geometries still have non-zero total charges

1. Vacuum solitons in gravity
2. Charged solitons in gravity
3. Solitons that resolve Schwarzschild horizon
4. Some gravitational properties

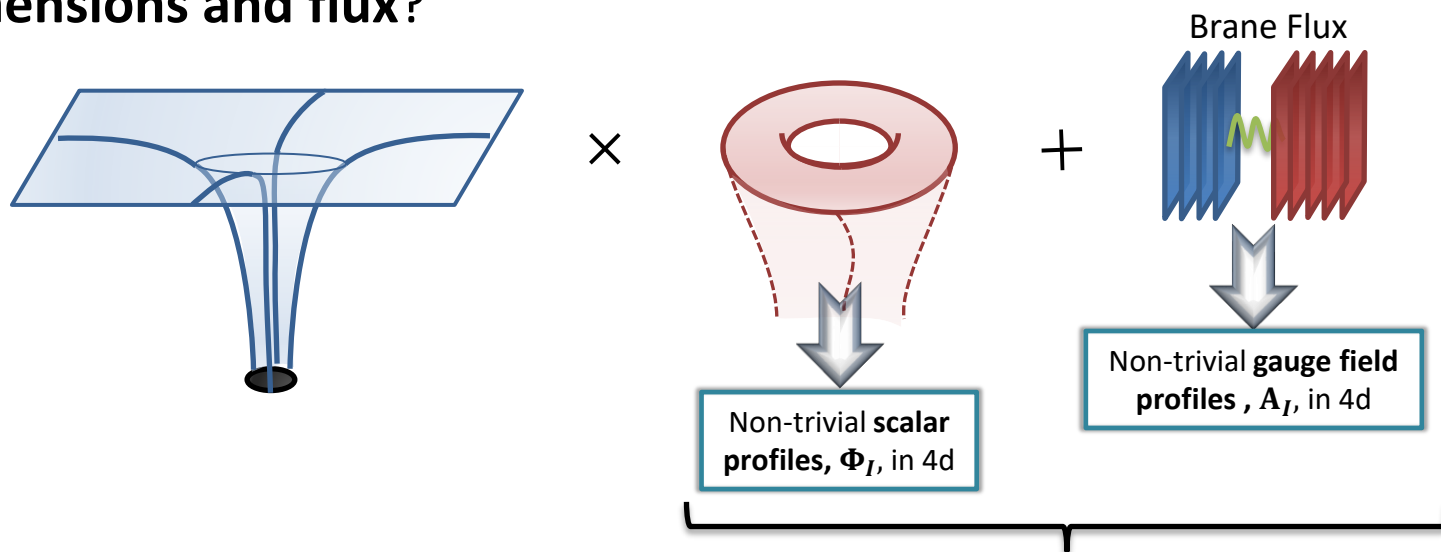
# Resolving Schwarzschild?

- How to **resolve Schwarzschild** arbitrarily close to its horizon using **extra dimensions and flux**?



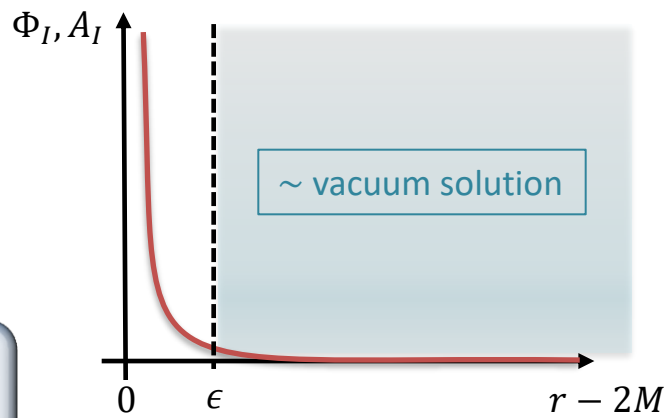
# Resolving Schwarzschild?

- How to **resolve Schwarzschild** arbitrarily close to its horizon using **extra dimensions and flux**?



- Extremal SUSY BH have already scalar and gauge field tails

$$\Phi_I, A_I \sim 1 + \frac{\alpha}{r} \rightarrow 1 + \sum_i \frac{\alpha_i}{r_i}$$



How to proceed for Schwarzschild?

