Wave Dark Matter

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In this talk

ultralight dark matter (ULDM) = wave dark matter

denotes any bosonic dark matter candidate whose mass is

$$m \lesssim 10\,\mathrm{eV}$$

What's special about wave dark matter is

the occupation number (particle number per wavelength) is much larger than 1

$$n_{\rm dm} = \frac{\rho_{\rm dm}}{m}$$

$$N_{\rm occ} \sim n_{\rm dm}\lambda^3 \sim \left(\frac{10\,{\rm eV}}{m}\right)^4$$

$$\rho_{\rm dm} \sim {\rm GeV/cm^3}$$

For $m < 10$ eV, the occupation number takes a gigantic value

this dark matter candidate is conveniently described by a collection of classical waves

Wave dark matter is a broad concept including numerous theoretical possibilities

In a pessimistic (minimalistic?) scenario where DM participates only in gravitational interaction

$$S = \int d^4x \sqrt{-g} \left[\frac{M_P^2}{2} R + \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - \frac{1}{2} m^2 \phi^2 \right]$$

the model is controlled by a single parameter, the mass ***m***

Even in this most minimalistic case
there's a landscape of possibilities

The mass smaller than what's shown may not be 100% DM in universe
the mass larger than what's shown is not wave DM in our definition

$$10^{-21}\,\mathrm{eV} \longrightarrow 10\,\mathrm{eV} \quad m$$

(almost) all mass range shown here is currently allowed*

it may also couple to SM through non-gravitationally interactions

$$\Delta\mathcal{L} = \sum g\phi J_{\mathrm{SM}} + g'\partial_\mu\phi J^\mu_{\mathrm{SM}} + \cdots$$

the model is now controlled by

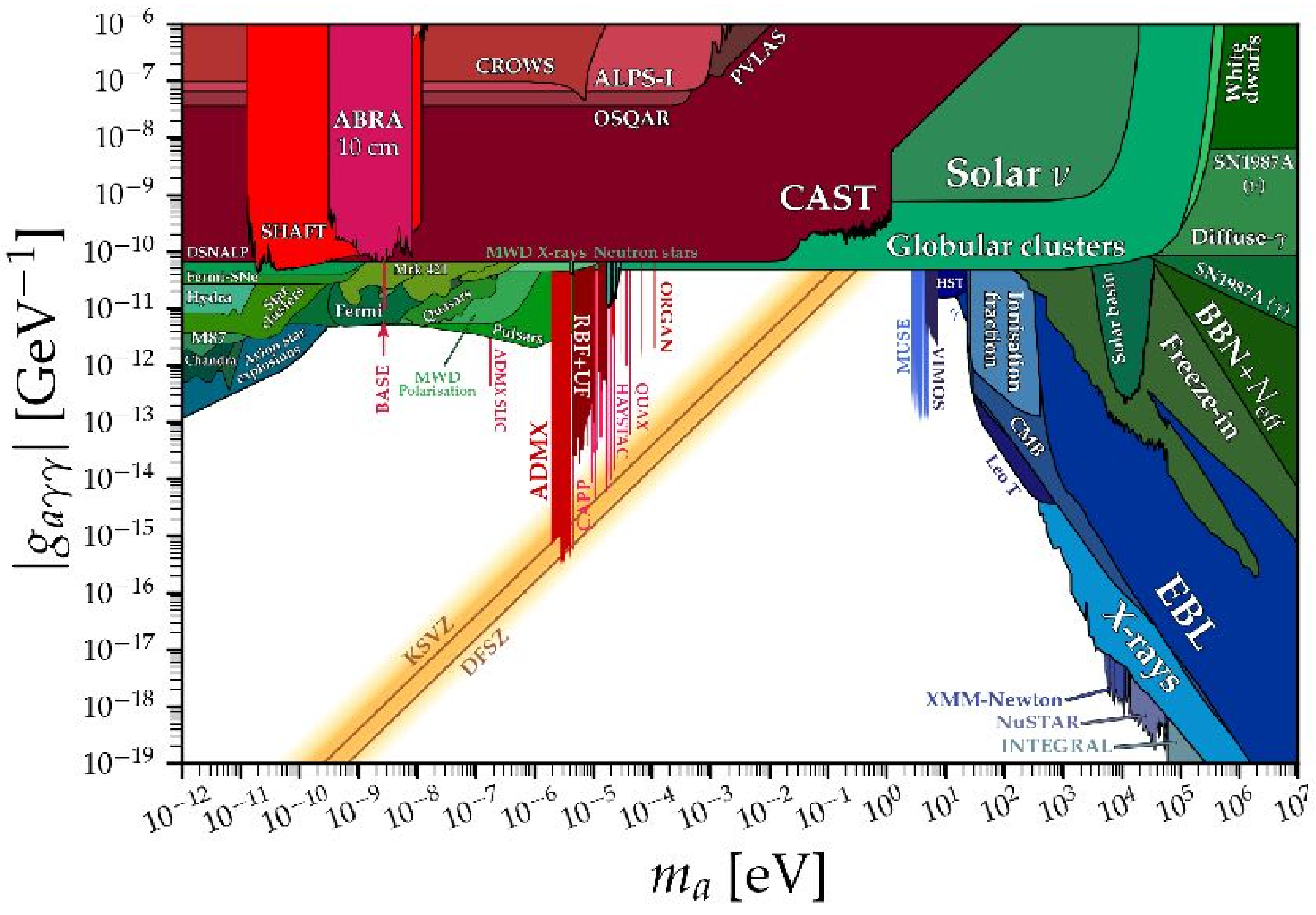
$$(m, g_i, g'_i, \cdots)$$

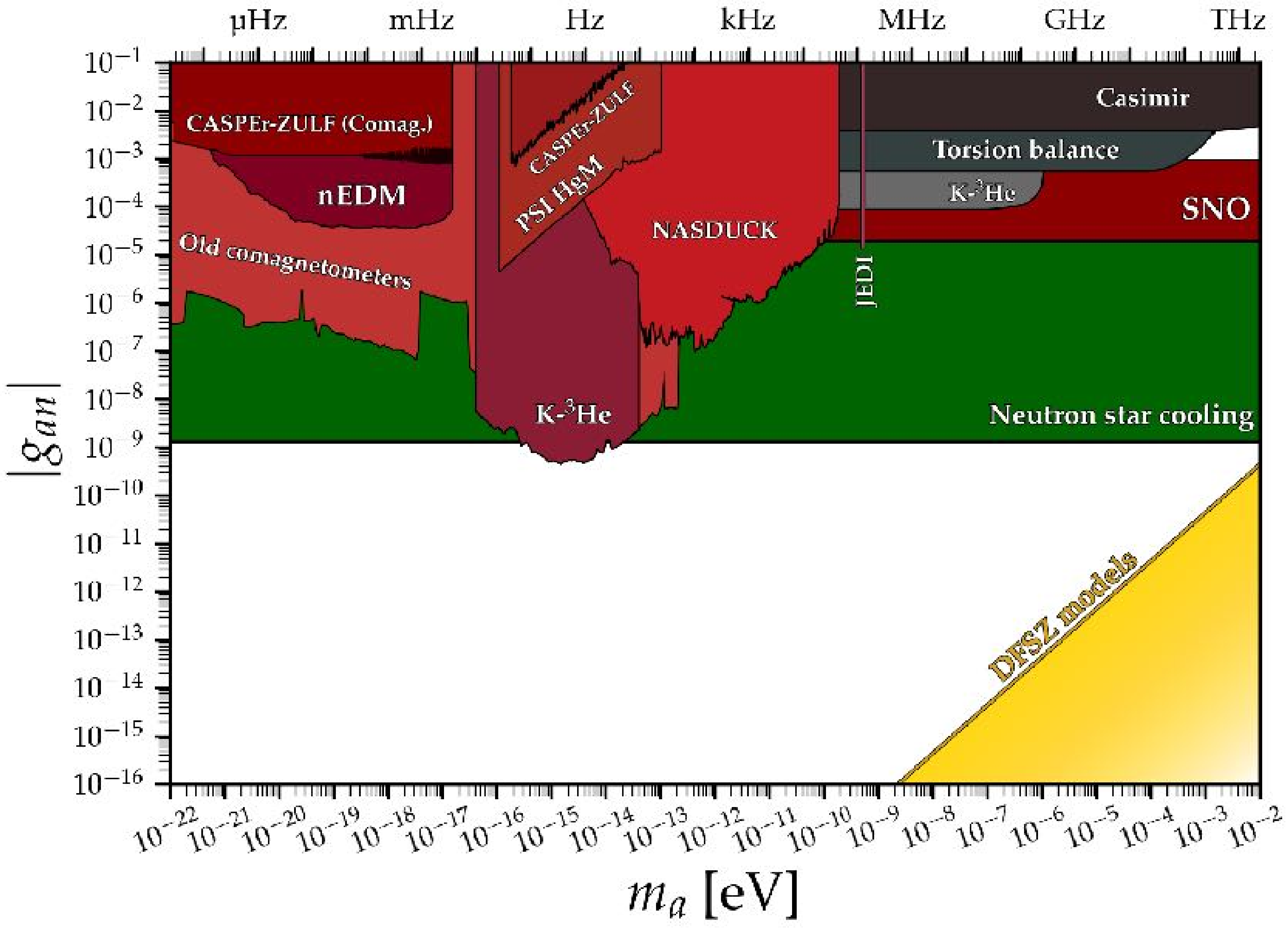
the landscape of possibilities becomes N-dim. space
posing challenges to wave dark matter searches

We may look at this from a different angle:
non-gravitational couplings & a wide range of mass
might provide more handles for us to search for ULDM

For instance for axions and axion-like particles

$$(\phi/f)F\widetilde{F} + (\partial_\mu\phi/f)\bar{\psi}\gamma^\mu\gamma_5\psi + (\phi/f)\bar{N}\sigma^{\mu\nu}\gamma_5 N F_{\mu\nu}$$



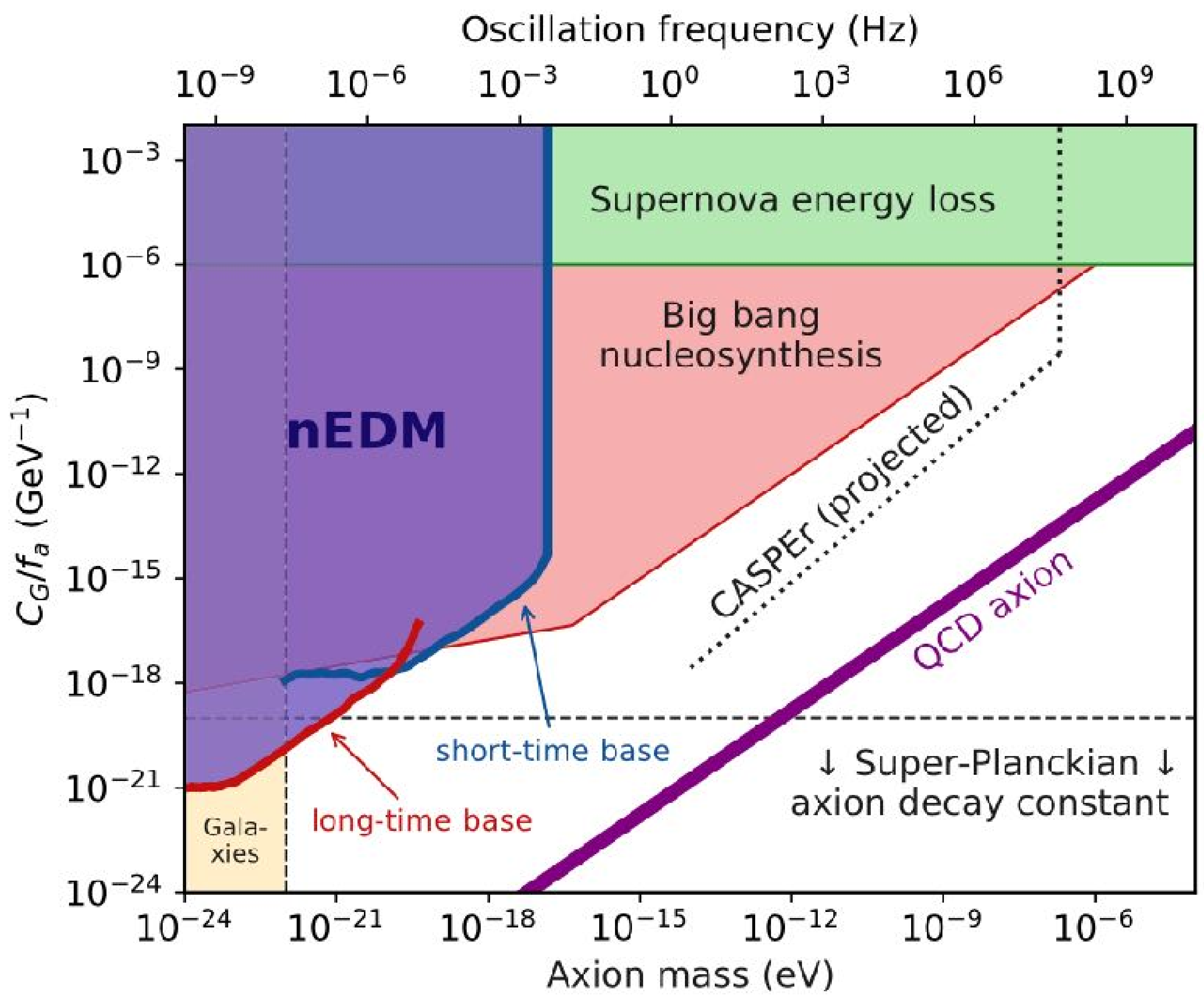


[O'Hare, AxionLimits]

For instance for axions and axion-like particles

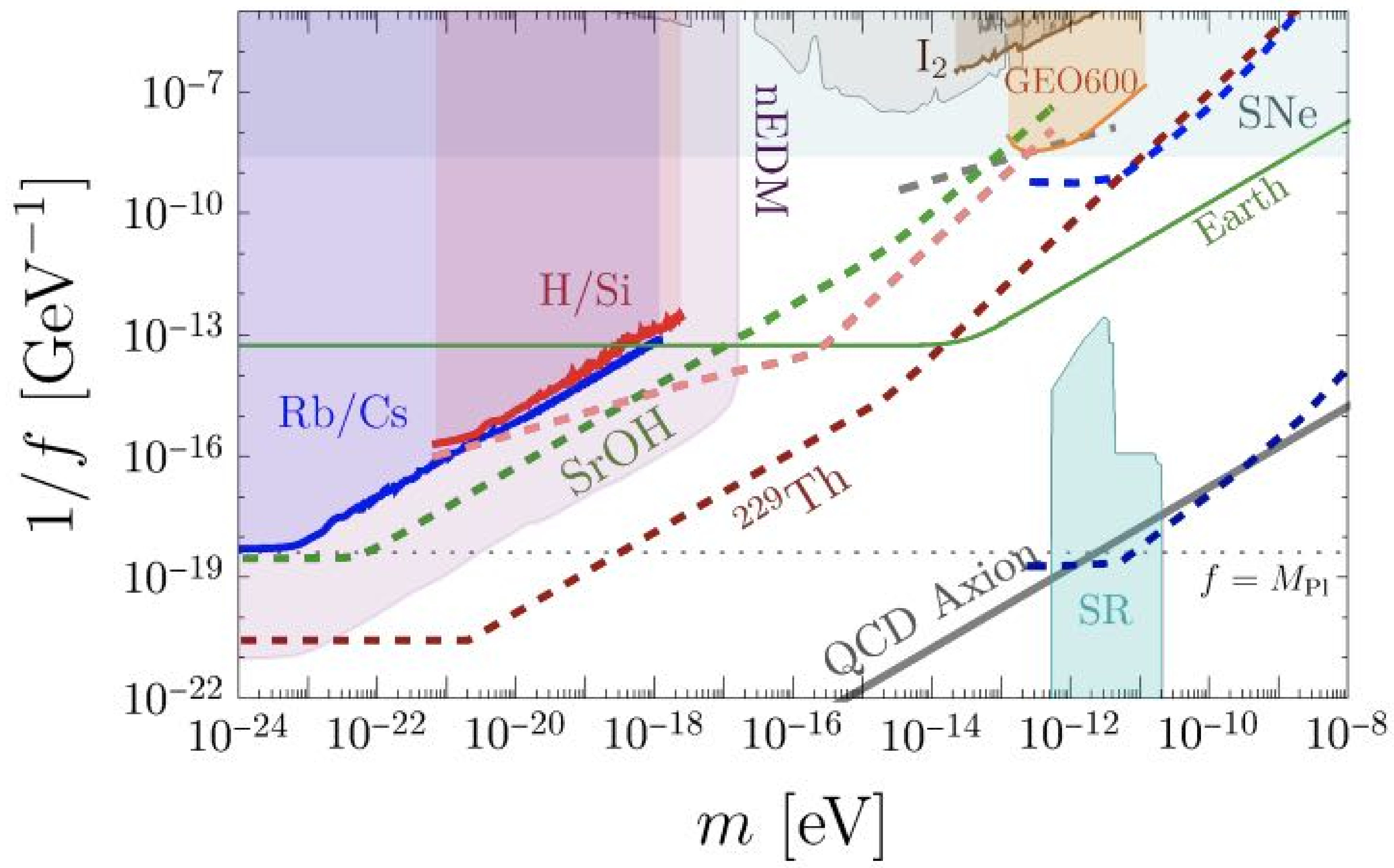
$$(\phi/f)F\widetilde{F} + (\partial_\mu \phi/f)\bar{\psi}\gamma^\mu\gamma_5\psi + (\phi/f)\bar{N}\sigma^{\mu\nu}\gamma_5 N F_{\mu\nu}$$

$$+\phi^2\pi^2 + \phi^2\bar{N}N + \cdots$$



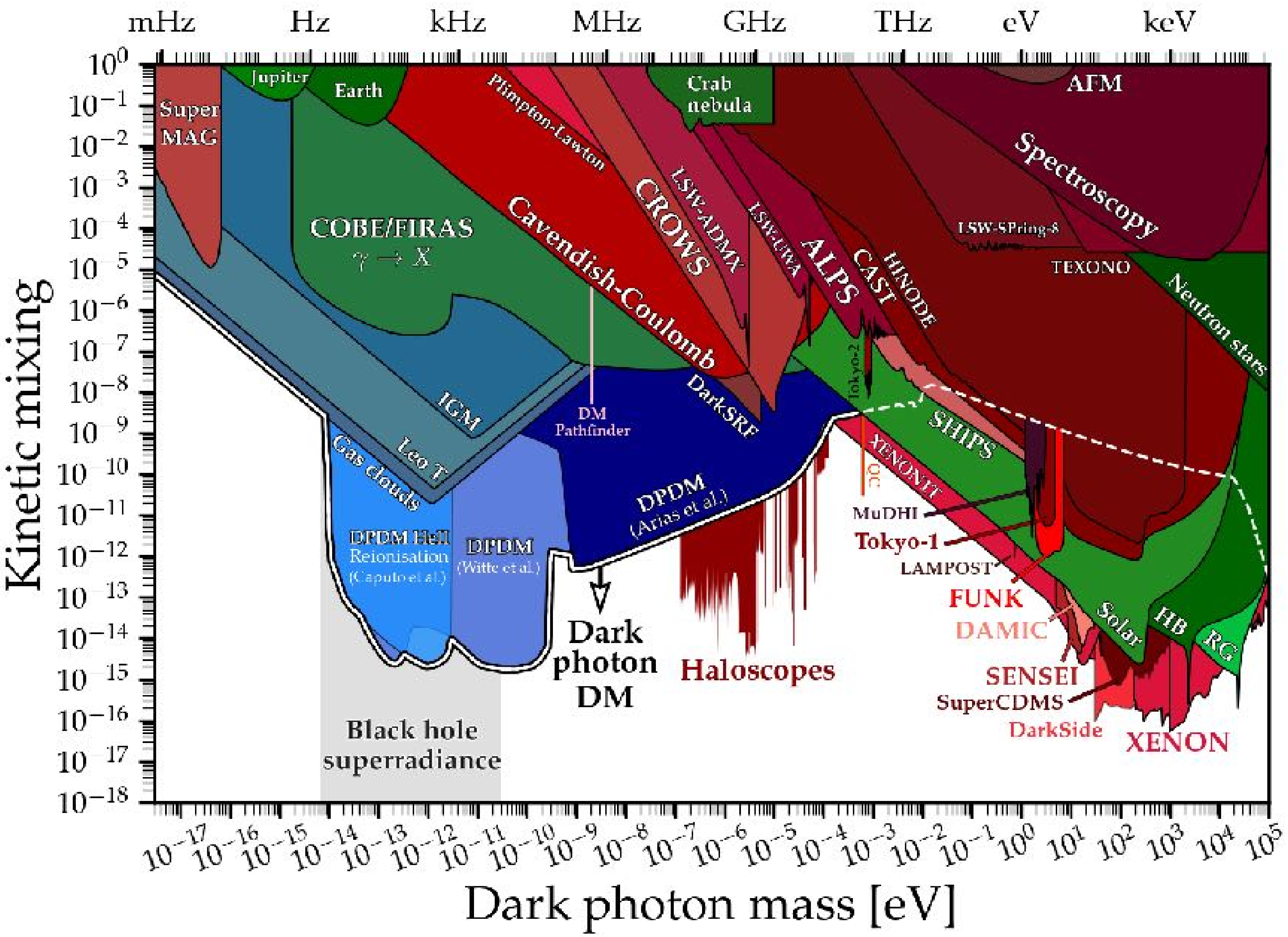
[Abel et al (17)]

[Kim and Perez (22)]



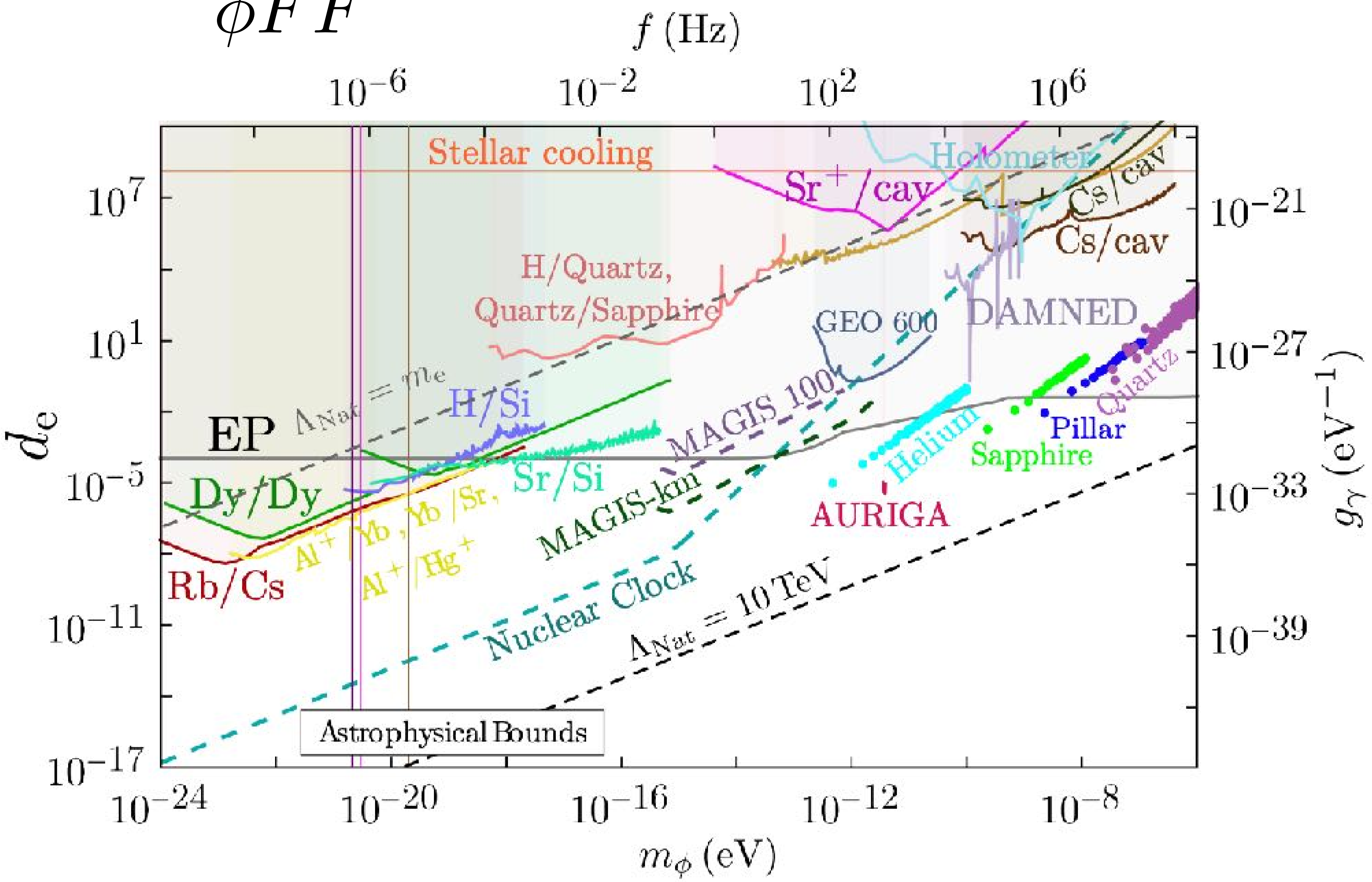
for dark photon dark matter or scalar dark matter

$\epsilon F_{\mu\nu} X_{\mu\nu}$



[Caputo et al (21)]

$\phi F F$

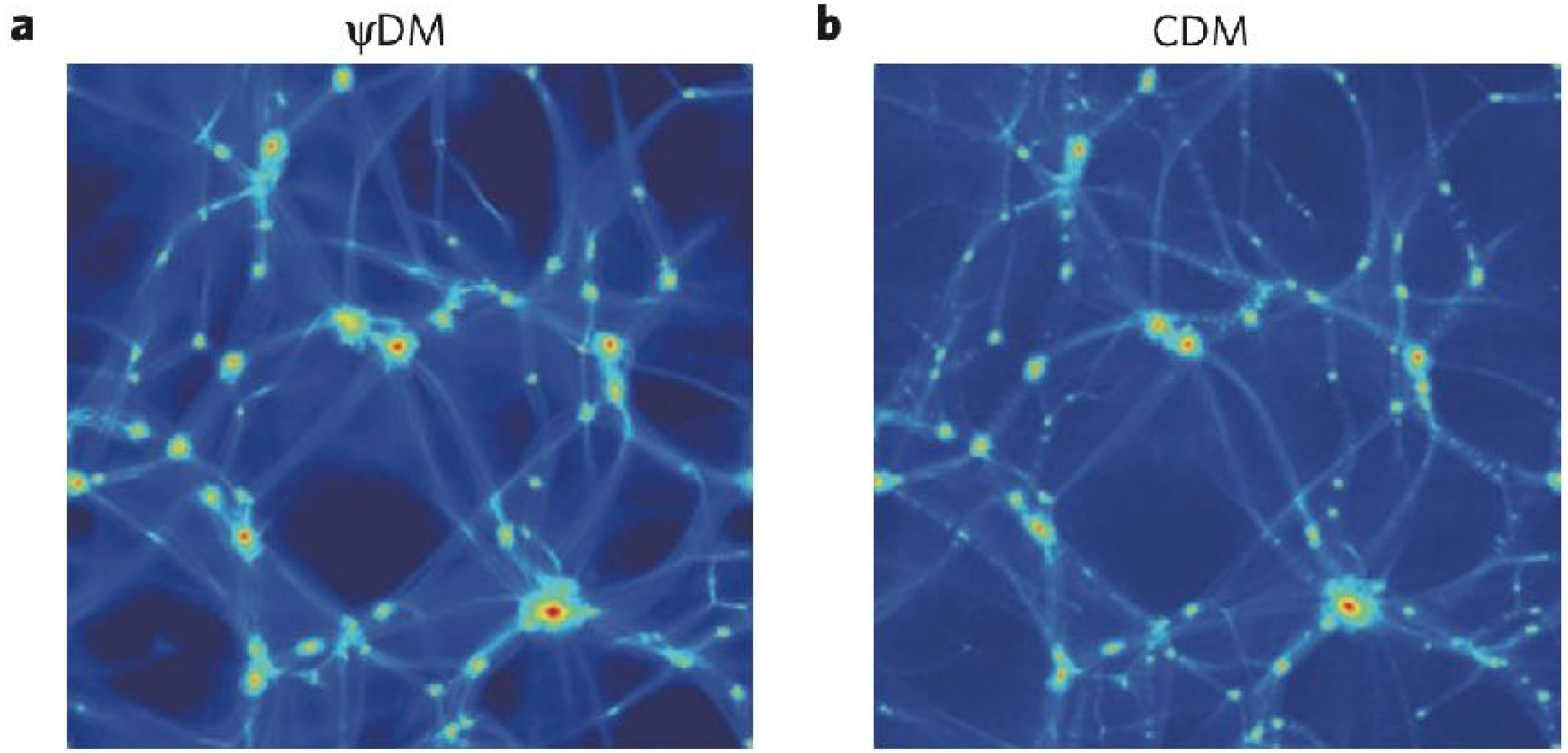


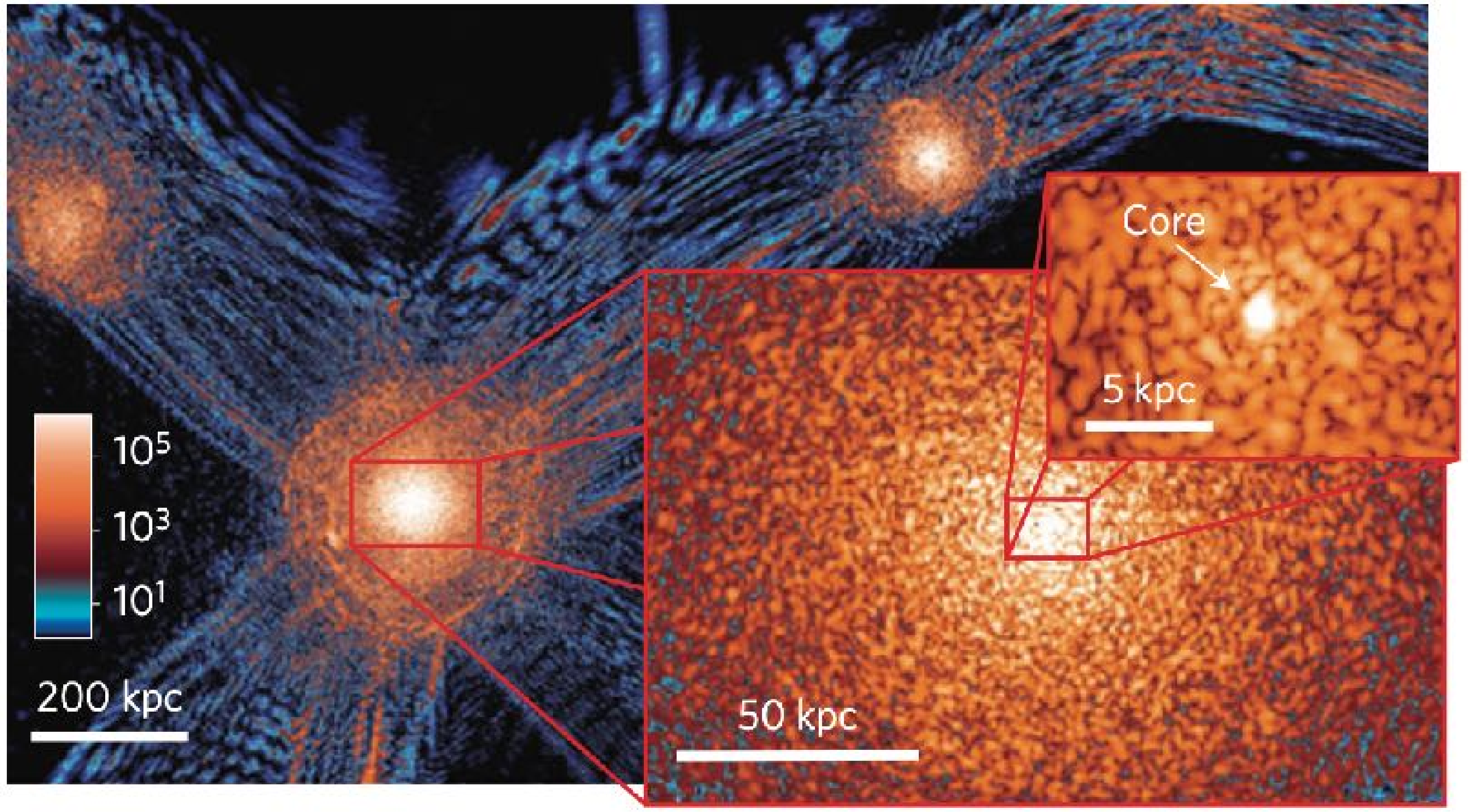
[Antypas et al, Snowmass (21)]

