Axion cosmology

GGI Axions across boundaries ...
Training week 26-28 Apr 2023
Javier Redondo

outline

- thermal axion production
- isocurvature fluctuations
- misalignment, pre/post inflationary
- post-inflationary scenario
- cosmic strings and walls
- axion miniclusters
- axion stars

QCD axion and cosmology

Key aspects for cosmology:

- Axion is a (pseudo) Goldstone Boson

- U(1) symmetry:
    - spontaneously broken at high-energy scale f_A >~ 10^8 GeV
    - Axial
    - colour anomalous,

- Axions are low E excitations of some HE theory
  (they are useful dof below f_A)
- "small" mass
- "feeble" interactions ~$1/f_A$

"Grand unified" axion spectrum

Most axions would be produced in the Big bang (DM, DR)

Irastorza 18

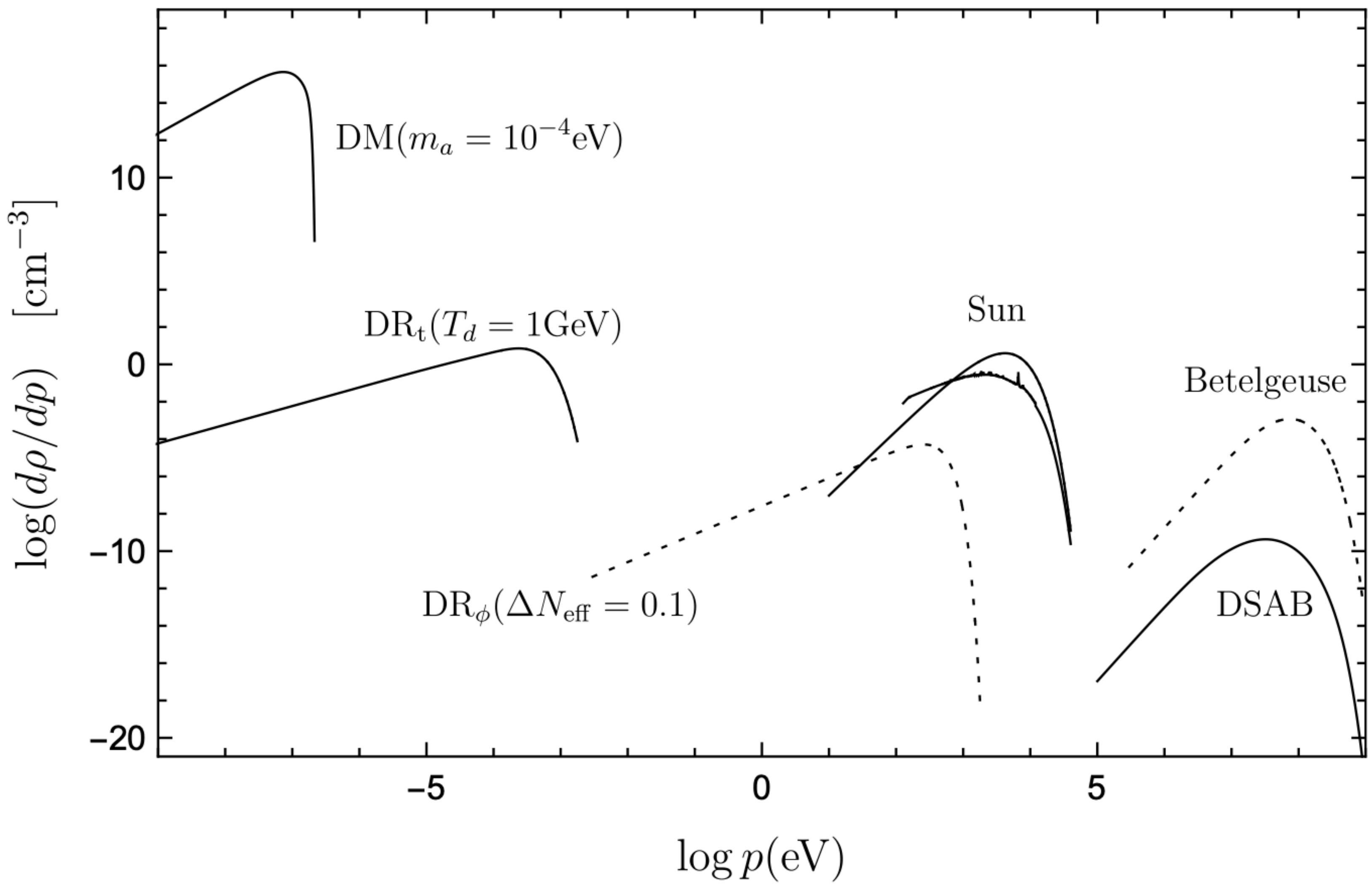
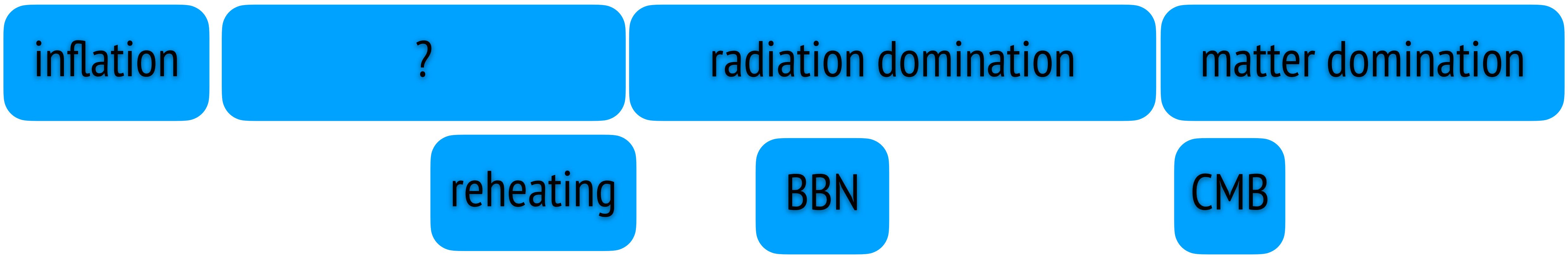


Figure 4: Energy spectrum of natural axions/ALPs as function of momentum at the Earth position. Galactic DM with $m_a = 10^{-4}$ eV, thermal DR (DR_t) and from modulus decay (DR_ϕ), solar Primakoff and ABC axions saturating the astrophysical bounds (from HB and WDs respectively) and maximum diffuse supernova axion background (DSAB) and axion pulse from Betelgeuse (50% of SN energy into axions).

Thermal axions

- Standard cosmological storyline



- After inflation, Universe reheats to a SM plasma (γ,e,μ,τ,g,q,W,H...)

- Thermal SM particles will produce axions

Thermal axions

- Most relevant production from strong-interactions (g, pi)
- Most important aspect is f_A suppression

$$\mathcal{M} \propto \frac{1}{f_A} \to \sigma(? \to ? + a) \propto \frac{1}{f_A^2}$$

Axion Production rate (relativistic, temperature T plasma)

$$\Gamma(? \to ? + a) \sim \langle n_? \sigma(? \to ? + a) v \rangle \sim \frac{T^3}{f_A^2}$$

Thermalisation effective?

$$\Gamma \gg H \sim g_* \frac{T^2}{m_{\rm Pl}} \qquad T \gg \frac{f_A^2}{m_{\rm Pl}}$$

Thermalisation

- Thermalisation is effective for $T \gg \frac{f_A^2}{m_{\rm Pl}}$

$$T \gg 10^5 \mathrm{GeV} \left(\frac{\mathrm{f_A}}{10^{12}\mathrm{GeV}} \right)^2$$

- Thermal Number density of axions, Bose-Einstein distributed in E

$$n_A = \frac{\zeta(3)}{\pi^2} T^3$$

- As Universe expands and cools down, temperature drops below critical, axions decouple $T_d \sim f_A^2 / m_{\rm Pl}$

- Number density today (assumes entropy conservation)

$$n_A(t_0) = n_A(T_d) \left(\frac{R(Td)}{R_0} \right)^3 = \frac{\zeta(3)}{\pi^2} T_d^3 \frac{g_S(T_0) T_0^3}{g_S(T_d) T_d^3} \sim \frac{\zeta(3)}{\pi^2} T_0^3 \frac{g_S(T_0)}{g_S(T_d)}$$

Abundance today

- Number density today (assumes entropy conservation)

$$n_A(t_0) = n_A(T_d)\left(\frac{R(Td)}{R_0}\right)^3 = \frac{\zeta(3)}{\pi^2}T_d^3\frac{g_S(T_0)T_0^3}{g_S(T_d)T_d^3} \sim \frac{\zeta(3)}{\pi^2}T_0^3\frac{g_S(T_0)}{g_S(T_d)}$$

R is FRW scale factor (Universe expansion)
Entropy "conservation" $g_S(T)T^3R^3 = \mathrm{cons.}$

Abundance today

- **Thermal Dark matter**

$$\rho = m_A n_A(t_0) \sim m_A T_0^3 \frac{g_S(T_0)}{g_S(T_d)}$$

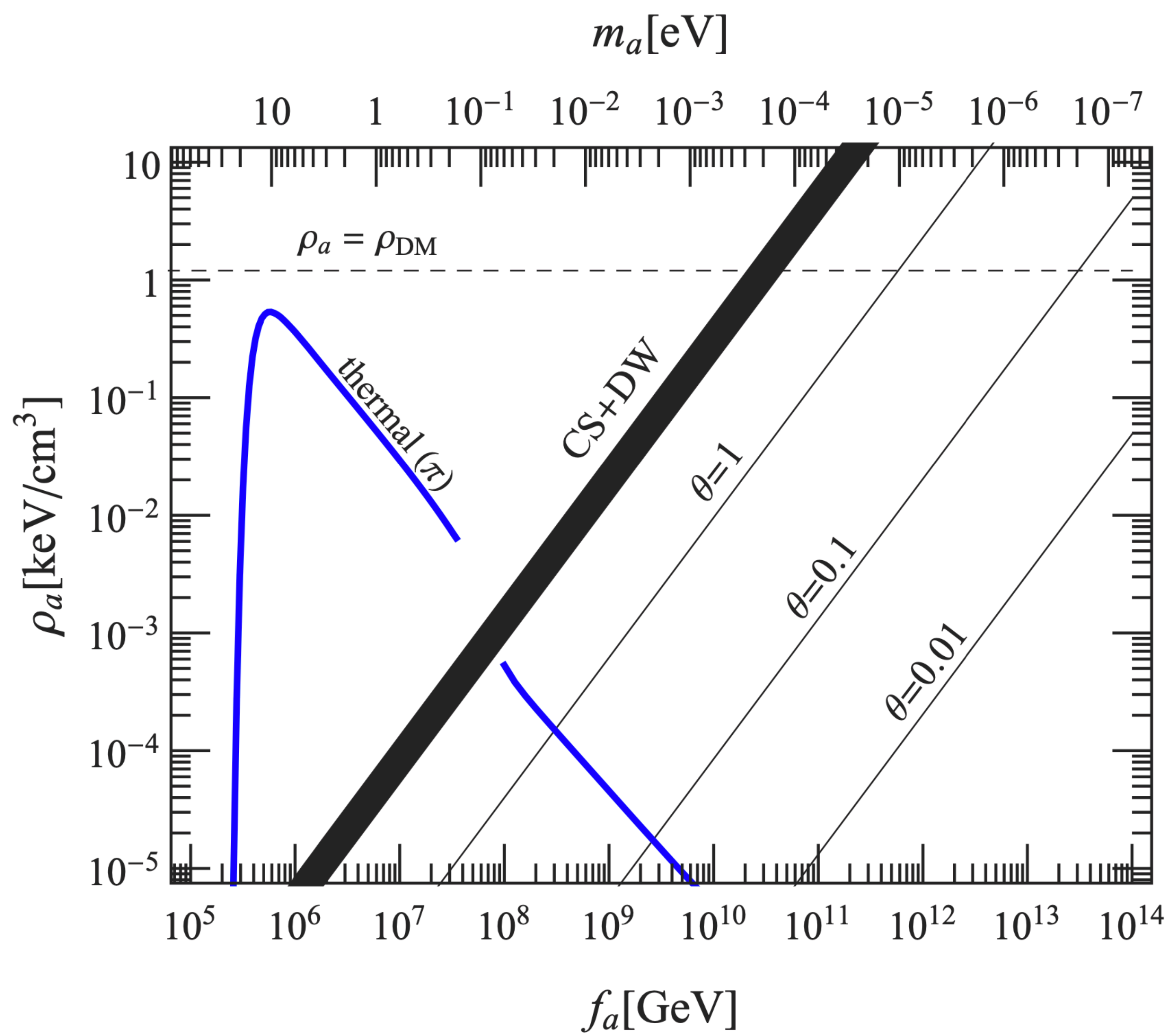
Can only be a subdominat component of DM

$$\rho_r \sim T_0^4 \sim \frac{\rho_m}{z_{\rm eq}} \sim 10^{-4}\rho_m \sim 10^{-4}\rho_c$$

$$\rho_{A,T} \sim \frac{m_A}{T_0}\frac{1}{g_S(T_d)} \times \rho_r \sim \frac{m_A < 0.06\text{eV}}{10^{-4}\text{eV}} 10^{-4}\rho_c < \rho_c$$

Abundance today

- Thermal Dark matter



Archidiacono 2015

Thermal Axion Dark matter is HOT

- Free streaming length

- comoving distance travelled up to CMB times

$$\lambda_{\rm FS} = \int_0^{\rm CMB} \frac{dt}{R(t)} v = \int_{z_{\rm CMB}}^{\infty} \frac{dz}{H} v$$

velocity is $v = \frac{p}{E} = \frac{p}{\sqrt{m^2+p^2}}$

momentum redshifts! $xT_0(1+z)$

axions become non-relativistic $v \sim 1 \rightarrow v \sim xT_0(1+z)/m$ **at** $z_{nr} \sim m/xT_0$

