(Benchmark) Axion Models

Andreas Ringwald

Lecture in Training Week of
Galileo Galilei Institute Workshop
Axions across Boundaries between Particle Physics, Astrophysics, Cosmology and Forefront Detection Technologies
Florence, Italy
April 26 - 28, 2023

Plan

- **Recap: Peccei-Quinn Solution of the Strong CP Problem**
- **KSVZ Model**
- **DFSZ Model**
- **GUT Axion Model**

Recap: Peccei-Quinn Solution of the Strong CP Problem

Promote theta parameter to a dynamical field

- Add to the Standard Model (SM) a Nambu-Goldstone field, $\theta(x) \equiv a(x)/f_a \in [-\pi, \pi]$, which interacts with the SM like a dynamical theta parameter,

$$\mathcal{L} \supset \overline{\theta}\,\frac{\alpha_s}{8\pi} G^b_{\mu\nu}(x)\tilde{G}^{b,\mu\nu}(x) + \frac{f_a^2}{2}\partial_\mu\theta(x)\,\partial^\mu\theta(x) + \theta(x)\,\frac{\alpha_s}{8\pi} G^b_{\mu\nu}(x)\tilde{G}^{b,\mu\nu}(x)$$

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  - Dimension 5 interaction; theory breaks down at scales of order f_a ; needs UV completion at scales above this scale
- $\overline{\theta}$ -parameter of QCD can be eliminated by the shift $\theta(x) \to \theta(x) - \overline{\theta}$:

$$\mathcal{L} \supset \overline{\theta}\, \frac{\alpha_s}{8\pi} G^b_{\mu\nu}(x)\tilde{G}^{b,\mu\nu}(x) + \theta(x) \frac{\alpha_s}{8\pi} G^b_{\mu\nu}(x)\tilde{G}^{b,\mu\nu}(x)$$

$$\to \theta(x) \frac{\alpha_s}{8\pi} G^b_{\mu\nu}(x)\tilde{G}^{b,\mu\nu}(x)$$

Recap: Peccei-Quinn Solution of the Strong CP Problem

Peccei-Quinn mechanism

[Peccei,Quinn 77]

- Dynamics of $\theta(x) \equiv a(x)/f_a$, at energy scales below f_a, but above $\Lambda_{\rm QCD}$, described by

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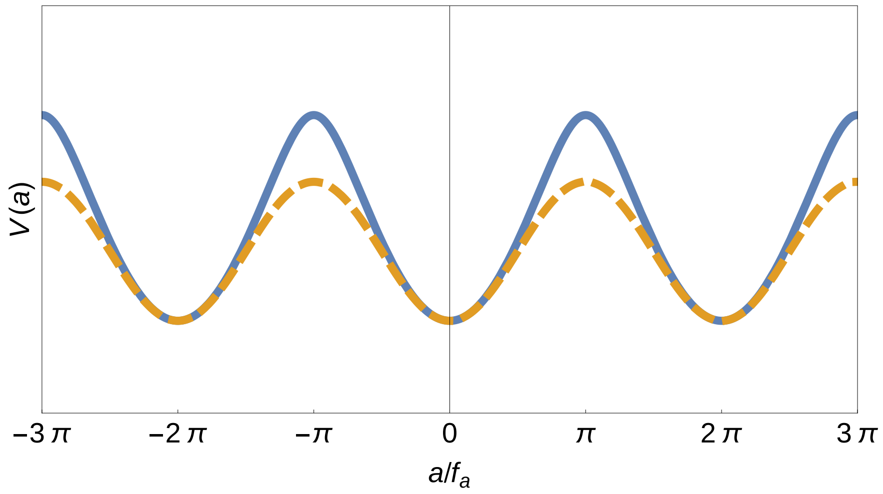
- At energies below $\Lambda_{\rm QCD}$,

[Di Vecchia,Veneziano `80; Leutwyler,Smilga `92]

$$\mathcal{L} \supset \frac{f_a^2}{2}\partial_\mu\theta(x)\partial^\mu\theta(x) - m_\pi^2 f_\pi^2\frac{\sqrt{1+z^2+2z\cos\theta}}{1+z}$$

$$z \equiv m_u/m_d \approx 1/2$$

  - Field dependence of effective potential coincides with theta-dependence of vacuum energy in QCD



[Grilli di Cortona et al. `16]

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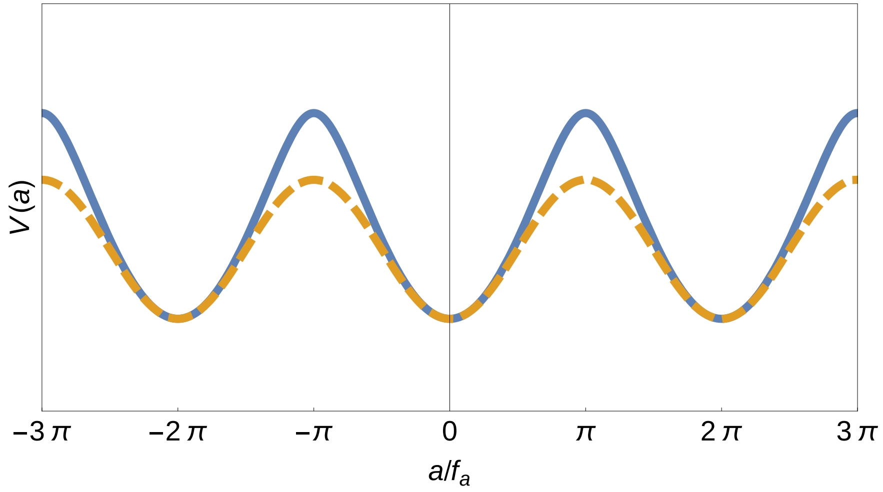

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- There is no strong CP violation in the vacuum: $\langle\theta\rangle_0 = 0$

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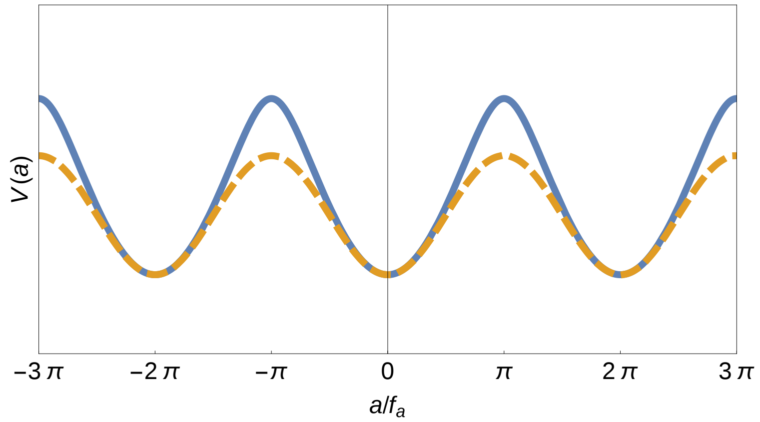
PQ mechanism predicts pseudo Nambu-Goldstone boson

[Weinberg 78; Wilczek 78]

$$\mathcal{L} \supset \frac{1}{2}\partial_\mu a(x)\partial^\mu a(x) - m_\pi^2 f_\pi^2 \frac{\sqrt{1+z^2+2z\cos\left(\frac{a(x)}{f_a}\right)}}{1+z}$$



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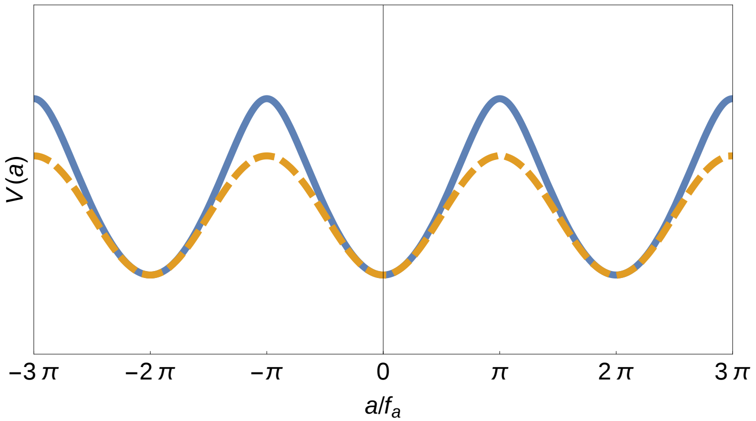

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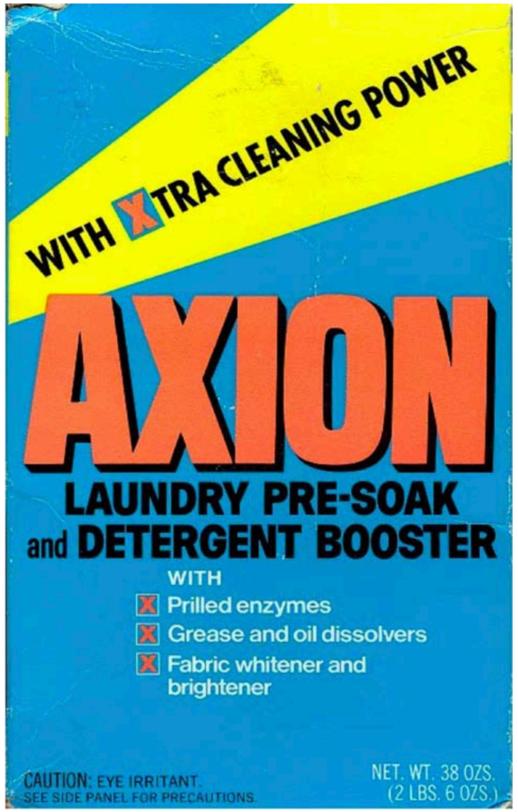
  - Cleans up a problem associated with an axial current



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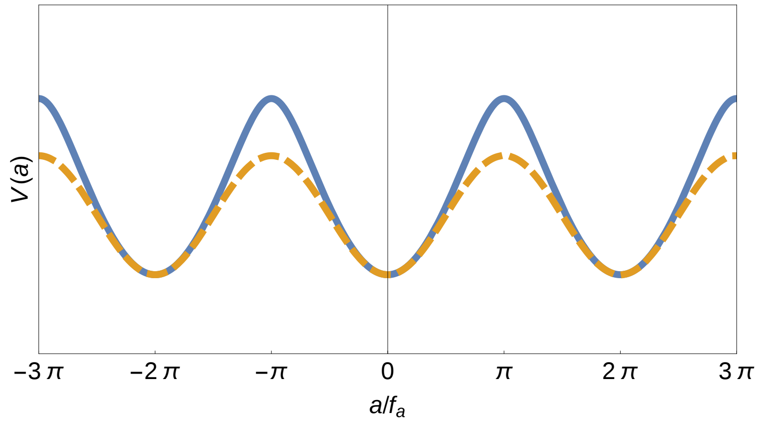

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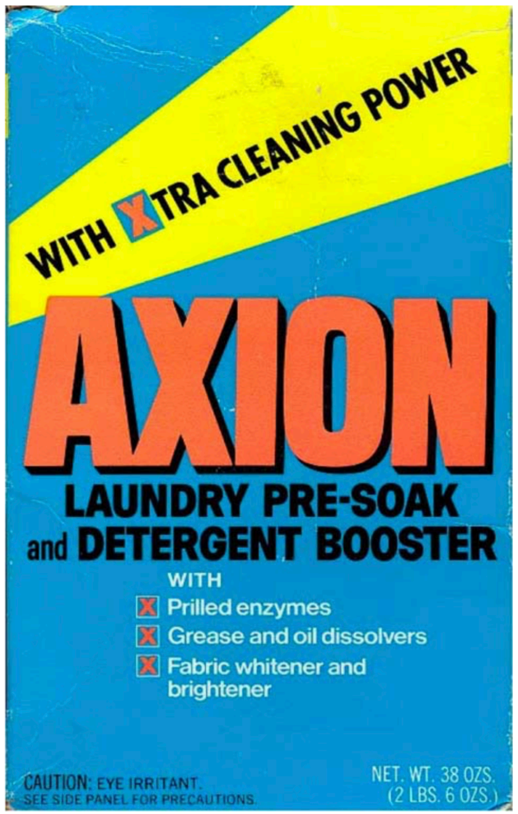

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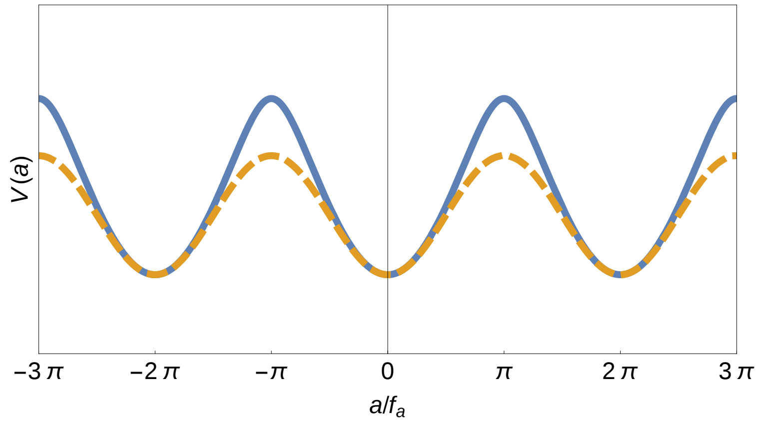

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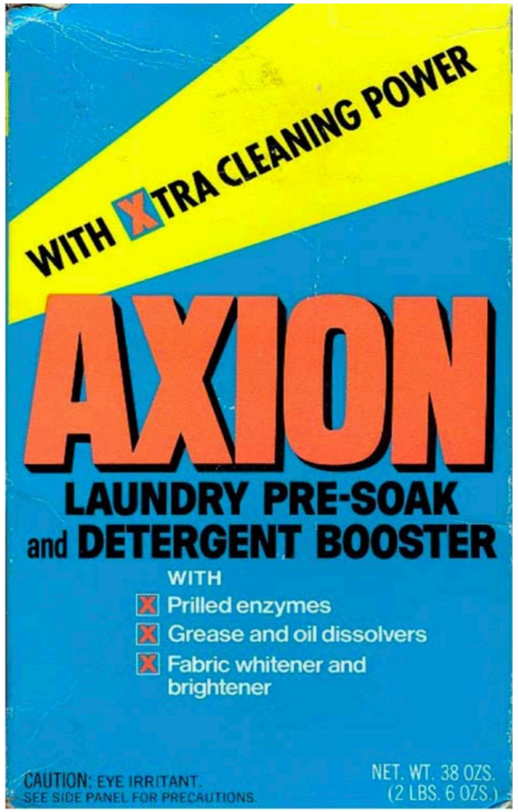
$$m_a \approx 6\ \text{meV}\left(\frac{10^9\ \text{GeV}}{f_a}\right)$$



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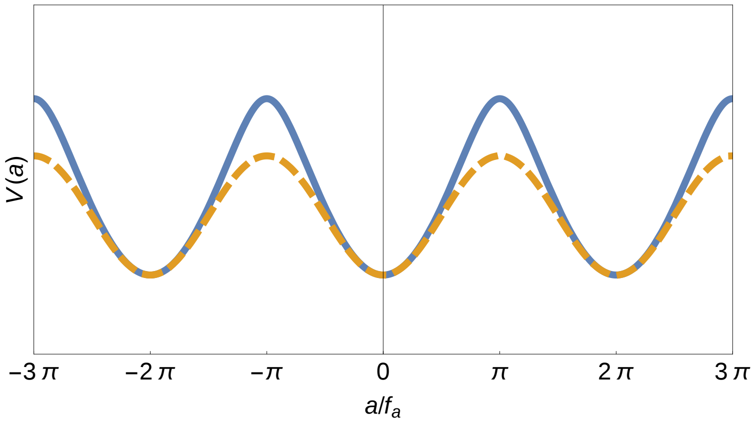

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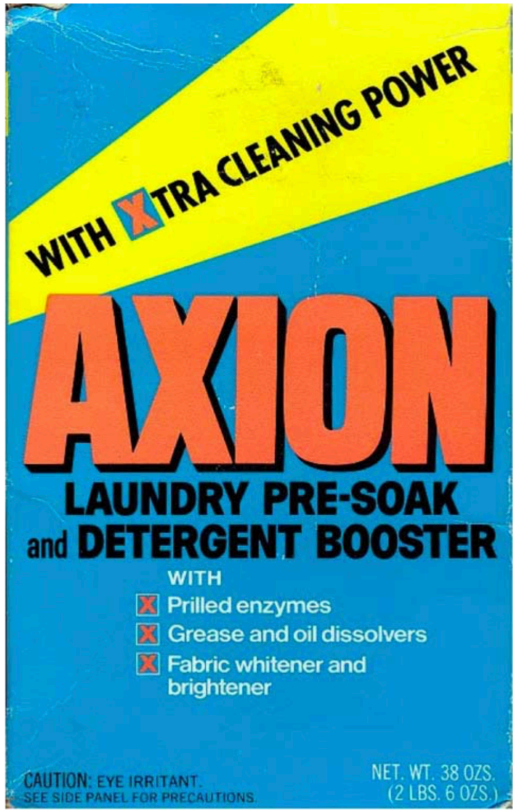
  - Precise calculation, by including $\mathcal{O}(\alpha)$QED corrections and NNLO corrections in chiral perturbation theory: [Gorghetto et al. 18]

$$m_a = 5.691(51)\left(\frac{10^9\ \text{GeV}}{f_a}\right)\text{meV}$$



KSVZ Axion Model

[Kim 79;Shifman,Vainshtein,Zakharov 80]

Minimal hadronic axion

- Add to SM a singlet complex scalar field σ, featuring a spontaneously broken global $U(1)_{\rm PQ}$ symmetry, and a vector-like fermion $\mathcal{Q} = \mathcal{Q}_L + \mathcal{Q}_R$ in the fundamental of colour, singlet under $SU(2)_L$ and neutral under hypercharge.