# Schwarzschild Scalarwalls

- Schwarzschild Scalarwalls:** Schwarzschild black hole with a backreacted singular scalar field Bah, PH '23

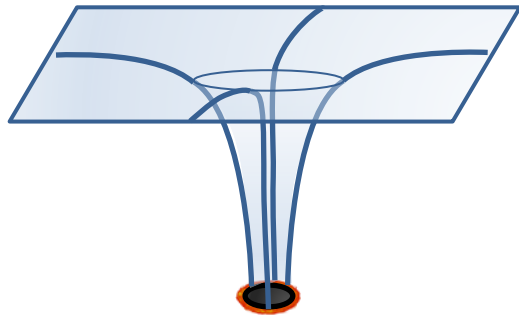
$$ds_4^2 = -\left(1 - \frac{2M}{r}\right) dt^2 + H(r, \theta) \left[ \frac{dr^2}{\left(1 - \frac{2M}{r}\right)} + r^2 d\theta^2 \right] + r^2 \sin^2 \theta d\phi^2,$$

$$e^{-\Phi} = 1 - \frac{2M}{r}, \quad H(r, \theta) = \left(1 + \frac{M^2 \sin^2 \theta}{r(r - 2M)}\right)^{-1}$$

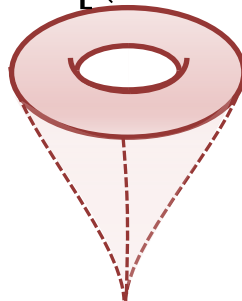
- Embedding in M-theory on  $T^7$  as a vacuum solution with  $T^7$  deformations

$$ds_{11}^2 = \left(1 - \frac{2M}{r}\right)^{\frac{2}{3}} [-dt^2 + dy_0^2 + dy_1^2 + dy_2^2] + \left(1 - \frac{2M}{r}\right)^{\frac{1}{3}} ds(T^4)^2$$

$$+ \left(1 - \frac{2M}{r}\right)^{-\frac{1}{3}} \left[ H(r, \theta) \left[ \frac{dr^2}{\left(1 - \frac{2M}{r}\right)} + r^2 d\theta^2 \right] + r^2 \sin^2 \theta d\phi^2 \right],$$



×




Can we resolve the  $r = 2M$  singular horizon?

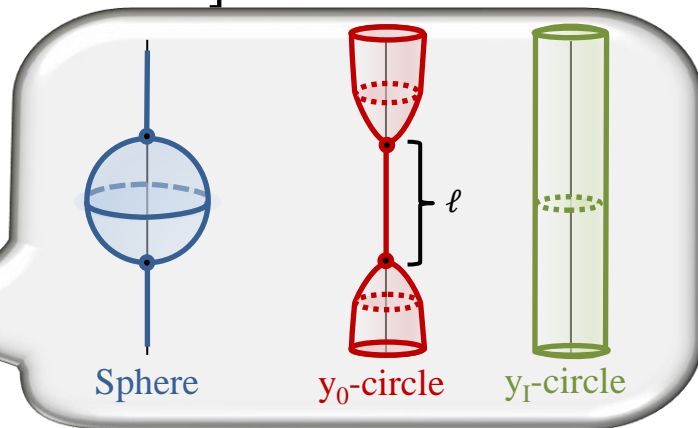
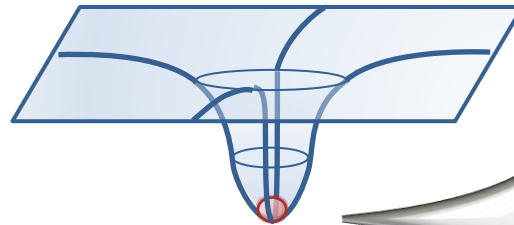
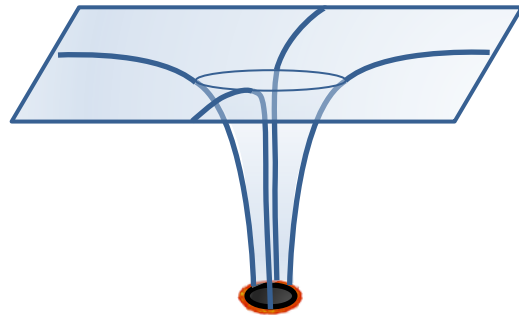
# Geometric Resolution