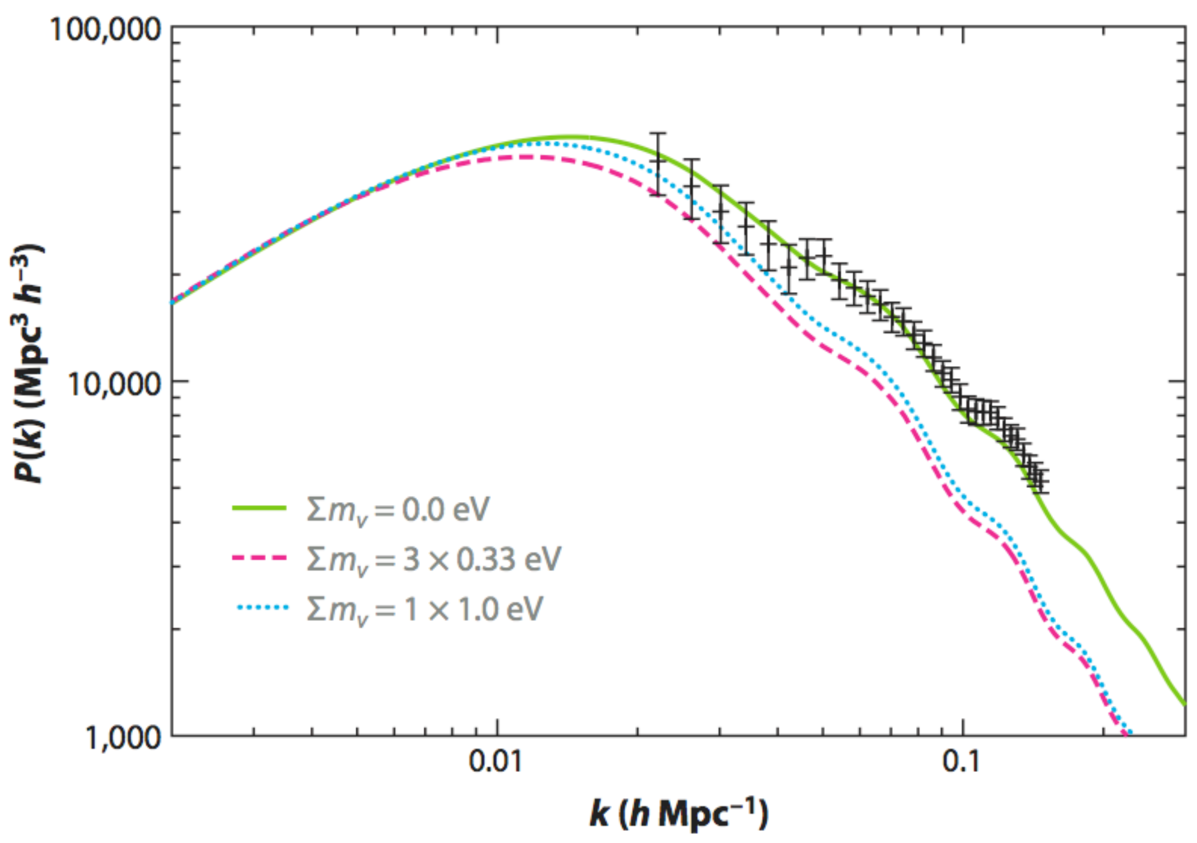
$$\lambda_{\rm FS} \sim \frac{1}{H_0\sqrt{\Omega_\gamma^0}} \left[\int \frac{dz}{(1+z)^2} v\right] \sim \frac{1}{H_0\sqrt{\Omega_\gamma^0}} \left(\frac{1}{1+z_{nr}} + \frac{xT_0}{m} \log(z_{nr}/z_{cmb})\right) \sim \frac{1}{H_0\sqrt{\Omega_\gamma^0}} \frac{xT_0}{m}$$

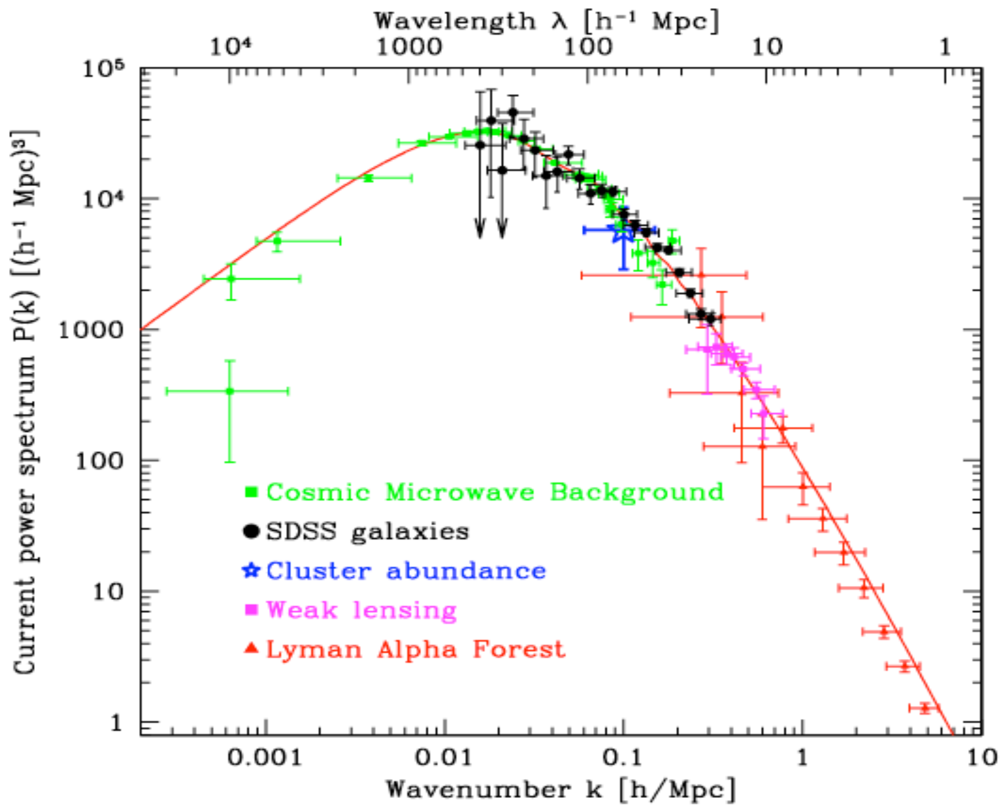
$$\lambda_{\rm FS} \sim 100\,{\rm Mpc} \left(\frac{1\,{\rm eV}}{m_\nu}\right) \times x$$

that is huge!!! axions cannot be ALL the dark matter...
<u>would free stream and erase density fluctuations</u> below ~100 Mpc/m!
effects are very similar to massive thermal neutrinos

Hot Dark matter is strongly constrained

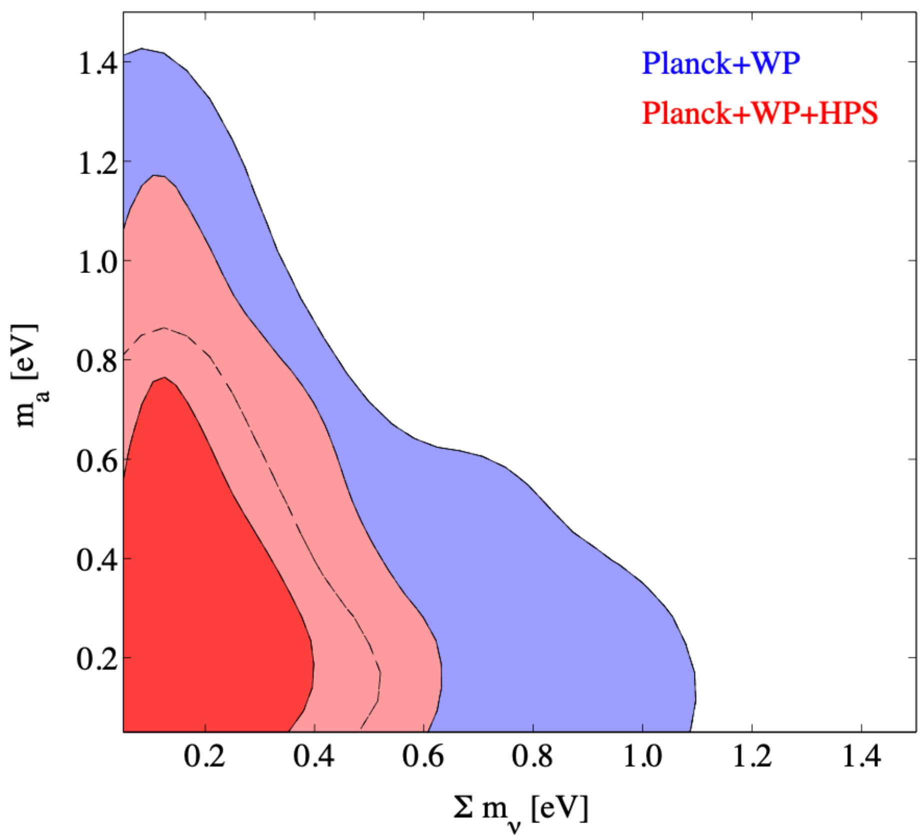
- Effects are vey similar to massive thermal neutrinos
  Neutrinos stream away from small-scale density fluctuations
  Matter power spectrum at small scales suppression (not observed)





Hot Dark matter is strongly constrained

- Planck data

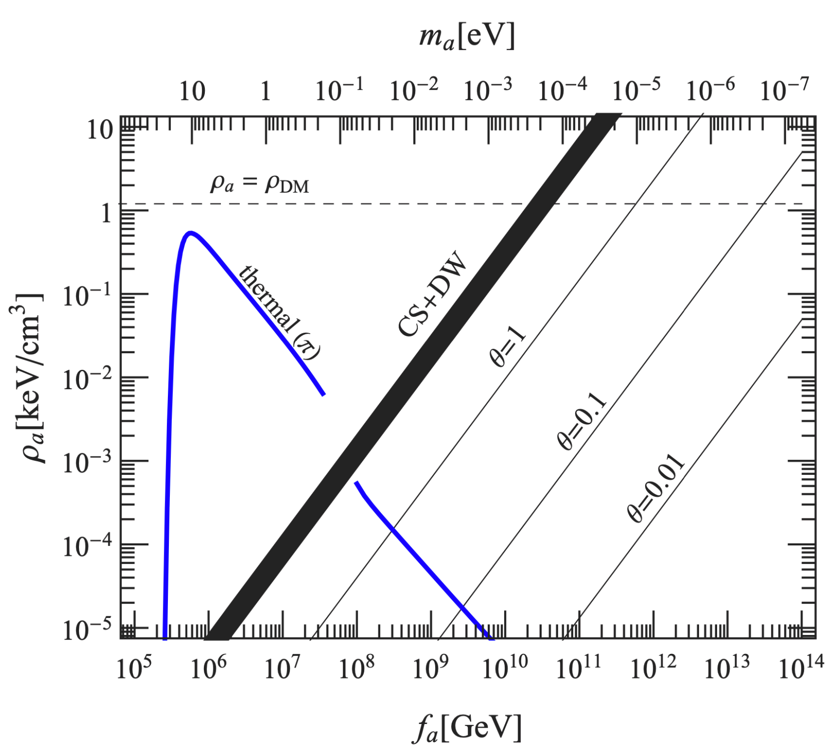


Wong 2013

- Future surveys

  - Euclid + Planck could pinpoint m_A>0.15 eV
  - Below, the axion densitty is too low

Archidiacono 2015



Dark radiation

- **Number density today (assumes entropy conservation)**

$$n_A(t_0) = n_A(T_d)\left(\frac{R(Td)}{R_0}\right)^3 = \frac{\zeta(3)}{\pi^2}T_d^3\frac{g_S(T_0)T_0^3}{g_S(T_d)T_d^3} \sim \frac{\zeta(3)}{\pi^2}T_0^3\frac{g_S(T_0)}{g_S(T_d)}$$

R is FRW scale factor (Universe expansion)

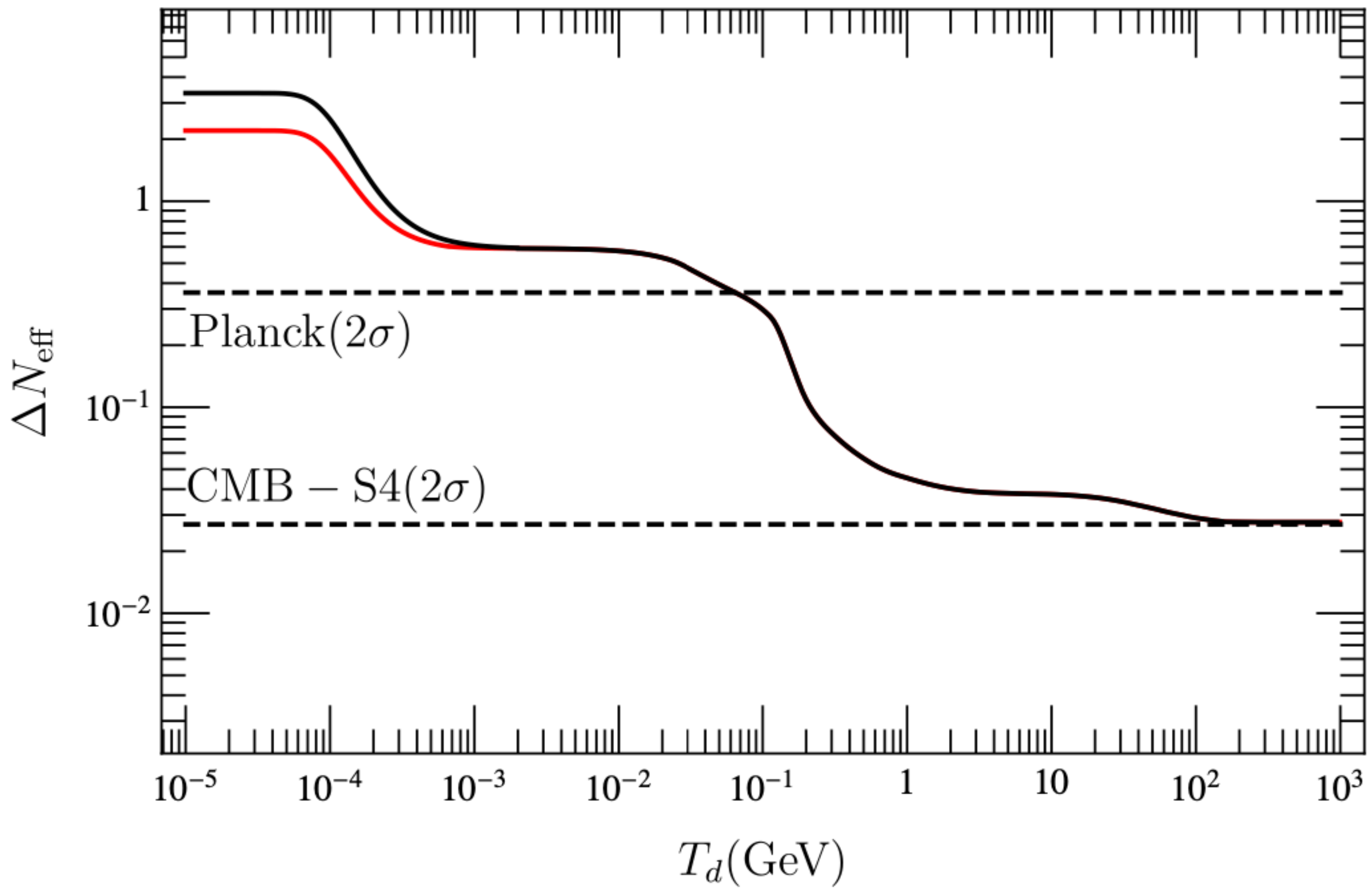
Entropy "conservation" $g_S(T)T^3R^3 = \text{cons.}$