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- Assuming that under $U(1)_{\rm PQ}$ the fields transform as

$$\sigma \to e^{i\alpha}\sigma\,, \qquad \mathcal{Q}_L \to e^{i\alpha/2}\mathcal{Q}_L\,, \qquad \mathcal{Q}_R \to e^{-i\alpha/2}\mathcal{Q}_R$$

the most general renormalizable Lagrangian can be written as

$$\mathcal{L}_{\rm KSVZ} = |\partial_\mu \sigma|^2 - \lambda_\sigma \left(|\sigma|^2 - \frac{v_\sigma^2}{2} \right)^2 + \overline{\mathcal{Q}}\, i\gamma_\mu D^\mu \mathcal{Q} - \left(y_{\mathcal{Q}} \overline{\mathcal{Q}}_L \mathcal{Q}_R \sigma + {\rm h.c.} \right)$$

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- Decomposing the scalar field in polar coordinates,

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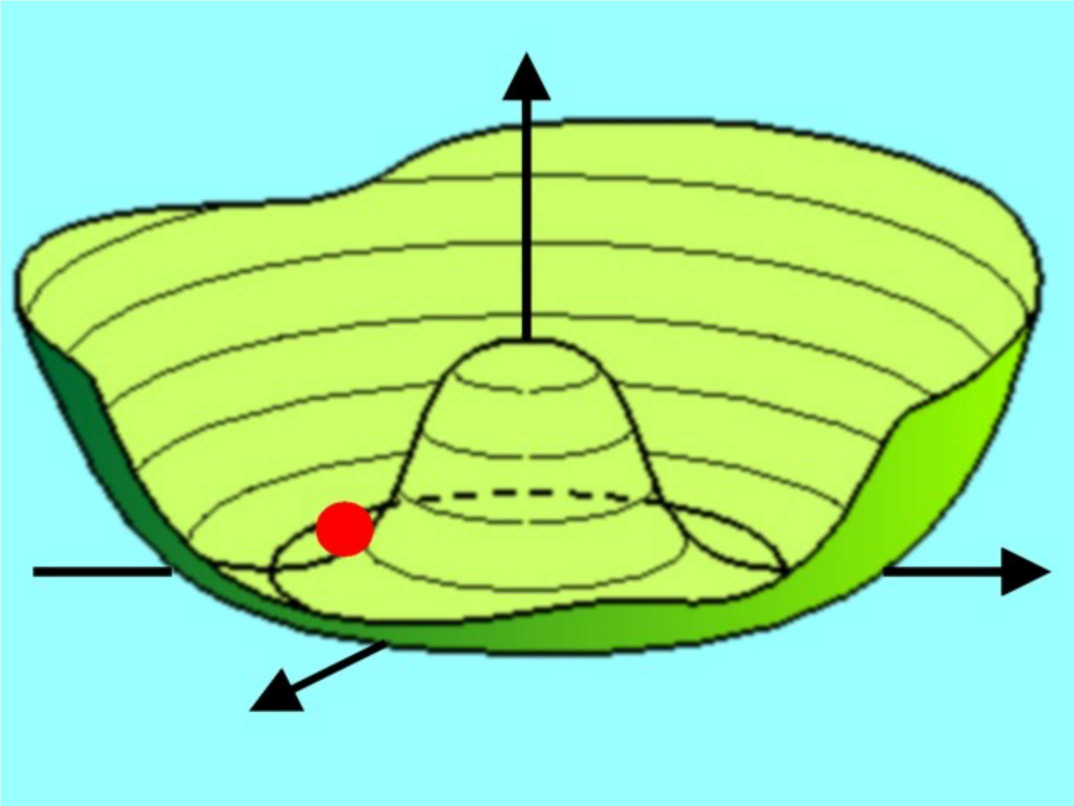
we see that this model features three BSM particles



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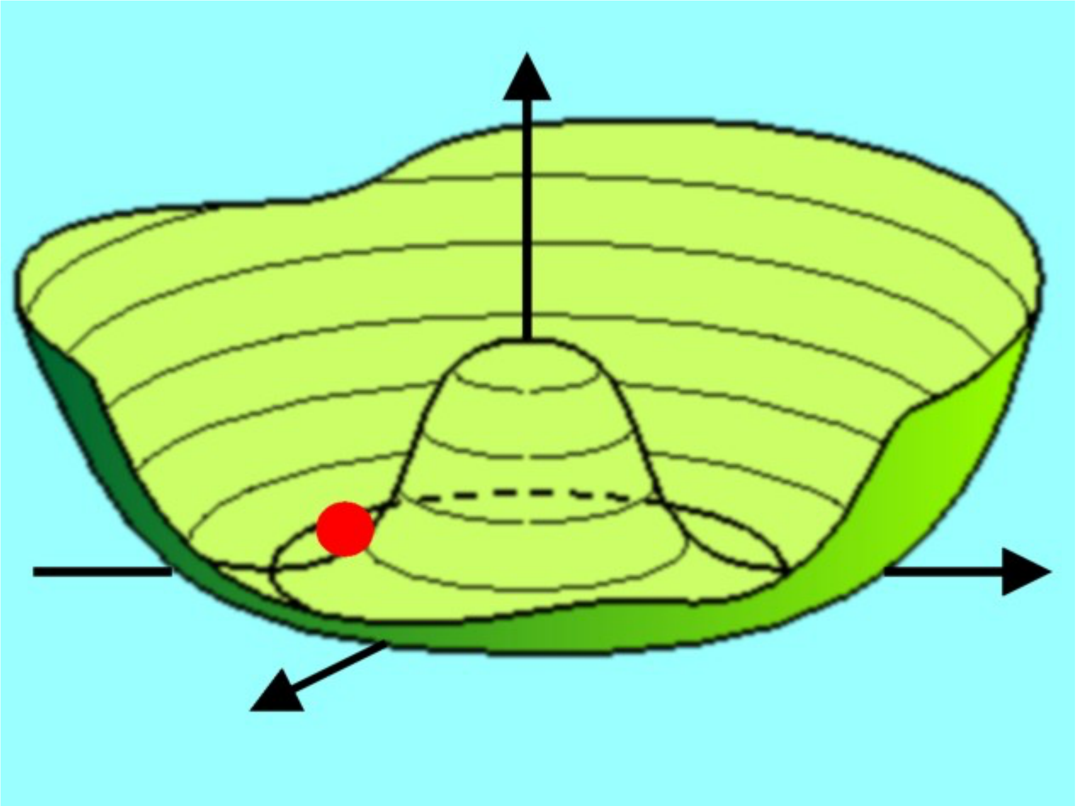

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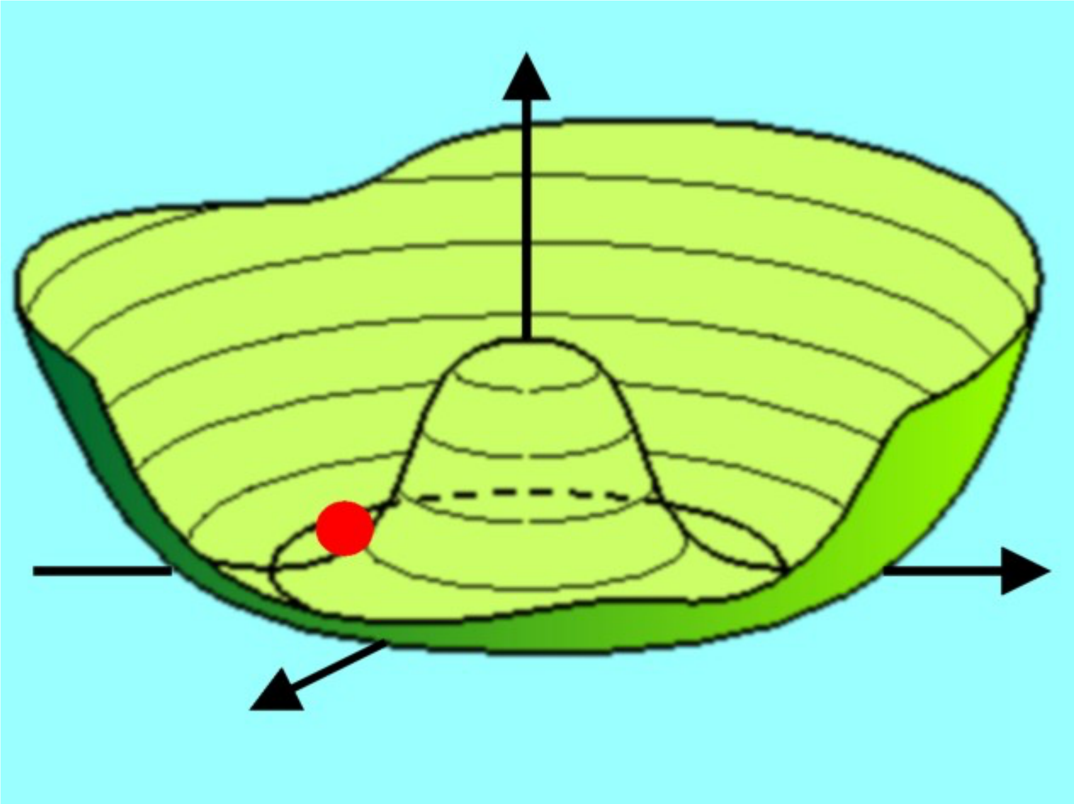

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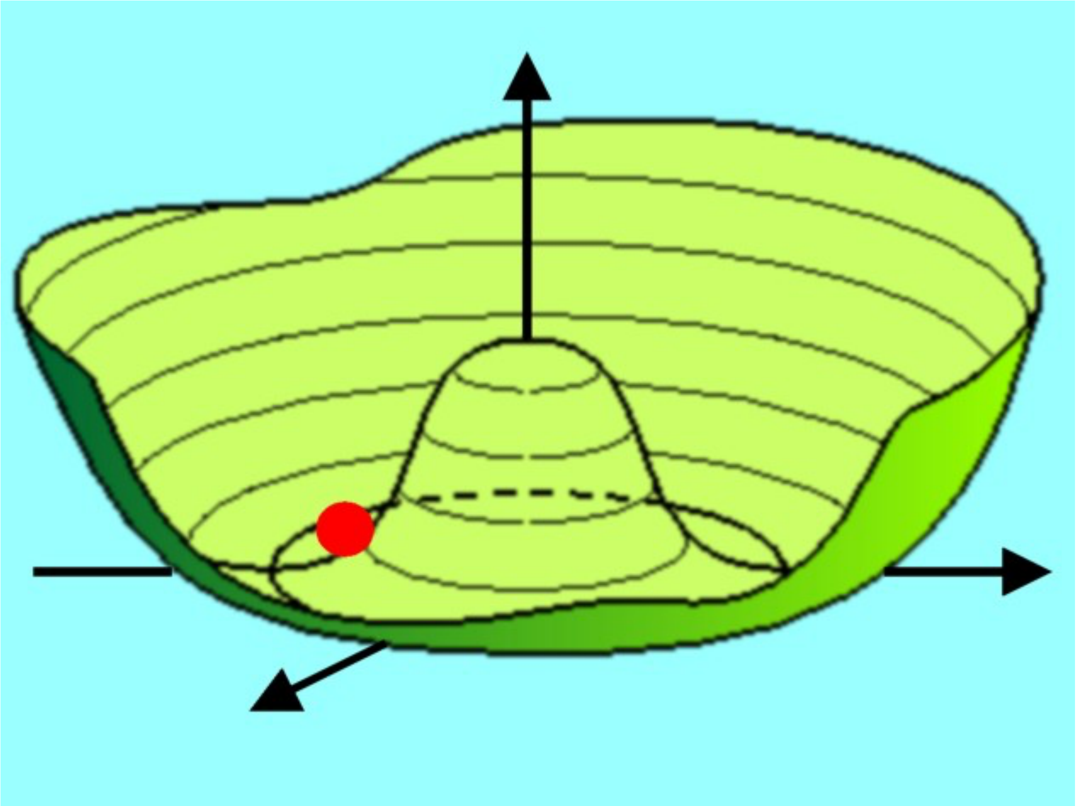

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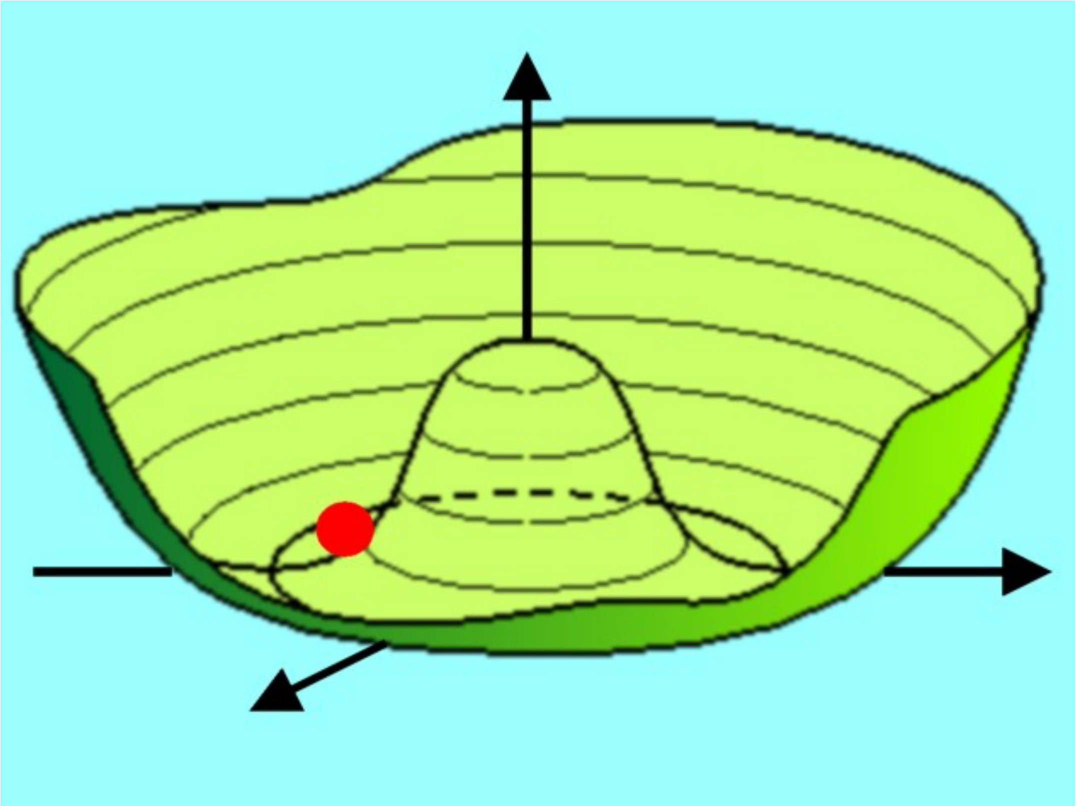
- For large PQ breaking scale v_σ, the latter two are heavy and may be integrated out, if we are only interested at the effective theory at energies much less than the breaking scale



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KSVZ Axion Model

[Kim 79;Shifman,Vainshtein,Zakharov 80]

Minimal hadronic axion

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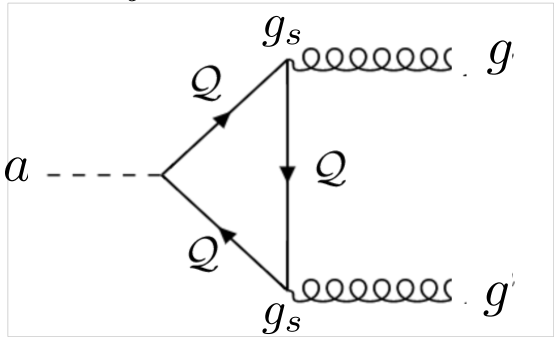

