Probing Quantum Effects at the Horizon Through Gravitational Waves

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Outline

- Alternative to Black holes.
- Probing quantum effects of black holes through gravitational waves.
- Various observables and implications of them.
- Looking forward.

Reference

- **SC**, Maggio, Mazumdar and Pani, PRD 106, 024041 (2022).
- Nair, **SC** and Sarkar, arXiv: 2208.06235.
- **SC**, Maggio, Pani and Silvestrini, Work in progress.

Why Black Holes?

- Black holes can be constructed from normal matter, using simple collapse scenarios.
- Black holes are unique and have universal properties.
[Heusler, Black Hole Uniqueness Theorem (Cambridge University Press)]
- Black holes behave as thermodynamic objects with temperature and entropy — Hint towards quantum gravity. **[Bekenstein, Phys. Rev. D 7, 2333 (1973)]**
- Black holes are stable under all possible perturbations.
- Observation of shadows from Event Horizon Telescope and the ringdown signals from LIGO and VIRGO are definitely consistent with the existence of Black Holes.
- Consistency with general relativity is another story.

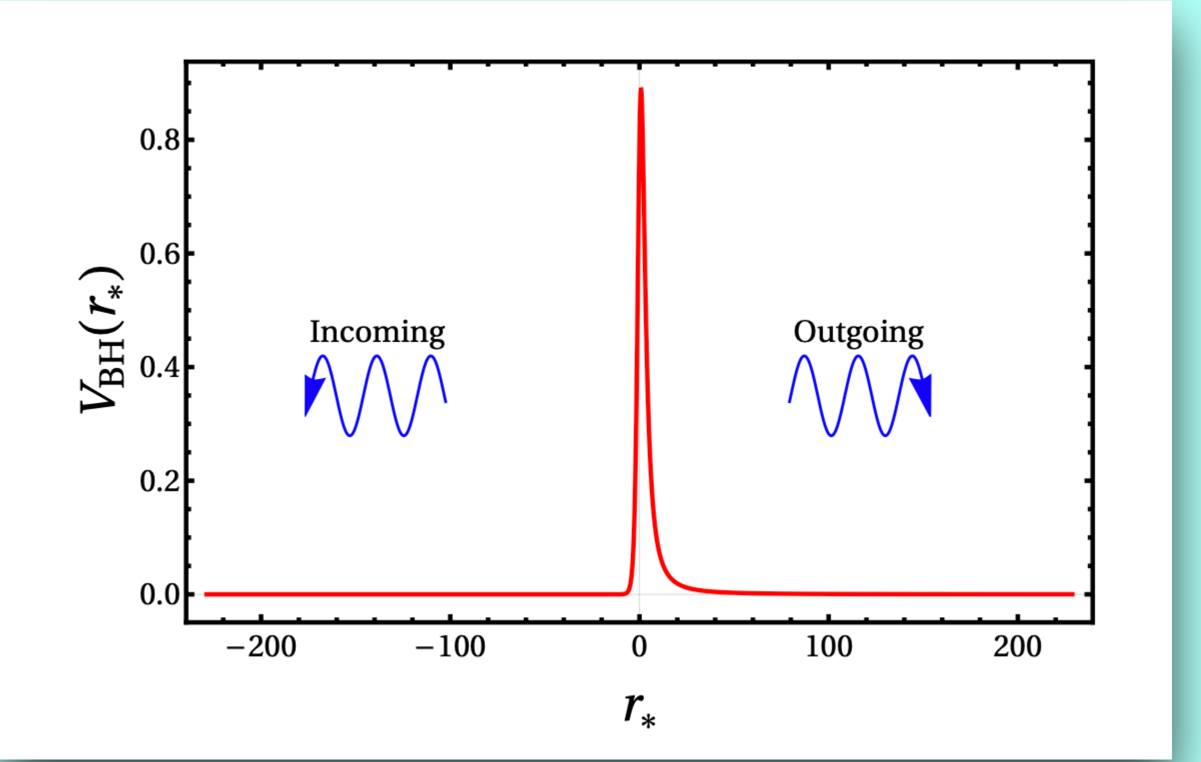
But …

- Despite being the simplest objects, there are issues.
- **Singularity**: All black hole spacetimes have a singular region/point → breakdown of the theory.
- **Loss of Predictability**: Most of the black holes inherit Cauchy horizon → future cannot be determined. [Cardoso et. al., Phys. Rev. Lett. 120, 031103 (2018)]
- **Information Loss Paradox**: The existence of thermal radiation results into loss of information. [Hawking, Commun. Math. Phys. 43, 199 (1975)] [SC and Lochan, Universe 3, 55 (2017)]
- All of these suggest that we may need to look for alternatives — curing these problems and yet remaining consistent with experiments.

Black Hole Hypothesis

- Does the existence of a photon sphere implies the existence of a black hole? [Cardoso et. al., Phys. Rev. Lett. 116, 171101 (2016)]

- The ringdown is governed by the photon sphere alone. [Cardoso et. al., Phys. Rev. D 79, 064016 (2009)]



- Structure beneath the photon sphere can not be probed directly.

- Can such objects exist? What will be their observational properties?

[Figure Courtesy: Biswas, Rahman and SC, Phys. Rev. D 106, 124003 (2022)]

Exotic Matter

- Raychaudhuri equation guarantees that normal matter cannot cure singularities → require exotic matter or, quantum effects.

[SC, Kothawala and Pesci, PLB 797, 134877 (2019)]

- The consistency with observations, require any alternatives to have

$$2M < R < 3M$$

- The limiting stellar configuration, with normal matter, being (Buchdahl limit):

$$\left(\frac{M}{R}\right) \leq \frac{4}{9}$$

- Recent shadow measurement argues that Buchdahl limit must be violated → exotic matter is necessary.