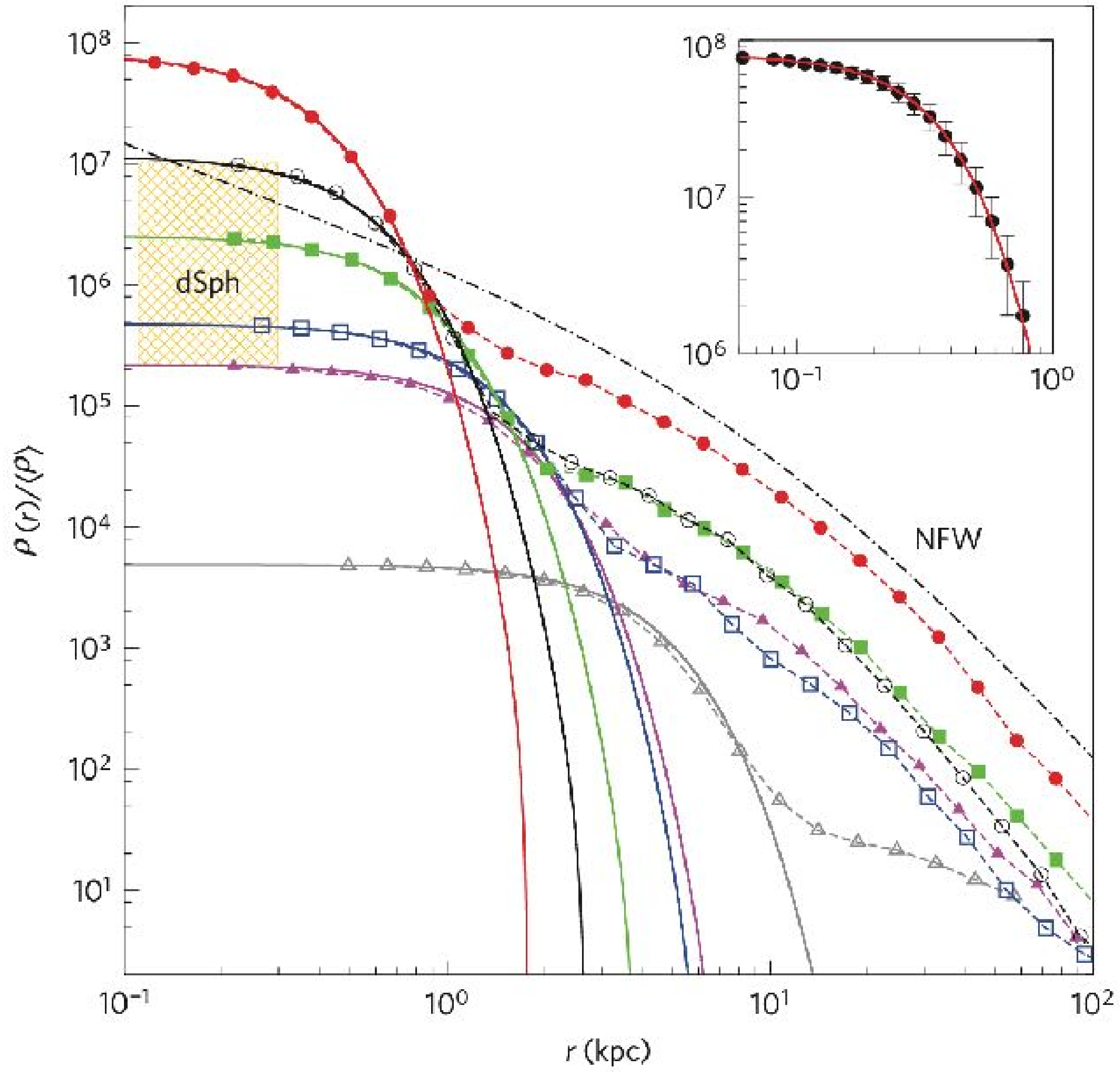
this summarizes the current efforts for wave dark matter searches
illustrating opportunities as well as challenges

Having discussed wave DM searches based on non-gravitational interactions with SM
let us switch a gear a little bit to discuss a few (less-discussed) topics in this workshop:
the behavior of ultralight dark matter on (sub)galactic scale ONLY with gravitational interaction

Schive, Chiueh, Broadhurst (14)

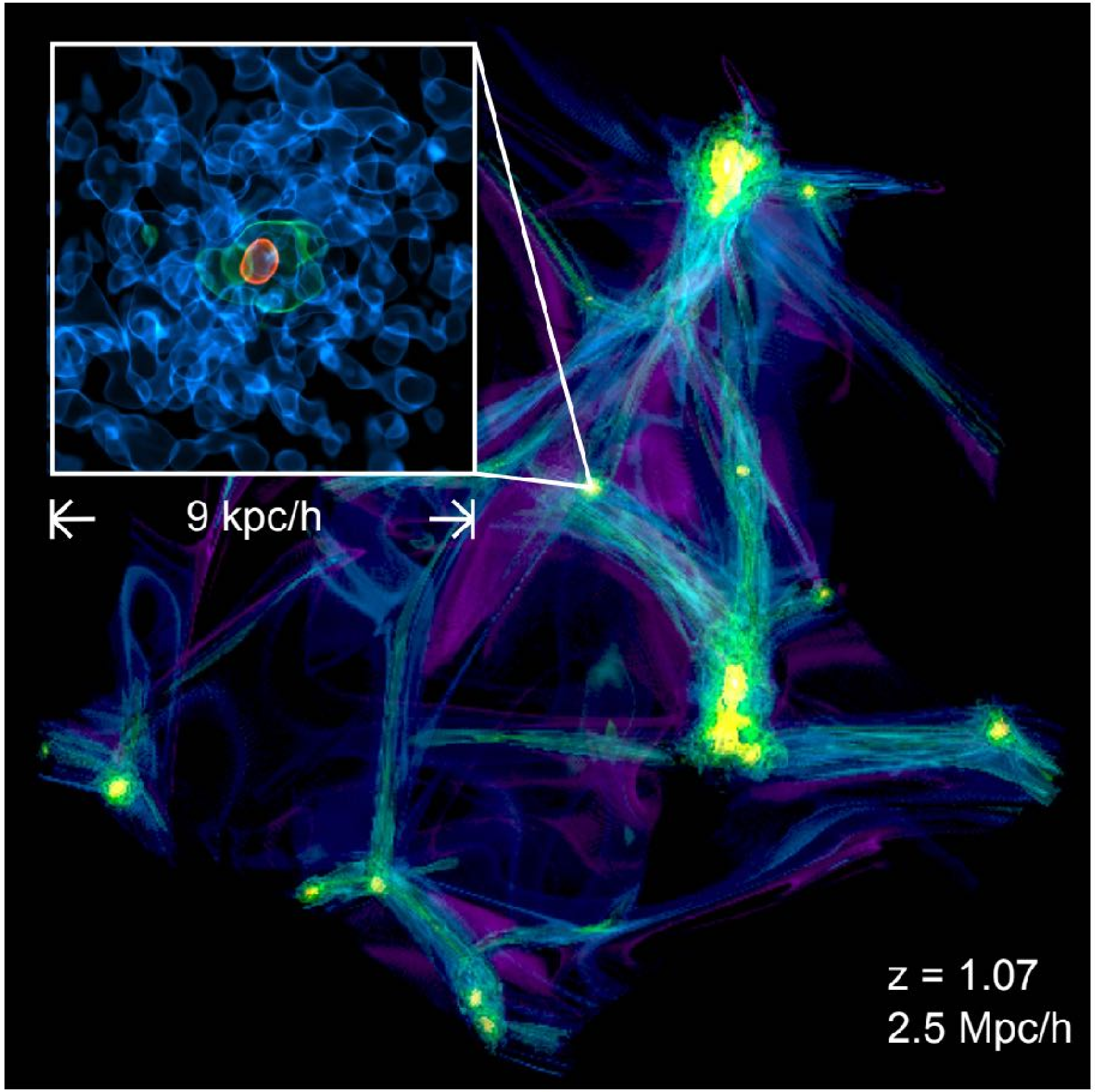




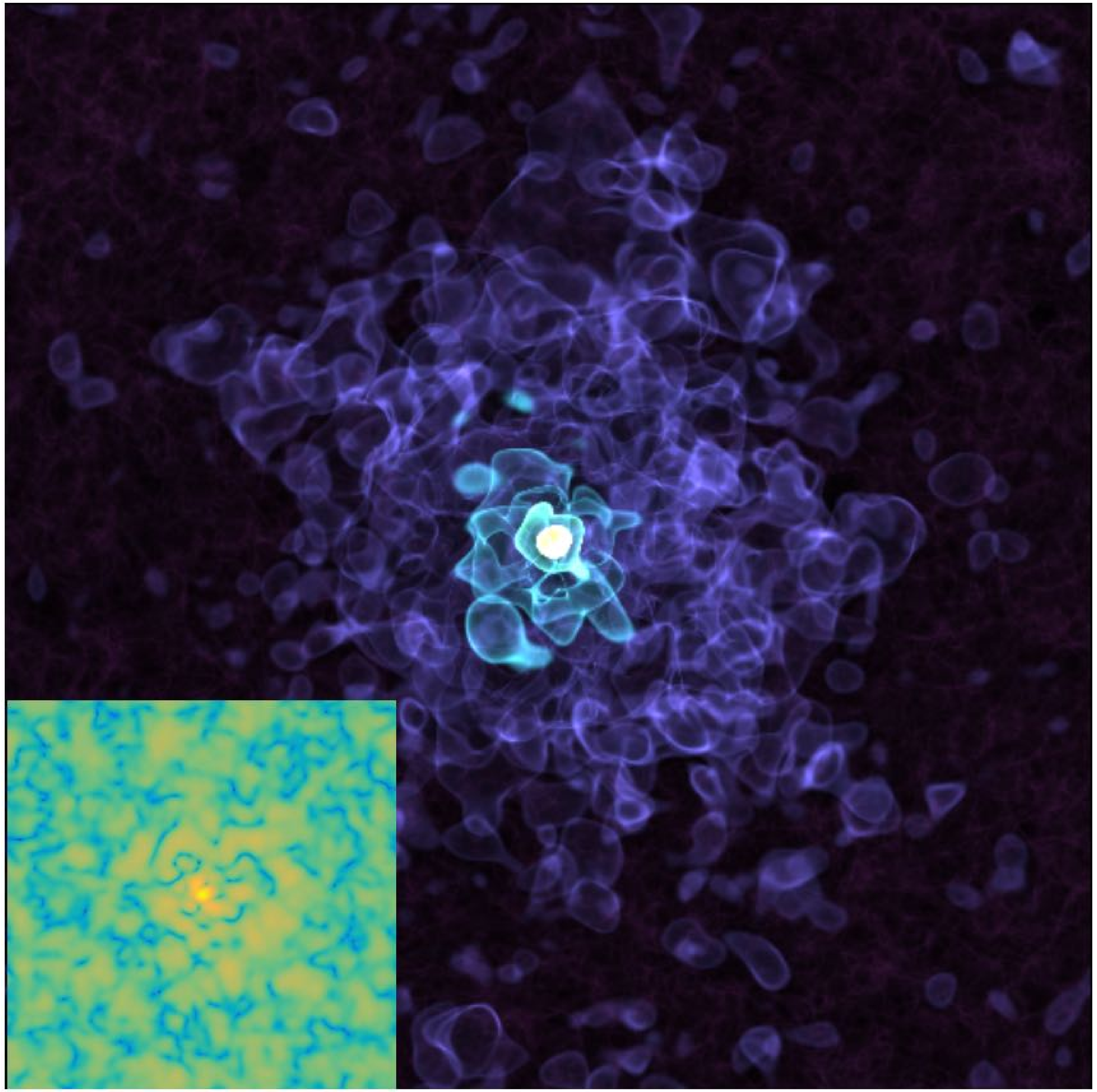


* characteristic *soliton* at the center has been observed

* small scale structures are erased



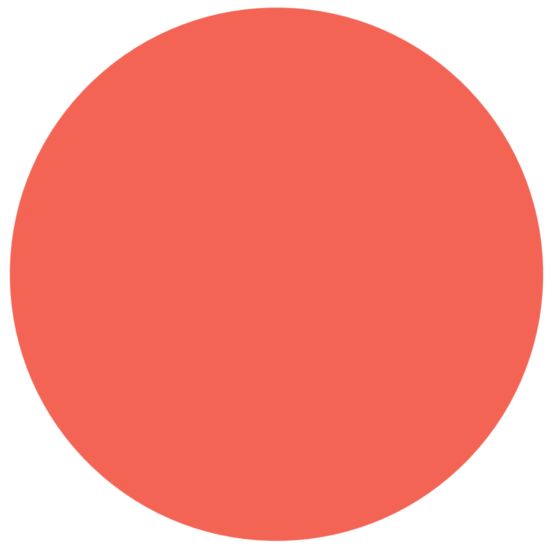
Veltmaat, Niemeyer, Schwabe (18)



this density fluctuation is another characteristic feature of ultralight dark matter
and some of interesting phenomenology on (sub)galactic scales follow from these fluctuations

One of the most intuitive ways to understand this fluctuation is to think of them as a sort of particles say, quasiparticles

Hui et al (17)

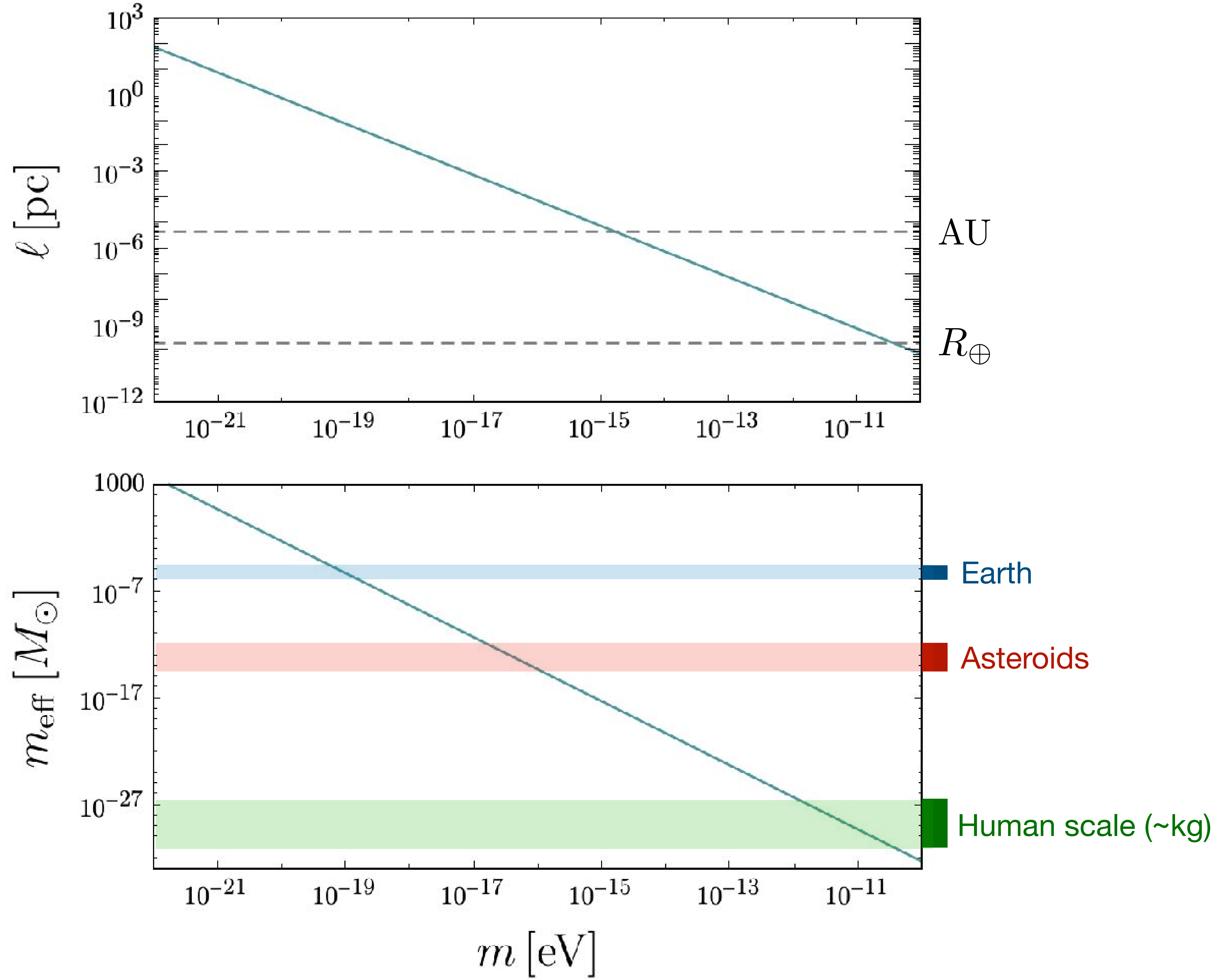


the size of the quasiparticle is given by its wavelength

$$\ell \sim \lambda = \frac{1}{mv}$$

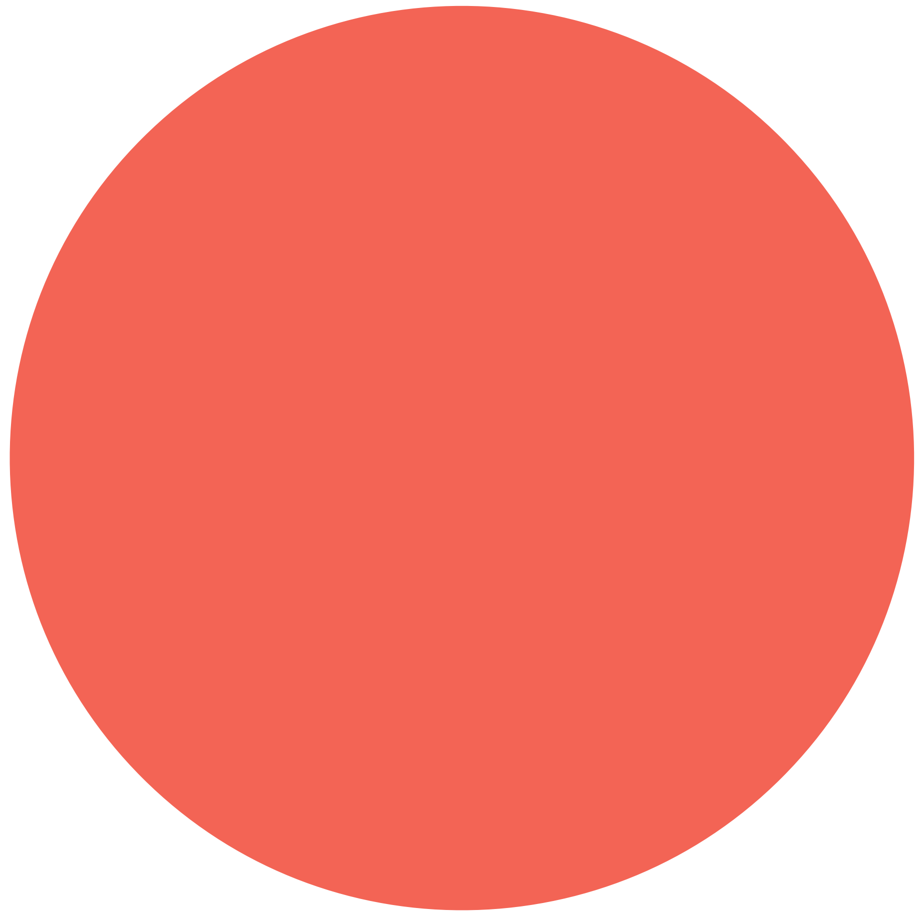
the effective mass of the quasiparticle is given by

$$m_{\rm eff} \sim \rho_{\rm DM} \ell^3$$



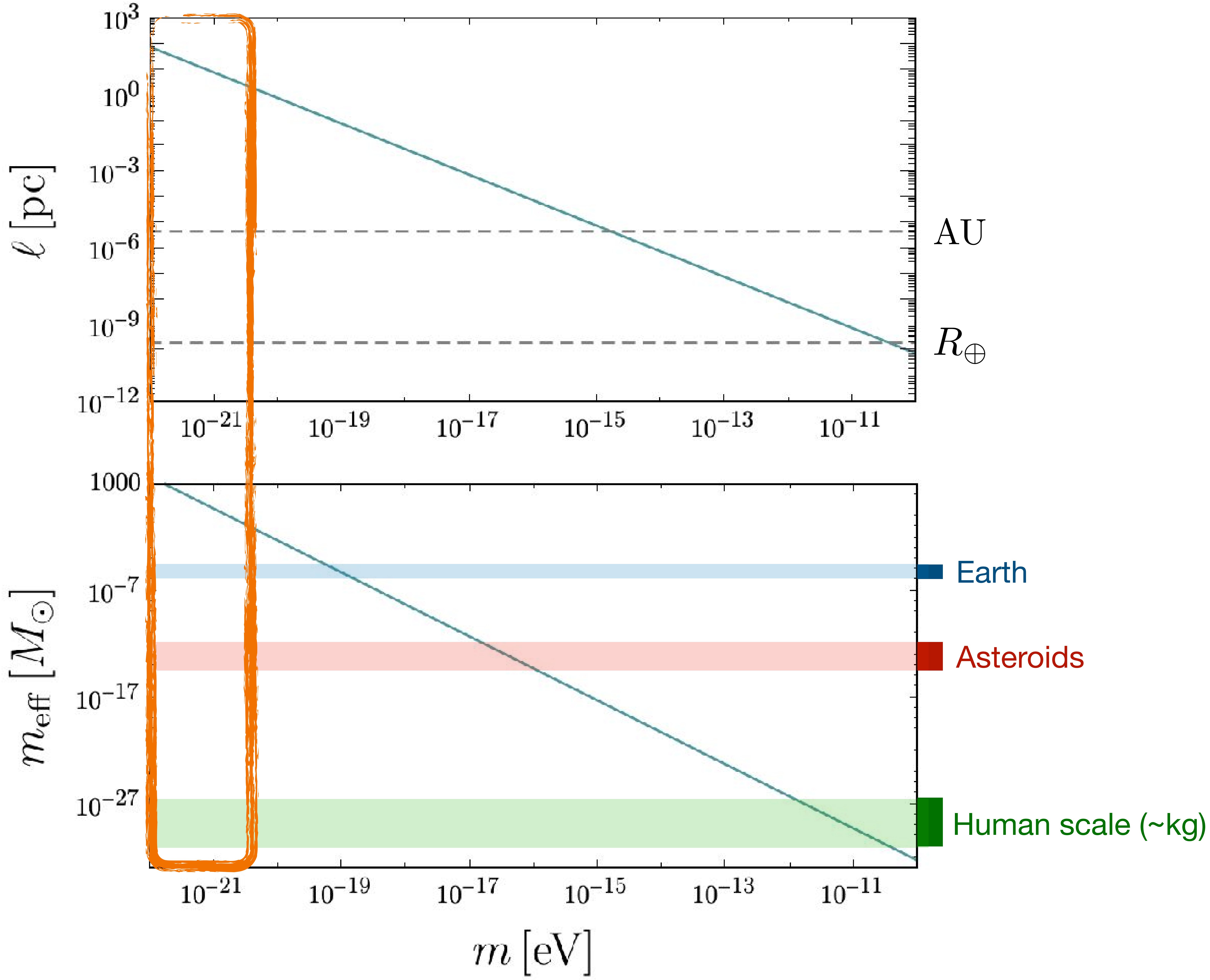
10^3
10^0
10^{-3}
10^{-6}
10^{-9}
10^{-12}
ℓ [pc]
AU
$R_\oplus$
10^{-21} 10^{-19} 10^{-17} 10^{-15} 10^{-13} 10^{-11}
1000
10^{-7}
10^{-17}
10^{-27}
$m_{\rm eff}\,[M_\odot]$
Earth
Asteroids
Human scale (~kg)
10^{-21} 10^{-19} 10^{-17} 10^{-15} 10^{-13} 10^{-11}
m [eV]