- **For much smaller masses, axions behave as dark radiation**

Dark radiation

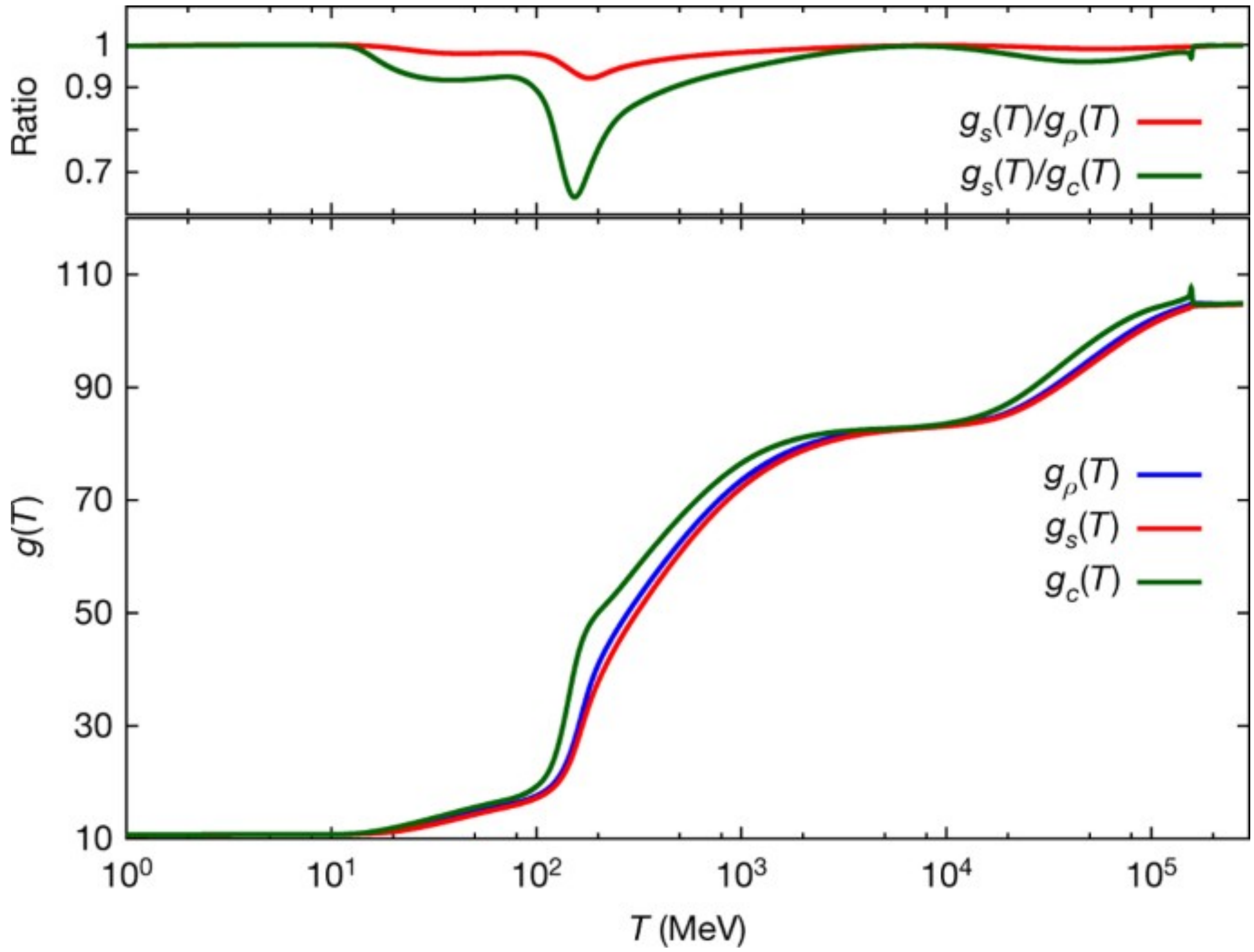
- **Energy density today (Effective number of neutrinos)** *(assumes negligible mass today)*

from IAXO Physics potential 2019



- **Generation 4 CMB satellite has the potential to be sensitive to ΔN_{eff} ~ 0.03**
- **Potential for axion discovery is (potentially) huge**

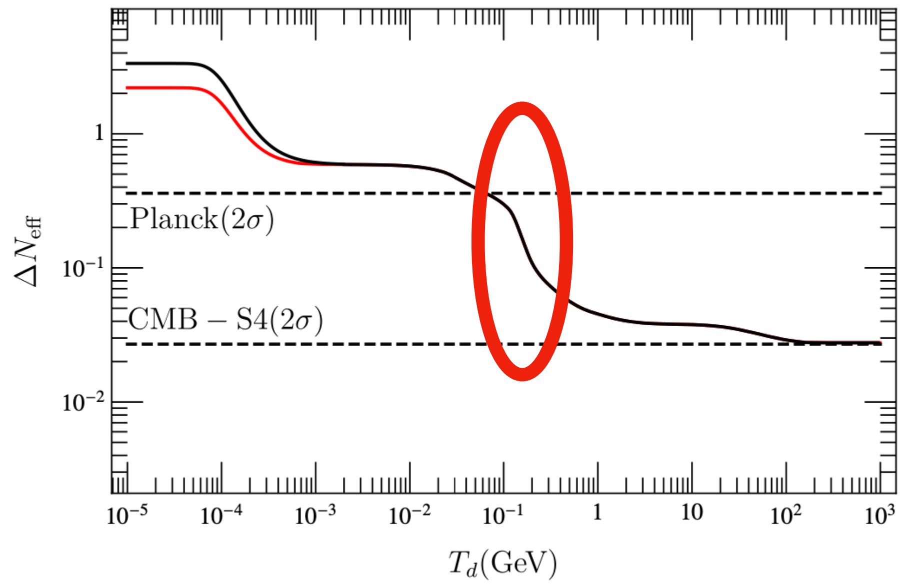
(degrees of freedom)

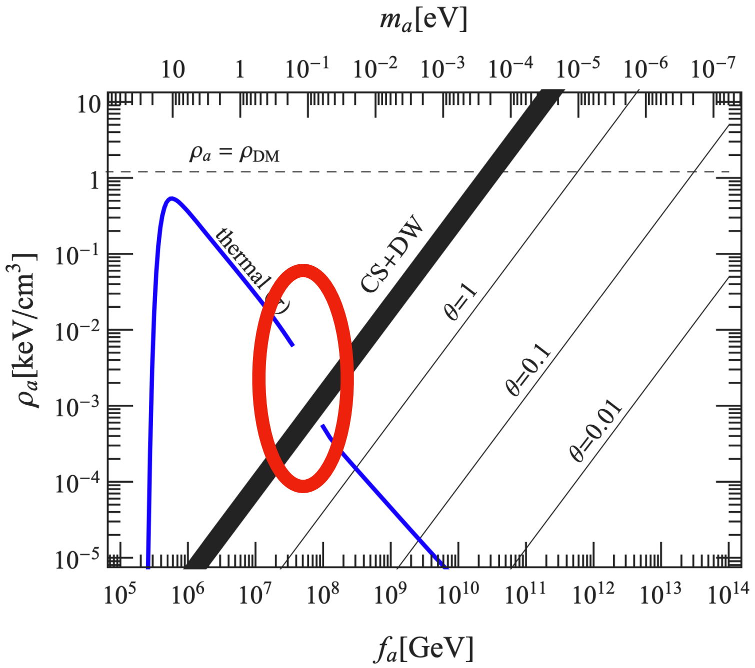


Borsanyi 2016

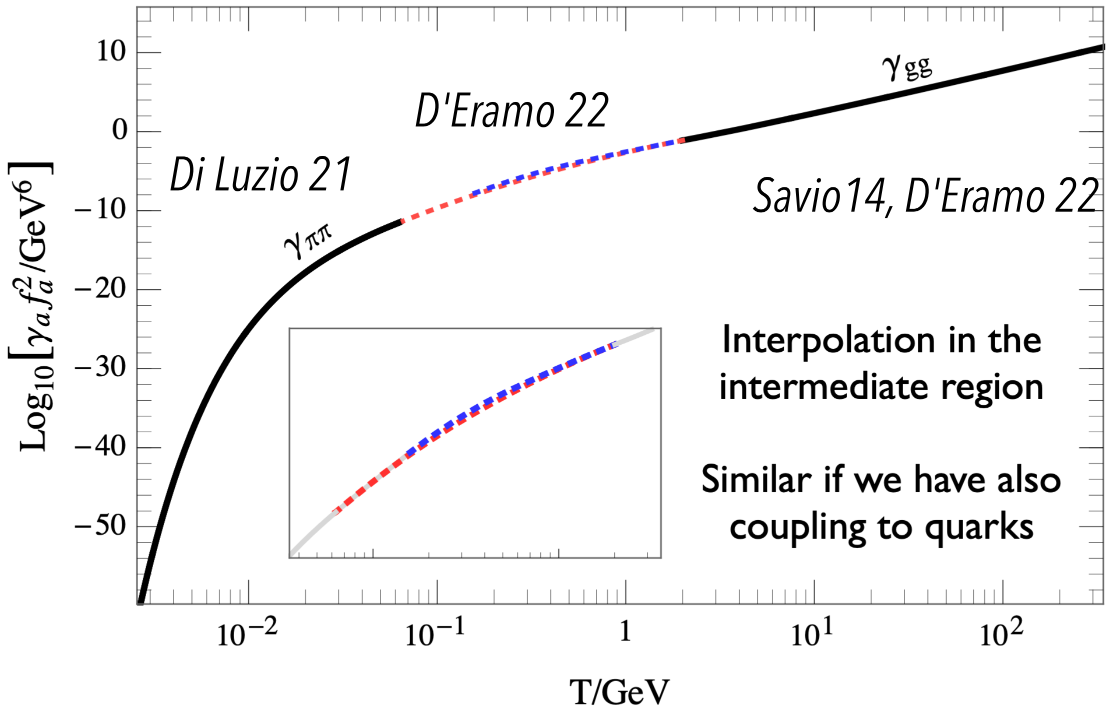
The QCD phase transition issue

- Hot DM bound was calculated in a region where ChiPT was not valid





- Recent study "patched" up the ChiPT and HE QCD reliable regions



D'Eramo 22

- Educated interpolation,
- Full lattice QCD required in this regime

Zero modes

0

$(k/R < H)$

Zero modes

- The axion thermalisation rate mentioned is for axions of energy E ~ T

- Axions are Goldstone bosons, Goldstone pole!

(most) Axion interactions are "derivative" ... they should vanish in the zero energy limit

$$\mathcal{M}(? \to ? + a(q^\mu)) \propto q_\mu \ldots \xrightarrow{q^\mu \to 0} 0$$

Zero modes

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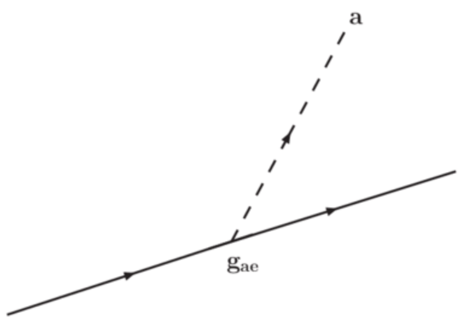
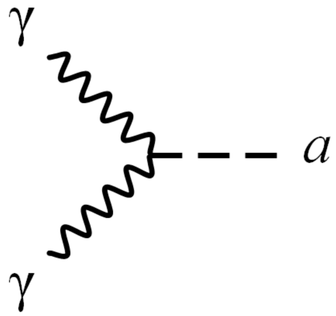
(most) Axion interactions are "derivative" ... they should vanish in the zero energy limit

$$\mathcal{M}(? \to ? + a(q^\mu)) \propto q_\mu \ldots \overset{q^\mu \to 0}{\longrightarrow} 0$$

Examples:



$$(\partial_\mu a)\bar{\psi}\gamma^\mu\gamma_5\psi \to \mathcal{M}(\psi \to \psi + a) \propto q_\mu \ldots$$



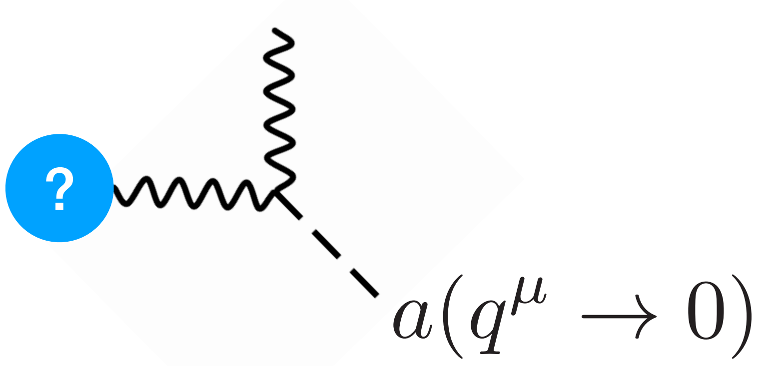
$$aF_{\mu\nu}\widetilde{F}^{\mu\nu} \to \mathcal{M}(\gamma_1 \to \gamma_2 + a) \propto k_1^\mu v_1^\nu k_2^\alpha v_2^\beta \epsilon^{\mu\nu\alpha\beta} = k_1^\mu v_1^\nu q^\alpha v_2^\beta \epsilon^{\mu\nu\alpha\beta}$$

Zero modes

- The axion thermalisation rate mentioned is for axions of energy E ~ T

- I have been cheating you badly (at low energies)

- Rate of absorbing(emitting) a ultralow-energy axion (mass m_ϕ) from a photon of energy ω



$$d\Gamma_{\phi C} = \frac{1}{2m_\phi}\Gamma_C(\omega)|\mathcal{M}(\gamma\phi \to \gamma^*)|^2 \frac{1}{(2\omega m_\phi)^2 + (\omega\Gamma_C(\omega))^2} dn_\gamma(\omega)$$
$$= g^2 m_\phi \beta^2 \frac{\omega^2 \Gamma_C(\omega)}{(2\omega m_\phi)^2 + (\omega\Gamma_C(\omega))^2} dn_\gamma(\omega)$$

$$\Gamma_A \sim \frac{T^3}{f_A^2}\frac{m_a}{\Gamma_\gamma} \qquad \Gamma_\gamma \sim \alpha^2 T$$

Cadamuro 2012

suppression of m_A/T with respect to previous estimates

Zero modes

- Neglecting g's, suppression of E/T, low-energy modes are E~H

$$\Gamma_A/H \sim \frac{T^2 H/f_A^2}{H} = \left(\frac{T}{f_A}\right)^2$$

- Below T ~ f_A, low-energy modes are expected to be decoupled ...
- This is much higher than f_A^2/m_{Pl} above which Thermal modes couple to SM

- Another interesting expression, using lowest energy axions, E ~ mass

$$\Gamma_A/H \sim \frac{T^2 m_A/f_A^2}{T^2/m_{\rm Pl}} \sim \frac{m_A m_{\rm Pl}}{f_A^2} \sim \frac{\Lambda_{\rm QCD}^2 m_{\rm Pl}}{f_A^3} \sim 10^{-10}\left(\frac{10^9\,{\rm GeV}}{f_A}\right)^3$$

Zero modes

0

(k/R < H)

Axion Zero modes

$$\mathcal{L}_A = \frac{1}{2}(\partial_\mu a)^2 + \partial_\mu a j^\mu_{\rm SM} + aF\widetilde{F} + aF\widetilde{F} + aG\widetilde{G} +$$

- Axion zero modes still interact with GGtilde
  GGtilde is not a pure-Goldstone interaction and does not vanish in the q->0 limit
  thanks to it we have the axion mass and potential

- Thus according to this low-energy theory, the most relevant axion zero mode interaction is due to the axion potential

$$V(a) = V_{\rm QCD}(\theta) = V_{\rm QCD}(a/f_A) \sim \chi_T(1-\cos\theta)$$

- But remember that QCD becomes poerturbative at high-T and $\chi_T \sim (\Lambda/T)^{8\ldots}$
  therefore, this interaction is also irrelevant at very high-T

Initial conditions

- Axion zero-modes are decoupled as far as the low-E theory is concerned

- Their abundance comes from their initial conditions

- Their initial conditions date back to the very early Universe… when the UV completion is active

UV

U(1) to the rescue

We do not know anything about the UV completion

We can assume however that it respects the U(1) symmetry

The axion is a relevant degree of freedom below E~f_A

Assume a phase transition at temperatures E~f_A (spontaneous PQ breaking)

