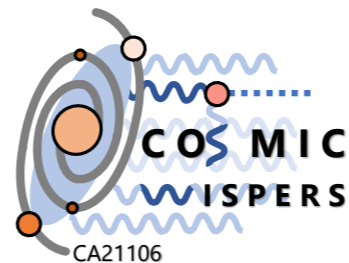


# Axion cosmology

GGI Axions across boundaries ...  
Training week 26-28 Apr 2023  
Javier Redondo



MAX-PLANCK-GESELLSCHAFT



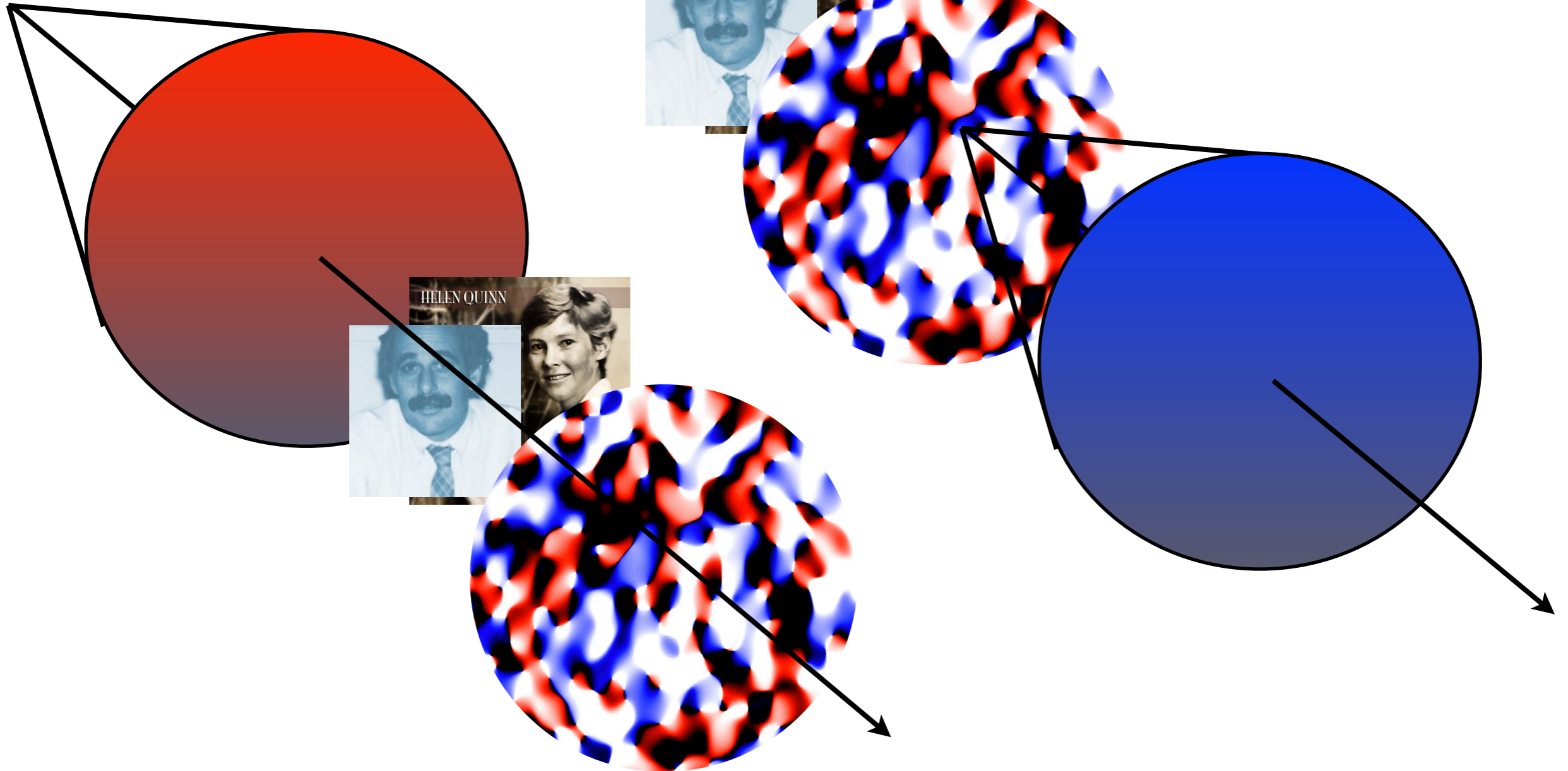
# outline

- thermal axion production
- misalignment, pre/post inflationary
- isocurvature fluctuations
- **post-inflationary scenario**
- **cosmic strings and walls**
- **axion miniclusters**
- axion stars

# Two (main) scenarios

- PQ breaking after inflation

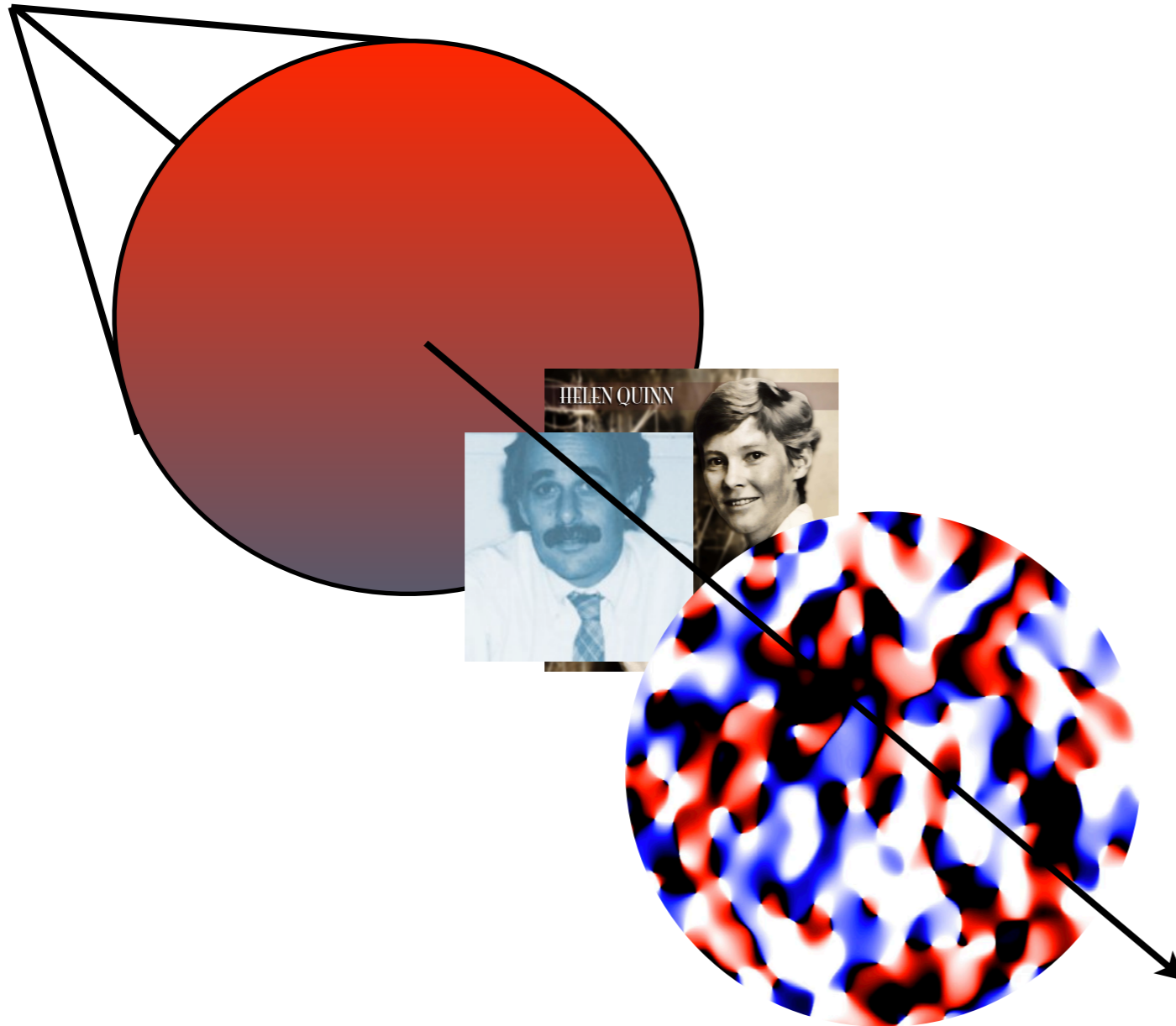
- PQ breaking before inflation



# Post inflationary PQ breaking scenario

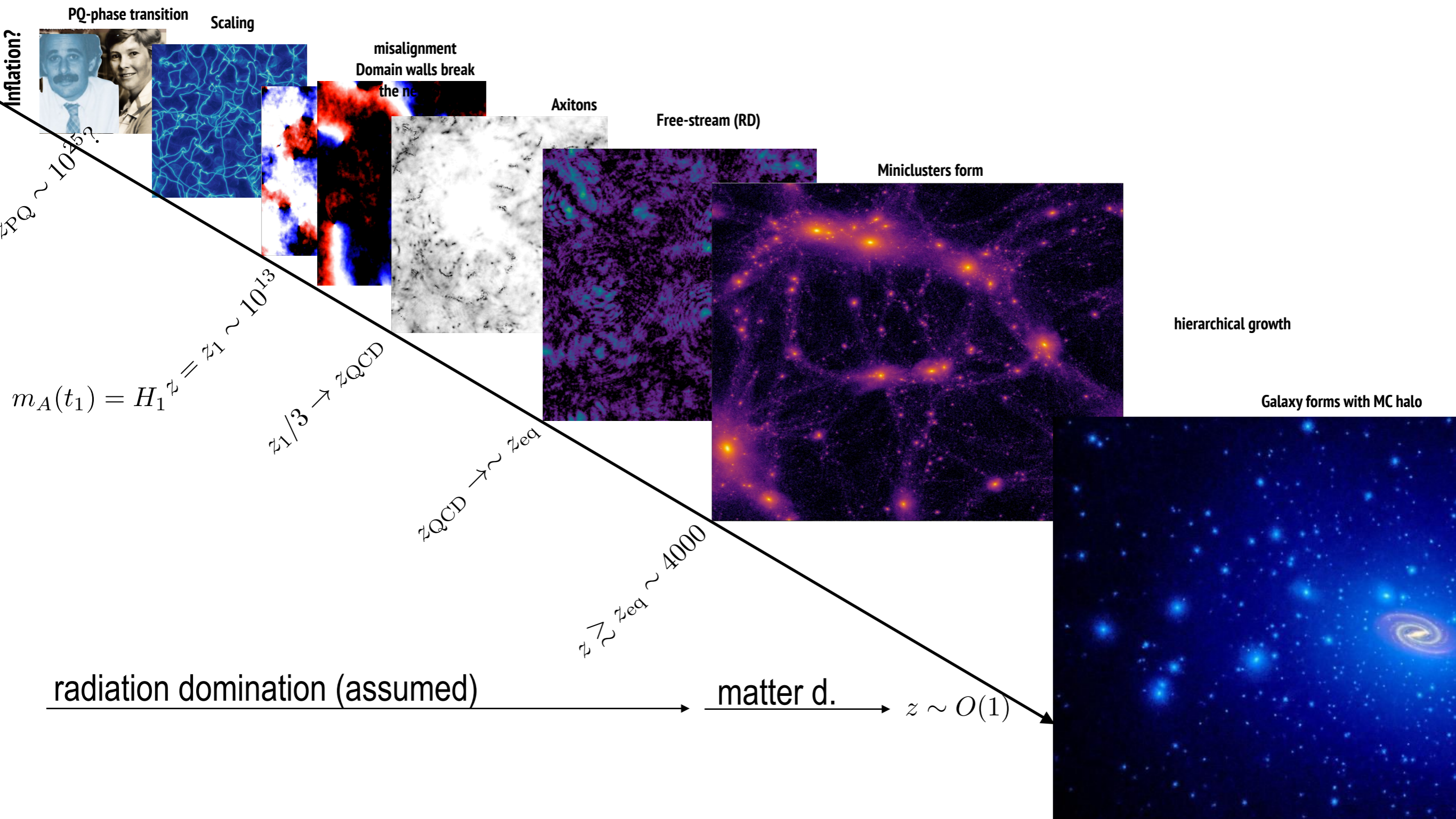
- PQ breaking after inflation

- No uncertainty over initial conditions
- All values of  $\theta_0$  realised in different early causally disconnected patches all inside our Universe today
- DM is inhomogeneous at small scales
- Small scale DM halos (Miniclusters)
- Problems with topological defects





# Post inflationary PQ breaking scenario



# axion dark matter abundance

- Misalignment contribution ... average over initial angles?

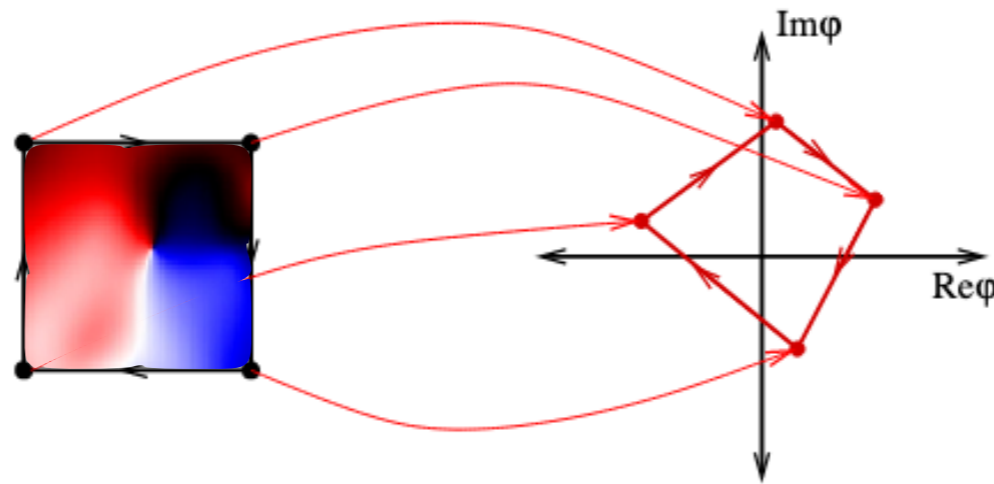
$$\rho_a(t) \simeq \theta_0^2 \chi(t_1) \frac{m_A}{m_A(t_1)} \left( \frac{R_1}{R(t)} \right)^3 \rightarrow \langle \theta_0^2 \rangle \chi(t_1) \frac{m_A}{m_A(t_1)} \left( \frac{R_1}{R(t)} \right)^3$$

this assumes exact zero mode, but correlation length is not infinite

- Most of the energy in the axion field is now NOT in the zero modes ...