- **Step 1:** blowing up a **bubble of nothing** of size  $\ell \sim 2M$
- **Step 2:** add  $(\ell/2)^2 \sim M^2$  M2 branes and anti-M2 branes at the poles
- **Step 3:** Replace the poles by small bubbles of size  $\sigma$  supported by the same M2 brane flux

$$ds_{11}^2 = [-dt^2 + ds(T^6)^2] + \left(1 - \frac{\ell}{r}\right) dy_0^2 + \left[ \frac{dr^2}{\left(1 - \frac{\ell}{r}\right)} + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2 \right]$$


 Red. on  $T^7$ 

$$ds_4^2 = -\sqrt{1 - \frac{\ell}{r}} dt^2 + \sqrt{1 - \frac{\ell}{r}} \left[ \frac{dr^2}{\left(1 - \frac{\ell}{r}\right)} + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2 \right] + \text{scalar} \sim 1 - \frac{\ell}{r}$$



# Geometric Resolution

- **Step 1:** blowing up a bubble of nothing of size  $\ell \sim 2M$
- **Step 2:** add  $(\ell/2)^2 \sim M^2$  M2 branes and anti-M2 branes at the poles
- **Step 3:** Replace the poles by small bubbles supported by the same M2 brane flux

$$ds_{11}^2 = Z^{-\frac{2}{3}}[-dt^2 + dy_1^2 + dy_2^2] + Z^{\frac{1}{3}} \left[ \left(1 - \frac{\ell}{r}\right) dy_0^2 + ds(T^4)^2 \right]$$

$$z \sim \left(1 - \frac{\ell}{r}\right)^{-1} \text{ for } r \gtrsim 2M + R_{y_0}$$

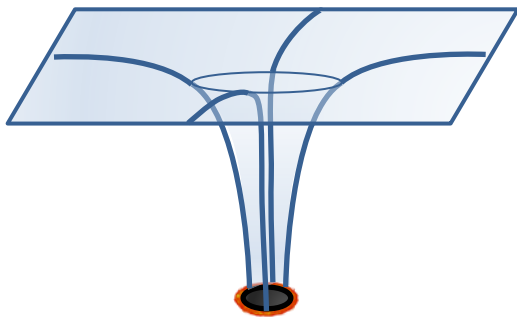
$$+ Z^{\frac{1}{3}} \left[ H \left[ \frac{dr^2}{\left(1 - \frac{\ell}{r}\right)} + r^2 d\theta^2 \right] + r^2 \sin^2 \theta d\phi^2 \right],$$

$$F_4 = dA \wedge dt \wedge dy_1 \wedge dy_2$$

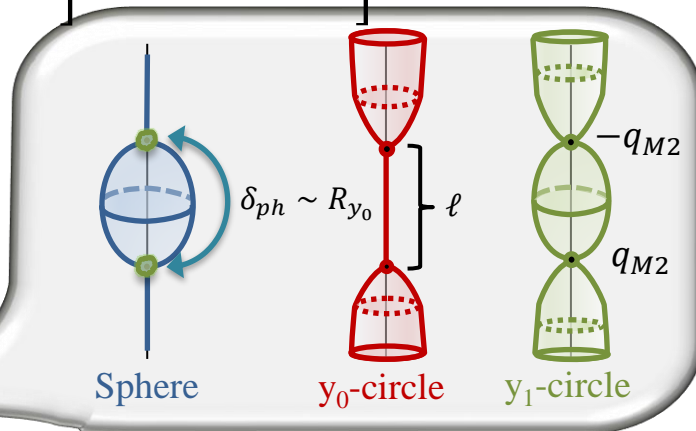
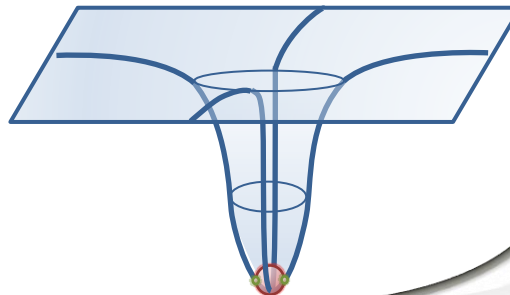
$$A \sim 0 \text{ for } r \gtrsim 2M + R_{y_0}$$