After the phase transition, the axion field takes a VEV

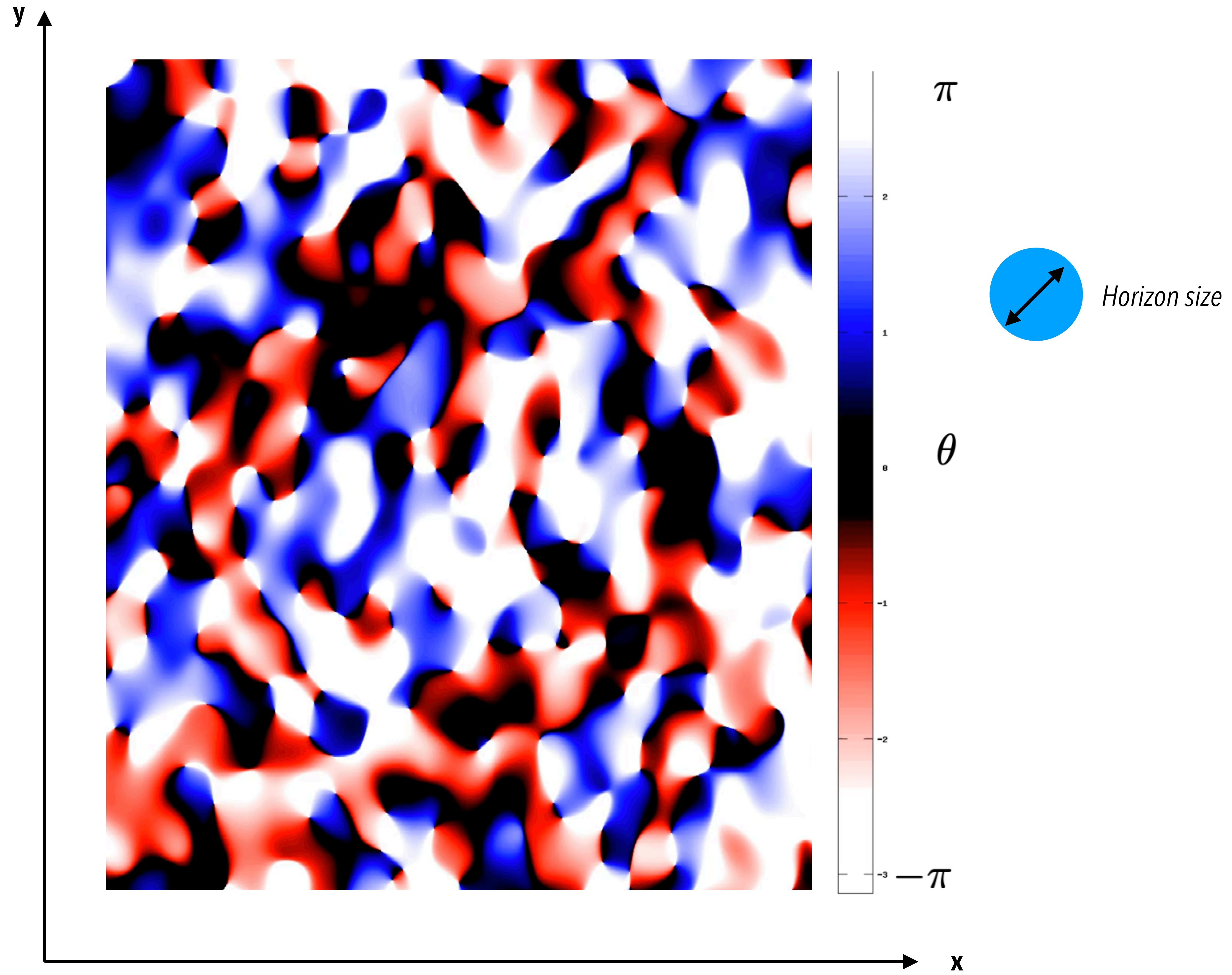
the VEV cannot be correlated beyond the horizon size

$$\theta(x) \neq \theta(x + d_H), d_H \sim 1/H$$

All values of the axion field are equally probable (U(1) symm)

After phase transition

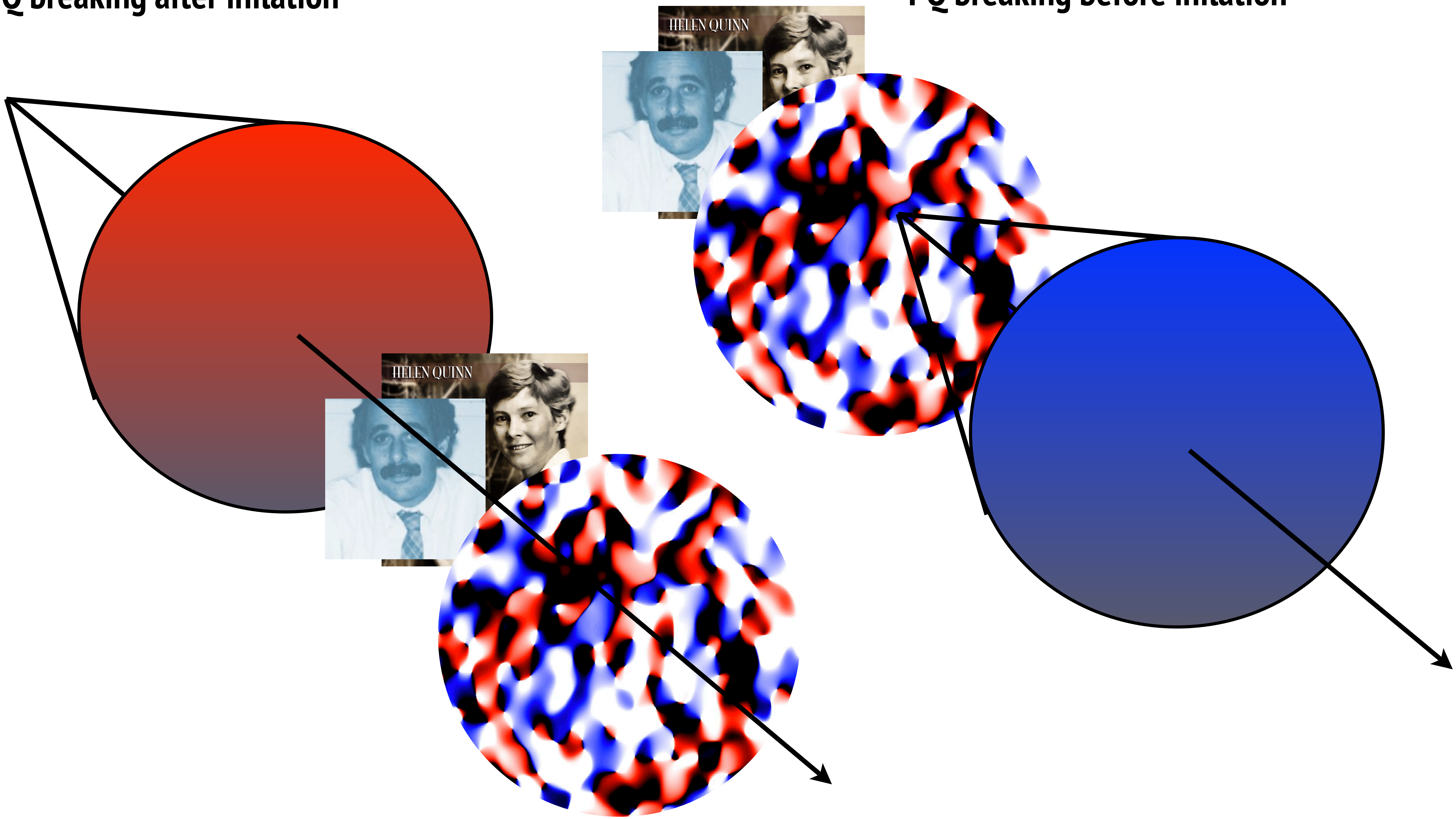
Typical slice of the Universe after phase transition*



Two (main) scenarios

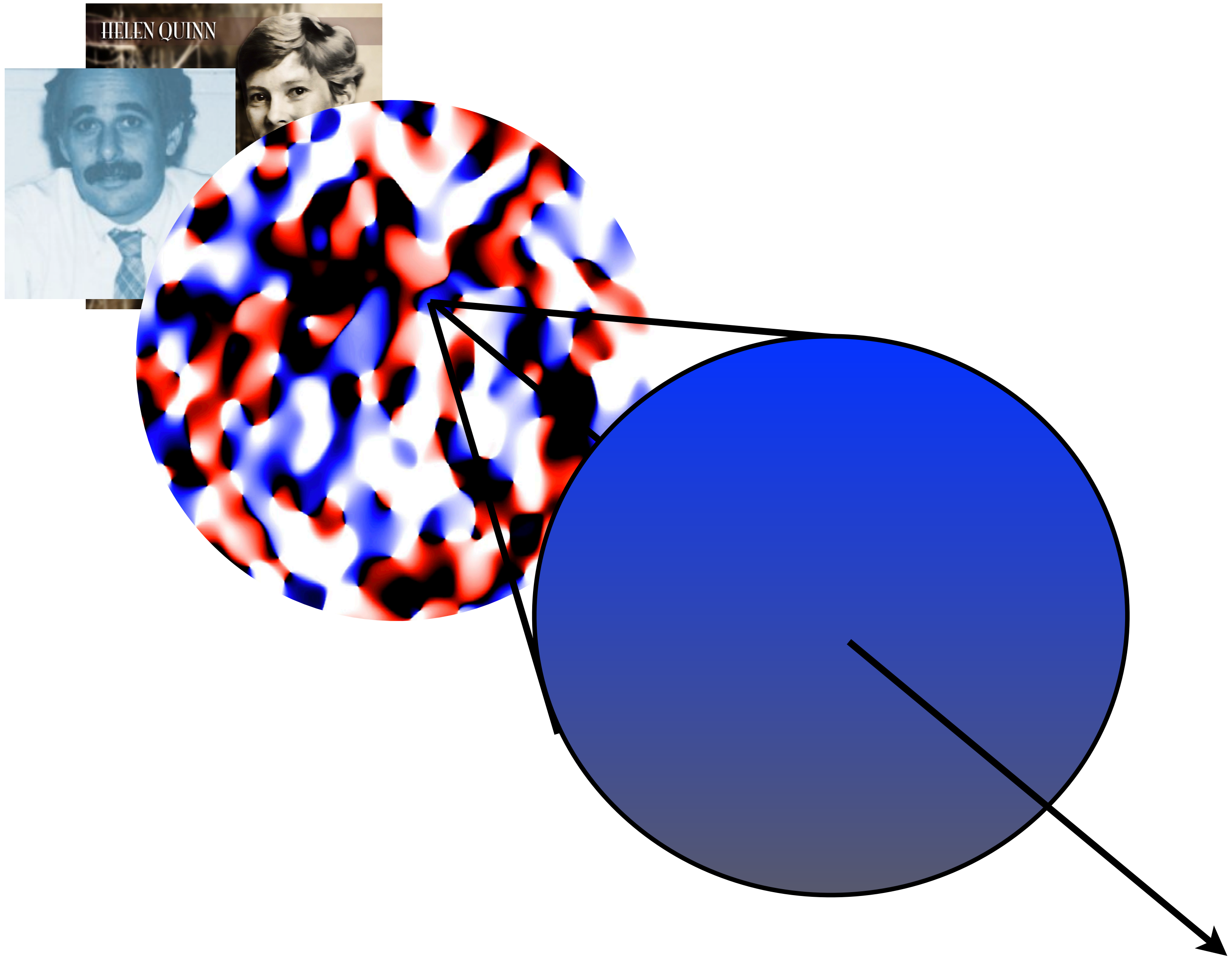
- PQ breaking after inflation

- PQ breaking before inflation



PQ breaking before inflation

- Inflation: One sub-causal region gets blown up to a size larger than our causal horizon TODAY
- The value of the axion field was ~homogeneous in that region
- The initial condition for the axion field is homogeneous (+ quantum fluctuations during inflation)



Evolution of zero mode

- The zero mode has huge occupation number and evolves classically*

Field evolution

$$S = \int d^4x\sqrt{-g}\mathcal{L} = \int d^4x\sqrt{-g}\left(\frac{f_a}{2}(\partial_\mu\theta)(\partial^\mu\theta) - V(\theta) + \mathcal{L}_{int}\right)$$

$$= \int d^4x\sqrt{-g}\left(\frac{1}{2}(\partial_\mu a)(\partial^\mu a) - V(a/f_a) + \mathcal{L}_{int}\right)$$

Equations of motion $\delta S = 0$ **Scale factor is** $R(t)$ **, Expansion rate** $H = \dot{R}/R$

$$\left(\frac{\delta\mathcal{L}}{\delta(\partial^\mu a)}\right)_{;\mu} - \frac{\delta\mathcal{L}}{\delta a} = 0$$

$$\ddot{a} + 3H\dot{a} - \frac{1}{R^2}\nabla^2 a + \frac{\partial V}{\partial a} = 0$$

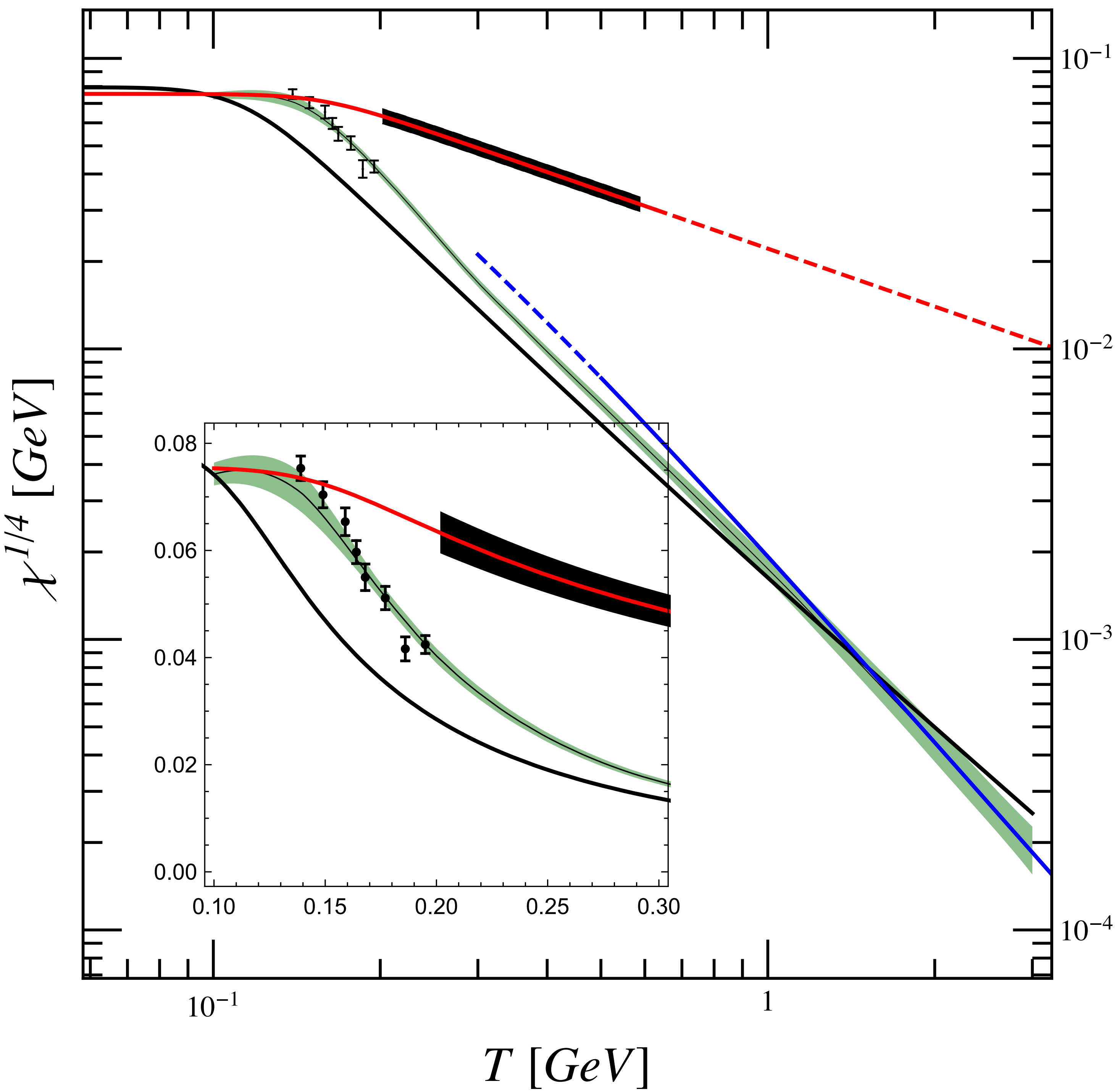
Effective mass, lattice calculations

Lattice QCD: we can compute axion mass

$$m_a^2 f_a^2 = \chi(T)$$

At high T (no mesons)
we can analytically compute potential (DIGA)

$$V(\theta) = -\chi(T)\cos\theta$$



Lattice QCD 2+1+1 [Borsanyi]
Lattice QCD 2+1 [Bonati]
Lattice QCD (DWF) 2+1 [Buchoff] (points)
DIGA (T>>Tc) [Borsanyi]
IILM [Wantz]