At PQ phase transition a network of GLOBAL COSMIC STRINGS FORM! *Kibble 76*

Global string is a 1D region that is topologically (i.e. forced by boundary conditions) to take all values of  $\theta$



Formally, energy would be infinity, practice is regulated by the UV completion of axion theory

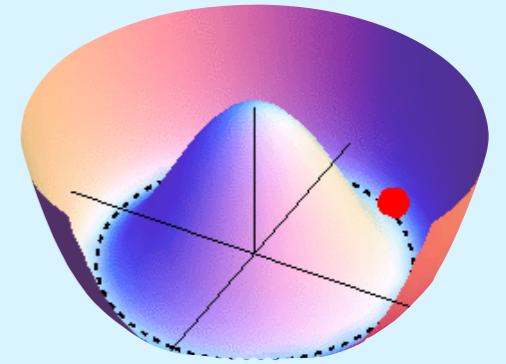


# Simple model KSVZ

- Peccei-Quinn symmetry, color anomalous, spontaneously broken at  $f_a$

$$\mathcal{L} \ni +i\bar{Q}DQ + \frac{1}{2}(\partial_\mu\phi)(\partial^\mu\phi^*) - (y\bar{Q}_L Q_R\phi + \text{h.c}) - \frac{1}{4}\lambda(|\phi|^2 - v^2)^2$$

$$\phi(x) = r(x)e^{ia(x)/f_A} \quad , \quad f_A = v \quad , \quad m_r = \sqrt{2\lambda}f_A$$



- At energies below  $f_a$  (SSB)

$$\mathcal{L} \in \frac{1}{2}(\partial a)^2 + \frac{\alpha_s}{8\pi} G\tilde{G} \frac{a}{f_a}$$

- At energies below  $\Lambda_{\text{QCD}}$ ,  $a - \eta' - \pi^0 - \eta - \dots$  mixing

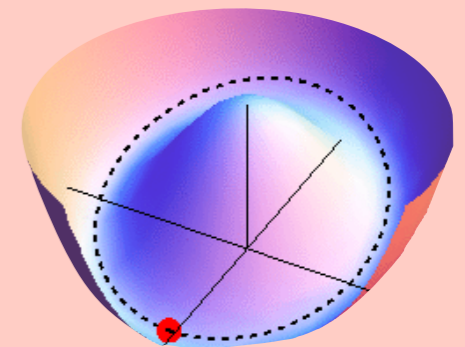
**axion mass**  $m_a \simeq \frac{m_\pi f_\pi}{f_a} \sim 6\text{meV} \frac{10^9\text{GeV}}{f_a}$

**couplings**  $\mathcal{L}_{a,I} = \sum_N c_{N,a} \bar{N}\gamma^\mu\gamma_5 N \frac{a}{f_a} + c_{a\gamma} \frac{\alpha}{2\pi} F_{\mu\nu}\tilde{F}^{\mu\nu} \frac{a}{f_a} + \dots$

nucleons ...

photons ...

mesons ...



ENERGY  $\sim fa$   $\sim \text{GeV}$

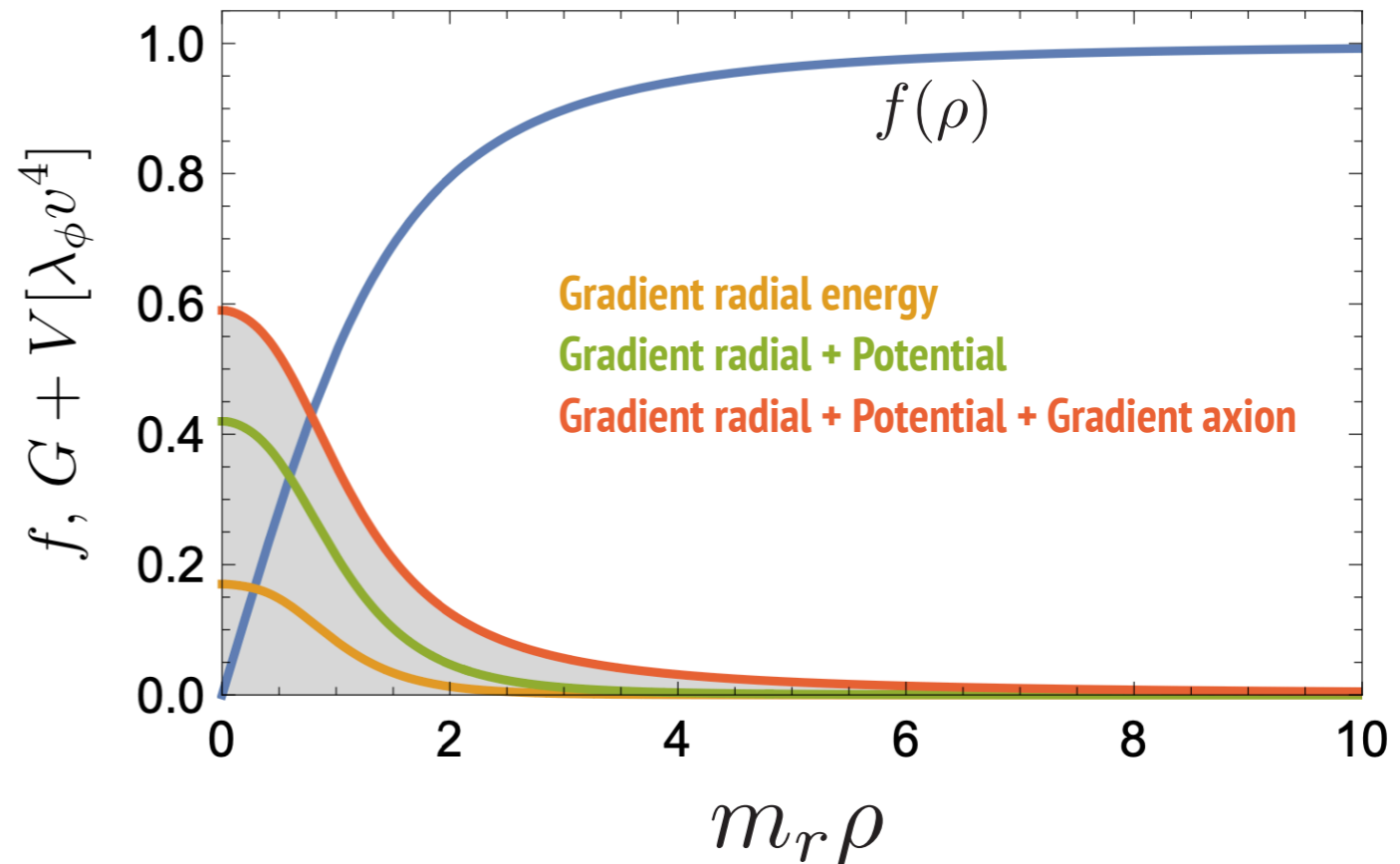
# Global strings solution

- Force  $\phi = v f(\rho) e^{i\varphi}$  ( $\rho, \varphi, z$ , cylindrical coordinates)

i.e.  $\theta$  wraps  $2\pi$  around the origin ...

- Static EOM for the radial field

$$\left[ \frac{f_\rho + \rho f_{\rho\rho}}{\rho} - \frac{f}{\rho^2} \right] = \lambda_\phi v^2 f (f^2 - 1)$$



- Energy/length (string tension)

$$\mu \sim 2\pi v^2 \int \rho d\rho \left( f_\rho^2 + m_r^2 (f^2 - 1)^2 + \frac{f^2}{\rho^2} \right) \sim v^2 \left( 2.6 + \pi \log \frac{m_r \rho}{1.55} \right)$$

logarithmic divergence from  $\frac{1}{\rho} \frac{\partial \phi}{\partial \varphi}$



# Global string network

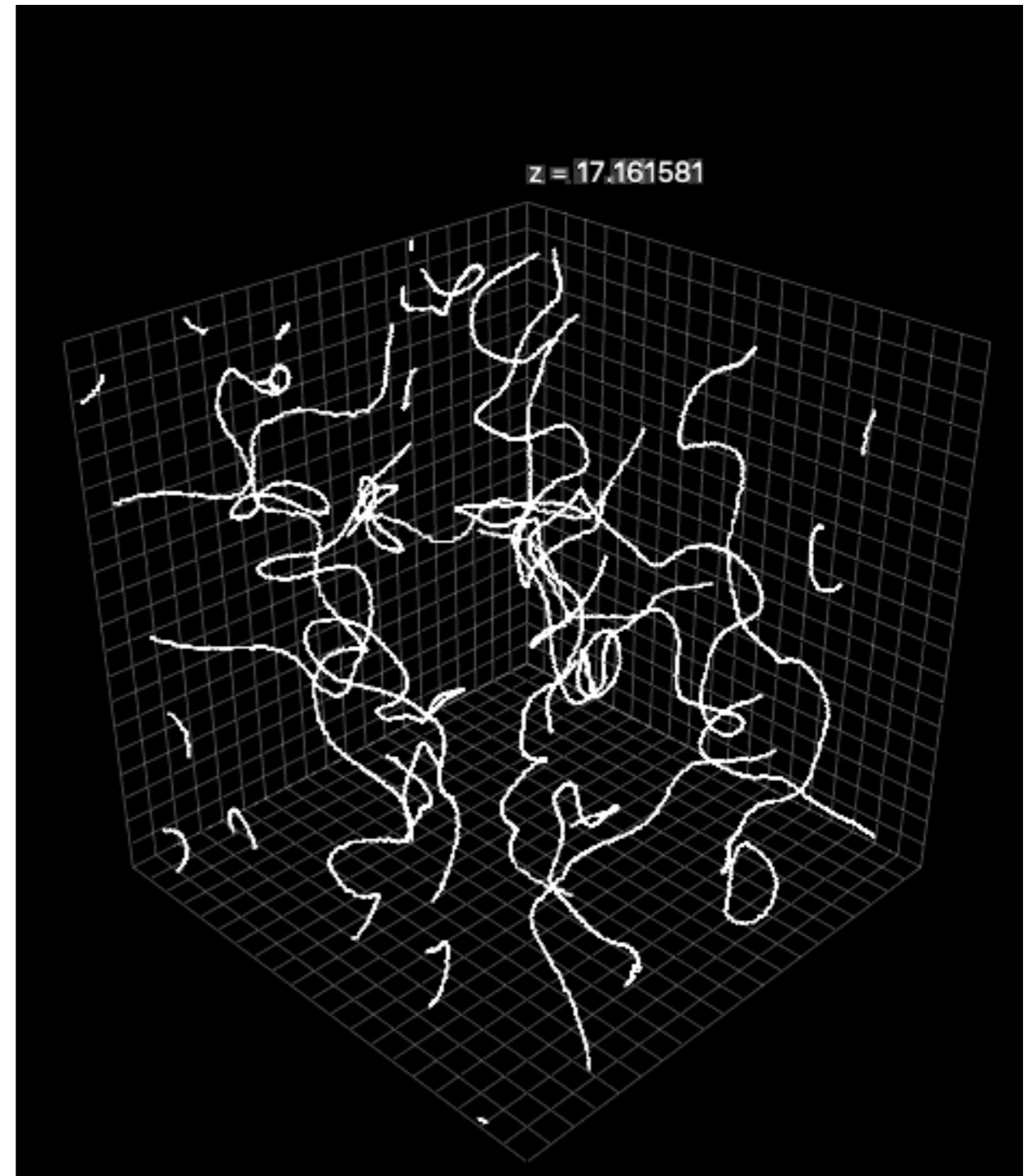
- Strings have no end, form loops
- Dynamics:
  - Strings straighten, intersect, loops collapse
  - Energy released in GWs, axions (mostly)
  - relativistic speeds
  - limited by causality
  - scaling solution

$$\frac{\text{string length}}{\text{causal volume}} \sim \frac{d_H}{d_H^3}$$

- energy density of network

$$\frac{\mu d_H}{d_H^3} \sim \mu H^2 \sim f_A^2 H^2 \log \frac{m_r}{H}$$

typical distance between strings  $\rho \sim d_H = 1/H$



# axion dark matter abundance at $t_1$

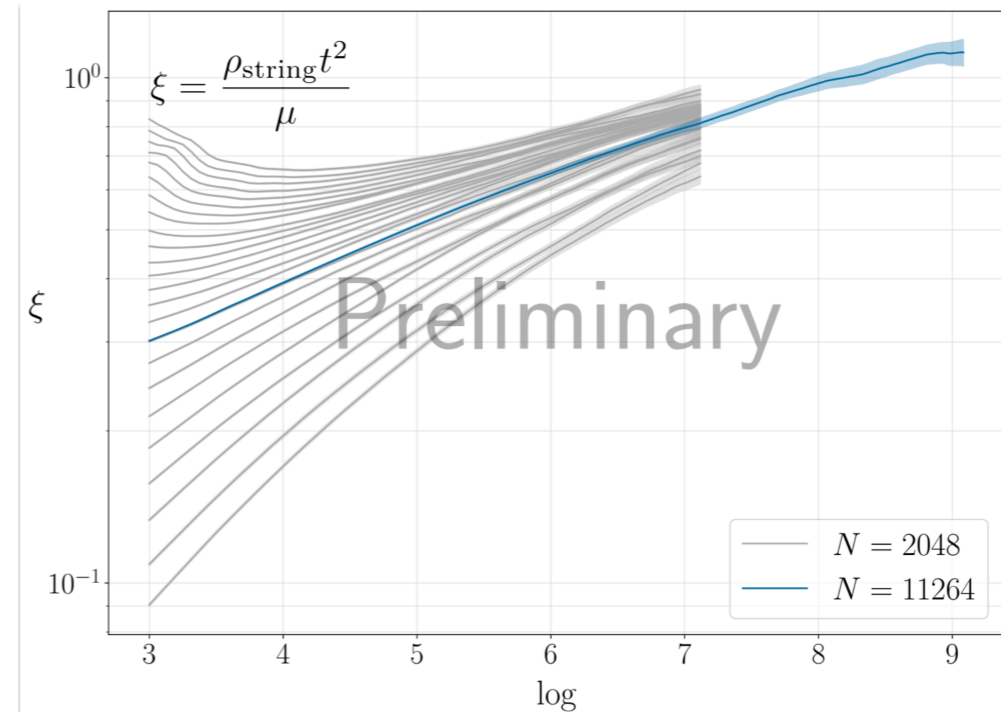
- Misalignment contribution...  $H_1 = m_A(t_1)$