Red. on  $T^7$

$$ds_4^2 = -\sqrt{\frac{1 - \frac{\ell}{r}}{Z}} dt^2 + \sqrt{Z \left(1 - \frac{\ell}{r}\right)} \left[ H \left[ \frac{dr^2}{\left(1 - \frac{\ell}{r}\right)} + r^2 d\theta^2 \right] + r^2 \sin^2 \theta d\phi^2 \right] + \text{scalars}$$



~



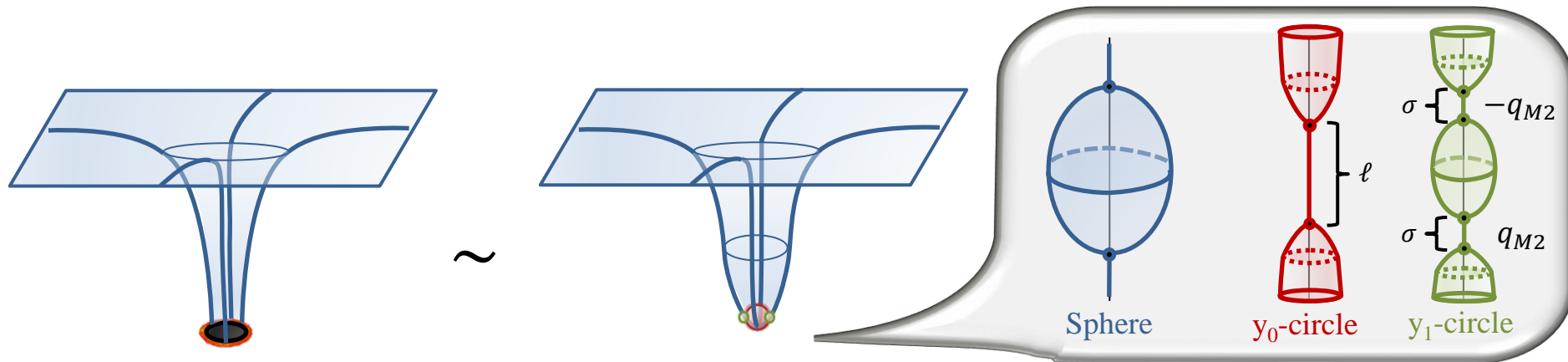


# Geometric Resolution

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- **Step 3:** Replace the poles by small bubbles supported by the same M2 brane flux

$$ds_{11}^2 = Z^{-\frac{2}{3}} \left[ -dt^2 + \left(1 - \frac{\sigma}{r_1}\right) \left(1 - \frac{\sigma}{r_3}\right) dy_1^2 + dy_2^2 \right] + Z^{\frac{1}{3}} \left[ \left(1 - \frac{\ell}{r_2}\right) dy_0^2 + ds(T^4)^2 \right] \\ + Z^{\frac{1}{3}} \left[ H \left[ \frac{dr^2}{\left(1 - \frac{\ell + 2\sigma}{r}\right)} + r^2 d\theta^2 \right] + r^2 \sin^2 \theta d\phi^2 \right], \quad F_4 = dA \wedge dt \wedge dy_1 \wedge dy_2$$

First example of topological solitons that resolve the horizon of a Schwarzschild black hole + a scalar tail



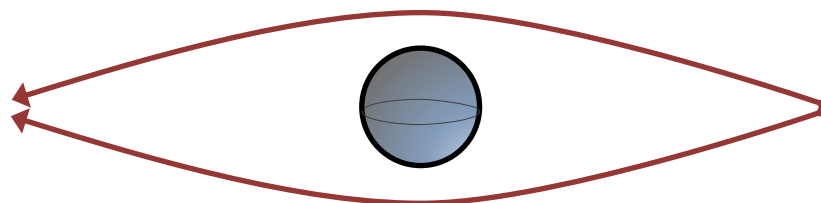
1. Vacuum solitons in gravity
2. Charged solitons in gravity
3. Solitons that resolve Schwarzschild horizon
4. Some gravitational properties

# Lensing and Gravitational Wave Signatures

- Two experiments allow to probe the near-environment of black holes.