Zero mode evolution

Damped harmonic oscillator (with a time-varing frequency/mass)

$$\ddot{a} + 3H\dot{a} + m_a^2 f_A \sin(a/f_A) \simeq 0$$

$$\ddot{\theta} + 3H\dot{\theta} + m_a^2 \sin\theta \simeq 0$$

H decreases and m_A increases in time, assuming radiation domination...

$$H \sim \frac{1}{2t}$$

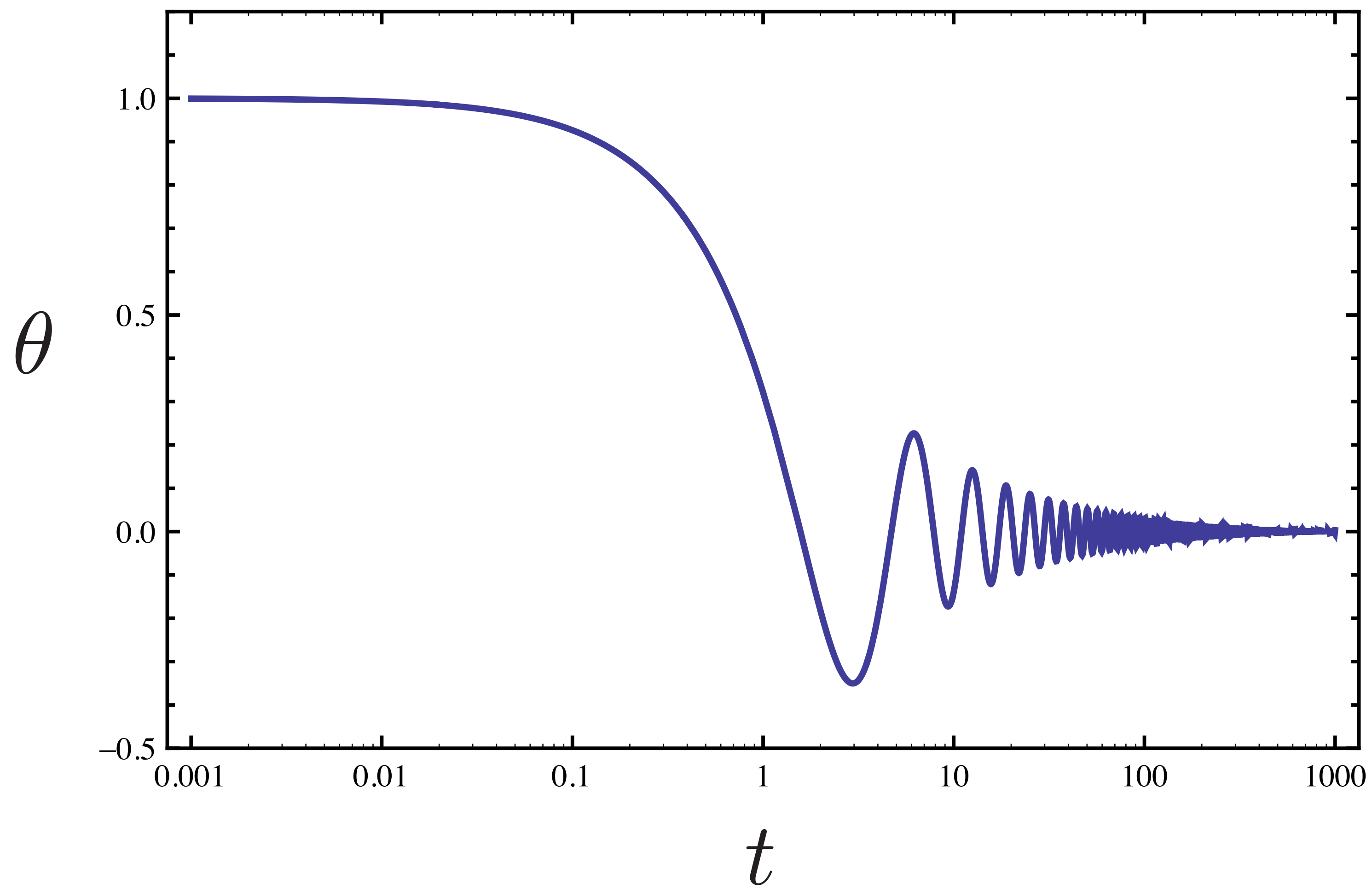
$$m_A \sim \frac{1}{T^{n/2}} \sim R^{n/2} \sim t^{n/4}$$

Zero mode evolution

Damped harmonic oscillator (with a time-varing frequency and damping)

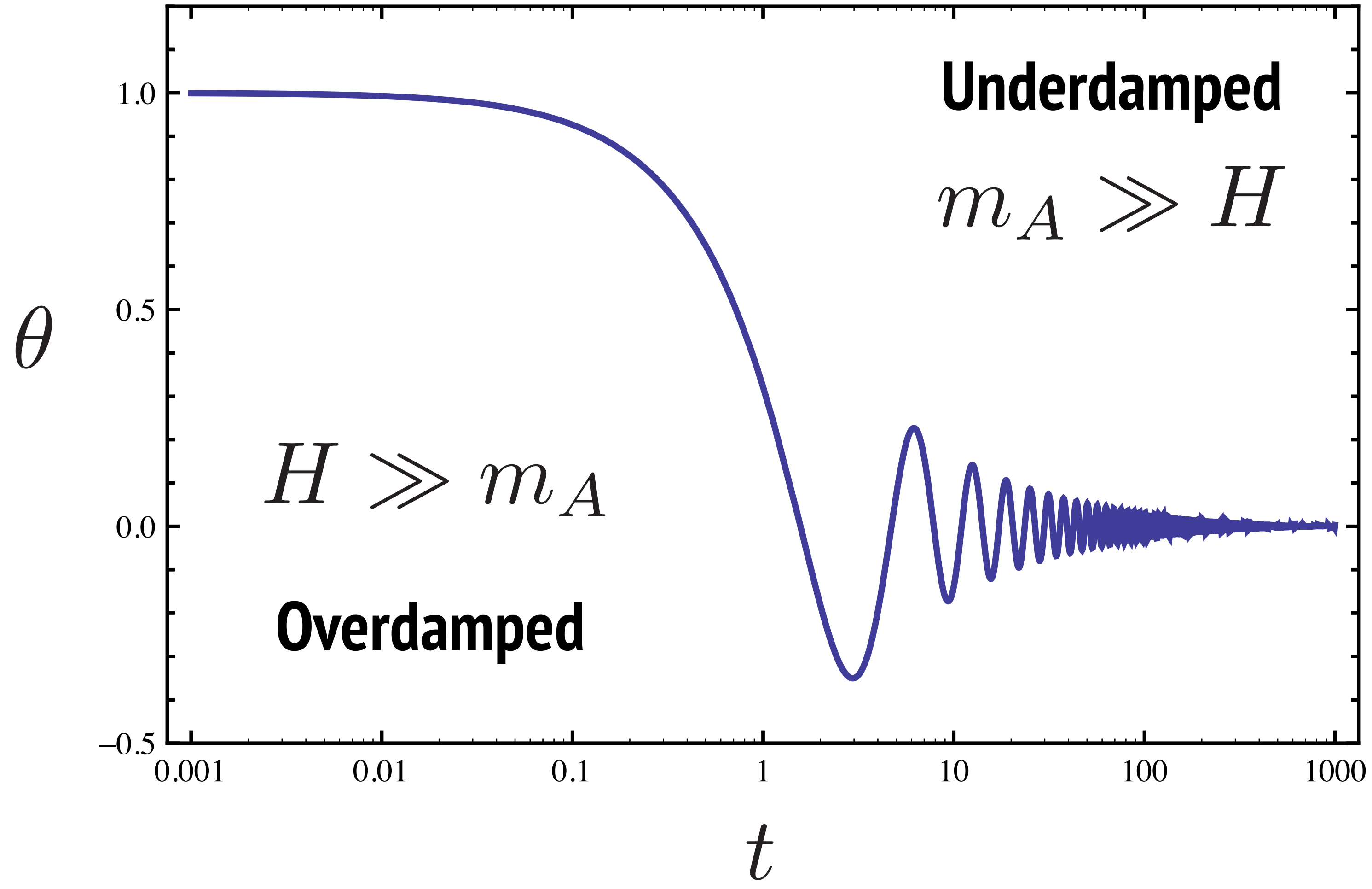
$$\ddot{a} + 3H\dot{a} + m_a^2 f_A \sin(a/f_A) \simeq 0$$

$$\ddot{\theta} + 3H\dot{\theta} + m_a^2 \sin\theta \simeq 0$$



Zero mode evolution

- Two regimes:
  - Overdamped: H>>mA
  - Underdamped mA << H
  - Critical time mA~H

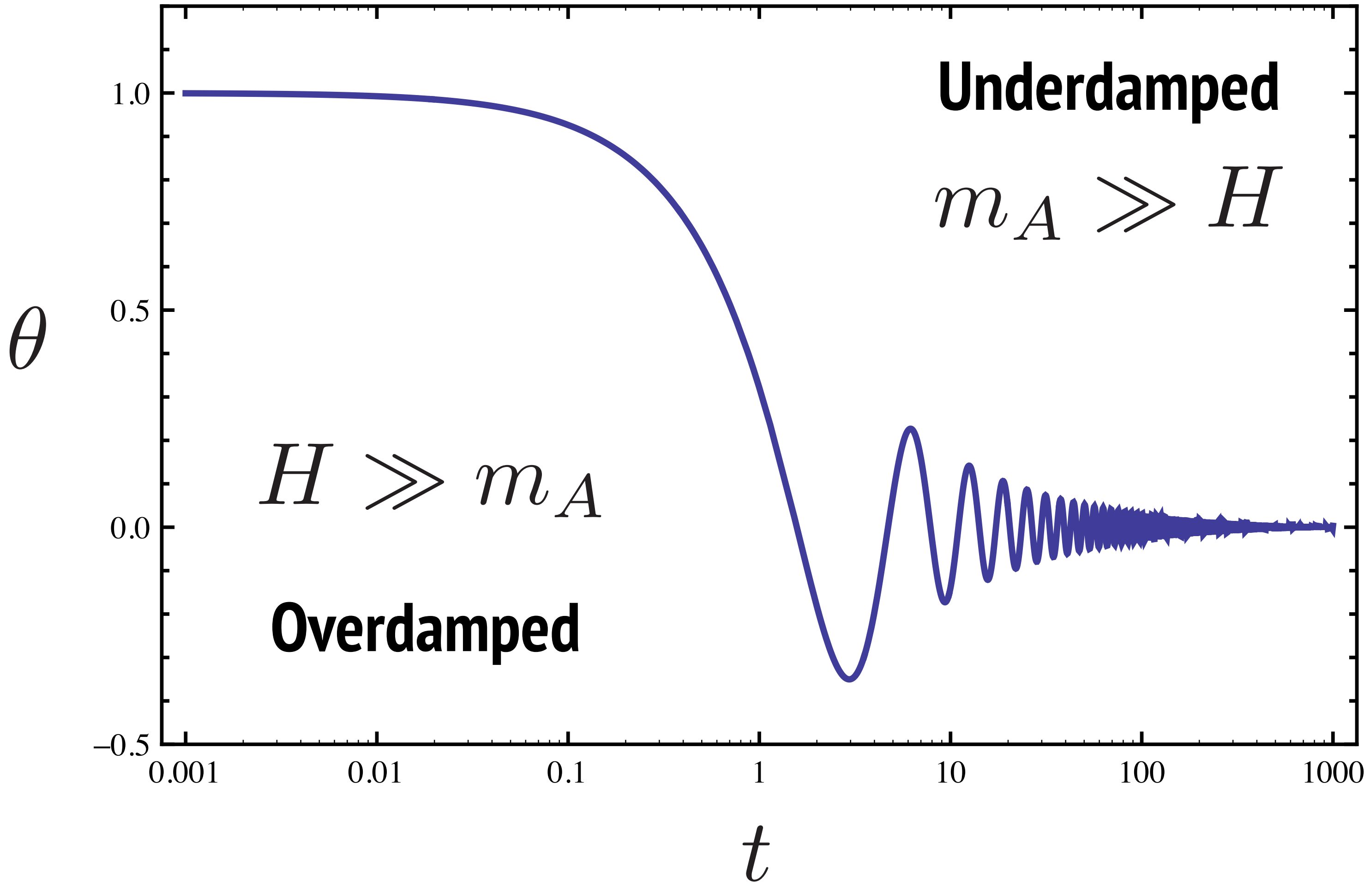


Zero mode evolution

- Two regimes:
  - Overdamped: H>>mA
  - Underdamped mA << H
  - Critical time mA~H

$$\ddot{\theta} + 3H\dot{\theta} \simeq 0$$

2 solutions: $$\begin{cases} \dot{\theta} = 0 \\ \dot{\theta} \propto 1/R^3 \end{cases}$$

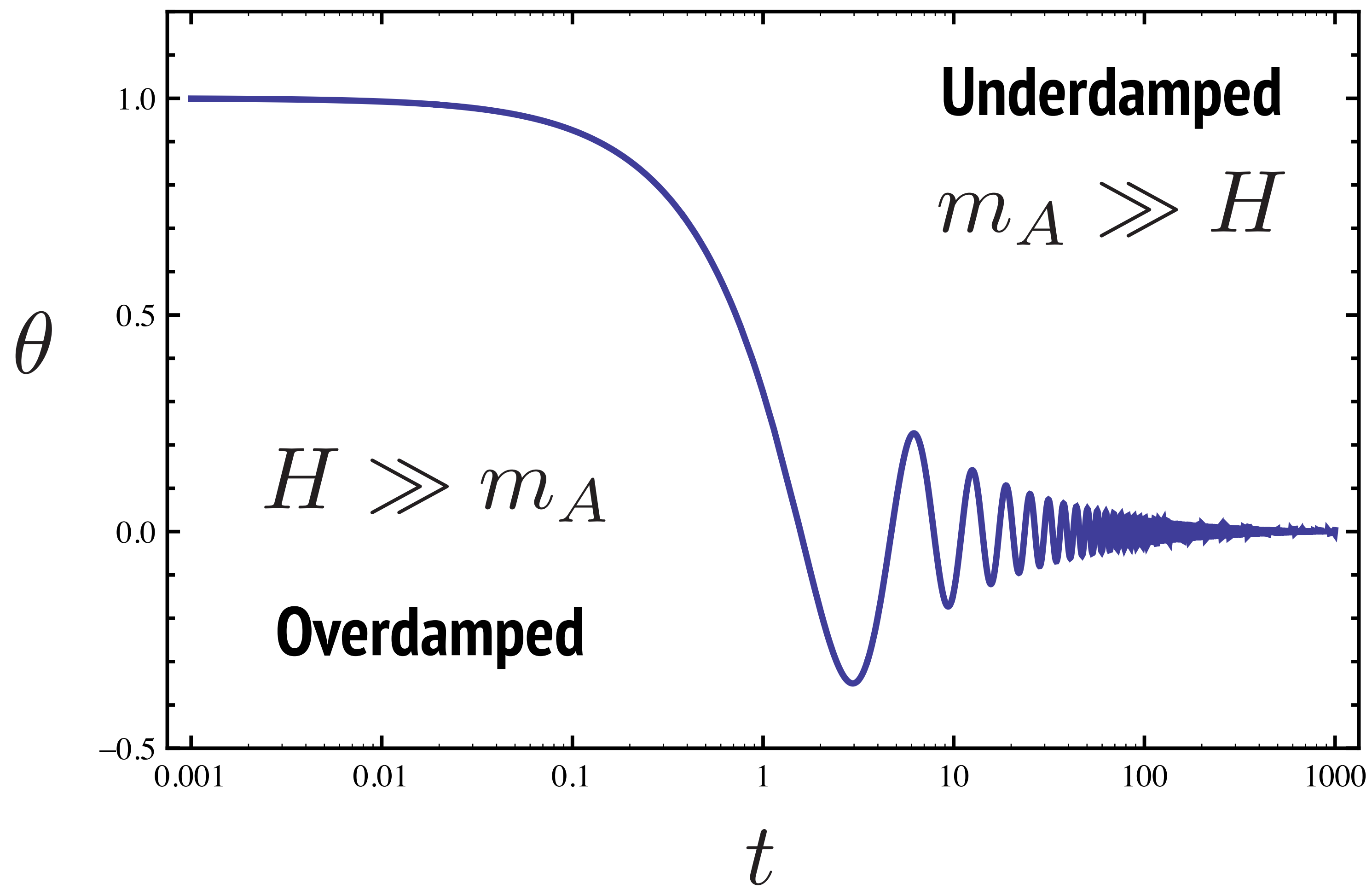


Zero mode evolution

- **Two regimes:**
  - Overdamped: H>>mA
  - **Underdamped mA << H**
  - Critical time mA~H