$m \sim 10^{-22}\,\mathrm{eV}$

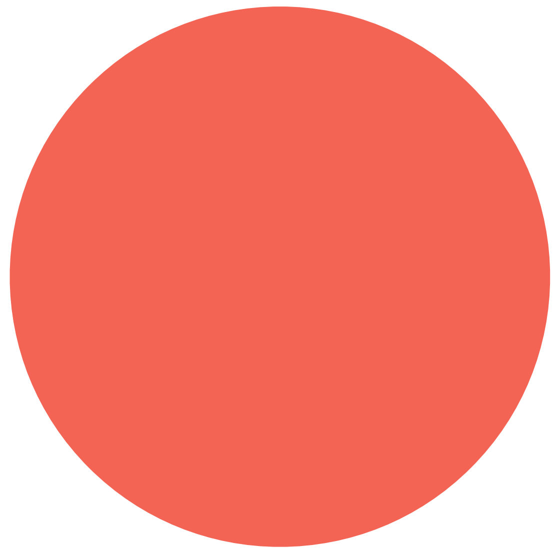


$m_{\mathrm{eff}} \sim \mathcal{O}(10^4) M_\odot$

$\ell \sim \mathcal{O}(10^2)\mathrm{pc}$

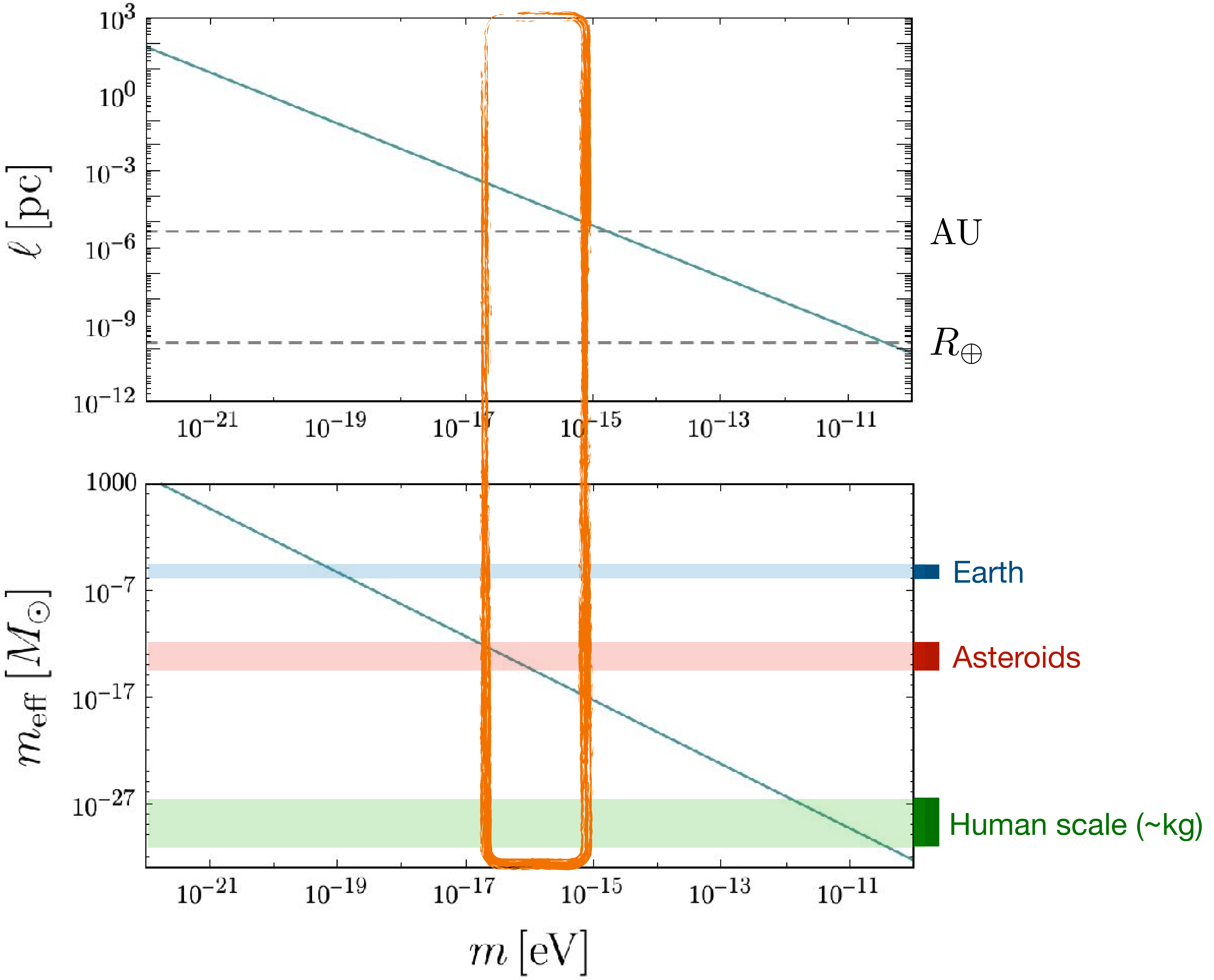


$m \sim 10^{-16}\,\mathrm{eV}$



$m_{\mathrm{eff}} \sim \mathcal{O}(10^{17})\,\mathrm{kg}$

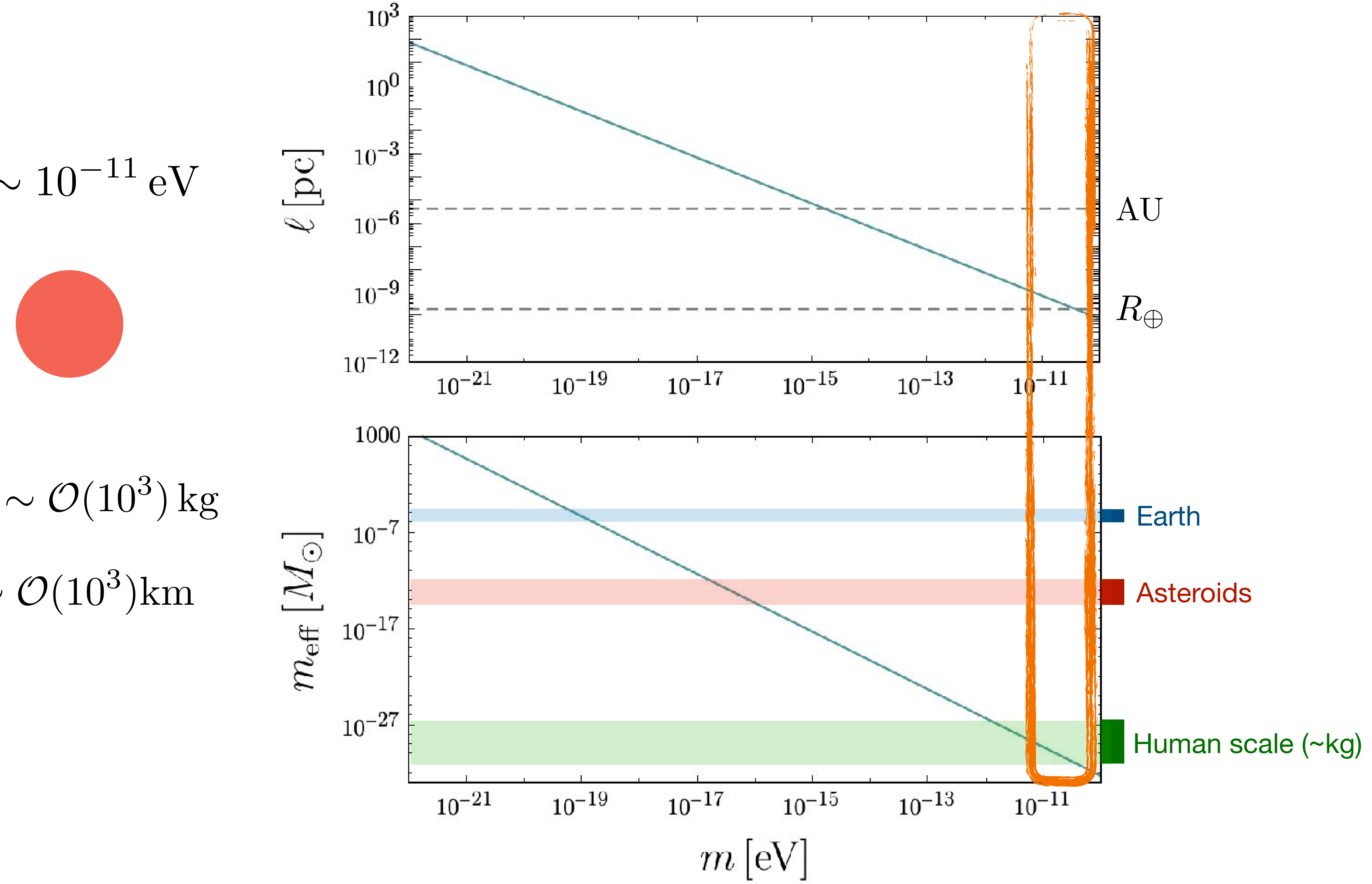
$\ell \sim \mathcal{O}(10)\mathrm{AU}$



$$m \sim 10^{-11}\,\mathrm{eV}$$

$$m_{\rm eff} \sim \mathcal{O}(10^3)\,\mathrm{kg}$$

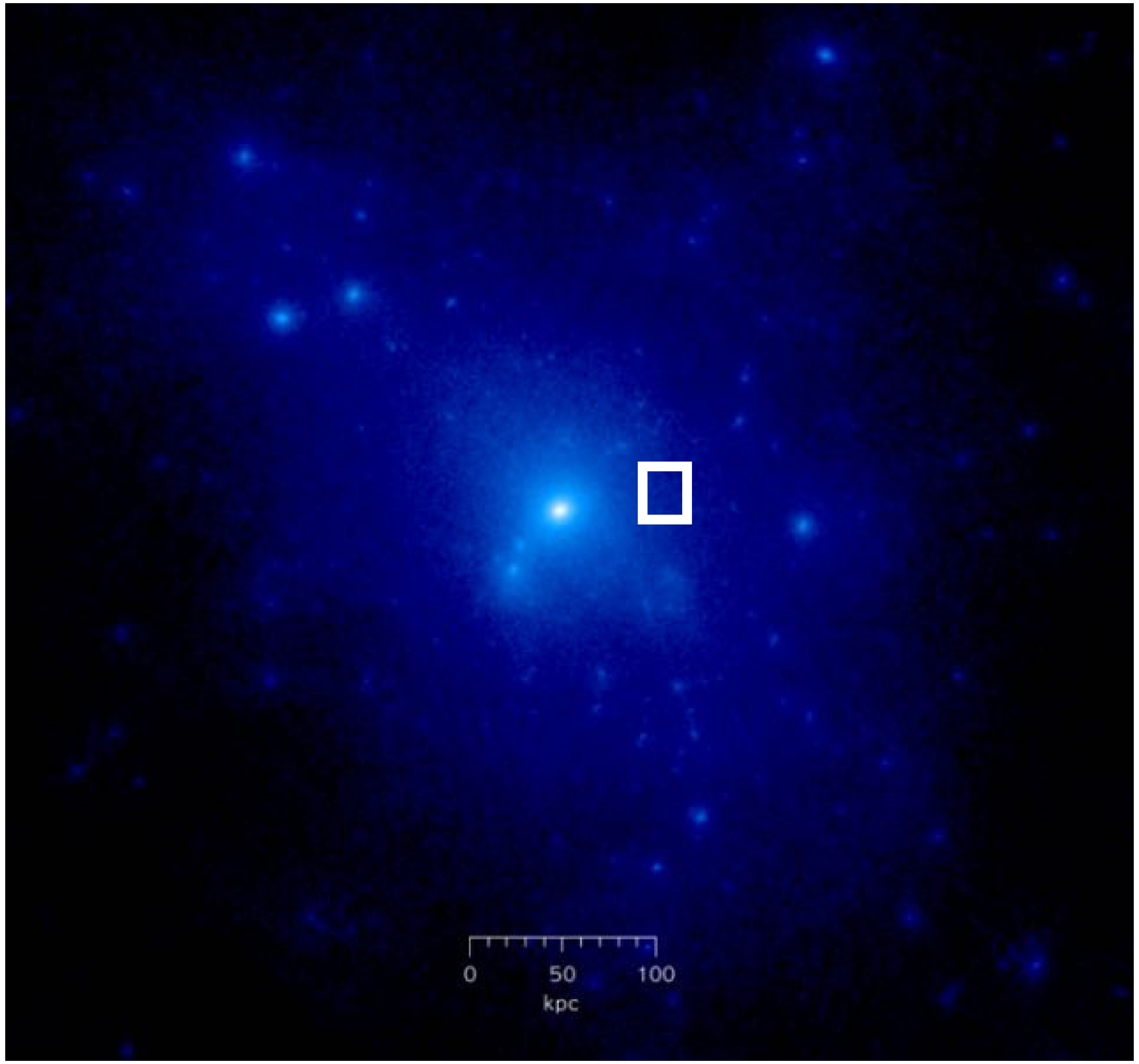
$$\ell \sim \mathcal{O}(10^3)\mathrm{km}$$

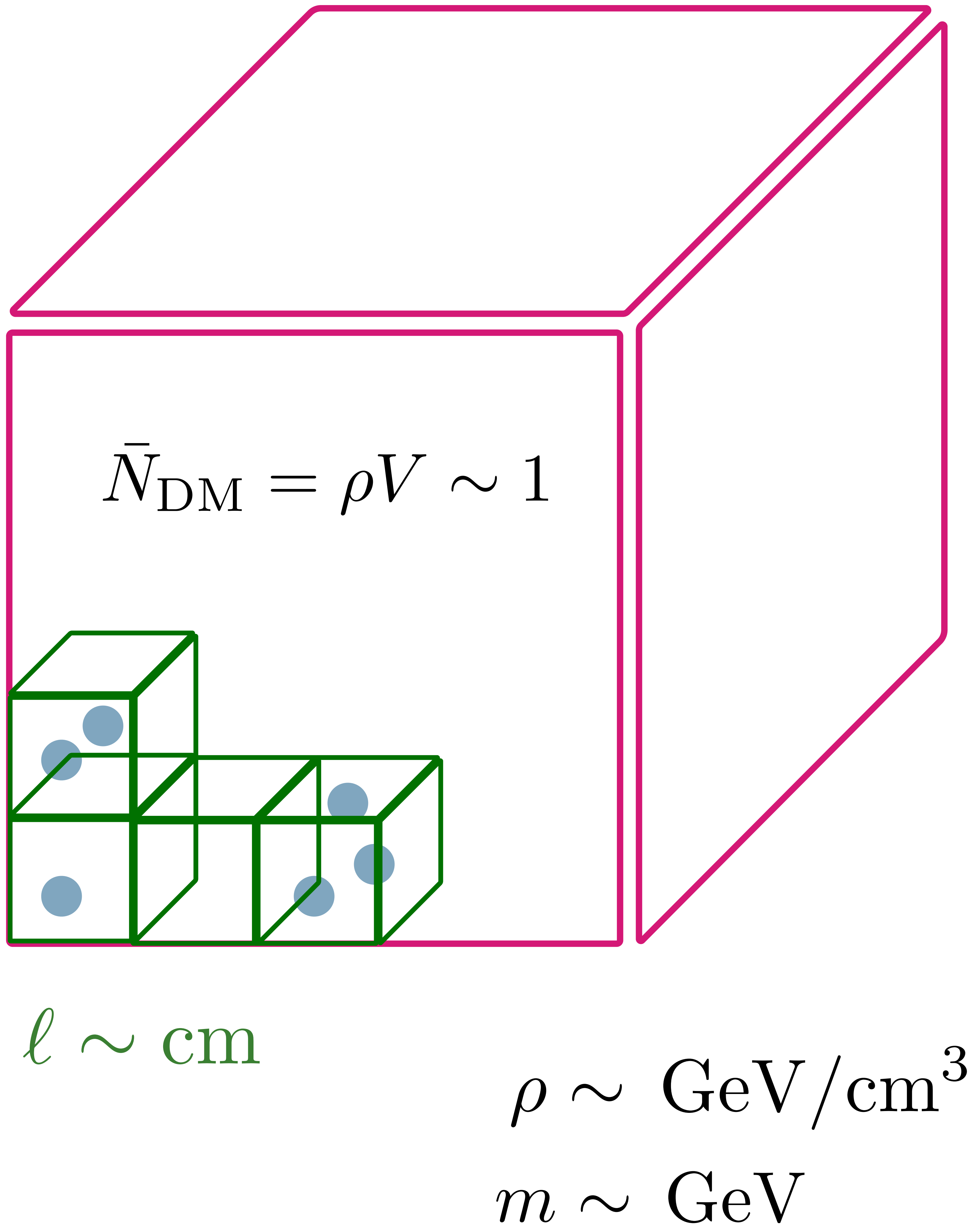


a concept of quasiparticle seems quite naive

but it provides a useful thinking tool to understand

the behavior of wave DM halo as well as the interaction with stellar objects *a large order one density fluctuation can naturally be understood in the particle dark matter halo*





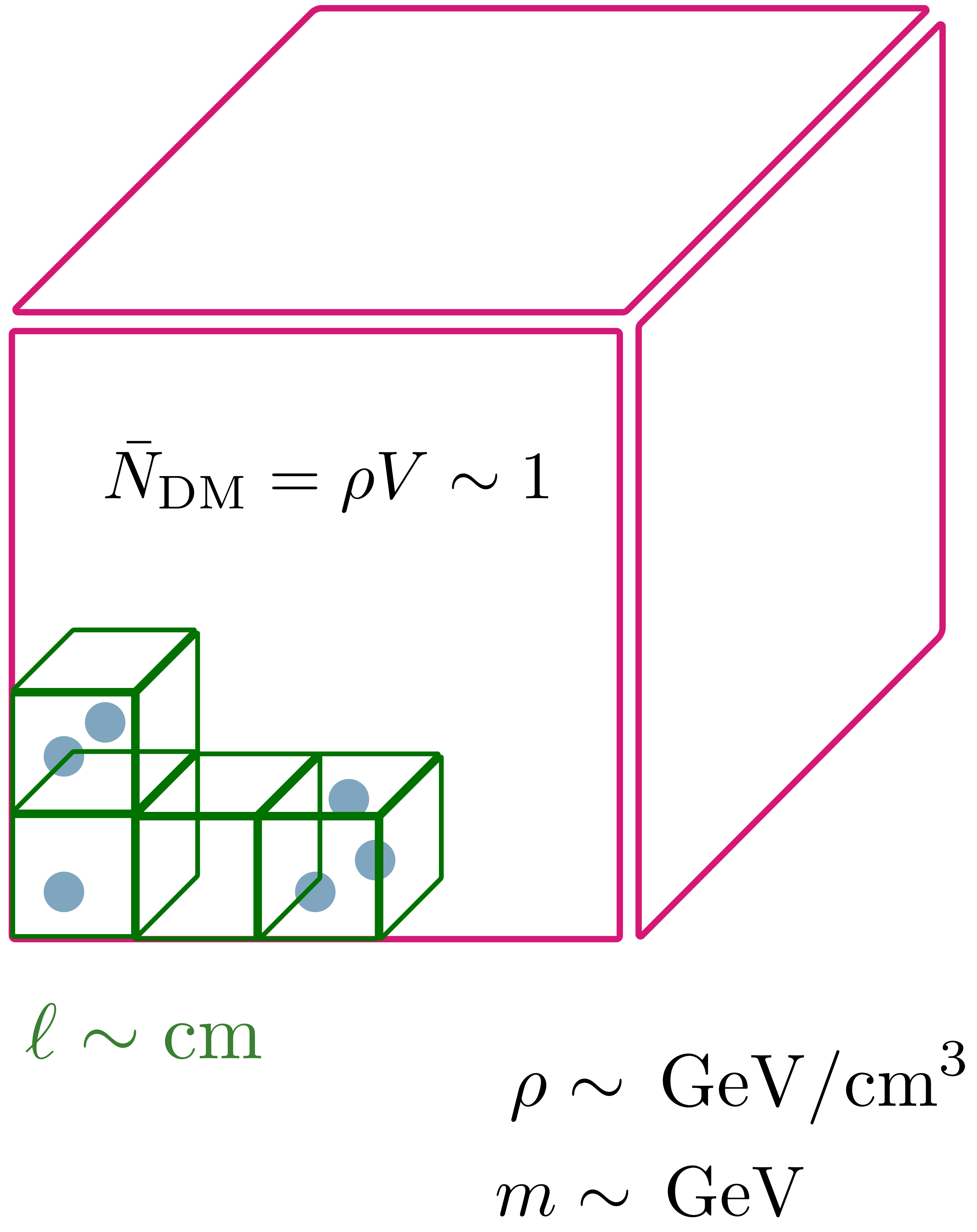
a large order one density fluctuation can naturally be understood in the particle dark matter halo

$$\langle N_{\rm DM}\rangle = \bar{N}_{\rm DM}$$

$$\langle \Delta N^2_{\rm DM}\rangle = \bar{N}_{\rm DM}$$

and therefore in this small volume

$$\frac{\delta\rho}{\rho} = \frac{1}{\sqrt{\bar{N}_{\rm DM}}} \sim \mathcal{O}(1)$$



if we instead choose larger volume

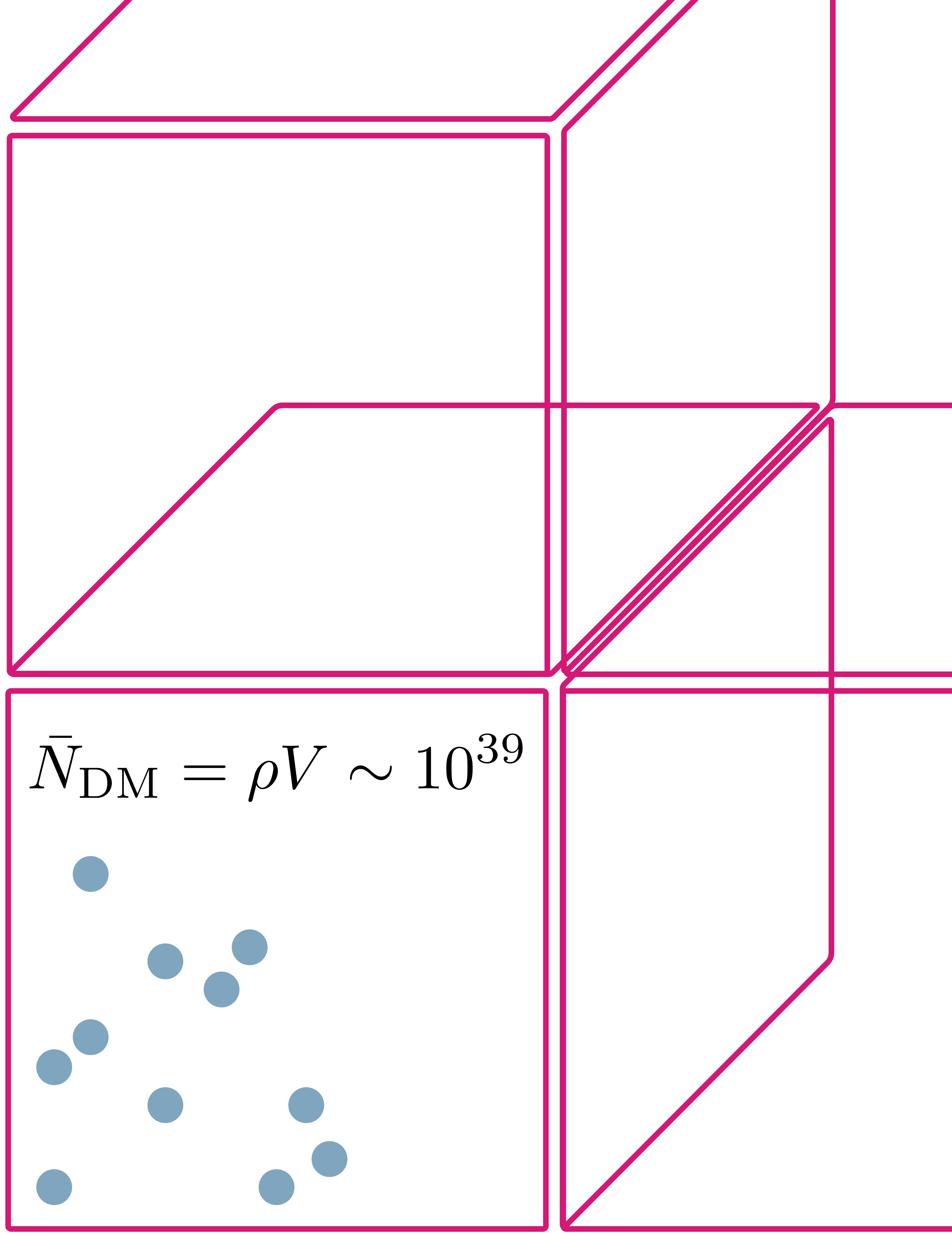
$$\langle N_{\rm DM}\rangle = \bar{N}_{\rm DM}$$

$$\langle \Delta N^2_{\rm DM}\rangle = \bar{N}_{\rm DM}$$

$$\gg 1$$

and therefore in this large volume

$$\frac{\delta\rho}{\rho} = \frac{1}{\sqrt{\bar{N}_{\rm DM}}} \ll 1$$



$\ell \sim {\rm AU}$

$\rho \sim {\rm GeV/cm^3}$

$m \sim {\rm GeV}$

we may apply the same argument for DM quasiparticles

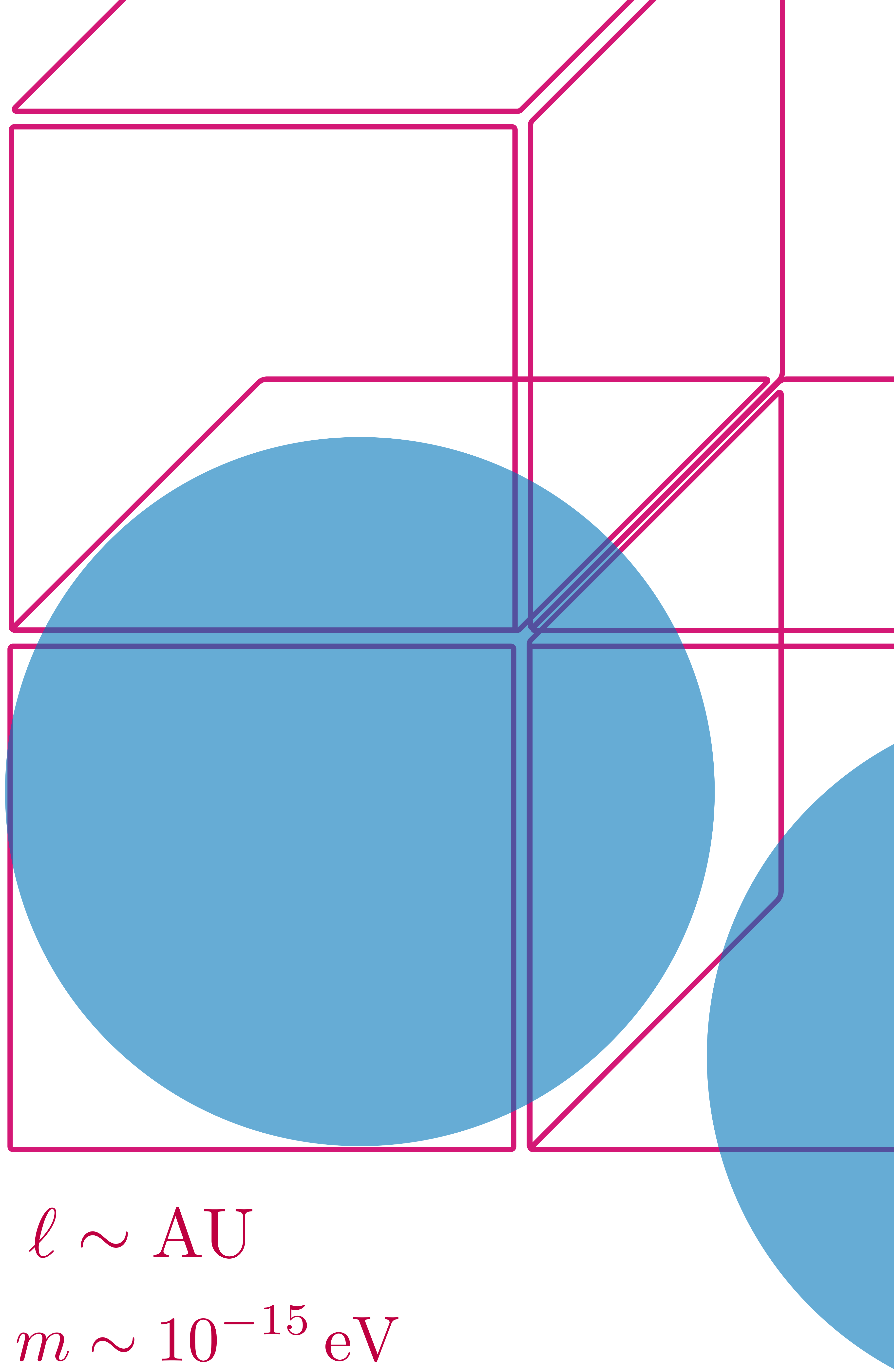
in particular over the scale of the wavelength

there's O(1) number of quasiparticle

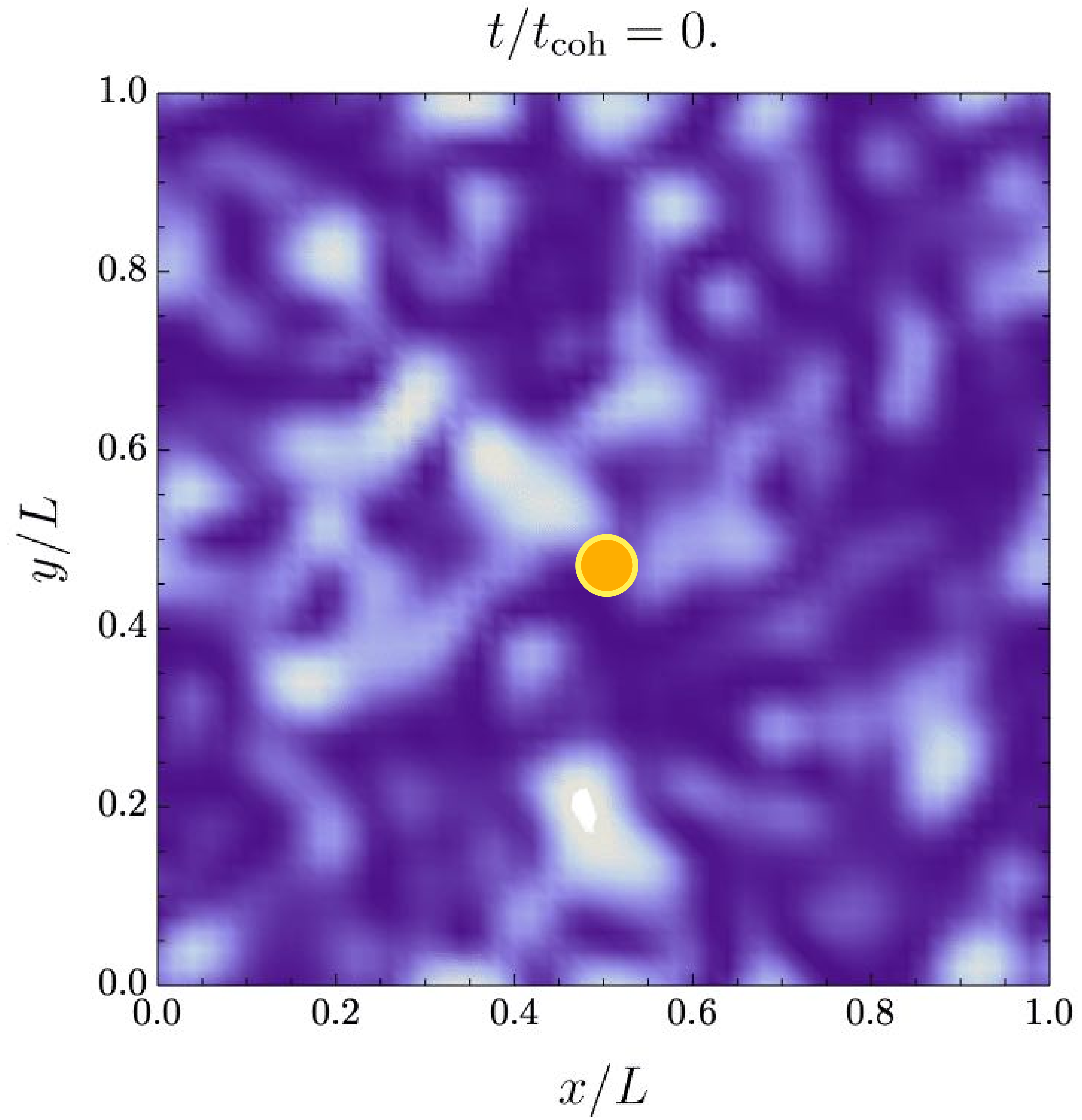
$$\frac{\delta\rho}{\rho} \sim 1 \qquad (\text{for } \ell \sim \lambda)$$

since the wavelength could be astronomical scales

it's natural to expect an order-one density fluctuation over such large spatial scales