$$\rho(t_1) \sim f_A^2 H_1^2, \quad n_1 \sim f_A^2 H_1$$

- Most of the energy in the axion field is in axion GRADIENTS around the strings

$$\rho_s \sim f_A^2 H^2 \log \frac{m_r}{H} \times \xi$$

$$\xi = \frac{\text{string length}}{\text{causal volume}} = \frac{d_H}{d_H^3}$$



Saikawa 23

density doubly log enhanced... xi grows logarithmically

**Axions emitted from cosmic strings can easily supersede misalignment average!!!**

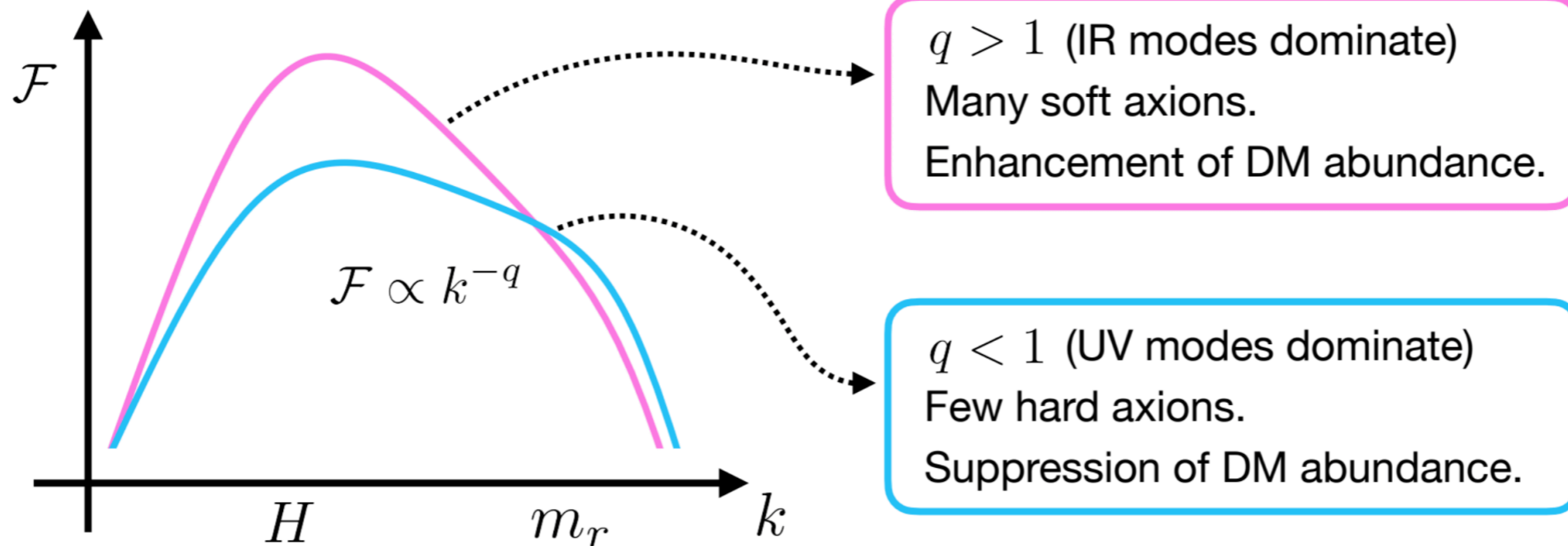


# axions from strings

- The AXION NUMBER radiated by strings depends on the AVERAGE RADIATED ENERGY

Differential energy transfer rate

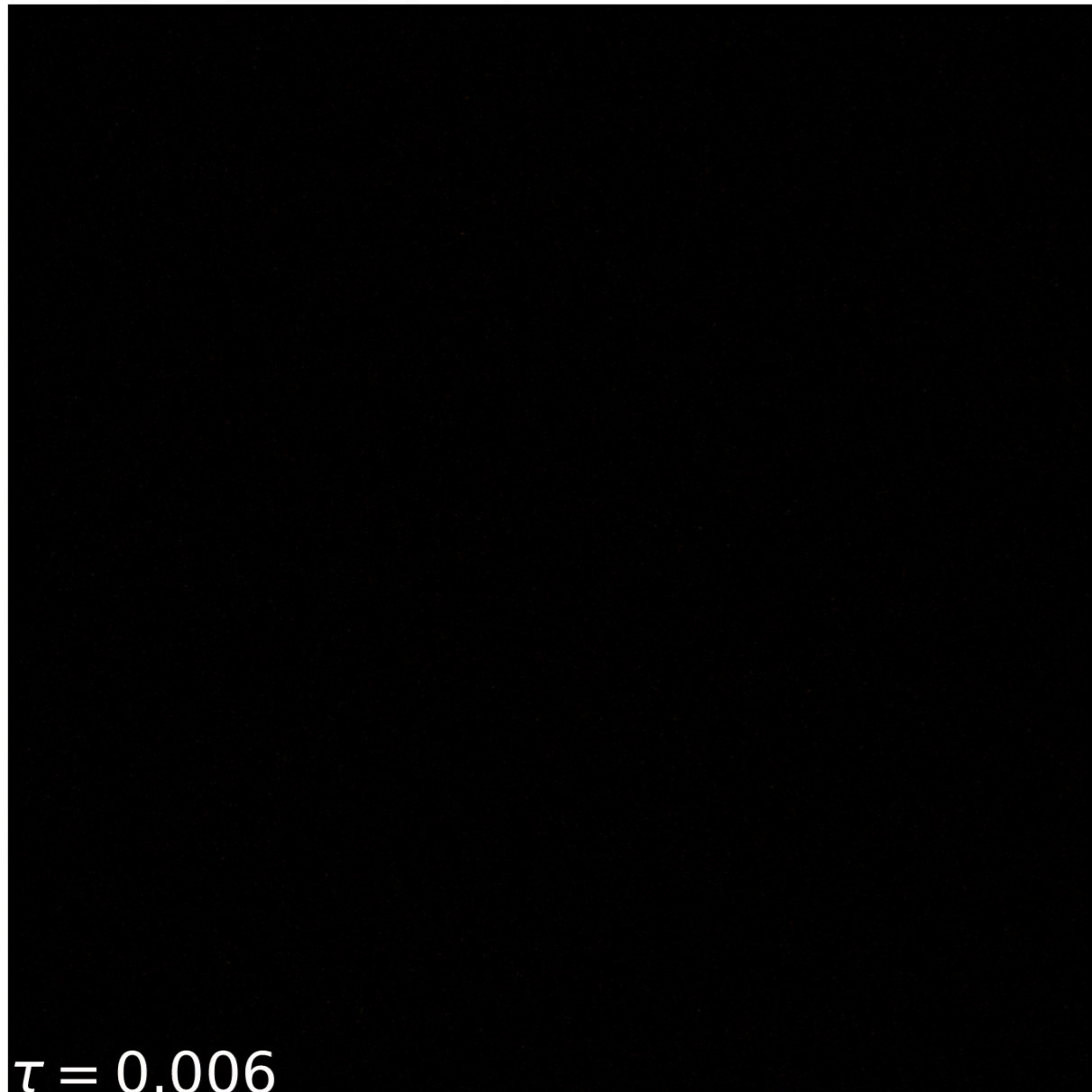
$$\mathcal{F} \left( \frac{k}{RH}, \frac{m_r}{H} \right) \equiv \frac{1}{(f_a H)^2} \frac{1}{R^3} \frac{\partial}{\partial t} \left( R^4 \frac{\partial \rho_a}{\partial k} \right)$$



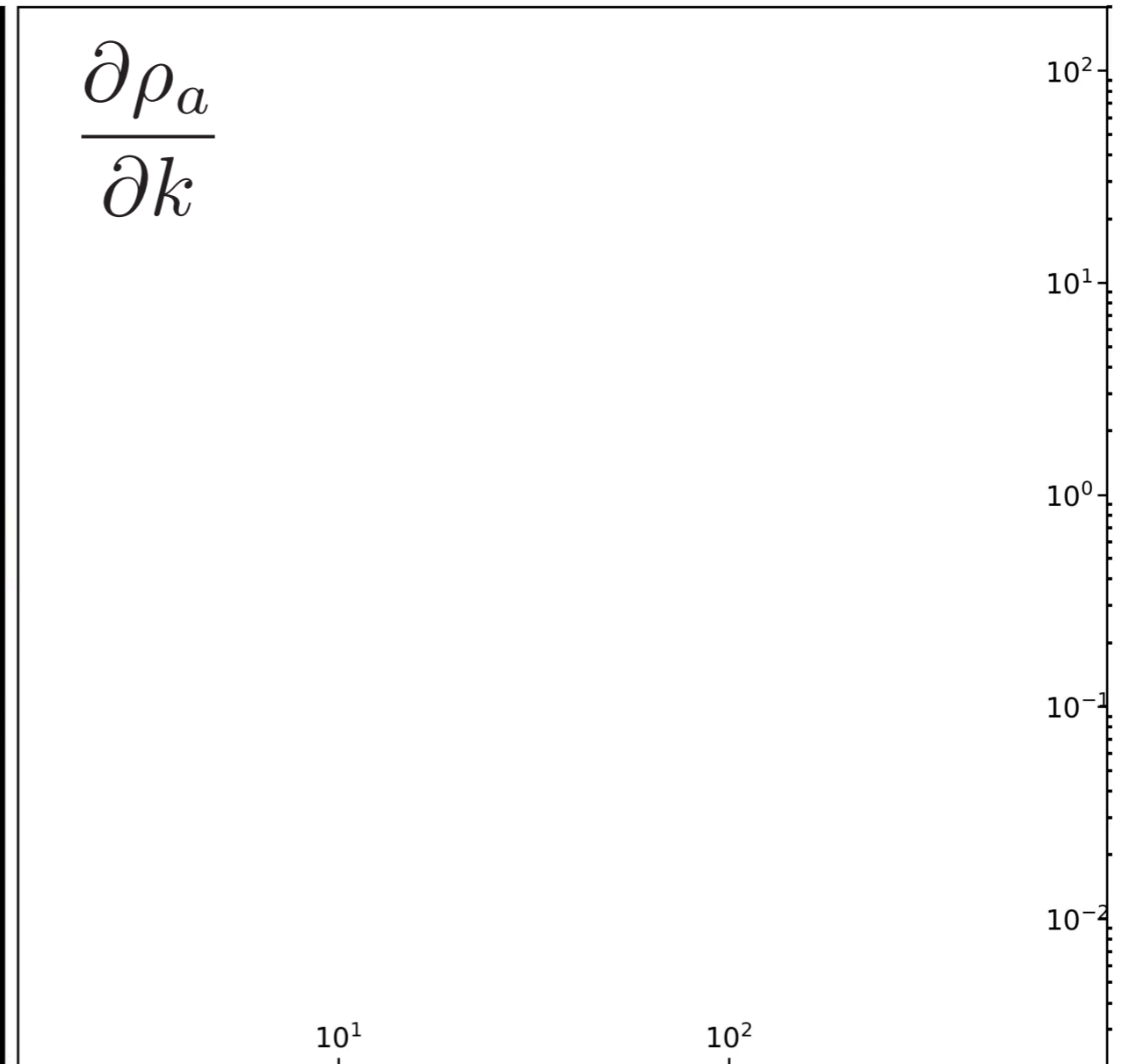
# numerical simulations

- Simulate complex scalar field in  $N^3$  cartesian grids ( $N \sim 11000$ )
- Calculate axion spectrum at several times
- Calculate time derivative and measure  $q$

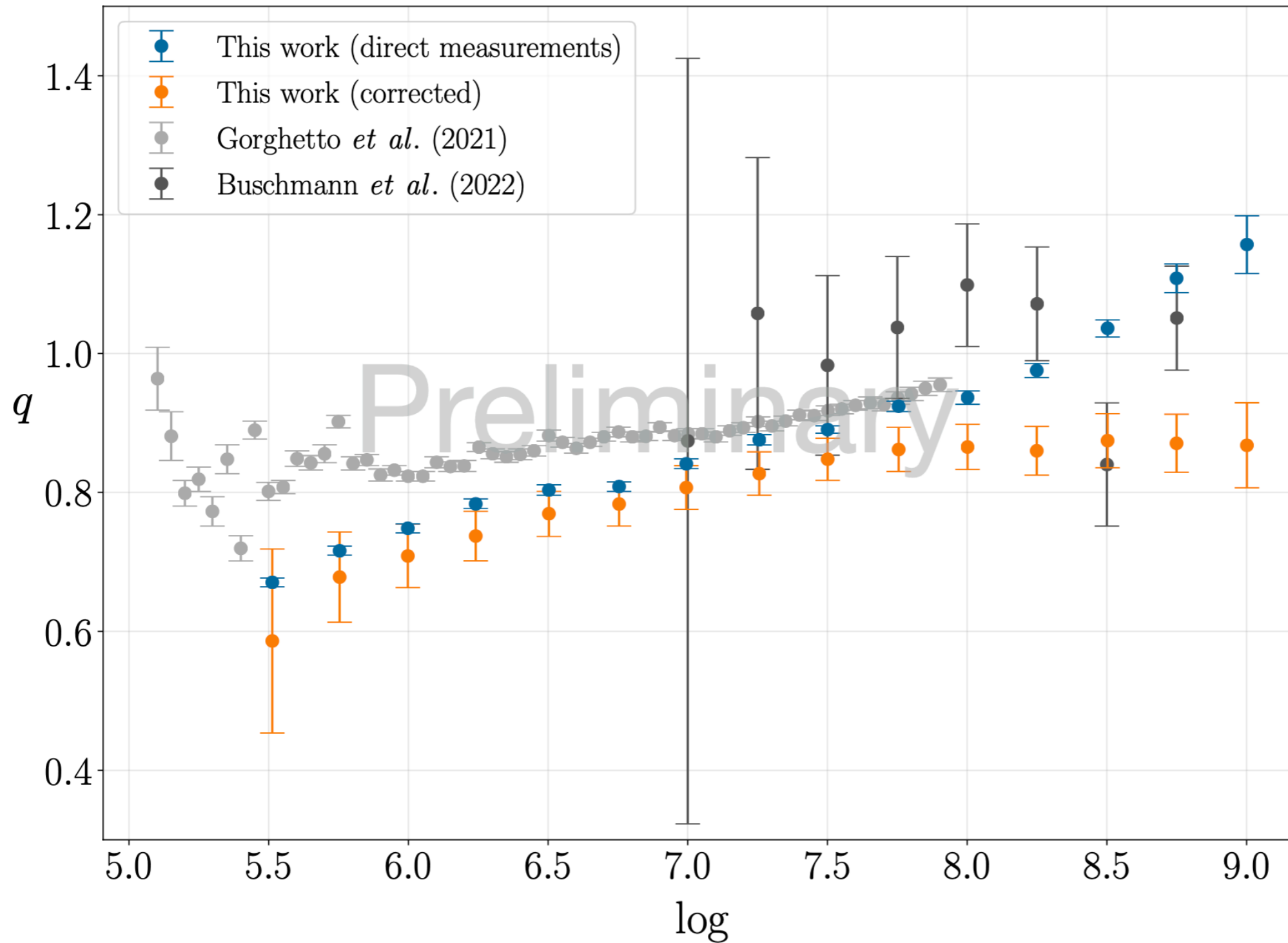
*Energy density projection plot*



*Spectrum (PRS strings)*



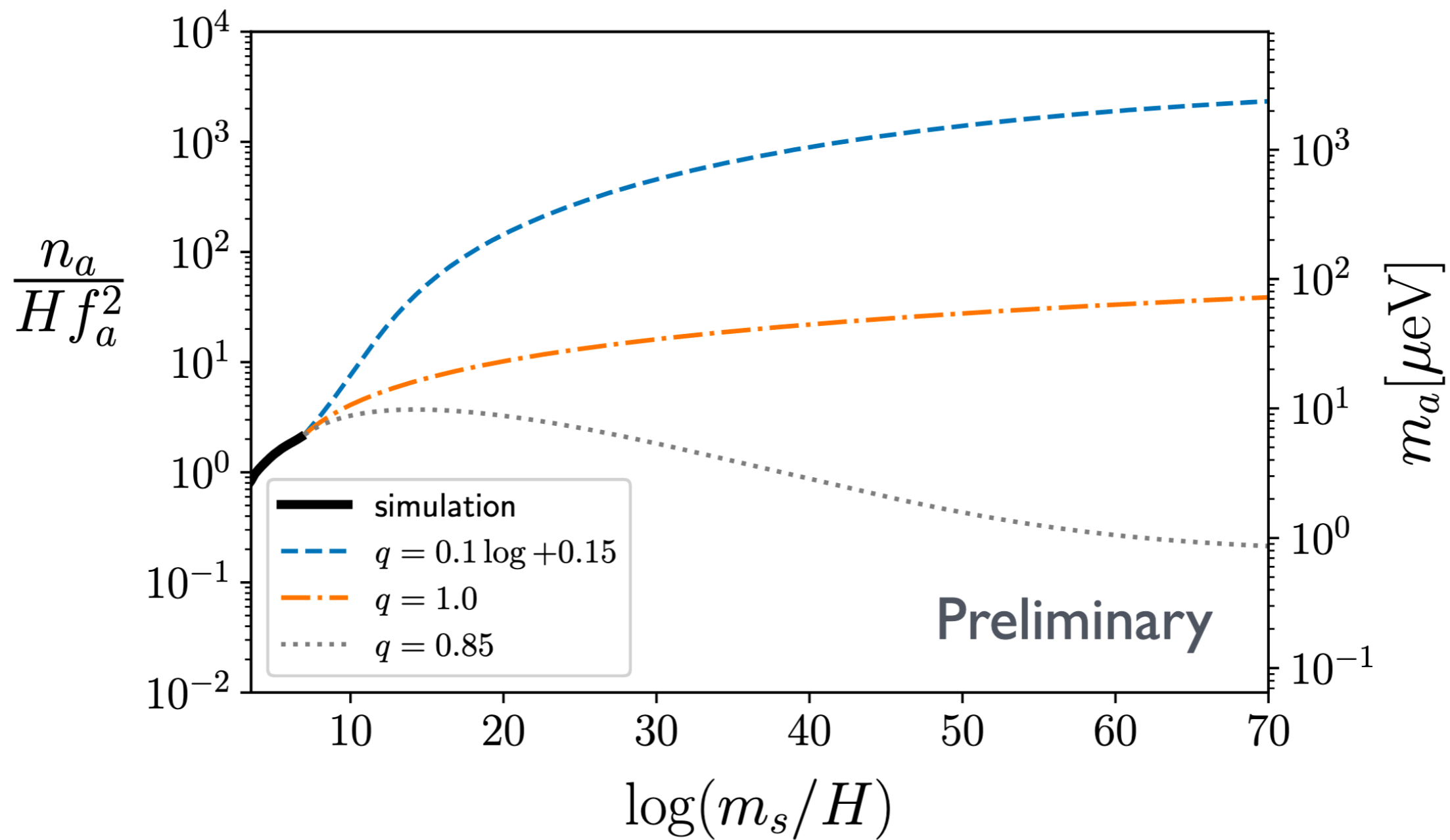
# state of the art numerical simulations