WKB approximation

$$\theta(t) = \frac{1}{\sqrt{m_A R^3}} e^{i \int^t m_A(t) dt'}$$



time-scales (radiation domination) linear regime

- **Two regimes:**
  - Overdamped: H>>mA
  - Underdamped mA << H
  - **Critical time mA~H**

$$H(t_1) = m_A(t_1)$$

$$\frac{T_1^2}{m_{\rm Pl}} \sim \chi_0 \left(\frac{\Lambda}{T_1}\right)^{n/2} \qquad \textbf{(RD)}$$

$$T_1 \sim (\chi_0 \Lambda^{n/2} m_{\rm Pl})^{\frac{2}{4+n}}$$

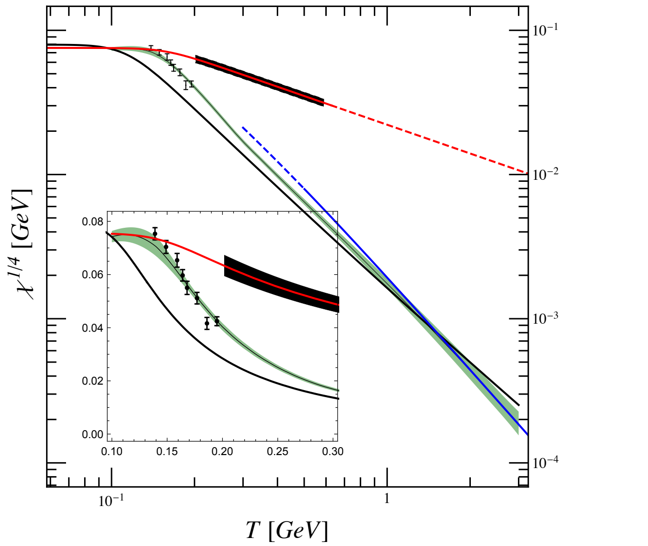
Vaquero 2019

Temperature $$T_1 \simeq 1.694\,{\rm GeV}\left(\frac{m_a}{50\,\mu{\rm eV}}\right)^{0.1638},$$

Exp. rate $$H_1 \simeq 3.45\times 10^{-3}\mu{\rm eV}\left(\frac{m_a}{50\,\mu{\rm eV}}\right)^{0.338},$$

Redshift $$1+z_1 \simeq R_1^{-1} = 1.956\times 10^{13}\left(\frac{m_a}{50\,\mu{\rm eV}}\right)^{0.1712},$$

Comoving horizon size $$L_1 \equiv \frac{1}{H_1 R_1} \simeq 1.116\times 10^{17}{\rm cm}\left(\frac{50\,\mu{\rm eV}}{m_a}\right)^{0.167} = 0.0362\,{\rm pc}\left(\frac{50\,\mu{\rm eV}}{m_a}\right)^{0.167}$$



time-scales non-linear regime

- Two regimes:
  - Overdamped: H>>mA
  - Underdamped mA << H
  - Critical time

Close to theta~pi, the QCD potential has a maximum,
acceleration decreases as $\sin\theta/\theta \to 0$
time-scales increase as $t \propto \sqrt{\theta/\sin\theta}$

In the limit theta=pi, the axion field evolves as vacuum energy,
quantum fluctuations drive the relaxation
Such fine-tuned initial conditions might be disfavoured

Energy density and pressure

$$T^{\mu}{}_{\nu} = (\partial^{\mu} a)(\partial_{\nu} a) - \mathcal{L}\delta^{\mu}{}_{\nu} = \mathrm{diag}\{\rho, p, p, p\}$$

$$\rho = \frac{1}{2}(\dot{a})^2 + \frac{1}{2}(\nabla a)^2 + V(a)$$

$$p = \frac{1}{2}(\dot{a})^2 - \frac{1}{2}(\nabla a)^2 - V(a)$$

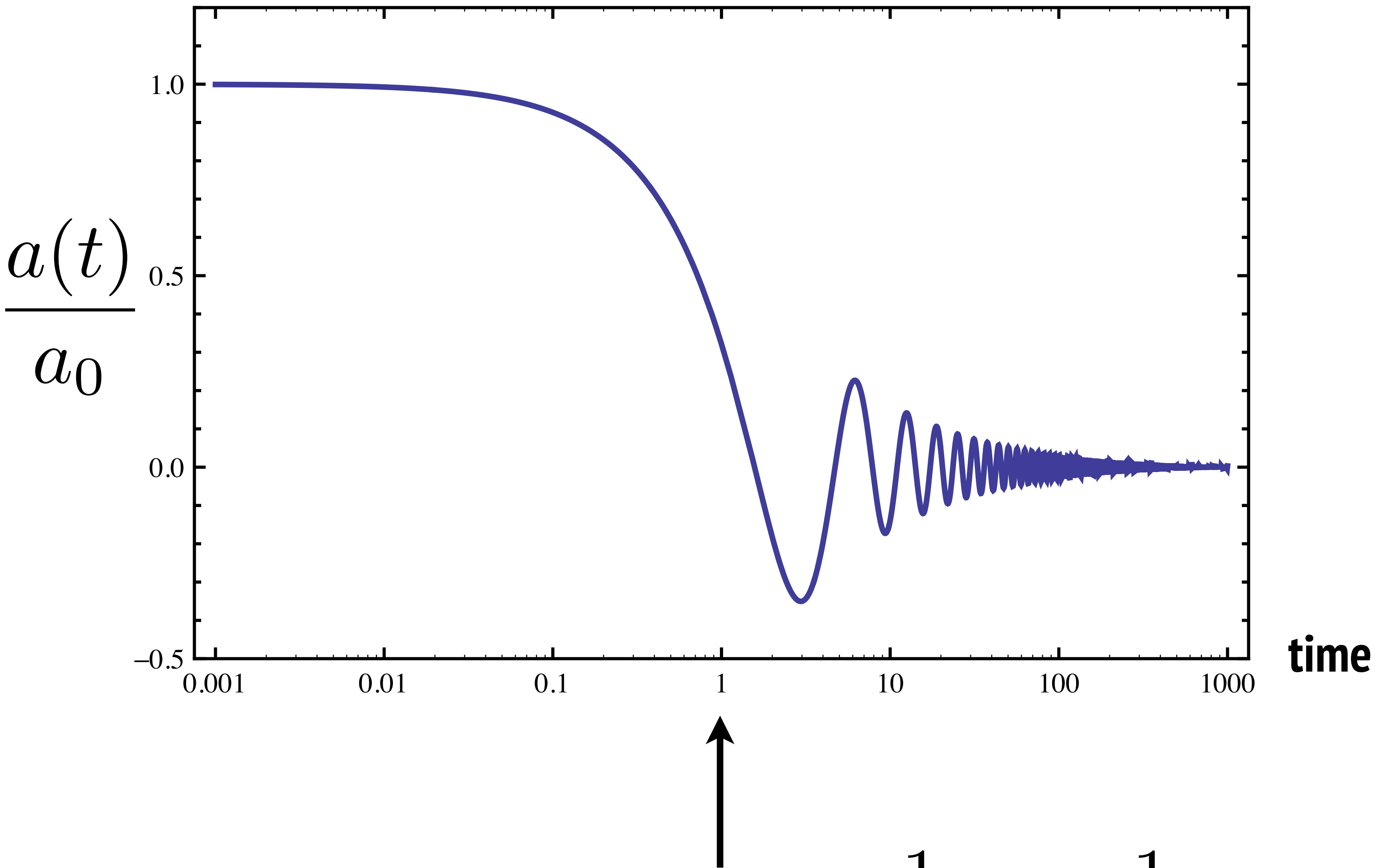
Zero mode evolution

Vacuum-like*

$\rho \sim V(\theta_0)$

DUST-like
(axion number conservation)

$\rho \sim m_A^2 \theta^2 \sim m_A / R^3$



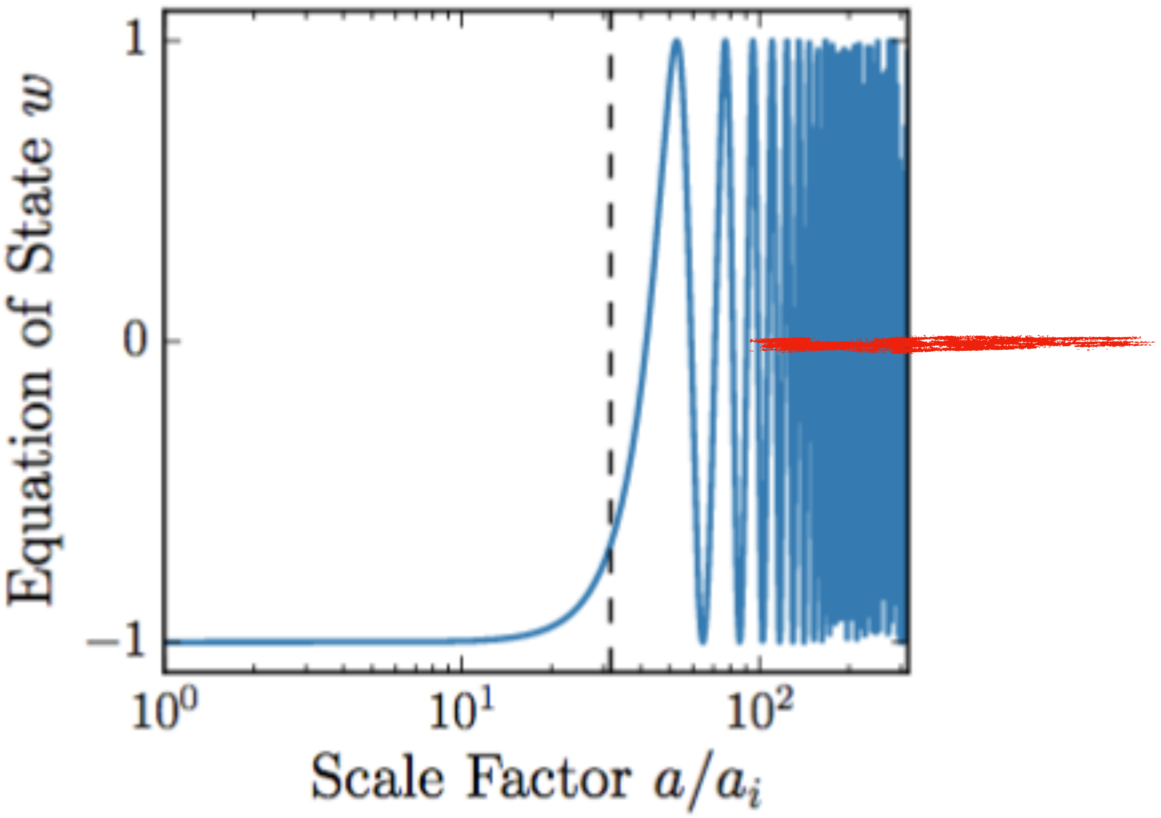
$$t_1 = \frac{1}{2H} \sim \frac{1}{m_a}$$

$$H \sim m_a$$

Equation of state

- The energy in vacuum axion oscillations behaves like cold-dark-matter

$$\ddot{\theta} + 3H\dot{\theta} + m_a^2 \sin\theta \simeq 0 \qquad (\theta < \pi)$$



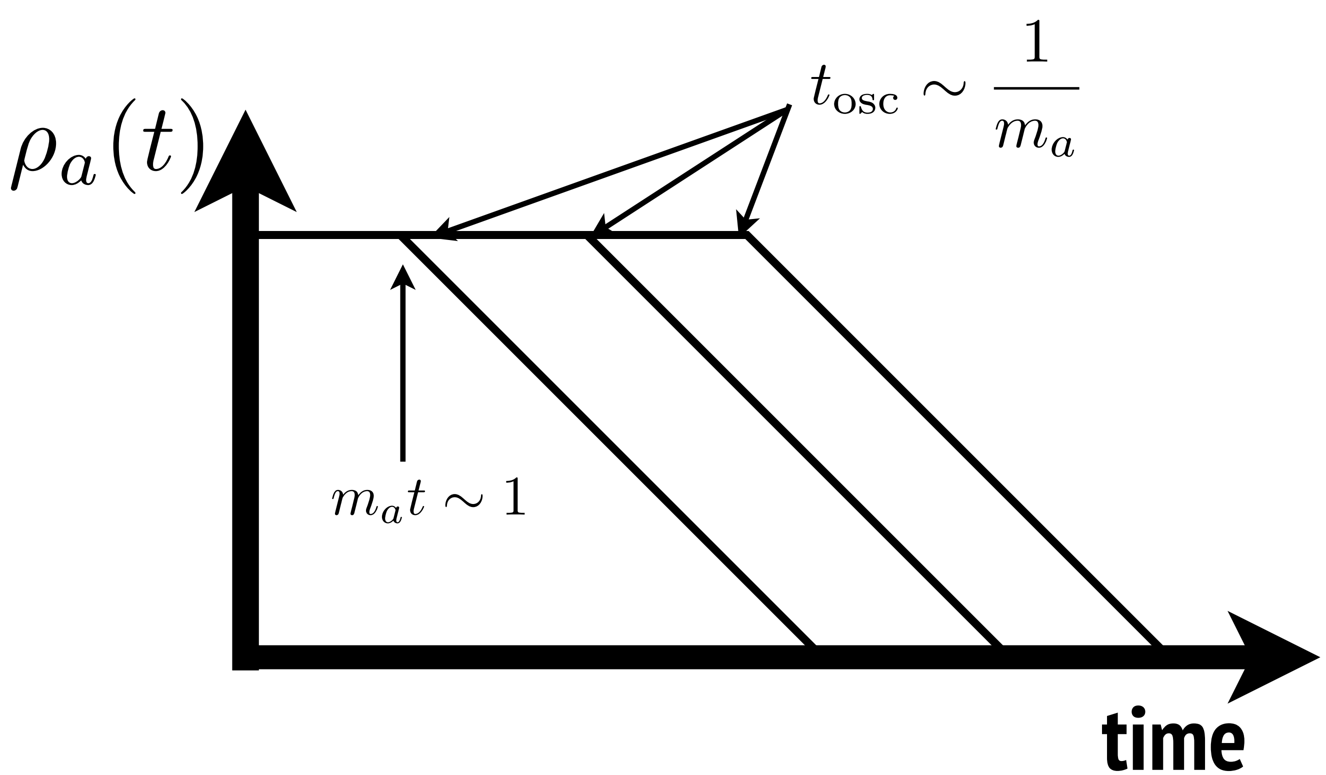
Energy density today

- **Energy density redshifts as matter, from the onset of oscillations** $H(t_1) \sim m_a$

$$\rho_a(t) \sim \theta_I^2 \chi \left(\frac{R_1}{R(t)}\right)^3 \propto \theta_I^2 \chi m_a^{-3/2}$$

- **dilution until today** $\left(\frac{R_1}{R_0}\right)^3 \sim \left(\frac{T_0}{T_1}\right)^3 \sim \left(\frac{T_0}{\sqrt{H_1 m_{\rm Pl}}}\right)^3 \sim \left(\frac{T_0}{\sqrt{m_a m_{\rm Pl}}}\right)^3 \propto m_a^{-3/2}$

Smaller mass axions, start oscillating later, and get less diluted ...