What are the **gravitational characteristics of topological solitons** compared to those of a black hole?

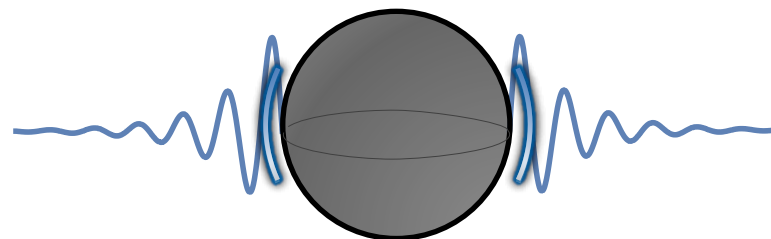
## 1. Gravitational lensing:



BH and TS do not emit light but **scatter light rays**

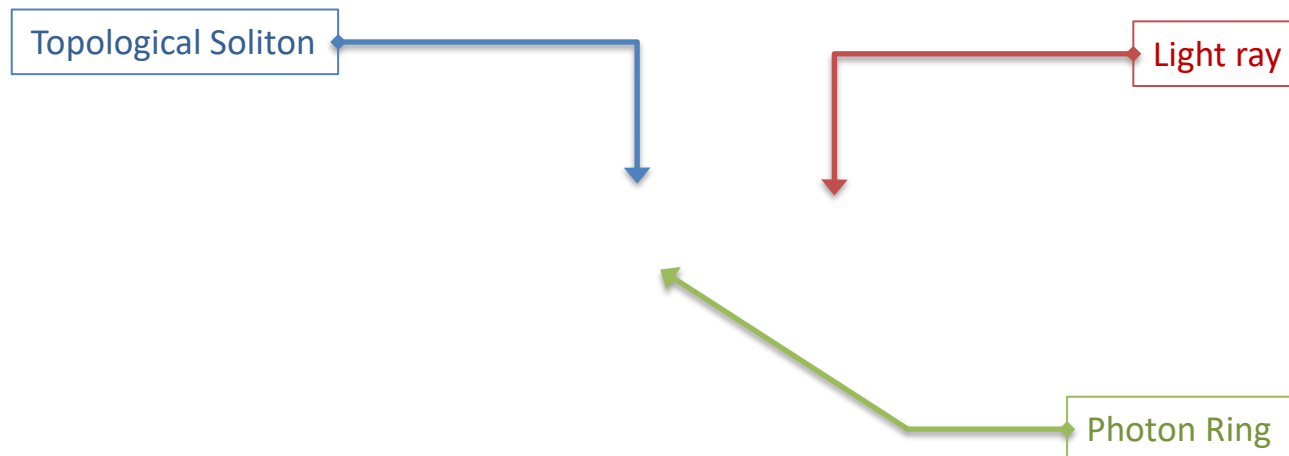
## 2. Gravitational wave emission:

After the gravitational merger, the final state relaxes by emitting a **ringdown gravitational wave signal**.



# Gravitational Lensing and Imaging

- Topological Soliton → **intense spacetime deformation** ⇒ **intense lensing**
- Photons follow **geodesic trajectories**
- Like black holes, they are circumscribed by an **photon ring** ( $\sim$  shadow) where light can be trapped.
- However, incoming photons do not fall into a horizon ( $\neq$  shadow) but **bounce on the soliton after a chaotic trajectory**



# Imaging Topological Solitons

## What do Topological solitons look like in the sky?

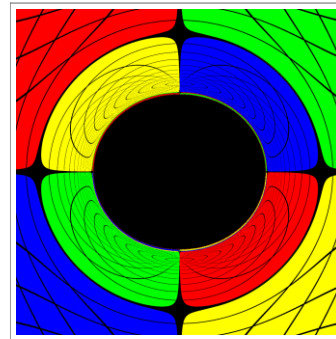
Bah, Berti, PH '22

1. Build a ray tracing code
2. Shoot  $10^6$  light rays from a celestial sphere
3. Reconstruct the picture as seen from a distant observer