Only Exotic Matter?

- Are these exotic matters stable → ergo-region instability, enhanced superradiant instability, for rotating objects.

[Cardoso et. al., Phys. Rev. D 77, 124044 (2008)]

- Can quantum effects play any role?

[Abedi et. al., Phys. Rev. D 96, 082004 (2017)]

- Area quantised black holes are generic predictions of theories of quantum gravity and these have non-trivial physics at horizons.

[Agullo et. al., Phys. Rev. Lett. 126, 041302 (2021)]

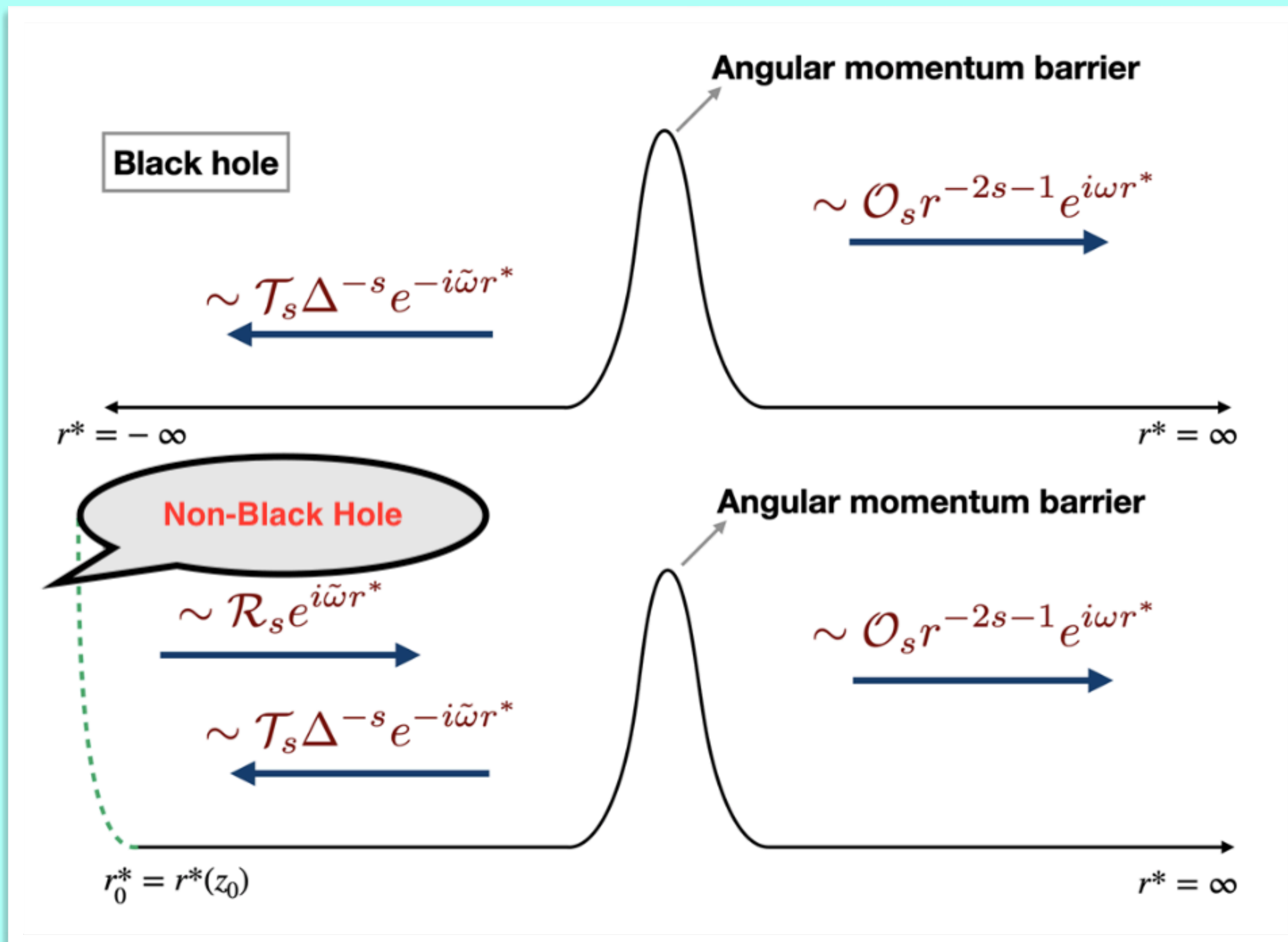
- The basic point is to modify the horizon itself by a reflective membrane as quantum effects are taken into account.

[Cardoso, Foit and Kleban JCAP 08, 006 (2019)]

[Maggio et. al., Phys. Rev. D 102, 064053 (2020)]

[Dey, SC and Afshordi, Phys. Rev. D 101, 104014 (2020)]