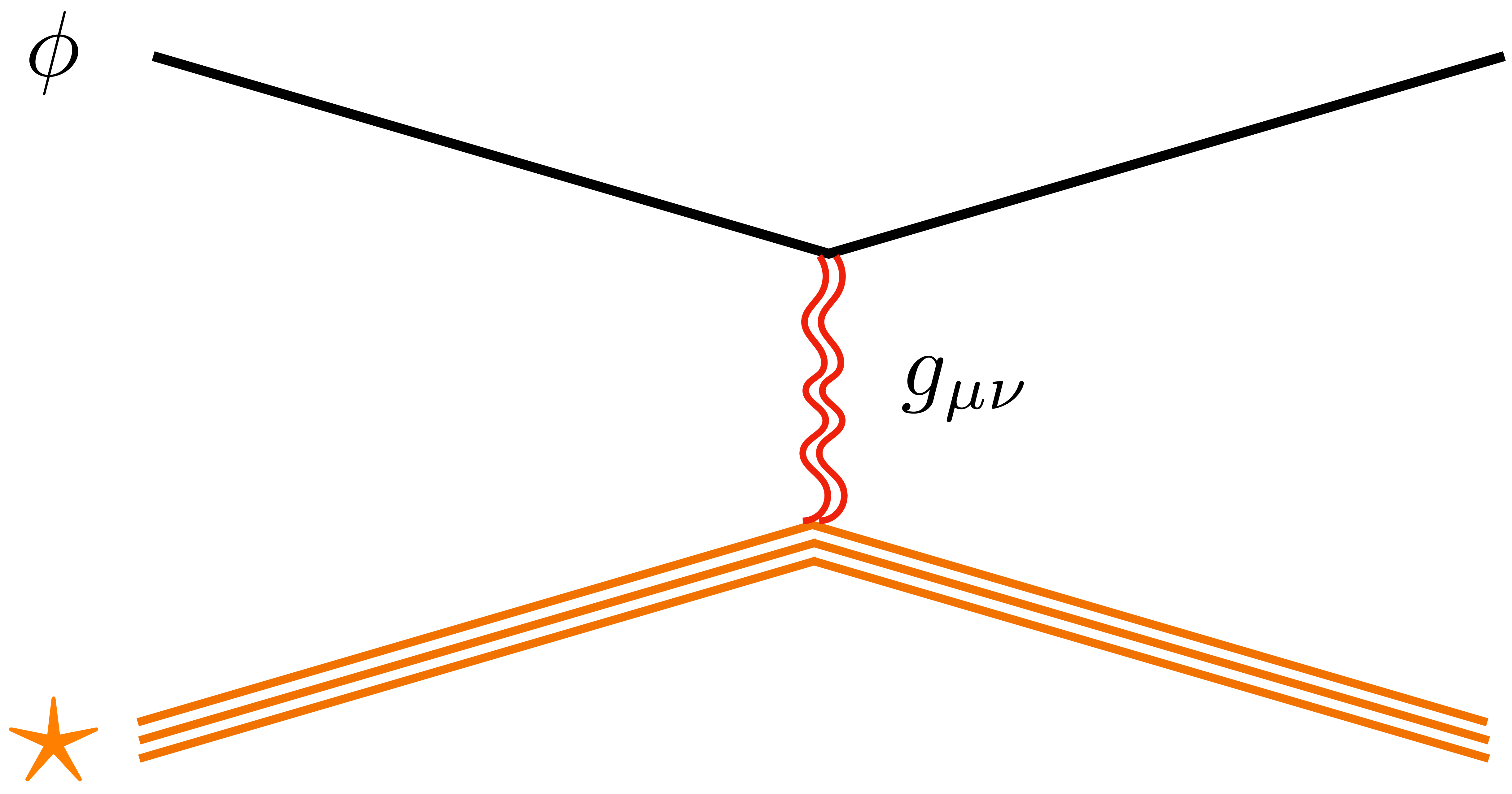


the usage of QP doesn't end here
it is also useful to understand the interaction
between wave DM and celestial objects



at the fundamental level

the interaction between wave DM and star might be represented by



$$\mathcal{M} \sim 16\pi G \frac{m^2 M_\star^2}{q^2}$$

$$V = \frac{GmM_\star}{r}$$

$$N_{\rm occ} \sim n_{\rm dm}\lambda^3$$

$$m_{\rm eff} = \rho_{\rm dm}\lambda^3$$

at the fundamental level

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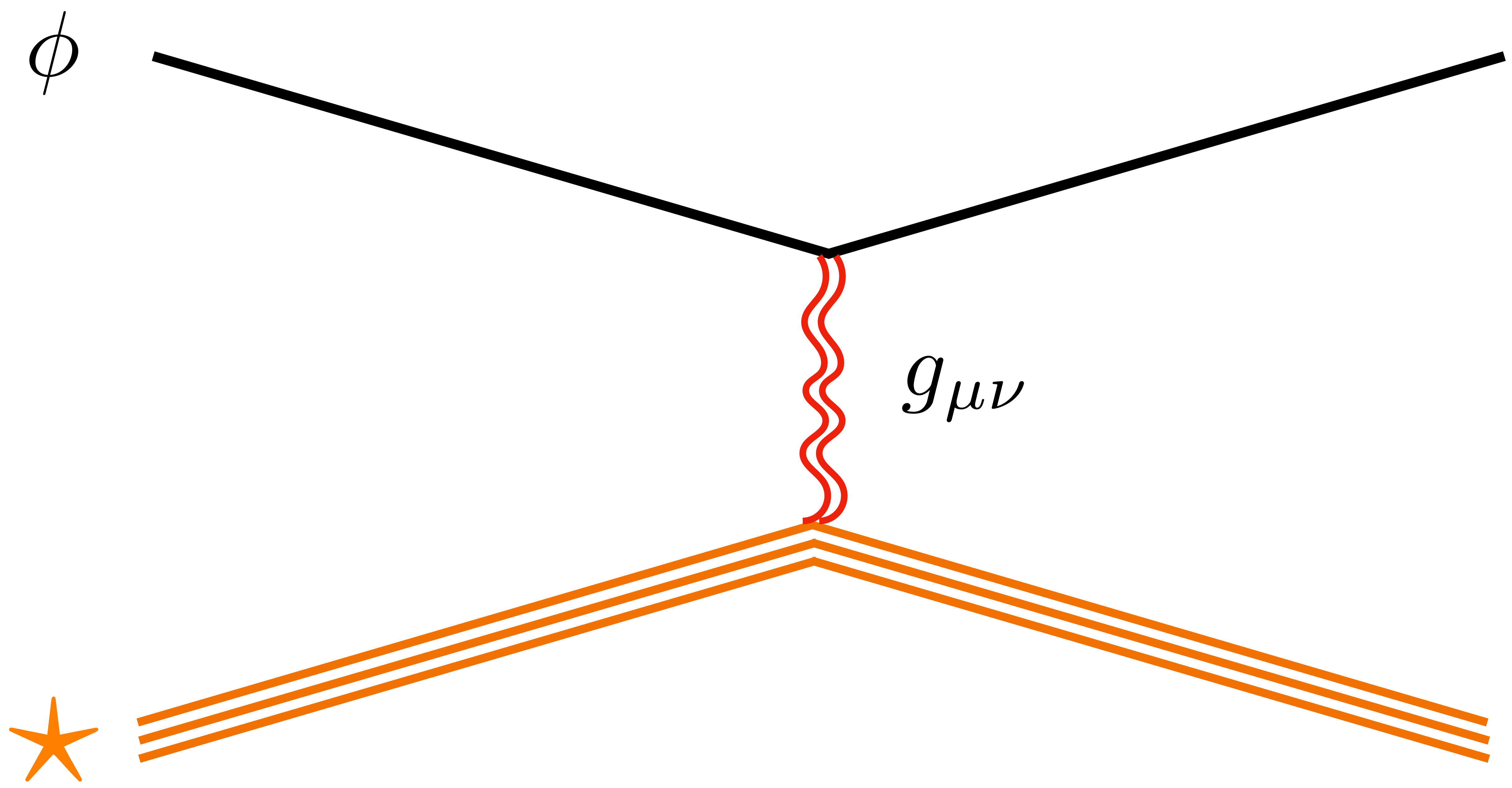


$$\mathcal{M} \sim 16\pi G \frac{m^2 M_\star^2}{q^2} \quad N_{\rm occ}$$

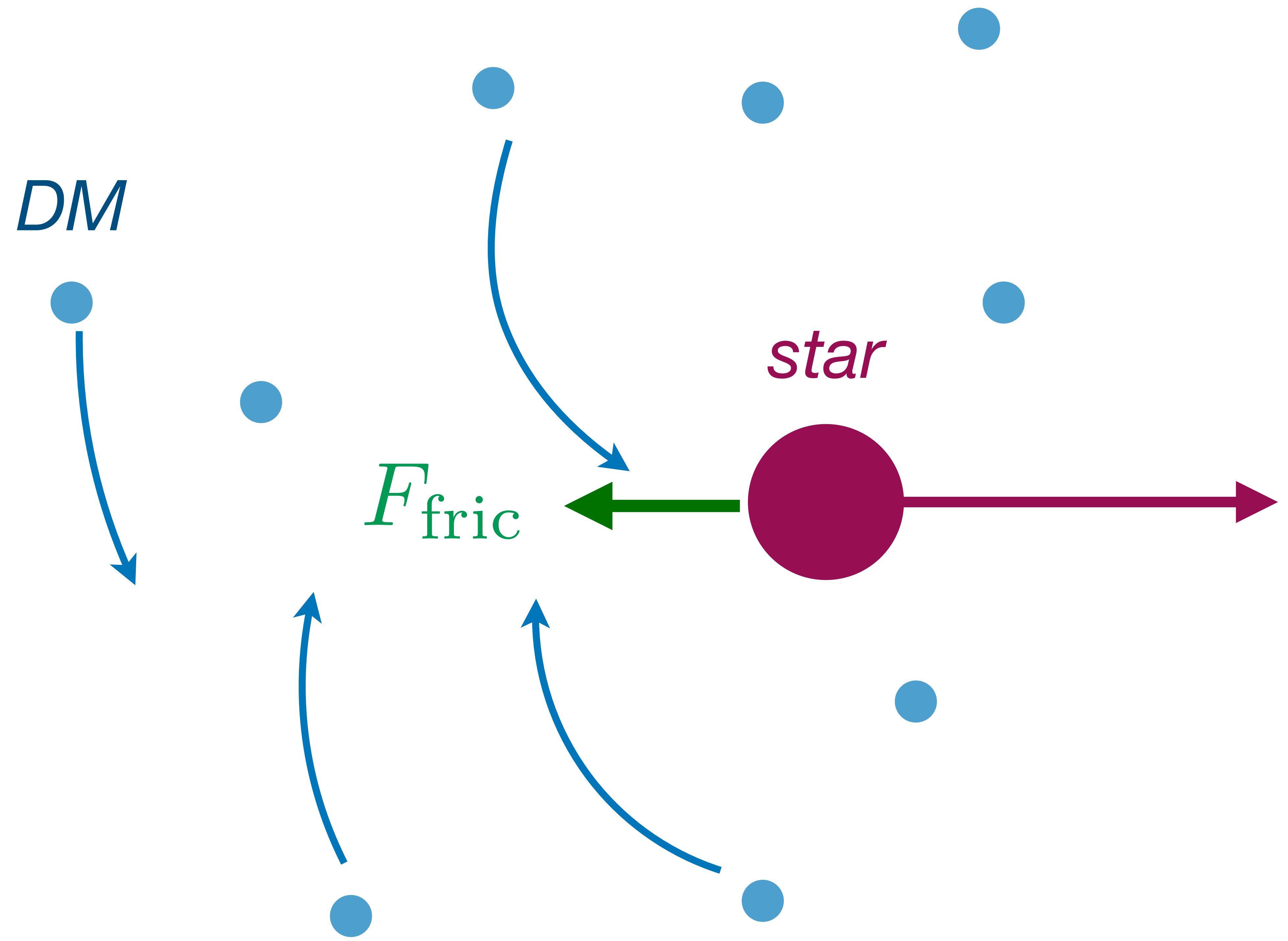
$$V = \frac{G m M_\star}{r} N_{\rm occ} = \frac{G M_\star m_{\rm eff}}{r}$$

while our argument are still at the heuristic level
it can be shown in more rigorously using kinetic equations (Boltzmann)
that this intuition generally holds for wave DM

[Bar-Or et al (19, 21); Chavanis (21); Bar, Blas, Blum, HK (21)] *phenomenologically, this interaction of wave DM may leave some interesting features in the motion of stars;*

the interaction between DM and stars

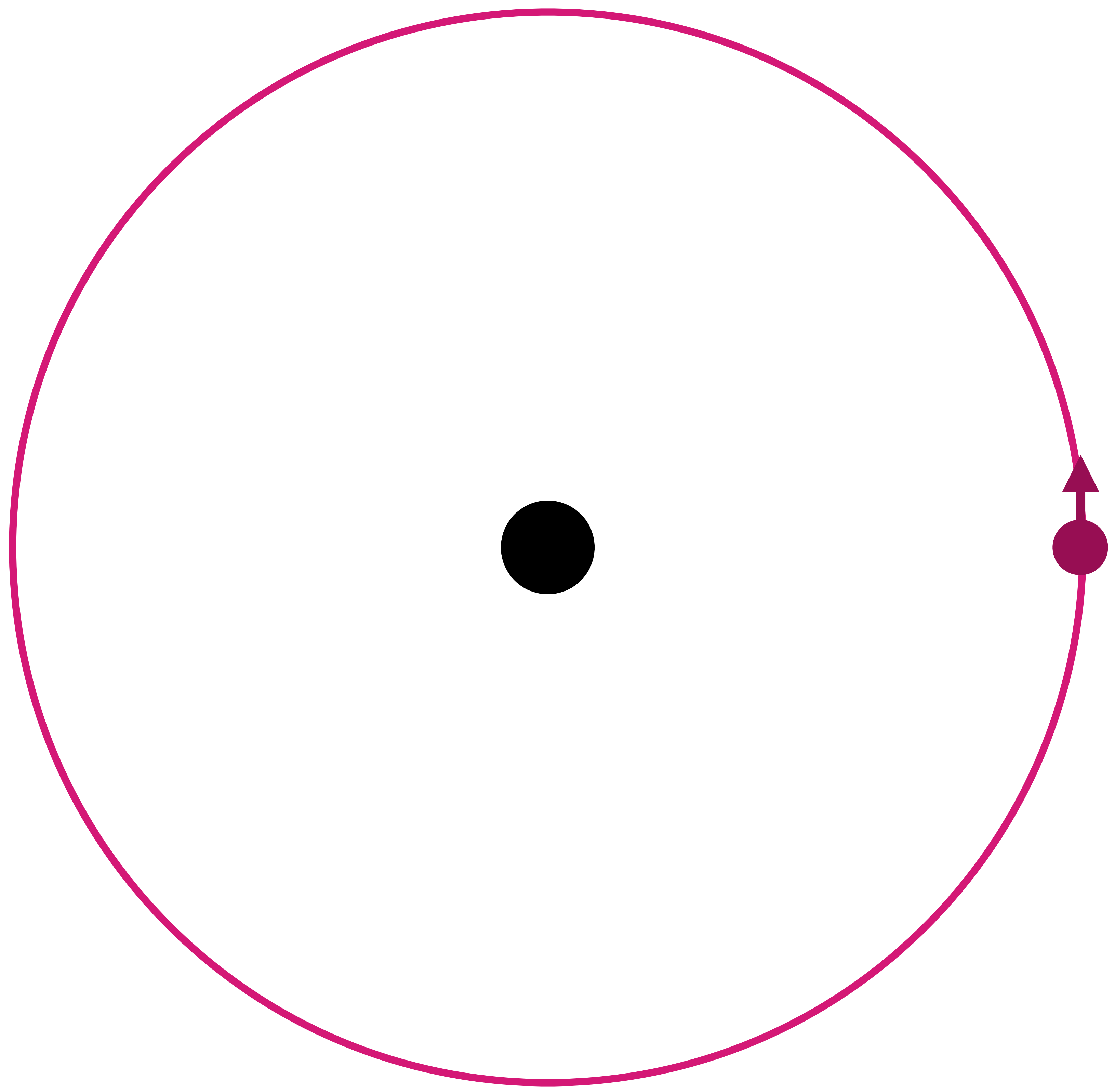
slows down stars, causing orbital decays

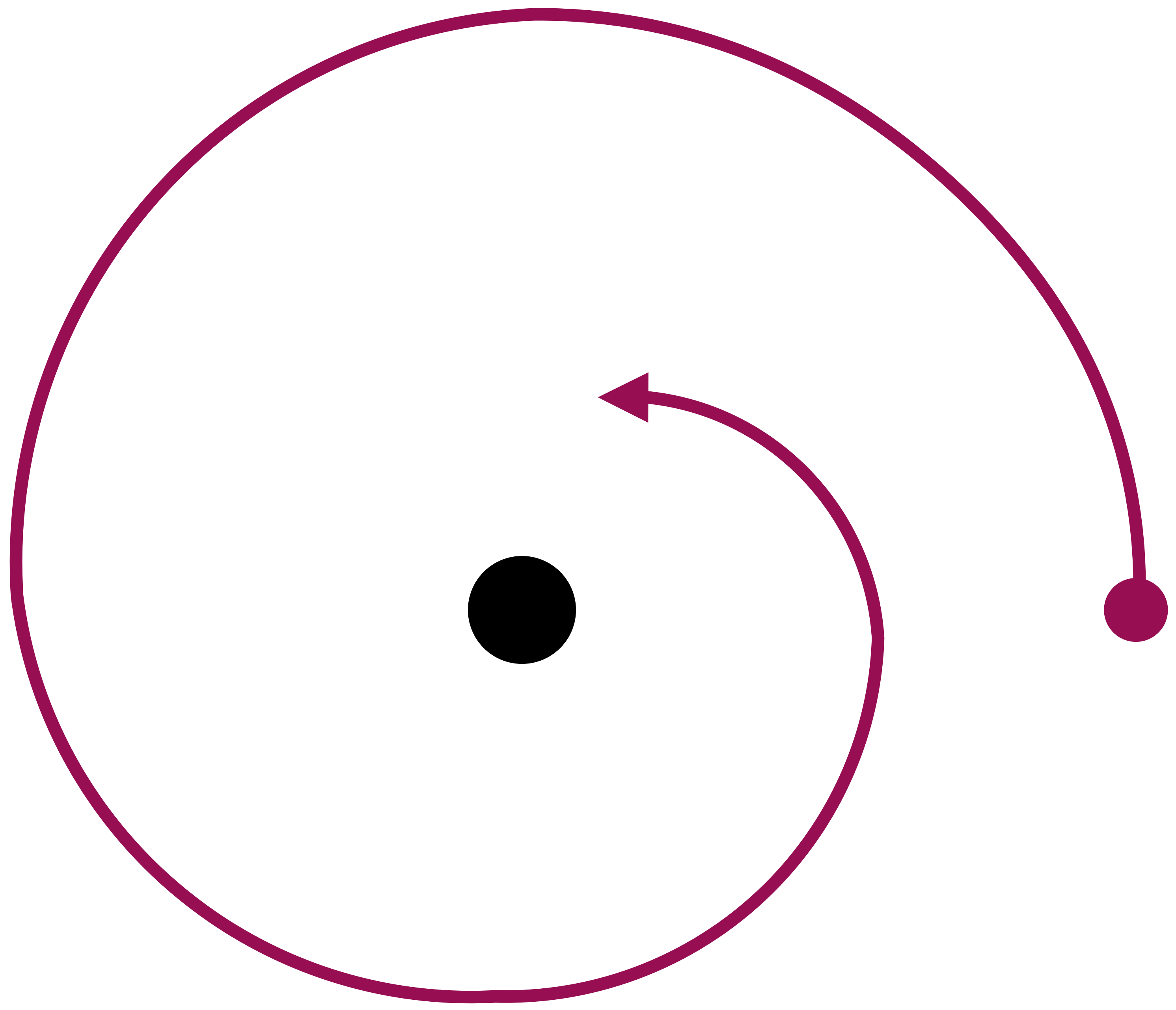


$$F \sim -\frac{4\pi G^2 \rho (m_{\rm DM} + m_\star)}{\sigma^2} \ln(b_{\rm max}/b_{\rm min}) G(X)$$

(classical) Dynamical friction

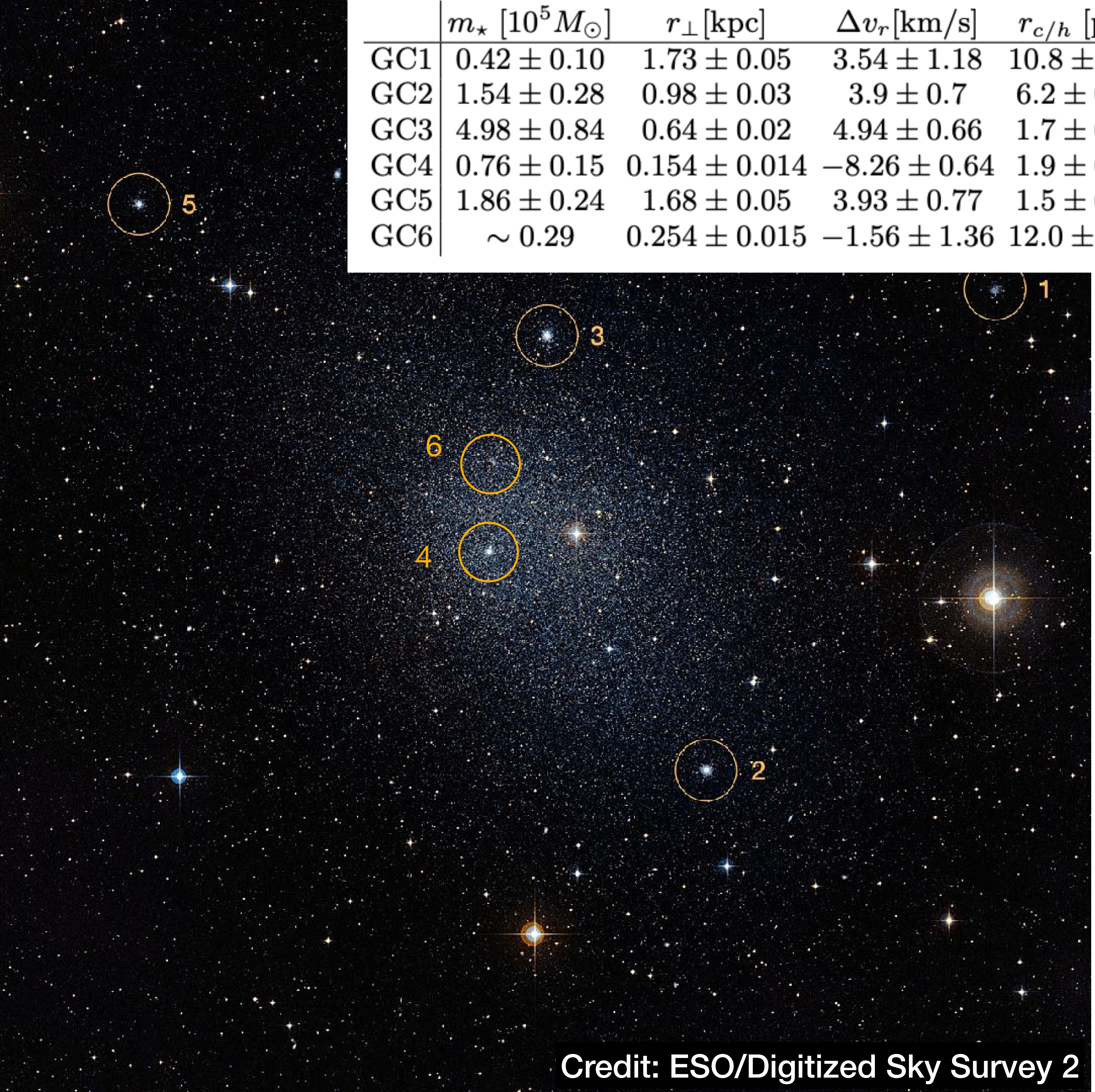
[Chandrasekhar (42)]





	$m_\star$ $[10^5 M_\odot]$	$r_\perp$[kpc]	Δv_r[km/s]	$r_{c/h}$ [pc]	Refs.	$\tau_{\rm CDM}$ [Gyr]	$\tau^{(135)}_{\rm DDM}$ [Gyr]	$\tau_{\rm SIDM}$ [Gyr]
GC1	0.42 ± 0.10	1.73 ± 0.05	3.54 ± 1.18	10.8 ± 0.3	[18, 19, 56–58]	119	122	79.3
GC2	1.54 ± 0.28	0.98 ± 0.03	3.9 ± 0.7	6.2 ± 0.2	[18, 19, 58, 59]	14.7	7.12	8.82
GC3	4.98 ± 0.84	0.64 ± 0.02	4.94 ± 0.66	1.7 ± 0.1	[18, 19, 60, 61]	2.63	1.48	2.21
GC4	0.76 ± 0.15	0.154 ± 0.014	-8.26 ± 0.64	1.9 ± 0.2	[18, 19, 60, 61]	0.91	10.7	14.8
GC5	1.86 ± 0.24	1.68 ± 0.05	3.93 ± 0.77	1.5 ± 0.1	[18, 19, 56, 60, 61]	32.2	30.1	20
GC6	~ 0.29	0.254 ± 0.015	-1.56 ± 1.36	12.0 ± 1.4	[15, 50]	5.45	16.1	22

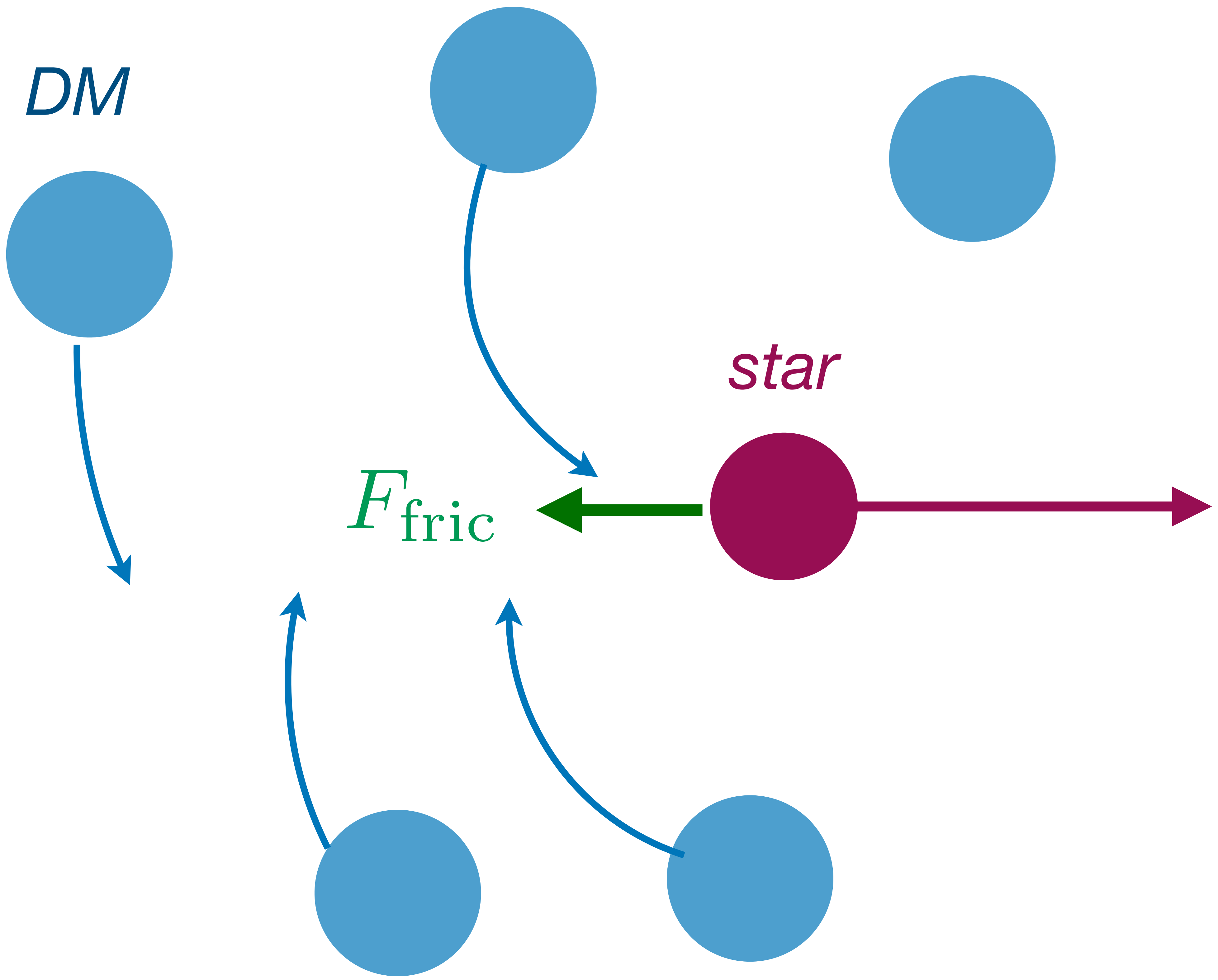
[Bar, Blas, Blum, HK (21)]



Fornax GC timing problem

[Tremaine (76)]

similarly wave DM (QP) particles will interact with stars and exert dragging force … …



$$F \sim -\frac{4\pi G^2 \rho (m_{\rm eff} + m_\star)}{\sigma^2} \ln(b_{\rm max}/\lambda) G(X)$$

	$m_\star$ [$10^5 M_\odot$]	$r_\perp$[kpc]	Δv_r[km/s]	$r_{c/h}$ [pc]	Refs.	$\tau_{\rm CDM}$ [Gyr]	$\tau^{(135)}_{\rm DDM}$ [Gyr]	$\tau_{\rm SIDM}$ [Gyr]
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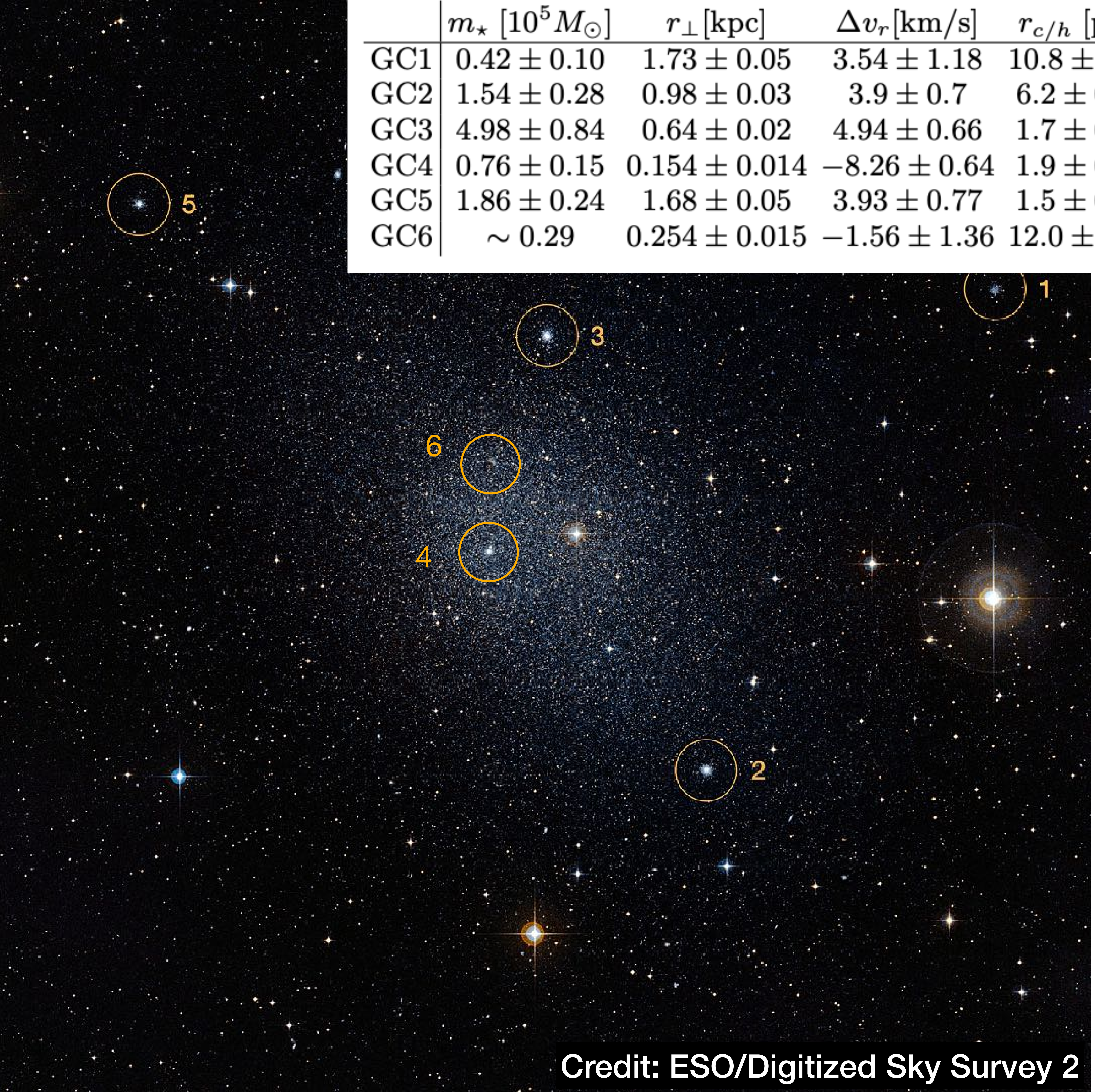
[Bar, Blas, Blum, HK (21)]