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- Now we can also safely integrate out the heavy exotic quark:

$$\mathcal{L}_{\rm KSVZ} \simeq \frac{1}{2}\partial_\mu a \partial^\mu a + \frac{g_s^2}{32\pi^2}\frac{a}{v_\sigma}G\tilde{G}$$

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- Now we can also safely integrate out the heavy exotic quark: $\theta = a/v_\sigma$ is indeed a dynamical $\bar{\theta}$-parameter!

$$\mathcal{L}_{\rm KSVZ} \simeq \frac{f_a^2}{2}\partial_\mu\theta\, \partial^\mu\theta + \theta\frac{\alpha_s}{8\pi} G\tilde{G} \qquad f_a = v_\sigma$$

Generalized KSVZ Axion Model

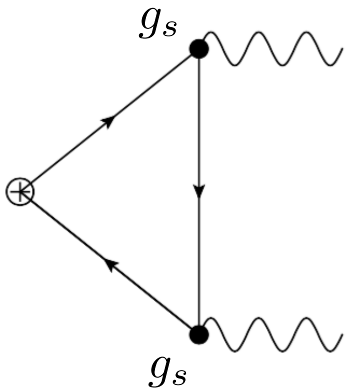
Hadronic axion with direct electromagnetic coupling

- Allowing for electric charge of the exotic quark, that is charged under $U(1)_E$, generalized KSVZ axion descri-bed by

$$\mathcal{L}_{\rm KSVZ} \simeq \frac{1}{2}\partial_\mu a \partial^\mu a + \frac{g_s^2}{32\pi^2}\frac{a}{f_a}G\tilde{G} + \frac{e^2}{32\pi^2}\frac{E}{N}\frac{a}{f_a}F\tilde{F}$$

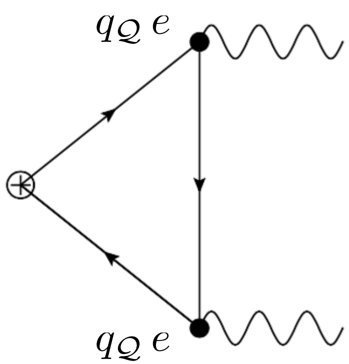
  - Axion decay constant: $f_a = v_\sigma / N$
  - Anomaly coefficients N and E:

$U(1)_{PQ}$ x $SU(3)_c$ x $SU(3)_c$



$$N = X_{Q_L} - X_{Q_R} = \frac{1}{2} - \left(-\frac{1}{2}\right) = 1$$

$U(1)_{PQ}$ x $U(1)_E$ x $U(1)_E$



$$E = 6\,(X_{Q_L} - X_{Q_R})\, q_Q^2 = 6\, q_Q^2$$

Generalized KSVZ Axion Model

Effective field theory below QCD scale

$$\mathcal{L}_{\rm KSVZ} \simeq \frac{1}{2}\partial_\mu a \partial^\mu a - \frac{1}{2} m_a^2 a^2 + \frac{\alpha}{8\pi}\frac{C_{a\gamma}}{f_a}\, a\, F_{\mu\nu}\tilde{F}^{\mu\nu} - \frac{i}{2}\frac{C_{a\gamma N}}{f_a}\, a\, \overline{\Psi}_N \sigma_{\mu\nu}\gamma_5 \Psi_N F^{\mu\nu}$$

$$m_a \approx \frac{\sqrt{z}}{1+z}\frac{m_\pi f_\pi}{f_a} \approx 6\ \mu{\rm eV}\left(\frac{10^{12}\,{\rm GeV}}{f_a}\right)$$

- Couplings to SM suppressed by powers of axion decay constant
  - KSVZ axion, for $f_a = v_\sigma/N \gg v \simeq 246\,{\rm GeV}$, is a benchmark “invisible axion”
- Wilson coefficients:
  - Electromagnetic coupling:

$$C_{a\gamma} = \frac{E}{N} - \frac{2}{3}\frac{4+z}{1+z}$$

$z \equiv m_u/m_d \approx 1/2$

[Kaplan 85;Srednicki `85; Grilli di Cortona et al. `16]

  - Nucleon electric dipole moment coupling:

$$C_{a\gamma N} = 2.4(1.0)\times 10^{-16}\, e\,{\rm cm}$$

[Pospelov,Ritz `00]

Generalized KSVZ Axion Model

Electromagnetic coupling

$$g_{a\gamma\gamma} \equiv \frac{\alpha}{2\pi f_a} C_{a\gamma} \simeq \frac{\alpha}{2\pi f_\pi} \frac{m_a}{m_\pi} \frac{1+z}{\sqrt{z}} \left(\frac{E}{N} - \frac{2}{3} \frac{4+z}{1+z} \right)$$

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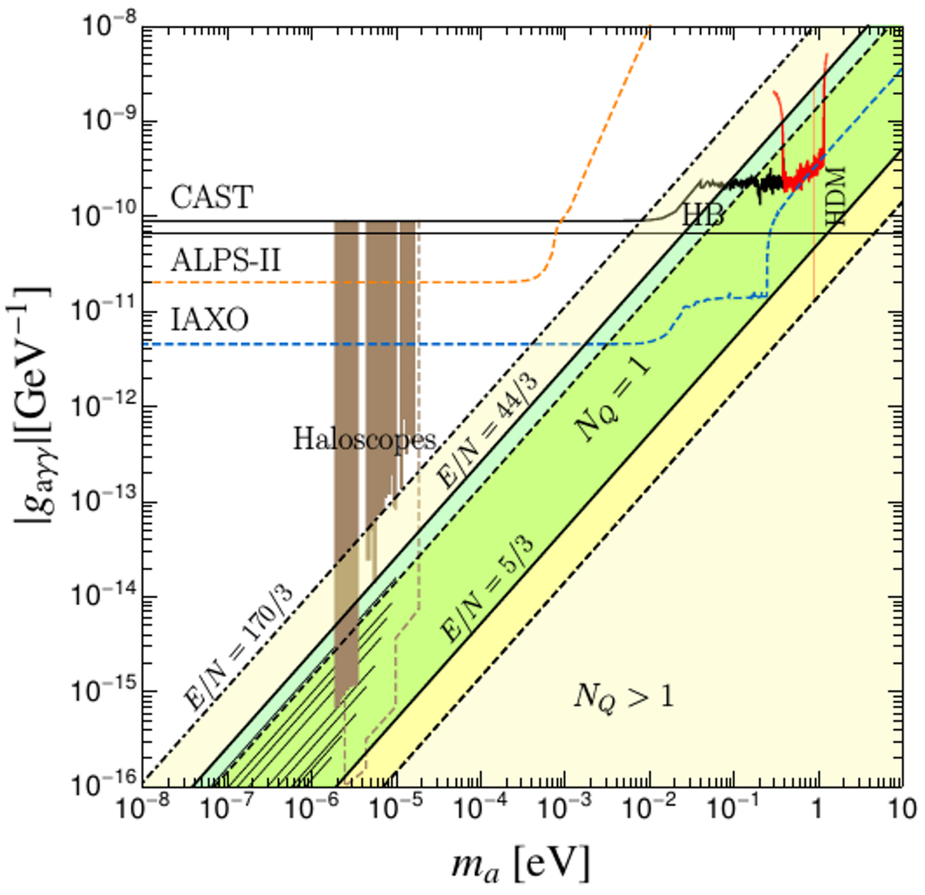

[Di Luzio,Mescia,Nardi 16, 18]

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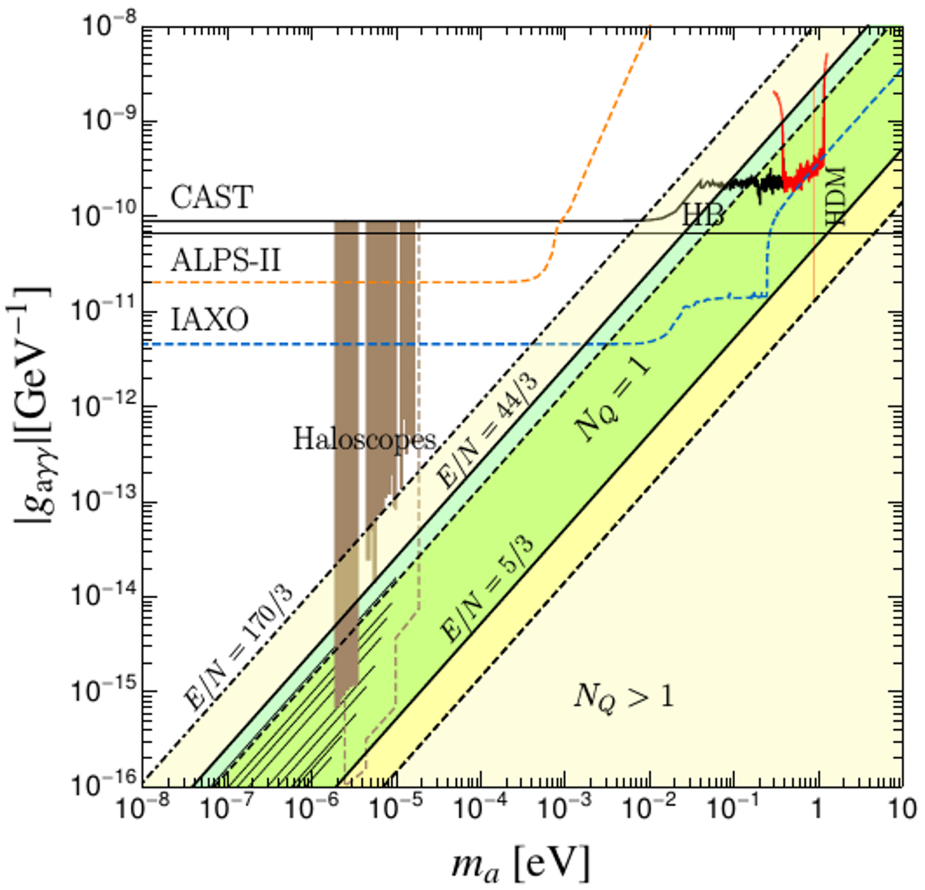
- What if exotic quark carries a magnetic charge? [Sokolov,AR 21, 22, 23]



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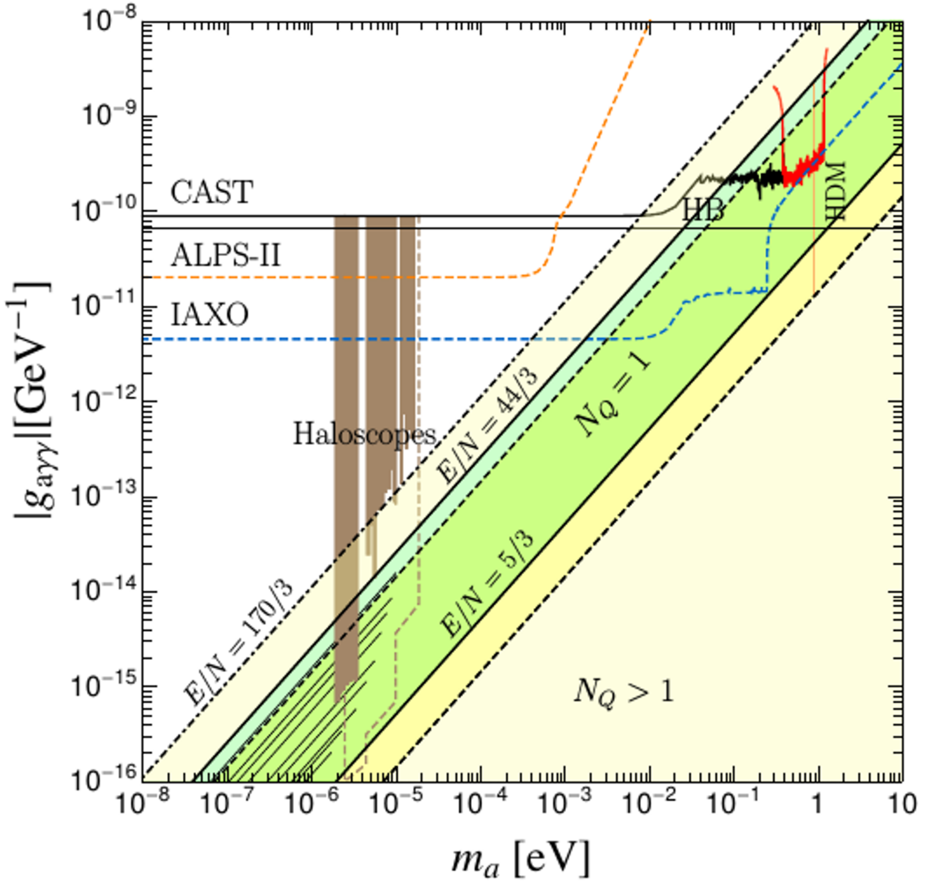

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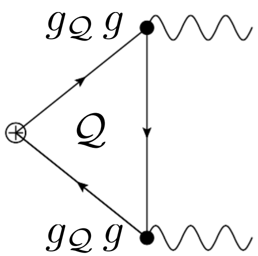
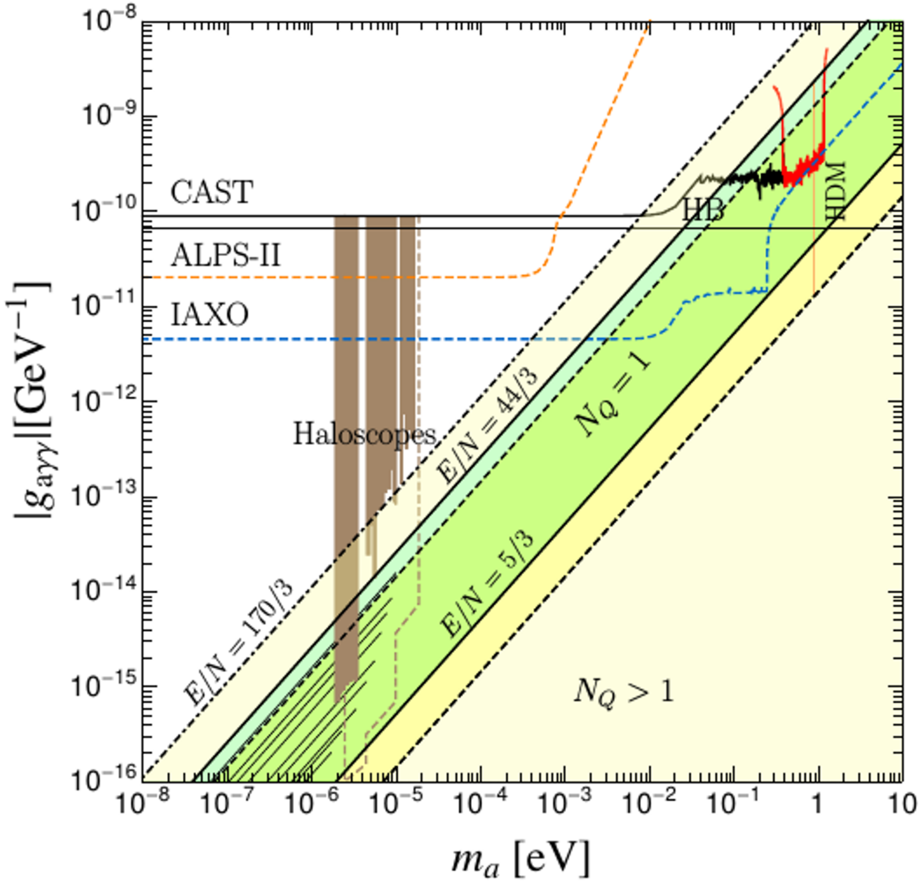
$$g_{aMM} \simeq \frac{\alpha_M}{2\pi f_\pi} \frac{m_a}{m_\pi} \frac{1+z}{\sqrt{z}} \frac{M}{N}$$



$$M = 6\, g_Q^2$$



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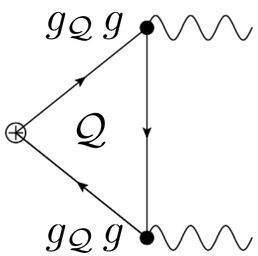