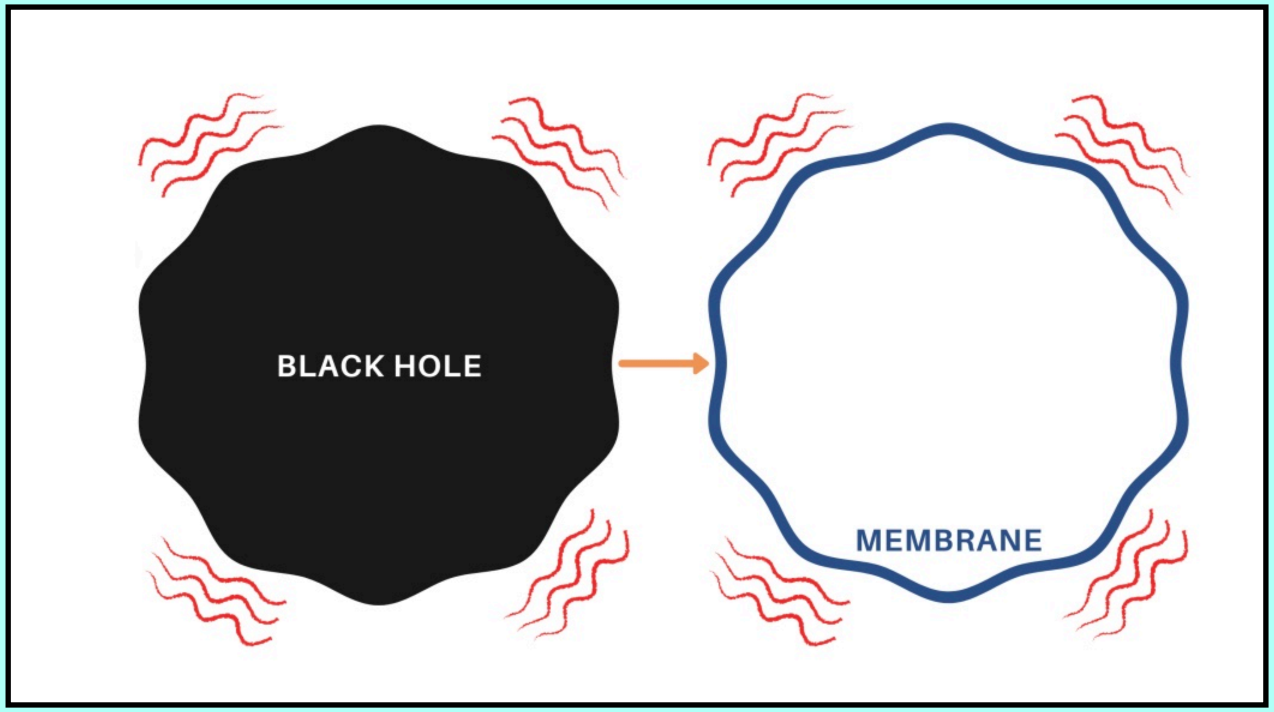
Reflective Horizon — Basics



[Figure Courtesy: Dey, Biswas and SC, Phys. Rev. D 103, 084019 (2021)]

Membrane Paradigm and GW

- Replacing the black hole horizon by a membrane is a natural assumption, giving rise to the membrane paradigm.
- Any reflective boundary, close to the horizon, arising due to some exotic compact object, can also be described by a similar membrane fluid.



$$T_{ab} = \rho u_a u_b + (p - \zeta\Theta)\,\gamma_{ab} - 2\eta\sigma_{ab}$$

$$[[K_{ab} - h_{ab}K]] = -8\pi T_{ab}$$

[Figure Courtesy: Maggio et. al., arXiv: 2006.14628]

[Price and Thorne, Phys. Rev. D 33, 915 (1986)]

Reflectivity of the Membrane

- The perturbation of the background spacetime, perturbs the energy momentum tensor of the membrane fluid as well.
- Thus one considers the perturbed junction conditions to relate gravitational perturbations to the perturbations of the membrane fluid.

$$\omega\psi(R) = i16\pi\eta\left(\left.\frac{\partial\psi}{\partial x}\right|_R + \frac{RV_{\text{axial}}(R)}{2f(R) - Rf'(R)}\psi(R)\right)$$

- The reflectivity of the membrane becomes,

$$|\mathcal{R}|^2 = \left(\frac{1-\eta/\eta_{\text{BH}}}{1+\eta/\eta_{\text{BH}}}\right)^2$$

[Maggio et. al., arXiv: 2006.14628]

"Quantum" Membrane

- The membrane is assumed exhibit a Gaussian profile, as if constructed out of a large number of harmonic oscillators at their ground states.

[SC et. al., arXiv: 2202.09111]

- The wave function of the membrane is governed by

$$\Psi(\epsilon) = A\exp\left(-\frac{\epsilon^2}{2\sigma^2}\right) \qquad |A|^2 = \frac{2}{\sigma\sqrt{\pi}}$$

- Classically the membrane will be located at $\langle\hat{\epsilon}\rangle$, with a fluctuation

$$\langle\hat{\epsilon}^2\rangle - \langle\hat{\epsilon}\rangle^2 = \sigma^2\left(\frac{1}{2} - \frac{1}{\pi}\right)$$

- Classical limit ($\hbar \to 0$), corresponds to $\sigma \to 0$.

- The quantum membrane will have an energy-momentum tensor

$$\hat{T}_{ab} = \rho\hat{u}_a\hat{u}_b + \left(p - \zeta\hat{\Theta}\right)\hat{\gamma}_{ab} - 2\eta\hat{\sigma}_{ab} \qquad \rho = \rho_0 + \delta\rho \qquad p = p_0 + \delta p$$

"Quantum" Matter = Geometry

- The properties of matter get related to the geometry by the junction conditions: $[[K_{ab} - K h_{ab}]] = -8\pi \langle \hat{T}_{ab} \rangle$ and $[[h_{ab}]] = 0$ on $R = r_+ + \langle \hat{\epsilon} \rangle$.
- In absence of perturbations, the energy density and pressure becomes,

$$\rho_0 = -\frac{f(R)^{3/2}}{4\pi R} \left[\frac{1}{f(R) + \frac{1}{2} f''(r_+) \left(\langle \hat{\epsilon}^2 \rangle - \langle \hat{\epsilon} \rangle^2 \right) + \mathcal{O}(\tilde{\sigma}^3)} \right]$$

$$p_0 = \frac{R\left[2f(R) + Rf'(R)\right]}{16\pi \sqrt{f(R)}\left[R^2 + \left(\langle \hat{\epsilon}^2 \rangle - \langle \hat{\epsilon} \rangle^2 \right)\right]}$$

- These must be perturbed due to perturbation of the metric and the governing equation will be $\delta K_{ab} - K \delta h_{ab} = -8\pi \langle \delta \hat{T}_{ab} \rangle$.
- For simplicity we will consider axial gravitational perturbation, which in the Regge-Wheeler gauge has only two independent components $\delta g_{t\phi}$ and $\delta g_{r\phi}$.