Fornax GC timing problem

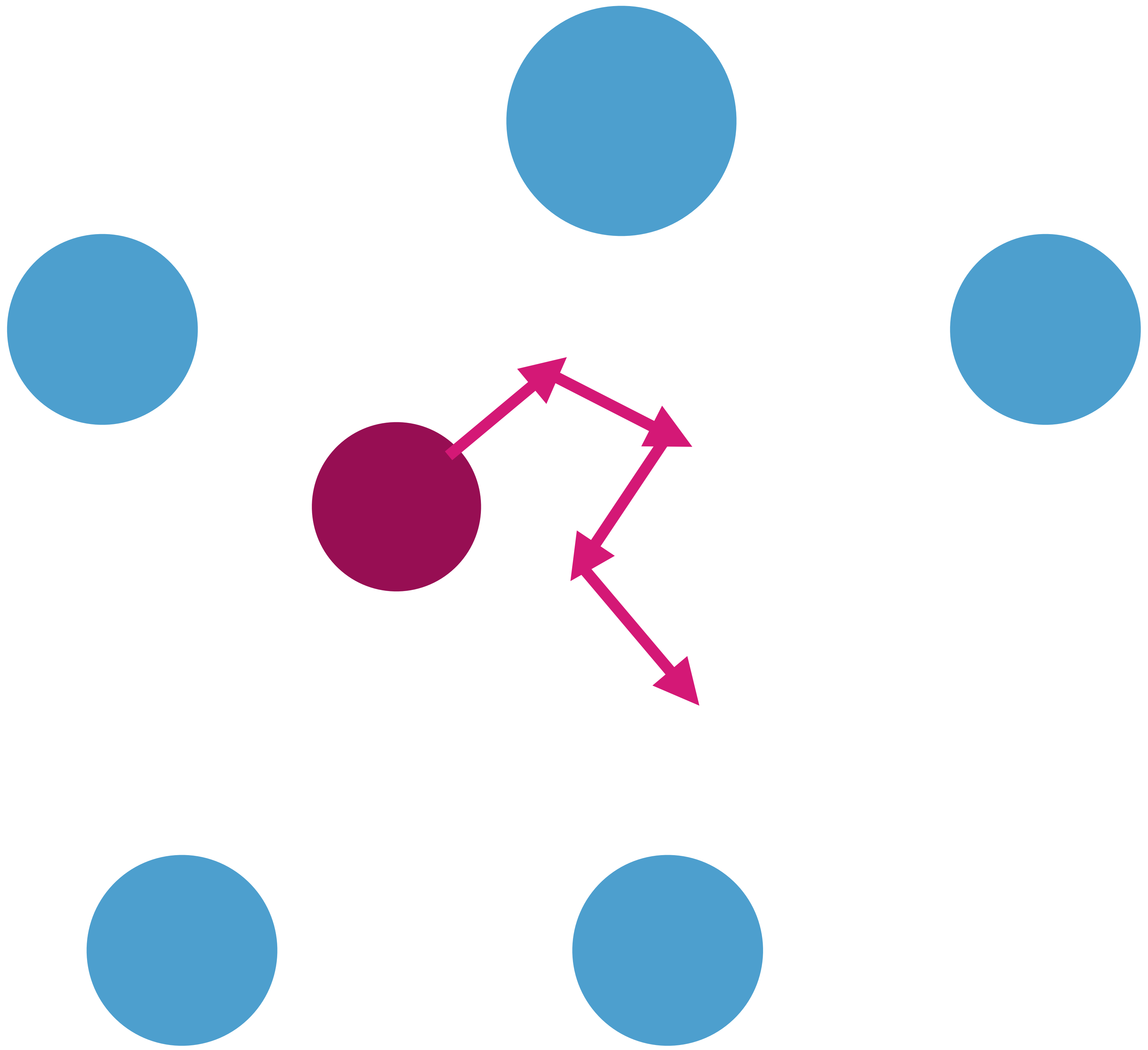
[Tremaine (76)]

	Projected radius	Cluster mass	CDM		FDM		
n	$r_\perp$ (kpc)	$m_{\rm cl}$ ($M_\odot$)	C	τ (Gyr)	kr	C	τ (Gyr)
1	1.6	3.7×10^4	4.29	112	8.90	2.46	215
2	1.05	1.82×10^5	3.32	9.7	5.04	1.88	12
3	0.43	3.63×10^5	2.45	0.62	0.97	0.29	2.2
4	0.24	1.32×10^5	2.50	0.37	0.31	0.033	10
5	1.43	1.78×10^5	3.46	21.3	7.79	2.32	31

[Hui et al (17)]

sometimes quasiparticles bombard stars

injecting additional energies into a stellar population



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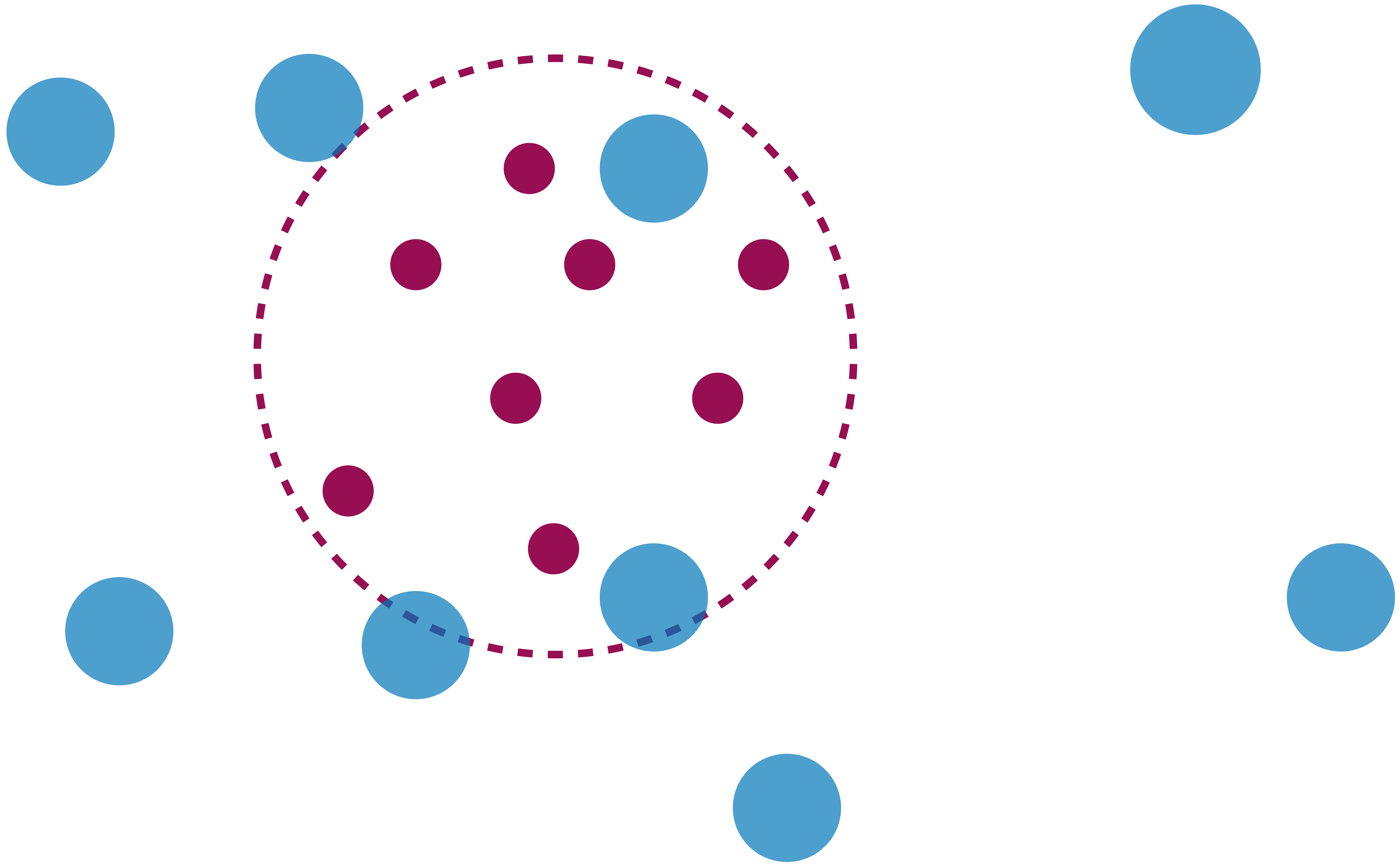

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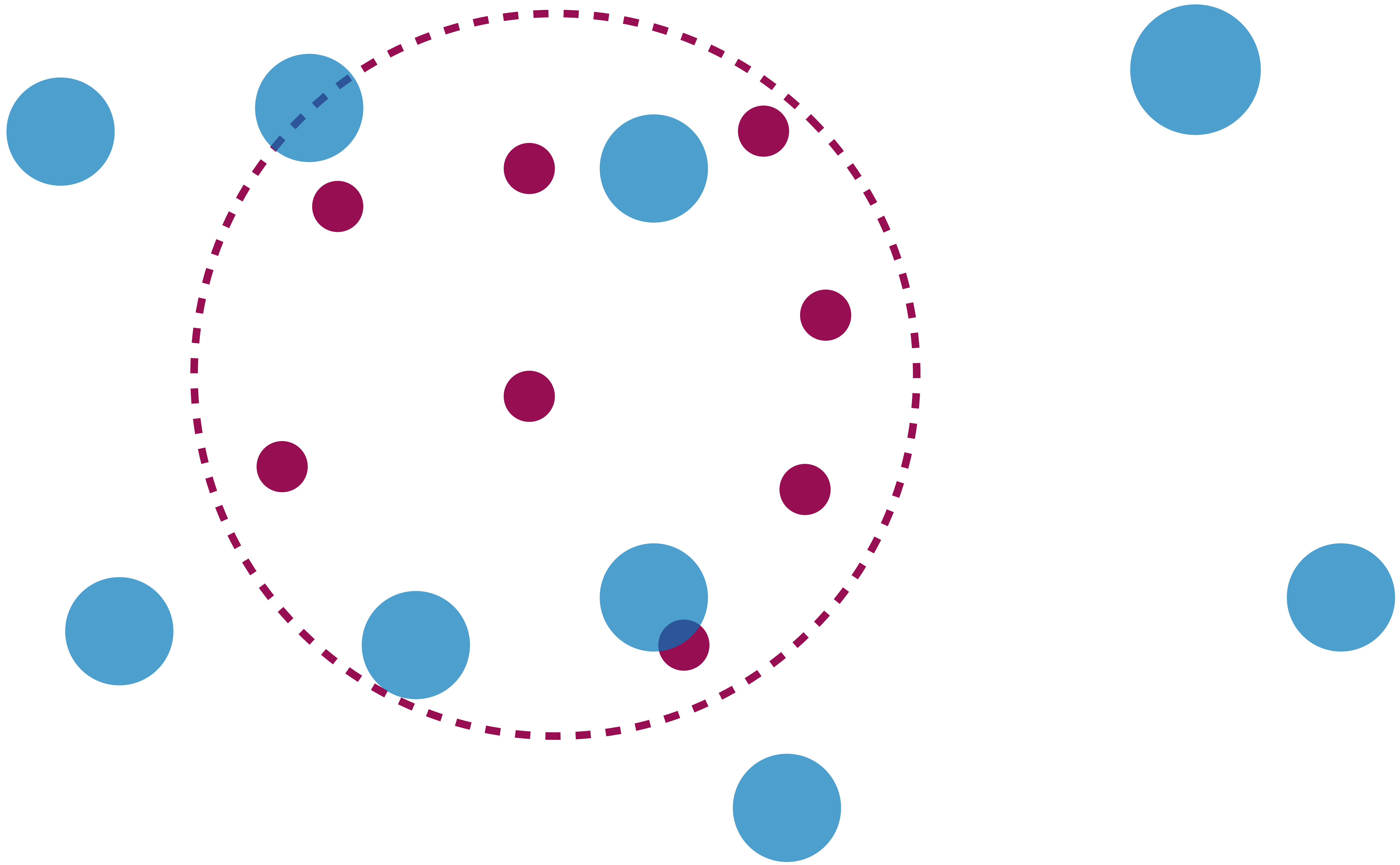
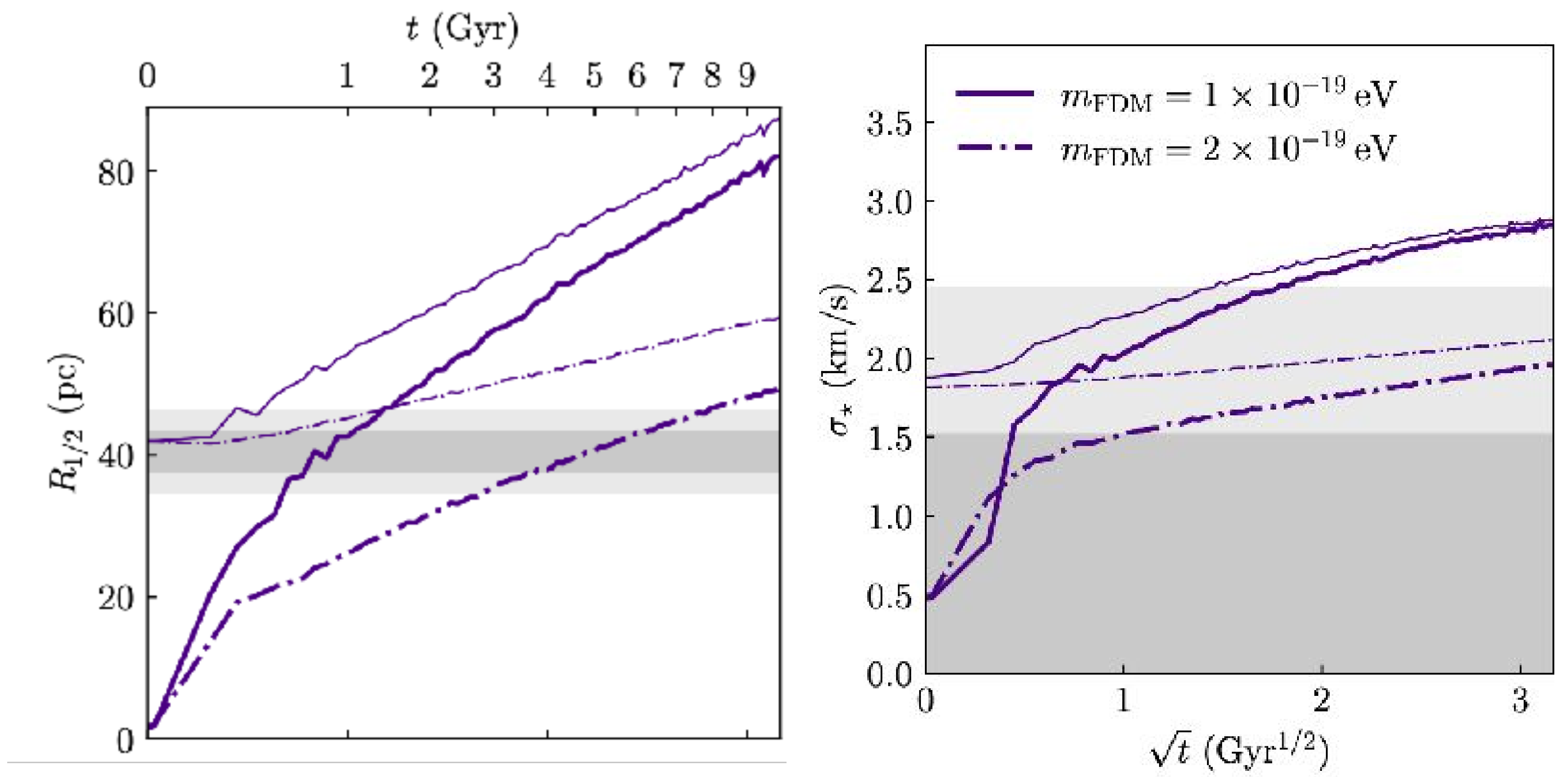
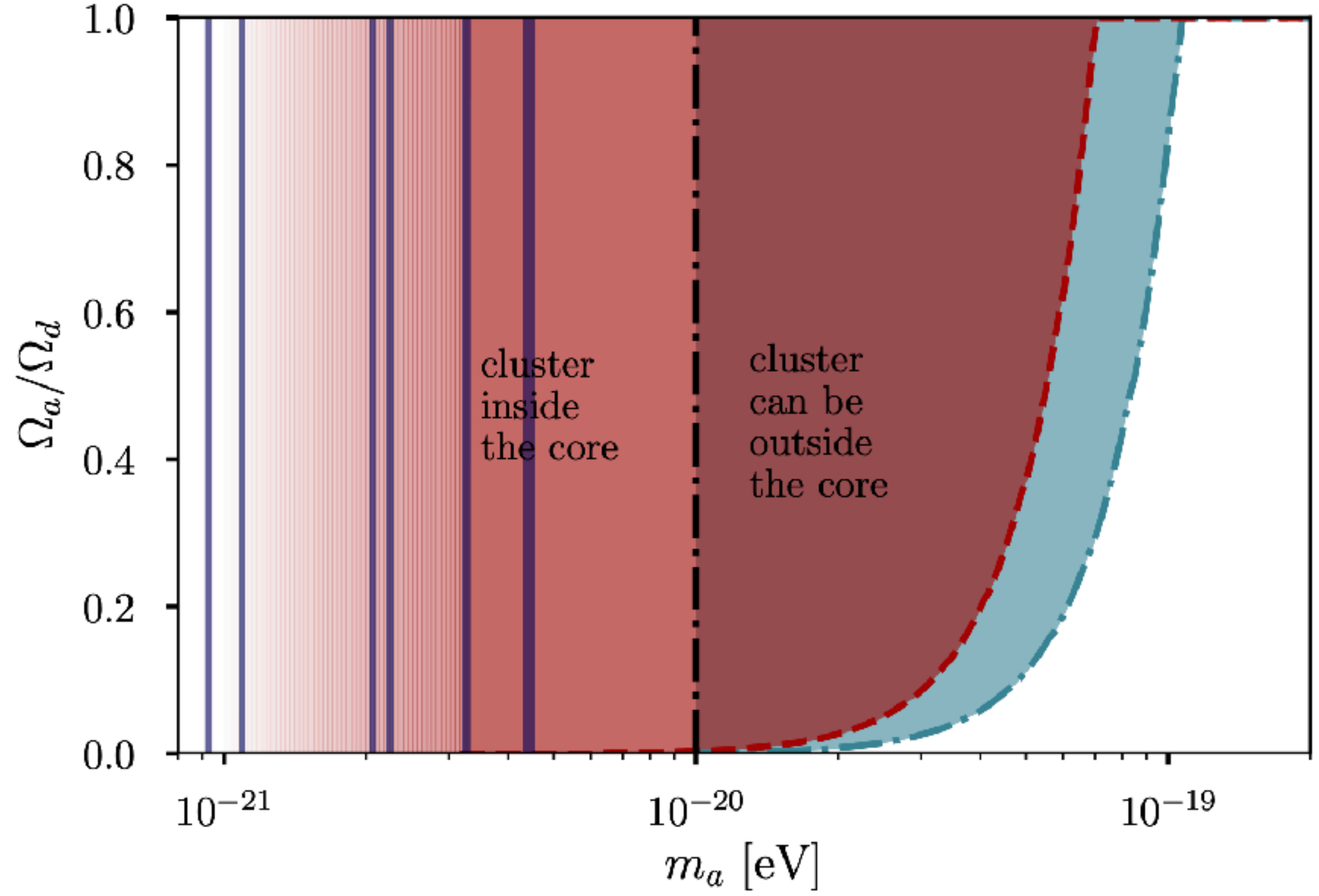


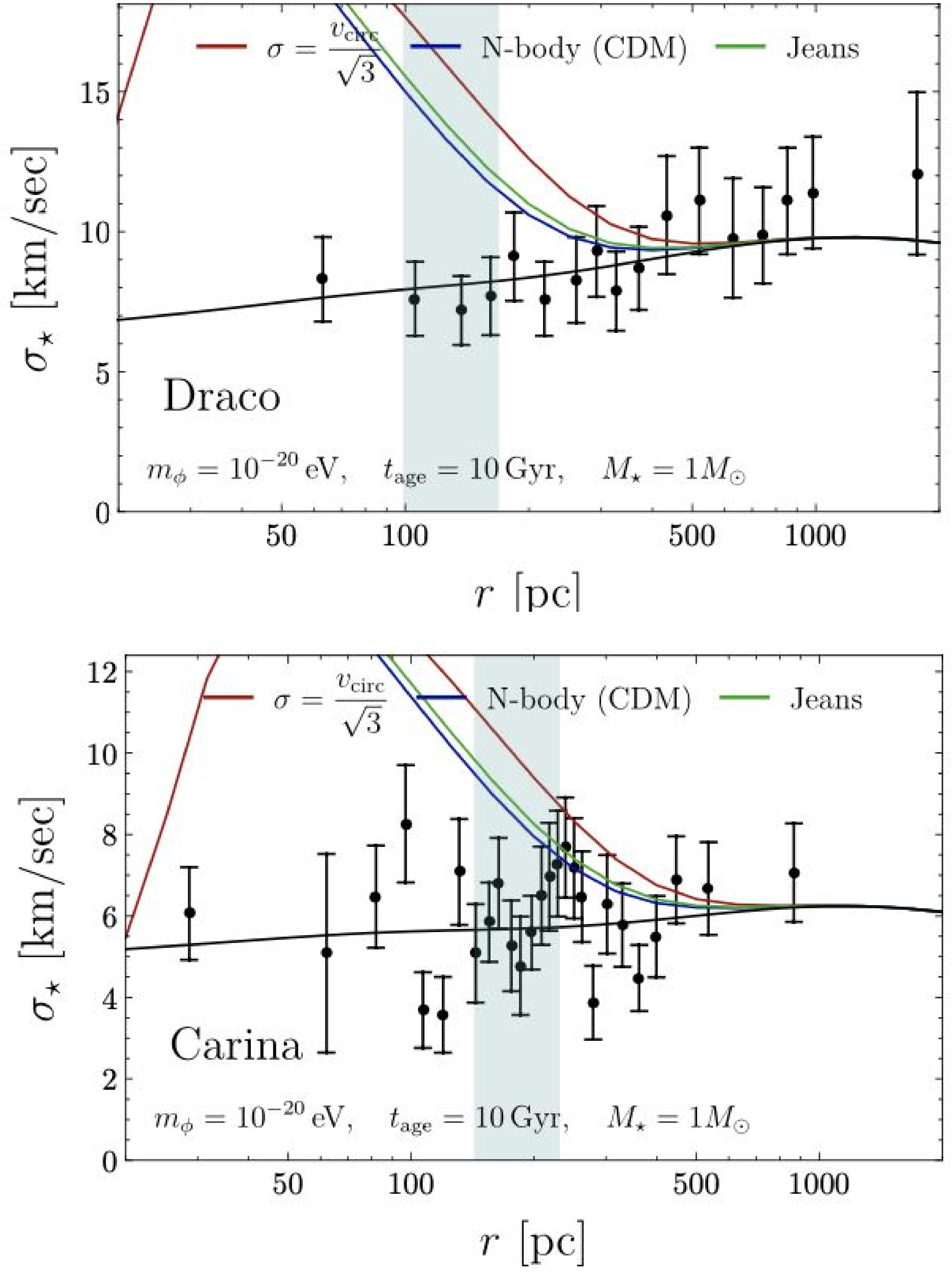
This ‘heating’ by quasiparticles may dissolve
star clusters, change velocity dispersion of stars,
perturb galactic disc as well as other stellar substructures …



[Dalal, Kravtsov (22)]



[Marsh and Niemeyer (19)]



[Blum, Eby, HK (unpublished)]

a deeper understanding of wave DM / QP

can be achieved by studying statistical properties of wave DM

let us begin with the dark matter field itself

$$\hat{\phi} = \sum_{i} \frac{1}{\sqrt{2\omega_i}} \left[a_i \psi_i + a_i^\dagger \psi_i^* \right]$$

Quantum # (green: i)

Operators (blue: a_i, $a_i^\dagger$)

Wave func. (red: ψ_i, ψ_i^*)

the wave function can be obtained
by solving the wave equation in the non-relativistic limit

$$\epsilon_i \Psi_i = H \Psi_i = \left[-\frac{\nabla^2}{2m} + \Phi \right] \Psi_i$$

$$\psi_i = e^{-i(m+\epsilon_i)t} \Psi_i$$

let us begin with the dark matter field itself

$$\hat{\phi} = \sum_i \frac{1}{\sqrt{2\omega_i}} \left[a_i \psi_i + a_i^\dagger \psi_i^* \right]$$

Quantum #

Operators

Wave func.

what to do with these operators?

to describe finite density system

we should specify the density operator

$$\hat{\rho} = \prod_i \hat{\rho}_i \otimes$$

$$\hat{\rho}_i = \int d^2\alpha_i \, P(\alpha_i) |\alpha_i\rangle\langle\alpha_i|$$

[Kim and Lenoci 21]

quasi-probability distribution

$$P(\alpha_i) = \frac{1}{\pi f_i} \exp\left[-\frac{|\alpha_i|^2}{f_i} \right]$$

$$a_i|\alpha_i\rangle = \alpha_i|\alpha_i\rangle$$

$$\alpha_i \in \mathbb{C}$$

$$\langle \hat{\mathcal{O}} \rangle = \mathrm{Tr}(\hat{\rho}\hat{\mathcal{O}})$$

let us begin with the dark matter field itself

$$\hat{\phi} = \sum_{i} \frac{1}{\sqrt{2\omega_i}} \left[a_i \psi_i + a_i^\dagger \psi_i^* \right]$$

Quantum #

Operators

Wave func.

more simplistically or conveniently

we can forget about this operators by replacing them with complex numbers

$$(a_i, a_i^\dagger) \to (\alpha_i, \alpha_i^*)$$

quasi-probability distribution

$$P(\alpha_i) = \frac{1}{\pi f_i} \exp\left[-\frac{|\alpha_i|^2}{f_i} \right]$$

[Derevianko 18]

[Foster, Rodd, Safdi 18]

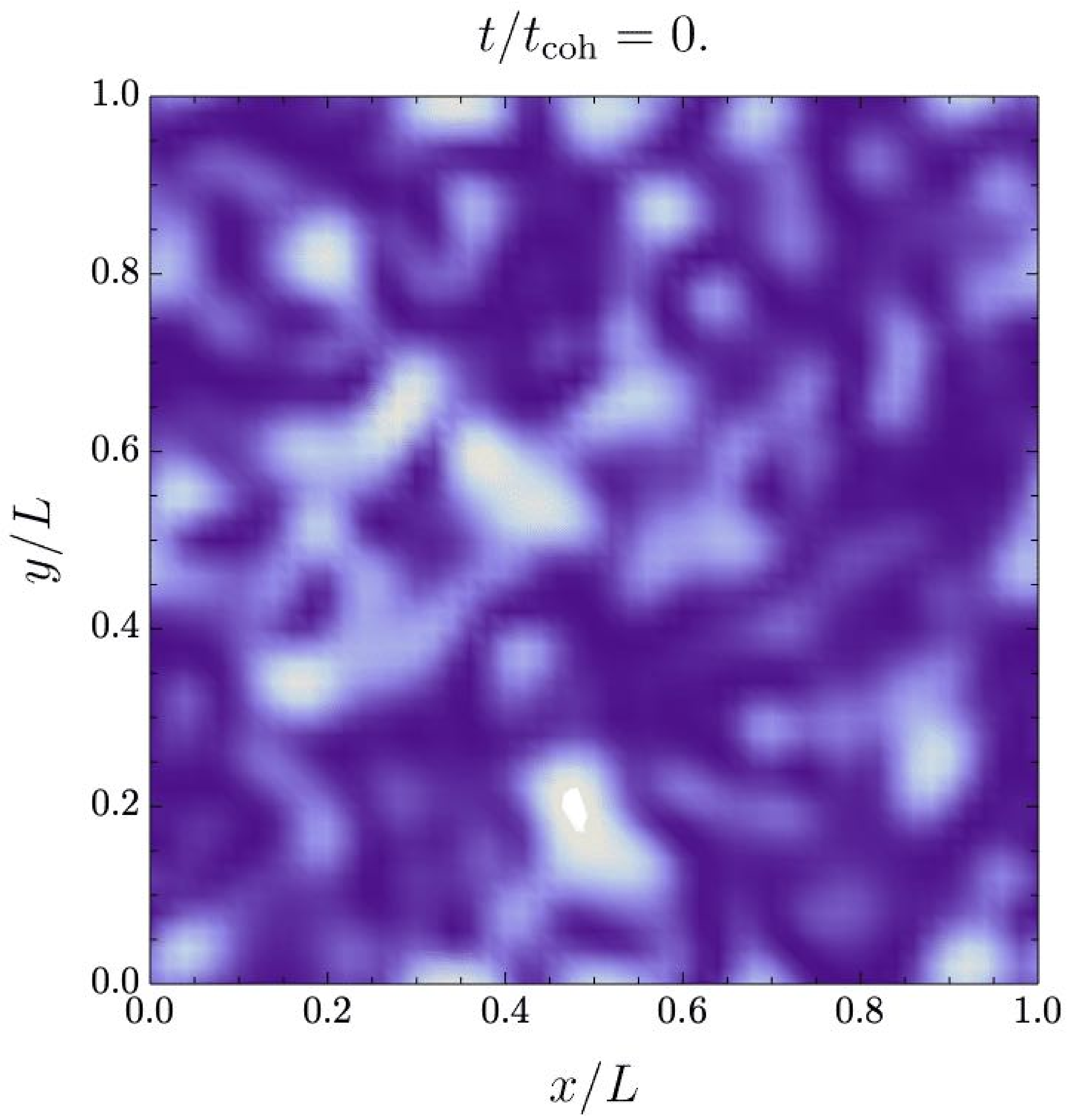
[Center et al 20]; [… …]