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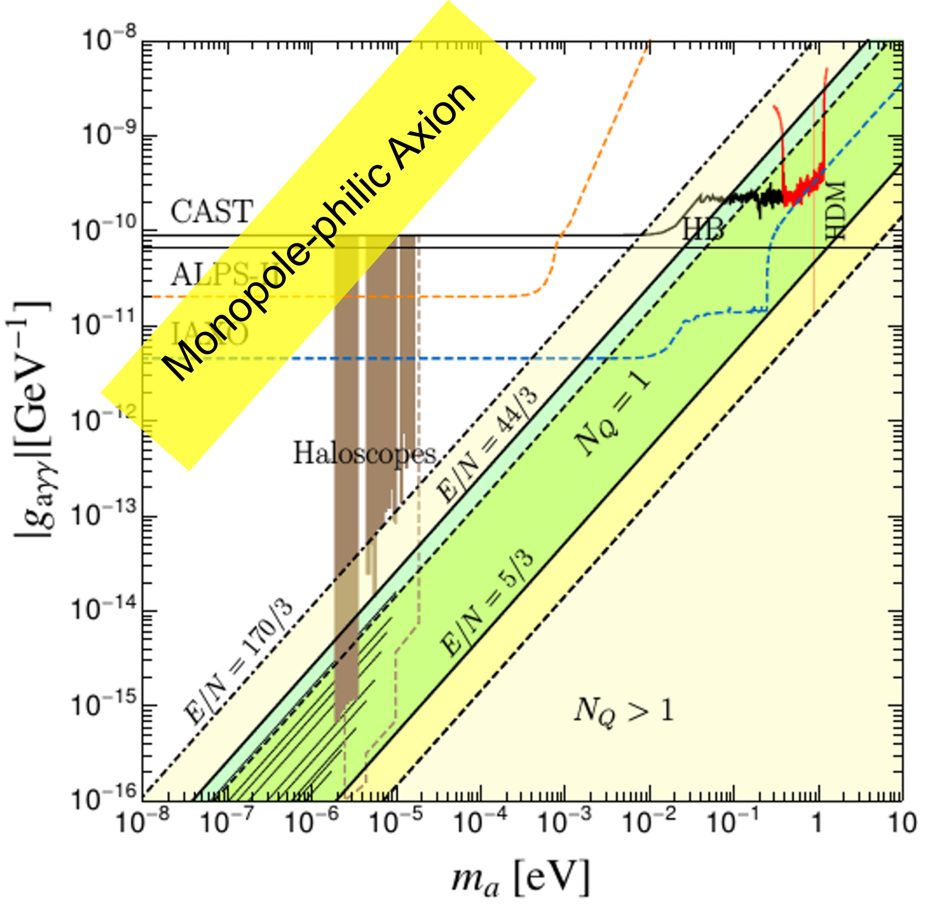
  - Parametrically enhanced due to charge quantisation:

$$\alpha_{\rm M} = \frac{g^2}{4\pi} \sim \frac{\pi^2/e^2}{4\pi} \sim \alpha^{-1} \quad \Rightarrow \frac{g_{aMM}}{g_{a\gamma\gamma}} \sim \alpha^{-2} \sim 10^4$$



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DFSZ Axion Model

[Zhitnitsky 80;Dine,Fischler,Srednicki 81]

Field content and PQ charges

- Add to SM a singlet complex scalar field σ, featuring a spontaneously broken global $U(1)_{\rm PQ}$ symmetry and extend the Higgs sector of the SM to a Type-II Two Higgs Doublet Model (2HDM)

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  - Most general Yukawa interactions (in Higgs literature dubbed “Type-II”):

$$-\mathcal{L}_Y = Y_u \bar{q}_L \tilde{H}_u u_R + Y_d \bar{q}_L H_d d_R + Y_e \bar{l}_L H_d e_R + \text{h.c.}$$

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  - Most general scalar field potential:

$$\begin{aligned} V(H_u, H_d, \sigma) = &\, M_{uu}^2\, H_u^\dagger H_u + M_{dd}^2\, H_d^\dagger H_d + M_{\sigma\sigma}^2\, \sigma^*\sigma \\ &+ \frac{\lambda_u}{2}\left(H_u^\dagger H_u\right)^2 + \frac{\lambda_d}{2}\left(H_d^\dagger H_d\right)^2 + \frac{\lambda_\sigma}{2}\left(\sigma^*\sigma\right)^2 \\ &+ \lambda_3 \left(H_d^\dagger H_d\right)\left(H_u^\dagger H_u\right) + \lambda_4 \left(H_d^\dagger H_u\right)\left(H_u^\dagger H_d\right) \\ &+ \lambda_{d\sigma}\left(H_d^\dagger H_d\right)\left(\sigma^*\sigma\right) + \lambda_{u\sigma}\left(H_u^\dagger H_u\right)\left(\sigma^*\sigma\right) - \lambda_{ud\sigma}\left(H_u^\dagger H_d \sigma^2 + h.c.\right) \end{aligned}$$

DFSZ Axion Model

[Zhitnitsky 80;Dine,Fischler,Srednicki 81]

Field content and PQ charges

- Add to SM a singlet complex scalar field σ, featuring a spontaneously broken global $U(1)_{\rm PQ}$ symmetry and extend the Higgs sector of the SM to a Type-II Two Higgs Doublet Model (2HDM)
  - Charge assignments:

Field	q_L	u_R	d_R	L	l_R	H_u	H_d	σ
PQ charge	0	X_u	X_d	0	X_d	X_u	$-X_d$	1

$$X_u + X_d = 2$$

  - Most general Yukawa interactions (in Higgs literature dubbed “Type-II”):

$$-\mathcal{L}_Y = Y_u \bar{q}_L \tilde{H}_u u_R + Y_d \bar{q}_L H_d d_R + Y_e \bar{l}_L H_d e_R + \text{h.c.}$$

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  - Most general scalar field potential:

$$\begin{aligned} V(H_u, H_d, \sigma) = & M_{uu}^2\, H_u^\dagger H_u + M_{dd}^2\, H_d^\dagger H_d + M_{\sigma\sigma}^2\, \sigma^* \sigma \\ & + \frac{\lambda_u}{2} \left(H_u^\dagger H_u\right)^2 + \frac{\lambda_d}{2} \left(H_d^\dagger H_d\right)^2 + \frac{\lambda_\sigma}{2} \left(\sigma^* \sigma\right)^2 \\ & + \lambda_3 \left(H_d^\dagger H_d\right) \left(H_u^\dagger H_u\right) + \lambda_4 \left(H_d^\dagger H_u\right) \left(H_u^\dagger H_d\right) \\ & + \lambda_{d\sigma} \left(H_d^\dagger H_d\right) \left(\sigma^* \sigma\right) + \lambda_{u\sigma} \left(H_u^\dagger H_u\right) \left(\sigma^* \sigma\right) - \lambda_{ud\sigma} \left(H_u^\dagger H_d \sigma^2 + h.c.\right) \end{aligned}$$

    - Last term responsible for explicit breaking of re-phasing symmetry,

$$U(1)_{H_u} \times U(1)_{H_d} \times U(1)_\sigma \to U(1)_Y \times U(1)_{\rm PQ}$$

DFSZ Axion Model

[Zhitnitsky 80;Dine,Fischler,Srednicki 81]

Vacuum structure, particle spectrum and masses

- Parameters in the scalar potential are chosen such that it attains minimum at the VEVs

$$\langle H_u^0 \rangle = v_u/\sqrt{2}, \quad \langle H_d^0 \rangle = v_d/\sqrt{2}, \quad \langle \sigma \rangle = v_\sigma/\sqrt{2}$$

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- For $v_\sigma \gg v \equiv \sqrt{v_u^2 + v_d^2} \simeq 246\,\mathrm{GeV}$: $\quad t_\beta \equiv \tan\beta \equiv v_u/v_d$
  - 5 heavy BSM scalars: $\quad m_H^2 \approx m_A^2 \approx m_{H^\pm}^2 \approx \lambda_{ud\sigma} v_\sigma^2 (1+t_\beta^2)/(2t_\beta) \qquad m_\rho^2 \approx \lambda_\sigma v_\sigma^2$

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  - SM-like Higgs boson: $m_h^2 \approx \left(\lambda_d + \lambda_u\, t_\beta^4 + 2\,(\lambda_3+\lambda_4)\, t_\beta^2\right) v^2 / \left(1+t_\beta^2\right)^2$

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  - Axion (NG boson): massless at tree level

DFSZ Axion Model

[Zhitnitsky 80;Dine,Fischler,Srednicki 81]

Low energy effective Lagrangian

- NG boson field eaten by the Z boson and axion field and can be parametrized in terms of phase directions:

$$H_u^0(x) \propto e^{i\left[\frac{\zeta(x)}{v} + X_u \frac{a(x)}{\tilde{v}_\sigma}\right]}, \qquad H_d^0(x) \propto e^{i\left[\frac{\zeta(x)}{v} - X_d \frac{a(x)}{\tilde{v}_\sigma}\right]}, \qquad \sigma(x) \propto e^{i \frac{a(x)}{\tilde{v}_\sigma}}$$

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  - Requiring orthogonality of those fields fixes the PQ charges:

$$X_u = 2\,(v_d/v)^2 \equiv 2\cos^2\beta, \qquad X_d = 2\,(v_u/v)^2 \equiv 2\sin^2\beta$$

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- Integrating out massive scalars and perform an axion field dependent chiral transformation on the fermions

$$u \to e^{i\gamma_5 X_u \frac{a}{2\tilde{v}_\sigma}} u, \qquad d \to e^{i\gamma_5 X_d \frac{a}{2\tilde{v}_\sigma}} d, \qquad e \to e^{i\gamma_5 X_d \frac{a}{2\tilde{v}_\sigma}} e$$

  to render the fermion mass terms axion field independent

$$-\mathcal{L}_Y = m_U \overline{u}_L u_R e^{-iX_u \frac{a}{\tilde{v}_\sigma}} + m_D \overline{d}_L d_R e^{-iX_d \frac{a}{\tilde{v}_\sigma}} + m_E \overline{e}_L e_R e^{-iX_d \frac{a}{\tilde{v}_\sigma}} + \text{h.c}$$

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  to render the fermion mass terms axion field independent provides anomalous couplings to gauge bosons:

$$-\mathcal{L}_Y = m_U \overline{u}_L u_R e^{-iX_u \frac{a}{\tilde{v}_\sigma}} + m_D \overline{d}_L d_R e^{-iX_d \frac{a}{\tilde{v}_\sigma}} + m_E \overline{e}_L e_R e^{-iX_d \frac{a}{\tilde{v}_\sigma}} + \text{h.c}$$

$$\to m_U \overline{u}_L u_R + m_D \overline{d}_L d_R + m_E \overline{e}_L e_R + \frac{\alpha_s}{8\pi}\, n_{\rm gen}\, (X_u + X_d)\frac{a}{\tilde{v}_a}\, G^c_{\mu\nu}\tilde{G}^{c,\mu\nu} + \frac{\alpha}{8\pi}\, n_{\rm gen}\, \frac{8}{3}\,(X_u + X_d)\frac{a}{\tilde{v}_a}\, F_{\mu\nu}\tilde{F}^{\mu\nu}$$

DFSZ Axion Model

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Low energy effective Lagrangian

$$\mathcal{L}_{\rm DFSZ} = \frac{1}{2}\partial_\mu a \partial^\mu a + \frac{\alpha_s}{8\pi}\frac{a}{f_a} G^c_{\mu\nu}\tilde{G}^{c,\mu\nu} + \frac{\alpha}{8\pi}\frac{E}{N}\frac{a}{f_a} F_{\mu\nu}\tilde{F}^{\mu\nu} + \frac{1}{2}\frac{\partial_\mu a}{f_a}\sum_f C_{af}\bar{f}\gamma^\mu\gamma_5 f$$

- Axion decay constant: $f_a = \tilde{v}_\sigma / N$ $\qquad \tilde{v}_\sigma = \sqrt{v_\sigma^2 + 4v^2/(2+t_\beta^2)^2}$

- Coefficients:
  - Coupling to gluons: $N = n_{\rm gen}(X_u + X_d) = 6 \qquad E = \frac{8}{3} n_{\rm gen}(X_u + X_d) = 16 \qquad \to \frac{E}{N} = \frac{8}{3}$
  - Coupling to fermions arise from field dependent chiral transformations via the fermion kinetic terms:

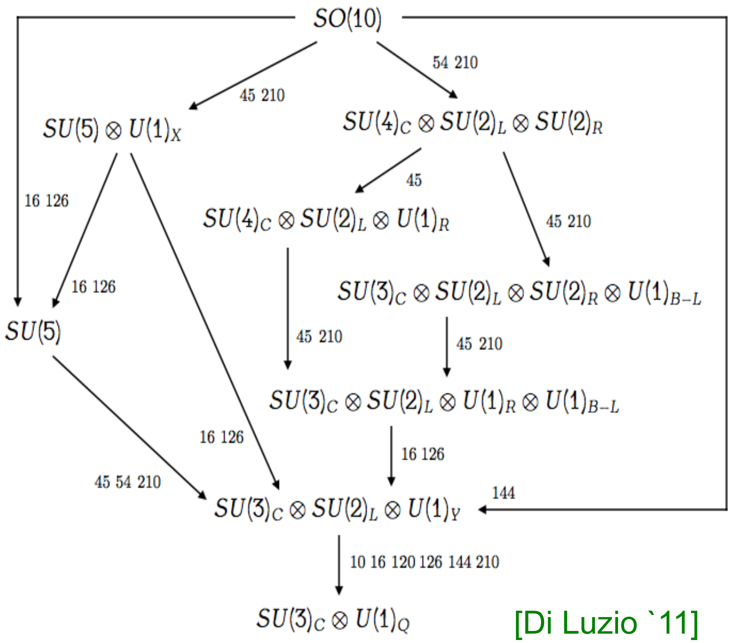
$$C_{ae} = C_{ad} = \frac{X_d}{N} = \frac{\sin^2\beta}{3}; \qquad C_{au} = \frac{X_u}{N} = \frac{\cos^2\beta}{3}$$

GUT Axion

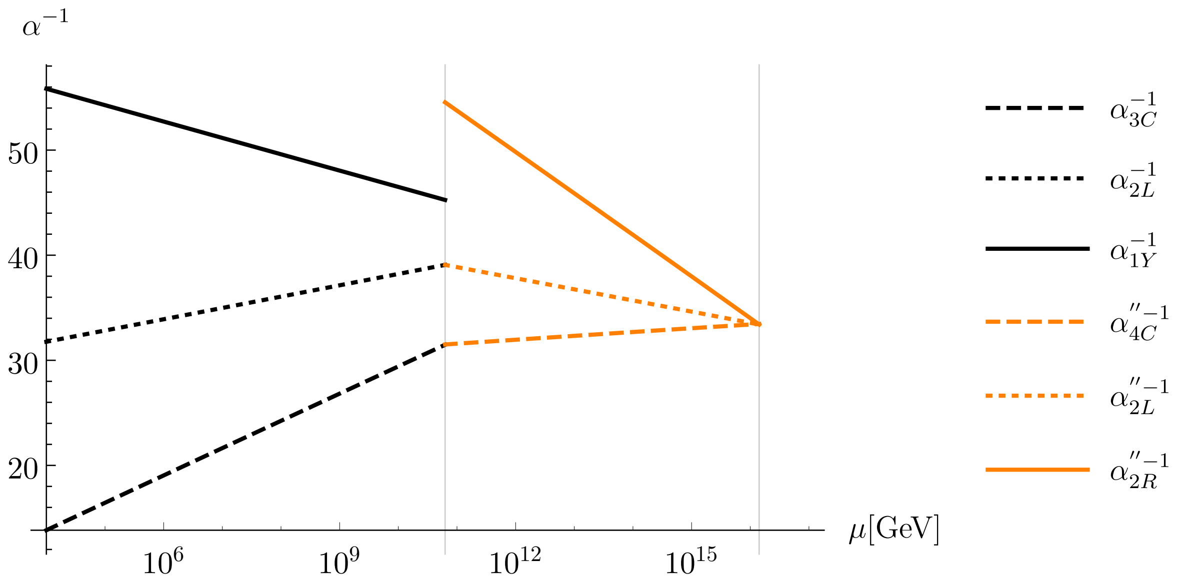
SO(10) x U(1)$_{PQ}$ GUT model

- Gauge coupling unification needs at least one intermediate scale; often discussed SSB chain:

$$SO(10) \overset{M_U - 210_H}{\longrightarrow} SU(4)_C \times SU(2)_L \times SU(2)_R$$
$$\overset{M_{BL} - 126_H}{\longrightarrow} SU(3)_C \times SU(2)_L \times U(1)_Y$$
$$\overset{M_Z - 10_H}{\longrightarrow} SU(3)_C \times U(1)_{em}$$



[Di Luzio `11]



[Ernst, AR, Tamarit, arXiv:1801.04906]

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- SO(10) GUT with three copies of 16_F automatically features
  - neutrino masses and mixing
  - baryogenesis via leptogenesis