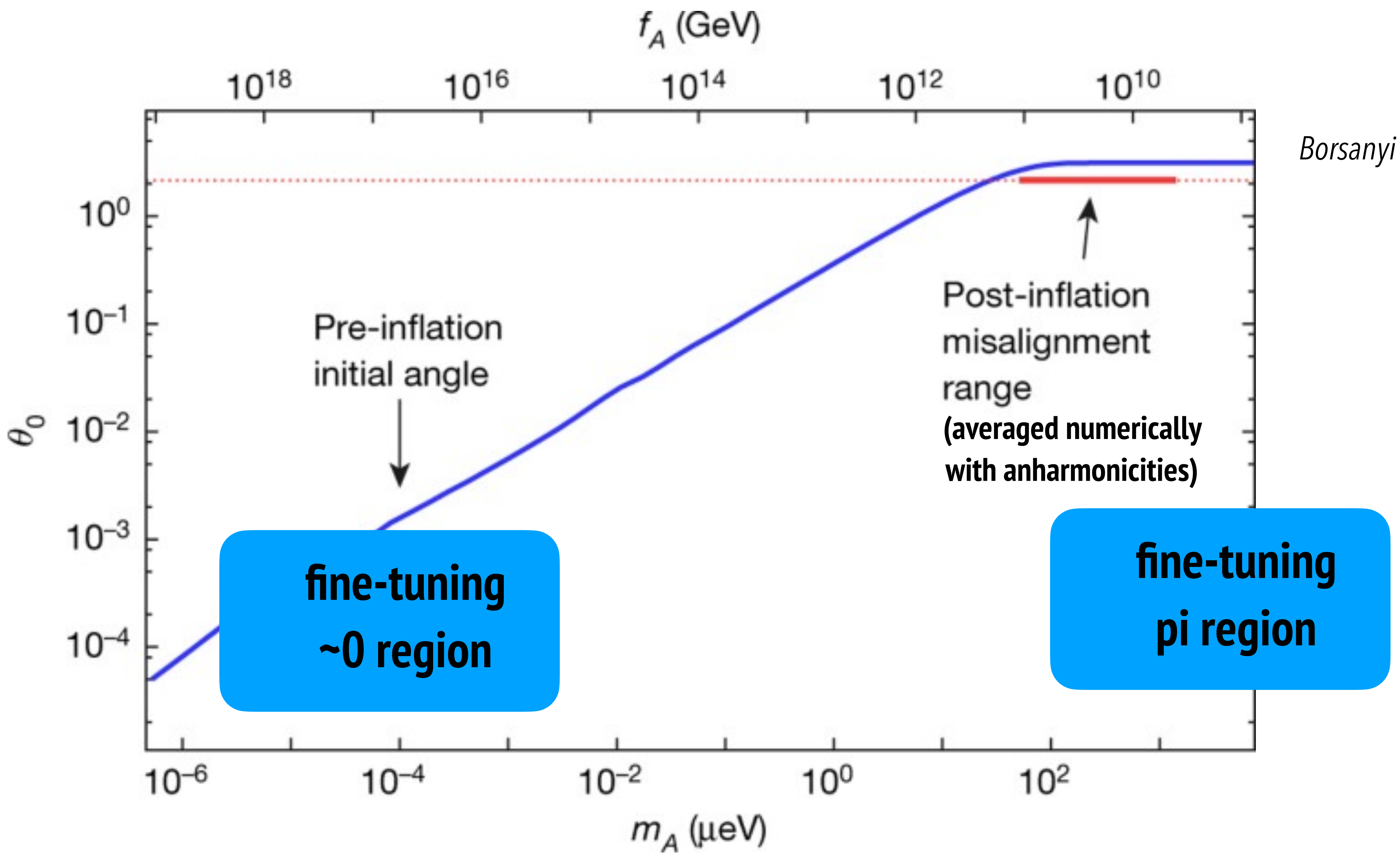


- **with** $\chi \propto T^{-n}$

$$\rho_a(t_0) \propto \theta_I^2 m_a^{-\frac{6+n}{4+n}}$$

Axion cold dark matter

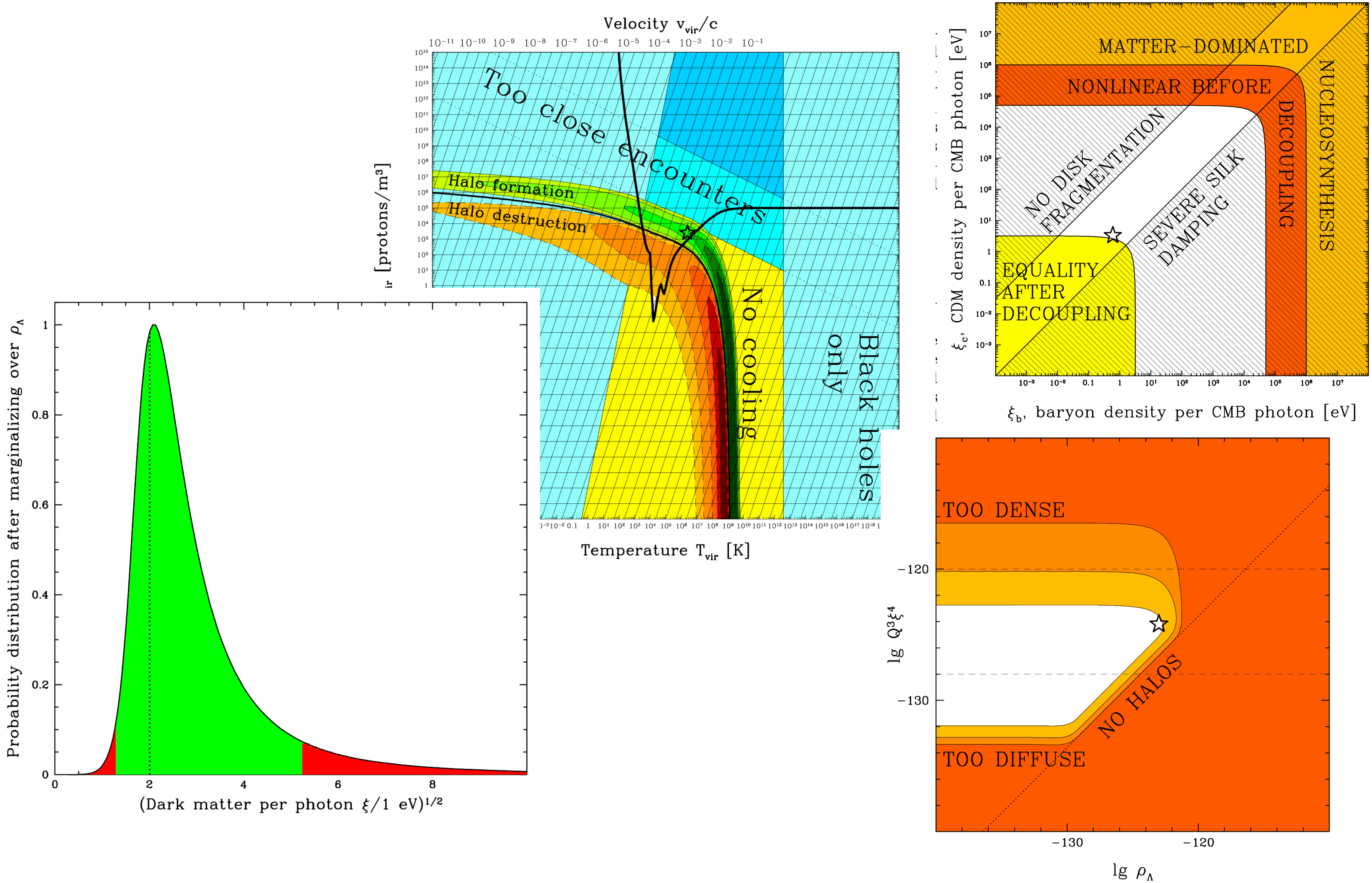
In the pre-inflation scenario, we can have essentially all CDM in axions
if we are lucky and live in the correct Universe with the right initial misalignment angle



Borsanyi 2016

Anthopic axion

- Anthropic selection arguments make "natural" or "viable" small θ_0 initial conditions *Tegmark 2006*



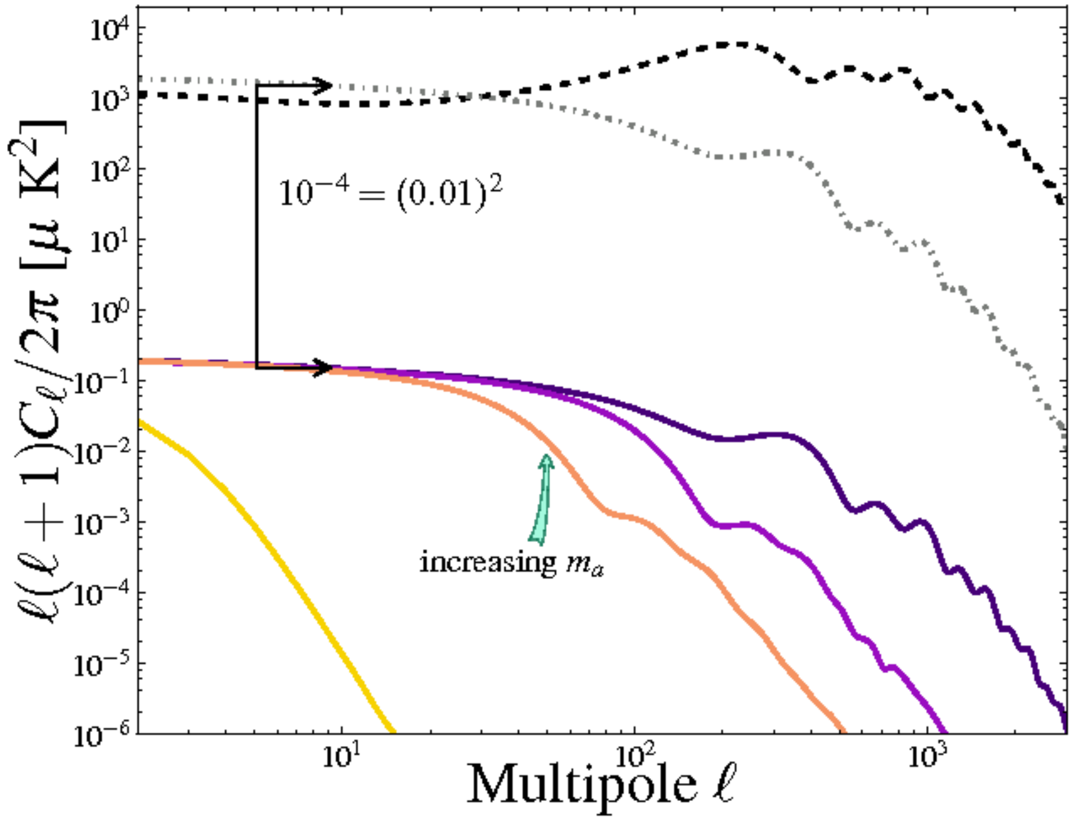
Quantum fluctuations ...

- During inflation, quantum fluctuations of the axion field grow and classicalise

- These fluctuations are "independent" of the inflaton fluctuations
  they are of ISOCURVATURE type

- At t_1, fluctuations in theta become fluctuations in the number density, i.e. CDM density

- Analysis of CMB anisotropies reveal that CDM fluctuations are ADIABATIC,
  i.e. correlated with the Temperature fluctuations



Adiabatic fluctuations

isocurvature component

Planck observational upper-limit

$$P_{\rm iso} < 0.039 P_s = 0.88 \times 10^{-10}$$

Quantum fluctuations …

- Size of axion fluctuations

$$P_{\rm iso} = \frac{d\langle n_a\rangle}{n_a} \sim \frac{d\langle a^2\rangle}{a_I^2} = \frac{H_I^2}{\pi^2 a_I^2} = \frac{H_I^2}{\pi^2 f_a^2 \theta_I^2} < 0.039 P_s = 0.88 \times 10^{-10}$$

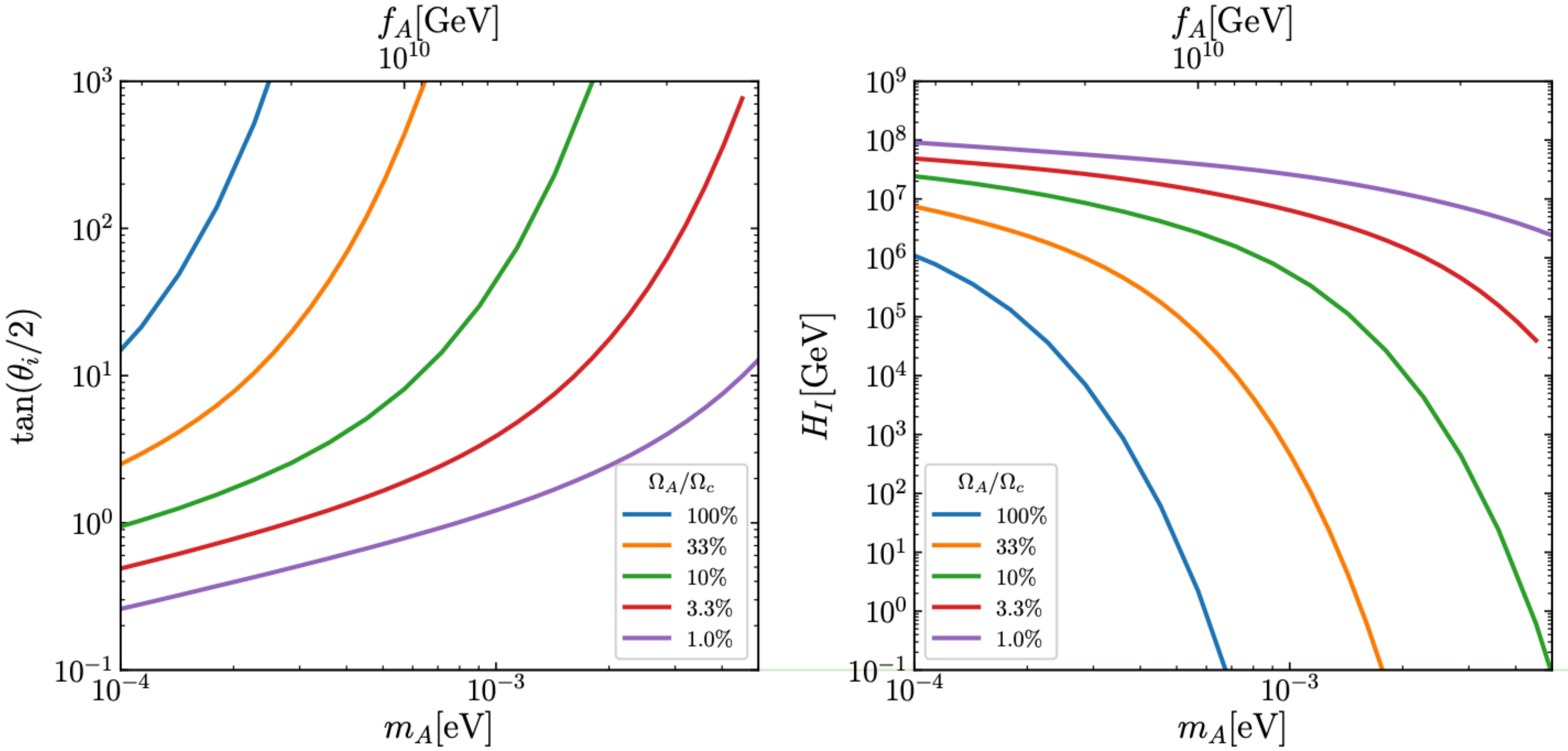
Fluctuations of a (can. normalised) massless field

$$\langle a^2\rangle = H_I^2/\pi^2$$

θ_I would be the average value during inflation

Isocurvature bounds

Assume θ_I to give a percentage of DM, we have a bound on H_I as a function of f_A