Not Purely Ingoing

- For axial perturbation of static and spherically symmetric spacetime, with $-g_{tt} = g^{rr}$, the Regge-Wheeler choice provides the following boundary condition at $\boxed{R = r_+ + \langle \hat{\epsilon} \rangle}$.

[SC et. al., arXiv: 2202.09111]

$$\boxed{\begin{aligned} i\omega\psi(R) = \frac{\eta}{(\rho_0 + p_0)\sqrt{f(R)}} \Bigg[& V_{\rm axial}(R)\psi(R) - \frac{1}{R}\frac{d\psi(R)}{dx}\left[Rf'(R) - 2f(R)\right] \\ & - \frac{4f(R)}{R}\left(\frac{d\psi(R)}{dx} + \frac{f(R)}{R}\psi(R)\right)\left(1 + \frac{4\pi\rho_0 R}{\sqrt{f(R)}}\right)\Bigg]\,. \end{aligned}}$$

- As $R \to r_+$, the above condition reduces to,

$$\boxed{i\omega\psi(R) = -16\pi\eta\frac{d\psi(R)}{dx}}$$

- This is equivalent to purely ingoing waves at the horizon, which will not be the case in general.

Reflecting "quantum" membrane

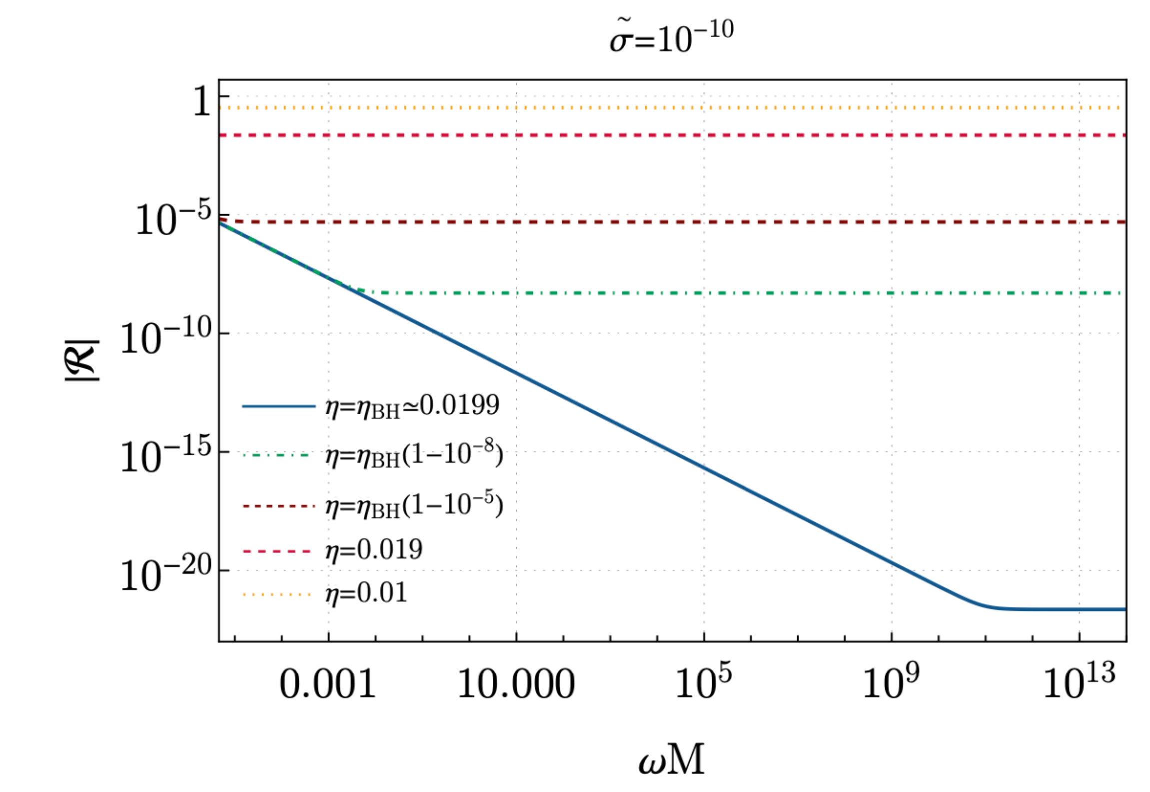
[SC et. al., arXiv: 2202.09111]