$SO(10)$	$4_C 2_L 2_R$	$4_C 2_L 1_R$	$3_C 2_L 1_R 1_{B-L}$	$3_C 2_L 1_Y$	scale
16_F	$(4,2,1)$	$(4,2,0)$	$(3,2,0,\frac{1}{3})$	$(3,2,\frac{1}{6}) := Q$	M_Z
			$(1,2,0,-1)$	$(1,2,-\frac{1}{2}) := L$	M_Z
	$(\bar{4},1,2)$	$(\bar{4},1,\frac{1}{2})$	$(\bar{3},1,\frac{1}{2},-\frac{1}{3})$	$(\bar{3},1,\frac{1}{3}) := d$	M_Z
			$(1,1,\frac{1}{2},1)$	$(1,1,1) := e$	M_Z
		$(\bar{4},1,-\frac{1}{2})$	$(\bar{3},1,-\frac{1}{2},-\frac{1}{3})$	$(\bar{3},1,-\frac{2}{3}) := u$	M_Z
			$(1,1,-\frac{1}{2},1)$	$(1,1,0) := N$	$M_{\rm BL}$

- Most general Yukawas:

$$\mathcal{L}_Y = 16_F \left(Y_{10} 10_H + \tilde{Y}_{10} 10^*_H + Y_{126} \overline{126}_H \right) 16_F$$

- SSB vevs:

$$v_L \equiv \langle (\overline{10},3,1)_{126} \rangle \,, \qquad v_R \equiv \langle (10,1,3)_{126} \rangle \,,$$
$$v^{10}_{u,d} \equiv \langle (1,2,2)^{10}_{u,d} \rangle \,, \qquad v^{126}_{u,d} \equiv \langle (15,2,2)^{126}_{u,d} \rangle$$

- Fermion masses/mixing:

$$M_u = Y_{10} v^{10}_u + \tilde{Y}_{10} v^{10^*}_d + Y_{126} v^{126}_u \,,$$
$$M_d = Y_{10} v^{10}_d + \tilde{Y}_{10} v^{10^*}_u + Y_{126} v^{126}_d \,,$$
$$M_e = Y_{10} v^{10}_d + \tilde{Y}_{10} v^{10^*}_u - 3 Y_{126} v^{126}_d \,,$$
$$M_D = Y_{10} v^{10}_u + \tilde{Y}_{10} v^{10^*}_d - 3 Y_{126} v^{126}_u \,,$$
$$M_R = Y_{126} v_R \,,$$
$$M_L = Y_{126} v_L \,.$$

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- PQ extension adds
  - predictivity of fermion masses/mixing
  - solution of strong CP problem
  - DM candidate: axion

[Bajc et al. 06; Altarelli,Meloni 13; Babu,Khan 15]

- PQ symmetry imposed:

$$\begin{aligned} 16_F &\to 16_F e^{i\alpha}, \\ 10_H &\to 10_H e^{-2i\alpha}, \\ \overline{126}_H &\to \overline{126}_H e^{-2i\alpha}, \\ 210_H &\to 210_H e^{4i\alpha} \end{aligned}$$

- Most general Yukawas:

$$\mathcal{L}_Y = 16_F \left(Y_{10} 10_H + Y_{126} \overline{126}_H \right) 16_F + \text{h.c.}$$

- SSB vevs:

$$v_L \equiv \left\langle (\overline{10}, 3, 1)_{126} \right\rangle , \qquad v_R \equiv \left\langle (10, 1, 3)_{126} \right\rangle ,$$
$$v_{u,d}^{10} \equiv \left\langle (1, 2, 2)_{u,d}^{10} \right\rangle , \qquad v_{u,d}^{126} \equiv \left\langle (15, 2, 2)_{u,d}^{126} \right\rangle$$

- Fermion masses/mixing:

$$\begin{aligned} M_u &= Y_{10} v_u^{10} + Y_{126} v_u^{126} , \\ M_d &= Y_{10} v_d^{10} + Y_{126} v_d^{126} , \\ M_e &= Y_{10} v_d^{10} - 3 Y_{126} v_d^{126} , \\ M_D &= Y_{10} v_u^{10} - 3 Y_{126} v_u^{126} , \\ M_R &= Y_{126} v_R , \\ M_L &= Y_{126} v_L . \end{aligned}$$

GUT Axion

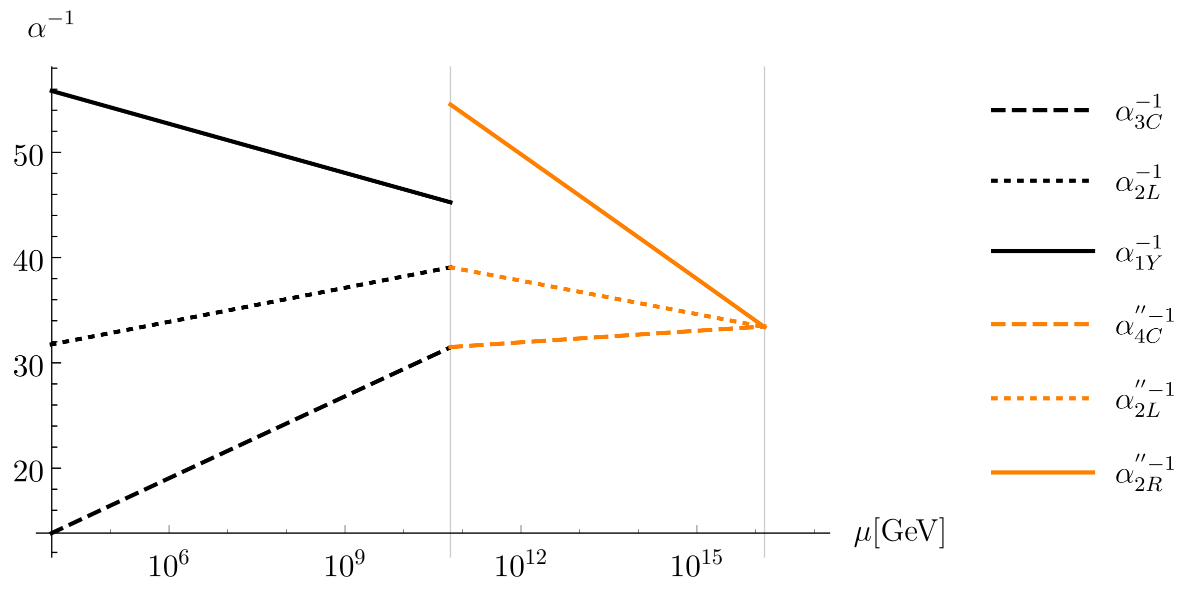
SO(10) x $U(1)_{PQ}$ GUT model

- Axion decay constant:

$$f_a \simeq \frac{1}{3}\frac{M_U}{g_U}$$

- From gauge coupling unification, assuming minimal scalar threshold corrections:

$$m_a = 5.691(51)\left(\frac{10^9\,\text{GeV}}{f_a}\right)\text{meV} \simeq 0.74\,\text{neV}$$



[Ernst, AR, Tamarit, arXiv:1801.04906]

$$M_\text{U} = 1.4\times10^{16}\,\text{GeV},\;\; \alpha_\text{U}(M_\text{U})^{-1} = 33.6$$

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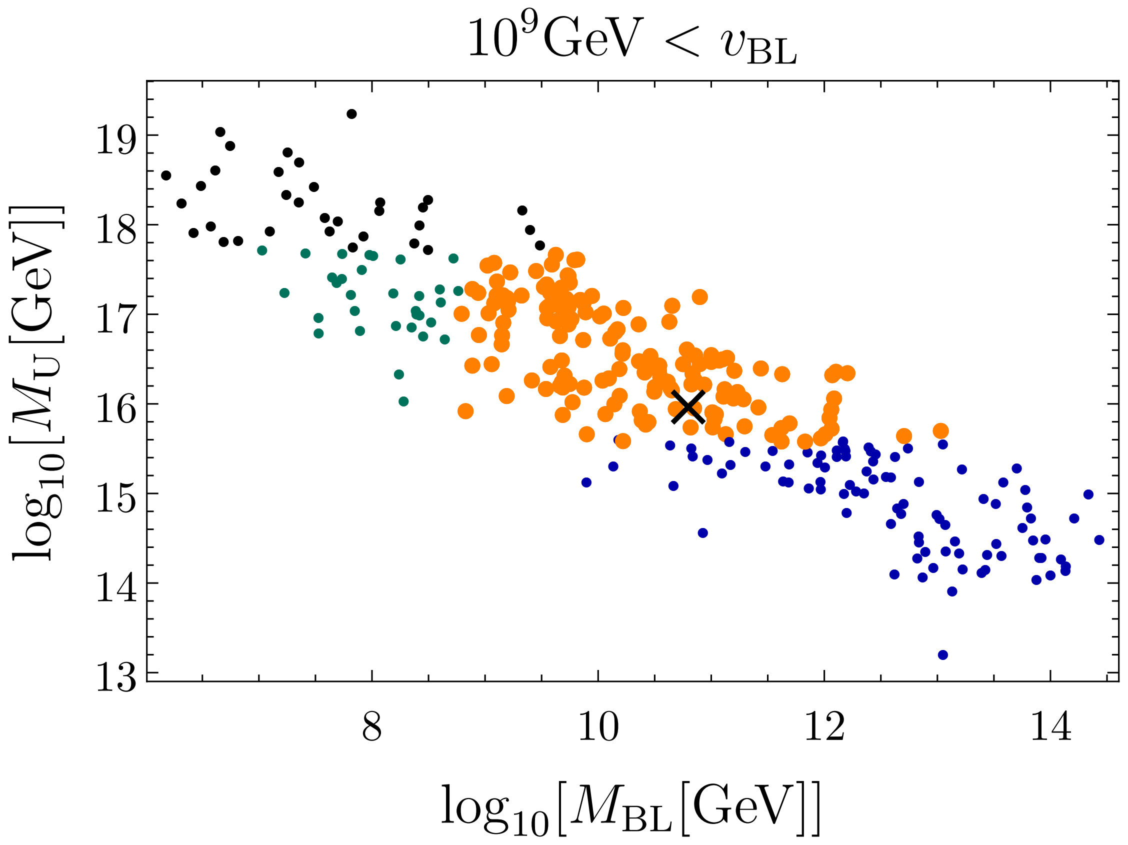
- Taking into account scalar threshold corrections and constraints from black hole superradiance and proton decay:

$$0.02\,\mathrm{neV} < m_a < 2.2\,\mathrm{neV}$$



[Ernst, AR, Tamarit, arXiv:1801.04906]

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- Low energy couplings to SM gauge bosons identical to DFSZ axion, but here decay constant fixed by GUT scale [Ernst, AR, Tamarit, arXiv:1801.04906]

$$\mathcal{L}_{\rm int}^{\rm QCD} = \frac{1}{2}\partial_\mu A \partial^\mu A - \frac{1}{2} m_A^2 A^2 + \frac{\alpha}{8\pi}\frac{C_{A\gamma}}{f_A} A\, F_{\mu\nu}\tilde{F}^{\mu\nu}$$
$$- \partial_\mu A \left[\frac{C_{AP}}{2 f_A} \overline{P}^\dagger \gamma^\mu \gamma_5 P + \frac{C_{AN}}{2 f_A} \overline{N}^\dagger \gamma^\mu \gamma_5 N + \frac{C_{AE}}{2 f_A} \overline{E}^\dagger \gamma^\mu \gamma_5 E \right],$$

$$C_{A\gamma} = \frac{8}{3} - 1.92(4),$$

$$C_{AP} = \ -0.62 + 0.43 \cos^2\beta \pm 0.03,$$

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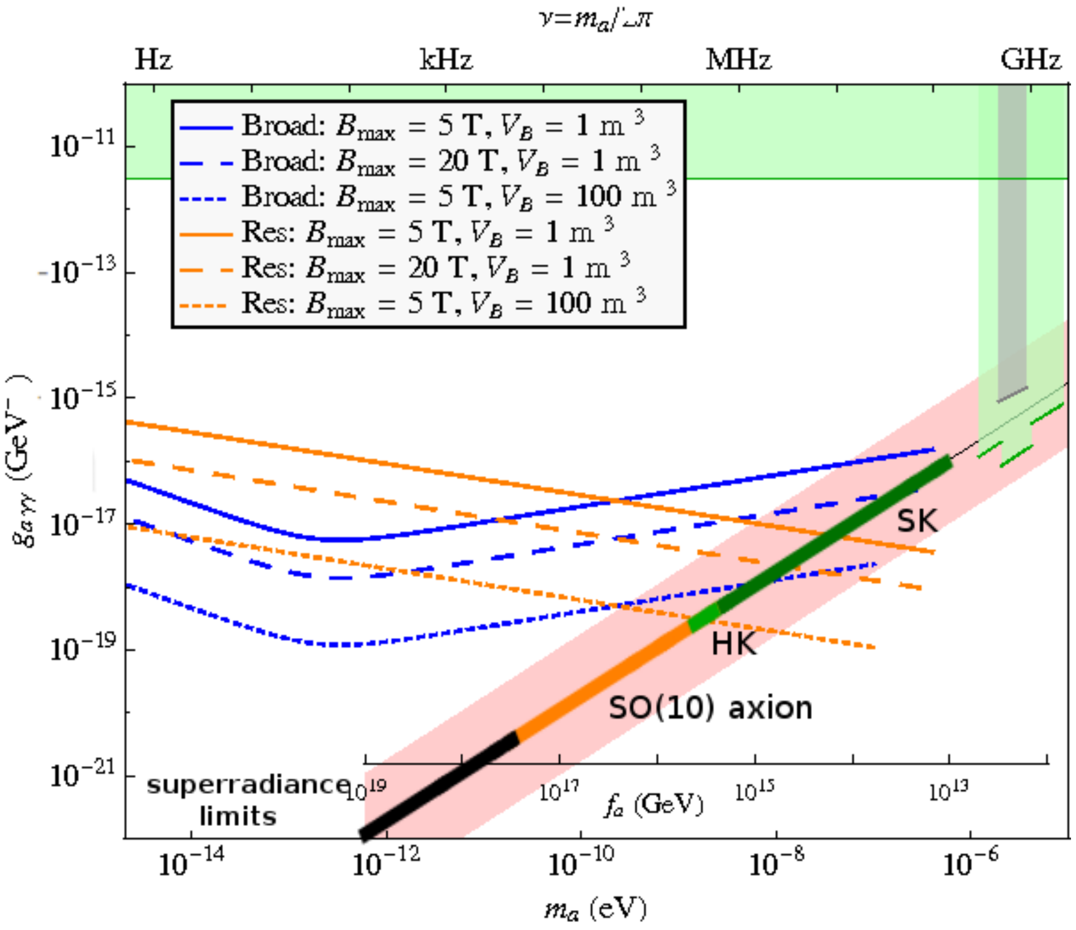
where we defined

$$\tan^2\beta \equiv \frac{v_1^2 + v_3^2}{v_2^2 + v_4^2}.$$

- Electromagnetic coupling may be probed by successor of ABRACADABRA: DMRadio-GUT



[Ernst 18; ABRACADABRA prospects from Kahn,Safdi,Thaler 16]