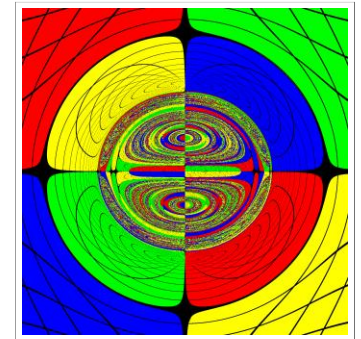
Flat Space:



Schwarzschild:



Schw. Scalarwall:



Smooth resolution:

Photon Rings **match with Schwarzschild Shadow** in size and scattering properties

# Imaging Topological Solitons

- Use a **star patch** for more “arty” imaging Bah, Berti, PH '22
- Incoming light endures **high redshift and chaotic trajectories**  
→ intensity should be suppressed beyond the probe limit

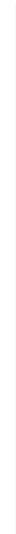
Flat Space:



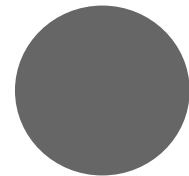
Schwarzschild:



Schw. Scalarwall:



Smooth resolution:



# Imaging Topological Solitons

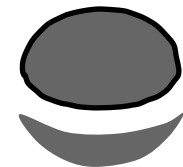
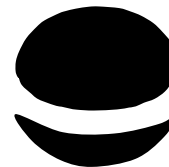
- Imaging from a **bright accretion disk**  $\rightarrow$   $\sim$ EHT Bah, Berti, PH '22

Flat Space:

Schwarzschild:

Schw. Scalarwall:

Smooth resolution:

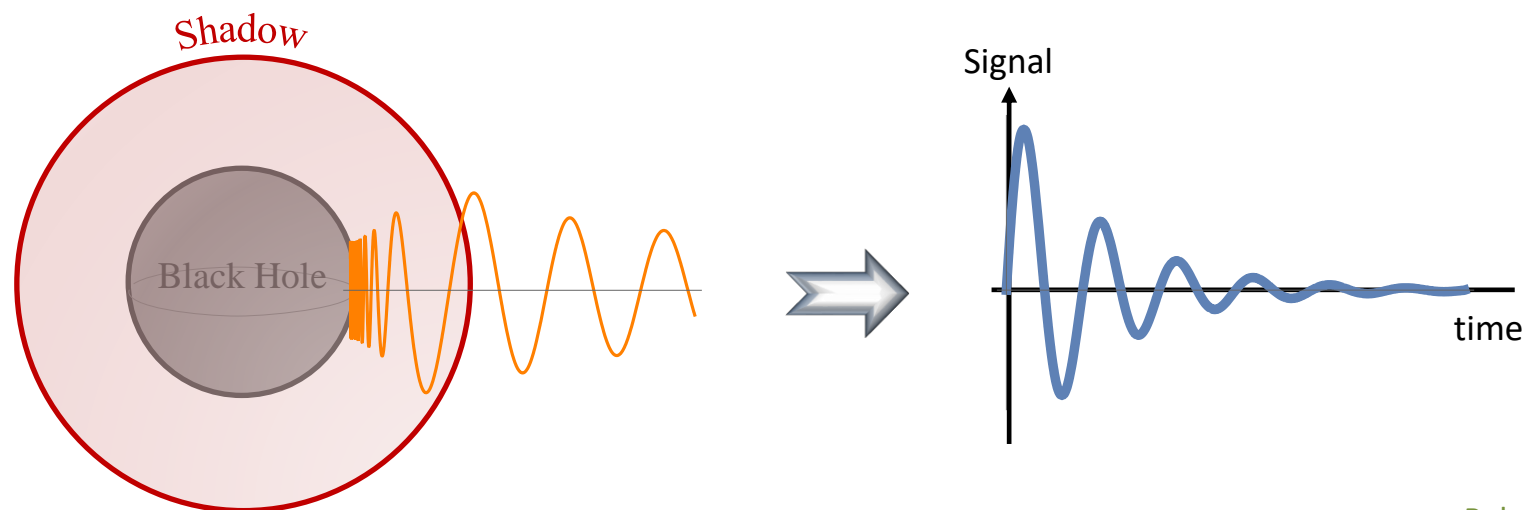


Expectation:

blurred and faded cloud should emerge from inside the shadow

# Gravitational Wave Signal

- Gravitational waves are governed by **quasi-normal**
- Static black hole: modes localize at the shadow and determined by its scattering properties
- Topological solitons replace the horizon by **stable photon rings**. How does it affect quasi-normal modes?
  - Fundamental modes localize at the microstructure
  - Black hole modes are part of the spectrum but cavity effect on their decay time