Saikawa 23

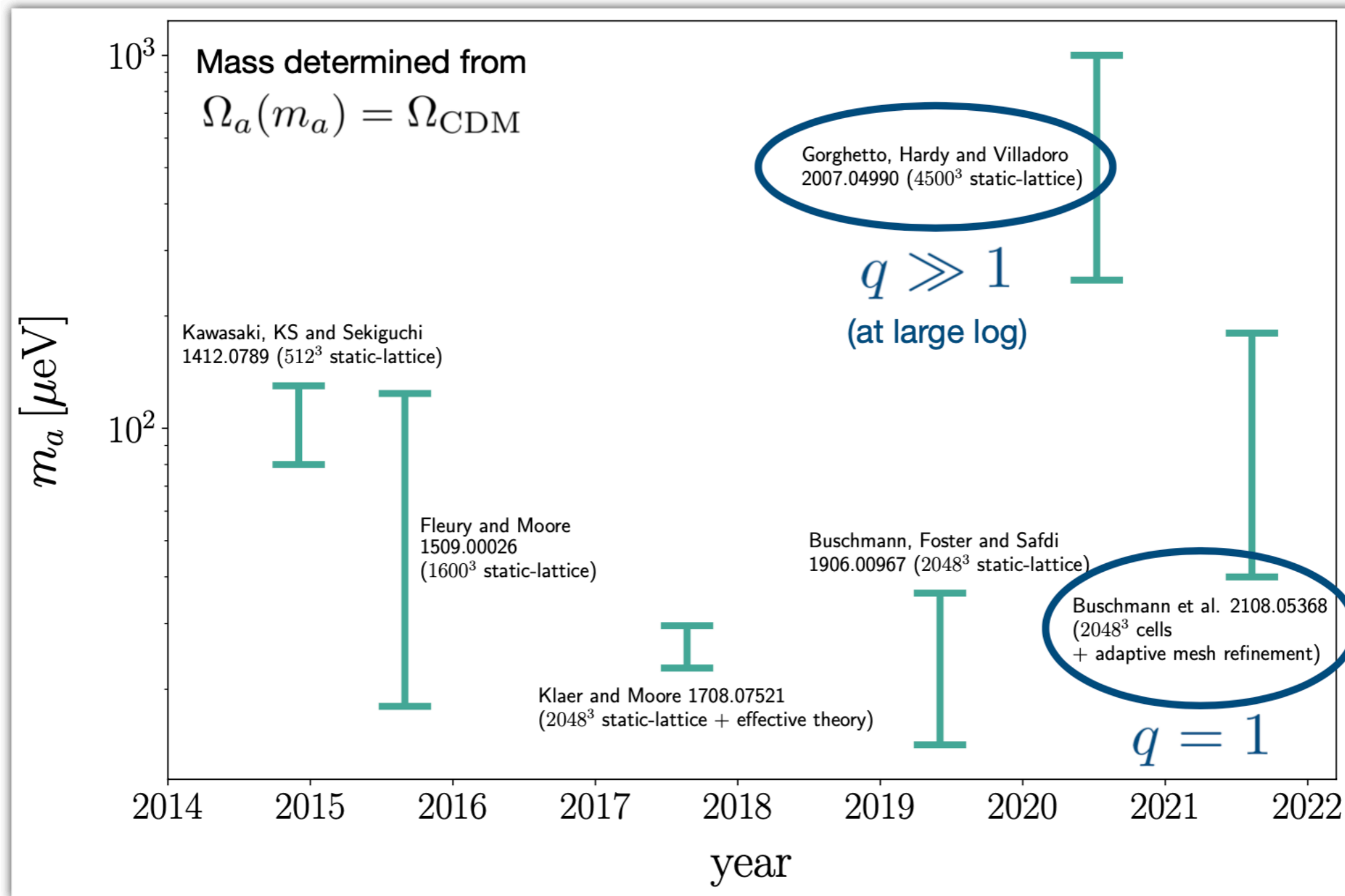


# extrapolation



# axion DM mass

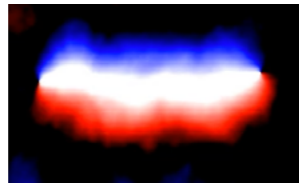
- For which value of  $m_A$  (fA) do we obtain the observed DM abundance in the post-inflationary scenario?



Saikawa 23

# Domain walls and the end of the network

- The scaling solution continues as long as the U(1) is a good symmetry
- Axion potential becomes relevant around  $t_1$  to favour  $\theta=0$
- Domain walls form attached to the strings



- Regions of  $\theta \sim \pi$ , trapped by boundary conditions  
( $\theta$  wraps  $2\pi$  around the string)

- Form membranes attached to strings

- Energy / Area  $\sim \frac{\chi \times A \times m_A^{-1}}{A} \sim m_A f_A^2$

- All DW energy density (1 DW per Horizon)

$$\rho_{\text{DW}} \simeq \frac{m_A f_A^2 d_H^2}{d_H^3} \sim m_A H f_A^2$$

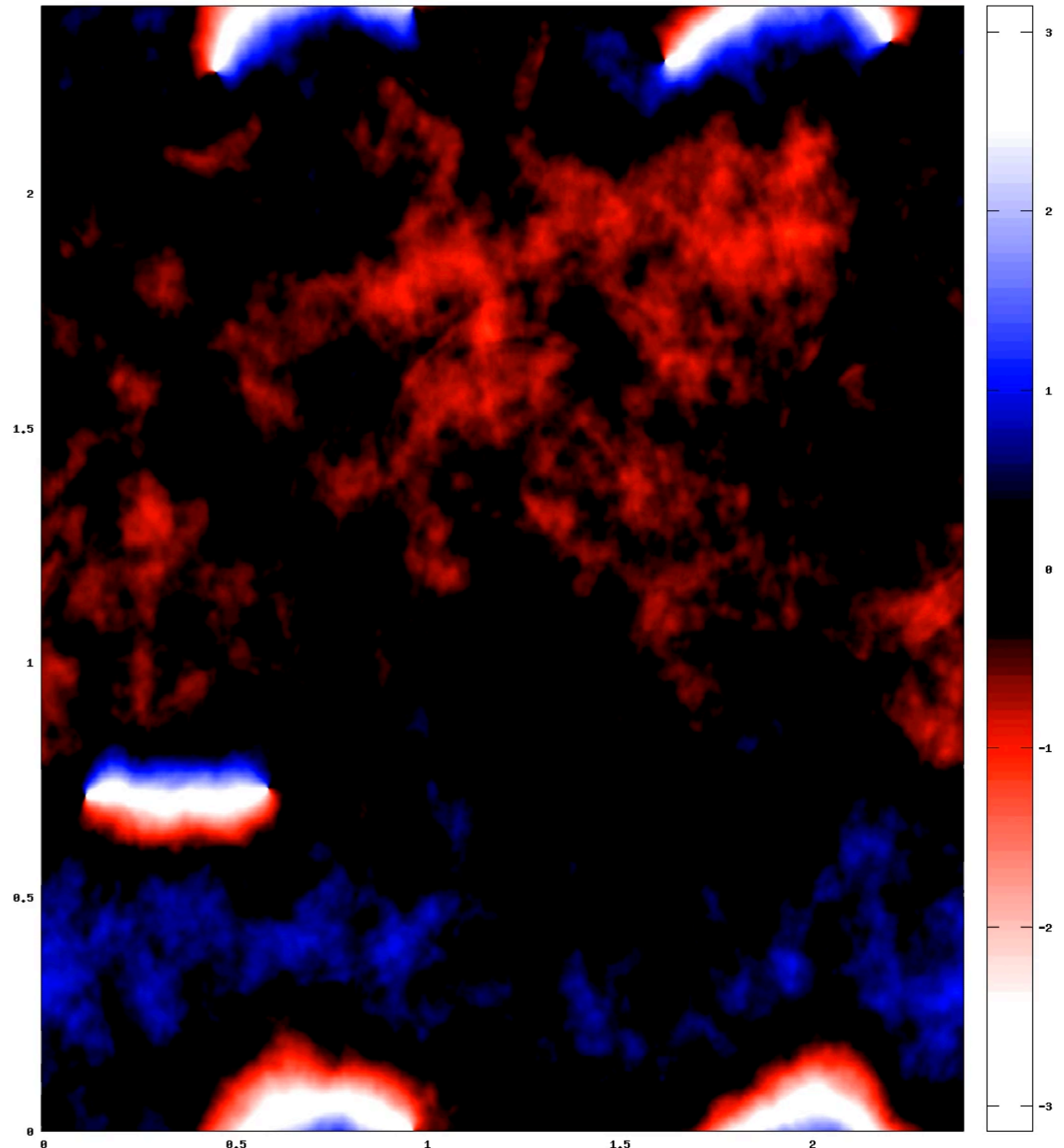
- DWs VS strings ...

$$\rho_s \sim H^2 f_A^2 \log^2$$

$$\rho_{\text{DW}} > \rho_s \rightarrow \frac{m_A}{H} \sim (t/t_1)^{n/4+1} \gg \log^2$$

$$\rho_{\text{DW}} \sim \rho_s \rightarrow \frac{m_A}{H} \sim (t/t_1)^{n/4+1} \sim \log^2$$

shortly after  $t_1$ , they dominate and pull the strings to destruction





# Domain Wall problem

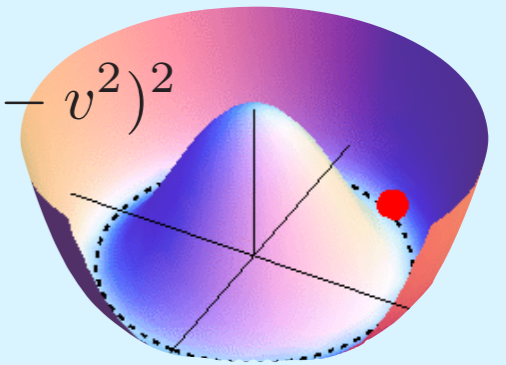
- All said is fair for the simplest KSVZ model with exactly 1 extra quark
- When  $N > 1$  the situation is VERY different

- Peccei-Quinn symmetry, color anomalous, spontaneously broken at  $f_a$

$$\mathcal{L} \ni +i \sum_Q \bar{Q} D Q + \frac{1}{2} (\partial_\mu \phi) (\partial^\mu \phi^*) - \sum_Q (y_Q \bar{Q}_L Q_R \phi + \text{h.c.}) - \frac{1}{4} \lambda (|\phi|^2 - v^2)^2$$

$$\phi(x) = r(x) e^{ia(x)/f_A}, \quad f_A = v, \quad m_r = \sqrt{2\lambda} f_A$$

physically  $\frac{a}{f_A} \equiv \theta \in [0, 2\pi), [0] \equiv [2\pi n]$  is the angle of a complex scalar !



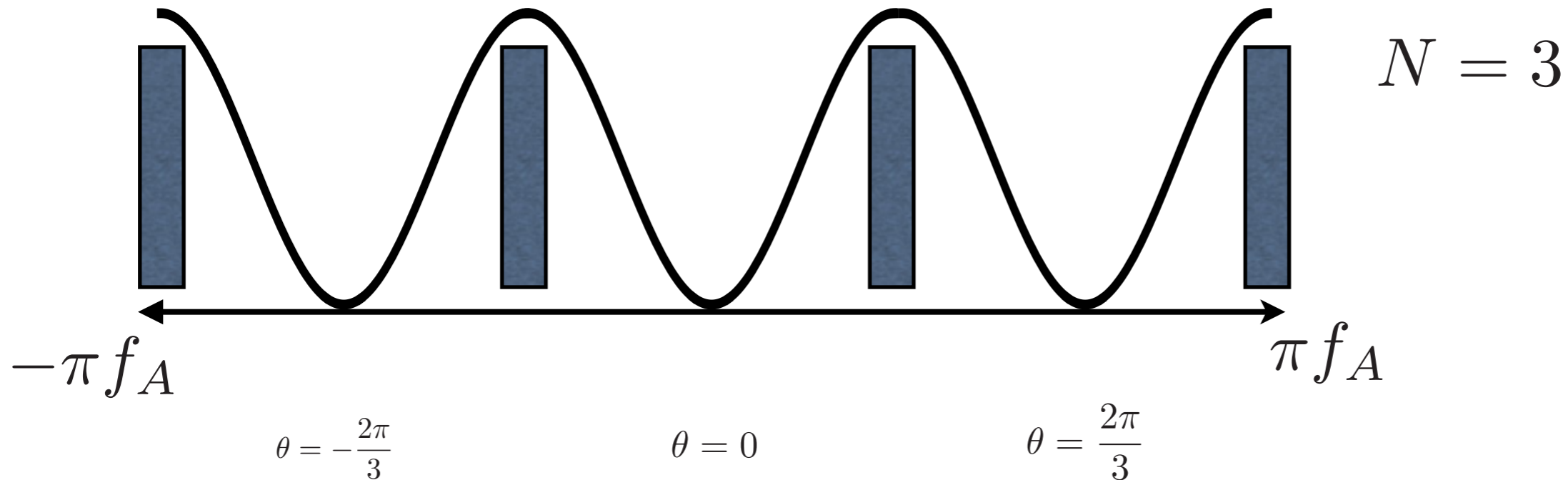
- At energies below  $f_a$  (SSB)  $\mathcal{L} \in \frac{1}{2} (\partial a)^2 + \frac{\alpha_s}{8\pi} G \tilde{G} \frac{a}{f_a} \times N$

$$V(\theta) = \chi_T (1 - \cos(N\theta))$$

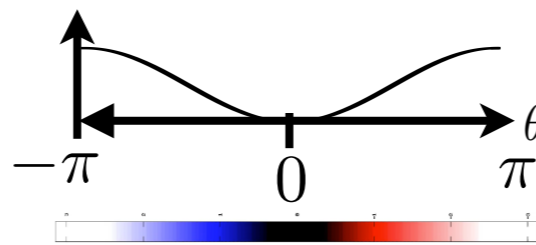
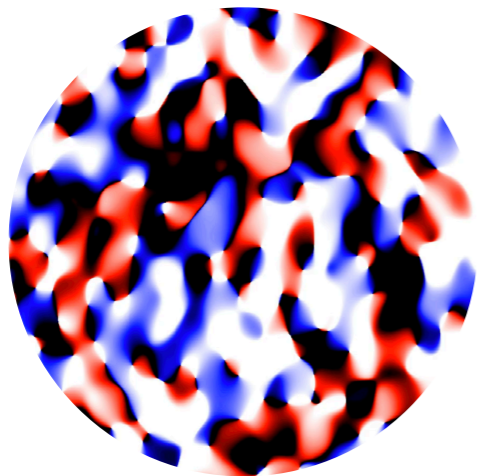
~GeV  
~f<sub>a</sub>

# Domain wall problem

- The physical field  $\theta$  can experience  $N$  times the QCD potential
- There are  $N$  degenerate CP conserving vacua
- There are  $N$  domain walls separating these physically different vacua!