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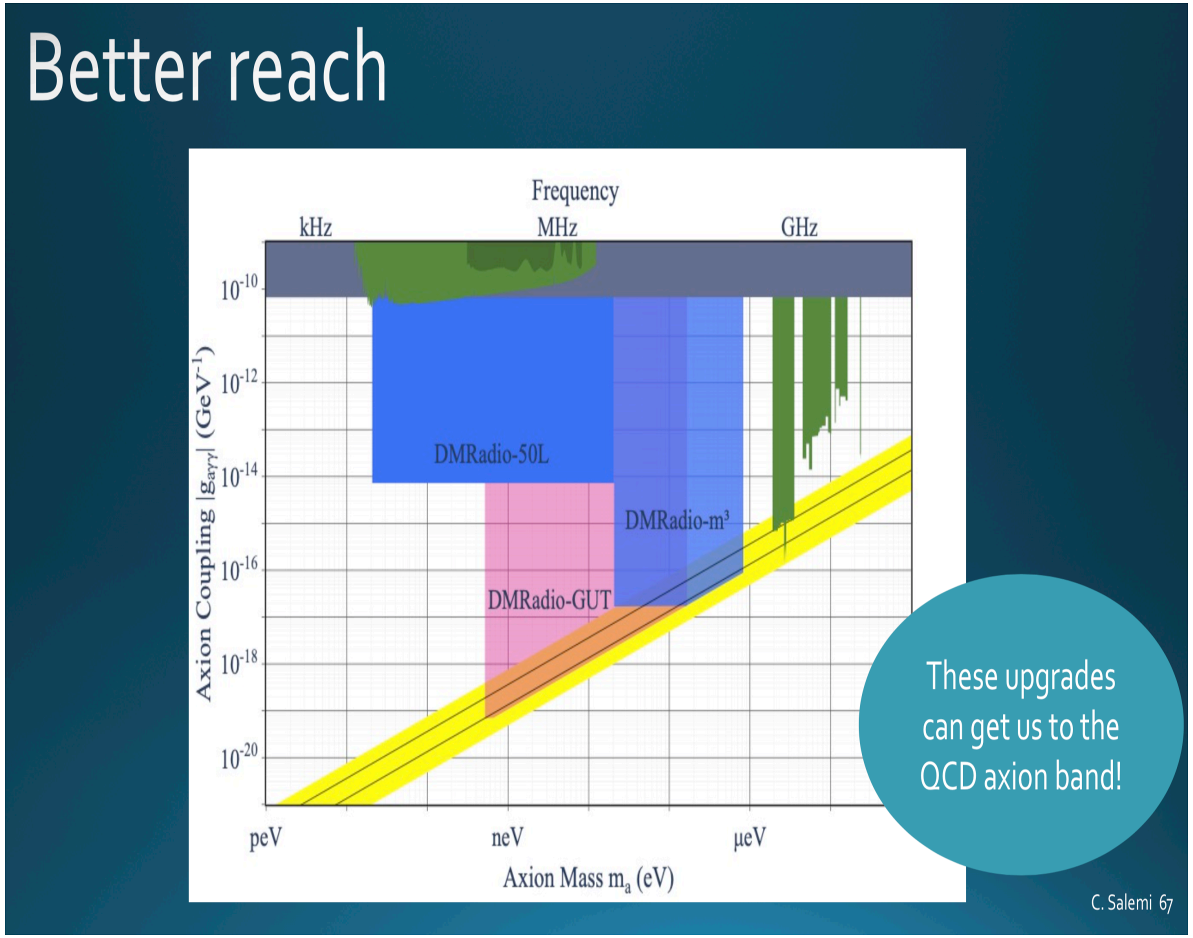

[Salemi '21]

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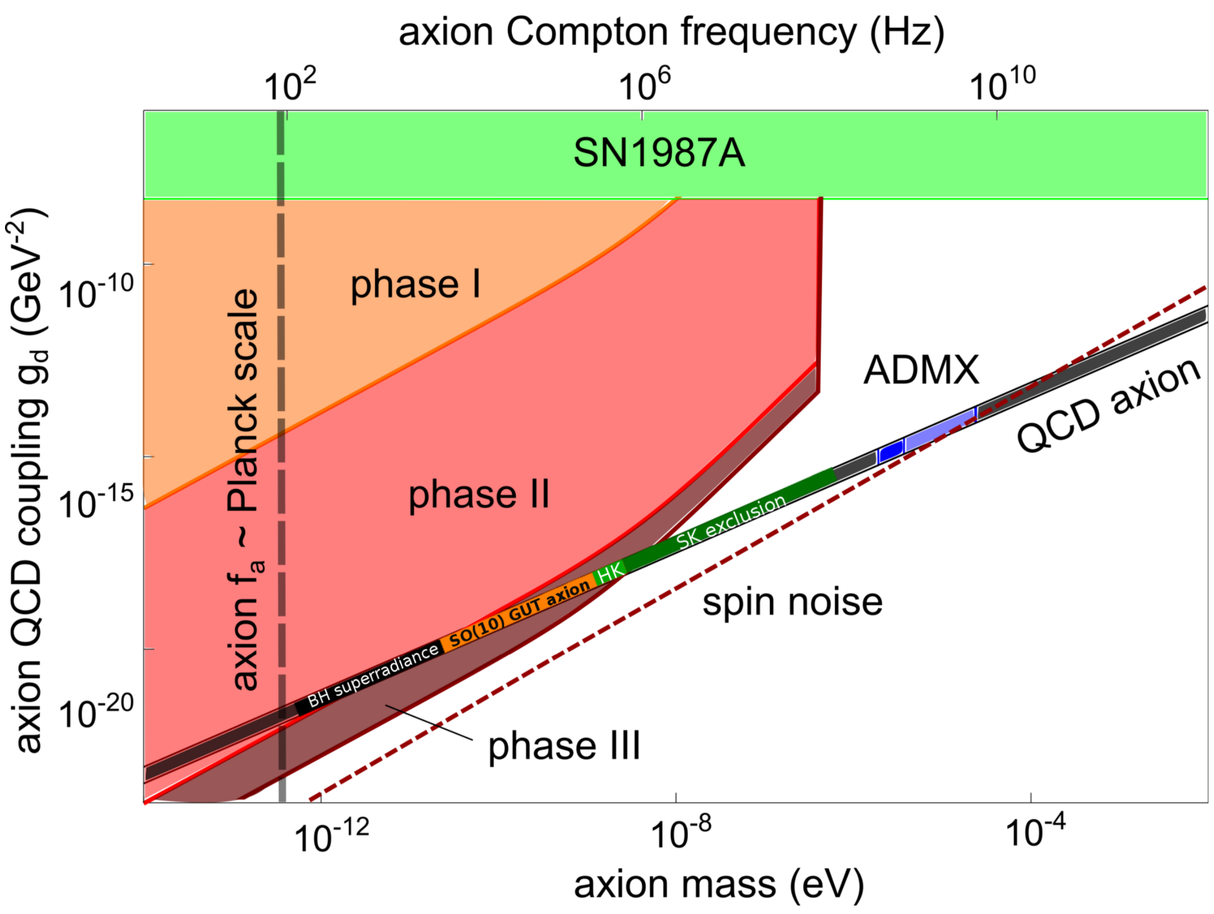
- Coupling to nucleon electric dipole moment may be probed by CAPEr-Electric



[Ernst 18; CASPEr prospects from Kimball et al. 17]

GUT Axion

Axion in non-SUSY SU(5) GUT

- Original non-SUSY SU(5) model comprised of
  [Georgi, Glashow 74]
  - three copies of 10_F and $\bar{5}_F$ representing chiral SM matter fermions
  - 24_H and 5_H , representing Higgs bosons

$$10_F = \underbrace{\left(\bar{3}, 1, -\frac{2}{3}\right)_F}_{u^c} \oplus \underbrace{\left(3, 2, +\frac{1}{6}\right)_F}_{q} \oplus \underbrace{(1, 1, +1)_F}_{e^c},$$

$$\bar{5}_F = \underbrace{\left(\bar{3}, 1, +\frac{1}{3}\right)_F}_{d^c} \oplus \underbrace{\left(1, 2, -\frac{1}{2}\right)_F}_{\ell},$$

$$24_H = \underbrace{(1, 1, 0)_H}_{S_H} \oplus \underbrace{(1, 3, 0)_H}_{T_H} \oplus \underbrace{(8, 1, 0)_H}_{O_H} \oplus \underbrace{\left(3, 2, -\frac{5}{6}\right)_H}_{X_H} \oplus \underbrace{\left(\bar{3}, 2, +\frac{5}{6}\right)_H}_{\bar{X}_H},$$

$$5_H = \underbrace{\left(3, 1, -\frac{1}{3}\right)_H}_{\mathcal{T}} \oplus \underbrace{\left(1, 2, +\frac{1}{2}\right)_H}_{h},$$

GUT Axion

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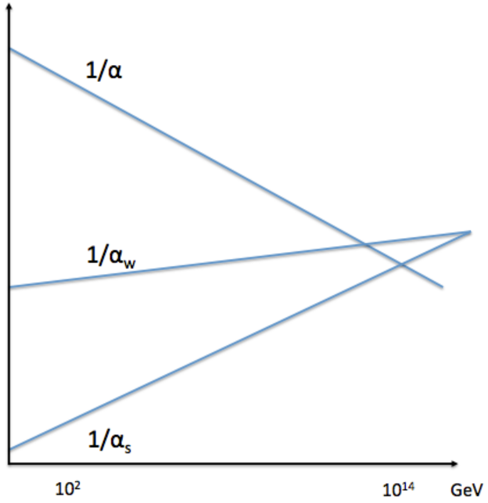

[StackExchange]

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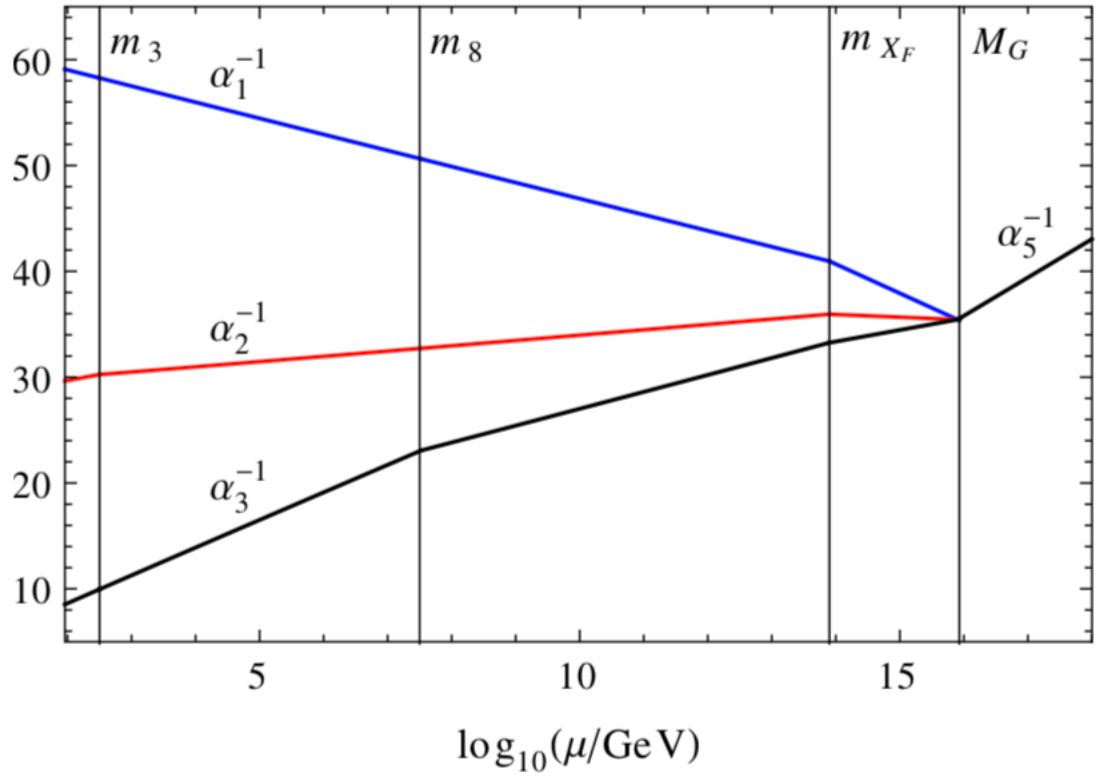

[Di Luzio, Mihaila 13]

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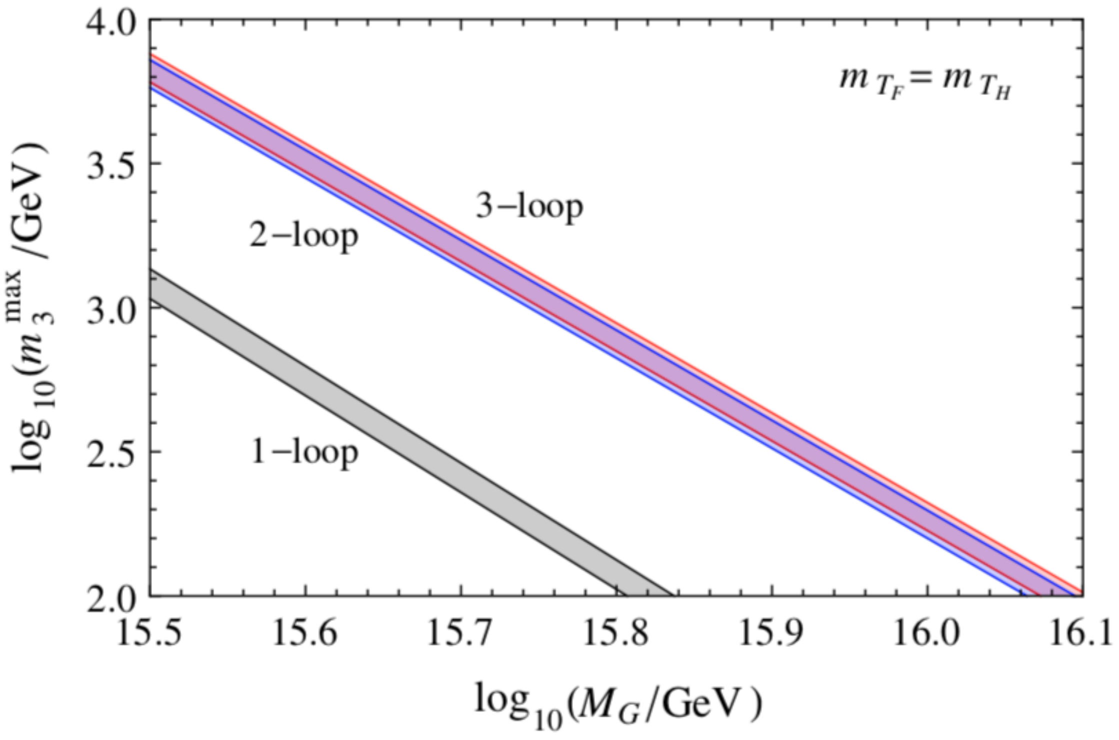
    - Clean correlation between effective electroweak triplet mass m_3 and unification scale M_G



[Di Luzio, Mihaila 13]

$$m_3 = \left(m_{T_F}^4 m_{T_H}\right)^{1/5}$$

GUT Axion

SU(5) GUT model

- Require that 24_H complex and add $5'_H$
- Impose PQ symmetry:

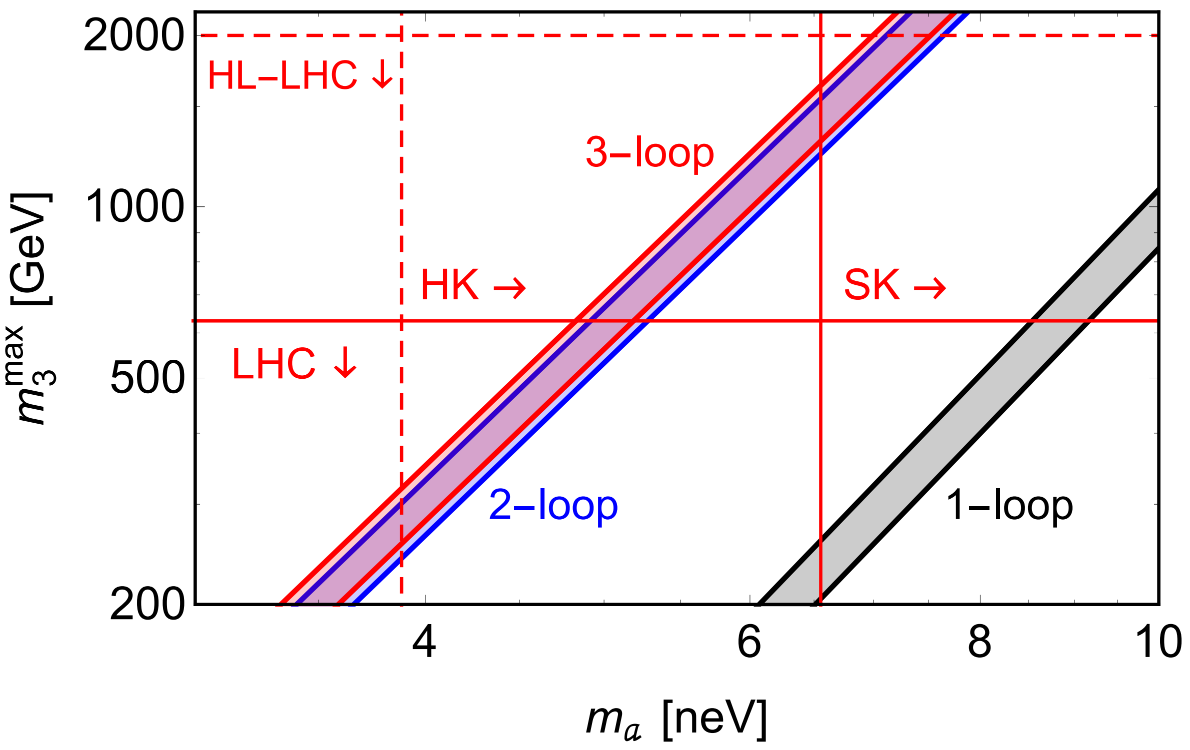
$$\bar{5}_F \to e^{-i\alpha/2}\bar{5}_F,$$
$$10_F \to e^{-i\alpha/2}10_F,$$
$$5_H \to e^{i\alpha}5_H,$$
$$5'_H \to e^{-i\alpha}5'_H,$$
$$24_H \to e^{-i\alpha}24_H,$$
$$24_F \to e^{-i\alpha/2}24_F$$

- Axion decay constant:

$$f_a \simeq \frac{1}{11}\sqrt{\frac{6}{5}}\frac{M_G}{g_5}$$

- Gauge coupling unification, taking into account LHC and Superkamiokande constraints:

$$m_a \in [4.8, 6.6]\ \mathrm{neV}$$



[Di Luzio, AR, Tamarit, arXiv:1807.09769]

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- Electromagnetic coupling coefficients as in DFSZ

Sensitivity of future axion DM searches. An axion in this mass range is extremely weakly coupled to SM particles, since its couplings to e.g. photons (γ), electrons (e), protons (p), and neutrons (n) are inversely proportional to the axion decay constant,

$$\mathcal{L}_a \supset \frac{\alpha}{8\pi}\frac{C_{a\gamma}}{f_a}\, a\, F_{\mu\nu}\tilde{F}^{\mu\nu} - \frac{1}{2}\frac{C_{af}}{f_a}\,\partial_\mu a\, \overline{\Psi}_f\gamma^\mu\gamma_5\Psi_f\,. \tag{12}$$

while the coefficients C_{ax} are of order unity. In the WGG+24_F model, we find:

$$\begin{aligned} C_{a\gamma} &= \tfrac{8}{3} - 1.92(4)\,, \qquad C_{ae} = \tfrac{2}{11}\sin^2\beta\,, \\ C_{ap} &= -0.47(3) \\ &\quad + \tfrac{6}{11}[0.288\cos^2\beta - 0.146\sin^2\beta \pm 0.02]\,, \\ C_{an} &= -0.02(3) \\ &\quad + \tfrac{6}{11}[0.278\sin^2\beta - 0.135\cos^2\beta \pm 0.02]\,, \end{aligned} \tag{13}$$

where we introduced the ratio of the electroweak VEVs, $\tan\beta = \langle 5_H\rangle / \langle 5_{H'}\rangle$. This makes the GUT axion clearly invisible for purely laboratory based experiments.