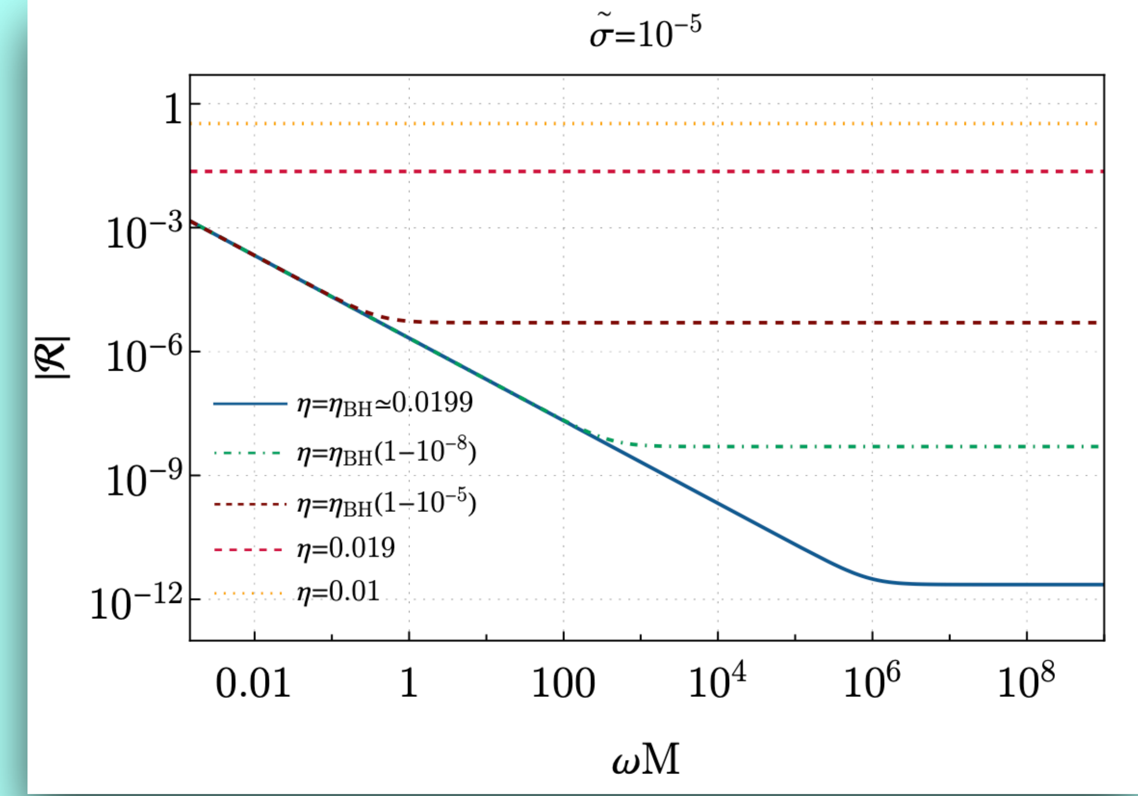
- The boundary condition allows existence of ingoing as well as outgoing waves near the membrane, such that,

$$\psi_{\rm M} = e^{-i\omega x} + \mathcal{R}e^{i\omega x}$$

- In appropriate limits,

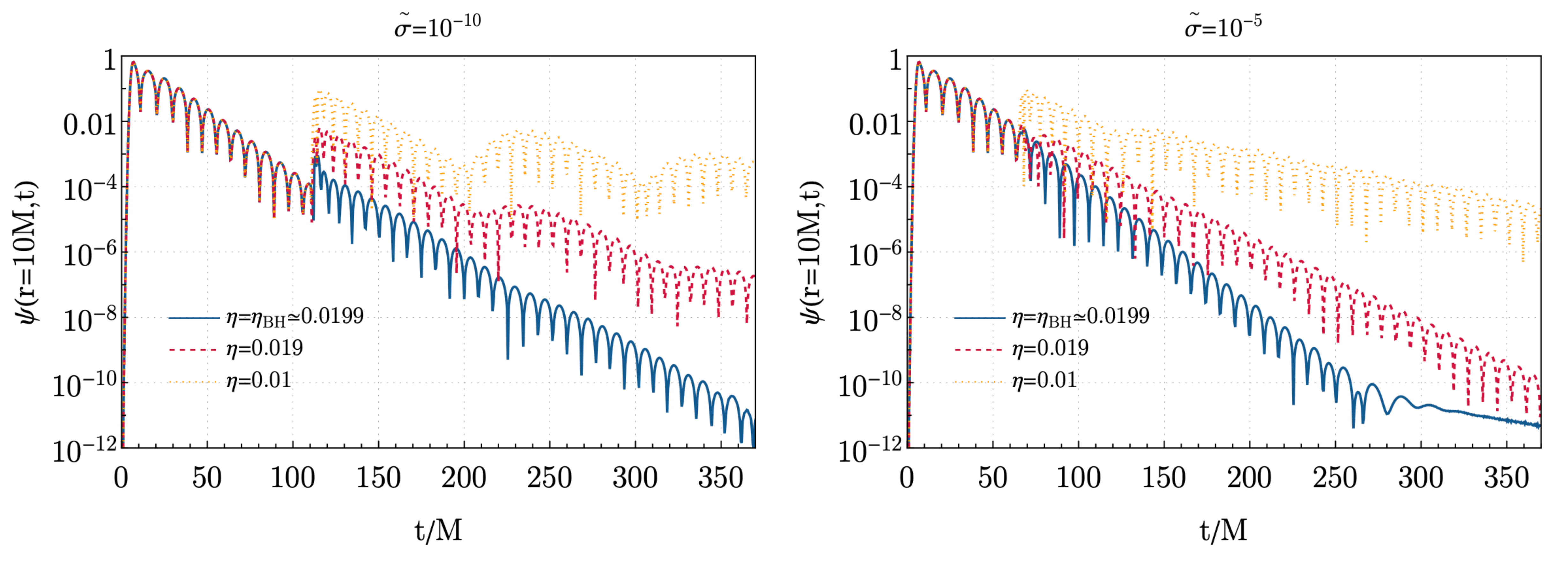
$$|\mathcal{R}|^2 \sim \left(\frac{1-\eta/\eta_{\rm BH}}{1+\eta/\eta_{\rm BH}}\right)^2 + \frac{16384\,[\ell(\ell+1)-3]^2\,\pi^3\eta^4\tilde{\sigma}^2}{(1+\eta/\eta_{\rm BH})^4\,\omega^2 M^2}$$