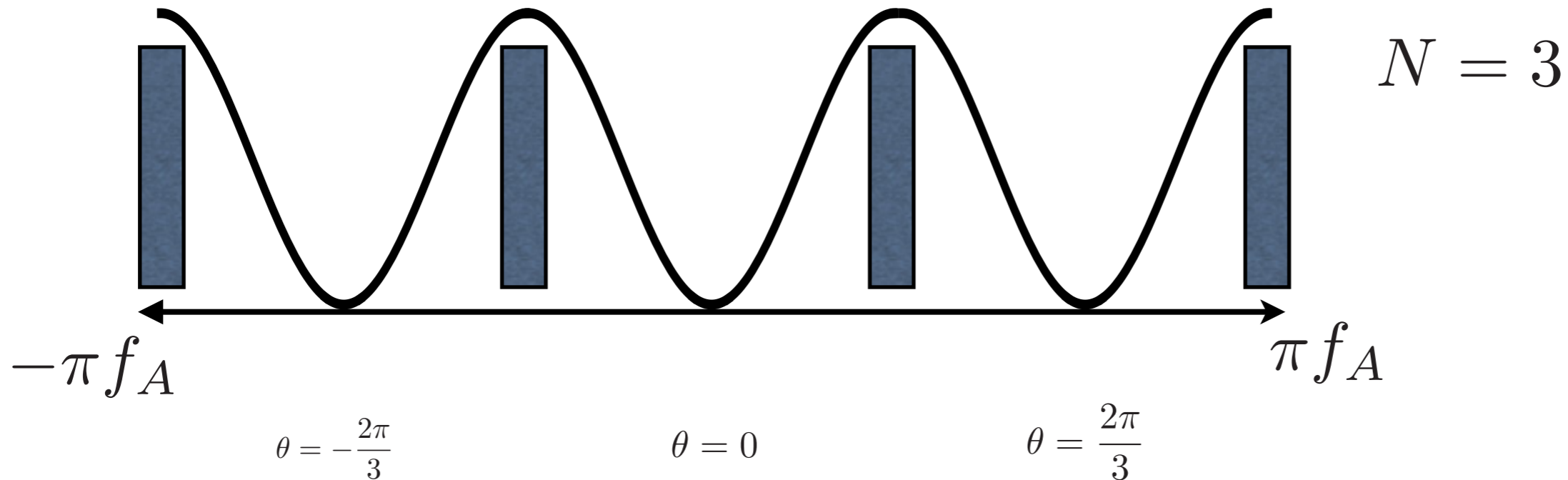
- All these values are populated after the PQ phase transition



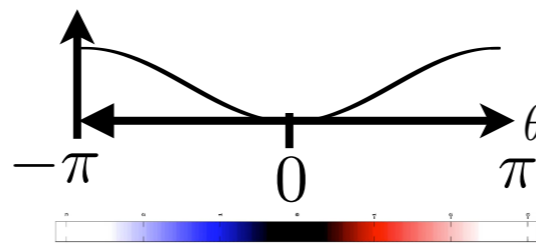
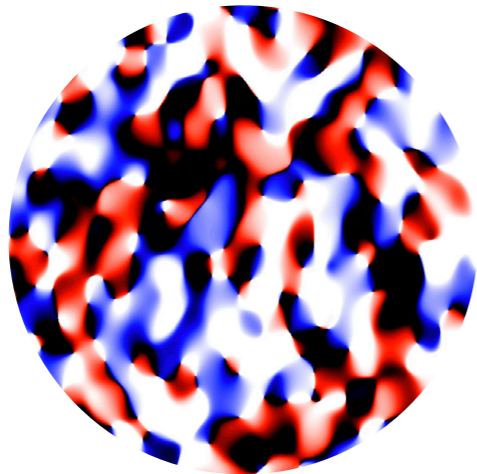
- But now cosmic strings have no preferred direction to collapse at  $t_2$ , no preferred vacuum!

# Domain wall problem

- The physical field  $\theta$  can experience  $N$  times the QCD potential
- There are  $N$  degenerate CP conserving vacua
- There are  $N$  domain walls separating these physically different vacua!



- All these values are populated after the PQ phase transition



- But now cosmic strings have no preferred direction to collapse at  $t_2$ , no preferred vacuum!



# Simulations $N=2$

**movie!**

# Domain wall problem

- String network cannot collapse
- Energy is dominated by domain walls

$$\rho_{\text{DW}} \simeq \frac{m_A f_A^2 d_H^2}{d_H^3} \sim m_A H f_A^2$$

$$\rho_{\text{DW}} \sim \frac{1}{t} \sim \frac{1}{R^2}$$

**Radiation domination**

$$\rho_{\text{DW}} \sim \frac{1}{t} \sim \frac{1}{R^{3/2}}$$

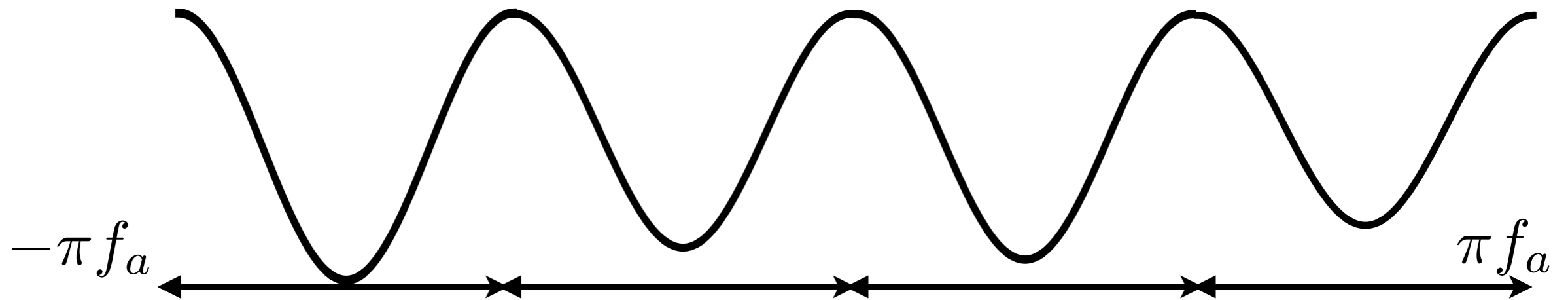
**Matter domination**

- Energy density in the DW network redshifts much slower than Rad or Matter and quickly dominates the Universe... (the problem is that we do not observe it!)

# Domain wall problem ... solutions

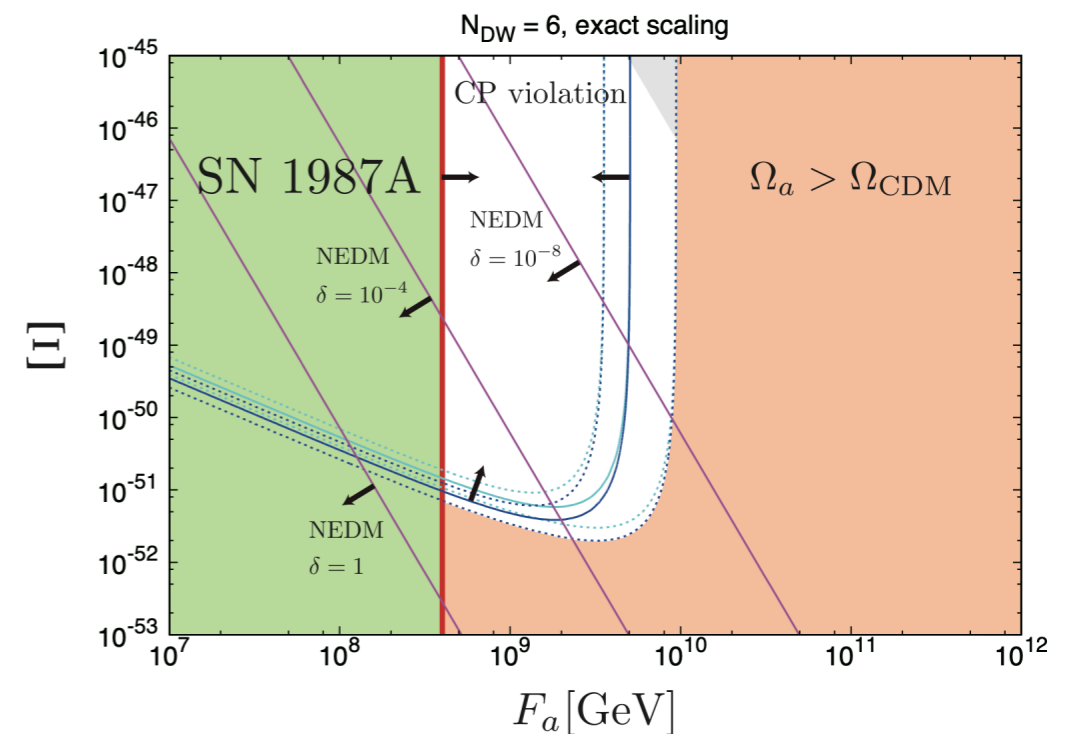
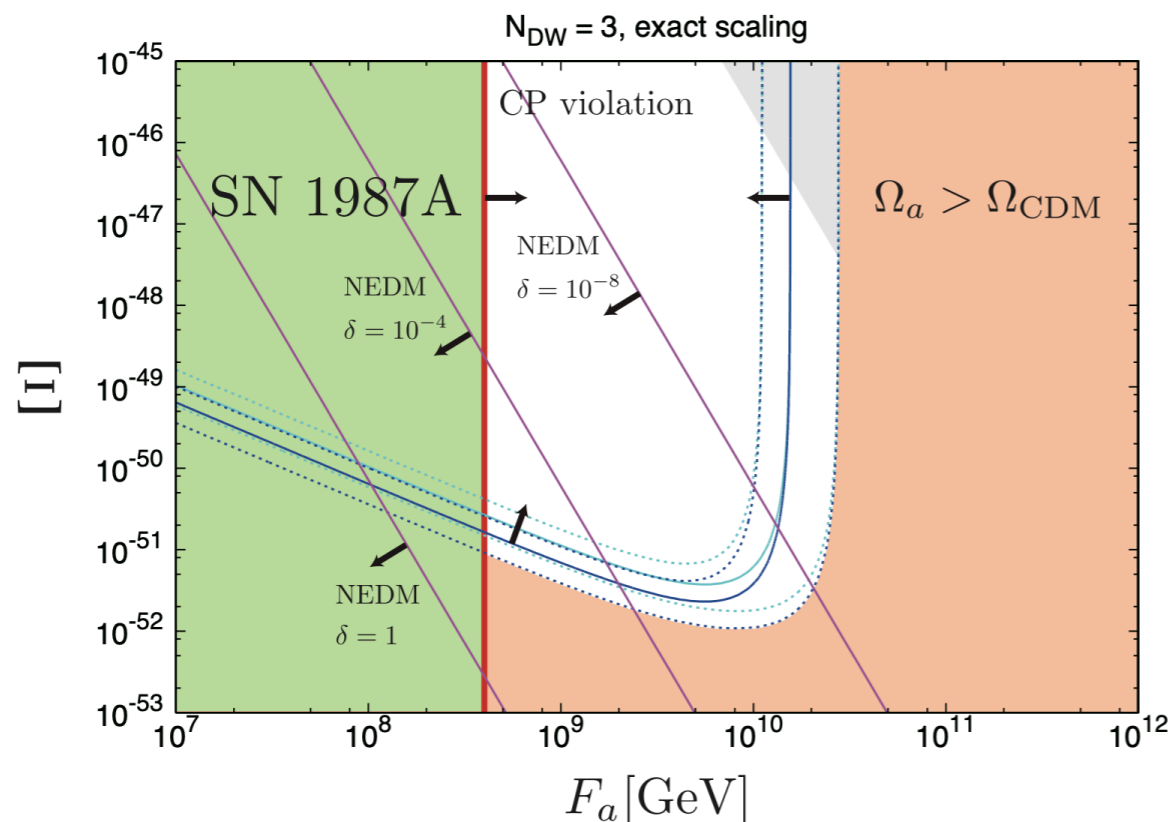
- Break degeneracy between vacua (break explicitly PQ... expected to some extent)

$$V_{\text{bias}}(\Phi) = -\Xi\eta^3 (\Phi e^{-i\delta} + \text{h.c.})$$



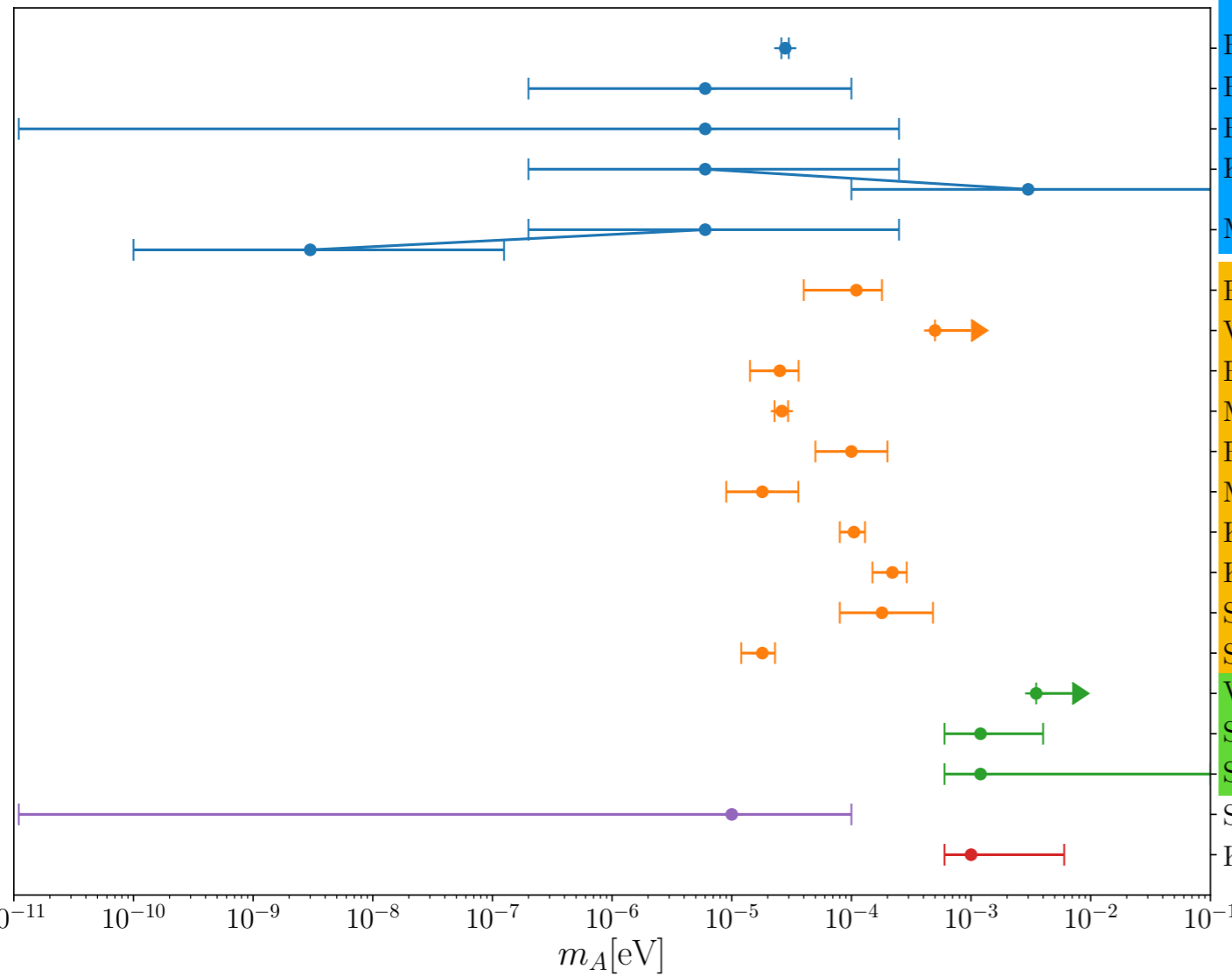
- But watch out not to spoil axion solution to strong CP problem (or overproduce DM)