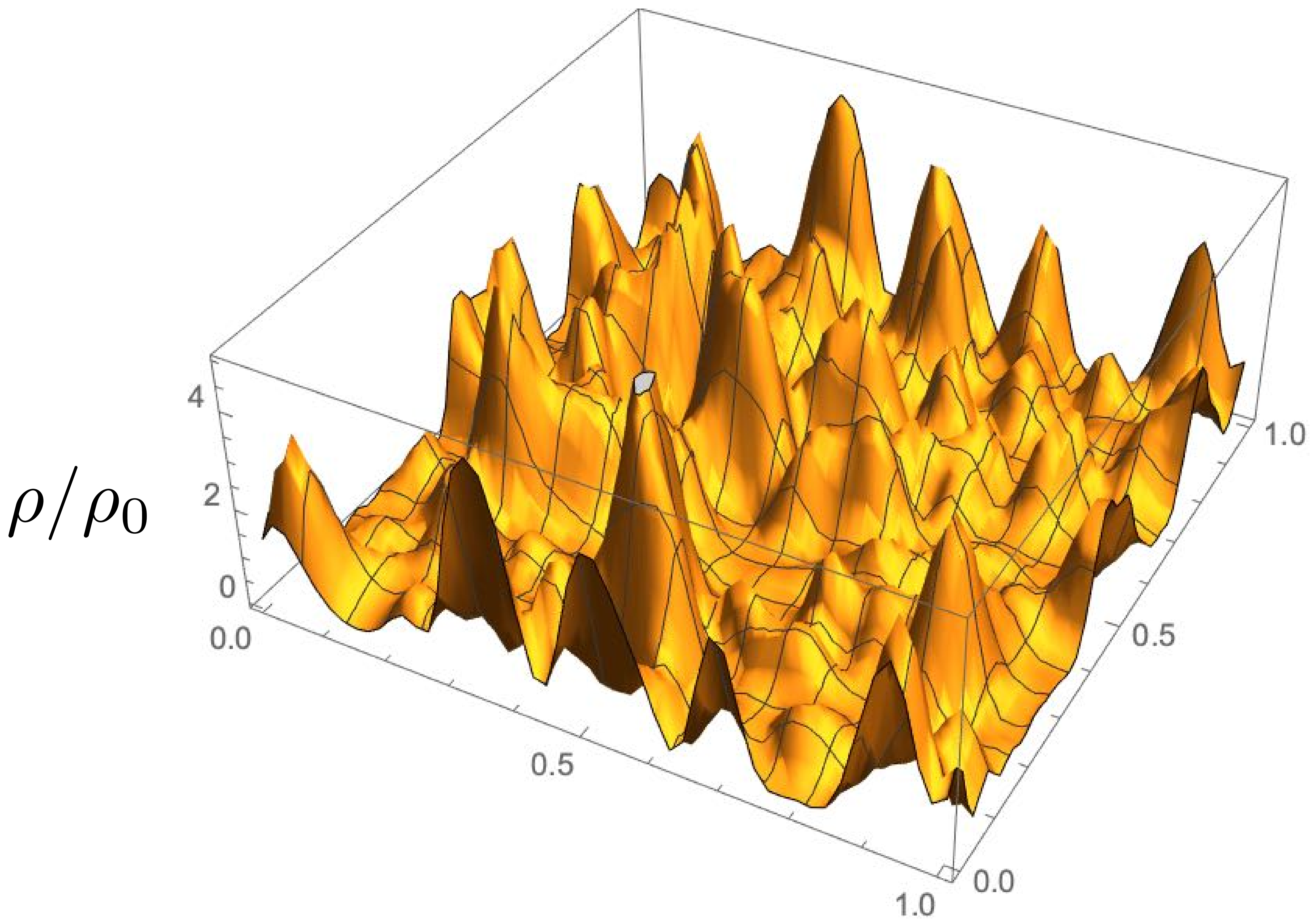
as a simple example

we may consider a homogeneous system where

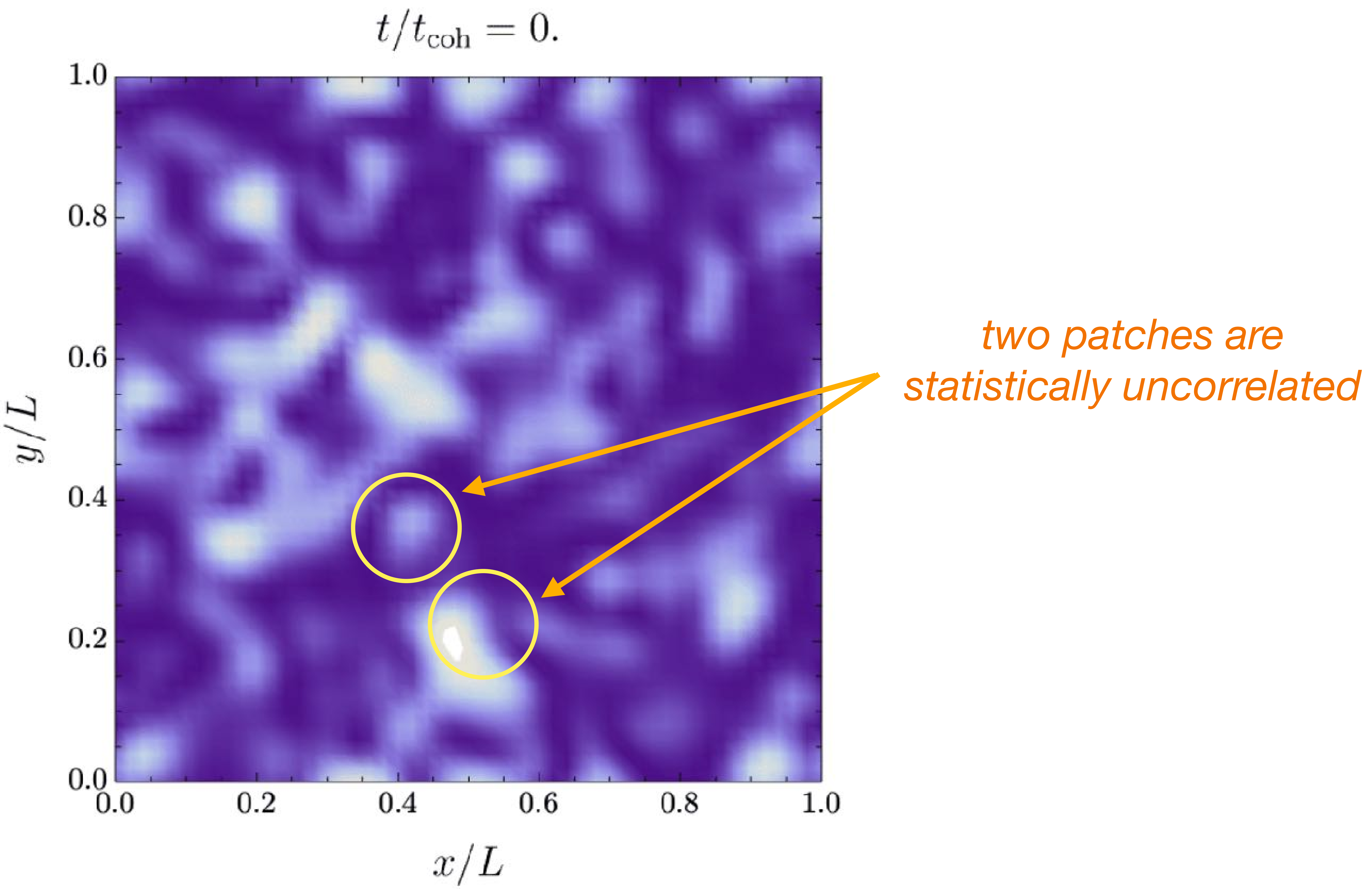
$$\psi_i = \frac{1}{\sqrt{V}} e^{-ik_i x}$$

this description provides a handy tool for analyzing wave DM without performing simulations

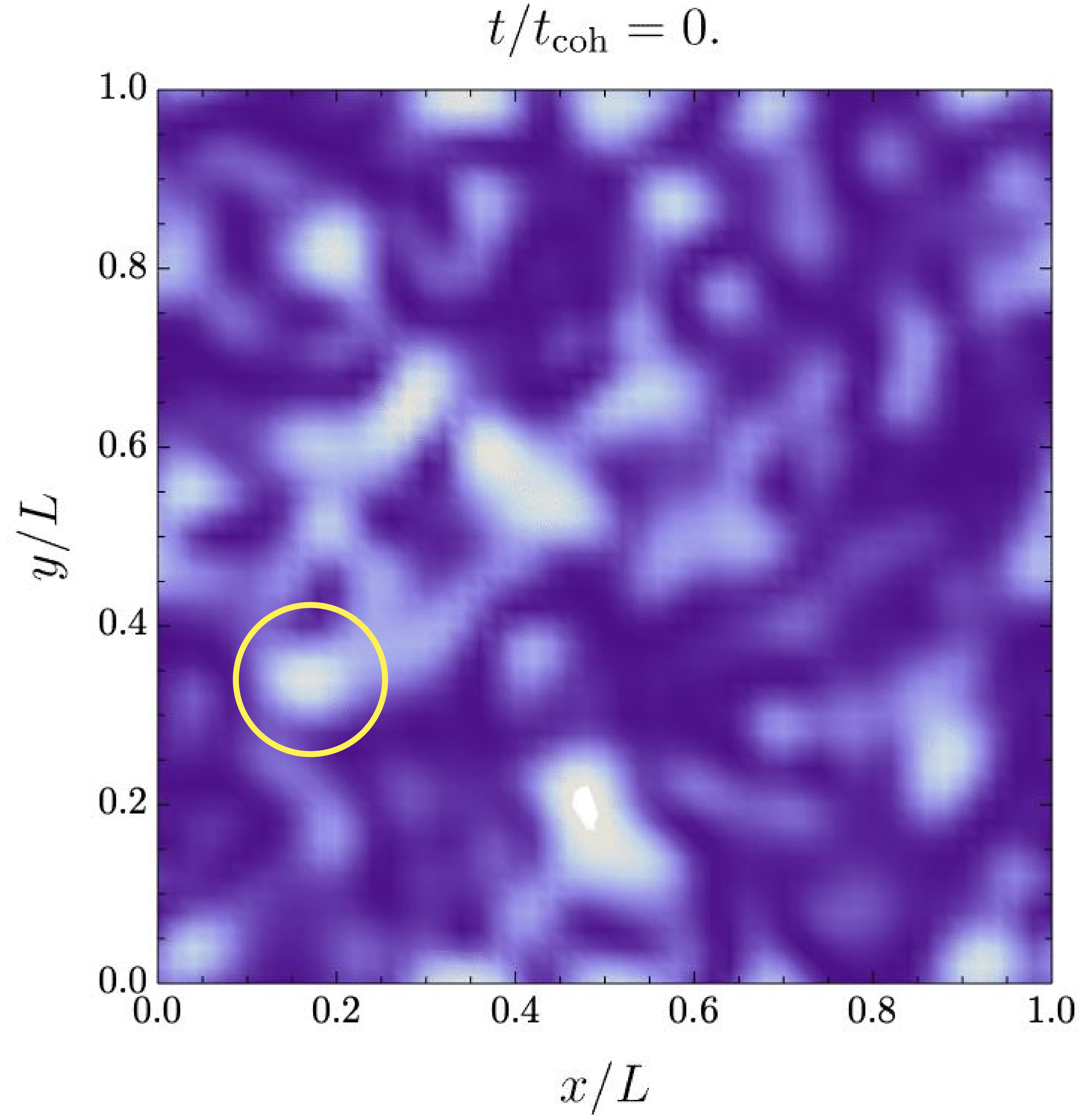




$$\langle \delta(t,x)\delta(t,y)\rangle \propto \exp\left[-|\Delta x|^2/\lambda^2\right]$$



$$\langle \delta(t,x)^2 \rangle$$



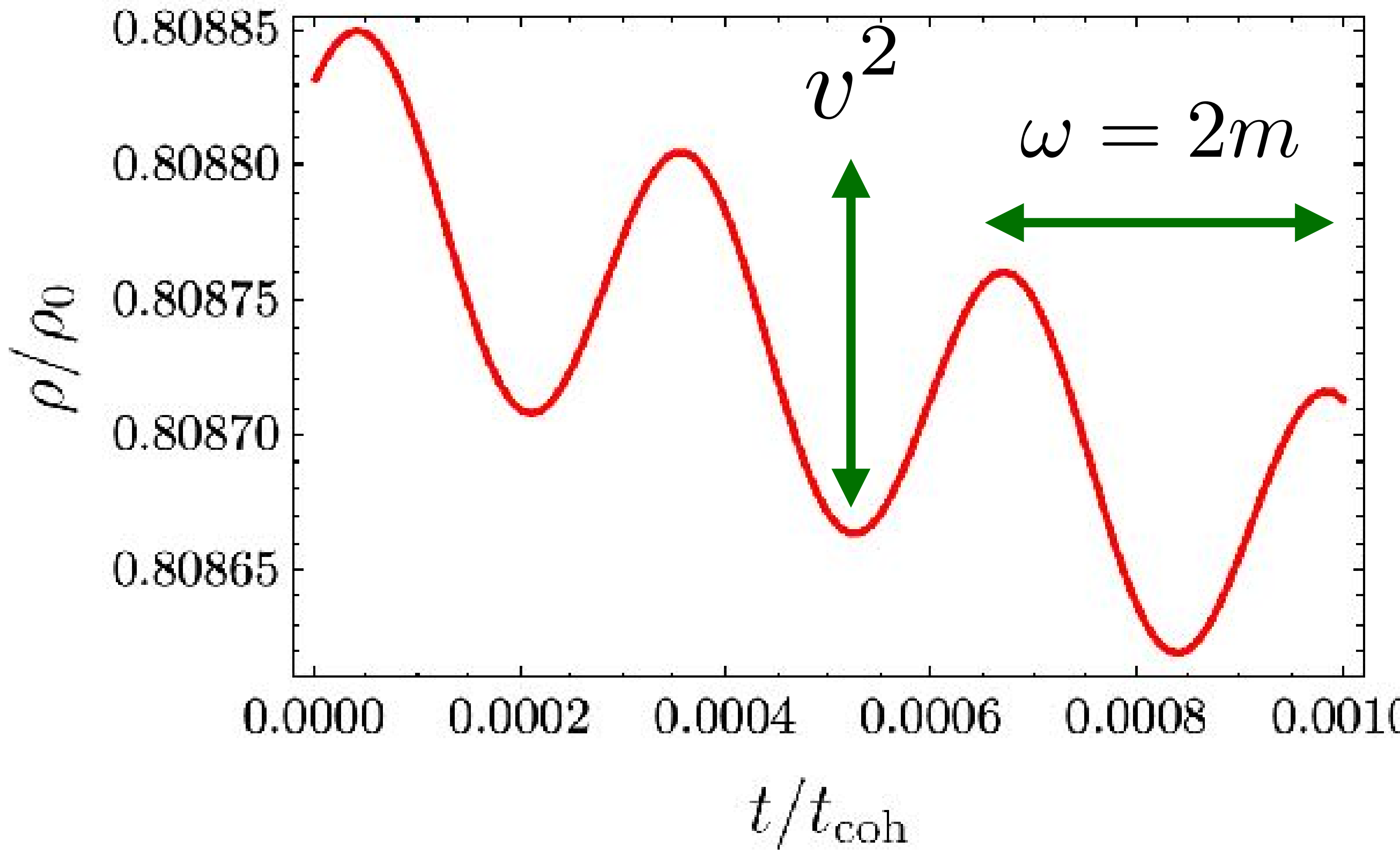
for a single mode

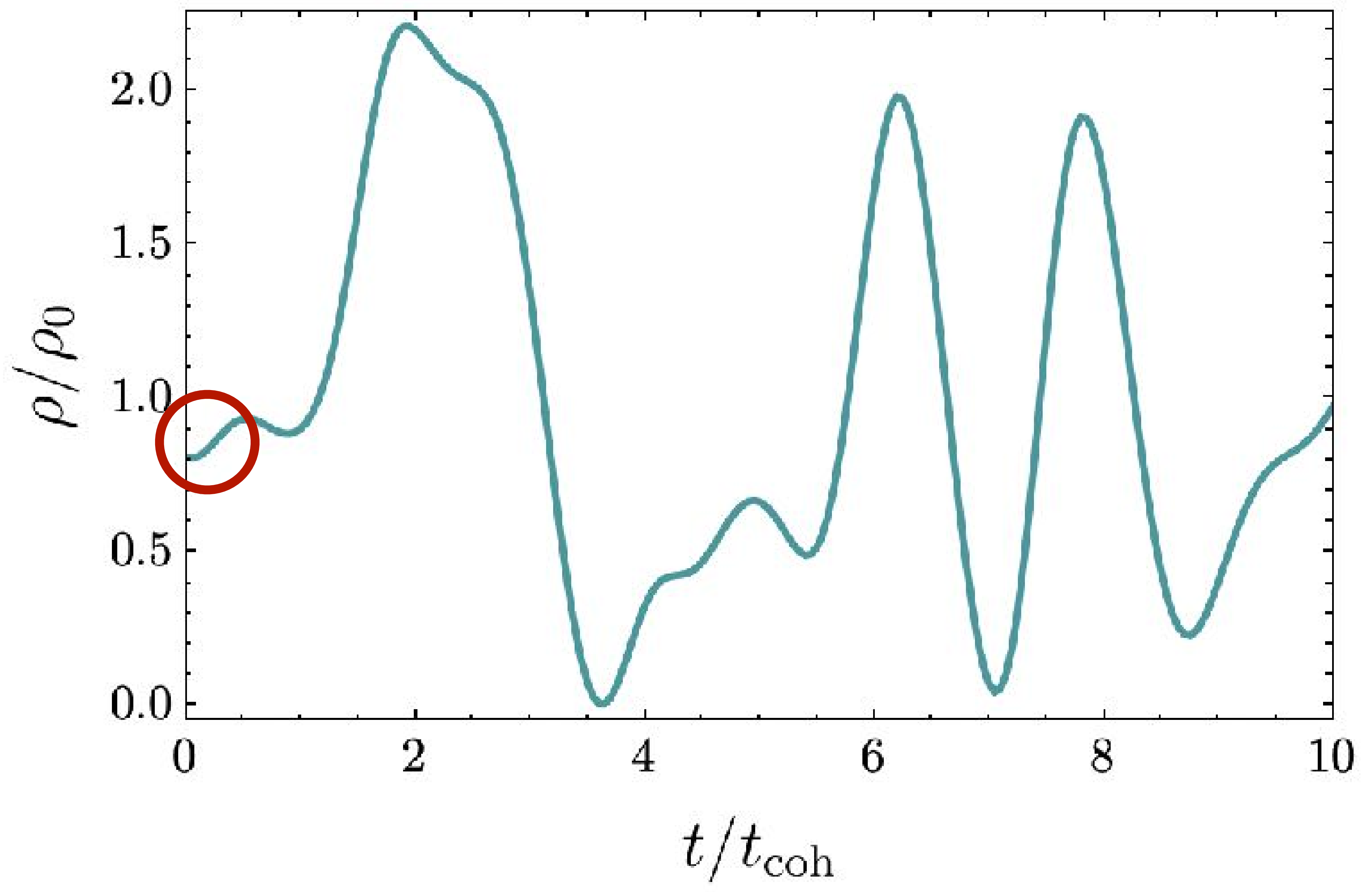
$$\phi = \phi_0 \cos(\omega t - kx)$$

the energy density is

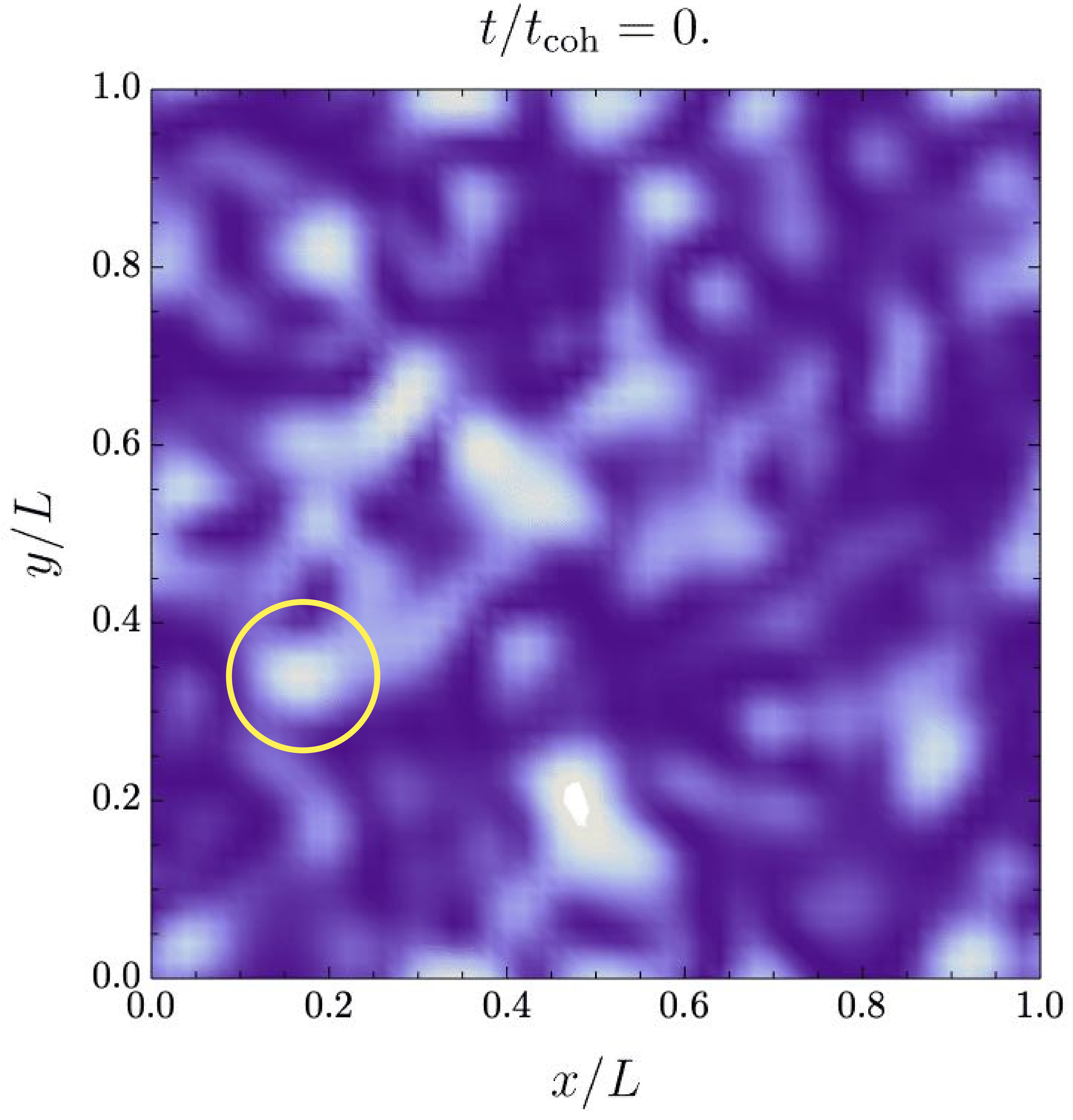
$$\rho = \frac{1}{2}\dot{\phi}^2 + \frac{1}{2}(\nabla\phi)^2 + \frac{1}{2}m^2\phi^2$$

$$\simeq \rho_0[1 - v^2 \cos(2(mt - kx))]$$

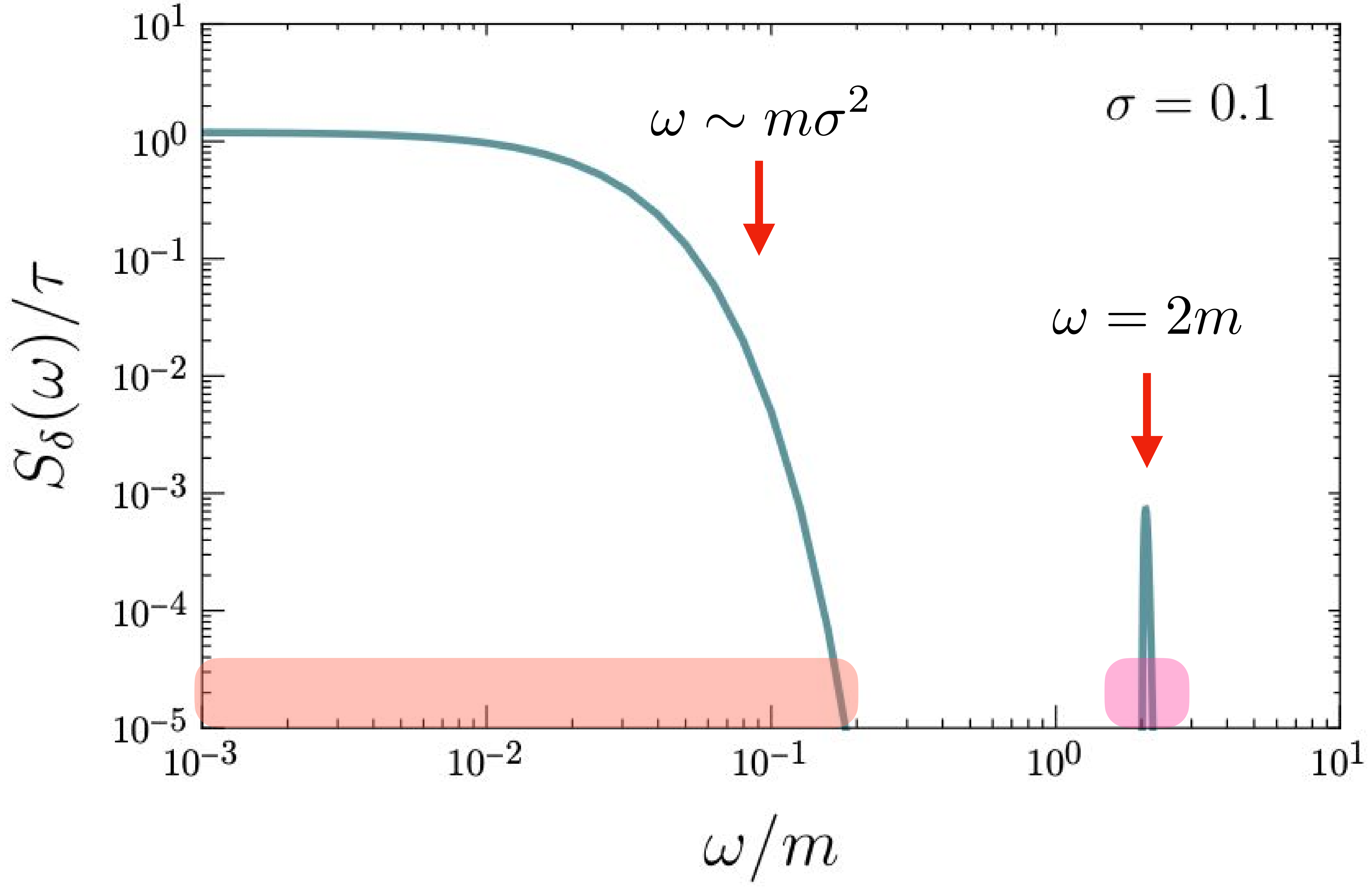


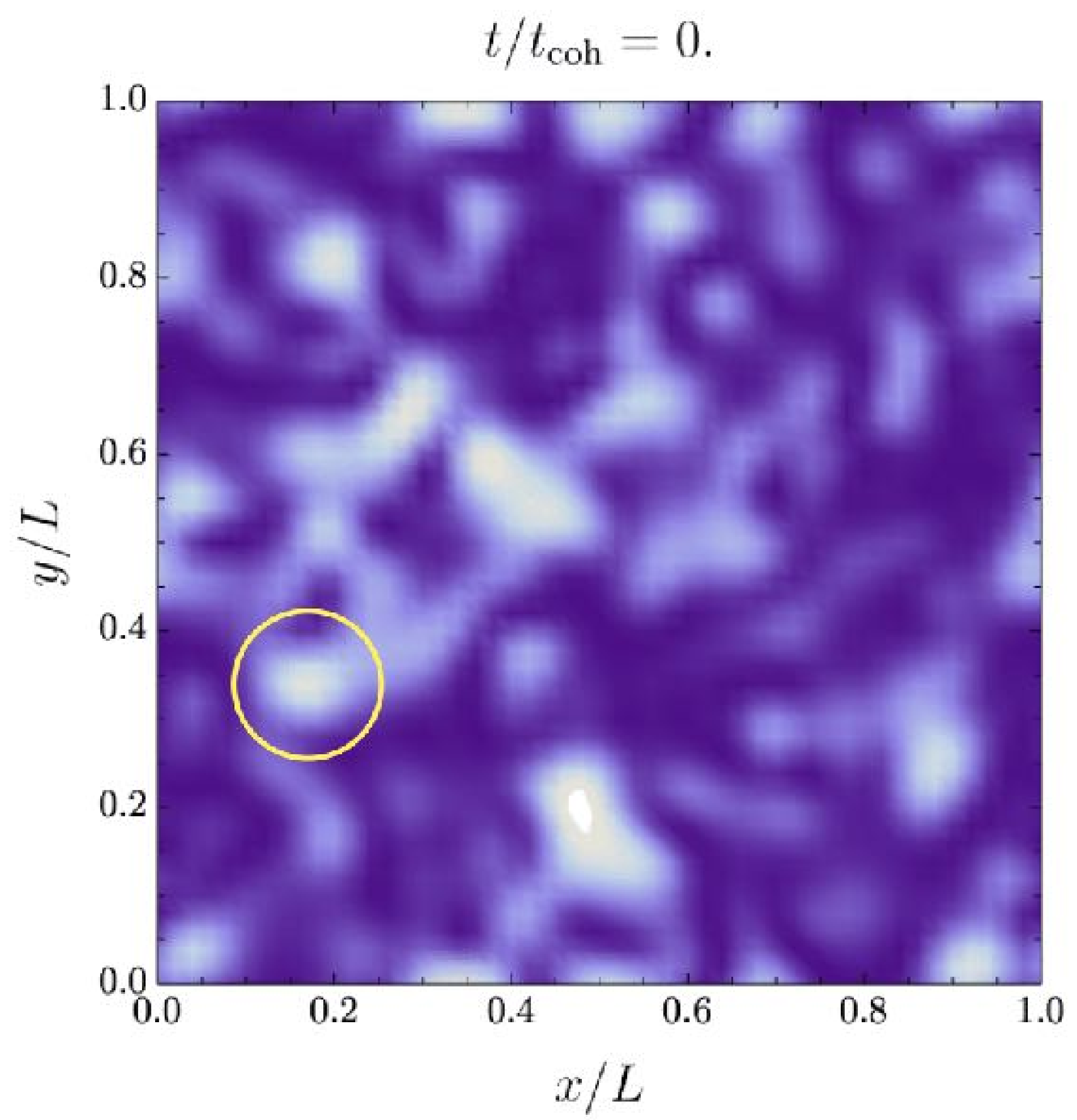


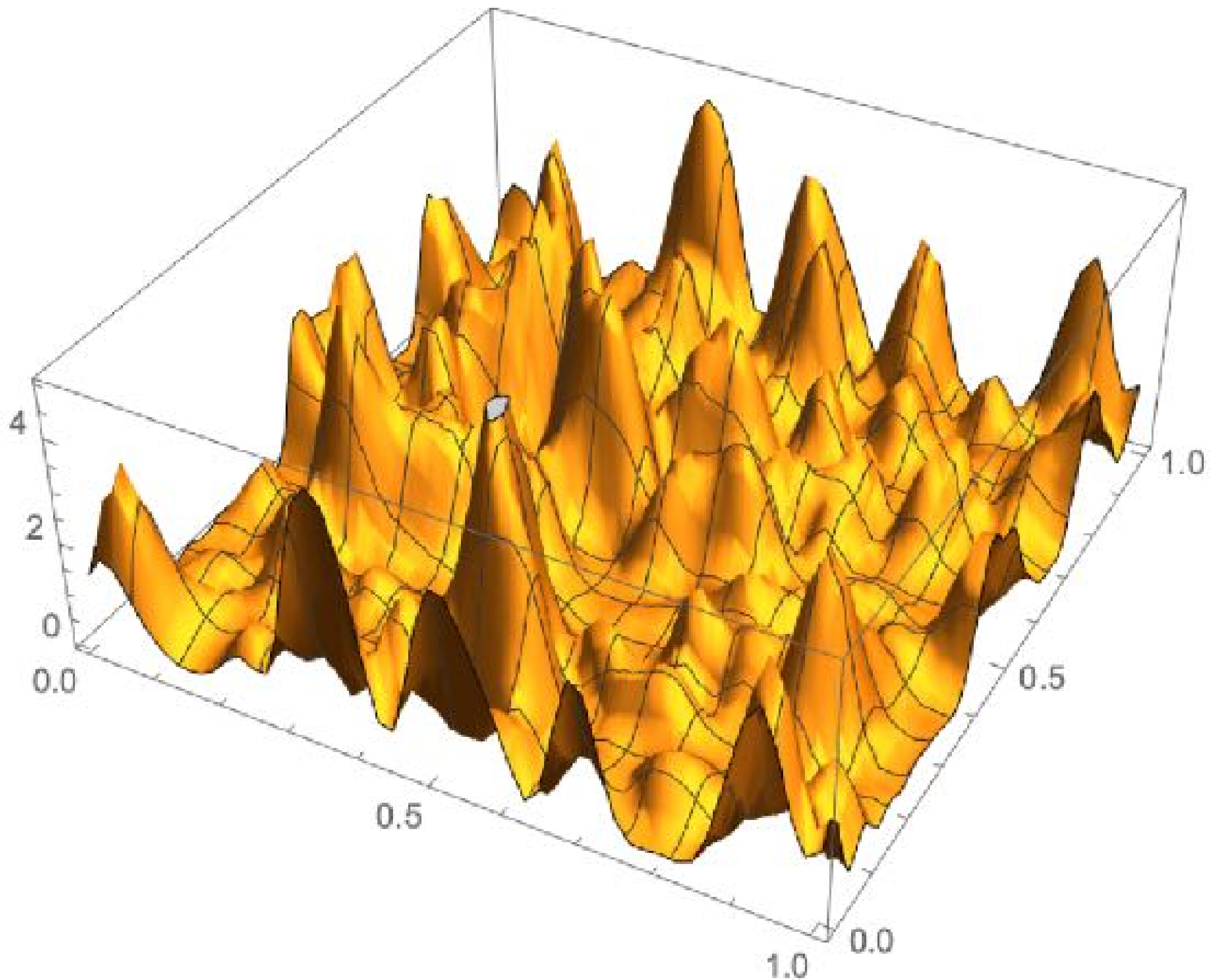
$$\langle \delta^2(x) \rangle = \int \frac{d\omega}{2\pi} S_\delta(\omega)$$