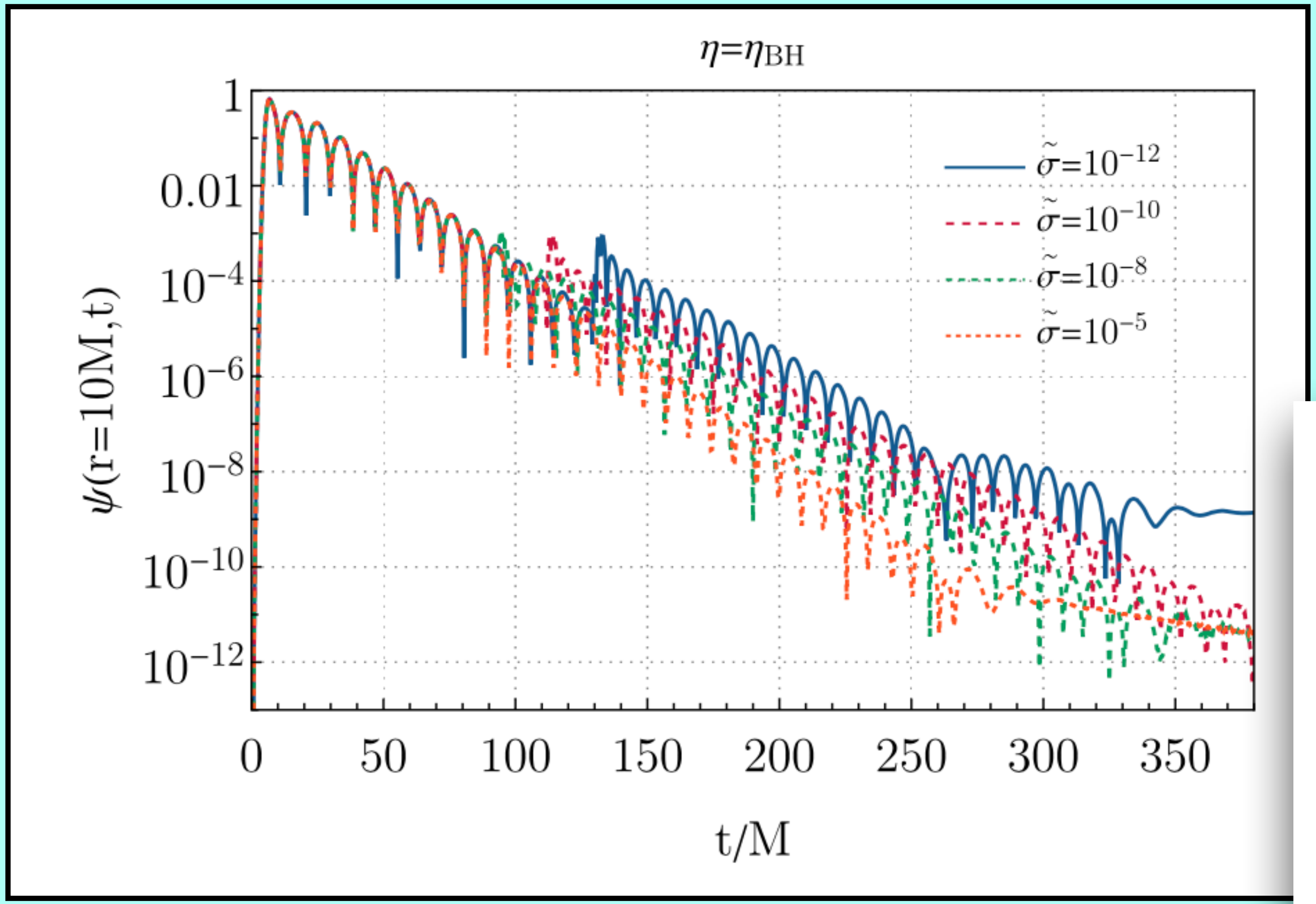


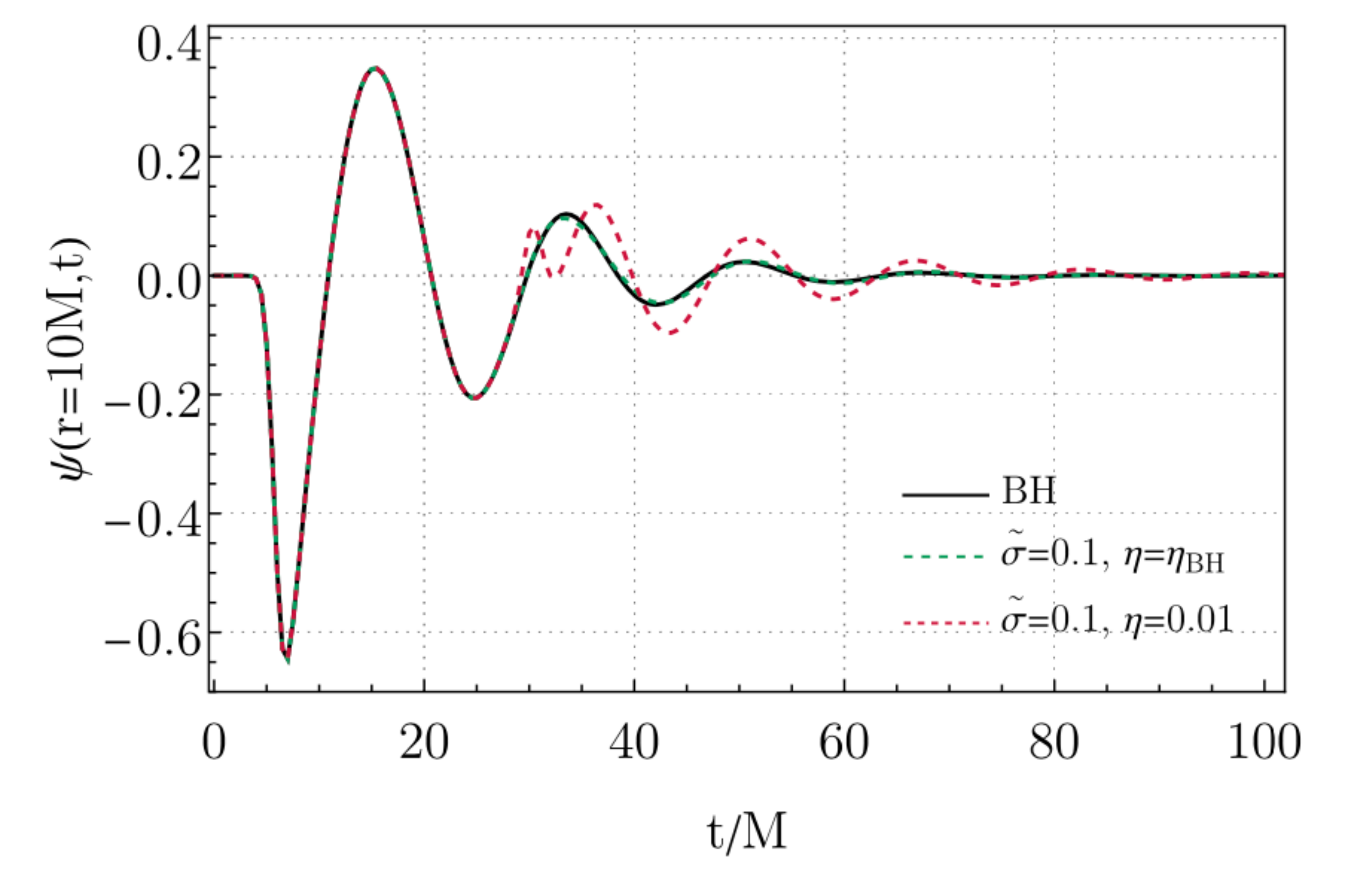
Ringdown Waveform

- As the effective classical membrane nears the horizon, there are pronounced echoes. [SC et. al., arXiv: 2202.09111]
- The time delay is consistent with size of the membrane.



Ringdown Waveform — Continued





[SC et. al., arXiv: 2202.09111]

Tidal Love Number

- There is another smoking gun test for classical black holes in general relativity — Tidal Love number is zero, i.e., to first order, black holes in general relativity cannot be deformed.

- However, there are ambiguities.

- Generally, one starts from the $-(1+g_{00})/2$ component and write it as

$$-\frac{1+g_{00}}{2}=\frac{1}{2}\mathcal{E}_{ij}x^i x^j+\cdots+\frac{-3}{2}\frac{Q_{ij}n^i n^j}{r^3}+O\left(\frac{1}{r^4}\right),$$

[Hinderer, ApJ 677, 1216 (2008)]

- The quadrupole moment can be identified as the coefficient of $1/r^3$ term and the tidal field as the coefficient of r^2 term.

$$Q_{ij}=-\left(\Lambda R^5\right)\mathcal{E}_{ij}$$

Tidal Love number — Ambiguities

- The relativistic setting has coordinate freedom. Thus choice of the r coordinate is not definitive and can be modified.

$$r' = r\left[1 + N\left(\frac{M}{r}\right)^5\right]$$

- Then the induced quadrupole moment will change by,

$$Q'_{ij} = Q_{ij} + \tfrac{2}{3} N M^5 \mathcal{E}_{ij},$$

- Therefore, the Love number becomes, $\Lambda' = \Lambda - \tfrac{2}{3}N.$

- Also while defining the Love number in the relativistic setting one uses a definite coordinate system, e.g., the Schwarzschild coordinate system and make Gauge choices, e.g., Zerilli and Regge-Wheeler gauge.