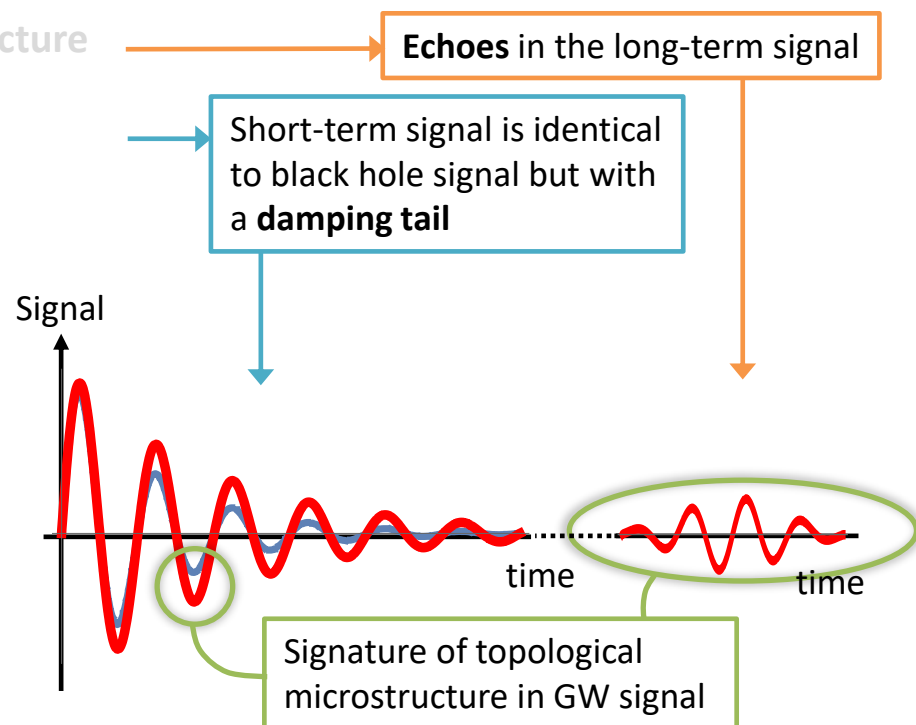
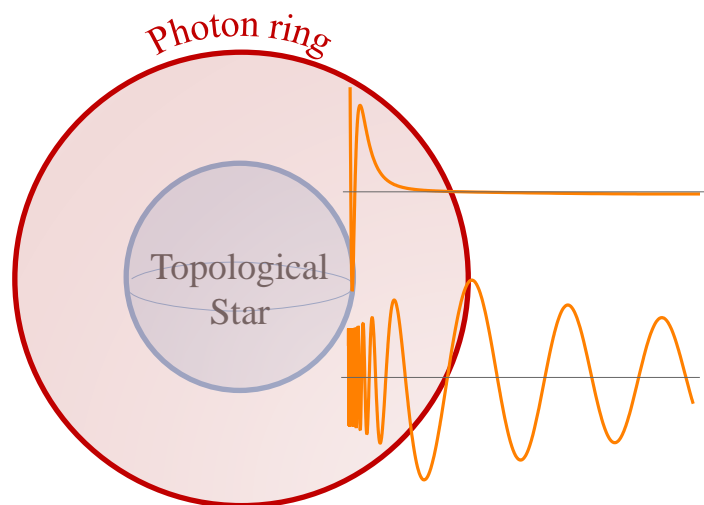