$$S_\delta(\omega) = \tau[\sigma^4 A_\delta(\omega) + B_\delta(\omega)]$$







density fluctuation at each patches follows the exponential distribution

$$p(\rho)d\rho = \frac{1}{\rho_0} e^{-\rho/\rho_0} d\rho$$

a probability to observe density fluctuation $\rho > \rho_c$ is

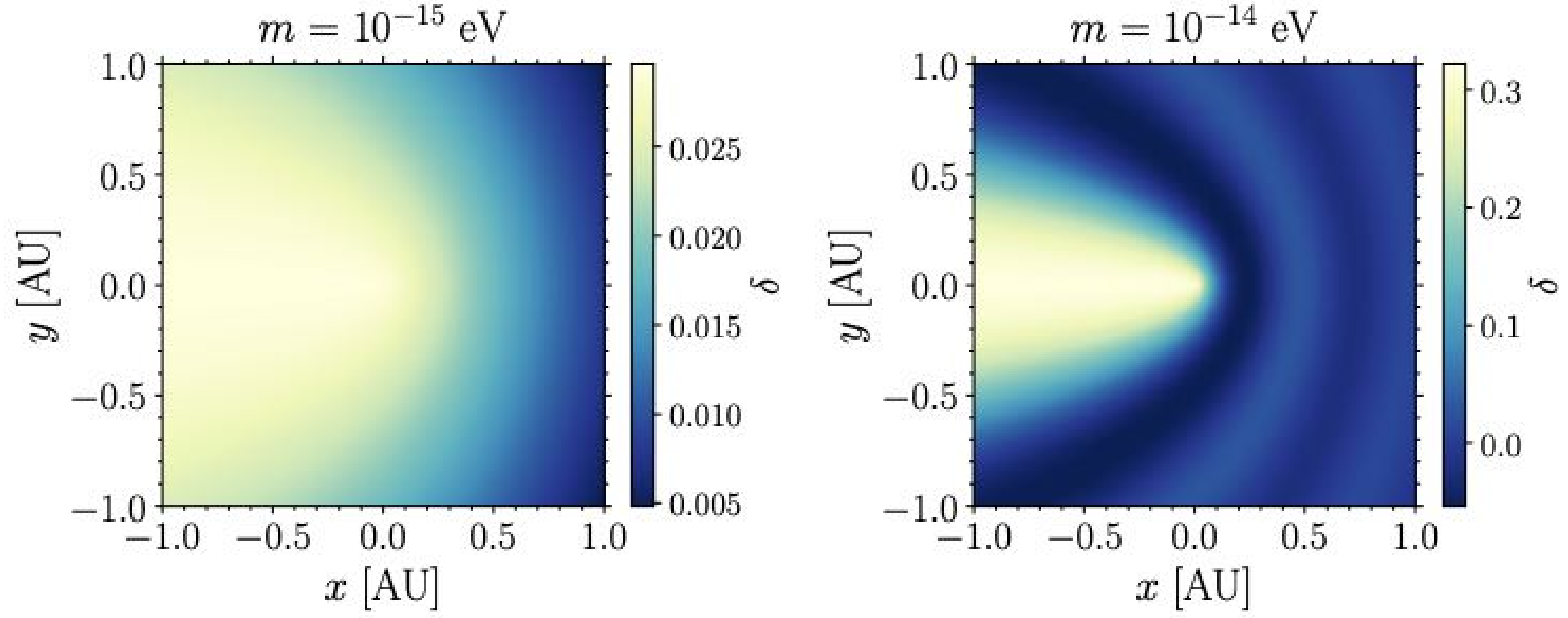
$$P(\rho > \rho_c) = e^{-\rho_c/\rho_0}$$

if there's Sun in the middle of the box

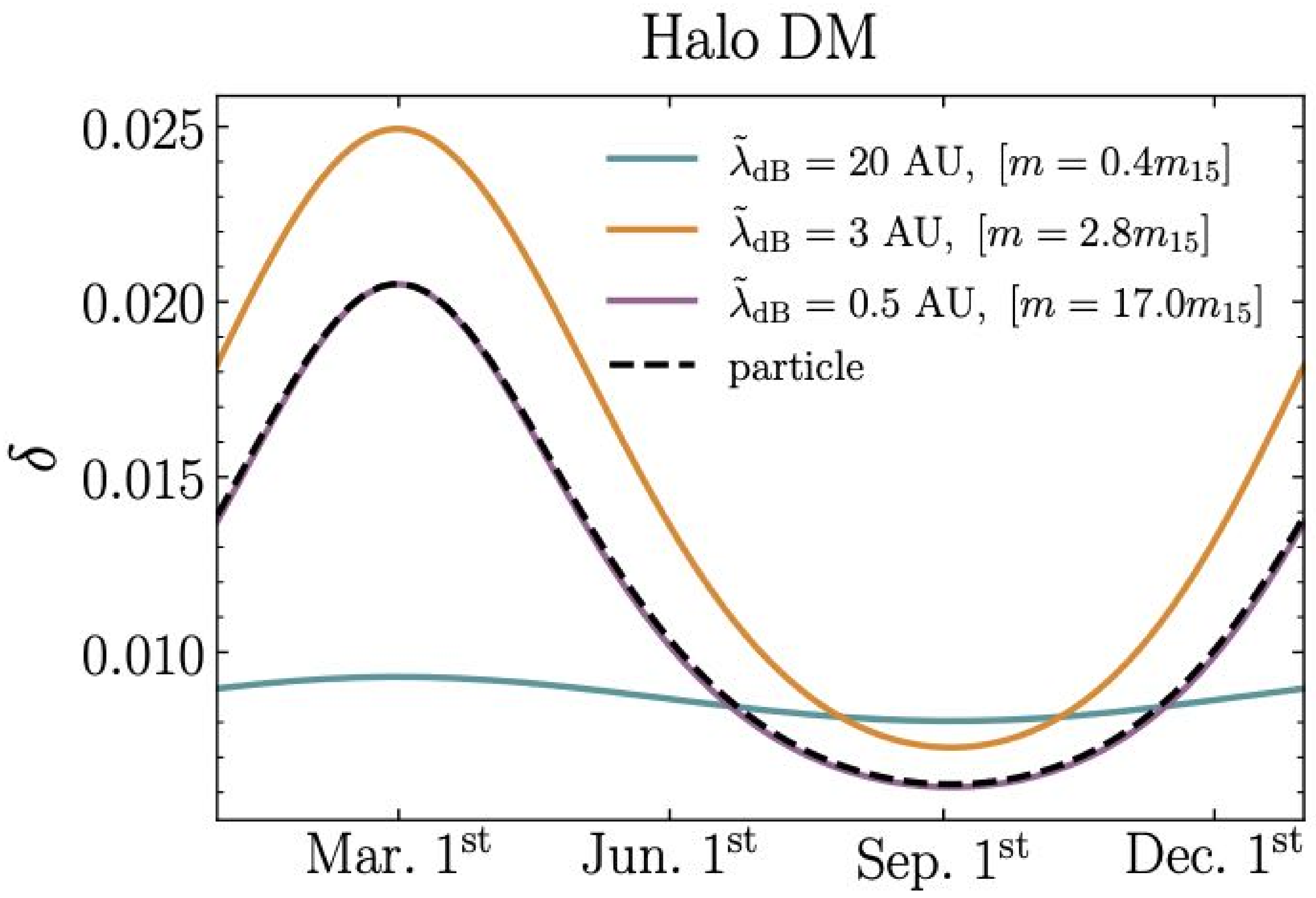
and dark matter has mean velocity (dark matter wind)

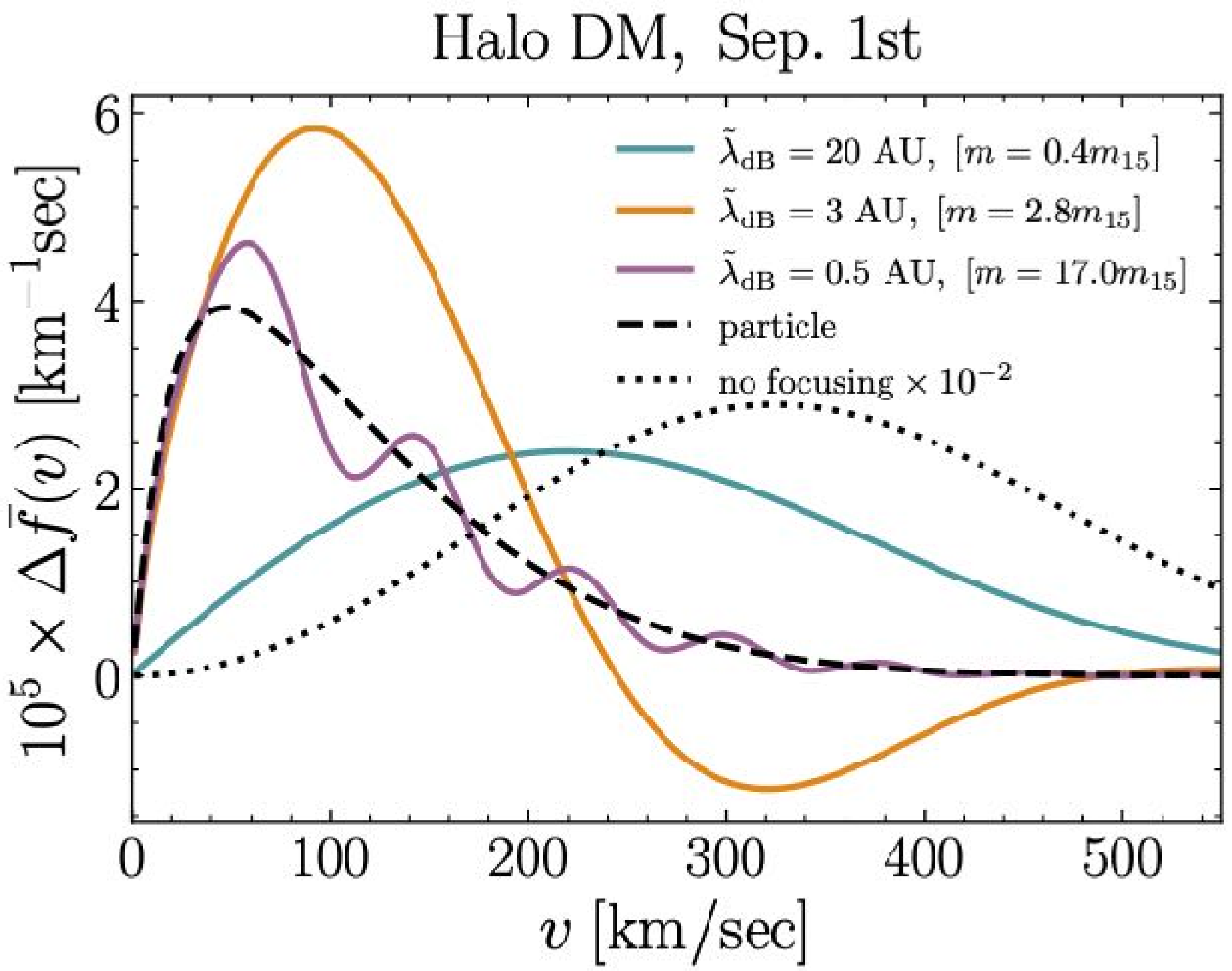
$$\psi_{\mathbf{k}}(\mathbf{x}) = e^{i\mathbf{k}\cdot\mathbf{x}}\Gamma(1 - i\beta)e^{\pi\beta/2}M\left[i\beta, 1, ikr(1 - \hat{k}\cdot\hat{x})\right].$$

mean density contrast <δ>
(for monochromatic case)



annual modulation





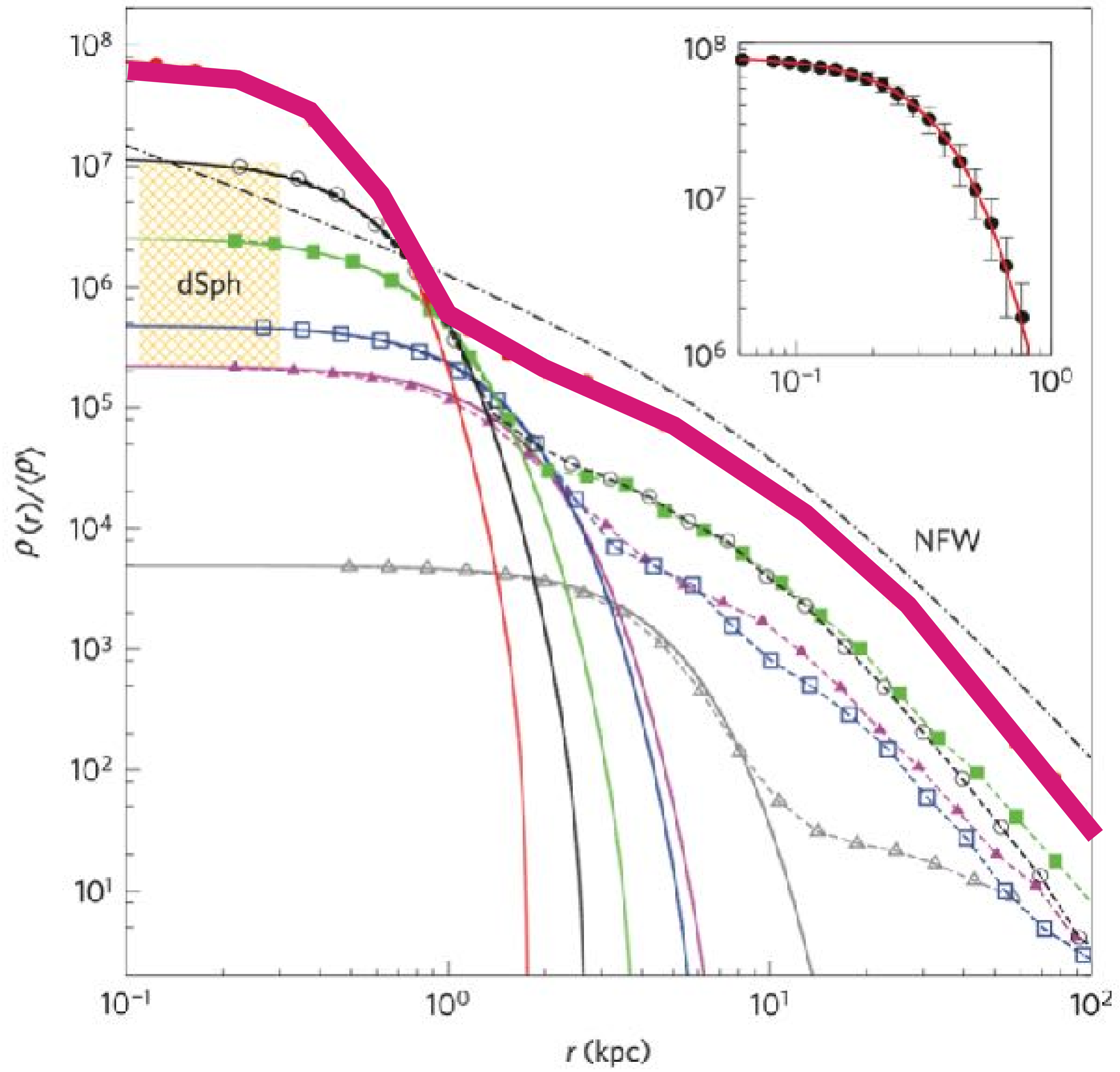
deformed spectral shape

[Kim and Lenoci 21]

it is also possible to construct
the whole wave DM halo in this way

[Widrow, Kaiser (93)]
[Lin, Schive, Wong, Chiueh (18)]
[Yavetz, Li, Hui (22)]

from results of numerical simulation
we make an educated guess for DM profile



ansatz:
soliton + NFW profile

$$\rho_t$$

this will fix the gravitational potential as well as density profile *since the gravitational potential is fixed*

wave equations to obtain eigenmodes of the system

$$\psi_{n\ell m} = R_{n\ell}(r) Y_\ell^m(\Omega)$$

$$(rR_{n\ell})'' + [2m^2(E_{n\ell} - \Phi) - \ell(\ell+1)/r^2](rR_{n\ell}) = 0$$

the field is expanded as

$$\hat{\phi} = \sum_{n\ell m} \frac{1}{\sqrt{2\omega}} \left[\alpha_{n\ell m} \psi_{n\ell m} + \alpha^\dagger_{n\ell m} \psi^*_{n\ell m} \right]$$

quasi-probability distribution

$$P(\alpha_{n\ell m}) = \frac{1}{\pi f_{n\ell m}} \exp\left[-\frac{|\alpha_{n\ell m}|^2}{f_{n\ell m}} \right]$$

the field is expanded as

$$\hat{\phi} = \sum_{n\ell m} \frac{1}{\sqrt{2\omega}} \left[\alpha_{n\ell m} \psi_{n\ell m} + \alpha^{\dagger}_{n\ell m} \psi^{*}_{n\ell m} \right]$$

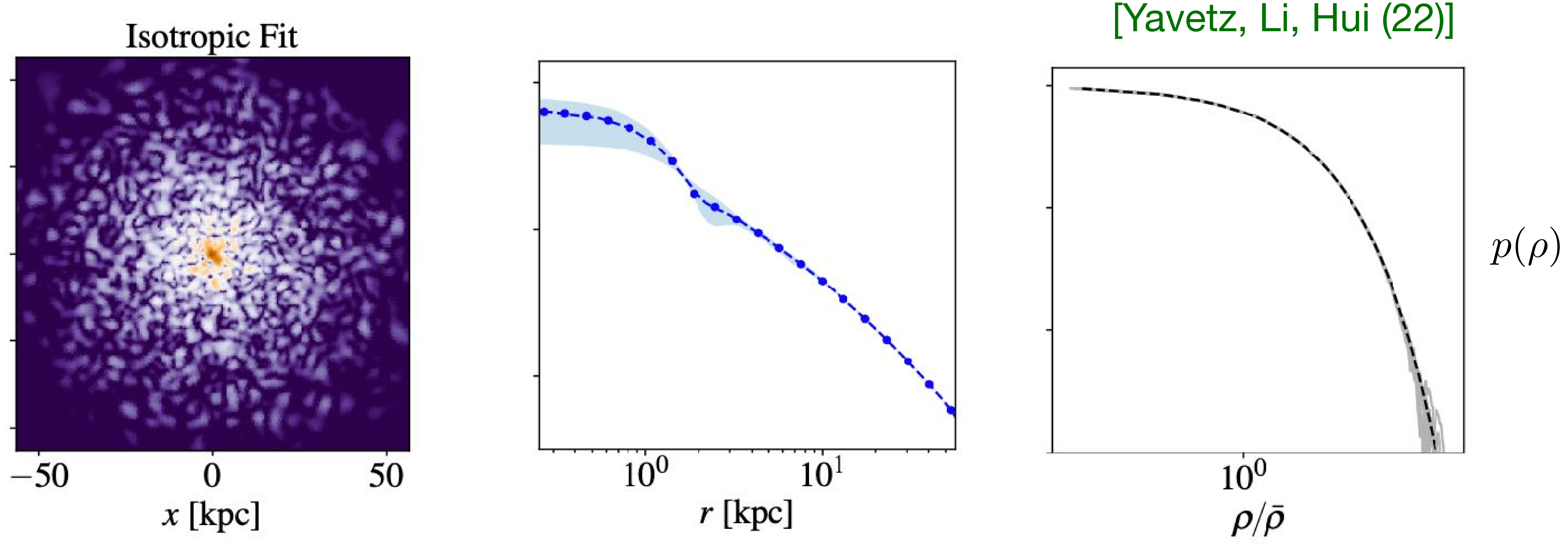
the occupation number f_{nlm} can be determined by comparing

$$\bar{\rho}(r) = \frac{m}{4\pi} \sum_{n\ell} (2\ell + 1) f_{n\ell} |R_{n\ell}(r)|^2 \quad \text{vs} \quad \rho_t$$

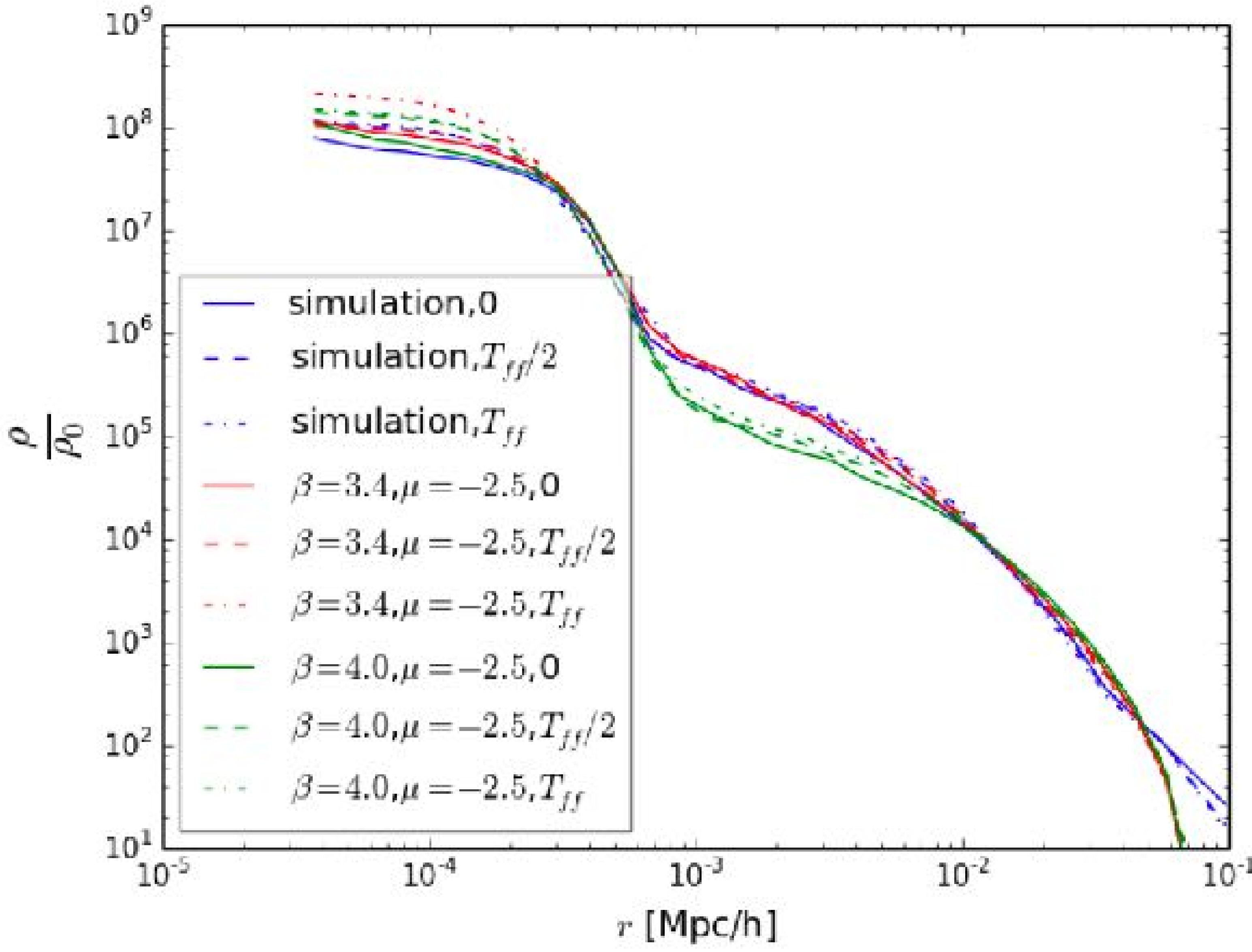
quasi-probability distribution

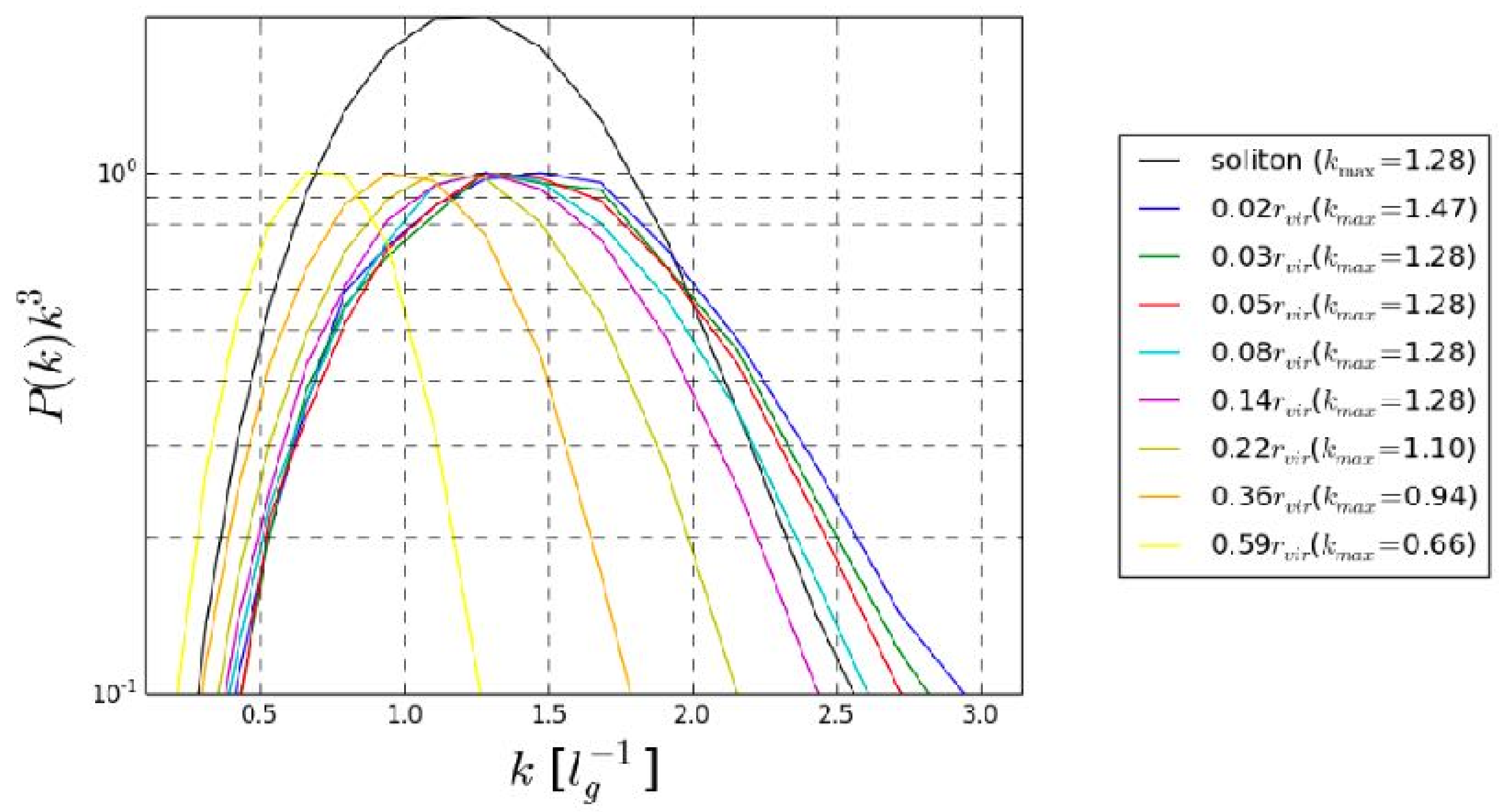
$$P(\alpha_{n\ell m}) = \frac{1}{\pi f_{n\ell m}} \exp\left[-\frac{|\alpha_{n\ell m}|^2}{f_{n\ell m}} \right]$$

[Yavetz, Li, Hui (22)]