[Gralla, Class. Quant. Grav. 35, 085002 (2018)]

Gauge Invariant Tidal Love Number

- For the Love number to be gauge invariant, one solves the Teukolsky equation for ψ_4 / ψ_0 and impose boundary conditions near the horizon.

$$\lim_{c\to\infty} c^2\psi_0 = \sum_{\ell m} \alpha_{\ell m}(t)\, r^{\ell-2}\left[1 + 2k_{\ell m}\left(\frac{R}{r}\right)^{2\ell+1}\right] {}_2Y_{\ell m}(\theta,\varphi)\,,$$

$$F_{\ell m}(\omega) = 2\,k_{\ell m} + i\omega\tau_0\,\nu_{\ell m} + \mathcal{O}(\omega^2)\,.$$

[Chia, arXiv:2010.07300]

[Le Tiec et. al., arXiv:2010.15795]

- For Kerr black hole, with $P_+ = (am - 2M\omega r_+)/(r_+ - r_-)$

$$F^{I,\mathrm{Kerr}}_{\ell m} = -iP_+ \frac{(\ell-2)!(\ell+2)!}{(2\ell)!(2\ell+1)!} \prod_{j=1}^{\ell} \left[j^2 + 4P_+^2\right]\,.$$

Compact Objects can be deformed

- Compact Objects having non-zero reflectivity, in general, have non-zero Love number. [Maggio, SC, Michella and Pani, Work in Progress]

$$k_{\ell m} = \mathrm{Re}\Bigg[- i\frac{P_+}{2}\left(\frac{(\ell+2)!(\ell-2)!}{(2\ell)!(1+2\ell)!}\right)\prod_{j=1}^{\ell}\left(j^2+4P_+^2\right) \times \left\{\frac{1+\frac{\mathcal{B}}{\mathcal{A}}\Gamma_2}{1+\frac{\mathcal{B}}{\mathcal{A}}\Gamma_1}\right\}\Bigg].$$

- Reflectivity is defined in terms of the Detweiler function.
- Detweiler function can be related to Teukolsky, Teukolsky can be expressed in terms of Regge-Wheeler and Zerilli and then either of them can be related to the metric perturbation.