Bah, Berti, Speeney, PH '23



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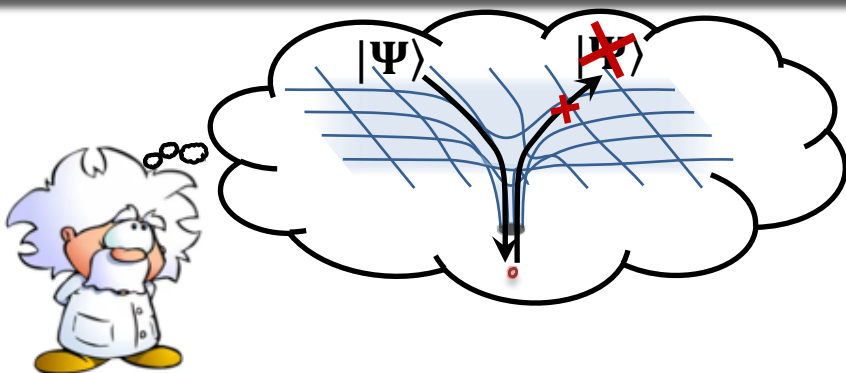
Bah, Berti, Speeney, PH '23

# Conclusion and Outlook



- Fundamental states of Quantum Gravity that manifest as **Topological Solitons in Gravity exist**. Their **phenomenology** and **fundamental role open a window into new astrophysics and black hole physics**.
  - Solitons in Anti-de Sitter spacetimes and application to **holography**? Bah, PH '22; Houppe, PH '22
  - General phase space of topological solitons? Spinning solitons? Chern-Simons interactions?
  - Physical observables for future experiments? Bah, Berti, PH '22; Bah, Berti, Speeney, PH '23
  - General aspect of stability and existence? Bah, Dey, PH '21
- Nucleation in the early universe (dark matter?) or late stage of matter collapse (BH)?

# Black Hole Puzzles

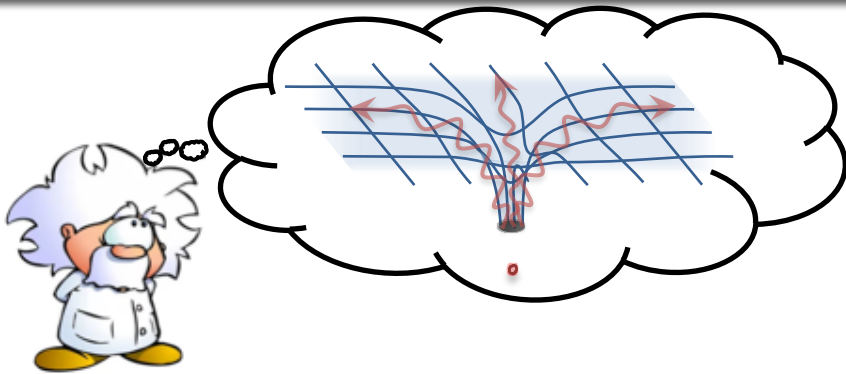


Black holes are at the center of  
**theoretical conflicts** between  
**GR and QM**

**Singularity**

**Horizon**  $\Rightarrow$  Information  
Paradox

# Black Hole Puzzles

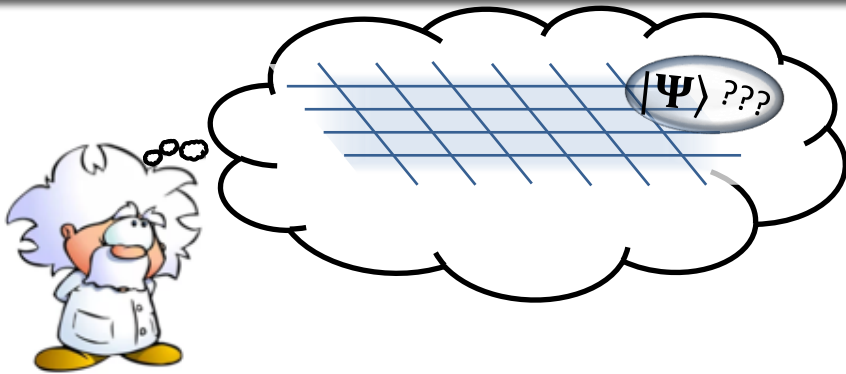


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# Black Hole Puzzles



Black holes are at the center of  
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