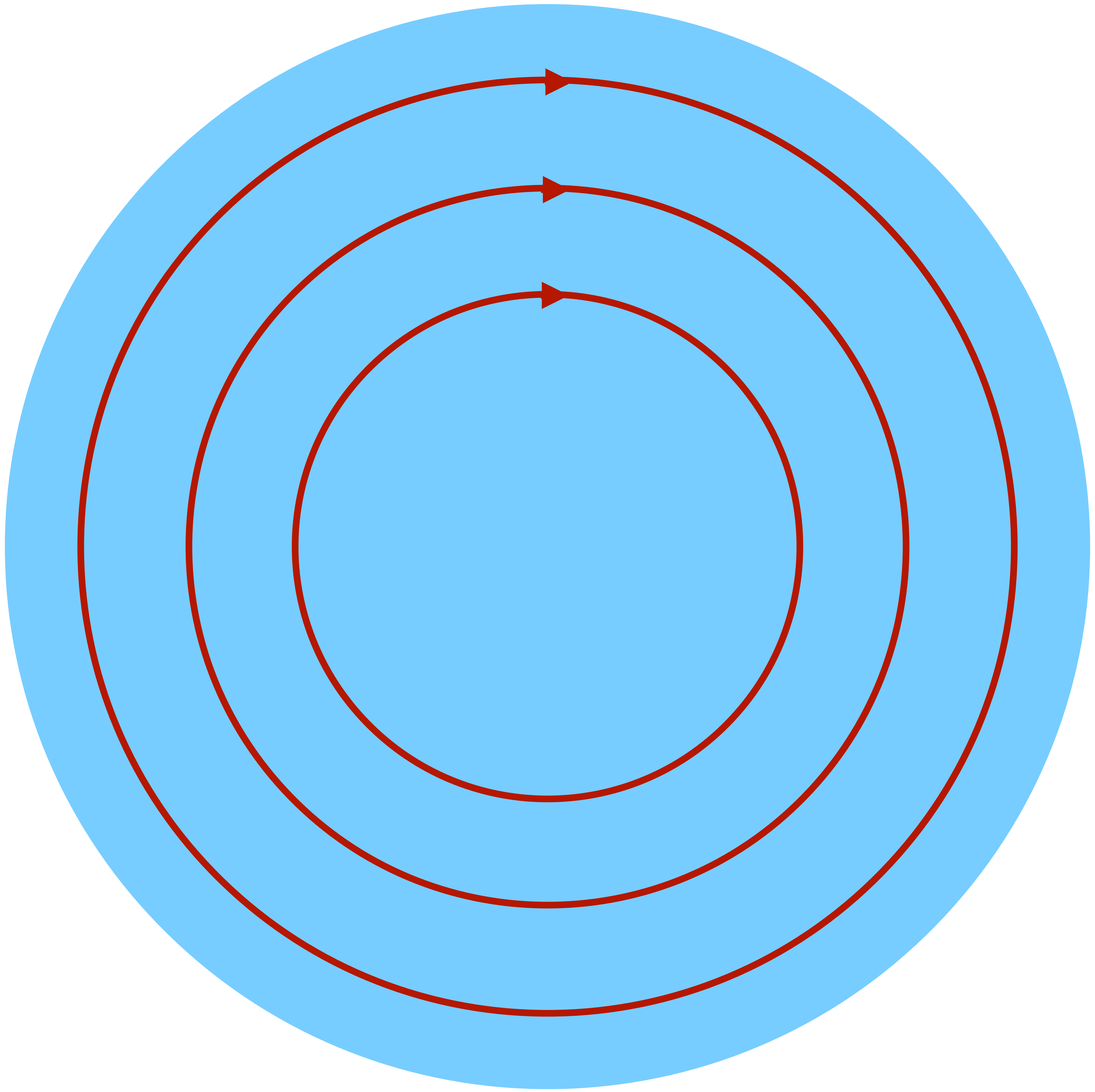
[Lin et al (18)]





furthermore, this tool could be useful
for studying the adiabatic evolution of the system

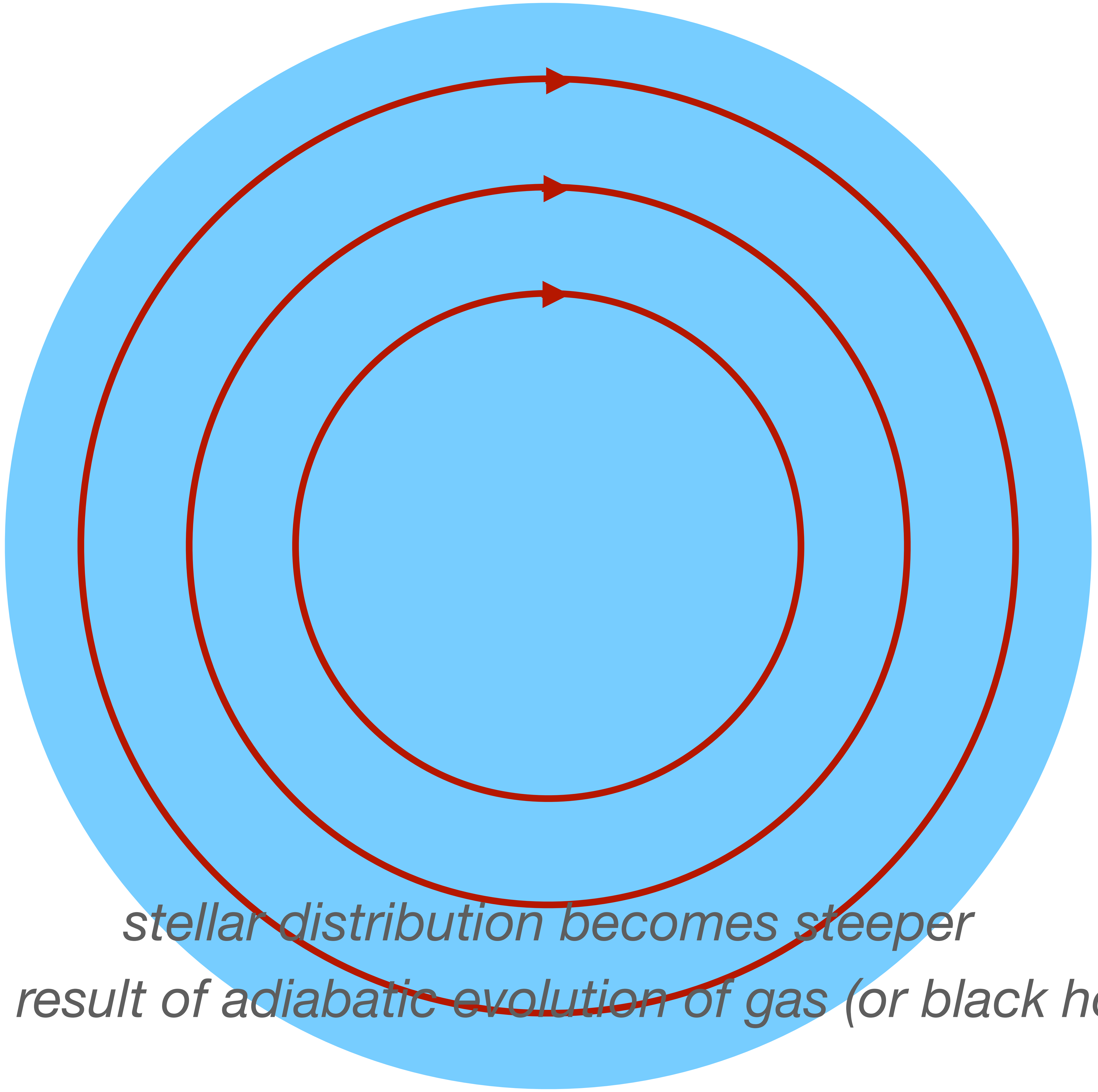
consider for instance



gas

star (or dark matter)

now imagine gas cloud contracts adiabatically
due to some cooling process



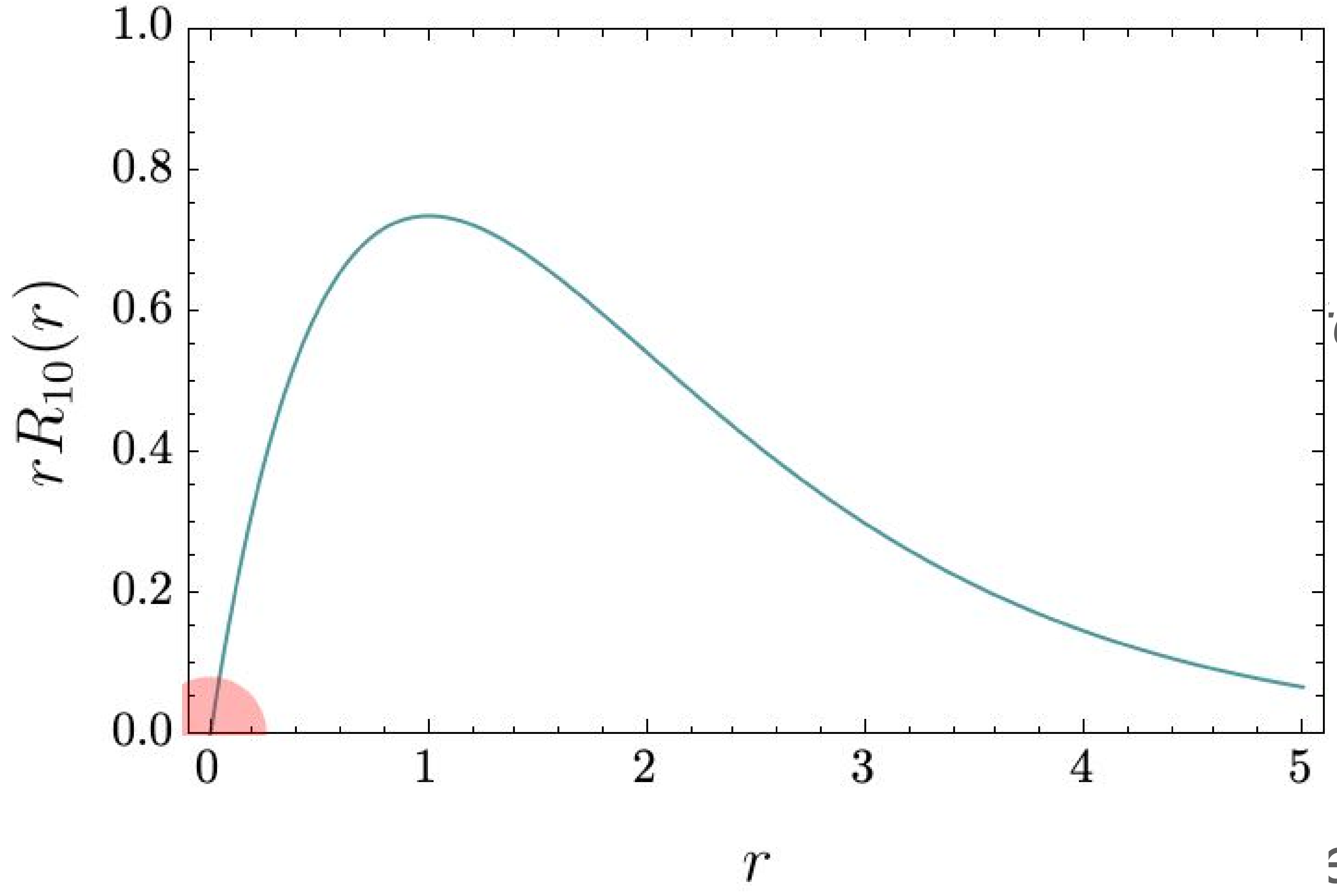
stellar distribution becomes steeper
as a result of adiabatic evolution of gas (or black hole)!

$$L = rv = \sqrt{GM_{\rm enc}(r) r}$$

[Young (90)]
[Blumenthal et al (86)]
[Quinlan et al (95)]
[ven der Marel (99)]

gas
star (or dark matter)

in r… …em



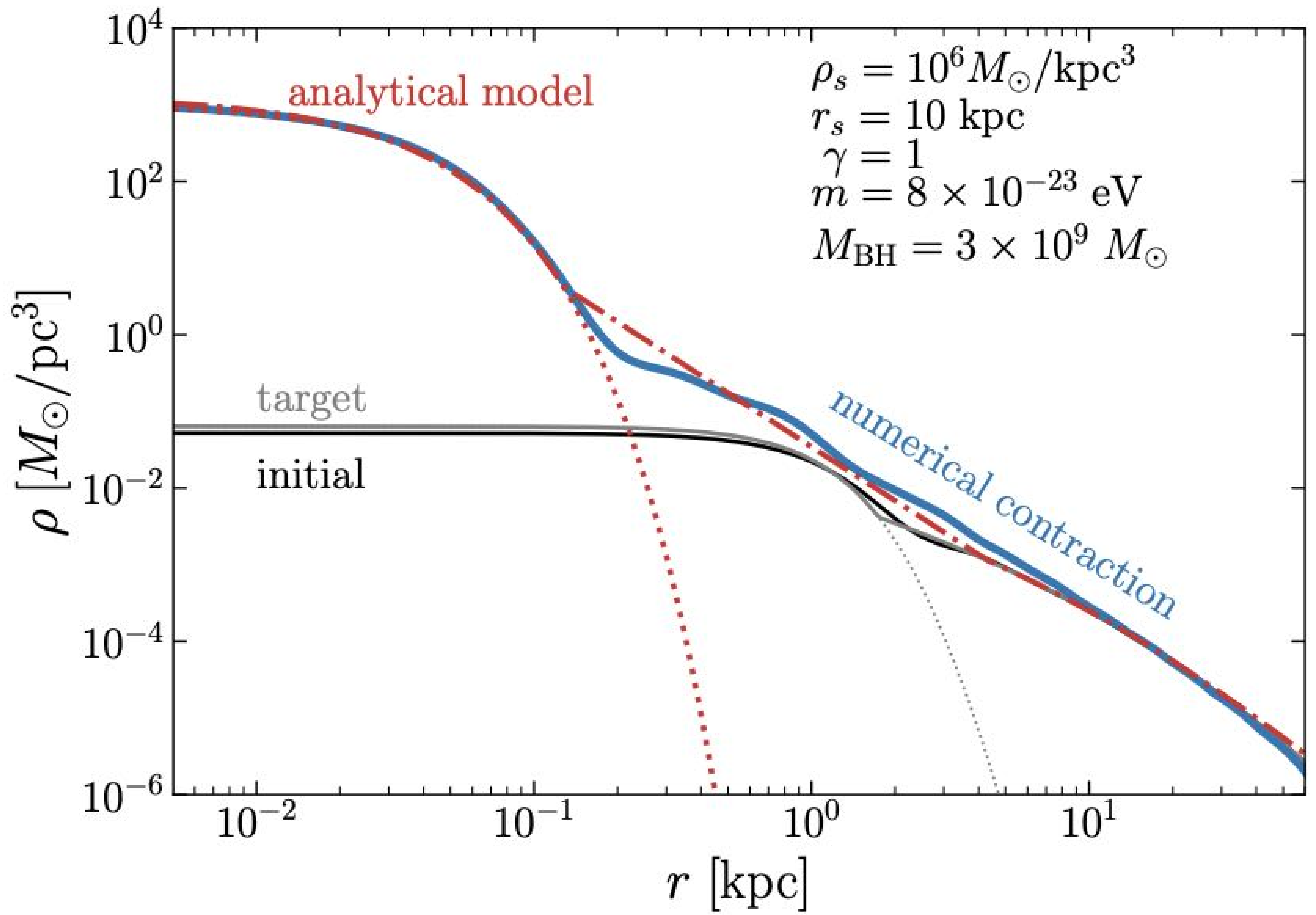
instantane… …e evolution

changes adiabatically

$$\bar{\rho}(r) = \frac{m}{4\pi}\sum_{n\ell}(2\ell+1)f_{n\ell}|R_{n\ell}(r)|^2$$

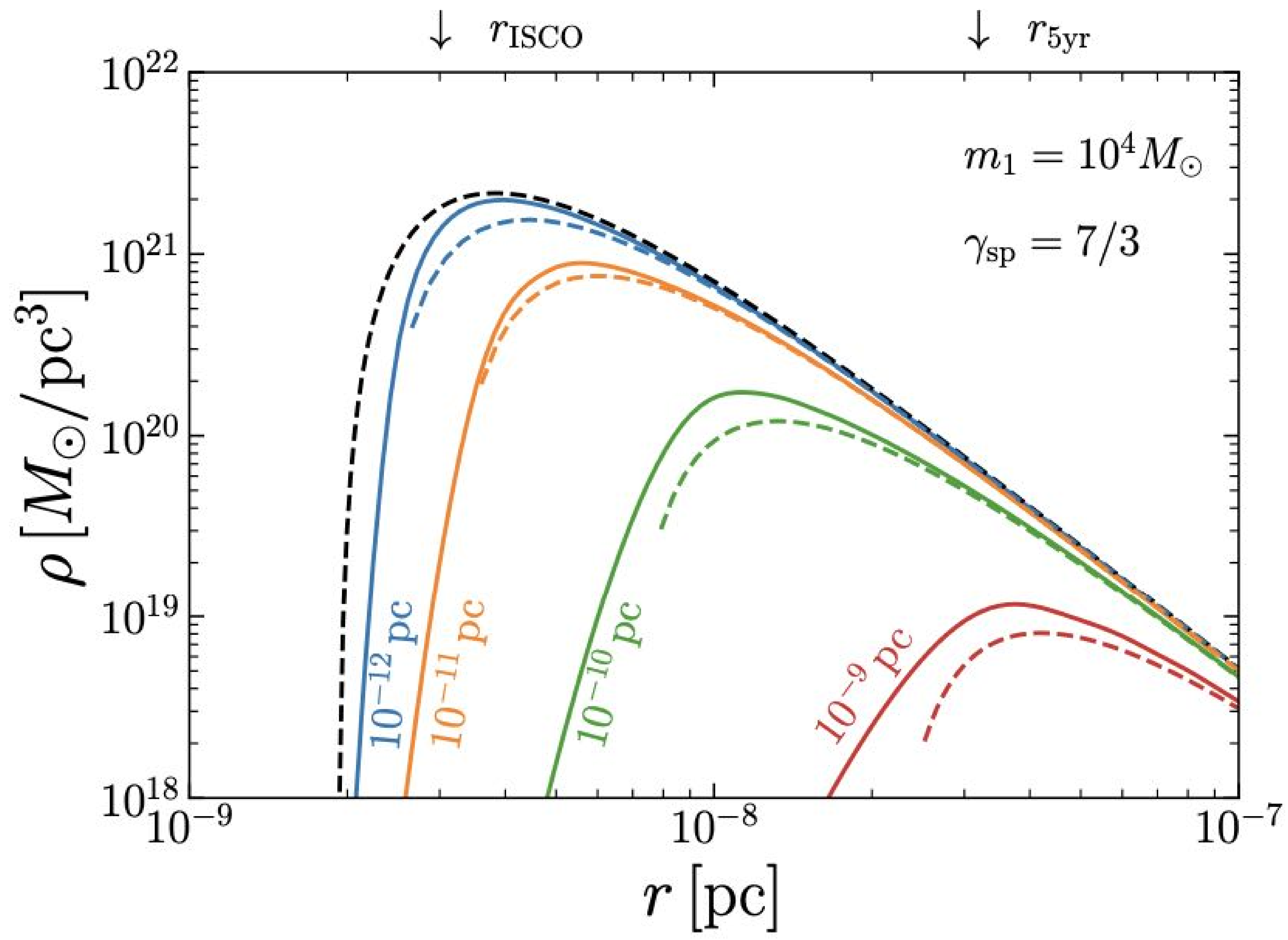
stays the same

[HK, Lenoci, Stomberg, Xue (22)]

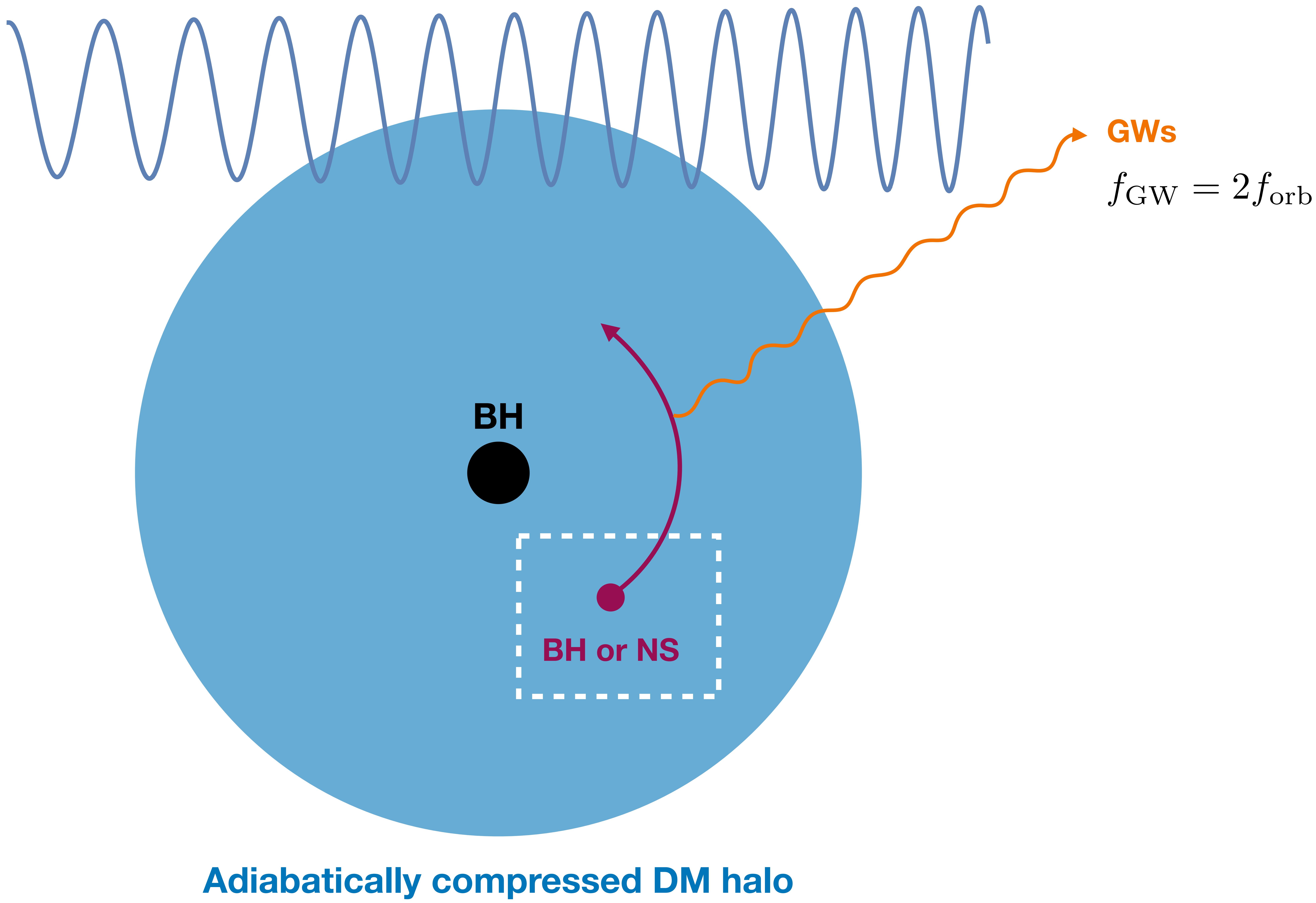


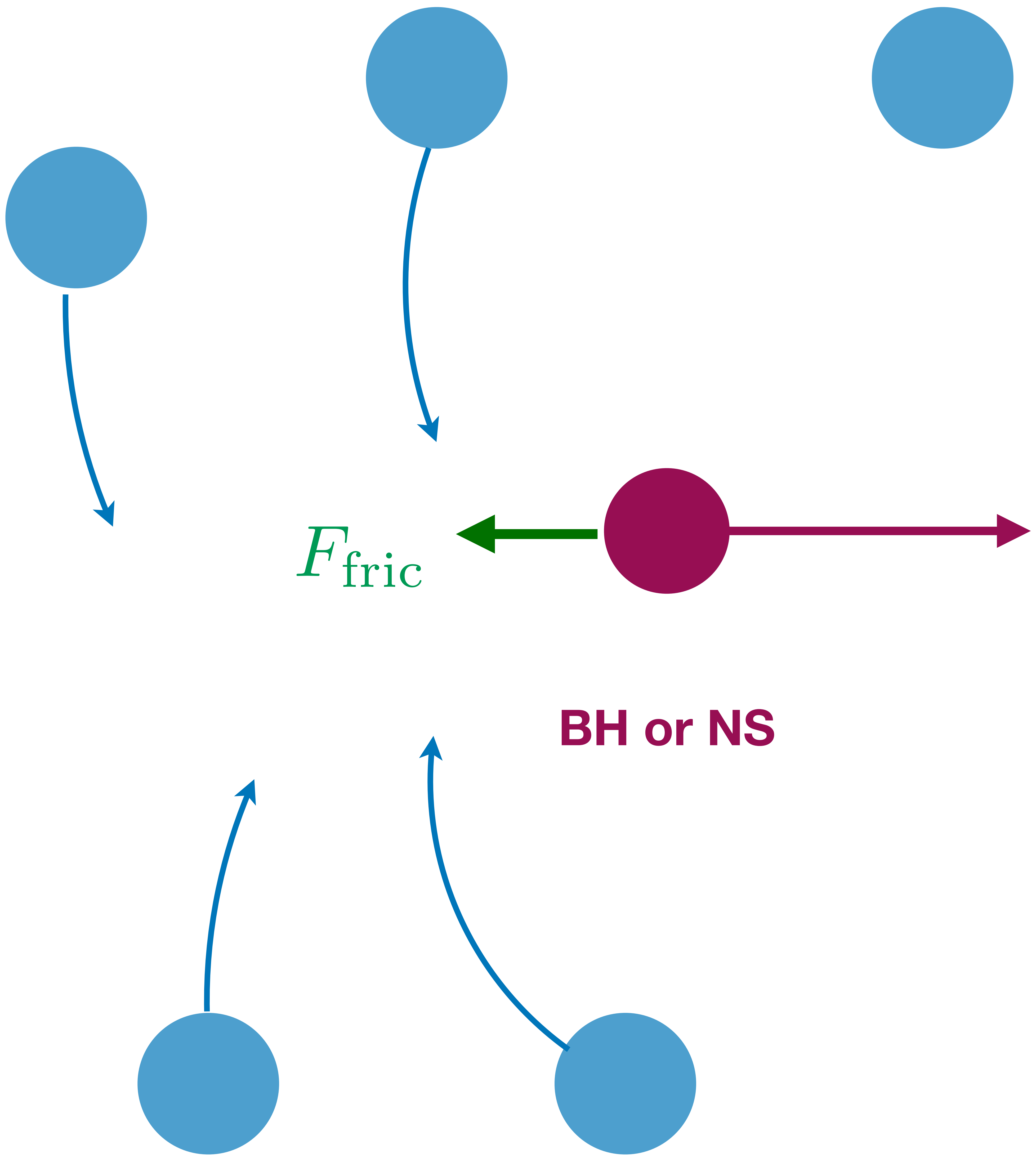
[HK, Lenoci, Stomberg, Xue (22)]

or if the central soliton & low angular moments are shallowed by the black hole over t~13 Gyr

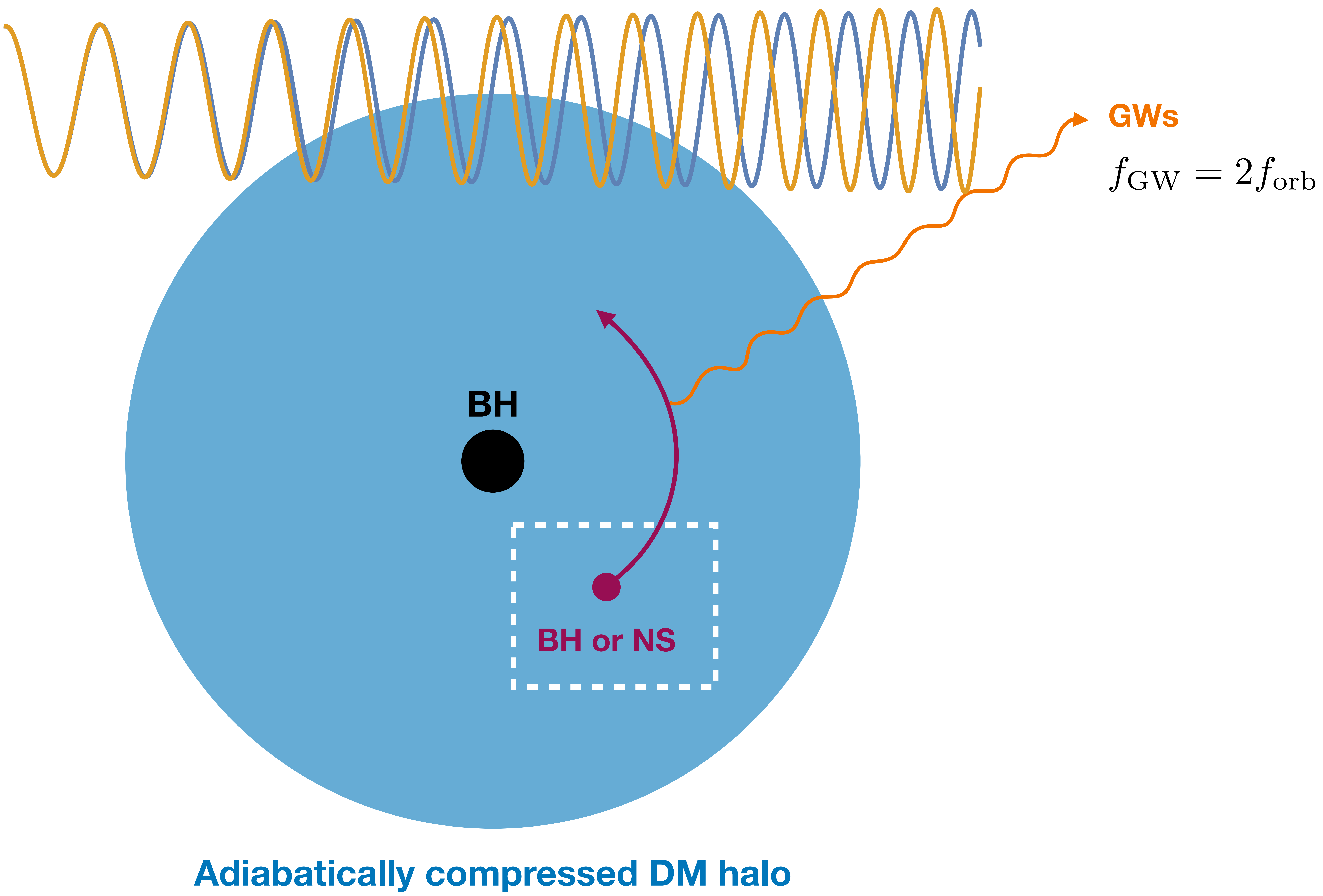


an interesting way to test this compressed wave DM halo is through the observation of GWs


GWs
$f_{\mathrm{GW}} = 2 f_{\mathrm{orb}}$
BH
BH or NS
Adiabatically compressed DM halo



DM overdensity forms
behind the BH or NS
exerting additional frictional force


GWs
$f_{\rm GW} = 2 f_{\rm orb}$
BH
BH or NS
Adiabatically compressed DM halo

$m = 10^{-13}\,\mathrm{eV}$

[HK, Lenoci, Stomberg, Xue (22)]