No Logarithmic Behaviour

[Maggio, SC, Michella and Pani, Work in Progress]

- The Dirichlet boundary condition:

$$\frac{h_0'}{h_0} = \frac{\left(f' + \frac{rV_{\rm RW}}{f}\right) + 2f\frac{\Psi'_{\rm RW}}{\Psi_{\rm RW}}}{f(r)\left(1 + r\frac{\Psi'_{\rm RW}}{\Psi_{\rm RW}}\right)}$$

- The Reflectivity in zero frequency limit:

$$R(\omega) = R_0 + i\omega R_1 + \mathcal{O}(M^2\omega^2)$$

$$X = e^{-i\omega r_*} + Re^{i\omega r_*}$$

$$\frac{\left(\frac{{}_{-2}X^{\rm axial}_{\ell m}}{dr_*}\right)}{{}_{-2}X^{\rm axial}_{\ell m}} \longrightarrow i\omega\left(1 - \frac{2}{1 + Re^{2i\omega r_*}}\right)$$

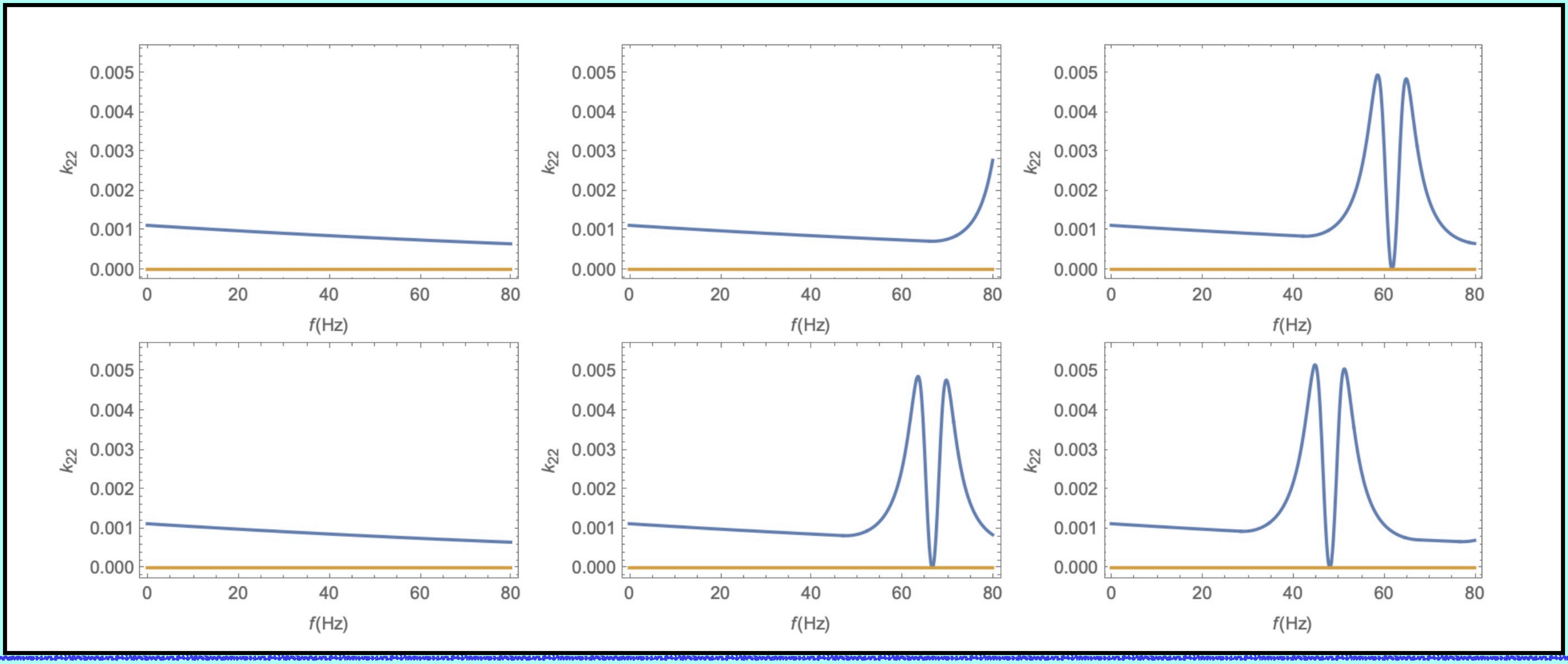
- The Dirichlet boundary condition can be retrieved only if $R_0 = -1$.

Area-Quantized Black Holes

- Area quantised black holes has non-trivial reflectivity and hence they have non-zero tidal Love number. [Nair, SC and Sarkar, arXiv:2208.06235]

- However the effects are too small and appears at 5 pN. [Krishnendu, Ajith, Kapadia, Datta, Ghosh and SC, Work in Progress]

- Significant implications for the tidal heating.



Future Directions

- Membrane fluid indeed affects the ringdown spectrum in a non-trivial manner. But effects in the in-spiral regime needs to be understood. This will be important for EMRIs and hence for LISA.

- Tidal Love number due to the deformation of the membrane depicts intriguing behaviour, both in the limit of zero rotation and zero frequency.

- It would be interesting to even get the viscosity coefficients from a microscopic perspective, in which case the whole problem will depend on quantities arising from quantum nature of spacetime.

- Hopefully, in future with more GW events and with more sensitivity, such quantum effects can be excavated.

Conclusion

- Models of "Quantum" black holes impose reflective nature on the black hole horizon.
- There are echoes in the ringdown signal, originating from the membrane and depends on the viscosity.
- The in-spiral part of the GW waveform also gets affected — tidal Love number of an exotic compact object is non-zero, but does not scale as Logarithm of ϵ, in general.
- Inclusion of rotation in the membrane paradigm and tidal heating need to be addressed.

Thank You