[Di Luzio, AR, Tamarit, arXiv:1807.09769]

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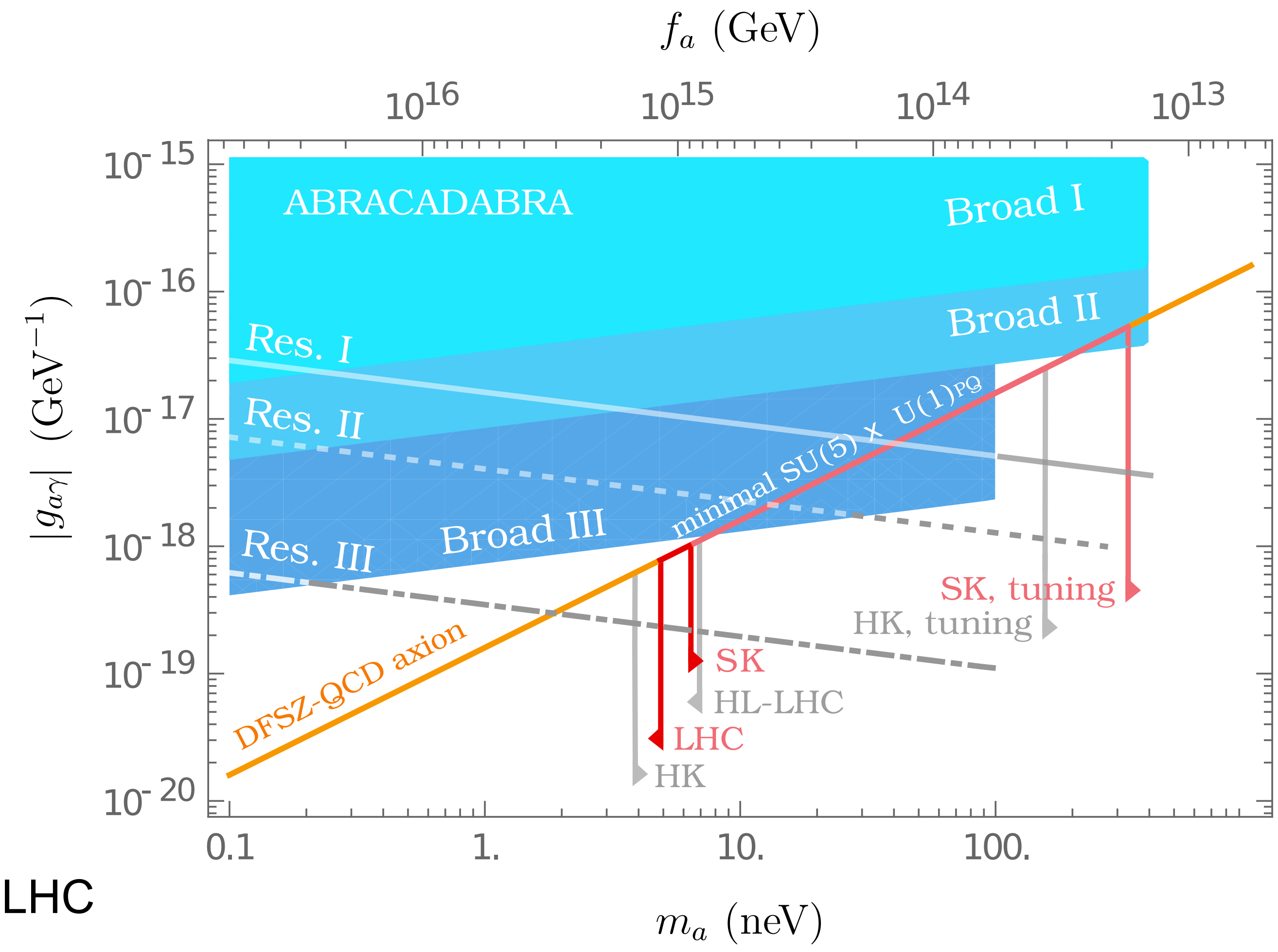
- Window can be explored by DMRadio-GUT



[Di Luzio, AR, Tamarit, arXiv:1807.09769]

The End

Questions?

Variant KSVZ Axion Model

Z_N axion in mirror world extension of SM

- We consider now $\mathcal{N}$ copies of the SM that are interchanged under a $Z_\mathcal{N}$ symmetry which is non-linearly realized by the axion field: [Hook, arXiv:1802.10093]

$$Z_\mathcal{N}: \quad \mathrm{SM}_k \longrightarrow \mathrm{SM}_{k+1\,(\mathrm{mod}\,\mathcal{N})}\,, \qquad a \longrightarrow a + \frac{2\pi k}{\mathcal{N}} f_a$$

- The most general Lagrangian implementing this symmetry describes $\mathcal{N}$ mirror worlds whose couplings take exactly the same values as in the SM, with the exception of the effective θ-parameter: for each copy the effective θ value is shifted by $2\pi/\mathcal{N}$ with respect to that in the neighbour k sector,

$$\mathcal{L} = \sum_{k=0}^{\mathcal{N}-1} \left[\mathcal{L}_{\mathrm{SM}_k} + \frac{\alpha_s}{8\pi} \left(\frac{a}{f_a} + \frac{2\pi k}{\mathcal{N}} \right) G_k \widetilde{G}_k \right] + \dots$$

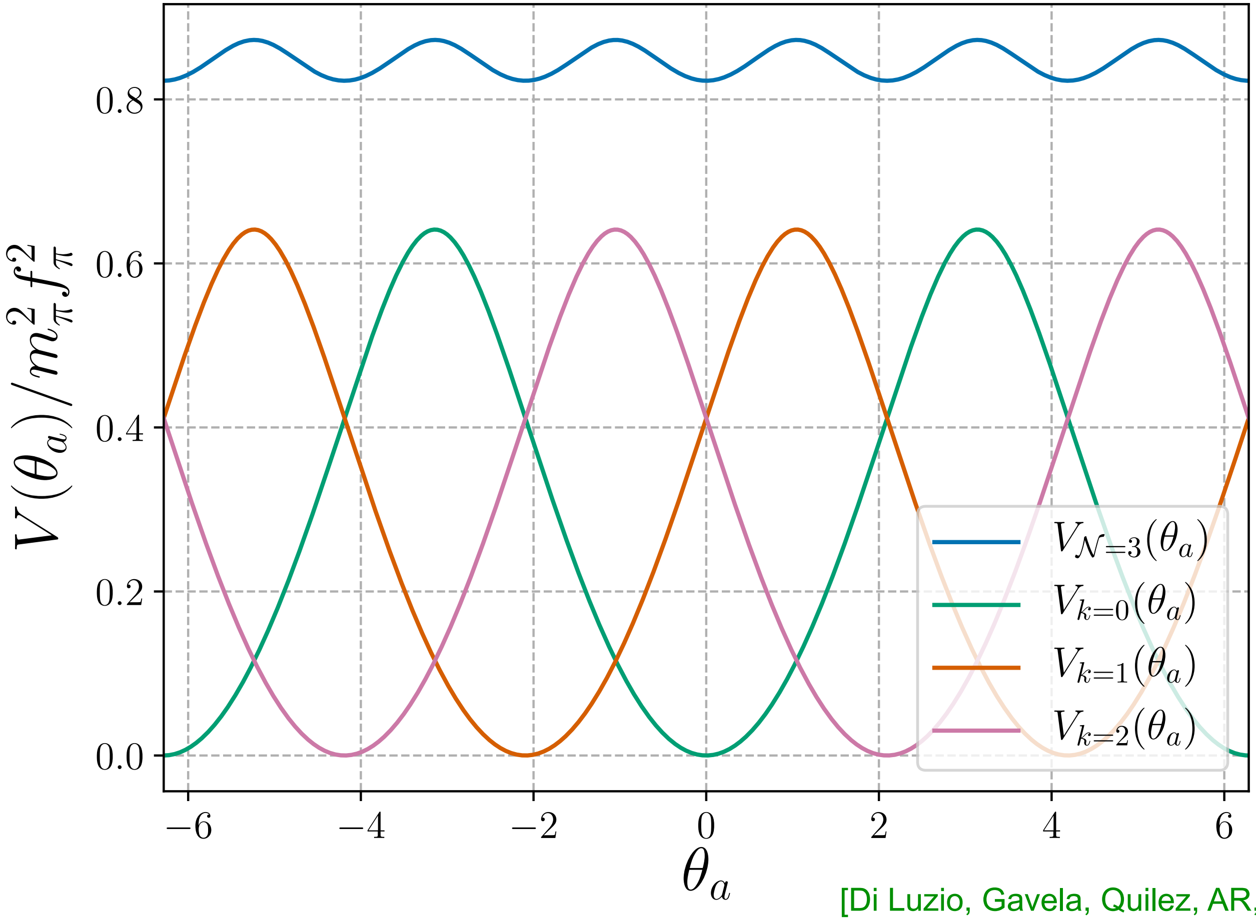
- Each QCD_k sector contributes to the axion potential, which in leading order chiral expansion reads

$$V_\mathcal{N}(a) = -\frac{m_\pi^2 f_\pi^2}{1+z} \sum_{k=0}^{\mathcal{N}-1} \sqrt{1 + z^2 + 2z \cos\left(\frac{a}{f_a} + \frac{2\pi k}{\mathcal{N}} \right)}$$

Variant KSVZ Axion Model

Z_N axion in mirror world extension of SM

- For $\mathcal{N}$ odd, strong CP problem solved: potential has $\mathcal{N}$ minima located at $a = \{\pm 2\pi\ell/\mathcal{N}\} f_a$, for $\ell = 0, 1, \ldots, (\mathcal{N}-1)/2$, including the origin, $a = 0$



[Di Luzio, Gavela, Quilez, AR, arXiv:2102.00012]

Variant KSVZ Axion Model

Z_N axion in mirror world extension of SM

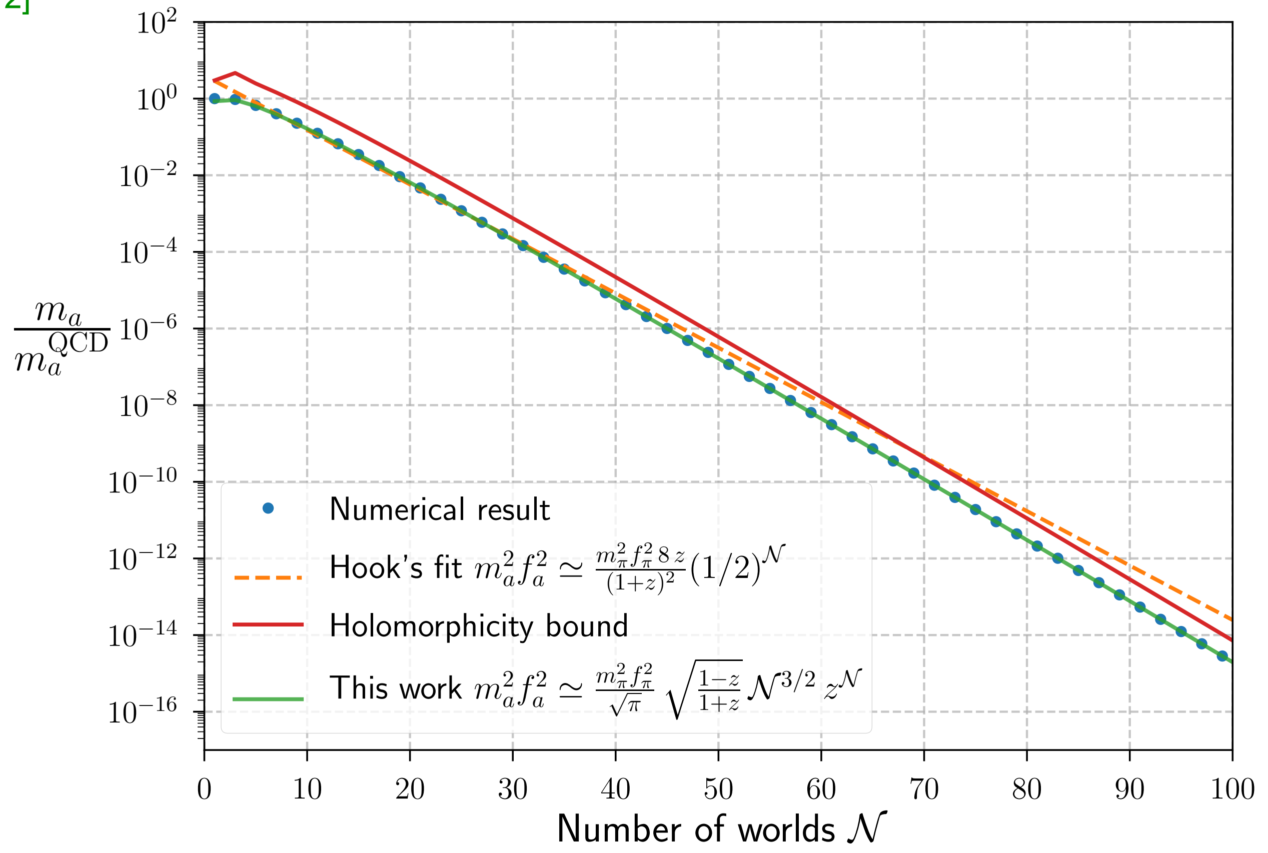
- In the large $\mathcal{N}$ limit: [Di Luzio, Gavela, Quilez, AR, arXiv:2102.00012]

$$V_{\mathcal{N}}(a) \simeq -\frac{m_\pi^2 f_\pi^2}{\sqrt{\pi}} \sqrt{\frac{1-z}{1+z}}\, \mathcal{N}^{-1/2}\, z^{\mathcal{N}} \cos\left(\mathcal{N}\frac{a}{f_a}\right)$$

  - In particular:

$$m_a^2 f_a^2 \simeq \frac{m_\pi^2 f_\pi^2}{\sqrt{\pi}} \sqrt{\frac{1-z}{1+z}}\, \mathcal{N}^{3/2}\, z^{\mathcal{N}}$$

  - Mass exponentially smaller by factor $z^{\mathcal{N}/2} \sim 2^{-\mathcal{N}/2}$ as compared to the canonical axion mass