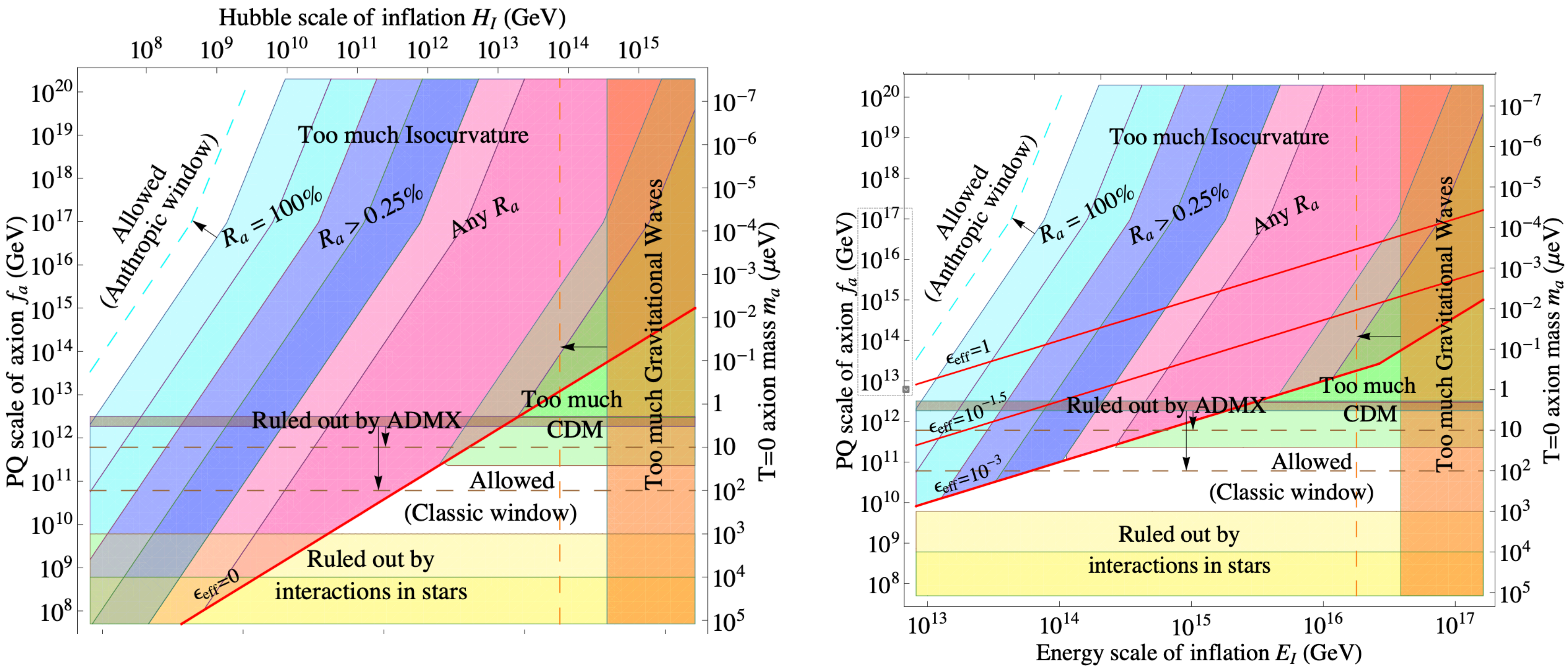


IAXO Physics potential 2019

Isocurvature bounds

Assume θ_I to give a percentage of DM, we have a bound on H_I as a function of f_A

Hertzberg 2010



Exciting prospects

A measurement of primordial gravitational waves from inflation pinpoints HI

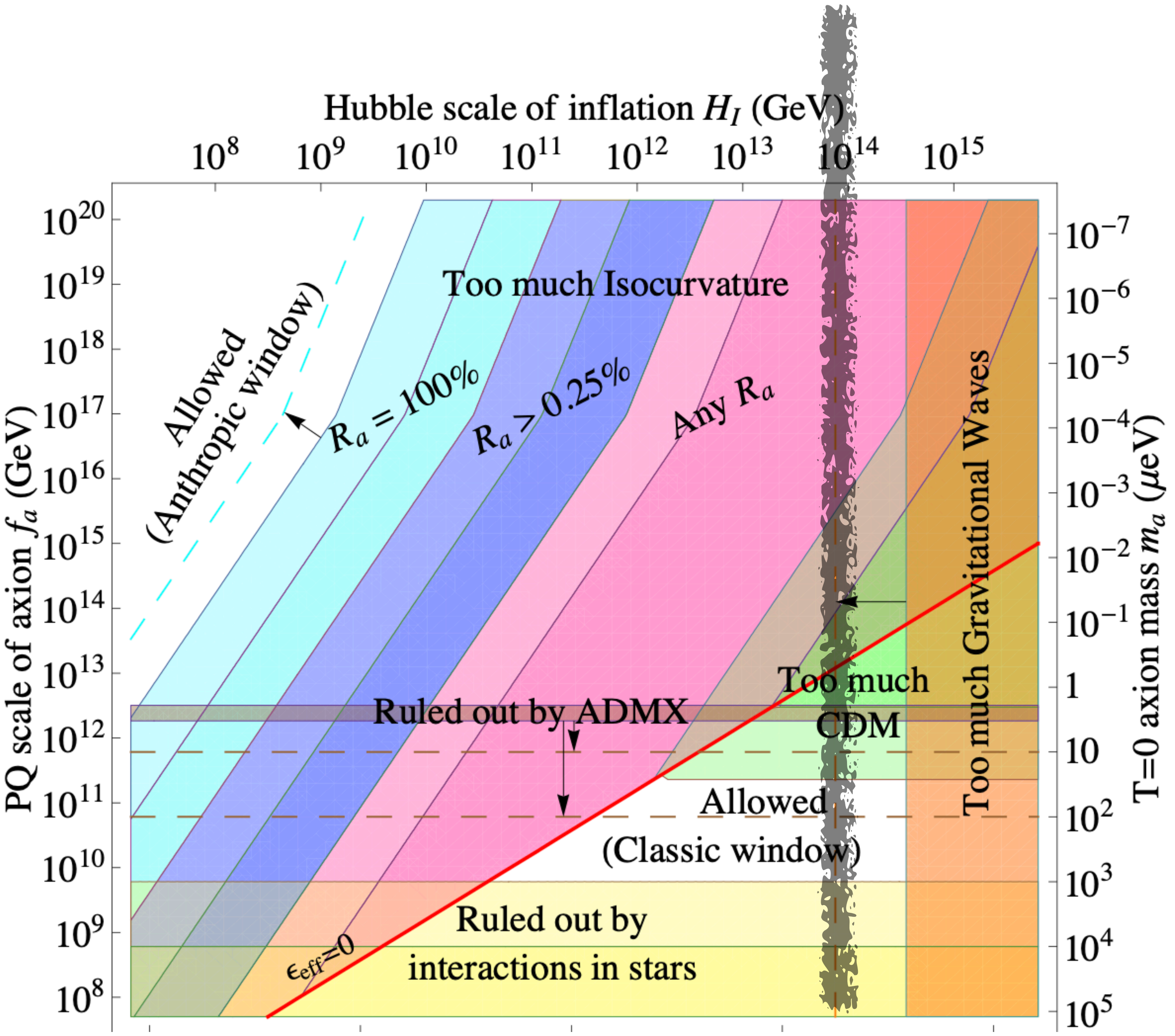
$$P_t \sim \frac{H_I^2}{m_{\rm Pl}^2}$$

Next generation experiments could measure down to $H_I \sim 10^{14}\mathrm{GeV}$

Exciting prospects

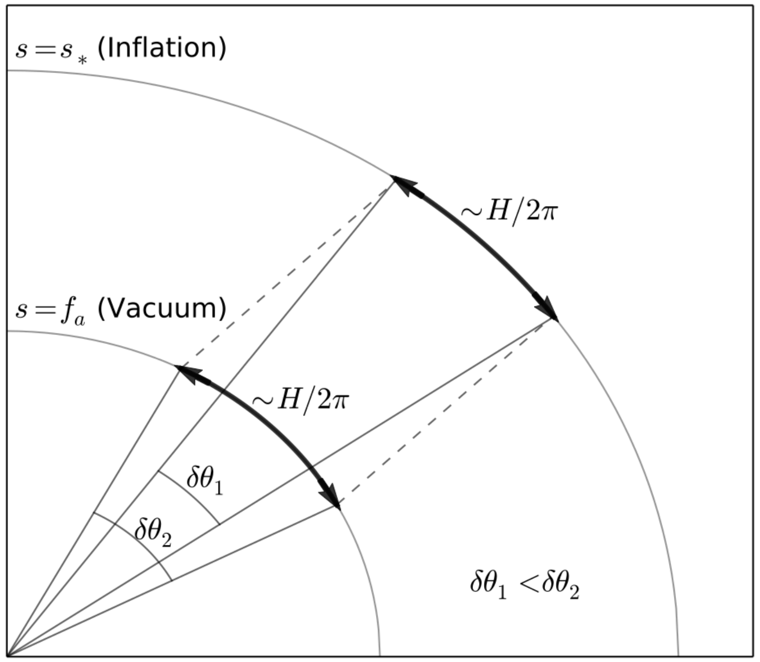
A positive measurement has potential to exclude very strongly this pre-inflation scenario

$$H_I \sim 10^{14}\text{GeV}$$



Ways out!

- Make $a_I \gg \theta_I f_A$



Fairbairn 2014

- In KDVZ, DFSZ models, fA is the vev of a scalar field, make it big during inflation! (you can make it the inflaton)

- Assume non-canonical kinetic terms, non-canonical couplings to gravity

$$\xi_\sigma |\sigma|^2 R$$

as in SMASH model, Ballesteros 2017

$$\frac{1}{2}\left(g^{\alpha\beta} - \frac{G^{\alpha\beta}}{M_a^2}\right)\partial_\alpha a \partial_\beta a$$

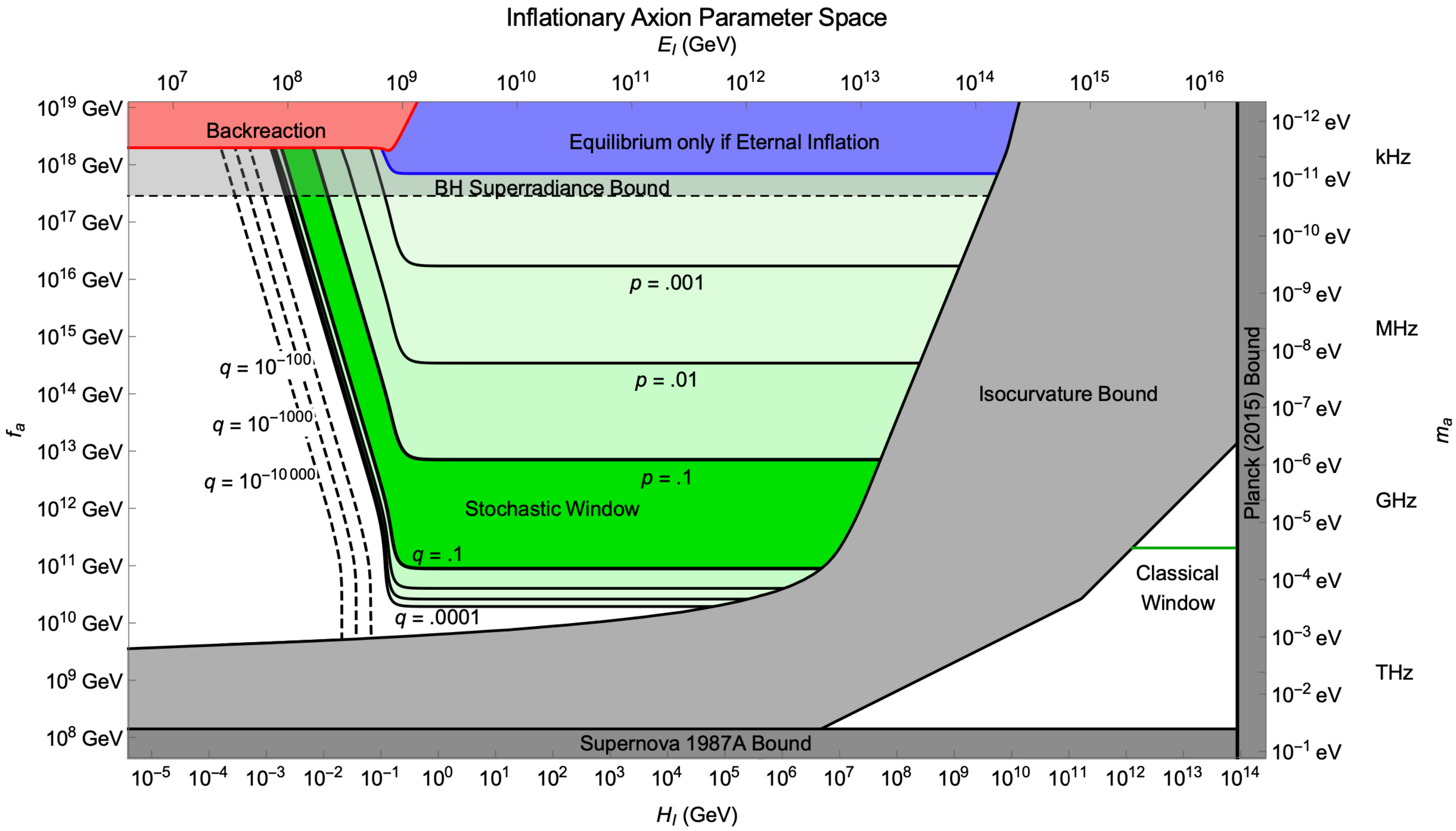
Folkerts 2013

Related production mechanisms

- Stochastic axion scenario

Graham 2018

Uses inflationary perturbations to generate DM axions



Related production mechanisms

- **kinetic misalignment** *Co 2019*
  - **Axion receives a huge kick around inflation, spins around longer than t_1**
  - **Tipically increases the axion DM yield for a given f_A**