Saikawa 2014





# Axion DM mass



RD  $\theta_0 = 2.155$

RD  $\theta_0 \in (0.1, 3)$

RD  $\theta_0 \in (10^{-3}, 3.14)$

Kination  $\theta_0 \in (0.1, 3.0), T_{RH} \in (5, 5 \times 10^3)$  MeV

Matter Decay  $\theta_0 \in (0.1, 3.0), T_{RH} \in (5, 5 \times 10^3)$  MeV

**PREINFLATION  
SCENARIO**

Buschmann 2021

Villadoro 2020

Buschmann 2019

Moore 2017

Redondo 2016

Moore 2015

Kawasaki 2014

Kawasaki 2012

Shellard 2010

Sikivie 2000

Villadoro 2020 ( $N_{DW} = 6$ )

Saikawa 2015 ( $N_{DW} = 10$ )

Saikawa 2015 ( $N_{DW} = 9$ )

Stochastic Graham 2018

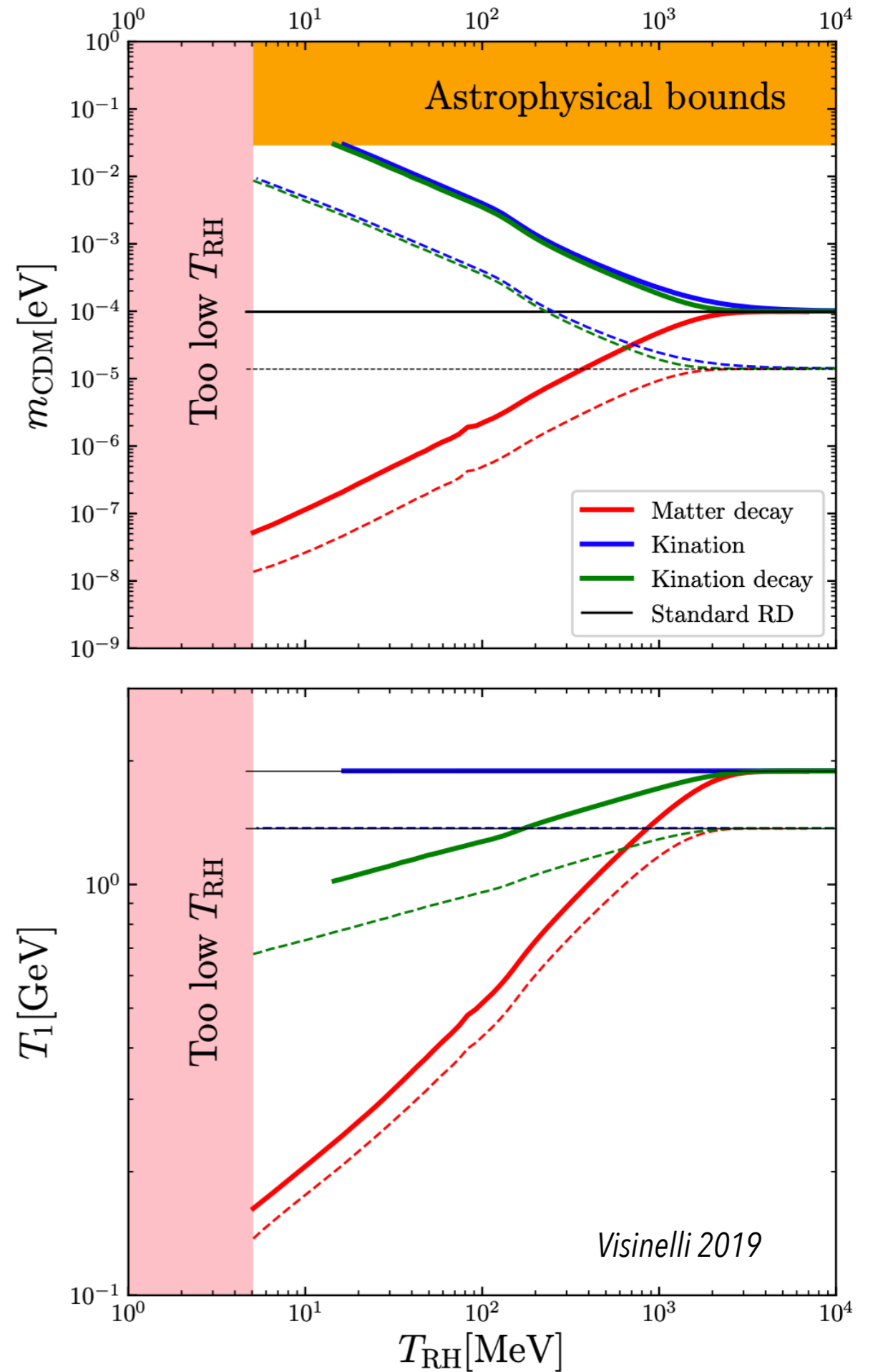
Kinetic mis/Parametric resonance Co 2020

**POSTINFLATION  
SCENARIO N=1**

**POSTINFLATION N>1**

# Alternative cosmologies

- Domination of other fluids at  $t_1$ , changes dilution factors



# Axion miniclusters

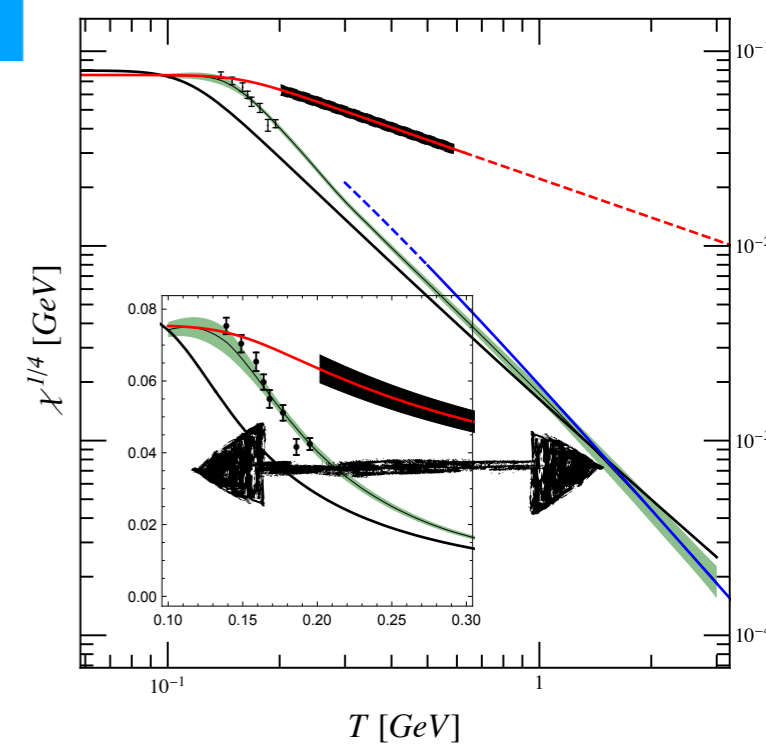
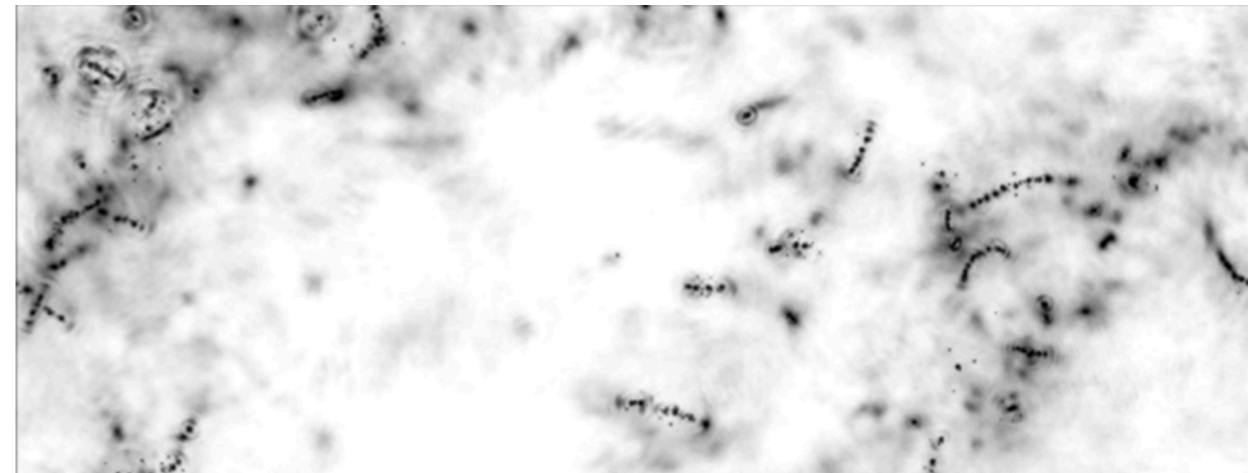
- Shortly after  $t_2$  "ZERO" mode axions ( $H_1$ ) are extremely non-relativistic

( $m_a$  continues to grow after  $H_1$  by a factor of  $10^4$  or so ...)

- The density field essentially freezes

\*Some non-linearities appear at very dense regions (axitons)

axions have attractive self-interactions  
 recurrently collapse and explode as bose-novas  
 until the density is low enough



- The density field essentially freezes but it has  $O(1)$  inhomogeneities at  $\sim L_1$  distances

$$L_1 \equiv \frac{1}{H_1 R_1} \simeq 1.116 \times 10^{17} \text{cm} \left( \frac{50 \mu\text{eV}}{m_a} \right)^{0.167} = 0.0362 \text{pc} \left( \frac{50 \mu\text{eV}}{m_a} \right)^{0.167}$$

# Gravitational collapse

## - $\mathcal{O}(1)$ overdensities collapse gravitationally very early

PHYSICAL REVIEW D

VOLUME 50, NUMBER 2

15 JULY 1994

### Large-amplitude isothermal fluctuations and high-density dark-matter clumps

Edward W. Kolb\*

NASA/Fermilab Astrophysics Center, Fermi National Accelerator Laboratory, Batavia, Illinois 60510  
and Department of Astronomy and Astrophysics, Enrico Fermi Institute, The University of Chicago, Chicago, Illinois 60637

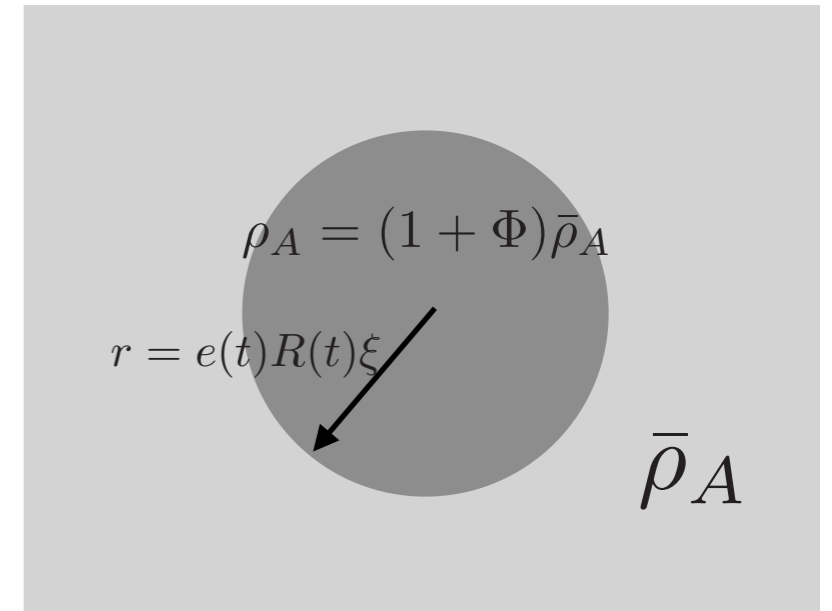
Igor I. Tkachev†

NASA/Fermilab Astrophysics Center, Fermi National Accelerator Laboratory, Batavia Illinois 60510  
and Institute for Nuclear Research of the Academy of Sciences of Russia, Moscow 117312, Russia

(Received 9 March 1994)

Large-amplitude isothermal fluctuations in the dark-matter energy density, parametrized by  $\Phi \equiv \delta\rho_{\text{DM}}/\rho_{\text{DM}}$ , are studied within the framework of a spherical collapse model. For  $\Phi \gtrsim 1$ , a fluctuation collapses in the radiation-dominated epoch and produces a dense dark-matter object. The final density of the virialized object is found to be  $\rho_F \approx 140\Phi^3(\Phi+1)\rho_{\text{eq}}$ , where  $\rho_{\text{eq}}$  is the matter density at equal matter and radiation energy density. This expression is valid for the entire range of possible values of  $\Phi$ , both for  $\Phi \gg 1$  and  $\Phi \ll 1$ . Some astrophysical consequences of high-density dark-matter clumps are discussed.

PACS number(s): 98.80.Cq, 05.30.Jp, 95.35.+d, 98.70.-f



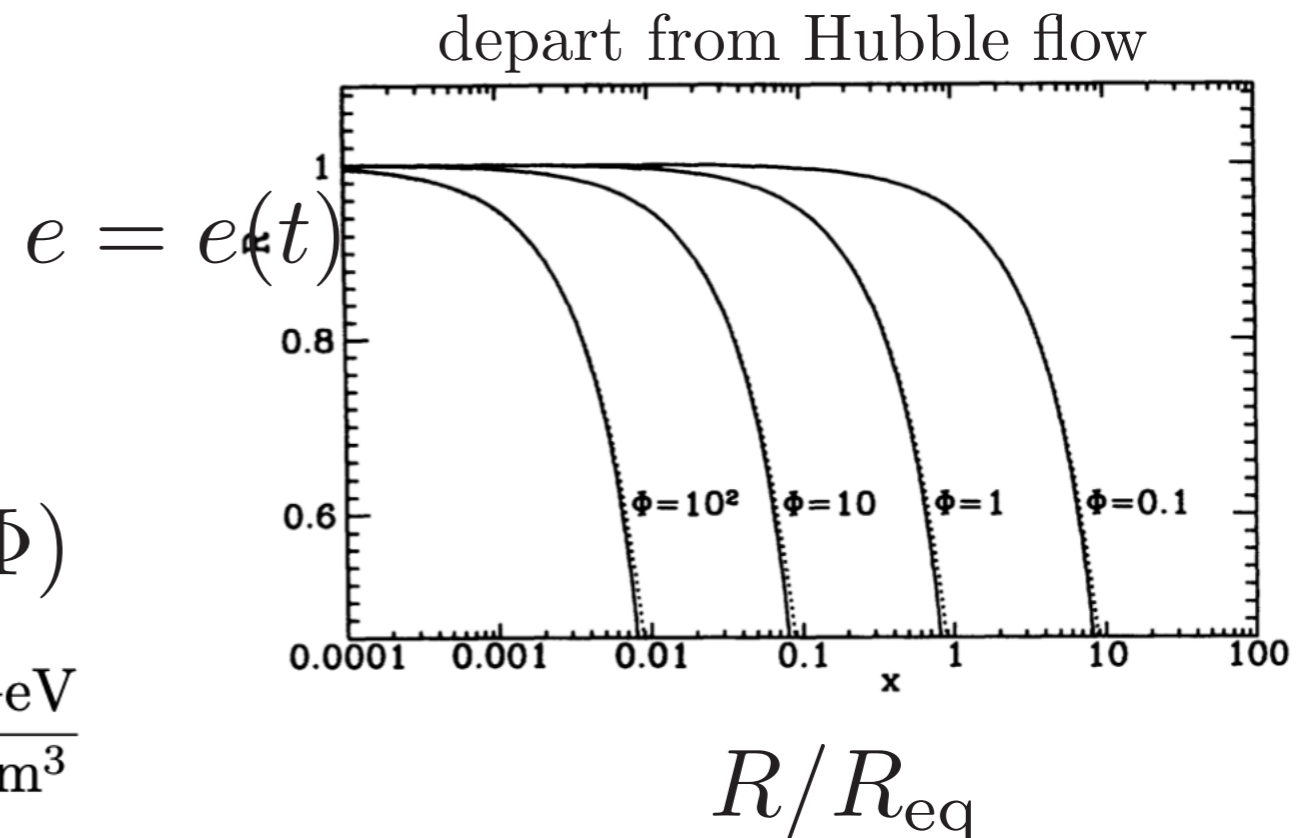
$$M_{\text{tot}} \equiv \frac{4\pi}{3} \rho_{\text{eq}} a_{\text{eq}}^3 [1 + \Phi(\xi)] \xi^3$$