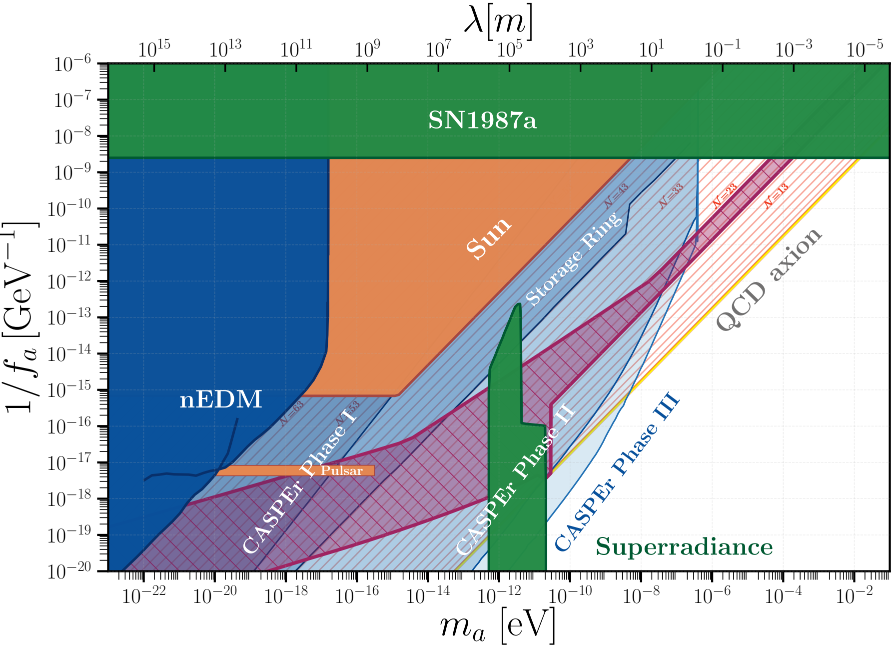


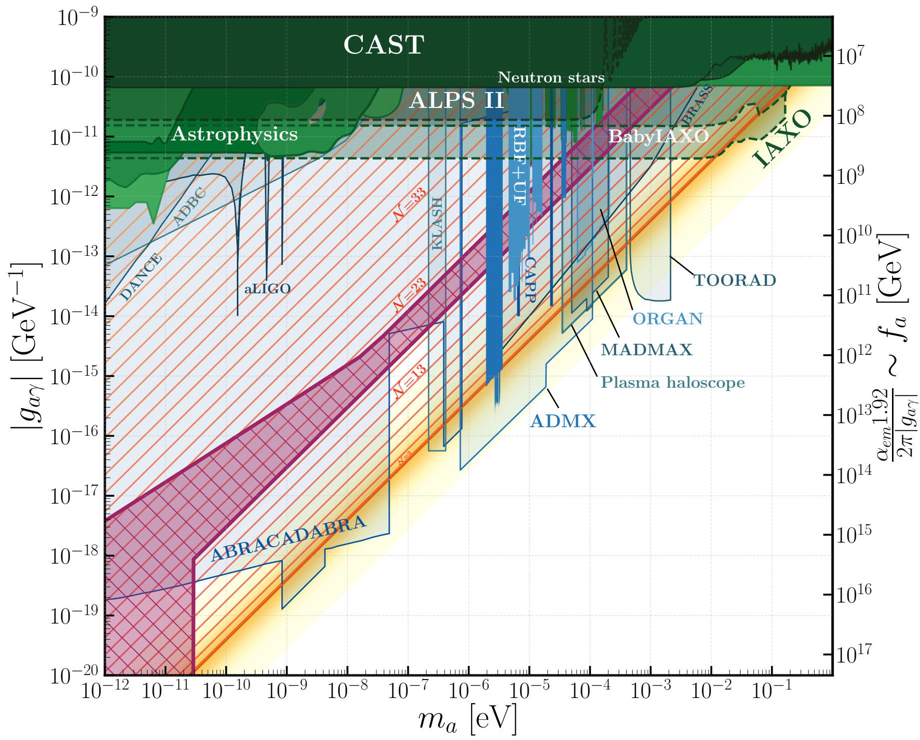
[Di Luzio, Gavela, Quilez, AR, arXiv:2102.00012]

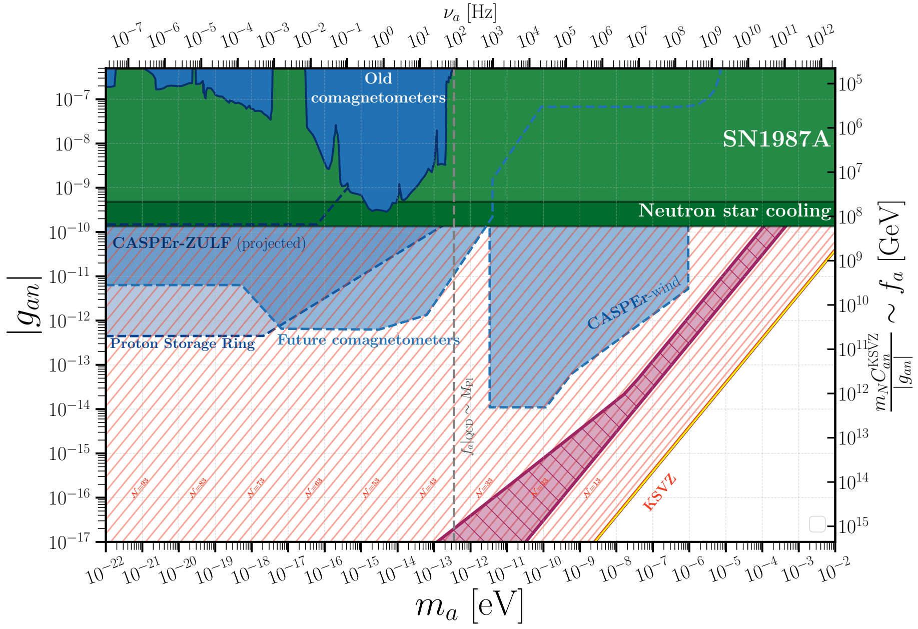
Variant KSVZ Axion Model

Z_N axion in mirror world extension of SM

- Universal increase of axion couplings to SM by factor $z^{-\mathcal{N}/2} \sim 2^{\mathcal{N}/2}$:







[Di Luzio, Gavela, Quilez, AR, arXiv:2102.01082]

Variant KSVZ Axion Model

What if the exotic quark carries also a magnetic charge?

- Have to extend Quantum Electrodynamics (QED) to Quantum Electromagnetodynamics (QEMD):

$$\mathcal{L} = \sum_k \bar{\psi}_k \left(i\partial\!\!\!/ - m_k - e_k A\!\!\!/^{(\mathrm{E})} - g_k A\!\!\!/^{(\mathrm{M})} \right) \psi_k$$

$$+ \frac{1}{8}\mathrm{tr}\left[(\partial \wedge A^{(\mathrm{E})}) \cdot (\partial \wedge A^{(\mathrm{E})}) \right] + \frac{1}{8}\mathrm{tr}\left[(\partial \wedge A^{(\mathrm{M})}) \cdot (\partial \wedge A^{(\mathrm{M})}) \right]$$

$$- \frac{1}{4n^2}\left\{ n \cdot \left[(\partial \wedge A^{(\mathrm{E})}) + (\partial \wedge A^{(\mathrm{M})})^d \right] \right\}^2 - \frac{1}{4n^2}\left\{ n \cdot \left[(\partial \wedge A^{(\mathrm{M})}) - (\partial \wedge A^{(\mathrm{E})})^d \right] \right\}^2$$

[Zwanziger `71]

- Notation: $a \cdot b = a_\mu b^\mu, (a \wedge b)^{\mu\nu} = a^\mu b^\nu - a^\nu b^\mu, \ (a \cdot G)^\nu = a_\mu G^{\mu\nu}$
- $U(1)_\mathrm{E} \times U(1)_\mathrm{M}$ gauge theory involving arbitrary fixed four vector n^μ - field theoretic counterpart of a frozen Dirac string
  - Variations of action with respect to matter and gauge fields gives classical equations:

$$\left(i\partial\!\!\!/ - m_k - e_k A\!\!\!/^{(\mathrm{E})} - g_k A\!\!\!/^{(\mathrm{M})} \right) \psi_k = 0$$

$$\partial_\mu F^{\mu\nu} = \sum_i e_i \bar{\psi}_i \gamma^\nu \psi_i \,,$$

$$\partial_\mu \tilde{F}^{\mu\nu} = \sum_i g_i \bar{\psi}_i \gamma^\nu \psi_i \,,$$

where

$$F = \frac{1}{n^2}\left\{ n \wedge [n \cdot (\partial \wedge A^{(\mathrm{E})})] - (n \wedge [n \cdot (\partial \wedge A^{(\mathrm{M})})])^d \right\}$$

$$\tilde{F} = \frac{1}{n^2}\left\{ (n \wedge [n \cdot (\partial \wedge A^{(\mathrm{E})})])^d + n \wedge [n \cdot (\partial \wedge A^{(\mathrm{M})})] \right\}$$

$$\partial^2 (n \cdot A^{(\mathrm{E})})^2 = \partial^2 (n \cdot A^{(\mathrm{M})})^2 = 0$$

Variant KSVZ Axion Model

What if the exotic quark carries also a magnetic charge?

- Have to extend Quantum Electrodynamics (QED) to Quantum Electromagnetodynamics (QEMD):

$$\mathcal{L} = \sum_k \overline{\psi}_k \left(i\partial\!\!\!/ - m_k - e_k A\!\!\!/^{(\mathrm{E})} - g_k A\!\!\!/^{(\mathrm{M})} \right) \psi_k$$

[Zwanziger `71]

$$+ \frac{1}{8}\mathrm{tr}\left[(\partial \wedge A^{(\mathrm{E})}) \cdot (\partial \wedge A^{(\mathrm{E})}) \right] + \frac{1}{8}\mathrm{tr}\left[(\partial \wedge A^{(\mathrm{M})}) \cdot (\partial \wedge A^{(\mathrm{M})}) \right]$$

$$- \frac{1}{4n^2}\left\{ n \cdot \left[(\partial \wedge A^{(\mathrm{E})}) + (\partial \wedge A^{(\mathrm{M})})^d \right] \right\}^2 - \frac{1}{4n^2}\left\{ n \cdot \left[(\partial \wedge A^{(\mathrm{M})}) - (\partial \wedge A^{(\mathrm{E})})^d \right] \right\}^2$$

- Notation: $a\cdot b = a_\mu b^\mu, (a \wedge b)^{\mu\nu} = a^\mu b^\nu - a^\nu b^\mu, \; (a\cdot G)^\nu = a_\mu G^{\mu\nu}$
- $U(1)_{\mathrm{E}} \times U(1)_{\mathrm{M}}$ gauge theory involving arbitrary fixed four vector n^μ - field theoretic counterpart of a frozen Dirac string
  - The two gauge fields are not independent:

$$\left[A_\mu^{(\mathrm{E})}(t,\vec{x}), A_\nu^{(\mathrm{M})}(t,\vec{y}) \right] = i\epsilon_{\mu\nu\rho 0}\, n^\rho \,(n\cdot\partial)^{-1}(\vec{x}-\vec{y}) \;,$$

$$\left[A_\mu^{(\mathrm{E})}(t,\vec{x}), A_\nu^{(\mathrm{E})}(t,\vec{y}) \right] = \left[A_\mu^{(\mathrm{M})}(t,\vec{x})\,, A_\nu^{(\mathrm{M})}(t,\vec{y}) \right] = -i\,(g_{0\mu}n_\nu + g_{0\nu}n_\mu)\,(n\cdot\partial)^{-1}(\vec{x}-\vec{y})$$

  - Only four independent phase space variables, corresponding to the two physical degrees of the photon
  - Has been shown by path integral techniques that time-ordered Green's functions of gauge-invariant local operators are independent of n^μ if the **Dirac-Schwinger-Zwanziger charge quantization condition** holds,

$$e_i g_j - e_j g_i = 4\pi n_{ij}, \quad n_{ij} \in \mathbb{Z}$$

[Brandt, Neri, Zwanziger `78]

Variant KSVZ Axion Model

What if the exotic quark carries also a magnetic charge?

- Integrate out $\rho(x)$:

$$\mathcal{L}'_{\rm KSVZ} = \frac{1}{2}\partial_\mu a \partial^\mu a + \overline{\mathcal{Q}}\, i\gamma_\mu D^\mu \mathcal{Q} - \left(m_{\mathcal{Q}} \overline{\mathcal{Q}}_L \mathcal{Q}_R e^{ia/v_\sigma} + \text{h.c.}\right)$$

- Last term can be brought to the form of a standard mass term by performing the field-dependent axial transformation, $\mathcal{Q} \to e^{-\frac{i}{2}\gamma_5 \frac{a}{v_\sigma}} \mathcal{Q}$, that is

$$\mathcal{Q}_L \to e^{\frac{i}{2}\frac{a}{v_\sigma}} \mathcal{Q}_L , \qquad \mathcal{Q}_R \to e^{-\frac{i}{2}\frac{a}{v_\sigma}} \mathcal{Q}_R$$

- However, fermionic measure in path integral is not invariant under axial transformations, cf.

$$\mathcal{D}\mathcal{Q}\mathcal{D}\bar{\mathcal{Q}} \to= \mathcal{D}\mathcal{Q}\mathcal{D}\bar{\mathcal{Q}}\; e^{i\int d^4x \mathcal{L}_F(x)} \quad \text{where} \quad \mathcal{L}_{\rm F} = \frac{\alpha_s}{8\pi}\frac{a}{2N f_a} G\tilde{G} + \mathcal{L}_{\rm F}^{\rm QEMD}$$

[Anton Sokolov, AR, arXiv:2205.02605]

$$\mathcal{L}_{\rm F}^{\rm QEMD} = \frac{a}{f_a}\left(\frac{\alpha}{8\pi}\frac{E}{N}\text{tr}\left\{ \left(\partial\wedge A^{(\rm E)}\right)\left(\partial\wedge A^{(\rm E)}\right)^d \right\} + \frac{\alpha_M}{8\pi}\frac{M}{N}\,\text{tr}\left\{ \left(\partial\wedge A^{(\rm M)}\right)\left(\partial\wedge A^{(\rm M)}\right)^d \right\} + \frac{\sqrt{\alpha\alpha_M}}{4\pi}\frac{D}{N}\,\text{tr}\left\{ \left(\partial\wedge A^{(\rm E)}\right)\left(\partial\wedge A^{(\rm M)}\right)^d \right\}\right)$$

  - If Q in fundamental of $SU(3)_{\rm color}$ then N=1/2 and $E = 3\,q_{\mathcal{Q}}^2\,,\ M = 3\,g_{\mathcal{Q}}^2\,,\ D = 3\,q_{\mathcal{Q}} g_{\mathcal{Q}}$
  - All three terms respect shift symmetry

Variant KSVZ Axion Model

Phenomenological implications of monopole-philic axion

- Axion Maxwell equations in terms of field strengths: [Anton Sokolov, AR, arXiv:2205.02605]

$$\left(\partial^2 - m_a^2\right) a = \left(g_{a\mathrm{EE}} + g_{a\mathrm{MM}}\right) \mathbf{E}_0 \cdot \mathbf{B}_0 + g_{a\mathrm{EM}} \left(\mathbf{E}_0^2 - \mathbf{B}_0^2\right) ,$$

$$\boldsymbol{\nabla} \times \mathbf{B}_a - \dot{\mathbf{E}}_a = g_{a\mathrm{EE}} \left(\mathbf{E}_0 \times \boldsymbol{\nabla} a - \dot{a}\mathbf{B}_0\right) + g_{a\mathrm{EM}} \left(\mathbf{B}_0 \times \boldsymbol{\nabla} a + \dot{a}\mathbf{E}_0\right) ,$$

$$\boldsymbol{\nabla} \times \mathbf{E}_a + \dot{\mathbf{B}}_a = -g_{a\mathrm{MM}} \left(\mathbf{B}_0 \times \boldsymbol{\nabla} a + \dot{a}\mathbf{E}_0\right) - g_{a\mathrm{EM}} \left(\mathbf{E}_0 \times \boldsymbol{\nabla} a - \dot{a}\mathbf{B}_0\right) ,$$

$$\boldsymbol{\nabla} \cdot \mathbf{B}_a = -g_{a\mathrm{MM}} \, \mathbf{E}_0 \cdot \boldsymbol{\nabla} a + g_{a\mathrm{EM}} \, \mathbf{B}_0 \cdot \boldsymbol{\nabla} a ,$$

$$\boldsymbol{\nabla} \cdot \mathbf{E}_a = g_{a\mathrm{EE}} \, \mathbf{B}_0 \cdot \boldsymbol{\nabla} a - g_{a\mathrm{EM}} \, \mathbf{E}_0 \cdot \boldsymbol{\nabla} a$$

$$g_{a\mathrm{MM}} = \frac{\alpha_M}{2\pi f_a} \frac{M}{N} \quad \gg \quad g_{a\mathrm{EM}} = \frac{\sqrt{\alpha \alpha_M}}{2\pi f_a} \frac{D}{N} \quad \gg \quad g_{a\mathrm{EE}} = \frac{\alpha}{2\pi f_a} \left(\frac{E}{N} - 1.92\right)$$