- Collapse "redshift" is  $z_c \sim z_{\text{eq}} \Phi$

- Virialised radius  $L_1/z_c \sim L_1/(\Phi z_{\text{eq}})$

- Virialised density  $\sim \bar{\rho}_A(z_{\text{eq}}) \Phi^3 (1 + \Phi)$

$$\rho_c \sim 6.7 \times 10^6 (1 + \Phi) \Phi^3 \frac{\text{GeV}}{\text{cm}^3}$$

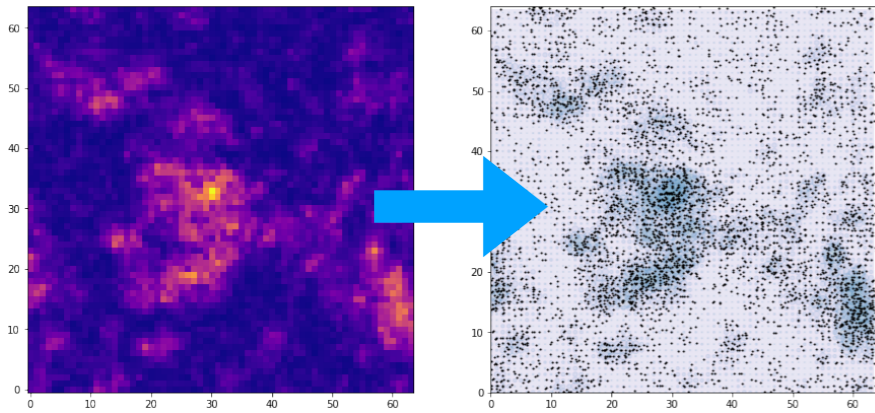




# Numerical simulations

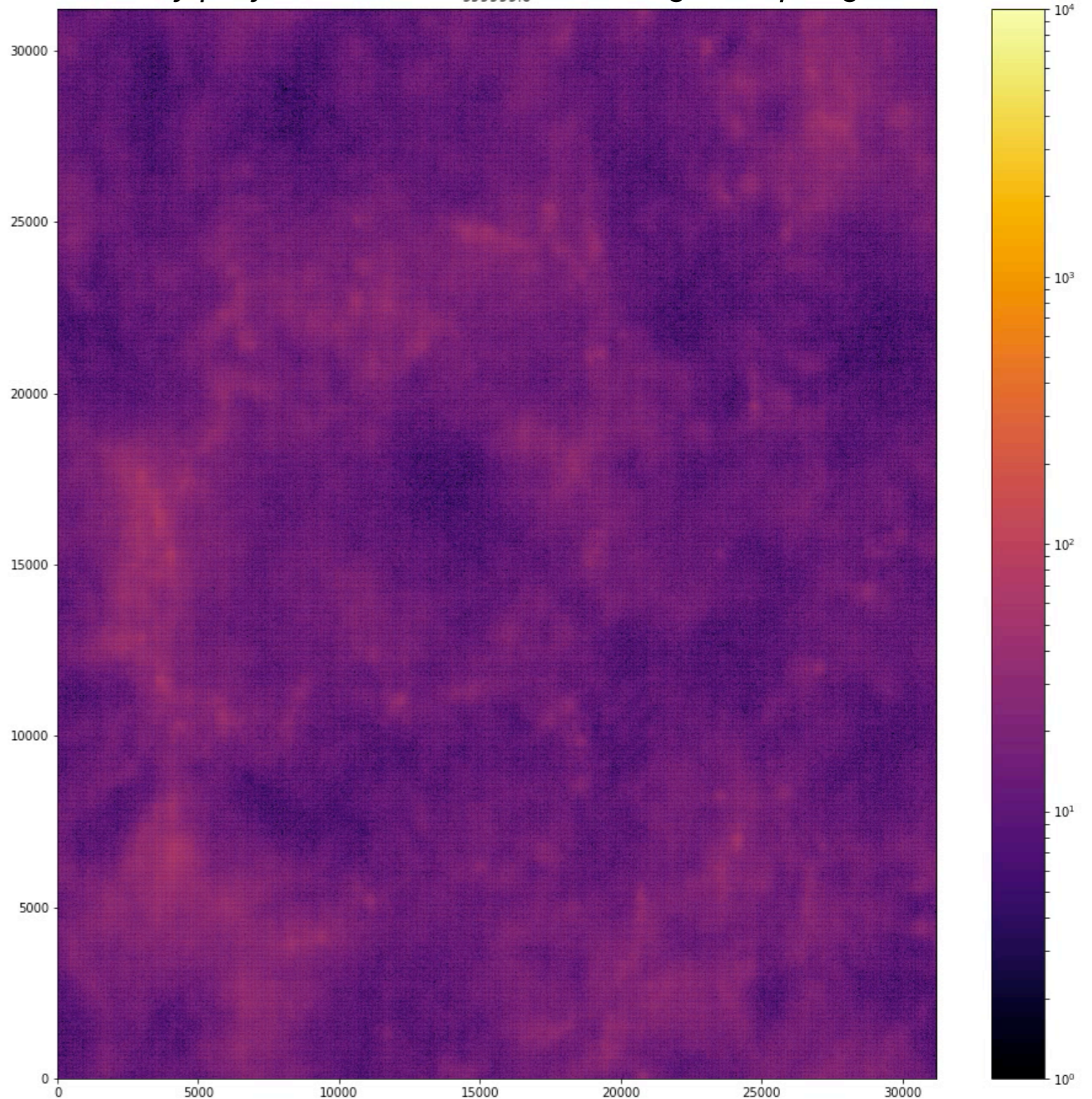
- MCs are not spherically symmetric ... use output of simulations

- Sample densities into particles



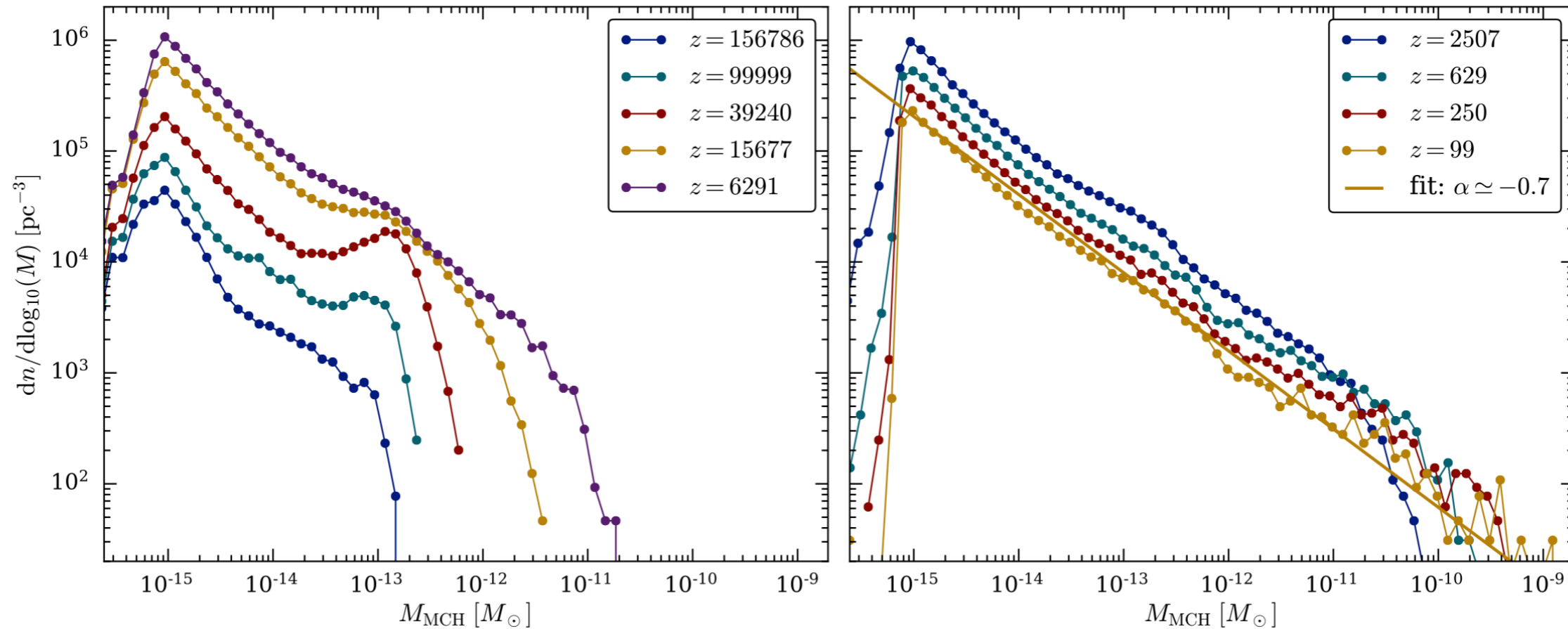
- Run N-body gravitational codes

*DM density projection (JR 2018) with Gadget2 (Springer 2005)*

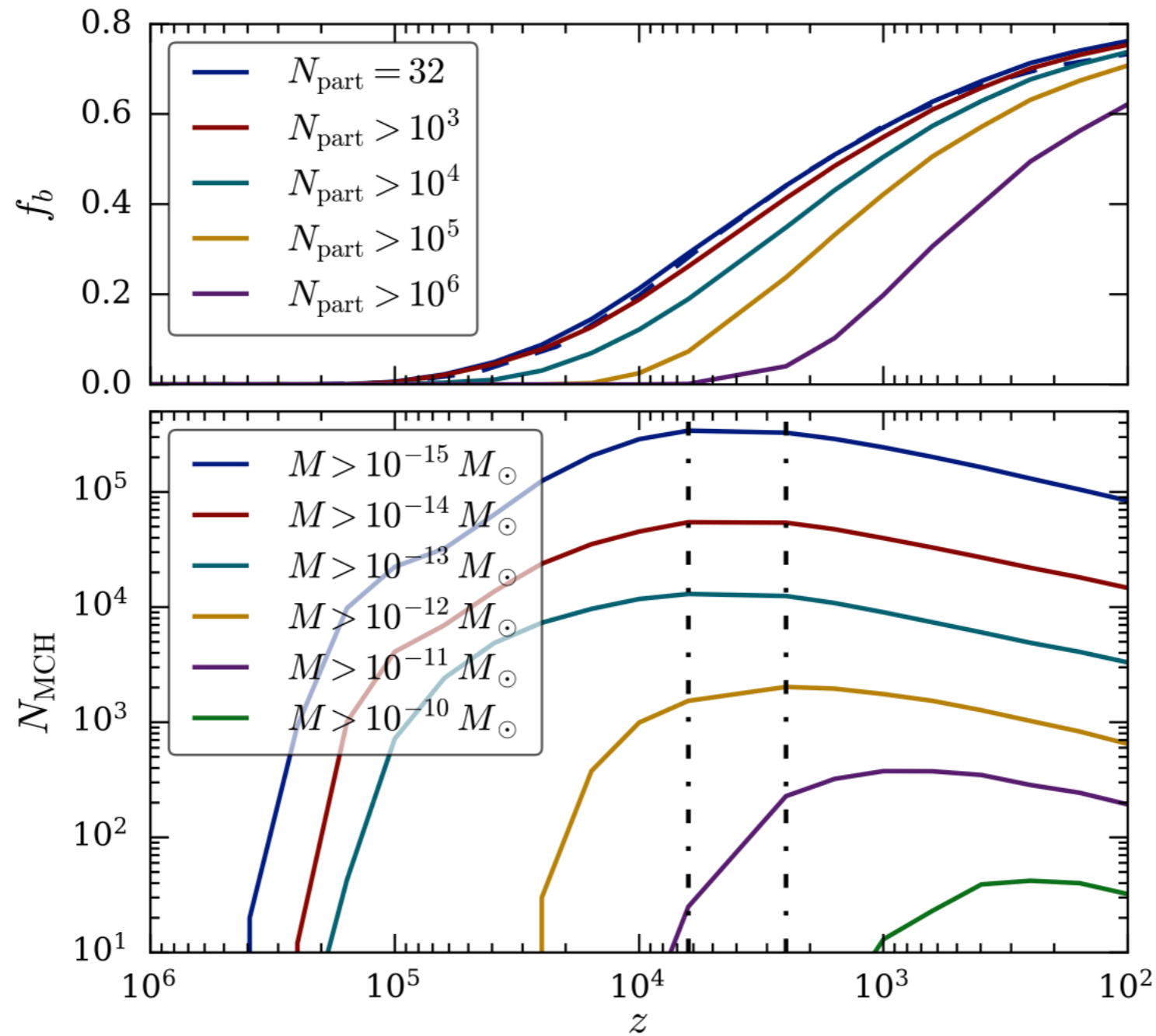




- Starting from results of Numerical simulations of QCD phase transition with strings+DWs ...
- Halo mass function (Mcs/mass/volume)

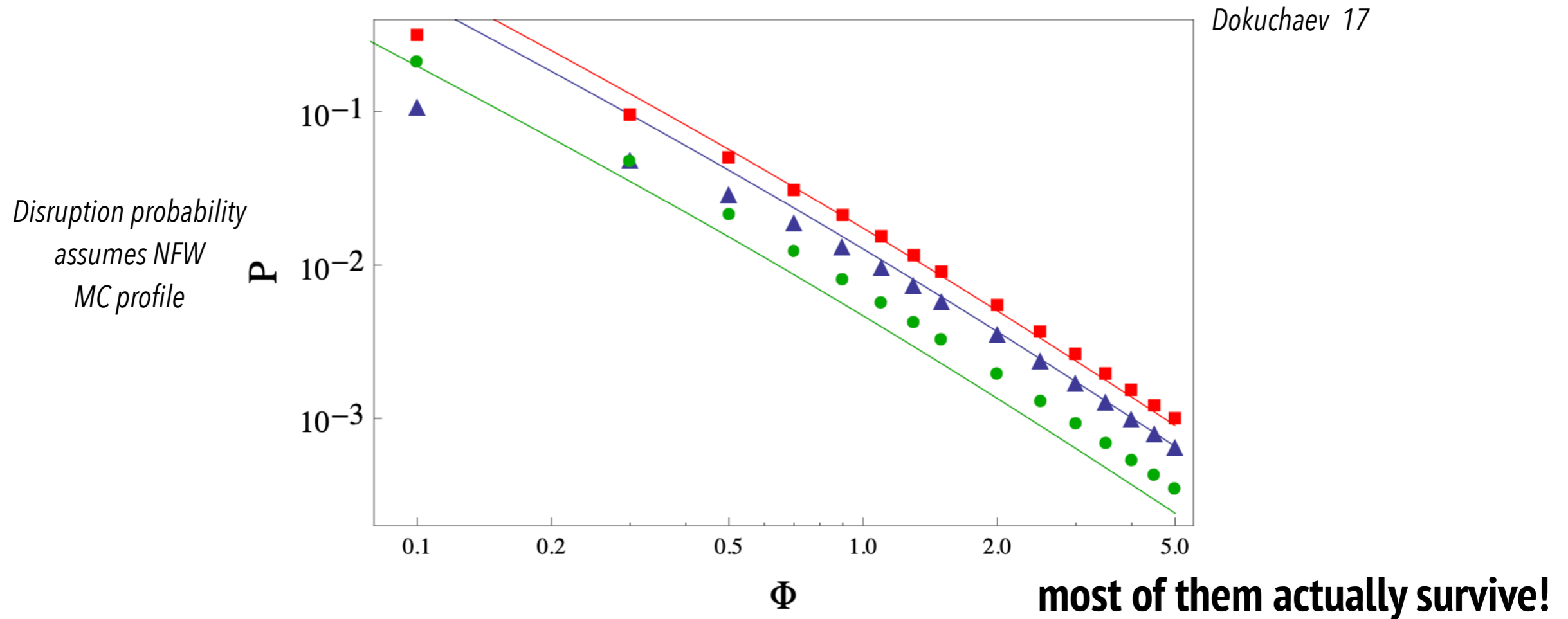


- Most axions are trapped in these high-density Mcs

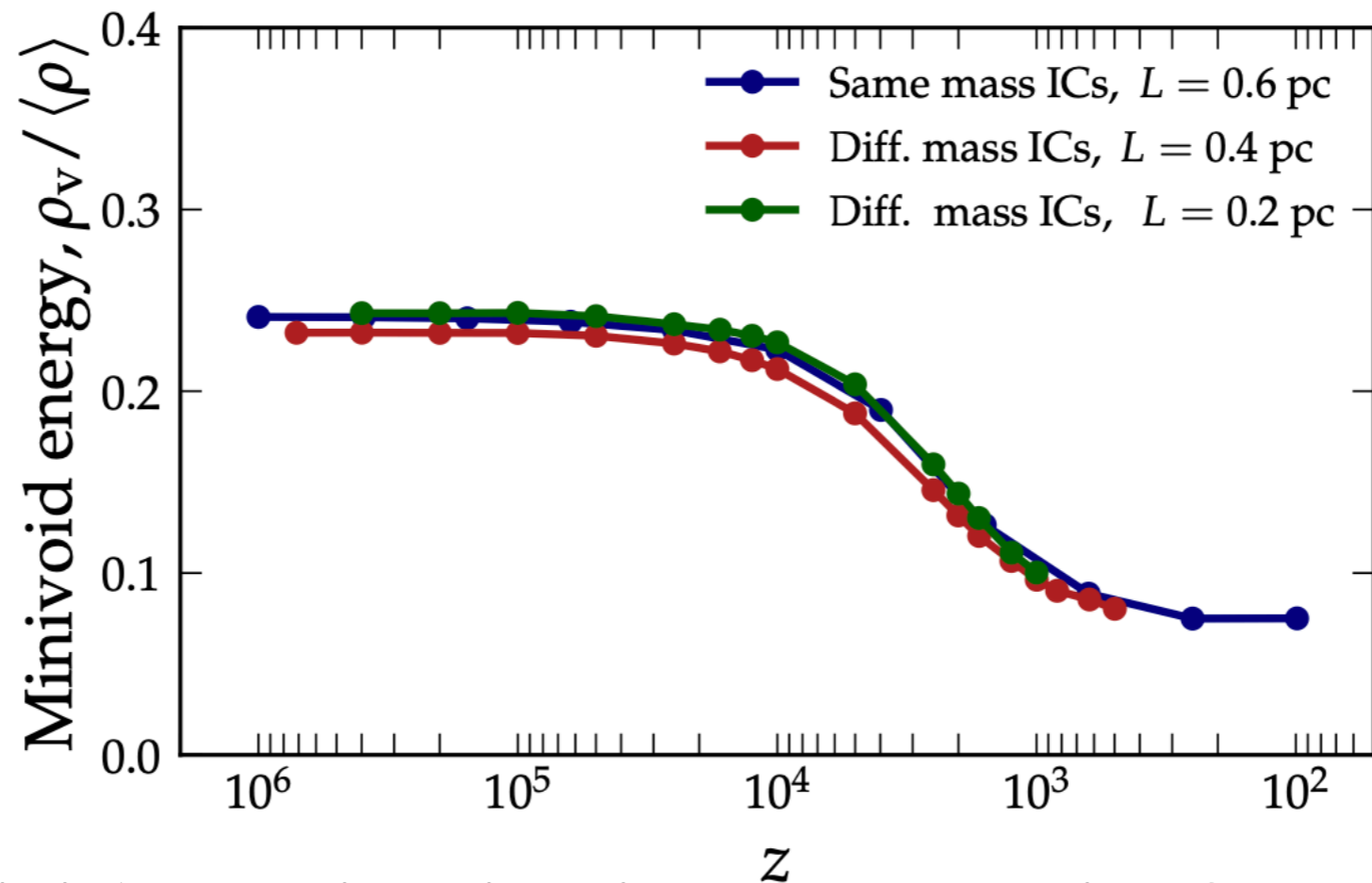


- This leaves very few axions to be discovered!

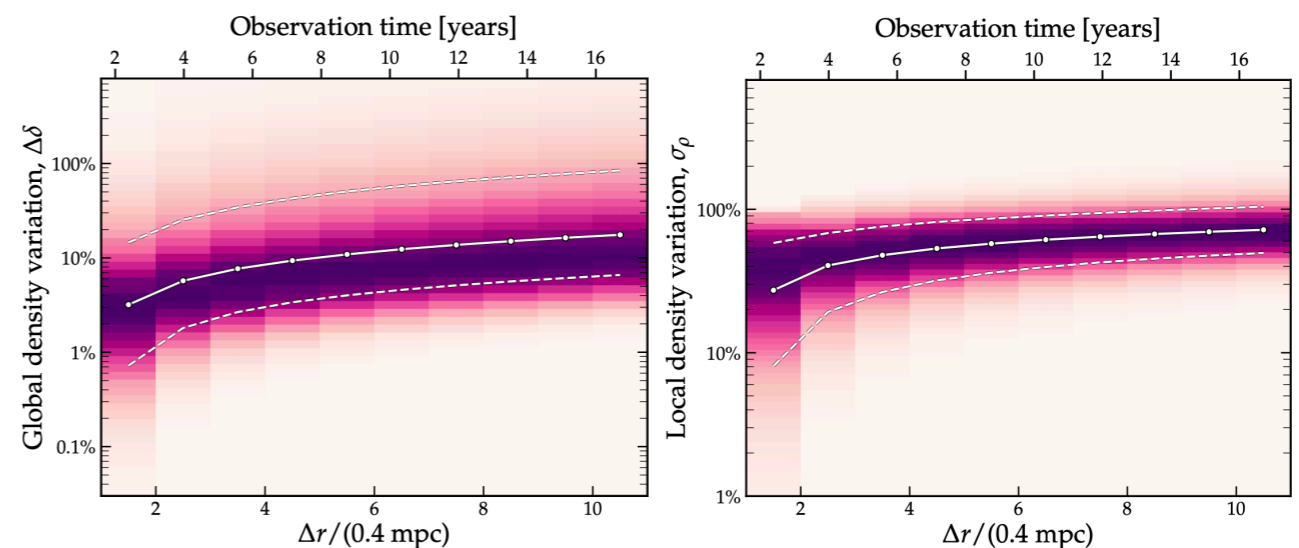
- Simulations end around  $z \sim 100$  for technical reasons
- Next important event is galaxy formation
- MCs can be disrupted by encounters with galactic stars



- The average void density seems to converge around few %

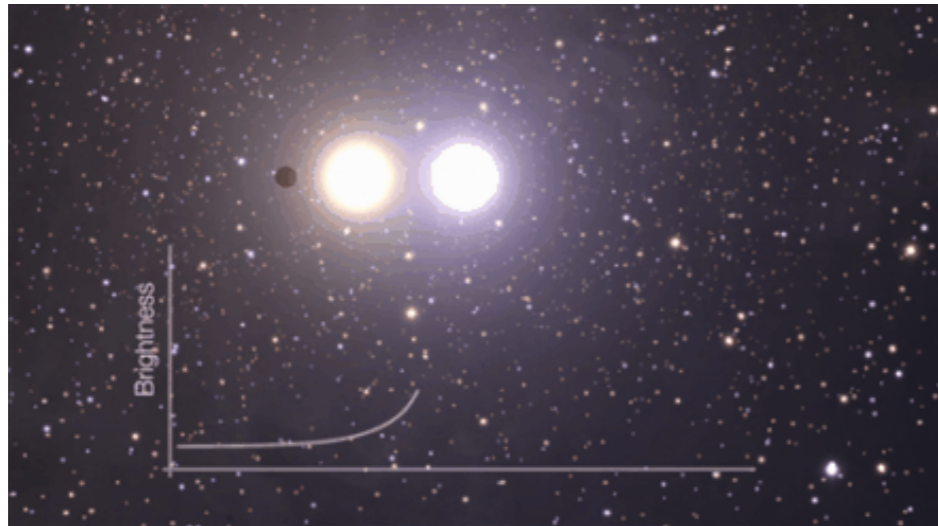


- This is a lower limit (galaxy disruption will release more axions from MCs into voids) that experiments can use

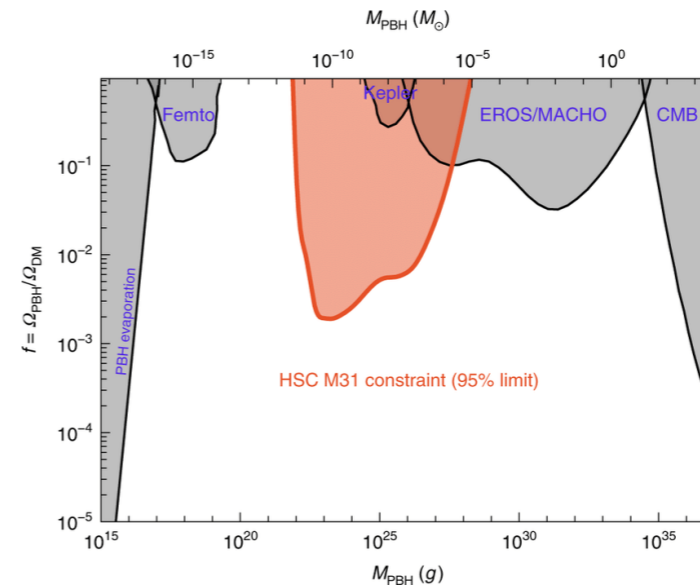


# Observing miniclusters?

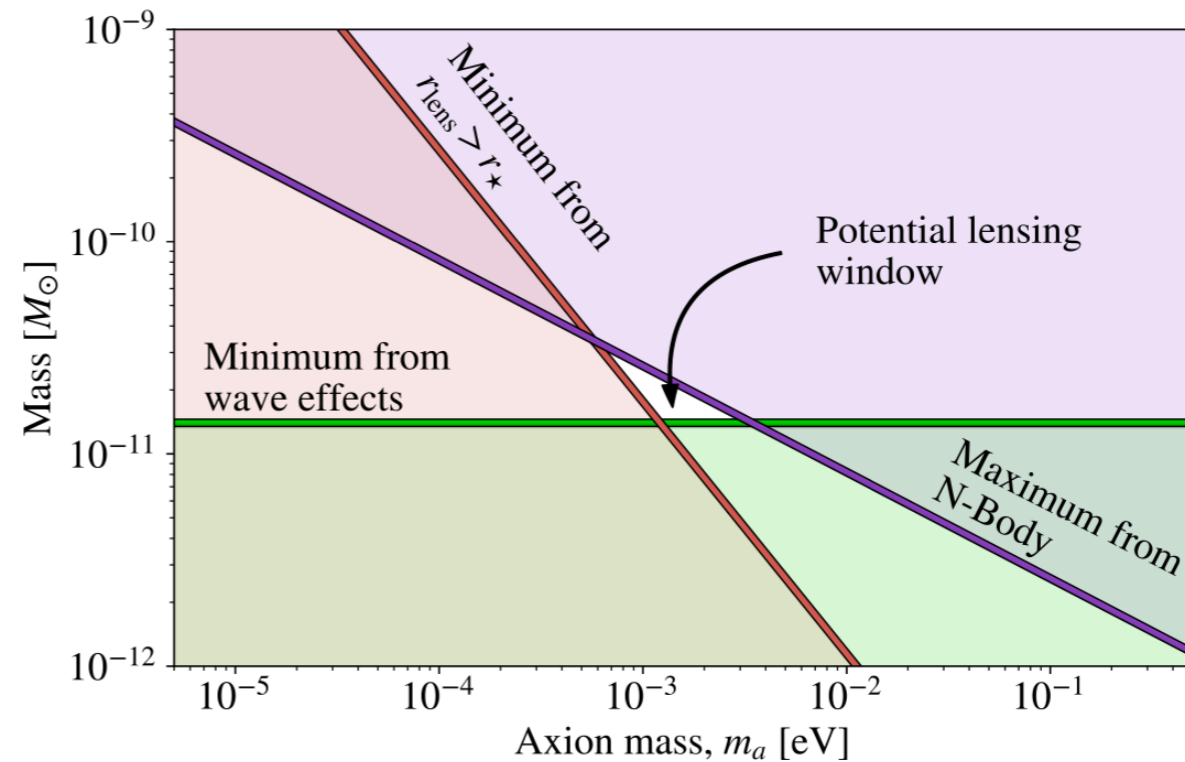
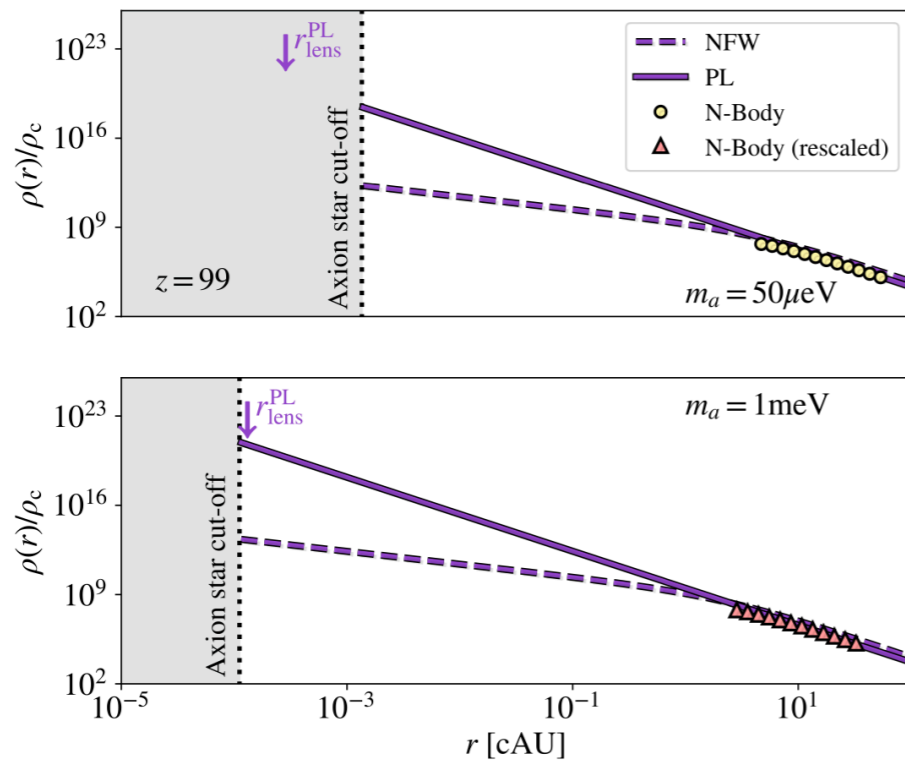
## - Microlensing



## Subaru HSC dedicated search of PHBs along M31



## - Axion MCs appear to be too fluffy (simulations do not resolve inner core)



Ellis 2022

only extreme assumptions offer a window of opportunity!



# More pheno

- **femtolensing**
- **minicluster encounters with NSs**
- **minicluster encounters with the Earth**
- **Bose-nova formation in miniclusters (and photon burst emission)**